Trade and Factor Prices in a Model of Capital Utilization*

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I. Introduction

In recent years there has been considerable work on the economics of capital utilization,[1] but the implications of endogenous capital utilization for trade and comparative advantage have not been worked out. Empirical research has revealed that there is a great deal of variation across countries in the willingness of employees to work during evening and night hours and on weekends, and the partial equlibrium theory of optimal capital utilization shows that a willingness on the part of employees to work during abnormal hours can reduce the cost of production, frequently by reducing the need for fixed capital. The effects on production costs will depend of course on the wage premium demanded by workers for abnormal-hours work. It stands to reason, therefore, that recognition of the varying degrees of willingness of employees to work during abnormal hours will have important implications for comparative advantage and trade.

This paper will be concerned with the introduction of endogenous capital utilization into one of the sectors of the standard two-factor two-sector model of trade. There are two fundamentally different ways in which this introduction can take place. In the first approach as each firm increases its level of capital utilization, it in effect imposes abnormal hours of work on its existing labor force and it must increase the wage premium for labor during abnormal hours. In this model the firm's optimal level of capital utilization occurs where the increased wage premium from the increase in abnormal-hours work balances the saving of capital costs from the increase in capital utilization.

In the second approach, the firm regards as given the wage rates for normal-hours and for abnormal-hours work, and therefore the wage premium for abnormal hours is also

* We are grateful to R. Jones whose comments on another paper were partially responsible for stimulating this one.

1. Two pioneers in this area are Marris [8] and Georgescu-Roegen [5]. Other contributions to the subject are discussed in the survey by Winston [12] and in the recent book by Betancourt and Clague [1].

given. The firm in effect buys labor from two separate labor markets and is a perfect competitor in each. In this separate-labor-markets model, the firm makes an all-or-nothing decision with respect to capital utilization: if the wage premium is above a certain level, the firm operates only during normal hours. If the wage premium falls below this level, the firm then increases utilization up to the maximum level. (The model assumes the different abnormal hours are all priced at the same wage rate.)

Two papers employing the first approach, or the single-labor-market model, are Betancourt, Clague, and Panagariya (BCP) [2; 3]. These papers show that the Stolper-Samuelson and Rybczynski theorems continue to hold in the presence of endogenous capital utilization. They also show that the Factor-Price-Equalization and Heckscher-Ohlin theorems continue to hold if the willingness to work abnormal hours is the same across countries, but these theorems break down if that willingness differs across countries.

The present paper will employ the second approach, or the separate-labor-markets model.[2] For ease of exposition we shall refer to normal-hours work as day work and to abnormal-hours work as night work. We shall show that in this model the willingness to work at night serves much like a third factor of production, and the model therefore resembles to some extent the specific-factors model in the literature [6; 9; 10]. In the specific-factors model, it is well known that the Stolper-Samuelson and Rybczynski theorems do not hold, and this statement applies to our model as well. At the same time factor proportions play a crucial role in our model (unlike the specific-factors model), so that our model can be said to have some Heckscher-Ohlin features as well as specific-factor features.

There is another striking difference between the single-labor-market models of BCP [2; 3] and the separate-labor-market model employed in the present paper. In the present paper it will be shown that if two countries have identical consumer tastes and identical amounts of capital and labor, the country with the greater willingness to work during abnormal hours will have a comparative advantage in the shift-working industry. This conclusion does not hold in the single-labor-market model, as is demonstrated in BCP [3]. In this paper we shall not take a position on whether the single-labor-market assumption or the separate-labor-markets assumption is a better approximation of reality. The purpose of our discussion here is to point out the strongly contrasting implications of the two approaches.

The organization of the paper is as follows. Section II lays out the equations of the model. Section III describes the solution of the model and derives some comparative statics results. Summary and conclusions are contained in Section IV.

II. The Model

Assume that our economy consists of two sectors, 1 and 2, of which only the former is subject to shift-work.[3] Let the outputs generated per unit of time be denoted by X_1 and X_2.

2. Svensson [11] has presented a related but somewhat different model of capital utilization in international trade. He assumes identical workers each of whom works both normal and abnormal hours. The implications for comparative advantage, price changes, and resource endowment changes are not worked out.

3. Sector 1 may be viewed as industry and sector 2 as agriculture. As Georgescu-Roegen [4] notes, agriculture is generally not subject to shift-work while industry is.

Suppose that from the viewpoint of the firm, two pools of labor exist, L^d and L^n, where L^d denotes the total supply of daytime labor (6 a.m. to 6 p.m.) and L^n denotes the total supply of night-time labor (6 p.m. to 6 a.m.). Sector 1 uses both daytime and nighttime labor (provided shift-work is profitable) while sector 2 uses the former only. There is another factor of production, capital, which is used by both sectors and whose supply in stock terms is denoted by K.

We denote by U the number of shifts operated in sector 1. If the capital stock is operated during the daytime only, U is defined to be 1, and if it is operated both day and night, U equals 2. As will be explained in footnote 5, it will turn out to be profitable for sector 1 to operate either at $U = 1$ or at $U = 2$. The units of capital services are defined so that when the stock of capital in sector 1, K_1, is employed for U shifts, it generates UK_1 units of capital services. Evidently, when only one shift is operated, the stock and service flows of capital in sector 1 both equal K_1. Letting L_1^d denote the amount of labor employed in sector 1 at the normal time, we can write the production function as

$$X_1 = F_1(K_1, L_1^d) + F_1((U-1)K_1, L^n), \tag{1}$$

where F_1 is assumed to be linear homogeneous in its arguments. It must be noted that $F_1(K_1, L_1^d)$ denotes the output of good 1 produced during the day shift while $F_1((U-1)K_1, L^n)$ is the output produced during the night shift.

As in BCP [2; 3] we assume that firms face a zero ex-post elasticity of substitution between capital and labor services. They therefore have to maintain the same capital-services-to-labor-services ratio during the day and night shifts.[4] Thus while this ratio can be altered between two different units of time, say, in response to a change in factor prices, it cannot be altered within the same unit of time. During any unit of time, we require that the same number of workers be working on the given capital stock at any given instant. Formally, we have

$$L^n = (U-1)L_1^d. \tag{2}$$

Substituting for L^n from (2) into (1), our production function assumes the form

$$\begin{aligned} X_1 &= F_1(K_1, L_1^d) + F_1((U-1)K_1, (U-1)L_1^d) \\ &= UF_1(K_1, L_1^d). \end{aligned} \tag{1'}$$

As noted earlier, sector 2 is assumed to be operating during the day only. Therefore, we can represent its output by a conventional production function

$$X_2 = F_2(K_2, L_2), \tag{3}$$

where F_2 is linear homogeneous in its arguments and K_2 and L_2, respectively, are capital and labor employed in sector 2. Observe that L_2 is the day labor and K_2 represents the stock as well as the service flow of capital per unit of time. Full employment of capital implies

$$K_1 + K_2 = K. \tag{4}$$

Denoting the rental on capital by r, the night wage by w^n, and the day wage by w^d, cost minimization by firms in sector 1 yields the following Lagrange expression:

4. This is a standard assumption in the capital utilization literature [1, 32-3].

$$\underset{K_1,\, L_1^d}{\text{Min. R}} = rK_1 + L_1^d w^d + (U-1)L_1^d w^n + \lambda(\bar{X}_1 - UF_1(K_1, L_1^d))$$

where λ is a Lagrange multiplier. Perfect competition will yield $\lambda = p_1$ where p_1 denotes the price of good 1. Minimization of R over K_1 and L_1^d yields [5]

$$p_1(\partial X_1/\partial K_1) = p_1 U(\partial F_1/\partial K_1) = r \quad (5)$$

$$p_1(\partial X_1/\partial L_1^d) = p_1 U(\partial F_1/\partial L_1^d) = w_d + (U-1)w^n. \quad (6)$$

According to equation (5), the value of marginal product of the capital stock is equated to the rental rate. To interpret equation (6), first note that we are holding the capital-services-to-labor-services ratio fixed between the normal- and abnormal- time shifts. Given the linear homogeneity of F_1 in capital and labor services, fixity of this ratio implies that the values of marginal products of labor during the two shifts will be equal.[6] Remembering that the day and night wages are different, it is evident that the value of marginal product of labor cannot be equated to both wage rates. Instead, according to condition (6), optimality requires that the value of marginal product of labor during either shift be equated to the average wage in sector 1 ($\bar{w}$), which equals $(1/U)\,[w^d + (U-1)\,w^n]$.[7,8]

In industry 2, cost minimization along with perfect competition yields the following standard conditions:

$$p_2(\partial F_2/\partial K_2) = r, \quad (7)$$

$$p_2(\partial F_2/\partial L_2) = w^d, \quad (8)$$

where p_2 denotes the price of good 2.

Next we turn to the supply of day and night labor. To keep matters simple we shall assume that the number of hours supplied is the same and fixed for all workers, whether they work during the day or at night. If a worker works at night, his utility is reduced thereby, but workers differ in the degree to which they find night work distasteful. Each worker must decide whether the extra wages of night work are sufficient to compensate him for the disutility of such work. As the night labor force increases, increasingly unwilling workers have to be drawn into night work and the required wage premium increases. Thus we write the supply of night labor as

$$\Psi(L^n) = (w^n - w^d)/w^d \quad (9)$$

5. The reader may wonder why U is not a choice variable in the firm's cost-minimization problem. The answer is that firms do maximize with respect to U but there is a corner solution with respect to this variable. Given a constant night wage, w^n, if it is profitable for firms to begin working during the night hours, it is profitable for them to operate throughout the night (i.e., $U = 2$).

6. Mathematically, the nighttime value of marginal product of labor can be written as $p_1\,\partial F_1((U-1)K_1, L^n)/\partial L^n = p_1\,\partial F_1((U-1)K_1, (U-1)L_1^d)/\partial((U-1)L_1^d) = p_1\,\partial F_1(K_1, L_1^d)/\partial L_1^d$. The last expression here denotes the value of marginal product of labor during the daytime shift.

7. The average wage in sector 1, by definition, equals $(w^d L_1^d + w^n L^n)/(L_1^d + L^n) = [w^d L_1^d + w^n(U-1)L_1^d]/[L_1^d + (U-1)L_1^d] = [w^d + (U-1)w^n]/U$. From equation (6), we have $p_1\,\partial F_1(K_1, L_1^d)/\partial L_1^d = [w^d + (U-1)w^n]/U$, the average wage.

8. An alternative interpretation of condition (6) can be given as follows. In postulating the cost-minimization problem, we have made use of the condition $L^n = (U-1)L_1^d$. Therefore, a unit increase in L_1^d is accompanied by an increase of $(U-1)$ units in L^n. Consequently, $p_1\,\partial X_1/\partial L_1^d$ in (6) represents the value of marginal product of labor when L_1^d increases by 1 unit and L^n increases by $(U-1)$ units. Evidently, it must be equated to $w^d + (U-1)w^n$.

where $\psi(0)$ and $\psi'(L^n)$ are both positive.[9] Assuming that each worker supplies one unit of labor, total labor supply will equal the total number of workers. We can write

$$L_1^d + L^n + L_2 = L \quad (10)$$

$L_1^d + L^n$ will sometimes be denoted L_1.

Our model is now completely specified. Given the small country assumption, we can specify $p(= p_1 / p_2)$ exogenously. Then, given fixed endowments of capital and labor, we can solve equations (1) - (10) for the ten endogenous variables

$$X_1, X_2, L^n, L_1^d, L_2, K_1, K_2, r, w^d \text{ and } w^n.$$

III. Solution of the Model

The model can be solved by juxtaposing a demand curve and a supply curve for night labor L^n. The supply curve is given in equation (9) and is represented by the curve *SS* in the southeast quadrant of Figure 1. We measure w^n / w^d downward along the vertical axis and L^n rightward along the horizontal axis. We shall now derive the demand curve for L^n. We start by picking a level of L^n. With $U = 2$ we have $L_1^d = L^n$ and we also have the level of $L_2 (= \bar{L} - L^n - L_1^d)$. For given levels of L^n, L_1^d, and L_2, and of p_1 and p_2, we can draw the value of marginal product of capital (VMP_K) curves. These curves are drawn in the northwest quadrant of Figure 1, where we measure capital stocks on the horizontal axis and r on the vertical line farthest to the left. The total capital stock is indicated by the distance from the origin to the point $\bar{K}$, and K_1 is measured to the right from $\bar{K}$ and K_2 is measured to the left from the origin. The intersection of the two VMP_K curves determines the levels of K_1 and K_2 and the value of r.

We can now draw the value of marginal product of labor (VMP_L) curves in the northeast quadrant of Figure 1, where the total availability of labor is indicated by the distance along the horizontal axis from the origin to the point $\bar{L}$. As mentioned above, L^n is measured to the right from the origin and L_2 is measured to the left from point $\bar{L}$. The horizontal distance between L^n and L_2 is L_1^d, which equals L^n. On the vertical axis from the origin we measure $\bar{w}$ and on the vertical line from point $\bar{L}$ we measure w^d. The *VMP* curves for L^n and for L_2, together with the values of L^n and L_2, determine $\bar{w}$ and w^d.[10] Then w^n can be found from $\bar{w} = (w^n + w^d)/2$.

We have now found the values of w^n and w^d, and hence (w^n / w^d), for a given value of L^n. We repeat this process for each value of L^n and trace out the demand curve for L^n, denoted *DD* in the southeast quadrant of Figure 1. An appendix available upon request shows that the *DD* curve has the normal shape for a demand curve (downward sloping if the graph were right side up, but upward sloping in Figure 1) provided the relative physical

9. One way to justify the assumption that the hours of work will be the same for all day and night workers is to assume that each worker has a Cobb-Douglas utility function in goods and leisure and that if the worker works at night, the utility function is reduced by being divided by $1 + \psi$, where ψ varies across workers. Let the utility function be $G(\phi(c_1, c_2), H)/(1 + \psi) = G(c, H)/(1 + \psi) = cH^b/(1 + \psi)$, where H equals hours of leisure. H for both day and night workers will be $b/(1 + b)$ times total time available and for the marginal worker who is indifferent between day and night work, we have $1 + \psi = w^n/w^d$.

10. The VMP curve for L^n is $p_1 \partial F_1 / \partial L^n$.

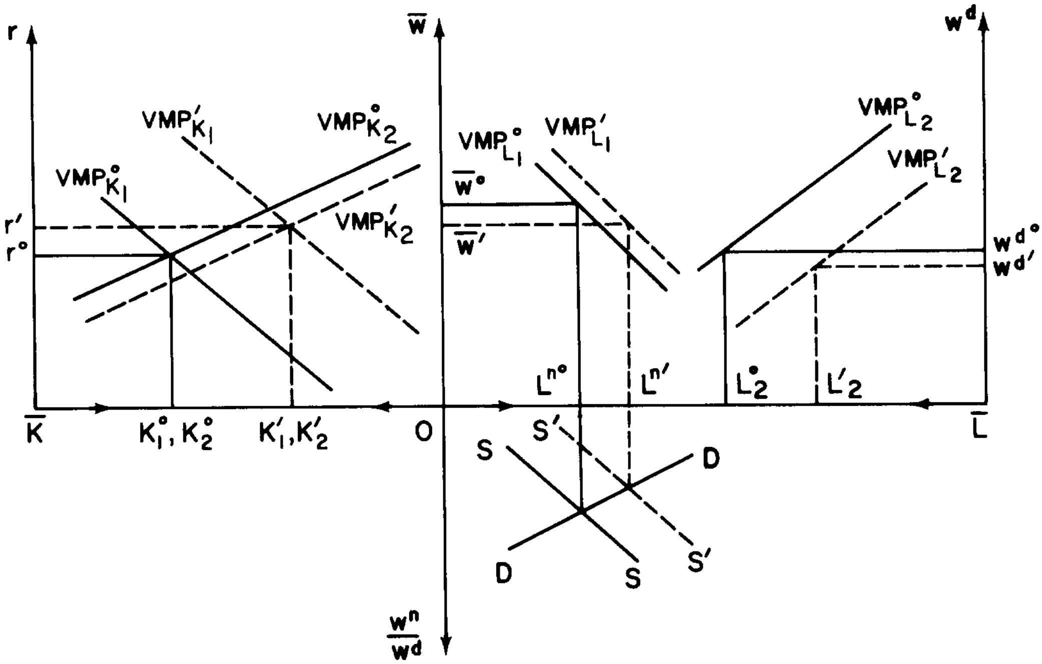

Figure 1

factor intensities of the two sectors correspond with their relative value factor intensities.[11] This is a condition that we would expect to be fulfilled.

Let us now consider some comparative statics of the model. We shall analyze in turn changes in the willingness of workers to work at night, changes in commodity prices, and changes in endowments.

Willingness to Work at Night and Comparative Advantage

An increased willingness to work at night would produce a shift to the right of the *SS* curve and the graph shows that L^n would increase and w^n/w^d would fall. It is clear that X_1 would increase and X_2 would fall,[12] and this result implies that if there were two otherwise identical economies, the one where workers were more willing to work at night would have a comparative advantage in the shift-working commodity.

An interesting feature of our model is that while a rightward shift in the *SS* curve unambiguously reduces w^n/w^d, it does not necessarily cause w^n to fall or w^d to rise; they might both rise or both fall. What happens to factor prices depends on factor intensities. If $K_1/L_1 > K_2/L_2$, then as sector 2 releases labor and capital at given factor prices, sector 1 tries to absorb the factors at a higher ratio of capital to labor than they are being released,

11. The physical factor intensities are K_1/L_1 and K_2/L_2. The value factor intensities are $rK_1/(rK_1 + w^d L_1^d + w^n L^n)$ and $rK_2/(rK_2 + w^d L_2)$. To say that the physical and value factor intensities correspond is to say that the same sector is capital or labor intensive under the two definitions.

12. L^n and L^d would rise. This change would shift up the VMP_{K_1} curve so that K_1 would rise. Hence X_1 would rise. Analogously, L_2 and K_2 must fall, and thus X_2 must fall.

and consequently there is upward pressure on r and downward pressure on w^d. On the other hand, if $K_1/L_1 < K_2/L_2$ the argument is reversed and there is downward pressure on r and upward pressure on w^d. This mechanism is reminiscent of the two-factor two-sector model and we shall refer to it as a Heckscher-Ohlin effect.

A graphical representation of the case in which $K_1/L_1 > K_2/L_2$ is shown by the dotted curves in Figure 1. The SS curve shifts to $S'S'$. L^n rises to $L^{n\prime}$. VMP_{K_1} shifts up and VMP_{K_2} shifts down in such a way that their intersection occurs at a higher level of r. VMP_{L_2} shifts down and VMP_{L_1} shifts up in such a way that at the new levels of L_2 and L^n, w^d and $\bar{w}$ both go down.[13] With (w^n/w^d) and w^d both falling, it is clear that w^n must also fall in this case. For more detail and in particular for a description of what happens to w^n under various cases, the reader is referred to the previously mentioned appendix.

Tariffs and Income Distribution

Suppose there is a change in either import or export duties such that the price of good 1 rises while the price of good 2 remains unchanged. The international trade literature has a long tradition of analyzing the effects of such a change on real factor returns. The Stolper-Samuelson results in a two-by-two model rely on Heckscher-Ohlin effects; the results are quite different in the specific-factors model [6; 9; 10]. We shall see that our model contains a combination of Heckscher-Ohlin and specific-factor effects.

Since the specific-factor effects are seen most clearly when we hold L^n constant, we shall let ψ' go to infinity initially. When the price of commodity 1 (p_1) rises while p_2 remains constant, the VMP_{K_1} curve shifts up and induces a rise in K_1 and a fall in K_2. The fall in K_2 shifts the VMP_{L_2} curve down and thereby reduces w^d. The rise in p_1 shifts the VMP_{L_1} up proportionally and the increase in K_1 shifts it still higher, so that $\bar{w}$ rises proportionally more than p_1. With $\bar{w}$ going up and w^d going down, w^n clearly goes up, and it goes up proportionally more than p_1.

To see the analogy between these results and those of the specific-factors model, we should think of capital as the mobile factor, night labor as the factor specific to sector 1 and day labor as the factor specific to sector 2. In the specific-factors model, a rise in p_1 raises the real return to the factor specific to sector 1, reduces the real return to the factor specific to sector 2, and causes the return to the mobile factor to fall in terms of good 1 and to rise in terms of good 2. These are exactly the effects we observe in our model.

When we let ψ' take on a positive but finite value, the results change. As the algebra of the above mentioned appendix shows, there is a Heckscher-Ohlin effect arising out of the change in L^n. The increase in p_1/p_2 causes sector 2 to release factors and sector 1 to absorb them. If $K_1/L_1 > K_2/L_2$, this transfer of factors puts downward pressure on w^d and w^n; this effect may be strong enough to make w^n/p_1 fall. Such a result is impossible in the specific-factors model.

The effects of a rise in p_1/p_2 on real factor returns are also ambiguous in the Jones and Easton [7] generalization of the specific-factors model, in which both sectors use all three factors and factors are defined as extreme and middle according to the intensity of use in the two sectors. The ambiguity in the Jones and Easton model arises out of substitutability

13. If $K_1/L_1 < K_2/\hat{L}_2$, the curves will shift in the same directions, but the intersection of VMP_{K_1} and VMP_{K_2} will occur at a lower level of r while the levels of w^d and $\bar{w}$ will be higher.

on the production side, whereas in our model the ambiguity arises out of substitutability in the *supplies* of day and night labor.

Capital Endowment Changes

Consider an increase in the endowment of capital. For simplicity we shall assume that the supply curve of L^n is vertical (that is, ψ' goes to infinity), so that L^n, L_1^d and L_2 are fixed. An increase in K (with product prices held constant) moves point $\bar{K}$ to the left in Figure 1 and causes the intersection of the two VMP_K curves to occur at a lower value of r. Both K_1 and K_2 must rise and hence the two VMP_L curves shift upward causing increases in $\bar{w}$ and w^d. What happens to w^n depends on whether $\bar{w}$ or w^d goes up proportionately more, since $\bar{w}$ is an equally weighted average of w^n and w^d. Specifically, if w^d rises (proportionately) more than $\bar{w}$, then w^n must be rising less than w^d and may even be falling. Which of these cases occurs depends on factor intensities in the two sectors, once again the reader is referred to the unpublished appendix for a demonstration. There we also show that these qualitative factor price effects ($\bar{w}$ and w^d rise, r falls, the change in w^n depends on factor intensities) continue to hold when the supply curve of L^n is not vertical.

Just as in the case of the commodity price change, the increase in capital endowment has both Heckscher-Ohlin and specific-factor effects. The fall in r and the rise in w^d are reminiscent of the capital endowment effects on factor prices in the specific-factors model, but the effects of a change in capital endowment on w^n and (w^n/w^d) depend on the relative capital-labor ratios in the two sectors, ratios which are crucial to the operation of the Heckscher-Ohlin model.

IV. Summary and Conclusions

We have presented a two-sector model in which there are essentially three factors of production, day labor, night labor, and capital. Workers differ in their distaste for day and night work and they move between the two times in response to the ratio of the two wages. Since night labor constitutes a third factor of production in this model, the Stolper-Samuelson and Rybcynski theorems are no longer valid. The striking contrast between these results and those of the single-labor-market model was noted in the introduction.

The willingness to work at night is a source of comparative advantage. We have shown that in otherwise identical economies, the one with the greater willingness to work at night must have a comparative advantage in the shift-working commodity. It is quite clear also that even where capital and labor endowments differ across counties, predictions of comparative advantage based on such endowments may be falsified by differences in the willingness to engage in night work.

Our model has certain features in common with the specific-factors model and certain features in comon with the (capital-labor version of the) Heckscher-Ohlin model. There are three factors of production as in the specific-factors model, but there are two factors which move between the sectors as in the Heckscher-Ohlin model. This mixture of ingredients yields a combination of Heckscher-Ohlin and specific-factor flavors in the results. As an example of the latter, an increase in capital endowment (at constant commodity prices) must reduce the price of capital and raise the wage of day labor. As an example of the former, the effect of a change in capital stock on the wage of night labor depends on the relative capital-labor intensities in the two sectors.

The effects of a commodity price change on real factor returns have a strong specific-factor flavor when the supply curve of night labor is vertical. In that case an increase in the price of the shift-working commodity must raise the real wage of night labor and lower the real wage of day labor. When the supply curve of night labor is no longer vertical, however, the factor price effects of a change in commodity prices become different. These effects now depend also on the capital-labor ratios in the two sectors, much as in the Heckscher-Ohlin model. Specifically, if sector 2 is sufficiently labor intensive, a rise in the price of the shift-working sector (1) may cause the real wage of night labor to fall.

Another result with a Heckscher-Ohlin flavor concerns the effects of an increased willingness of workers to work at night. A rightward shift in the supply curve of night labor must reduce the relative night wage (w^n / w^d), but, since it causes an expansion in sector 1, if that sector is sufficiently labor intensive the real wage of night labor (along with that of day labor) will rise.

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