AN ECONOMIC ANALYSIS OF CAPITAL UTILIZATION*

ROGER R. BETANCOURT and CHRISTOPHER K. CLAGUE

University of Maryland

Why is it a common occurrence for the capital stock of factories to be utilized much less than twenty-four hours a day? A frequent answer to this question is, "insufficient demand." But if the entrepreneur had correctly anticipated demand, he could have built a smaller factory and have utilized it more intensively. Under conditions of perfect foresight, then, appeal must be made to other factors to explain low levels of capital utilization.

The pioneering analysis of this subject has been conducted by Marris [7]. Briefly, Marris found that the extra wages which must be paid to night workers were not sufficient to render night-time operation unprofitable; he discussed many factors which discourage shift-work, but in his view the most important was what he called "output restraint." He assumed that, as a result of an imperfectly competitive market, the level of output which the firm could sell was given and the decision to utilize capital intensively involved constructing a smaller factory, with attendant diseconomies of small-scale operation. Finally, among the factors which influence the existence of shift-work positively, Marris stresses the capital intensity of the production process under single-shift operation. Winston has forcefully brought the subject of capital utilization to the attention of economists [13–17]. (See also Georgescu-Roegen [5].) In so doing he has shown the important role of the elasticity of substitution in determining the effect of factor prices on shift-work under the assumption of a CES production function and constant returns to scale [14; 17].

This paper integrates the work of Marris and Winston in two important ways. First, by formulating the CES case in a different manner from Winston, we are able to analyze the impact of capital intensity on the profitability of shift-work separately from that of the elasticity of substitution; and, we are also able to assess the joint impact of these two factors on the profitability of shift-work. Secondly, by introducing economies of scale into a CES framework, we formulate a model which incorporates as special cases the contributions of both Winston and Marris under the assumption of output restraint. In addition, we generalize their studies by analyzing the case in which the rigid output constraint is replaced by a demand curve facing the firm. This generalization is of importance for a proper evaluation of the policy implications of increased capital utilization. For, shift-work is increasingly mentioned in connection with the employment problem in less developed countries; but, the effects of shift-work on employment depend critically on the nature of the output restraint. If it is rigid, we find that in a wide variety of cases total employment will actually fall under shift-work. When the firm faces a demand curve, on the other hand, it may or may not choose to produce a larger level of output under shift-work than under single-shift operation. If it does, total employment will usually increase, with the magnitude of the increase depending on demand conditions and on the nature of economies of scale. Finally, we correct an error in Winston's analysis which obscures one of the most interesting consequences of increased utilization: contrary to his analysis

* The authors would like to thank Lloyd Atkinson, Paul Wonnacott, and an anonymous reviewer for comments on an earlier draft of this paper. Computer time was provided by the Computer Science Center of the University of Maryland. This paper is a somewhat condensed version of a November 1973 paper, which is available from the authors on request.

[14], and to Marris' assertion [7], we find that it is quite possible for capital productivity to decline when utilization increases.

The next section sets forth our general model of the capital utilization decision under the assumption of a fixed output. The second develops the conditions for profitable shift-work and the implications of shift-work for employment and for capital productivity under a CES production function. In the third section the firm is free to choose the most profitable level of output, which need not be the same under the two systems. Finally, a conclusion highlights the main results of the paper and suggests some of its implications.

I. THE BASIC MODEL

Let us assume that an entrepreneur who wants to maximize profits is constrained to produce in a single-product plant a given level of output which is to be sold at a known price. He must select a system of operation, i.e., a single-shift or double-shift system,[1] and construct the appropriate plant in such a way as to minimize costs, and hence maximize profits, for the given level of output.

More precisely, the entrepreneur will choose the system with the highest profit function. Under the first system this profit function will be

$$\pi^1 = pf(S^1, L^1) - rK^1 - w_1L^1$$

and under the second it will be

$$\pi^2 = p[f(S_1^2, L_1^2) + f(S_2^2, L_2^2)] - rK^2 - w_1L_1^2 - wL_2^2,$$

where π represents daily profits, f is the production function, K is the stock of capital, S is the daily flow of capital services, L is the daily flow of labor services, r is the cost of owning the capital stock for one day, and w is the price of a unit of labor services. Throughout, a superscript will be used to identify the system of operations and the subscript will be used to identify the shift. We assume both marginal products are declining but positive. The production functions are assumed to be the same on both shifts.

The price of capital, r, equals $P_c(i + d)$ where P_c is the price of a standard machine, i is the daily interest rate and d is the rate of depreciation. Implicitly, we have assumed that depreciation is independent of the rate of utilization by allowing the daily cost of owning a unit of capital to be the same under the two systems. Capital services are assumed to be related to the capital stock in the following manner: $S^1 = U^1K^1$, $S_1^2 = U_1^2K^2$, $S_2^2 = U_2^2K^2$, and $U_1^2 + U_2^2 = U^2$. With the assumptions made thus far, i.e., a given level of output, no demand uncertainty, and no depreciation in use, it will always be optimal for the firm to produce within a shift at the maximum rate of capital utilization (the "full capacity" rate [16; 5]). Therefore, $U_1^1 = U_2^2 = U^1 = U^*$ and $U^2 = 2U^*$. The capital utilization rate is assumed to include a constant which defines the period of analysis and the maintenance requirements.

Our analysis will neglect certain other relevant factors which have been discussed by Marris and Winston and on which we have nothing to add. We shall merely note their implications here. The existence of economies in prime costs due to continuous processing tends to increase the profitability of shift-work in some industries and in certain cases such as petroleum refining, steel, and glass becomes a dominant factor in the decision. On the other hand, entrepreneurs may have strong preferences against the inconveniences of shift-work and may thus be willing to forego some profits to avoid them. In addition, there may be "indivisibilities in management" (apart from economies of scale in production) which require a certain establishment size for an efficient division of labor within the management group when shifts are employed [7, 92–94]. Finally, depreciation in use would discourage both capital and capacity utilization by making

[1] The analysis is easily extended to three- or four-shift systems, but the economic reasoning is the same and therefore these cases will be ignored.

the price of capital higher as utilization increases.

Returning to the original problem, we can state the choice of the system of operations in terms of the condition for the higher profitability of the two-shift system, i.e., $\pi^2 \geq \pi^1$:

$$rK^1 + w_1L^1 \geq rK^2 + w_1L_1^2 + w_2L_2^2.$$

While we shall allow for capital-labor substitution before the plant is built (*ex ante* substitution), we shall assume that there is no substitutability *ex post*.[2] This, together with our full capacity assumption, implies that under the double-shift system the same amount of labor will be used on the two shifts: $L_1^2 = L_2^2$. Dividing both sides by costs under system 1 and defining $\alpha = w_2/w_1 - 1$ as the percentage shift differential, we obtain after manipulation:

$$1 > \left[\frac{rK^2}{w_1L_1^2} + (2+\alpha)\right]\frac{L_1^2}{L_1}\left[\frac{rK^1}{w_1L^1} + 1\right]^{-1},$$

where the R.H.S. is the ratio of costs under system 2 to those under system 1. Defining θ as the share of capital costs in combined labor and capital costs under system 1, we have:

$$1 > \left[\frac{rK^2}{w_1L_1^2} + (2+\alpha)\right]\frac{L_1^2}{L^1}(1-\theta). \quad (1)$$

We shall refer to (1) as the shift-work profitability condition and to its R.H.S. as the "cost ratio." The lower the cost ratio, the greater the gains (or the smaller the losses) from shift-work.

II. THE CES CASE

We shall assume that the *ex ante* production function is of the CES form. Production under system 1 is given by

$$X^1 = \gamma^\beta[\delta(S^1)^{-\rho} + (1-\delta)(L^1)^{-\rho}]^{-\beta/\rho}$$

and under system 2 by

$$X_1^2 = \gamma^\beta[\delta(S_1^2)^{-\rho} + (1-\delta)(L_1^2)^{-\rho}]^{-\beta/\rho} = X_2^2.$$

[2] The case in which the *ex post* elasticity of substitution is equal to the *ex ante* has been treated by Baily [1, 31–33].

Cost minimization implies:

$$\frac{K^1}{L^1} = \left[\frac{\delta}{1-\delta}\frac{w_1}{r}\right]^\sigma (U^*)^{\sigma-1} \quad (2)$$

$$\frac{K^2}{L_1^2} = \left[\frac{\delta}{1-\delta}\frac{w_1}{r}(2+\alpha)\right]^\sigma (U^*)^{\sigma-1} \quad (3)$$

$$L_1^2/L^1 = 2^{-1/\beta}[\theta(2+\alpha)^{\sigma-1} + (1-\theta)]^{\sigma/(1-\sigma)}. \quad (4)$$

Hence the profitability condition becomes:

$$1 > 2^{-1/\beta}(2+\alpha)[\theta(2+\alpha)^{\sigma-1} + (1-\theta)]^{1/(1-\sigma)}. \quad (5)$$

It would be more conventional to write the condition (5) using δ and w/r instead of the share θ. (This is what Winston does [14].) There are several advantages, however, to writing the profitability condition and other conditions (see below) in terms of θ. In the first place, δ is not calculable from observed data but must be estimated; θ, on the other hand, is either observable or can readily be calculated.[3] Secondly, both w/r and δ are arbitrary; they depend on the units in which the inputs are measured, and in the case of machines the units are arbitrary. Thirdly, we would like to be able to ascertain the impact of a change in the elasticity of substitution on the profitability of shift-work; but the question arises as to what should be held constant when σ is changed. It seems to us to be appropriate to hold the capital-labor ratio constant at the initial set of factor prices when σ is changed. (There is considerable uncertainty about the values of σ in the real world; new information on these values would not be accompanied by any change in our estimates of observed capital-labor ratios at existing factor prices.) Our procedure accomplishes this by holding θ constant as σ is changed in equation (5). On the other hand, the alternative procedure would lead us to consider a change in σ with δ constant. As can be seen from equation (3),

[3] For firms working one shift θ is observable. For firms working more than one shift, it is necessary to assume a value of σ in order to calculate θ (see Betancourt and Clague [4]).

this implies a change in the capital-labor ratio at the initial set of factor prices. Thus, in order to separate the role of capital intensity from that of the elasticity of substitution, it is desirable to write the condition terms of θ.

The effect on the cost ratio of changes in parameter values can be derived by differentiating equation (5) with respect to the various parameters. The derivatives with respect to α and β can easily be shown to be positive, and the derivative with respect to θ is clearly negative.[4] The sign of the derivative with respect to σ is not obvious from the expression given in footnote 4, but experimentation with a variety of numerical values revealed that it was always negative. Rather than present the results of those calculations here, we present the value of the cost ratio itself under various parameter values (see Table I). It can be seen there that the cost ratio falls as σ increases, with the other parameters held constant.

The signs of the partial derivatives are consistent with economic reasoning. The higher is α, the less profitable is shift-work, since night-time labor is more expensive. Economies of scale ($\beta > 1$) discourage shift-work, in the presence of the output restraint, by requiring the construction of a smaller plant under double-shift operation. The higher is σ (holding constant the capital share), the more profitable is shift-work. This phenomenon can be explained intuitively as follows. If wear-and-tear depreciation is ignored,[5] the price of capital services per hour is only half as great under system 2 as under system 1. The average price of labor is also more expensive under system 2. If possible, then, the entrepreneur will select a higher instantaneous capital-labor ratio under system 2 than system 1, as can be seen in comparing equations (2) and (3). The higher the elasticity of substitution, the more the entrepreneur will be able to take advantage of cheaper capital services and avoid the use of more expensive labor, and the greater will be the profitability of shift-work.[6] Finally, the higher is θ, the capital share, the more profitable is shift-work, since shift-work involves paying more for a unit of labor and less for a unit of capital services.

Policymakers frequently wish to know the effect of changes in factor prices on the profitability of shift-work. The formulation of the condition in terms of θ makes it easy to calculate these effects: it is necessary only to translate factor price changes into changes in θ; as is well-known, this depends on σ. If $\sigma < 1$, a fall in the wage rate leads to a rise in θ and increased profitability of shift-work. If $\sigma > 1$, a fall in the wage rate leads to the opposite result.

Shift-work has been suggested as a means of ameliorating the employment problem in less developed countries. The longrun effects of shift-work on employment per unit of output can be calculated from equation (4). If L_1^2/L^1 is less than $\frac{1}{2}$, less total employment will be required (for given output) on system 2 than on system 1. L_1^2/L^1 will be smaller, the smaller is β, the larger is α, the

[4] Denote $\theta(2+\alpha)^{\sigma-1} + (1-\theta)$ by B and note that $B > 0$. Taking the log of the RHS of (5) and partially differentiating yields:

$$\frac{\partial \log CR}{\partial \beta} = \frac{\log 2}{\beta^2} > 0$$

$$\frac{\partial \log CR}{\partial \alpha} = \frac{1-\theta}{(2+\alpha)B} > 0$$

$$\frac{\partial \log CR}{\partial \theta} = \frac{(2+\alpha)^{\sigma-1} - 1}{(1-\sigma)B} < 0$$

$$\frac{\partial \log CR}{\partial \sigma} = \frac{1}{1-\sigma}\,\frac{\theta(2+\alpha)^{\sigma-1}\log(2+\alpha)}{B} + \log B\,\frac{1}{(1-\sigma)^2}\,.$$

[5] If wear-and-tear depreciation is incorporated into the analysis, the substitution phenomenon described in this paragraph will be smaller but will not disappear.

[6] As a general proposition, whenever a firm experiences a decline in the price of a factor, the firm's profits will go up by more if that factor can be more readily substituted for other factors.

TABLE I

COST RATIO[1] AND EMPLOYMENT RATIO[2]

	θ	σ = 0 C.R.	σ = 0 L_1^2/L^1	σ = .4 C.R.	σ = .4 L_1^2/L^1	σ = 1.0 C.R.	σ = 1.0 L_1^2/L^1	σ = 1.4 C.R.	σ = 1.4 L_1^2/L^1
I. β = 1., α = .5									
	.1	1.175	.500	1.163	.486	1.141	.456	1.122	.430
	.2	1.100	.500	1.079	.471	1.041	.416	1.011	.372
	.3	1.025	.500	.997	.457	.950	.380	.915	.323
	.4	.950	.500	.918	.442	.866	.347	.832	.283
II. β = 1.15, α = .5									
	.1	1.286	.548	1.273	.532	1.249	.499	1.228	.470
	.2	1.204	.548	1.181	.516	1.139	.456	1.107	.407
	.3	1.122	.548	1.091	.500	1.039	.416	1.002	.354
	.4	1.040	.548	1.005	.484	.948	.379	.910	.309
III. β = 1.15, α = .2									
	.1	1.138	.548	1.129	.534	1.113	.506	1.099	.482
	.2	1.073	.548	1.057	.519	1.028	.467	1.007	.426
	.3	1.007	.548	.986	.505	.951	.432	.925	.378
	.4	.941	.548	.917	.491	.878	.399	.852	.337

[1] The ratio of costs under system 2 to those under system 1 (C.R.). α is the shift differential, β is the scale parameter, θ the share of capital costs in combined labor and capital costs on the first shift, and σ is the *ex ante* elasticity of substitution between labor and capital.

[2] The ratio of employment in the first shift of system 2 to employment in the first shift of system 1. (L_1^2/L^1).

larger is θ, and the smaller is σ.[7] Illustrative values of L_1^2/L^1 are shown in Table I, which shows that when shift-work is profitable, L_1^2/L^1 is less than ½ in a rather wide variety of cases.[8] Of course, these calculations understate the effects of shift-work on total employment because they do not allow for the tendency of firms working shifts to choose a larger volume of output. This topic is taken up in the next section. Nevertheless, since firms are frequently not in a position to increase output substantially, the figures in Table I suggest that shift-work may not have the effects on employment which it is widely hoped and believed to have. Finally, shift-work has usually been thought of as a means of saving capital.[9] But our model shows that shift-work may actually reduce

[7] Taking logs of (4) and partially differentiating yields (where B is defined in footnote 4):

$$\frac{\partial \log L_1^2/L^1}{\partial \beta} = \frac{1}{\beta^2} \log 2 > 0$$

$$\frac{\partial \log L_1^2/L^1}{\partial \alpha} = -\frac{\sigma}{B}\,\theta(2+\alpha)^{\sigma-2} < 0$$

$$\frac{\partial \log L_1^2/L^1}{\partial \theta} = \frac{\sigma}{1-\sigma}\,\frac{1}{B}\,[(2+\alpha)^{\sigma-1} - 1] < 0$$

$$\frac{\partial \log L_1^2/L^1}{\partial \sigma} = \frac{\sigma}{1-\sigma}\,\frac{1}{B}\,\theta(2+\alpha)^{\sigma-1}\log(2+\alpha) + \log B(1-\sigma)^{-2}.$$

The sign of the last expression was always found to be negative under a wide variety of parameter values.

[8] A slightly paradoxical result emerging from our model is that a decline in the night-shift premium, sufficient to tip the balance in favor of shift-work, may actually cause employment per unit of output to fall. For example, comparing parts II and III of Table I for $\sigma = .4$, $\theta = .4$, if $\alpha = .5$ total employment equals L^1; but, if $\alpha = .2$, total employment equals $.982L^1$.

[9] In his opening paragraph, Marris [7, 1] states, "shift-work attracts attention, and is widely practised because it in many cases reduces prime costs, and in all cases saves capital."

capital productivity, or the output per unit of capital stock. Combining (2), (3), and (4) results in:

$$K^2/K^1 = (2+\alpha)^\sigma(L_1^2/L^1) = (2+\alpha)^\sigma 2^{-1/\beta} [\theta(2+\alpha)^{\sigma-1} + (1-\theta]^{\sigma/(1-\sigma)}. \quad (6)$$

Since output is assumed equal for the two systems, a decline in capital productivity under shift-work merely requires the R.H.S. of (6) to exceed 1. This will come about if σ and β are "large enough." A large σ increases the capital intensity of the technique selected under system 2, and a large β decreases capital productivity through diseconomies of small-scale operation. This can be more easily seen by rewriting (6) in terms of the cost ratio as follows:

$$K^2/K^1 = (C.R)^\sigma 2^{(\sigma-1)/\beta}. \quad (7)$$

This expression also reveals that a $\sigma > 1$ is a necessary (but not sufficient) condition for shift-work to be profitable and at the same time reduce capital productivity.[10]

III. ALLOWING OUTPUT TO DIFFER UNDER THE TWO SYSTEMS

We now assume that the firm faces a downward-sloping demand curve. If the marginal cost curves under system 1 and system 2 do not coincide, the firm will normally select different outputs under the two systems. To avoid confusion, let us emphasize the differences in the assumptions employed in different sections of this paper. In Sections I and II, the firm's output and price were both outside the firm's control and were constant over time. These assumptions were adopted for simplicity, but for the sake of a convenient label we might call this the case of oligopoly with stable demand. The case of pure competition with stable price can be analyzed with the model developed in Sections I and II, provided β is set equal to one. In the present section the firm is treated as a monopolist, with some control over both its output and its price. This is monopoly with stable demand.[11] We shall assume that the demand curve is smooth, but this does not rule out the possibility that the curve incorporates the price responses of the competition. If, however, the demand curve is kinked, and the marginal-cost curves intersect in such a way that output would be the same under the two systems, then the model of Sections I and II can be applied without modification.

Under monopoly with stable demand, the condition for choosing system 2 over system 1, $\pi^2 \geq \pi^1$, becomes:

$$TR(X_2) - TC^2(X_2) > TR(X_1) - TC^1(X_1),$$

where X_1, X_2 = the optimal levels of output under systems 1 and 2

TC^1, TC^2 = total costs under systems 1 and 2

TR = total revenue.

Rewriting this conditions, we get:

$$1 > \frac{TR(X_1)}{TC^1(X_1)} - \frac{TR(X_2) - TC^2(X_2)}{TC^1(X_1)}. \quad (8)$$

[10] Winston's [14] equation (10) and Table III imply that capital productivity is always higher under shift-work. This is due to an error in his analysis. Correctly formulated, his equation (10) should read:

$$\frac{Q_0/\bar{K}_2}{Q_0/\bar{K}_1} = 2\left[\frac{1 + R^{1-\sigma}\Delta^{-\sigma}}{1 + (2BR)^{1-\sigma}\Delta^{-\sigma}}\right]^{\sigma/(1-\sigma)}.$$

If this equation is used, his results are quite consistent with ours. Parenthetically, the possibility that shift-work may lower capital productivity is contained in [12, Table 1], but the authors did not mention it.

[11] In an earlier version of the paper we investigated the anticipated unstable demand case under perfect competition and oligopoly. Since the analysis was somewhat informal, we merely report the main conclusion emerging from our discussion: under both oligopoly and perfect competition, the impact of anticipated demand fluctuations on the average amount of time the factory was designed to work shifts depended on the outcome of the stable demand case; if the factory was designed to work shifts under stable demand, the introduction of anticipated fluctuations in demand reduced the average amount of time during which the factory would be designed to operate shifts; if the factory was designed to work a single shift under stable demand, then anticipated demand fluctuations would increase the average amount of time during which the factory would be designed to operate shifts.

The R.H.S. of (8) will be called the "value of condition."

In discussing the optimal level of output under the two systems, it is convenient to define economies of scale in a different way. Let us define the "cost elasticity," denoted by ϕ, as the percentage increase in average costs when output under single-shift operation is cut in half. This elasticity arises naturally in a discussion of shifting from one shift to two when output is fixed. Rather than deriving our previous results with this concept, we shall simply show the relationship between ϕ and β, the scale parameter used earlier. Letting ϕ and β be functions of output (X), and letting AC and TC stand for average and total costs,[12]

$$1 + \phi(X) = \frac{AC(X/2)}{AC(X)} = \frac{2TC(X/2)}{TC(X)} = 2^{1-1/\beta}.$$

To find the value of condition in (8), we must develop an expression for the relationship of $TC^2(X)$ to $TC^1(X)$. This relationship clearly depends on α, β, θ and the form of the production function. In the case of the CES production function, we can rewrite equation (5) as:

$$\frac{TC^2(X_1)}{TC_1(X_1)} = \frac{(2 + \alpha)\ [\theta(2 + \alpha)^{\sigma-1} + (1 - \theta)]^{1/(1-\sigma)}}{2^{1/\beta}}.$$

Replacing β by an expression in ϕ, we get:

$$\frac{TC^2(X_1)}{TC^1(X_1)} = \frac{(2 + \alpha)\ [\theta(2 + \alpha)^{\sigma-1} + (1 + \theta)]^{1/(1-\sigma)}}{2} \cdot [1 + \phi(X_1)] = C^*[1 + \phi(X_1)],$$

where C^* represents the value of the cost ratio when economies of scale are absent and is independent of X. The following expression then holds for all values of X:

$$TC^2(X) = C^*[1 + \phi(X)]TC^1(X).$$

Hence,

$$MC^2(X) = C^*\{[1 + \phi(X)] \cdot MC^1(X) + TC^1(X)\phi'(X)\}, \quad (9)$$

where MC^1 and MC^2 refer to marginal costs under systems 1 and 2.

In order to illustrate the model, we shall assume that when constrained to produce at output X_1, the firm is indifferent between working and not working shifts. This implies that $C^* = 1/[1 + \phi(X_1)]$. To calculate the value of condition when the output constraint takes the form of a demand curve, we need to make assumptions about the functional form of the demand curve and the cost curves. We assume the demand curve has a constant elasticity e. Average costs are assumed to decline smoothly and asymptotically toward 1 according to the function

$$AC^1(X) = AX^{-f} + 1; \quad A > 0, \quad 0 < f \leq 1. \quad (10)$$
$$AC^2(X) = A(X/2)^{-f} + 1.$$

An important parameter in the analysis is $\phi'(X)$, or the rate at which the cost elasticity declines as output expands. Referring back to (9), we can see there that if $\phi'(X) = 0$, which is to say that $\phi(X)$ is a constant, and if $C^*[1 + \phi(X)] = 1$, then the marginal costs of system 1 and 2 coincide, so that the level of output selected would be identical under the two systems. It follows that the replacement of a rigid demand constraint by a demand curve would have no effect on the choice of systems.[13]

Is it possible that $\phi(X)$ is constant? The .6

[12] $X = [\mu\ TC(X)]^\beta$ and $X/2 = [\mu\ TC(X/2)]^\beta$ where μ is a constant which depends on factor prices. Hence $TC(X/2)/TC(X) = 2^{-1/\beta}$.

[13] This result also follows from the analysis in the previous sections; i.e., as long as the scale parameter (β) is not made a function of the level of output, the profitability condition is independent of the level of output (see equation (5)). Therefore, if one system of operations is more profitable at one level of output, it will also be more profitable at all levels of output.

rule, employed by engineers to estimate costs, implies that it is. Some studies of cost functions find the data broadly consistent with that hypothesis, especially in "process industries" such as chemicals, glass and steel [6; 8]. The constancy of ϕ, however, implies that average costs asymptotically approach zero as output expands. Studies based on accounting and interview data typically find that costs level out at some point [2; 3; 9; 10; 11].

Without attempting to resolve this empirical question, we shall analyze the case in which $\phi(X)$ does decline with output. Since $\phi'(X_1)$ depends on the level of X_1, which is arbitrary, we shall work with the elasticity of ϕ, or

$$\lambda(X) = \phi'(X)[X/\phi(X)].$$

When output is increased by 1 percent, ϕ changes by λ percent. It can be shown, if equation (10) is assumed to hold, that $0 > \lambda(X_1) > -1$.

We select values of $\phi(X_1)$, $\lambda(X_1)$ and e. For an arbitrary X_1 we can then solve for the parameters of the cost function and the demand function. We then locate X_2 and calculate the value of condition in (8).

Since λ must lie between 0 and 1, we have selected values of $-.67$, $-.33$ and 0 for this important parameter. We selected .285, .1667, and .1 for ϕ. The elasticity of demand e was set at 2 and at its maximum value was consistent with the firm's breaking even under system 1. Table II shows the results.

Several interesting points emerge from Table II. When economies of scale are entirely absent, condition takes on the value C^*. This situation would obtain in a perfectly competitive industry, since all firms would expand, at least until they reached their minimum optimal size. This, then, is the incentive to utilize capital when the output constraint is ignored. This corresponds to the economic analysis of shift-work prior to Marris. If the output constraint takes the form of a rigid limit, the value of condition becomes 1. This represents Marris' (and Winston's) analysis. When the rigid output constraint is replaced by a smooth demand curve, condition takes on the values shown in the rest of the table.

Always assuming that when constrained to output X_1, the firm is indifferent between working and not working shifts, the table shows, as expected, that the larger the elasticity of demand, the greater the incentive to work shifts. Given e, the larger the absolute value of $\lambda(X_1)$ (the closer it is to -1), the stronger the inducement to work shifts.

TABLE II

VALUE OF CONDITION WHEN OUTPUT IS ALLOWED TO VARY

$\phi(X_1)$	$\lambda(X_1)$	C*	Output Fixed	e=2	e=emax	emax[1]	Per cent increase in output[2]
.285	-.67	.778	1.00	.958	.915	3.4	138
.285	-.33	.778	1.00	.984	.958	3.1	131
.285	0	.778	1.00	1.00	1.00	n.a.	0
.1667	-.67	.857	1.00	.986	.939	5.7	145
.1667	-.33	.857	1.00	.996	.970	5.0	136
.1667	0	.857	1.00	1.00	1.00	n.a.	0
.100	-.67	.909	1.00	.995	.959	9.2	148
.100	-.33	.909	1.00	.999	.979	8.2	138
.100	0	.909	1.00	1.00	1.00	n.a.	0

[1] $emax$ = maximum value of $e = AC(X_1)/[AC(X_1) - MC(X_1)]$.
[2] Per cent by which output under system 2 exceeds output under system 1 when $e = emax$.

Finally, given e and $\lambda(X_1)$, the greater the value of $\phi(X_1)$, the greater the incentive to utilize capital.

The increases in output resulting from the decision to work shifts are relevant to the calculation of the employment effects of shift-work. The percentage increase in output need only be multiplied by the ratio of employment per unit of output under system 2 to that under system 1 to find the percentage increase in employment resulting from shift-work. Table II shows the percentage increase in output when $e = e\text{max}$. These increases are quite large. As a qualification of these results it should be pointed out that the percentage increase in output is sensitive to the assumption of a constant-elasticity demand curve; a straight-line demand curve would generate a much smaller increase.

IV. CONCLUSIONS AND IMPLICATIONS

This paper has presented a neoclassical analysis of the firm's decision to utilize its capital stock. In line with previous work, we found the profitability of intensive capital utilization to be related positively to the capital share and negatively to the shift differential and to economies of scale. In going beyond previous work, we showed that a high elasticity of substitution favors shift-work by enabling the firm to fake full advantage of the cheaper capital and more expensive labor which shift-work implies. Moreover, we also showed how these four factors interact in determining the profitability of shift-work. The replacement of Marris' rigid output restraint by a demand curve facing the firm increased modestly the profitability of shift-work (under most circumstances). There is a tendency for firms deciding to work shifts to produce a larger level of output. (Recall that the level of output is determined before the plant is built; we are not talking about adding an extra shift to an existing plant.) The magnitude of the increase in output under shift-work depends in part on the nature of economies of scale, in particular how rapidly they diminish as output expands. A rapid diminution of economies of scale (λ close to -1) is conducive to a larger increase in output.

To conclude, let us highlight some of the specific results obtained which have been ignored in the earlier literature and which may have significant policy implications. Shift-work does not always save capital, but on the contrary in a range of cases it reduces the productivity of the capital stock. In a broader range of cases shift-work reduces employment per unit of output. Moreover, it is quite possible for a decline in the nightly wage to make shift-work profitable and thereby decrease employment per unit of output. In all of these unorthodox results, a critical role is played by the elasticity of substitution. The effects of shift-work on total employment depend importantly on what happens to output: under conditions of oligopoly, a rigid output constraint may be reasonable; if not, consideration must be given to the shape of the demand curve and the nature of economies of scale.

REFERENCES

1. Baily, Mary Ann, "Capital Utilization in Kenya Manufacturing Industry." Ph.D. dissertation, Massachusetts Institute of Technology, January 1974.
2. Bain, Joe S., "Economies of Scale, Concentration, and the Condition of Entry in Twenty Manufacturing Industries." *American Economic Review*, March 1954, 15–39.
3. ———, *Barriers to New Competition*. Cambridge, Massachusetts: Harvard University Press, 1956.
4. Betancourt, Roger R. and Christopher K. Clague, "An Econometric Analysis of Capital Utilization." Mimeographed. College Park Maryland. University of Maryland, May 1974.
5. Georgescu-Roegen, Nicholas, "The Economics of Production." *American Economic Review*, May 1970, 1–9.
6. Haldi, John and David Whitcomb, "Economies of Scale in Industrial Plants." *Journal of Political Economy*, August 1967, 373–385.
7. Marris, Robin. *The Economics of Capital Utilization*. Cambridge, England: Cambridge University Press, 1964.
8. Moore, Frederick T., "Economies of Scale: Some Statistical Evidence." *Quarterly Journal of Economics*, May 1959, 232–245. (See also comments and reply in August 1960 issue.)
9. Pratten, C. F. *Economies of Scale in Manufacturing Industry*. Cambridge, England: Cambridge University Press, 1971.
10. Scherer, F. M. *Industrial Market Structure and*

Economic Performance. Chicago: Rand McNally, 1970.

11. ———, "The Determinants of Industrial Plant Size: An International Comparison Study." Mimeographed. Ann Arbor, Michigan, University of Michigan, November 1971.
12. Willmore, L. N. and Keith Acheson, "Capital Utilization in Economic Development: A Comment." *Economic Journal*, March 1974, 159–167.
13. Winston, Gordon C., "Capital Utilization and Economic Development." *Economic Journal*, March 1971, 36–60.
14. ———, "Capital Utilization and Optimal Shift-Work." *Bangladesh Economic Review*, April 1974, 515–558.
15. ———, "On the Inevitability of Factor Substitution." *Journal of Development Economics*, September 1974, 145–163.
16. ———, "The Theory of Capital Utilization and Idleness." *Journal of Economic Literature*, December 1974, 1301–1320.
17. ——— and Thomas O. McCoy, "Investment and the Optimal Idleness of Capital." *Review of Economic Studies*, forthcoming.