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AN ECONOMETRIC ANALYSIS OF CAPITAL UTILIZATION*

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After many years of neglect by economists, the decision to work shifts in a factory or to utilize capital intensively has recently become the object of theoretical analysis. The resulting theoretical advances, however, have not been subjected to thorough empirical tests. One reason for this situation is that severe estimation problems arise out of the nature of the theory and the econometrician is reduced to ignoring many of the theoretical results in the course of empirical implementation. Thus, previous empirical work on the subject has been mainly limited to confirming a positive correlation between capital intensity and shift-work (Marris [7], Winston [14], Kim and Kwon [5], Baily [1], Thoumi [10]) using various definitions of both variables. Moreover, a relationship between economies of scale and shift-work has not been empirically confirmed even though some authors have stressed the importance of this factor at the theoretical level (Marris [7], Betancourt and Clague [2]).

The purpose of this paper is to apply what we believe to be an appropriate framework for estimation, given the nature of the dependent variable suggested by the theory, and to illustrate how some of the implications derived from the theory can be incorporated into this framework. In particular, the theory suggests a functional form for the relationship between the explanatory variables and the profitability of shift-work. This functional form is determined primarily by the specific form one assumes for the production function, and the incorporation of this phenomenon explicitly into the analysis is one of the main contributions of the study. For purposes of comparison the results will be contrasted with those obtained, within the same estimation framework, by an equation that makes no attempt to incorporate the implications of the theory other than to suggest appropriate variables to include as explanatory variables. Plant data for four countries (France, India, Israel, Japan) make up the data base.

The next section presents the theoretical model and the various specifications that will be tested against the data. Subsequently, the estimation techniques are discussed and a new measure of predictive performance for probabilistic models is introduced. Section 3 describes the data and other aspects of the empirical

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implementation. Section 4 presents the results and a concluding section highlights the implications of these results.

1. RECENT THEORETICAL DEVELOPMENTS

Economists have long been aware of the fact that shift-work will be more profitable, the greater the capital intensity of the production process and the smaller the night-time wage differential. Marris [7] formalized these ideas and pointed out that, on the basis of these two factors alone, shift-work should have been more prevalent in British manufacturing than it was. To account for this anomaly, Marris introduced the notion of output restraint. On the assumption that the plant's level of output is given, the decision to work shifts involves producing a smaller instantaneous rate of output, and this will increase average costs if there are economies of scale.

More recently several writers (Baily [1], Winston [15], and Betancourt and Clague [2]) have shown how various aspects of the production function affect the profitability of shift-work. The key theoretical results will now be briefly summarized; the reader is referred to Betancourt and Clague [2] for elaboration.

The entrepreneur is assumed to maximize profits under certainty for a given and stable level of output. He simultaneously makes decisions about the type of plant to be constructed and the number of shifts to be worked. He could build the factory in such a way that the entire output would be produced in one shift, or he could build it so that two shifts would be required to produce the output. Incidentally, this conceptualization of the problem already implies that the outcome of the decision will be captured by a dichotomous variable; hence, the appropriate estimation framework will be that for dichotomous dependent variables. In the development of the theory the assumption is made that the ex-ante production function is of the constant-elasticity-of-substitution (CES) form. Even when the ex-ante function is CES, however, it is assumed that once the factory is installed, no possibilities remain for capital-labor substitution. Within each shift the plant is operated at full capacity. Finally, capital is assumed to depreciate with time but not with use.

On these assumptions an expression can be developed for the Cost Ratio, or the ratio of costs under double-shift operation to costs under single-shift operation. Profitable shift-work requires that the Cost Ratio be less than one. For the special case of the CES where the elasticity of substitution equals zero, i.e., the Leontief production function, the condition is[2]

[2] In order to simplify the exposition, we will anticipate the results by noting that the CES Cost Ratio was empirically inferior to the Leontief one in every comparison. This result holds for comparisons that use predictive power as the criterion as well as for comparisons that use the absolute size of the t-ratio as the criterion. Moreover, the difference in performance increased as the assumed value of the elasticity of substitution increased. Therefore, the subsequent discussion in the text will be limited to the Leontief case, and the reader interested in the CES case is

(Continued on next page)

$$1 > [\theta + (2 + \alpha)(1 - \theta)]\left(\frac{1 + e_{cx}}{2}\right) \tag{1}$$

where θ = the share of capital costs in combined labor and capital costs under a single-shift system

α = the proportion by which the night-time hourly wage rate exceeds the day-time hourly wage rate, or the night-time wage differential

e_{cx} = the proportional increase in average costs when output under single-shift operation is cut in half, or the cost elasticity

The RHS of the inequality in (1) is the Cost Ratio. Shift-work will be more profitable, or less unprofitable, the smaller the Cost Ratio; that is, the smaller is α, the larger is θ, and the smaller is e_{cx}.

In confronting the theory with the data two alternative specifications of the theory are followed: a restricted form specification, which incorporates the information about the functional form of the Cost Ratio contained in (1); and a free form specification, which ignores this functional form and simply relates shift-work to variables that purport to capture the theoretical factors appearing in (1). Incidentally, the latter approach is the one used in previous empirical studies on shift-work. The two approaches need not always be viewed as substitutes. The possible complementarity between the two approaches can be brought out by discussing, briefly, the differences between them with respect to the empirical definition of variables, the incorporation of alternative hypotheses, and the influence of a variable suggested by the theory but unavailable in the data.

Let us consider first, for example, the definition of capital intensity. In the restricted form specification, θ must be calculated for each factory in order to be able to calculate the Cost Ratio. In the free form specification, the investigator has the flexibility of using other measures of capital intensity which are more directly available in the data, e.g., the capital-to-wages ratio. Secondly, the specification of economies of scale leads to two alternative hypotheses: economies of scale are independent of size (A); economies of scale are a function of size (B). In the restricted form specification, these two hypotheses are implemented through two different empirical definitions of e_{cx}. In particular, for the second hypothesis (B) it is necessary to specify precisely how e_{cx} varies as size varies for each firm. However, there is only one explanatory variable in both cases, the Cost Ratio. On the other hand, in the free form specification hypothesis (A) is implemented by simply not including size as an explanatory variable, whereas hypothesis (B) is implemented by introducing a linear and a quadratic measure of size. The four versions of the theory that are tested with the data are summarized in Table 1

(Continued)

referred to an earlier version of this paper, which is available from the authors upon request. It should be noted that the main source of differences between the CES and the Leontief case is in the functional form of the Cost Ratio, but the sign of the impact of the variables which determine the Cost Ratio on the profitability of shift-work is the same in both cases.

TABLE 1

EXPLANATORY VARIABLES AND THEIR EXPECTED IMPACT ON THE LIKELIHOOD OF SHIFT-WORK UNDER ALTERNATIVE VERSIONS OF THE THEORY

Functional Form	Economies of Scale Independent of Size	Economies of Scale a Function of Size
I. Restricted Form	A) Cost Ratio (e_{cx} independent of size) Negative Impact	B) Cost Ratio (e_{cx} a function of size) Negative Impact
II. Free Form	A) Capital Intensity Positive Impact	B) Capital Intensity Positive Impact Size Positive (but decreasing) Impact

for future reference. If the theory is robust, we expect the same hypothesis, either A or B, to be consistent with a given body of data under both the restricted and the free form specification.

At this point the reader may be wondering what happened to the shift differential, α. This variable is not available in our data, or in the previous empirical studies cited in the introduction. In the restricted form specification, however, a value must be assumed, and the selection procedure for this assumed value is described in Section 3. In the free form specification, the usual practice, also followed here, is to ignore this variable. The two treatments of this variable lead to two different types of econometric problems. In the free form case, the traditional analysis of specification error suggests that a bias will exist unless the omitted variable, α, is uncorrelated with the included ones. On the other hand, in the restricted form specification we have an error in the independent variable due to the departures of the assumed value of α from the true ones. In contrast to the classical error-in-variables problem, however, the variance of the true Cost Ratio can be larger, smaller, or the same as the measured one (note that the independent variable is the Cost Ratio and the measurement error is in one of its components (α)); therefore, the classical downward-bias conclusion is not necessarily applicable. Since the biases, if any, in the restricted form and the free form specifications are not necessarily in the same direction (and normally not of the same magnitude), the use of both functional form specifications provides a safeguard against unusual results that may arise with either specification on account of the absence of information about α.

It is true, of course, that a number of relevant factors have been omitted from this analysis. For example, the empirical work assumes that the observed number of shifts in a plant is what had been planned when the factory was constructed, but perhaps the observed number of shifts is different from the planned number because of short-run fluctuations in demand or in input supplies. Shift-work may also be affected by managerial preferences[3] and market structure.[4] These variables are omitted from our empirical analysis for lack of appropriate data.[5] Although shift-work is influenced by seasonality of demand or supply and by the continuous-process nature of some manufacturing activities, these variables will not affect our empirical tests, for factories with these characteristics have been excluded from the sample.

2. THE ESTIMATION TECHNIQUE

In light of the preceding discussion and of the binary nature of the dependent variable, the estimation problem may be conceptualized as follows (see Goldberger [3, (250)]):

$$S_i = \begin{cases} 1 & \text{if} \quad Y_i \geq Y_i^* \\ 0 & \text{if} \quad Y_i < Y_i^* \end{cases}$$

where S_i takes on the value of 1 if the plant works shifts and the value of zero otherwise; Y_i is an index which is a linear function of the regressors; and Y_i^* is an unobservable threshold value which plays a role analogous to that of a disturbance term in the classical model, i.e., it incorporates errors of measurement in the dependent variable, other influences, and firm's preferences. This approach allows us to formulate the probability that the i-th plant works shifts as a function of the profitability of working shifts for the i-th plant, or

$$(2) \qquad P(S_i = 1|Y_i) = \text{Prob}(Y_i^* \leq Y_i|Y_i) = f(Y_i) = f(X_i'\beta)$$

where X_i' is defined in four alternative ways corresponding to the four versions of the theory presented in Table 1; and associated with each version is a vector of parameters β, each element of which measures the impact of the corresponding variable on the probability of working shifts. A constant will be included in each version so that the first element in X_i' is always unity, and the first element of β is β_0.

Several approaches have been suggested to deal with this type of estimation problem (reviewed by Warner in [12]). The simplest method is to assume that

[3] For example, in Colombia the most important variable exlaining shift work was a dummy for incorporated enterprises. Corporations tended to work shifts (Thoumi [10]).

[4] It is shown in [2] that if the output restraint is replace by a smooth demand curve, shift-work is more likely to be profitable, the higher the elasticity of demand. Moreover, if managers regard shift-work as a means of purchasing higher profits at the expense of some of their leisure, competitive pressures on profits may make shift-work a necessity.

[5] Once more the impact of these omitted variables is likely to differ for the restricted and the free form specification. To see this note that, when the omitted factors enter additively into both specifications, in the restricted form specification there will be no bias if the omitted factors are uncorrelated with the cost ratio; on the other hand, in the free form specification there will be no bias if the omitted factors are uncorrelated with each of the other included variables, capital intensity and/or size.

the probability is linearly related to the conditioning variables, i.e., f is the identity function, and then it appears possible to run a simple regression on the binary dependent variable. The two main difficulties with this approach are that the assumption of homoskedasticity cannot be satisfied, and that the estimated probabilities will not in general be in the zero-to-one interval (thereby impairing their interpretation as probabilities). Goldberger [**3**] and subsequently others, e.g., Lave [**6**], have recommended the use of weighted least squares to correct for the former problem. This recommendation does not seem to have much practical value, however, for it also transforms the estimation problem from one involving a qualitative dependent variable into one involving a limited dependent variable, i.e., the zeros will remain zeros after the transformation to correct for heteroskedasticity has been applied to the dependent variable. Furthermore, there is no satisfactory solution to the second difficulty in the context of the simple regression model.

For our purposes the interpretation of the predicted values as probabilities is important. Therefore it is necessary to use another approach which restricts the probabilities to lie in the (0, 1) interval. From the many possible functions (f) which impose this restriction, the following one was selected:

$$P(S_i = 1 | Y_i) = e^{X'_i\beta}/(1 + e^{X'_i\beta}) \tag{3}$$

Since the probability that the i-th plant does not work shifts is given by the logistic function, this approach is referred to in the literature as logit analysis[6] (see, for example, Theil [**9**, (632)]). Equation (3) is nonlinear in its parameters, but the maximum likelihood estimators of these parameters can be obtained by iterative methods (see, for example, Warner [**12**]), and either the likelihood ratio test or the t-ratio test based on the estimates of the standard errors obtained from the estimated inverse of the information matrix can be employed to test hypotheses on the β coefficients.

In practice the results from using both methods of estimation should be reported because recent Monte Carlo results lead to the following conclusion (Goldfeld and Quandt [**4**, (134)]): if the data are generated by the uniform model (f is the identity function) OLS performs reasonably well and probit analysis yields inferior results; if the data are generated by a probit model, OLS does as well as other methods and probit analysis is unambiguously better only for a sample size of at least 200. Since our sample sizes range from 51 to 76 and since logit analysis is very similar to probit analysis, both OLS and logit analysis will be used in order to check the sensitivity of the results to the choice of estimation procedure.

Our first interest is to establish the consistency or inconsistency of the theory with the data, and the usual statistical tests will suffice for that purpose. It is

[6] The main alternative available in the literature is probit analysis, but logit analysis was selected because of its simplicity and lower computational cost. For a comparison, see Warner [**12**,(Ch. 2)].

also desirable, however, to discriminate between the two functional form specifications and between the alternative hypotheses about economies of scale. For these purposes, it will be necessary to compare the predictive performance of the different versions of the theory. The R^2 statistic has frequently been used for this purpose and will be employed here.[7] But in models where the predicted value is a probability, an alternative measure of predictive performance can be constructed on the basis of the concept of entropy (E) (Theil [**9**]).[8]

We proceed by defining formally the concepts of entropy and information, and developing from their properties a summary measure of predictive performance; an intuitive motivation for this measure is provided below. The concept of entropy is defined for an individual factory in a sample as

$$E_i = -[\hat{p}_i \log \hat{p}_i + (1 - \hat{p}_i) \log (1 - \hat{p}_i)] \tag{4}$$

where $\hat{p}_i$ is the estimated probability that the i-th factory works shifts. Entropy, which may be interpreted as a measure of the amount of uncertainty associated with the distribution ($\hat{p}_i$, $1-\hat{p}_i$), takes its maximum value in our case at $\hat{p}_i=1/2$ and its minimum value at $\hat{p}_i=1$ or $\hat{p}_i=0$. We seek a measure of the amount of information contained in a predicted probability $\hat{p}_i$. The measure will be defined by $I_i=1-E_i/E_{\text{MAX}}$ where E_{MAX} is the maximum amount of entropy associated with the distribution, e. g., $E_{\hat{p}_i}=1/2$ in the dichotomous case, $E_{\hat{p}_i}=1/3$ for three possible outcomes, etc. I_i thus takes on its maximum value at $\hat{p}_i=1$ or $\hat{p}_i=0$ and its minimum value in our case at $\hat{p}_i=1/2$.

Entropy has the attractive property that the joint entropy of two independent random variables is the sum of the entropies for each random variable. The amount of information I_i has the same property. Therefore, defining a correct prediction as $\hat{p}_i>.5$ when the factory actually works shifts and $\hat{p}_i<.5$ when it does not work shifts, we may define a summary measure of the amount of information contained in a set of predictions as $\bar{I}=(I_1-I_2)/N$ where I_1 is the sum of information for all the correct predictions, I_2 is the sum of misinformation for all the incorrect predictions, and N is the number of observations. $\bar{I}$ ranges from -1 (all probabilities incorrectly predicted as 1 or 0) to $+1$ (all probabilities correctly predicted as 1 or 0).

Intuitively, this summary measure scores each prediction in a given set by giving it points not only in accordance with whether the prediction is right (positive points) or wrong (negative points), but also in a way which reflects the degree of certainty of the prediction. In other words, more credit (discredit) should be and is given to a correct (incorrect) prediction if a high probability underlies the prediction, i.e., close to 1 or 0, than a low one, i.e., close to .5. Parenthetically,

[7] That is, $R^2=1-\Sigma(Yi-\hat{Y}i)^2/\Sigma(Yi-\bar{Y})^2$. Note that the usual decompostition of total variation is not applicable in the nonlinear case. The degrees of freedom correction can be applied as in the linear case [3, (217)], to define $\bar{R}^2$.

[8] The authors would like to thank D. Madan for a very stimulating discussion on the concept of entropy.

a degrees-of-freedom correction identical to the one for R^2 can also be defined for the positive range of $\bar{I}$, which is the relevant one in practice.

3. EMPIRICAL IMPLEMENTATION[9]

The sample actually used consisted of 231 plants in France, Japan, India, and Israel (UNIDO [**11**]). Continuous-process factories were excluded from the sample. Since continuous-process factories tend to be highly capital-intensive and invariable work shifts, their exclusion is quite desirable. Failure to exclude them clouds the interpretation of the capital-intensity variable and perhaps other variables in the regression analyses of Winston [**14**], Kim and Kwon [**5**], and Thoumi [**10**].

The published information for each factory includes the number of production workers on each shift, total wages paid to production workers, and the book value of machinery and buildings. The free form specification of capital intensity was the ratio of the book value of machinery and buildings to the wages of direct production workers on the first shift. The restricted form required calculation of θ, the capital share under single-shift operation. θ is defined as $rK/(rK+wL)$, where K is the book value of buildings and machinery, L is the number of workers on the first shift, w is the annual wage rate for day-time workers, and r is the cost of owning a unit of capital stock for a year, i.e., $r=(i+d)P_c$, where i is the annual interest rate, d is the rate of depreciation, and P_c is the price of a standard machine. i and d were approximated by using U.S. data for the corresponding two-digit U.S. industrial classification.

Factory size was measured as a geometric average of employment and value added on the first shift for each firm in the sample. This hybrid number was used in order to correct for the bias involved in using either measure by itself. The actual size variable used in the empirical analysis, however, was relative size (RS). That is, a relative size variable was constructed by using as a base the factory size, defined as above, of a U.S. plant in the 30-th percentile of the same four-digit industry classification. Thus, the relative size variable incorporates differences across industries in the minimum optimal size of plant. In the free form, RS and RS^2 are the independent variables. In the restricted form, e_{cx} rather than relative size enters the Cost Ratio. In one version e_{cx} was set equal to .10 for all firms; in the other e_{cx} was made to decline as RS increases, according to $e_{cx}=a+b\log RS$. The value of a was set at .10 so that where RS equals unity, e_{cx} equals .10. e_{cx} was constrained to have a minimum value of zero and b was obtained by assuming that e_{cx} is half of .10 at $RS=4$.

Finally, in the absence of information about the shift differential, α, alterna-

[9] This section has been condensed to economize on space. A detailed description of empirical procedures is contained in an earlier version of this paper, which is available on request, and will be included in a book now being prepared by the authors.

tive values were assumed.[10] On the basis of preliminary investigations, a high and a low value of α were selected, far enough apart to give reasonable confidence that the optimal α lay between the high and low values. Then, since the predictive performance of the restricted form specification was very similar in all cases for the two different values of α (the estimated coefficient on the Cost Ratio adjusts as α changes), we simply selected the "better" value of α from these two for inclusion in the text. (The results for the other value are available from the authors upon request.) Of course, the demonstrated insensitivity of our regression coefficients to the choice of a single α does not guarantee that having the correct α for each observation would make no difference.

4. RESULTS

Since the results for the developed countries (France and Japan) are different from the results for the less developed countries, they will be discussed separately.

4.1. *Developed Country Results: France, Japan.* The results are surprisingly similar for both countries. As would be expected from the previous empirical work, using a one tail test the coefficient of the variable which is related to a measure of capital intensity, $\hat{\beta}_1$, is found to be (virtually) significantly different from zero at the 1% level for both versions of the free form specification and with both estimation methods in both countries. The same result holds, without the qualification, for the coefficient of the Cost Ratio $\hat{\beta}_1$ in both versions of the restricted form specification (see Tables 2 and 3).

In addition, the results establish the lack of importance of the relative size variable in determining the probability that a firm works shifts in both France and Japan. Consider the results for the free form specification (version II) first. In both France and Japan, the coefficients of the relative size variable are not significantly different from zero at either the 5% or the 1% level. This is true for a joint test on both coefficients as well as for a single test. Moreover, it is true for both estimation methods. Consider now the results for the restricted form specification (version I). While a rigorous statistical test is not available, the results are unquestionably favorable to the specification of economies of scale as independent of the measure of size. For in both countries and with both estimation methods, there are substantial increases in predictive performance,[11] using either R^2 or $\bar{I}$, from using version IA (economies of scale independent of relative size) rather than version IB. For example, the minimum increase in predictive performance is about 15% (France, logit analysis: .3331/.2808) and the largest

[10] It is interesting to note that in a recent controversy over the uses and abuses of simulation, the critics of simulation suggest this type of procedure as one of the "acceptable" uses (Mirer and Peck [8]).

[11] Unless specified otherwise in the text, the comparisons are done in terms of the uncorrected measures of predictive performance.

TABLE 2

RESULTS FOR ALTERNATIVE VERSIONS OF THE THEORY: FRANCE; $n = 52$, $(\alpha = .40)$

Version of the Theory	$\hat{\beta}_0^f$	$\hat{\beta}_1^f$	$\hat{\beta}_2^f$	$\hat{\beta}_3^f$	$\#^a$, L^b	R^{2d}	$\bar{I}^{d,e}$
Statistical Model: OLS Regression							
I. Restricted Form							
A, $e_{c,x} = 0.10$	2.3532	−1.9526			5	.3509	.2981
	(.3934)	(.3756)				(.3379)	(.2841)
B, $e_{c,x} = g(RS)$	2.0429	−1.6863			3	.2939	.2462
	(.3803)	(.3697)				(.2798)	(.2311)
II. Free Form							
A, $\beta_2 = \beta_3 = 0$	.0893*	.0477			1	.2231	.1778
	(.0857)	(.0126)				(.2076)	(.1614)
B	.1339*	.0471	−.0111*	.0008*	1	.2374	.1977
	(.1068)	(.0128)	(.0249)	(.0010)		(.1897)	(.1476)
Statistical Model: Logit Regression							
I. Restricted Form							
A, $e_{c,x} = 0.10$	11.0083	11.6046			20.2930[c]	.3231	.3056
	(3.2665)	(3.2588)				(.3096)	(.2917)
B, $e_{c,x} = g(RS)$	9.0670	−9.9087			17.1715[c]	.2808	.2621
	(2.8387)	(2.9053)				(.2664)	(.2473)
II. Free Form							
A, $\beta_2 = \beta_3 = 0$	−2.1027	.2671			12.1985[c]	.2084	.1806
	(.5798)	(.0949)				(.1926)	(.1642)
B	−1.8364	.2759	−.0838*	.0001*	1.2712**[g]	.2265	.1996
	(.6398)	(.1015)	(.1703)	(.0100)		(.1782)	(.1496)

a OLS regressions: number of predictions outside the 0–1 range
b Logit regressions: observed value of $-2 \ln \lambda$ where λ is a likelihood ratio statistic
c $\lambda = L(\hat{\omega} \mid \beta_1 = 0)/L(\hat{\Omega})$
d The numbers in parentheses are the measures of predictive performance corrected for degrees of freedom.
e In the calculation of $\bar{I}$ predicted values greater than or equal to unity and less than or equal to zero were set at .99 or .01, respectively.
f The numbers in parentheses are the estimated standard errors of the coefficients.
g $\lambda = L(\hat{\omega} \mid \beta_2 = \beta_3 = 0)/L(\hat{\Omega})$
* statistically insignificant coefficient at the 1% level, i.e., $|t| < 2.4$
** statistically insignificant restriction at the 1% level, i.e., $\chi^2 < 6.635$ for Models I, IIA or $\chi^2 < 9.210$ for Model IIB.

TABLE 3

RESULTS FOR ALTERNATIVE VERSIONS OF THE THEORY: JAPAN; $n=52$, $(\alpha=.40)$

Version of the Theory	$\hat{\beta}_0^f$	$\hat{\beta}_1^f$	$\hat{\beta}_2^f$	$\hat{\beta}_3^f$	$\#^a$, L^b	R^{2d}	$\bar{I}^{d,e}$
			Statistical Model: OLS Regression				
I. Restricted Form							
A, $e_{cx}=0.10$	2.3685	−1.9202			3	.2977	.2550
	(.4509)	(.4171)				(.2837)	(.2401)
B, $e_{cx}=g(RS)$	2.0287	−1.6016			3	.2403	.2188
	(.4365)	(.4028)				(.2251)	(.2032)
II. Free Form							
A, $\beta_2=\beta_3=0$	.1935	.0225			1	.1657	.1153
	(.0698)	(.0072)				(.1490)	(.0976)
B	.2262*	.0222	−.0123*	−.0001*	2	.1844	.1435
	(.0953)	(.0073)	(.0580)	(.0029)		(.1334)	(.0900)
			Statistical Model: Logit Regression				
I. Restricted Form							
A, $e_{cx}=0.10$	11.6468	−11.7411			16.5459[c]	.3022	.2576
	(4.0829)	(3.8301)				(.2882)	(.2428)
B, $e_{cx}=g(RS)$	7.3251	−7.8549			9.7546[c]	.1823	.1656
	(3.0002)	(2.9065)				(.1659)	(.1489)
II. Free Form							
A, $\beta_2=\beta_3=0$	−2.4475	.3952			14.9656[c]	.2635	.2320
	(.6948)	(.1641)				(.2488)	(.2166)
B	−2.5422	.3974*	.4441*	−.1250*	1.9871**[g]	.2814	.2721
	(1.0114)	(.1715)	(1.0719)	(.2159)		(.2365)	(.2266)

a OLS regressions: number of predictions outside the 0–1 range

b Logit regressions: observed value of $-2 \ln \lambda$ where λ is a likelihood ratio statistic

c $\lambda = L(\hat{\omega} \mid \beta_1=0)/L(\hat{\Omega})$

d The numbers in parentheses are the measures of predictive performance corrected for degrees of freedom.

e In the calculation of $\bar{I}$ predicted values greater than or equal to unity and less than or equal to zero were set at .99 or .01, respectively.

f The numbers in parentheses are the estimated standard errors of the coefficients.

g $\lambda = L(\hat{\omega} \mid \beta_2=\beta_3=0)/L(\hat{\Omega})$

* statistically insignificant coefficient at the 1% level, i.e., $|t|<2.4$

** statistically insignificant restriction at the 1% level, i.e., $\chi^2<6.635$ for Models I, IIA or $\chi^2<9.210$ for Model IIB increase is about 65% (Japan, logit analysis: .3022/.1823). In this connection, it should be stressed that model IB incorporates two sources of data variation into a single variable through the functional form specified from the theory; yet, both measures of predictive performance are lower in every single case. This suggests that measures of predictive performance should be given a greater weight in assessing results in this context than in the standard analysis where the introduction of additional variables must increase the value of the measure of predictive performance.

Since in both functional form specifications the data support the version which makes economies of scale independent of relative size (A), a comparison of the two specifications will be undertaken by comparing versions IA and IIA. In every one of the four possible comparisons, the restricted form (IA) performs better than the free form (IIA) according to both measures of predictive performance. These increases range from a low of 11% (.2576/.2320) for the increase in $\bar{I}$ with logit analysis in Japan to an increase of more than 100% (.2550/.1153) in $\bar{I}$ with ordinary least squares, also in Japan. Thus these results provide strong evidence in favor of the restricted form. This conclusion is important because any attempt to predict the impact of changes in policy parameters (e. g., taxes and subsidies with respect to capital or labor) on the probability that a factory works shifts would involve very different procedures depending on which specification one accepts as appropriate. More specifically, predictions using IA require incorporation of the changes into the Cost Ratio, the *RHS* of (1), first and then use of the resulting change in the Cost Ratio with the estimated coefficient to obtain the prediction. Parenthetically, a similar remark applies to the use of the two estimation procedures for prediction, i.e., account must be taken of the nonlinearity in logit analysis (Westin [**13**]).

4.2. *Less Developed Country Results*: *India, Israel*. As in the developed countries, the coefficient of the variable related to capital intensity, $\hat{\beta}_1$, in the free form specification is again found to be (virtually) significantly different from zero at the 1% level in all instances, and the coefficient of the Cost Ratio, $\hat{\beta}_1$, in the restricted form is significantly different from zero at the 1% level in all cases. See Tables 4 and 5.

Turning to the differences between the developed and less developed country results, the empirical evidence for India and Israel indicates that relative size plays a role in determining the probability that a factory works shifts. Consider the results for the free form specification first. Using logit analysis, a likelihood ratio test rejects the null hypothesis that the coefficients of the relative size variable are equal to zero at the 1% level of significance in both countries; and, using OLS, a t-test at the 5% level of significance rejects the null hypothesis that three out of the four coefficients of the relative size variable are equal to zero although the same test at the 1% level leads to the opposite result. In the restricted form specification, the version of the theory which allows relative size to determine economies of scale (IB) performs better than the one which does not (IA) in all the possible

TABLE 4

RESULTS FOR ALTERNATIVE VERSIONS OF THE THEORY: INDIA; $n=76$, $(\alpha=.20)$

Version of the Theory	$\hat{\beta}_0^f$	$\hat{\beta}_1^f$	$\hat{\beta}_2^f$	$\hat{\beta}_3^f$	$\#^a$, L^b	R^{2d}	$\bar{I}^{d,e}$
			Statistical Model: OLS Regression				
I. Restricted Form							
A, $e_{cx}=0.10$	1.7376	−1.1657			3	.1265	.1626
	(.3258)	(.3561)				(.1147)	(.1513)
B, $e_{cx}=g(RS)$	1.9044	−1.3349			9	.1871	.1979
	(.2997)	(.3235)				(.1761)	(.1871)
II. Free Form							
A, $\beta_2=\beta_3=0$	.5741	.0113			4	.1005	.1287
	(.0640)	(.0039)				(.0884)	(.1169)
B	.4287	.0108	.1389	−.0085*	6	.1888	.1785
	(.0819)	(.0038)	(.0548)	(.0044)		(.1550)	(.1442)
			Statistical Model: Logit Regression				
I. Restricted Form							
A, $e_{cx}=0.10$	6.8956	−6.6014			10.6958[c]	.1175	.1424
	(2.1811)	(2.2832)				(.1056)	(.1124)
B, $e_{cx}=g(RS)$	8.5323	−8.2069			16.4236[c]	.1808	.1972
	(2.3637)	(2.4207)				(.1697)	(.1864)
II. Free Form							
A, $\beta_2=\beta_3=0$	−.6399*	.2592			16.3212[c]	.1689	.2156
	(.8197)	(.1063)				(.1577)	(.2050)
B	−1.3799*	.2573*	.6781*	n. a.[h]	8.3943[g]	.2610	.2942
	(.6136)	(.1109)	(.3143)			(.2408)	(.2749)

a OLS regression: number of predictions outside the 0–1 range

b Logit regression: observed value of $-2 \ln \lambda$ where λ is a likelihood ratio statistic

c $\lambda=L(\hat{\omega} \mid \beta_1=0)/L(\hat{\Omega})$

d The numbers in parentheses are the measures of predictive performance corrected for degrees of freedom.

e In the calculation of $\bar{I}$ predicted values greater than or equal to unity and less than or equal to zero were set at .99 or .01, respectively.

f The numbers in parentheses are the estimated standard errors of the coefficients.

g $=\lambda L(\hat{\omega} \mid \beta_2=0)/L(\hat{\Omega})$

h Introduction of the square term led to nonconvergence of the iterative procedure.

* statistically insignificant coefficient at the 1% level, i. e., $|t|<2.38$

TABLE 5

RESULTS FOR ALTERNATIVE VERSIONS OF THE THEORY: ISRAEL; $n=51$, $(\alpha=.15)$

Version of the Theory	$\hat{\beta}_0^f$	$\hat{\beta}_1^f$	$\hat{\beta}_2^f$	$\hat{\beta}_3^f$	#[a], L[b]	R^2[d]	$\bar{I}$[d, e]
Statistical Model: OLS Regression							
I. Restricted Form							
A, $e_{cx}=0.10$	2.5360 (.4191)	−2.0161 (.4431)			8	.2970 (.2827)	.2239 (.2081)
B, $e_{cx}=g(RS)$	2.4325 (.3689)	−1.8875 (.3855)			7	.3285 (.3148)	.2371 (.2215)
II. Free Form							
A, $\beta_2=\beta_3=0$	.4262 (.0844)	.0355 (.0095)			6	.2213 (.2054)	.1667 (.1497)
B	.4743 (.0939)	.0305 (.0095)	.0877* (.0611)	−.0097* (.0049)	6	.3002 (.2556)	.2018 (.1509)
Statistical Model: Logit Regression							
I. Restricted Form							
A, $e_{cx}=0.10$	13.5144 (4.0559)	−13.3163 (4.0608)			18.4633[c]	.3258 (.3120)	.2545 (.2393)
B, $e_{cx}=g(RS)$	12.5120 (3.6028)	−12.1723 (3.5847)			20.0185[c]	.3594 (.3463)	.2788 (.2641)
II. Free Form							
A, $\beta_2=\beta_3=0$	−1.3178* (.6261)	.5062 (.1871)			19.5287[c]	.3188 (.3049)	.2893 (.2748)
B	−3.2931 (1.1954)	.6623 (.2726)	3.1873 (1.2673)	−.5686 (.2335)	15.8098[g]	.5738 (.5466)	.5113 (.4801)

a OLS regressions: number of predictions outside the 0–1 range

b Logit regressions: observed value of $-2 \ln \lambda$ where λ is a likelihood ra tio statistic

c $\lambda = L(\hat{\omega} \mid \beta_1=0)/L(\hat{\Omega})$

d The numbers in parentheses are the measures of predictive performance corrected for degrees of freedom.

e In the calculation of $\bar{I}$ predicted values greater than or equal to unity and less than or equal to zero were set at .99 or .01, respectively.

f The numbers in parentheses are the estimated standard errors of the coefficients.

g $\lambda = L(\hat{\omega} \mid \beta_2=\beta_3=0)/L(\hat{\Omega})$

* statistically insignificant coefficient at the 1% level, i.e., $|t| < 2.4$

comparisons according to both criteria for predictive performance. These increases range from a low of about 5% for $\bar{I}$ using OLS in Israel to a high of about 34% for R^2 using logit analysis in India.

Since in both functional form specifications the empirical evidence supports the hypothesis which allows economies of scale to depend on relative size, these versions (B) will be used to compare the two specifications. At this point, however, a substantial discrepancy arises between the results obtained using logit analysis and those obtained using OLS. Incidentally, since there are three regressors in IIB and only one in IB, all the comparisons will be made in terms of the measures of predictive performance corrected for degrees of freedom. While the restricted form (IB) outperforms the free form (IIB) when OLS is used as the estimation method, the free form does better (in terms of both R^2 and $\bar{I}$, and in both India and Israel) when logit analysis is employed.

What is the source of these contradictory results obtained from the two estimation methods? If one tabulates the proportion of plants that work shifts for various relative size categories,[12] it becomes apparent that there is a nonlinear relation between the proportion of factories that work shifts and relative size for both India and Israel. Thus, the relatively free form in which relative size is introduced in IIB combined with the intrinsic nonlinearity in logit analysis fits the observed pattern much better than IIB with OLS or IB with either logit analysis or OLS. This information, however, sheds no light on how these results should be interpreted.

Any attempt at interpretation must take account of the fact that the nature of the nonlinearity is not the same in India as in Israel. In India, the proportion of plants that work shifts rises at a decreasing rate to a maximum of 1 for plants with relative sizes larger than 4. Thus, it is quite consistent with the formulation in IB which makes e_{cx} a declining function of relative size; but it suggests that specifying e_{cx} to decline more rapidly, as relative size increases, would improve predictive performance. On the other hand, in Israel the proportion of plants that work shifts rises to a maximum of 1 for relative sizes between 2 and 4, and it declines to .20 for relative sizes greater than 4. Clearly, this pattern is inconsistent with what would be expected on the basis of the prior literature on economies of scale about the behavior of e_{cx} as relative size increases. One possible explanation is that in less developed countries institutional constraints may lead some firms to build large plants.[13] For example, if the amount of imports permitted is tied to the size of plant as measured by the capital stock in place, a powerful incentive is provided for firms that depend critically on imported inputs to build large plants. Given a large domestic market like India's, this restric-

[12] Proportion and number of firms that work shifts for various relative size categories:

	$< .25$	$.25 - 2$	$2 - 4$	> 4	$\widehat{RS}$
India $\sum_t S_t/n(n)$	.42(19)	.70(40)	.83(8)	1.0(9)	
Israel $\sum_t S_t/n(n)$	.09(11)	.85(27)	1.(8)	.20(5)	

[13] We would like to thank J. Behrman and J. Nugent for this suggestion.

tion might be circumvented by using lower quality or higher cost substitutes and producing for a protected domestic market without affecting the number of shifts. However, with a small domestic market like Israel's a considerable part of the output of these very large plants would have to be placed abroad under competitive conditions and the firm may simply choose to satisfy whatever demand it can while operating a single shift.

5. CONCLUDING REMARKS

The main results which have been established in this study are the following: 1) The specification which incorporates the restrictions derived from production theory into the statistical analysis of shift-work has been confirmed by the data for the two developed countries; moreover, it performs much better than the alternative which does not incorporate these restrictions, with both OLS and logit analysis, according to both criteria of predictive performance, R^2 and $\bar{I}$. 2) In the two developed countries the data support conclusively the hypothesis that shift-work is independent of relative size, for both specifications of functional form and with both estimation methods; on the other hand, the same hypothesis is rejected by the data for the two less developed countries for both specifications and with both estimation methods. 3) While the specification which incorporates the restrictions derived from production theory is also consistent with the data for the two less developed countries using both methods of estimation, it performs much worse, using logit analysis, according to both criteria for predictive performance than one which does not incorporate these restrictions. In so doing, it raises the question of specification error which may eventually be resolved by a better specification of the relation between e_{cx} and relative size in India and by the incorporation of other factors into the theory in a form suitable for estimation in Israel. 4) The measure of predictive performance introduced in this study, $\bar{I}$, behaves in a manner similar but not identical to the R^2. Together with the measure's conceptually appealing properties, these results suggest the development of the statistical properties of $\bar{I}$ as a worthwhile area for further research. They might allow, in the context of logit analysis, the formulation of rigorous statistical tests to choose between models that incorporate different restrictions into a regressor.

Finally, a diversity of formulations and estimation methods yielded rewarding results in this study: it allowed us to discover a striking feature of the data for the less developed countries which could have gone unnoticed otherwise; and, it enhanced our confidence in the robustness of those results which came through in both models and/or with both estimation methods. We hope to have contributed a useful starting point, both substantively and methodologically, for the empirical analysis of the long run determinants of shift-work.

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