Capital Utilization and Factor Specificity

Roger Betancourt, Christopher Clague, Arvind Panagariya

The Review of Economic Studies, Volume 52, Issue 2 (Apr., 1985), 311-329.

Review of Economic Studies (1985) LII, 311-329
0034-6527/85/00210311$02.00

Capital Utilization and Factor Specificity

ROGER BETANCOURT
CHRISTOPHER CLAGUE
and
ARVIND PANAGARIYA
University of Maryland

In this study a model of firm behavior that allows the level of capital utilization to be optimally chosen by cost-minimizing firms is embedded into the standard specific-factors model employed in the international trade literature. The resulting generalization of the specific-factors model provides several new insights. For instance, allowing for variable utilization in either or both sectors gives rise to a greater variety of possible trade patterns than forcing utilization to remain constant. Similarly, international differences in the willingness to work during abnormal hours generate a wider variety of trade patterns than are possible in the standard specific-factors model. Finally, this model allows a reconciliation of the "dual scarcity" explanation of the nineteenth century Anglo-American pattern of trade with the historical evidence on levels of utilization.

1. INTRODUCTION

Capital utilization has become a subject of increasing interest to economists and policy makers in the last twenty years. This interest has resulted in a substantial body of knowledge on the subject in the context of one sector growth models and as a partial equilibrium phenomenon;[1] however, the exploration of the general equilibrium implications of varying levels of utilization has barely started.

Betancourt, Clague, and Panagariya (1983) have recently investigated the implications of varying levels of utilization using the standard two factor-two sector general equilibrium model. Their approach consists of allowing the level of capital utilization to be an endogenous variable, i.e. optimally chosen by cost minimizing firms, in one of the two sectors while allowing the level of capital utilization to remain exogenously given to the firms in the other sector. The variable utilization sector can be thought of as the industrial sector while the fixed utilization sector can be thought of as the agricultural sector. The model of firm behaviour in the variable utilization sector is taken from the partial equilibrium literature referred to in footnote 1. The main finding from BCP (1983) is that the basic results of the standard model are robust to the incorporation of capital utilization, in the sense that the main theorems of trade theory continue to be valid. On the other hand, if one allows workers' willingness to work during abnormal hours to vary across countries, as in BCP (1982), neither factor price equilization nor the Heckscher-Ohlin pattern of trade will hold in the two-factor two-sector model with variable utilization. A striking result emerges in this context, namely international capital mobility normally enhances rather than substitutes trade.

In this paper, we allow capital utilization to be endogenously determined in one of the two sectors while remaining exogenously given in the other sector, but we assume, in contrast to the above model, that capital is specific to each of the two sectors. Thus, we will analyse the general equilibrium implications of varying levels of utilization in the context of the specific-factors model.

Our interest in this subject arises from several considerations. First, since capital utilization is variable in one sector (industrial) and not in the other (agricultural), the assumption that capital is specific to each sector seems particularly suited to the analysis of capital utilization. Second, recent interest in variants of the specific-factors model among international trade theorists has led to a number of new results, e.g. Jones and Easton (1983), Grossman (1983). Finally, the topic bears directly on a controversy over the explanation of nineteenth century Anglo-American trade.

In the next section, we describe the structure of the specific-factors model when the level of capital utilization is optimally chosen by cost minimizing firms in one of the two sectors. Subsequently, in Section 3, the major changes in the operation of the model due to the endogeneity of capital utilization are highlighted. An important restriction on the nature of these changes derives from a stability analysis which is undertaken in Appendix A. Indeed, this restriction also limits the range of effects of changes in commodity prices and factor endowments which are analysed in Sections 4 and 5, respectively. To facilitate the exposition, the algebraic derivations underlying the substantive results in Sections 4 and 5 are presented in Appendix B. In Section 6 we apply our generalization of the specific-factors model to the analysis of the historical controversy mentioned above. We conclude the paper by showing that our results generalize when utilization is variable in both sectors and by analysing the implications for comparative advantage of intercountry differences in the willingness to work abnormal hours.

Our substantive findings fall into three categories, corresponding to the three considerations mentioned earlier. With respect to international trade theory, one of our two main findings is that the qualitative effects of changes in commodity prices and endowments on factor returns and outputs are the same as in the standard specific-factors model with one important exception. Namely, an increase in the endowment of capital specific to the variable utilization sector can lead to a decrease in the output of this sector. Consequently, even with identical tastes with respect to work during abnormal hours, our model generates a greater diversity of trade patterns than the standard specific-factors model. Our other main finding is that differences in the willingness to work during abormal hours can be a source of comparative advantage. With respect to the role of capital utilization in general equilibrium, our main finding is that the qualitative effect of changes in commodity prices and factor endowments on the level of utilization in a small open economy is determined solely by whether the elasticity of substitution between capital services and labour services is greater or less than unity. These results are substantially different from those of the two-factor two-sector model with variable utilization; moreover, the differences are shown to be due to the differences in the general equilibrium nature of the two models rather than to the role of utilization in them. With respect to the nineteenth century Anglo-American pattern of trade, our main finding is the reconciliation of the "dual scarcity" explanation based on the standard specific-factors model, as presented by Jones (1971) and Temin (1966), with the historical evidence on levels of utilization emphasized by Brito and Williamson (1973).

Before proceeding to the next section, mention must be made of an alternative line of inquiry which also seeks to examine the general equilibrium implications of varying levels of capital utilization. The basic difference in the two approaches lies in the modelling of the labour market. In the literature previously cited, and in this paper, firms are perceived as operating in a single labour market where they recognize that high utilization levels lead to high wage costs, because workers demand a premium for work performed during abnormal hours. In the alternative approach, firms are viewed as operating in two labour markets, one for work during normal hours and one for work during abnormal hours; and, more to the point, these two labour markets are completely separate in the sense that firms do not perceive any connection between the wages that are paid during normal and abnormal hours. This approach has been developed in BCP (1984) and Svenson (1982). A detailed comparison of the two approaches is available in BCP (1982). Here we simply note that the modelling of the labour market adopted in this paper is more apt to capture the institutional realities of modern shift-working systems than the alternative, and variations in the amount of shift-work are the typical source of variations in long-run levels of capital utilization, e.g. Foss (1981).

2. THE MODEL

Assume that our economy produces two goods, 1 and 2, using three primary factors of production, capital specific to sectors 1 and 2 and labour. Denote by X_i, L_i, and K_i, respectively, the flow of output, the flow of labour services and the stock of capital in industry i $(i=1,2)$ during a given period of calendar time, say, a week or a day. Define the units of capital services so that the capital stock K_i, when used for U_i shifts, generates U_iK_i amount of capital services per unit of calendar time. If the capital stock is operated during all normal hours but not beyond, then $U_i=1$ and the flow of capital services during the calendar time equals the stock of capital.[2] In line with the usual practice, let the production functions for the two goods be linear homogeneous in labour and capital services.

As noted in the introduction, we assume that capital utilization is a choice variable in only one of the two industries. Let this industry be 1 so that $U_1 \equiv U \geqq 1$ and $U_2 \equiv 1$. We also assume a putty-clay technology for both industries. Thus, while the capital-services-to-labour-services ratio can be varied *between* any two units of calendar time, it cannot be changed *within* any *one* unit of calendar time. In other words, we assume zero ex-post and positive ex-ante substitutability between the services of the factors of production. Given this description of the technology, we can write the flow of output of good 1 in any one period as

$$X_1 = F_1\left(K_1, \frac{L_1}{U}\right) + F_1\left[(U-1)K_1, \frac{U-1}{U}L_1\right] \equiv F_1(UK_1, L_1), \tag{1}$$

where F_1 determines the form of the production function and is linear homogeneous in capital and labour services. Observe that $F_1(K_1, L_1/U)$ and $F_1[(U-1)K_1, (U-1)L_1/U]$, respectively, denote the outputs of good 1 produced during the normal and abnormal hours in a given calendar period. The industry employs L_1/U units of labour services during the normal hours and $(U-1)L_1/U$ units during the abnormal hours. The ratio of capital services to labour services is the same at UK_1/L_1 at all times.

Next, let the output of good 2 be represented by

$$X_2 = F_2(K_2, L_2), \tag{2}$$

where F_2 is linear homogeneous and K_2 represents both the stock and services of capital in industry 2 employed in any one period. Denoting the economy's endowment of labour services by L and its endowments of the specific factors in the two sectors by K_1^* and K_2^*, respectively, the full employment of resources assumption yields

$$L_1 + L_2 = L \tag{3}$$

$$K_1 = K_1^* \tag{4}$$

$$K_2 = K_2^*. \tag{5}$$

Work during abnormal hours must be compensated by a higher wage. Thus, denoting the nominal wage in the normal or day shift by W per unit of time and the abnormal shift wage by W_a per unit of time, we can write

$$W_a = (1+\beta(U))W, \tag{6}$$

where β is the shift premium and $\partial\beta/\partial U \equiv \beta_U > 0$. The average wage in industry 1 is

$$W_1 = \frac{1}{L1}\left[\frac{L_1}{U}W + \frac{U-1}{U}L_1 W_a\right] = \left[1+\frac{U-1}{U}\beta(U)\right]W = [1+\alpha(U)]W \tag{7}$$

where, for convenience, we have replaced the average shift premium $(U-1)\beta/U$ by $\alpha(U)$. For future reference, we note the following properties of the α function: $\alpha_U \equiv \partial\alpha/\partial U > 0$; $\alpha_{UU} \equiv \partial\alpha_U/\partial U \gtreqless 0$.

Let us denote the price of commodity i by p_i and the rental rate on specific capital by r_i. Given the normal shift wage W in equation (7), the total cost of producing output X_1 will be given by $r_1K_1 + [1+\alpha(U)]WL_1$. Therefore, cost minimization by firms in sector 1 leads to the following first-order conditions:

$$p_1\frac{\partial F_1}{\partial L_1} = [1+\alpha(U)]W \tag{8}$$

$$p_1\frac{\partial F_1}{\partial K_1} \equiv p_1\frac{\partial F_1}{\partial(UK_1)}U = r_1 \tag{9}$$

$$p_1\frac{\partial F_1}{\partial U} \equiv p_1\frac{\partial F_1}{\partial(UK_1)}K_1 = \alpha_U WL_1. \tag{10}$$

Equations (8) and (9) are the usual conditions for the optimal employment of labour and capital, respectively, except that the right-hand side of (8) represents the average wage rate in sector 1. Equation (10) provides the condition for the optimal utilization of capital. If we were to view capital utilization as a third factor of production, according to (10), we must equate the value of marginal product of capital utilization to its marginal cost ($= WL_1\, \partial\alpha/\partial U$). The latter is incurred in the form of the additional wage paid to the workers exposed to working the abnormal hours. Finally, the second order conditions of the cost minimization problem imply the following[3]

$$(\theta_{1L} - \sigma_1) + \theta_{1L}(1+R) > 0, \tag{11}$$

where σ_1 is the elasticity of substitution between capital services and labour services in industry 1; θ_{1L} is the share of labour in the value of the product in sector 1, i.e. $\theta_{1L} = ([1+\alpha(U)]WL_1/p_1X_1)$; and $R = U\alpha_{UU}/\alpha_U$. This new term R is simply the percentage change in the marginal costs of increased utilization, given W and L in (10), that results from a percentage change in utilization. Remembering that $\sigma_1 \geqq 0$, (11) implies that $R > -2$.

Cost minimization by firms, together with perfect competition in industry 2, yields the standard first-order conditions

$$p_2\frac{\partial F_2}{\partial L_2} = W \tag{12}$$

$$p_2 \frac{\partial F_2}{\partial K_2} = r_2. \tag{13}$$

Finally, the homogeneity of the production functions and the cost minimizing conditions (8)-(10), (12), and (13) imply

$$r_1 K_1 + [1 + \alpha(U)] W L_1 = p_1 X_1 \tag{14}$$

$$r_2 K_2 + W L_2 = p_2 X_2. \tag{15}$$

Our model is now completely specified. It contains ten independent equations (1-5, 8-10, and 12-13), and ten endogeneous variables (X_1, X_2, L_1, L_2, K_1, K_2, r_1, r_2, W, U).

3. THE ROLE OF UTILIZATION IN THE MODEL

Allowing capital utilization to be determined endogeneously introduces two important changes in the structure of the specific-factors model. These changes are highlighted in this section where we develop our model along the lines pioneered by Jones (1971).

Indicating by a circumflex, ^, a proportionate change in a variable, and through the use of appropriate substitutions, the main equations in the system can be written in terms of proportionate changes as follows:

$$\hat{X}_1 = \theta_{1K}\hat{K}_1^* + \theta_{1K}\hat{U} + \theta_{1L}\hat{L}_1, \tag{1)'$$

$$\hat{X}_2 = \theta_{2K}\hat{K}_2^* + \theta_{2L}\hat{L}_2, \tag{2)'$$

$$\lambda_{1L}\hat{L}_1 + \lambda_{2L}\hat{L}_2 = \hat{L}, \tag{3)'$$

$$\hat{K}_1 = \hat{K}_1^*, \tag{4)'$$

$$\hat{K}_2 = \hat{K}_2^*, \tag{5)'$$

$$\hat{U} = \frac{1}{(1+R)}[(\hat{r}_1 - \hat{W}) + (\hat{K}_1^* - \hat{L}_1)], \tag{10'}$$

$$\theta_{1L}\hat{W} + \theta_{1K}\hat{r}_1 = \hat{p}_1, \tag{14)'$$

and[4] (15′)

$$\theta_{2L}\hat{W} + \theta_{2K}\hat{r}_2 = \hat{p}_2.$$

The following notation will be used throughout the paper, $\theta_{1K} = r_1K_1/p_1X_1$, $\theta_{2K} = r_2K_2/p_2X_2$, $\theta_{1L} = W(1+\alpha)L_1/p_1X_1$, $\theta_{2L} = WL_2/p_2X_2$, $\lambda_{1L} = L_1/L$, and $\lambda_{2L} = L_2/L$.

In order to proceed, it is necessary to define the elasticities of substitution in the two sectors in terms of proportionate changes.

$$\sigma_1 = \frac{\hat{U} + \hat{K}_1^* - \hat{L}_1}{\hat{U}(1 + U\alpha_U/1+\alpha) + \hat{W} - \hat{r}_1} = \frac{\hat{U} + \hat{K}_1^* - \hat{L}_1}{\hat{U}(1/\theta_{1L}) + (\hat{W} - \hat{r}_1)} \tag{16}$$

and

$$\sigma_2 = \frac{\hat{K}_2^* - \hat{L}_2}{\hat{W} - \hat{r}_2}. \tag{17}$$

When utilization varies, the appropriate definition of the elasticity of substitution is in terms of substitution between capital services (S), i.e. $S = UK$, and labour services. Thus, by definition, $\sigma_1 = (\hat{S}_1 - \hat{L}_1)/(\hat{W}_1 - \hat{r}_1^*)$, where the price of capital services in sector 1 is $r_1^* = r_1/U$ and W_1 (the price of labour services in sector 1) is defined by equation (7).

Manipulation of this definition leads to (16). Finally, if utilization is constant ($\hat{U}=0$), (16) collapses to the same form as (17).

At this point, it is straightforward to identify the two main changes introduced into the model by allowing utilization to be determined endogenously. First, given the capital stock and the amount of labour services available to sector 1, whatever increases (decreases) utilization will increase (decrease) the output of this sector. This effect is easily observed by looking at (1′). Secondly, cost minimization by firms in sector 1 will lead to the joint determination of the capital stock to labour services ratio ($\hat{K}_1^* - \hat{L}_1$) and the level of utilization for any given normal or day wage rental rate facing firms in sector 1. Endogenous utilization modifies the usual response of the capital stock to labor services ratio to changes in the normal wage rental rate. This effect can be seen from (10)′ and (16), which form a subsystem of two equations in two variables for a given wage rental rate in sector 1. Eliminating the level of utilization from (16), through the use of (10)′, we have

$$(\hat{K}_1^* - \hat{L}_1) = \frac{T_1}{T_2}(\hat{W} - \hat{r}_1) = \tilde{\sigma}_1(\hat{W} - \hat{r}_1), \tag{18}$$

where $T_1 = [(\theta_{1L} - \sigma_1) + \sigma_1\theta_{1L}(1+R)]$ and $T_2 = [(\theta_{1L} - \sigma_1) + \theta_{1L}(1+R)]$.

The ratio T_1/T_2 on the R.H.S. of (18) measures the response of the capital-stock to labour-services ratio to changes in the day wage rental ratio. Hence, it can be interpreted as an elasticity of substitution between the capital stock in sector 1 and labour services, $\tilde{\sigma}_1$. Its values will be determined by the interaction between labour's share in sector 1 (θ_{1L}), the elasticity of substitution between capital services and labor services (σ_1) and the nature of the shift premium function (R). When utilization is not variable, as in the standard factor specific model, this stock elasticity becomes identical to σ_1. From (11), we know that $T_2 > 0$, but in principle it would seem possible that $T_1 < 0$. It turns out, however, that the stability of the model requires $T_1 \geqq 0$, as is demonstrated in Appendix A. Therefore, $\tilde{\sigma}_1 = T_1/T_2 \geqq 0$. These restrictions become critical in determining the range of effects of changes in commodity prices and endowments on factor returns, outputs, and utilization.

To conclude this section, we note some characteristics of T_1/T_2 and their main substantive implications. If $\sigma_1 > 1$, $T_1/T_2 > 1$ because $(1+R)$ must be positive to ensure $T_2 > 0$; if $\sigma_1 = 1$, $T_1/T_2 = 1$. If $\sigma_1 < 1$, then $T_1/T_2 < 1$ if $(1+R) > 0$ and $T_1/T_2 > 1$ if $(1+R) < 0$, which requires $\sigma_1 < \theta_{1L}$ to ensure $T_2 > 0$. If $\sigma_1 = 0$, $T_1/T_2 = 1/(2+R) > 0$. The main substantive result that emerges from these characteristics is the determination of the effect of changes in the day wage rental ratio in sector 1 on the level of utilization. From (10)′, we see that this ratio has a direct effect on utilization as well as an indirect one, through ($\hat{K}_1^* - \hat{L}_1$). The net effect is easily obtained by using (18) in (10)′ to yield $\hat{U} = (\tilde{\sigma}_1 - 1)(\hat{W} - \hat{r}_1)/(1+R)$. Utilization increases (decreases) when the day wage rental ratio in sector 1 increases if the factor services elasticity of substitution (σ_1) is greater (less) than unity. If $\sigma_1 = 1$, utilization is constant. Incidentally, these substantive results are exactly the same as the ones in the partial equilibrium literature, Winston and McCoy (1974).[5]

4. THE EFFECTS OF CHANGES IN COMMODITY PRICES

In this section we consider the effects of changes in relative commodity prices, $p = p_1/p_2$, on real factor returns, outputs, and utilization given factor endowments, i.e. $\hat{K}_1^* = \hat{K}_2^* = \hat{L} = 0$. The formal results are presented in Appendix B, Section 1.

Our most important result for trade theory is that the qualitative effects of changes in relative prices on real factor returns and outputs are the same in our model as in the standard factor specific model. Equations (B2)-(B4) and (B6)-(B7) in Appendix B lead to this conclusion. Thus, whether the real return to the mobile factor increases or decreases as prices change depends on the commodity used to measure changes in real income. The real returns to the specific factors always have opposite signs. The magnification effect is preserved for the specific factors but not for the mobile factor. Parenthetically, a key concept driving these qualitative results is $\tilde{\sigma}_1(=T_1/T_2)$. For, this concept plays a role in our model similar to that of σ_1 in the standard model with respect to the determination of the response of factor returns to changes in commodity prices. Finally, just as in the standard specific-factors model, the output effects are always such that the output of the sector experiencing the increase (decrease) in relative prices does not contract (does not expand). An interesting feature of this last result is that it differs from the one obtained for the two-factor model with variable utilization. In that model, BCP (1983), it is possible to have perverse output effects, e.g. an increase in p decreasing X_1, without violating the stability condition.[6] This finding provides additional evidence in favour of Neary's conjecture that "... intersectoral capital mobility is the source of all the paradoxes which are peculiar to international trade theory ...", Neary (1978*a*, p. 507).

Notwithstanding the previous results, it is worth noting that there will be quantitative differences between our model and the standard specific-factors model. An intuitively appealing way of bringing out these differences is to contrast the two models under special assumptions. If one assumes that the elasticity of substitution in sector 1 is zero, substantial differences emerge between our model and the standard model. In the standard model, when $\sigma_1 = 0$, there is no sectoral labor reallocation possible and outputs cannot change (Jones (1971)). Hence, an increase in the price of good 1, given p_2, would lead to the following effects on factor returns: $\hat{r}_1 = \hat{p}_1 > \hat{W} = 0 > \hat{r}_2 = -\hat{p}_1$. By contrast in our model, when $\sigma_1 = 0$, changes in commodity prices will lead to changes in the outputs of both sectors and reallocation of labour between sectors. The crucial mechanism driving these results is that in our case $\tilde{\sigma}_1 \neq 0$ and utilization increases as a result of an increase in the relative price of good 1, given $\hat{p}_2 = 0$ and $\sigma_1 = 0$, as can be seen from (B5). These changes in the real variables of our model are associated with changes in factor returns that differ from those of the standard model, i.e. $\hat{r}_1 = \hat{p}_1 = \hat{W} > 0 > \hat{r}_2 \gtreqless -\hat{p}_1$.

A unique feature of our model is, of course, the variability of the level of utilization in sector 1. Therefore, it is of interest to consider explicitly the effect of commodity prices on utilization. For ease of interpretation it is convenient to note how commodity prices affect utilization through their effect on relative factor returns in sector 1, that is, from (B4) and (B2) we have

$$\hat{W} - \hat{r}_1 = -(\sigma_2 \lambda_{2L} / \theta_{1K} \theta_{2K} \Delta)(\hat{p}_1 - \hat{p}_2), \tag{19}$$

where

$$\Delta = \lambda_{1L}(\tilde{\sigma}_1 / \theta_{1K}) + \lambda_{2L}(\sigma_2 / \theta_{2K}) > 0.$$

These changes in relative factor returns are the ones that affect utilization, as we saw in the previous section. Hence, the qualitative effect of changes in relative commodity prices on utilization will be entirely determined by the elasticity of substitution. For instance, as can be seen from (B5), if $\sigma_1 > 1(<1)$, an increase in the relative price of commodity 1 will decrease (increase) the level of utilization. A useful perspective on this result is obtained by noting how it differs from the one obtained in the two-factor model with variable utilization. In that case, BCP (1983) found the following relation

$$\hat{U} = (\theta_{1L}/\theta)[(1-\sigma_1)/T_2](\hat{p}_1 - \hat{p}_2), \tag{20}$$

where $\theta = \theta_{1K} - \theta_{2K}$ and all other symbols are the same as in this paper. Thus the effects of changes in commodity prices on utilization will be critically affected by relative factor intensities in the two sectors. This difference arises solely as a result of the difference in the way commodity prices relate to factor returns in the two models.

5. THE EFFECTS OF CHANGES IN FACTOR ENDOWMENTS

In this section we consider the effects of changes in factor endowments on factor returns, outputs and utilization given commodity prices. As in the previous section, the formal results are presented in Appendix B (section 2).

One of the principal results of our analysis for trade theory is that the qualitative effect of changes in factor endowments on factor returns will be the same in this model as in the standard factor specific model. For instance, the effect of changes in factor endowments on the return to the mobile factor is always opposite in sign to the effect on the specific factors, as can be seen from (B10)-(B12) in Appendix B. Perhaps more importantly for trade theory, the qualitative effects of changes in factor endowments on outputs will be the same as in the standard model for labour and capital specific to sector 2 *but not* for capital specific to sector 1. That is, the outputs of both sectors increase when the labour endowment increases and the output of sector 1(2) decreases (increases) when the endowment of capital specific to sector 2 increases, but the output of both sectors can decrease when the endowment of capital specific to sector 1 increases. These results can be seen from (B14) and (B15) and the subsequent discussion in Appendix B. Here, we merely note that the unusual result of a decrease in the output of both sectors is possible because, when $\sigma_1 < 1$, the negative output effect on X_1 induced by a lowering of the utilization level when K_1 increases can dominate the positive output effect on X_1 due to the increase in K_1.[7] As a result, it becomes feasible for a country to have more capital specific to the variable utilization sector than another and to produce a smaller amount of this sector's commodity. This possibility enhances the variety of trade patterns that can be generated by our model relative to the standard one.

An explicit discussion of the implications of the last result is useful for understanding its importance. First, note that it is possible for an increase in K_1 to result in a decrease in X_1/X_2. This result follows from the discussion of (B16) in Appendix B, where it is shown, for example, that X_1 can decrease while X_2 remains constant when $\sigma_1 < 1$ and $\sigma_2 = 0$. In this situation let us consider a two country model where country A is identical to country B except that it has more capital specific to sector 1. At the same commodity prices country A would produce less X_1 and the same X_2 as country B. If consumer preferences are identical and homothetic in both countries, commodity prices will bear the following relation under autarky: $(p_1/p_2)^A > (p_1/p_2)^B$. Therefore the opening of trade will lead country A to export good 2 and import good 1 even though capital specific to sector 1 is more abundant in country A, contrary to what would happen in the standard specific-factors model.

Of course, there are also quantitative differences between the results of our model and the results of the standard one. Once again, it is easier to see these differences by considering a special case. If one assumes that $\sigma_2 = 0$ in our model, the effect of changes in the labour endowment, for example, on the day wage and on the output of sector 1 will be, from (B12) and (B14),

$$\hat{W} = -(\theta_{1K}/\lambda_{1L})(1/\tilde{\sigma}_1)\hat{L}; \qquad \hat{X}_1 = (\theta_{1L}/\lambda_{1L})(1 + \theta_{1K}(1 - \sigma_1)/T_1)\hat{L}.$$

If one assumes that $\sigma_2 = 0$ in the standard model, the same effects are given by, e.g. Jones (1971),

$$\hat{W} = -(\theta_{1K}/\lambda_{1L})(1/\sigma_1)\hat{L}; \qquad \hat{X}_1 = (\theta_{1L}/\lambda_{1L})\hat{L}.$$

This comparison clearly illustrates the source of the differences between the two models. The effect on the day wage will differ due to the difference in the elasticity of substitution between the capital *stock* and labour services in the two models. The effect on the output of sector 1 in the two models will differ due to the output effect of utilization, captured by $\theta_{1L}\hat{U}$ in equation (1)′ of our model.

Let us now consider a unique feature of our model, namely, the effect of changes in factor endowments on utilization. This effect is given by equation (B13). Increases in the endowment of the mobile factor will have the opposite effect on utilization as increases in the endowment of the specific factors. More importantly for our present purposes, the effect of factor endowments on utilization will also be critically affected by whether or not the elasticity of substitution between factor services (σ_1) is greater than, equal to, or less than unity. For example, increases in capital specific to the variable utilization sector will increase (decrease) utilization if $\sigma_1 > 1 (< 1)$. Therefore in contrast to the two-factor model with variable utilization, there is a relationship between endowment changes and capital utilization. Once again the difference is entirely due to the different nature of the two general equilibrium models. That is, changes in K_1 affect the day wage rental ratio in sector 1 in the factor specific model whereas changes in K have no effect on factor returns in the two factor model; and, in both general equilibrium contexts, changes in factor returns are the ultimate determinants of changes in utilization.

To conclude this section, we note an unusual characteristic of our specific-factors model. Perhaps the most well established proposition of the partial equilibrium literature on capital utilization is the positive association between the capital services to labour services ratio (UK_1/L_1) and the level of utilization, Betancourt and Clague (1981, Chapter 1). This positive association need not hold in our model[8]. Since a change in endowments affects both of these endogeneous variables, one can use (18), (B10), (B12) and (B13) to obtain

$$(\hat{U} + \hat{K}_1 - \hat{L}_1)/\hat{U} = 1 + \frac{[-\tilde{\sigma}_1 T_2]}{\theta_{1L}(1-\sigma_1)} = 1 - \frac{T_1}{\theta_{1L}(1-\sigma_1)}. \qquad (21)$$

If $\sigma_1 > 1$, the R.H.S. of (21) will be positive; but if $\sigma_1 < 1$, the R.H.S. of (21) will be positive or negative as $T_1 \lesseqgtr \theta_{1L}(1-\sigma_1)$. The argument at the end of Section 2 in Appendix B establishes that either inequality may prevail. While factor returns are exogenous in partial equilibrium, they are endogenous in general equilibrium; hence, the possible disparity of results should not be surprising upon reflection. Nevertheless it is emphasized here because it contradicts the conjecture that, in contrast to the two-factor model, the specific-factors model provides a general equilibrium framework in which partial equilibrium results are validated, Neary (1978*a*, p. 507).

6. AN APPLICATION

One of the most interesting applications of the standard specific-factors model is to the explanation of nineteenth-century trade between Britain and the United States. This issue was originally raised by Temin (1966) and the results were developed in detail by Jones (1971). We proceed by showing how our generalization of the specific-factors model sheds additional light on the explanation of this historical experience.

If one views land as the factor specific to agriculture (sector 2), and capital as the factor specific to industry (sector 1), the specific-factors model with constant utilization explains why wage rates, measured in terms of agricultural goods, and interest rates, measured as the ratio of the rental rate to the price of manufacturing goods, were both higher in America than in the U.K. even if technology were the same in both countries. As Jones (1971) has argued, these conditions are consistent with the model if the U.S. had a more abundant endowment of land and capital relative to labour than the U.K. What's not explained by this "dual scarcity" argument, however, is why American manufacturing firms had a higher rate of capital utilization during this period than British firms. This behavioral difference between firms in the two countries was substantial enough that, for example, Brito and Williamson (1973) developed a partial equilibrium model of the firm directed to the explanation of this difference. Moreover, they cast doubts on the so-called "dual scarcity" explanation of the history of the period partially because of its failure to account for this difference between firms on both sides of the Atlantic.

Our generalization of the specific-factors model can reconcile Jones' argument with the historical evidence on utilization levels. For instance, if the factor services elasticity of substitution in manufacturing is greater than unity, a higher price of manufacturing goods in America than in the U.K. would lead to a lower level of capital utilization in America than in the U.K.; however, the larger endowment of capital and land relative to labour in the U.S. than in the U.K. would lead to a higher level of capital utilization in America than in the U.K. Therefore, if the endowment effect on utilization dominates the relative price effect on utilization, interest rates, real wages, and utilization would all be higher in America than in the U.K. A similar argument can be developed when the factor services elasticity of substitution in manufacturing is less than unity, but in this case the reconcilation with the historical facts would require the price effect on utilization to dominate the endowment effect on utilization.

7. ADDITIONAL CONSIDERATIONS

One question that arises naturally with respect to the previous results is: Are they a consequence of the assumption that utilization is fixed in one of the two sectors? This question is specially pertinent when factor specificity is attributed to the short-run immobility of capital. For if the reason that factors are specific is that capital is immobile in the short-run, why should capital utilization be variable in one sector and fixed in the other? Fortunately, all of the previous results generalize when utilization is endogenously determined in both sectors.

The main algebraic results are contained in Section 3 of Appendix B. Here, we will highlight the essence of the results. Allowing utilization in sector 2 to be variable introduces an additional equation into the model (analogous to (10)). Not surprisingly, this generalization introduces a divergence between the capital-stock-to-labour-services elasticity of substitution ($\tilde{\sigma}_2$) and the capital-services-to-labour-services elasticity of substitution (σ_2) in sector 2 which is exactly analogous to the one discussed earlier with respect to sector 1. The workings of the model are hardly affected.

All of the qualitative effects of changes in commodity prices on factor returns, utilization in sector 1 and outputs are the same as before. The quantitative effects will differ for two reasons. First, whenever σ_2 appears it is now replaced by $\tilde{\sigma}_2$ and the two concepts take on different values, unless $\sigma_2 = 1$; second, the output effect on sector 2 will now have an additional term that incorporates the consequences of changes in utilization on the output of this sector. Thus, the main novel feature of changes in commodity prices is their effect on the level of utilization in sector 2. More specifically, as can be seen from equation (B.17), an increase in the relative price of commodity 1 will decrease (increase) utilization in sector 2 if σ_2 falls short of (exceeds) unity. This result implies, for example, that if $\sigma_1 < 1$ and $\sigma_2 > 1$, an increase in p_1/p_2 will raise utilization in both sectors. Yet, it will remain true that X_1 expands and X_2 contracts.

With respect to changes in factor endowments, all of the qualitative results of changes in labour and capital specific to sector 1 are the same as before. The qualitative effects of changes in capital specific to sector 2 are the same as before with one exception. It is now possible for an increase in K_2^* to decrease X_2 (and to increase X_1/X_2), as can be seen from equation (B.20) of the Appendix. This effect is exactly analogous to the one discussed in section 5, where increases in K_1^* could lead to a decrease in X_1. Consequently, this effect further enhances the variety of trade patterns that can be generated by the model. Finally, the effects of endowment changes on utilization in sector 2 are analogous to those described in Section 5 for sector 1, as can be seen by comparing equations (B19) and (B13) in the Appendix.

Another question that arises naturally in the context of our model is—what are the effects on the patterns of trade if preferences toward work during abnormal hours differ between countries? The answer to this question can be obtained in terms of the following experiment. Consider a two country model where country *A* is identical to country *B*, except that workers in *A* have a stronger aversion than those in *B* to work during abnormal hours. If preferences are identical and homothetic in the two countries, the effect on the pattern of trade can be determined by whether, at the same commodity prices, country *A* produces more X_1 relative to X_2 than *B*. Under autarky commodity market equilibrium in each country requires $(p_1/p_2)^A < (p_1/p_2)^B$ if $(X_1/X_2)^A > (X_1/X_2)^B$, or $(p_1/p_2)^A > (p_1/p_2)^B$ if $(X_1/X_2)^A < (X_1/X_2)^B$. If the latter relation prevails, the opening of trade leads country *A* to import X_1; if the former relation prevails, the opening of trade leads country *A* to export X_1.

To simplify the exposition, we assume, as in previous sections, that utilization is endogeneously determined in only one of the two sectors. As before we relegate the algebraic derivations to Section 4 of Appendix B. In our model an increase in aversion towards work during abnormal hours can be captured by an upward shift in the shift-premium function, $\beta(U, \delta)$ where δ is the shift parameter. Such a shift has two effects on the average shift-premium function $\alpha(U, \delta)$: An intercept effect (α_δ), which increases the average cost of labour in sector 1; and a slope effect ($\alpha_{U\delta}$), which increases the marginal cost of utilization in sector 1. These two changes in the average shift premium function will affect the optimal level of utilization in sector 1 and the allocation of labour between sectors, as can be seen from (B22) and (B23) in the Appendix. These two effects, in turn, will determine the changes in outputs of the two sectors, equations (B24) and (B25) of the Appendix. The results are in general ambiguous. While one would expect the country with the stronger aversion towards work during abnormal hours (*A*) to produce less X_1 relative to X_2, hence import the commodity produced under shift-work, it is possible for country *A* to produce more X_1 relative to X_2 and thus export the commodity produced under shift-work.

In order to understand the mechanisms underlying these results, it is desirable to consider three special cases. First, suppose that $\sigma_2 = 0$. In this situation, no labour reallocation between sector is possible and the output of sector 2 cannot change. Thus, X_1 increases (decreases) if utilization increases (decreases). Utilization increases, remains constant or decreases as the proportionate increase in the average cost of labour in sector

1 $(\alpha_\delta/(1+\alpha))$ is greater than, equal to or less than the proportionate increase in the marginal cost of utilization, $\alpha_{U\delta}/\alpha_U$, as can be seen from (B26). The economics of this result is clear. The stronger aversion to work during abnormal hours increases the marginal cost of utilization, which lowers the incentive to utilize, but it also increases the average cost of labour in sector 1, which raises the incentive to utilize. That is an increase in the average cost of labour induces firms in sector 1 to choose techniques with a higher capital-services-to-labour-services ratio. This increse in UK_1/L_1 can only be accomplished by raising the level of utilization, since K_1/L_1 is fixed. The net effect on utilization and on the pattern of trade depends on the relative magnitude of these two forces.

As a second special case consider a situation in which $\sigma_2 \neq 0$ but the magnitudes of the above two forces are the same, i.e. $\alpha_\delta/(1+\alpha) = \alpha_{U\delta}/\alpha_U$. In this case labour moves from sector 1 to sector 2 in the country with the greater aversion to work during abnormal hours, as can be seen from (B23); utilization increases (decreases) if $\sigma_1 > 1 (\sigma_1 < 1)$, as can be seen from (B22). Even if $\sigma_1 > 1$, the labour allocation effect dominates the utilization effect and it will always be the case that the output of sector 1 falls relative to the output of sector 2, as can be seen from (B27) in the Appendix. Hence, country A will always import the commodity produced under shift-work. Finally, consider a situation where $\sigma_2 \neq 0$, $\alpha_\delta/(1+\alpha) \neq \alpha_{U\delta}/\alpha_U$, but $\sigma_1 = 0$. Utilization falls, regardless of the magnitude of the average cost of labour effect, labour moves to sector 2, and, consequently, X_1/X_2 will always decrease, as can be seen from (B28). Hence, country A will always import the commodity produced under shift-work.

To conclude, the incorporation of endogenous capital utilization into the specific-factors model substantially enriches the analysis of the patterns of trade that can be generated by the model. This statement holds whether the source of comparative advantage is differences in factor endowments or differences in preferences toward work during abnormal hours.

APPENDIX A

The stability of the model

In accordance with the assumptions in the text, the stability analysis is developed in terms of a small open economy. Adapting the approach in, for example, Neary (1978b) to our model, we assume labour and utilization to be instantaneously fixed while in the long run they both change in the direction of the higher return. The other assumptions of our model are unaltered. Thus, full employment and commodity market equilibrium are maintained at all times and returns adjust continuously so as to ensure zero profits.

The adjustment mechanisms just described can be written as

$$\dot{L}_1 = A\left(\frac{W^1}{W^2} - 1\right) \tag{A1}$$

$$\dot{U} = B\left(\frac{r_1 K_1}{U_{\alpha U} W^1 L_1} - 1\right) \tag{A2}$$

where the superscript indicates the day wage that prevails in each sector, A and B are positive speeds of adjustment and the "$\cdot$" over a variable indicates a time derivative.

A particular equilibrium position, $(\bar{L}_1, \bar{U})$, will be locally stable in a dynamic sense if $\lim_{t\to\infty} L_1(t) = \bar{L}_1$ and $\lim_{t\to\infty} U(t) = \bar{U}$, which implies and is implied by the stability of the Jacobian matrix of the system associated with (A1) and (A2) e.g. Mayer (1974).

Local dynamic stability then requires that the Routh-Hurwitz conditions hold for J, where

$$J=\begin{bmatrix}\partial\dot{L}_1/\partial L_1 & \partial\dot{L}_1/\partial U\\ \partial\dot{U}/\partial L_1 & \partial\dot{U}/\partial U\end{bmatrix}. \tag{A3}$$

A general statement of these conditions is given in Quirk and Saposnik (1968); below, we will note their implications for our model.

In order to obtain the terms in the Jacobian, we first find the total differential of (A1) and (A2), i.e.

$$d\dot{L}_1=A(\hat{W}^1-\hat{W}^2) \tag{A4}$$

$$d\dot{U}=B[-(\hat{W}^1-\hat{r}_1)-(1+R)\hat{U}-\hat{L}_1], \tag{A5}$$

where the partial derivatives are evaluated at equilibrium values of the variables, e.g. $W^1=W^2=W$.

Secondly, we solve the equations of the model for $\hat{W}^1-\hat{W}^2$ and $\hat{W}^1-\hat{r}_1$ in terms of $\hat{U}$ and $\hat{L}_1$ given the assumptions of the stability analysis, e.g. full employment.

$$L_2=-\frac{\lambda_{1L}}{\lambda_{2L}}\hat{L}_1 \tag{A6}$$

$$\theta_{1L}\hat{W}^1+\theta_{1K}\hat{r}_1=0 \tag{A7}$$

$$\theta_{2L}\hat{W}^2+\theta_{2K}\hat{r}_2=0 \tag{A8}$$

$$(\hat{W}^1-\hat{r}_1)=\frac{1}{\sigma_1\theta_{1L}}(\theta_{1L}-\sigma_1)\hat{U}-\frac{1}{\sigma_1}\hat{L}_1 \tag{A9}$$

$$(\hat{W}^2-\hat{r}_2)=-\frac{1}{\sigma_2}\hat{L}_2=\frac{\lambda_{1L}}{\sigma_2\lambda_{2L}}\hat{L}_1. \tag{A10}$$

The solution for $\hat{W}^1-\hat{r}_1$ is given by (A9). To find $\hat{W}^1-\hat{W}^2$, note that from (A8), $\hat{r}_2=-(\theta_{2L}/\theta_{2K})\hat{W}^2$; hence (A10) becomes

$$\hat{W}^2=\frac{\lambda_{1L}\theta_{2K}}{\sigma_2\lambda_{2L}}\hat{L}_1. \tag{A10$'$}$$

Similarly, (A7) implies $\hat{r}_1=-(\theta_{1L}/\theta_{1K})\hat{W}^1$; therefore, using (A9), we obtain $\hat{W}^1=(\theta_{1K}/\sigma_1\theta_{1L})(\theta_{1L}-\sigma_1)\hat{U}-(\theta_{1K}/\sigma_1)\hat{L}_1$, which yields

$$\hat{W}^1-\hat{W}^2=\frac{\theta_{1K}}{\sigma_1\theta_{1L}}(\theta_{1L}-\sigma_1)\hat{U}-\Delta'\hat{L}_1 \tag{A11}$$

where

$$\Delta'=\frac{\sigma_1\lambda_{1L}\theta_{2K}+\sigma_2\theta_{1K}\lambda_{2L}}{\sigma_1\sigma_2\lambda_{2L}}>0.$$

Consequently, the Jacobian Matrix in our model will be

$$J=\begin{bmatrix}-\dfrac{A\Delta'}{L_1} & -\dfrac{A}{U}\dfrac{\theta_{1K}}{\sigma_1\theta_1}(\theta_{1L}-\sigma_1)\\ \dfrac{B}{L_1}\left[\dfrac{1}{\sigma_1}-1\right] & -\dfrac{B}{U}\dfrac{1}{\sigma_1\theta_{1L}}(T_1)\end{bmatrix}, \tag{A12}$$

where T_1 is defined as in the text.

In terms of our model, the Routh–Hurwitz conditions are that the trace of J in (A12) must be negative and its determinant must be positive, i.e.

$$\operatorname{tr}(J) = -\frac{A}{L_1}\Delta' - \frac{B}{U}\frac{1}{\sigma_1\theta_{1L}}T_1 < 0, \tag{A13}$$

and

$$|J| = \frac{AB}{\sigma_1\theta_{1L}L_1U}\left[\frac{\lambda_{1L}\theta_{2k}}{\sigma_2\lambda_{2L}}T_1 + \theta_{1K}T_2\right] > 0. \tag{A14}$$

All the characteristics of our model in these two expressions, except T_1, are known to be positive. Thus, $T_1 \geqq 0$ is sufficient to ensure that (A13) and (A14) hold for all speeds of adjustment and, consequently, that the model is D-stable. Additionally, if the speed of adjustment of utilization is large relative to the speed of adjustment in the labour market, $T_1 \geqq 0$ is also necessary for stability.

APPENDIX B

1. *The effects of changes in relative prices*

Equations (3)′, (10)′, (16), and (17) are used to express changes in the day wage as a function of changes in the rental rates of the specific factors. To wit,

$$\hat{W} = \frac{\lambda_{1L}\tilde{\sigma}_1}{S}\hat{r}_1 + \frac{\sigma_2\lambda_{2L}}{S}\hat{r}_2 \tag{B1}$$

where $S = \lambda_{1L}\tilde{\sigma}_1 + \sigma_2\lambda_{2L}$ and $\tilde{\sigma}_1$ was defined in the text. Next, equations (B1), (14)′, and (15)′ can be used to express changes in the rental rates of the specific factors as functions of changes in commodity prices. Considerable manipulation results in

$$\hat{r}_1 = (1/\Delta)\{[\lambda_{1L}(\tilde{\sigma}_1/\theta_{1K}) + (1/\theta_{1K})\lambda_{2L}(\sigma_2/\theta_{2K})]\hat{p}_1$$
$$-[(\theta_{1L}/\theta_{1K})\lambda_{2L}(\sigma_2/\theta_{2K})]\hat{p}_2\} \tag{B2}$$
$$\hat{r}_2 = (1/\Delta)\{-[(\theta_{2L}/\theta_{2K})\lambda_{1L}(\tilde{\sigma}_1/\theta_{1K})]\hat{p}_1$$
$$+[(1/\theta_{2K})\lambda_{1L}(\tilde{\sigma}_1/\theta_{1K}) + \lambda_{2L}(\sigma_2/\theta_{2K})]\hat{p}_2\} \tag{B3}$$

where $\Delta = \lambda_{1L}(\tilde{\sigma}_1/\theta_{1K}) + \lambda_{2L}(\sigma_2/\theta_{2K})$. Substituting (B2) and (B3) into (B1) allows us to express changes in the day wage as a function of changes in commodity prices, i.e.,

$$\hat{W} = (1/\Delta)[\lambda_{1L}(\tilde{\sigma}_1/\theta_{1K})\hat{p}_1 + \lambda_{2L}(\sigma_2/\theta_{2K})\hat{p}_2]. \tag{B4}$$

It is worth noting here the resemblance between our equations (B2)–(B4) and the analogous equations in Jones (1971). Essentially, Jones' equations can be obtained from ours by substituting $\tilde{\sigma}_1$ by σ_1 in appropriate places. It is also of interest to note the crucial role played by the elasticities of marginal physical product curves of labour in the two sectors, $\tilde{\sigma}_1/\theta_{1K}$ and σ_2/θ_{2K}, given capital *stocks.*

In contrast to Jones (1971), in our model changes in factor returns due to changes in commodity prices leads to changes in the rate of capital utilization. Hence, equations (B2), (B3) and (B4), together with (18), can be used in (16) to express the change in utilization that results from changes in commodity prices. Specifically, we have

$$\hat{U} = (1/\Delta)(\theta_{1L}/\theta_{1K})(1/T_2)\lambda_{2L}(\sigma_2/\theta_{2K})(1-\sigma_1)(\hat{p}_1 - \hat{p}_2). \tag{B5}$$

Moreover, changes in commodity prices (and the consequent change in utilization) also lead to changes in outputs. Therefore, use of (18), (B4), (B2), and (B5) in (1)′ leads to

$$\hat{X}_1 = (\theta_{1L}/\Delta)\lambda_{2L}(\sigma_2/\theta_{2K})[(\tilde{\sigma}_1/\theta_{1K}) + (1-\sigma_1)/T_2](\hat{p}_1 - \hat{p}_2). \quad \text{(B6)}$$

Similarly, use of (18), (3′), (B4) and (B2) in (2)′ yields

$$\hat{X}_2 = -(\theta_{2L}/\Delta)\lambda_{1L}(\tilde{\sigma}_1/\theta_{1K})(\sigma_2/\theta_{2K})(\hat{p}_1 - \hat{p}_2). \quad \text{(B7)}$$

These equations yield all of the results described in Section 4. It is not apparent, however, why the bracketed expression in (B.6) is necessarily positive when $\sigma_1 > 1$. To show this note that given $\tilde{\sigma}_1 = T_1/T_2$ and $T_1 = (\theta_{1L} - \sigma_1) + \sigma_1\theta_{1L}(1+R)$, this expression may be rewritten as

$$(1/\theta_{1K}T_2)\{(\theta_{1L} - \sigma_1) + \sigma_1\theta_{1L}(1+r) + \theta_{1K}(1-\sigma_1)\} \quad \text{(B8)}$$

which, after addition and subtraction of $\sigma_1(\theta_{1L} - \sigma_1)$, can be shown to be equivalent to

$$(1/\theta_{1K}T_2)\{(1-\sigma_1)^2 + \sigma_1 T_2\}. \quad \text{(B8)}'$$

Remembering that T_2 is necessarily positive, the bracketed expression in (B8)′, and hence in (B6), is shown to be unambiguously positive.

2. *The effects of changes in factor endowments*

Equations (3)′, (10)′, (16), and (17) are used to express the day wage as a function of the changes in factor endowments and the rental rates to the specific factors. That is,

$$\hat{W} = (1/S)[\lambda_{1L}\hat{K}_1^* + \lambda_{2L}\hat{K}_2^* - \hat{L}] + (\lambda_{1L}\tilde{\sigma}_1/S)\hat{r}_1 + (\lambda_{2L}\sigma_2/S)\hat{r}_2 \quad \text{(B9)}$$

where $\tilde{\sigma}_1$ and S are defined as before. Substitution of (B9) into (14)′ and (15)′ allows us to express changes in the rental rates as functions of changes in endowments. Straightforward but tedious algebra results in

$$\hat{r}_1 = (1/\Delta)(\theta_{1L}/\theta_{1K})[\hat{L} - \lambda_{1L}\hat{K}_1^* - \lambda_{2L}\hat{K}_2^*] \quad \text{(B10)}$$

$$\hat{r}_2 = (1/\Delta)(\theta_{2L}/\theta_{2K})[\hat{L} - \lambda_{1L}\hat{K}_1^* - \lambda_{2L}\hat{K}_2^*] \quad \text{(B11)}$$

where Δ is also defined as in the previous section. Substitution of (B10) and (B11) into (B9) results in

$$\hat{W} = -(1/\Delta)[\hat{L} - \lambda_{1L}\hat{K}_1^* - \lambda_{2L}\hat{K}_2^*] \quad \text{(B12)}$$

Note once again the similarity between our expressions (B10)-(B12) and the analogous equations in Jones (1971). Just as in the previous section, and in contrast to Jones (1971), changes in factor returns lead to changes in utilization. Equations (16), (17), and (3)′ are used to eliminate $\hat{L}_1$ from (10)′. Substituting (B10), (B11), and (B12) into the resulting expression leads to (B13) below.

$$\hat{U} = (1/\Delta)(\theta_{1L}/\theta_{1K})(1/T_2)(1-\sigma_1)[\hat{L} - \lambda_{1L}\hat{K}_1^* - \lambda_{2L}\hat{K}_2^*] \quad \text{(B13)}$$

Finally, all of the previous changes generate changes in the outputs of the two sectors. Equations (3)′ and (17) can be used to find a reduced form for $\hat{L}_1$; insertion of the result and (B13) into (1)′ yields the following solution for the output of sector 1

$$\begin{aligned}\hat{X}_1 = {} & (1/\Delta)(\theta_{1L}/T_2)(1-\sigma_1)(\hat{L} - \lambda_{1L}\hat{K}_1^* - \lambda_{2L}\hat{K}_2^*) \\ & + (1/\Delta)\{\theta_{1L}(\tilde{\sigma}_1/\theta_{1K})(\hat{L} - \lambda_{2L}\hat{K}_2^*) + [\tilde{\sigma}_1 + (\lambda_{2L}/\lambda_{1L})(\sigma_2/\theta_{2K})]\lambda_{1L}\hat{K}_1^*\}. \quad \text{(B14)}\end{aligned}$$

A similar procedure leads to the following solution for the output of sector 2

$$\hat{X}_2=(1/\Delta)\{\theta_{2L}(\sigma_2/\theta_{2K})(\hat{L}-\lambda_{1L}\hat{K}_1^*)+[(\lambda_{1L}/\lambda_{2L})(\tilde{\sigma}_1/\theta_{1K})+\sigma_2]\lambda_{2L}\hat{K}_2^*\}. \quad (B15)$$

These equations generate all the results described in Section 5 of the text. Two of these results, however, are far from obvious. For instance, it is not immediately apparent from (B14) that when L or K_2 change and $\sigma_1>1$ the direct output effect (the second term in (B14)) will dominate the indirect output effect due to the change in utilization (the first term in (B14)). Remembering that $\tilde{\sigma}_1=T_1/T_2$, this will always be true if $T_1+\theta_{1K}(1-\sigma_1)=(\theta_{1L}-\sigma_1)+\sigma_1\theta_{1L}(1+R)+\theta_{1K}(1-\sigma_1)$ is positive. But in view of the positivity of the expression in (B8), this last expression is necessarily positive.

Another instance where the result is not transparent also arises with reference to expression (B14). If the endowment of K_1 changes and σ_1 is less than unity, there are two terms opposite in sign that will determine the effect on X_1. Specifically, setting $\hat{L}=\hat{K}_2^*=0$ and substituting $\tilde{\sigma}_1=T_1/T_2$ in (B14), we have

$$\hat{X}_1=(1/\Delta)(1/T_2)\{[T_1-\theta_{1L}(1-\sigma_1)]+(\lambda_{2L}/\lambda_{1L})(\sigma_2/\theta_{2K})\}\lambda_{1L}\hat{K}_1^*. \quad (B16)$$

If $\sigma_1<1$, $\hat{X}_1/\hat{K}_1^*$ can be positive or negative. To see this consider the special case when $\sigma_2=0$. In this case, the sign of (B16) is determined entirely by the term in square brackets of (B16) which can be shown to equal $\sigma_1\theta_{1L}[(1+R)-(\theta_{1K}/\theta_{1L})]$. This last expression is necessarily positive if θ_{1K} is small and $1+R$ is positive. Similarly, it is necessarily negative if θ_{1L} is small. Thus, the ambiguity of the sign of $\hat{X}_1/\hat{K}_1^*$ is established.

3. *The effects of variable utilization in both sectors*

Denote by $U_i (i=1,2)$ the rate of utilization in sector i. Moreover, define $R_i=U\alpha_{UU}/\alpha_U$ evaluated at $U=U_i$. Since U_1 and U_2 will differ in general, R_1 and R_2 will also differ despite the fact that the α function relevant to the two sectors is the same. Analogously to T_1, T_2 and $\tilde{\sigma}_1$, we also need to define V_1, V_2 and $\tilde{\sigma}_2$ such that $V_1=(\theta_{2L}-\sigma_2)+\sigma_2\theta_{2L}(1+R_2)$, $V_2=(\theta_{2L}-\sigma_2)+\theta_{2L}(1+R_2)$ and $\tilde{\sigma}_2=V_1/V_2$.

The endogeneity of utilization in sector 2 yields an additional equation analogous to (10′), i.e.

$$\hat{U}_2=[1/(1+R_2)][(\hat{r}_2-\hat{W})+(\hat{K}_2^*-\hat{L}_2)]. \quad (10a)$$

This equation, together with (17) suitably modified, leads to

$$\hat{K}_2-\hat{L}_2=(V_1/V_2)(\hat{W}-\hat{r}_2)=\tilde{\sigma}_2(\hat{W}-\hat{r}_2). \quad (18a)$$

If we now derive the effects of commodity price changes on factor returns and output and utilization in sector 1, retracing the steps in Section 1 of this Appendix, we obtain the same expressions as before with $\tilde{\sigma}_2$ replacing σ_2 wherever it appears. Therefore, these expressions are not reproduced here. Using these results, the final expression for $\hat{U}_2$ can be shown to be

$$\hat{U}_2=-(1/\tilde{\Delta})(\theta_{2L}/\theta_{2K})(1/V_2)\lambda_{1L}(\tilde{\sigma}_1/\theta_{1K})(1-\sigma_2)(p_1-\hat{p}_2) \quad (B17)$$

where $\tilde{\Delta}=\lambda_{1L}(\tilde{\sigma}_1/\theta_{1K})+\lambda_{2L}(\tilde{\sigma}_2/\theta_{2K})$.

Using (18a), (B17) and the reduced forms for the factor returns, we obtain from (2)′,

$$\hat{X}_2=-(\theta_{2L}/\tilde{\Delta})\lambda_{1L}(\tilde{\sigma}_1/\theta_{1K})[(\tilde{\sigma}_2/\theta_{2K})+(1-\sigma_2)/V_2](\hat{p}_1-\hat{p}_2). \quad (B18)$$

Incidentally, the expression within the brackets will be unambiguously positive. The proof is similar to that given to establish the positivity of the term in brackets in (B6) and is therefore suppressed.

Turning to the effects of changes in factor endowments, the expressions for factor returns and utilization and output in sector 1 remain unchanged except that $\tilde{\sigma}_2$ replaces σ_2 everywhere it appears. Therefore, we only present the expressions for changes in U_2 and X_2. We have

$$\hat{U}_2=(1/\tilde{\Delta})(\theta_{2L}/\theta_{2K})(1/V_2)(1-\sigma_2)(\hat{L}-\lambda_{1L}\hat{K}_1^*-\lambda_{2L}\hat{K}_2^*) \tag{B19}$$

$$\begin{aligned}\hat{X}_2=&(1/\tilde{\Delta})(\theta_{2L}/V_2)(1-\sigma_2)(\hat{L}-\lambda_{1L}\hat{K}_1^*-\lambda_{2L}\hat{K}_2^*)\\&+(1/\tilde{\Delta})\{\theta_{2L}(\tilde{\sigma}_2/\theta_{2K})(\hat{L}-\lambda_{1L}\hat{K}_1^*)\\&+[\tilde{\sigma}_2+(\lambda_{1L}/\lambda_{2L})(\tilde{\sigma}_1/\theta_{1K})]\lambda_{2L}\hat{K}_2^*\}.\end{aligned} \tag{B20}$$

Note that the first term in (B20) is due to the variability of utilization in sector 2; moreover it is this term that leads to the possibility of an increase in K_2^* lowering X_2 and raising X_1/X_2, for the same reasons that were given in connection with equation (B14).

4. *The effects of different preferences toward work during abnormal hours*

We consider a vertically upward shift in the β function. Rembering that α equals $(U-1)\beta/U$ by definition, this shift leads to a change in both the intercept and slope of the α function. Specifically, letting δ be a shift parameter in the β function, we have $\alpha_\delta=(U-1)\beta_\delta/U$ and $\alpha_{U\delta}=\beta_\delta/U^2$. Assuming that a higher δ implies grater aversion towards work during abnormal hours, β_δ, α_δ and $\alpha_{U\delta}$ will all be positive.

Shifts in the α function imply that equations (10)′, (14)′ and (16) must now be modified as follows:

$$\hat{U}=[1/(1+R)][(\hat{r}_1-\hat{W})+(\hat{K}_1^*-\hat{L}_1)-(\alpha_{U\delta}/\alpha_U)d\delta] \tag{10″}$$

$$\theta_{1L}\hat{W}+\theta_{1L}\alpha_\delta/(1+\alpha)]d\delta+\theta_{1K}\hat{r}_1=\hat{p}_1 \tag{14″}$$

$$\sigma_1=(\hat{U}+\hat{K}_1^*-\hat{L}_1)/\{(\hat{U}/\theta_{1L})+(\hat{W}-\hat{r}_1)+[\alpha_\delta/(1+\alpha)]d\delta\}. \tag{16′}$$

Equations (10)″ and (16)′ lead to a modified version of (18),

$$\hat{K}_1^*-\hat{L}_1=\tilde{\sigma}_1(\hat{W}-\hat{r}_1)+(A/T_2)d\delta \tag{18′}$$

where $A=(\theta_{1L}-\sigma_1)(\alpha_{U\delta}/\alpha_U)+\sigma_1\theta_{1L}(1+R)[\alpha_\delta/(1+\alpha)]$.

Use of (18)′, (3)′, (16′) and (17) with the assumption of fixed factor endowments leads to

$$\hat{W}=(\lambda_{1L}\tilde{\sigma}_1/S)\hat{r}_1+(\lambda_{2L}\sigma_2/S)\hat{r}_2-(\lambda_{1L}A/S)d\delta \tag{B21}$$

where S is defined in Section 1 of this Appendix.

Since our focus is on the output effects, we merely note that (B21), (14)″ and (15)′ can be used, given commodity prices, to obtain the reduced forms for the changes in rental rates. In turn, these results and (B21) can be used to obtain the reduced form for the changes in utilization and in the capital-stock-to-labour-services ratio, i.e.

$$\begin{aligned}\hat{U}=&-(\theta_{1L}/\Delta)(1/T_2)\{[\lambda_{1L}(\sigma_1/\theta_{1K})+\lambda_{2L}(\sigma_2/\theta_{2K})][(\alpha_{U\delta}/\alpha_U)\\&-\alpha_\delta/(1+\alpha)]+(1/\theta_{1K})(1-\sigma_1)\lambda_{2L}(\sigma_2/\theta_{2K})[\alpha_\delta/(1+\alpha)]\}d\delta\end{aligned} \tag{B22}$$

$$\begin{aligned}\hat{K}_1^*-\hat{L}_1=&(1/\Delta)(1/T_2)\lambda_{2L}(\sigma_2/\theta_{2K})\{[T_2(\tilde{\sigma}_1/\theta_{1K})\alpha_\delta/(1+\alpha)]\\&+(\theta_{1L}-\sigma_1)[(\alpha_{U\delta}/\alpha_U)-\alpha_\delta/(1+\alpha)]\}d\delta.\end{aligned} \tag{B23}$$

Finally, (1)′, (2)′, (3)′, (B22) and (B23) can be used, given factor endowments, to obtain the output effects presented below:

$$\hat{X}_1 = -(\theta_{1L}/\Delta)(1/T_2)\{[\lambda_{1L}(\sigma_1/\theta_{1K})+\lambda_{2L}(\sigma_2/\theta_{2K})]\theta_{1K}[(\alpha_{U\delta}/\alpha_U) - \alpha_\delta/(1+\alpha)]+[\lambda_{2L}(\sigma_2/\theta_{2K})(1-\sigma_1)\alpha_\delta/(1+\alpha)] + \lambda_{2L}(\sigma_2/\theta_{2K})[\{T_2(\tilde{\sigma}_1/\theta_{1K})\alpha_\delta/(1+\alpha)\} + (\theta_{1L}-\sigma_1)[(\alpha_{U\delta}/\alpha_U)-\alpha_\delta/(1+\alpha)]]\}d\delta. \tag{B24}$$

$$\hat{X}_2 = (\theta_{2L}/\Delta)(1/T_2)\lambda_{1L}(\sigma_2/\theta_{2K})\{[T_2(\tilde{\sigma}_1/\theta_{1K})\alpha_\delta/(1+\alpha)] + (\theta_{1L}-\sigma_1)[(\alpha_{U\delta}/\alpha_U)-\alpha_\delta/(1+\alpha)]\}d\delta. \tag{B25}$$

As indicated in the text, these expressions simplify considerably in three specific cases.

I. $\sigma_2 = 0$,

$$\hat{X}_1 - \hat{X}_2 = -(\theta_{1L}/\Delta)(1/T_2)\lambda_{1L}\sigma_1[(\alpha_{U\delta}/\alpha_U)-\alpha_\delta/(1+\alpha)]d\delta. \tag{B26}$$

II. $\alpha_{U\delta}/\alpha_U = \alpha_\delta/(1+\alpha)$,

$$\hat{X}_1 - \hat{X}_2 = -(1/\Delta)(1/T_2)(\sigma_2/\theta_{2K})\{\theta_{1L}\lambda_{2L}[T_2(\tilde{\sigma}_1/\theta_{1K})+(1-\sigma_1)] + T_2\theta_{2L}\lambda_{1L}(\tilde{\sigma}_1/\theta_{1K})\}[\alpha_\delta/(1+\alpha)]d\delta. \tag{B27}$$

Note that, as shown in the case of (B6), the term in the first set of square brackets of (B27) is necessarily positive.

III. $\sigma_1 = 0$,

$$\hat{X}_1 - \hat{X}_2 = -(1/\Delta)(\theta_{1L}/\theta_{1K})(\sigma_2/\theta_{2K})(1/T_2)\{\theta_{1L}[\lambda_{2L}(1+\theta_{1K}) + \lambda_{1L}\theta_{2L}][\alpha_\delta/(1+\alpha)]+\theta_{1K}[\lambda_{2L}+\lambda_{1L}\theta_{2L}](\alpha_{U\delta}/\alpha_U)\}d\delta. \tag{B28}$$

This expression is unambiguously negative.

First version received February 1983; *final version accepted September* 1984 (*Eds.*).

We would like to thank a referee and an Editor of the Review for their very useful comments, which among other things led to the development of the material in Section 7 of the paper. Errors and omissions, of course, remain our responsibility.

NOTES

1. Early contributions to the literature were made by Marris (1964) and N. Georgescu-Roegen (1970). During the 1970's, a major survey was published, Winston (1974). More recently, a book-length treatment of the topic has become available, Betancourt and Clague (1981), as well as an overview which stresses the relationship of this topic to other aspects of firm behaviour, Oi (1981). Moreover, in the last ten years substantial data gathering efforts focusing on this subject have been undertaken by international organizations, e.g. the ILO, N. Phan-Thuy *et al* (1981), and the World Bank, Bautista, *et al.* (1981), as well as national governments, e.g. the surveys of plant capacity initiated by the U.S. Department of Commerce in 1973.

2. While it is easier to think in terms of a day as the unit of time and to view an eight-hour day shift, for example, as normal hours of operation, the model can be applied equally well to the week by viewing, for example, five eight-hour day shifts as normal hours of operation and all other shifts, including those that take place on the weekends, as abnormal hours.

3. The derivation of (11) is straightforward but very lengthy. Thus, it is presented in an Appendix available upon request.

4. In order to obtain (14)′ use must be made of the first order conditions for cost minimization which imply that $(U\alpha_u/(1+\alpha)]\theta_{1L} = \theta_{1K}$.

5. It is also worth noting that these substantive results are the same as those of the two-factor two-sector model with variable utilization.

6. Changes in utilization are only the proximate cause of the perverse output effects in this model. The basic reason for these effects is the relation between factor returns and commodity prices in the two-factor

model. For, a necessary condition for the existence of perverse output effects is that a rise in p_1 increases W/r_1, which in turn affects utilization. In the specific-factors model, on the other hand, an increase in p_1 always lowers W/r_1, as can be seen from (19), thereby preventing the possibility of a perverse output effect arising from the changes in utilization.

7. Since this result may be viewed as surprising, at least initially, it should be stressed that it is consistent with the stability condition of the model as analyzed in Appendix A, i.e. $T_1 \geqq 0$. Furthermore, it is also consistent with the additional stability condition that emerges in a variable utilization model when capital is allowed to move across sectors. In that situation stability requires $T_1 \geqq 0$ and $\theta\lambda > 0$, where $\lambda = \lambda_{1K} - \lambda_{1L}$, BCP (1983). This additional stability condition, which is also the one applicable in the standard two-factor model with fixed utilization in both sectors (Neary (1978*b*)), is fully consistent with our result.

8. Incidentally, the same possible divergence between the general equilibrium result and the partial equilibrium result is found in the two-factor model with variable utilization, BCP (1983).

REFERENCES

BAUTISTA, R. *et al.* (1981), *Capital Utilization in Manufacturing: Colombia Israel, Malaysia and the Philippines* (New York: Oxford University Press).

BETANCOURT, R. and CLAGUE, C. (1981), *Capital Utilization: A Theoretical and Empirical Analysis* (New York: Cambridge University Press).

BETANCOURT, R., CLAGUE, C. and PANAGARIYA, A. (1982), "Capital Utilization, Factor Returns and Comparative Advantage" (Unpublished paper, University of Maryland, College Park).

BETANCOURT, R., CLAGUE, C. and PANAGARIYA, A. (1983), "Capital Utilization in General Equilibrium" (Unpublished paper, University of Maryland, College Park).

BETANCOURT, R., CLAGUE, C. and PANAGARIYA, A. (1984), "Trade and Factor Prices in a Model of Capital Utilization", *Southern Economic Journal*, **50**, 734-742.

BRITO, D. and WILLIAMSON, J. (1973), "Skilled labor and Nineteenth Century Anglo-American Managerial Behavior", *Explorations in Economic History*, **10**, 235-52.

FOSS, M. (1981), "Long-Run Changes in the Workweek of Fixed Capital", *American Economic Review Paper and Proceedings*, **71**, 51-63.

GEORGESCU-ROEGEN, N. (1970), "The Economics of Production", *American Economic Review*, **60**, 1-9.

GROSSMAN, G. (1983), "Partially Mobile Capital: A General Approach to Two-Sector Trade Theory", *Journal of International Economics*, **15**, 1-17.

JONES, R. (1971), "A Three Factor Model in Theory, Trade and History," in Bhagwati, J. *et al.* (eds.) *Trade, Balance of Payments and Growth* (Amsterdam: North-Holland Publishing Company) 3-21.

JONES, R. and EASTON, S. (1983), "Factor Intensities and Factor Substitution in General Equilibrium", *Journal of International Economics*, **15**, 65-99.

MARRIS, R. (1964), *The Economics of Capital Utilization* (Cambridge: Cambridge University Press).

MAYER, W. (1974), "Variable Returns to Scale in General Equilibrium Theory: A Comment", *International Economic Review*, **15**, 225-235.

NEARY, J. (1978*a*), "Short-Run Capital Specificity and the Pure Theory of International Trade", *Economic Journal*, **88**, 488-510.

NEARY, J. (1978*b*), "Dynamic Stability and the Theory of Factor Market Distortions", *American Economic Review*, **68**, 671-682.

OI W. (1981), "Slack Capacity: Productive or Wasteful", *American Economic Review Papers and Proceedings*, **71**, 64-69.

PHAN-THUY, N. *et al.* (1981), *Industrial Capacity and Employment Promotion.* (Westmead: Gower Publishing Company Limited).

QUIRK, J. and SAPOSNIK, R. S. (1968), *Introduction to General Equilibrium Theory and Welfare Economics* (New York: McGraw-Hill Book Co).

SVENSSON, L. (1982), "On Variable Capital Utilization and International Trade Theory", *Economic Letters* (forhcoming).

TEMIN, P. (1966), "Labour Scarcity and the Problem of American Industrial Efficiency in the 1850's", *Journal of Economic History*, **26**, 277-298.

U.S. DEPARTMENT OF COMMERCE (Various years), "Surveys of Plant Capacity", *Current Industrial Reports*, MQ-C1, Department of Commerce, Bureau of the Census.

WINSTON, G. (1974), "The Theory of Capital Utilization and Idleness", *Journal of Economic Literature*, **12**, 1301-1320.

WINSTON, G. and MCCOY, T. (1974), "Investment and the Optimal Idleness of Capital", *Review of Economic Studies*, **41**, 419-428.