A Mega Millions Anomaly

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Abstract

The interstate lottery game Mega Millions introduced a new product in October 2017 called Just the Jackpot. Sales of this product have been anemic. The Standard option accounts for over 90% of sales even though it is never the expected value maximizer for consumers among ticket options at any jackpot level. Several popular decision theoretic models predict Just the Jackpot should have strong appeal, while interest in the Standard option should be low. I show that consumers' choice of product is not due to inattentiveness, liquidity constraints or lags in the adjustment of consumption to new product introduction. I argue that the data trends are due to differences in ex post outcome feedback on foregone choices depending on which option is selected, as well as minimal winner regret, something not accounted for in most models. I propose a Feedback Weighted Regret Minimax model that incorporates a feedback parameter as well as a novel winner-loser regret feature that captures the data trends significantly better. It is puzzling that lottery managers chose to introduce Just the Jackpot, as existing decision models predict negligible increases in Mega Millions participation on the extensive margin. I show that inducing players to switch from another Mega Millions option to Just the Jackpot maximizes neither lottery revenue nor lottery profits. Finally, I argue that the seemingly irrational inverse relationship between jackpot size and the Just the Jackpot sales percentage can be explained by changes in player demographics, as a larger share of players at bigger jackpots are likely unaware of the existence of the Just the Jackpot option.

1 Introduction

This paper provides novel empirical evidence of the inability of the most popular models of decision making under risk to capture actual lottery player behavior. An analysis of the ticket options and sales for Mega Millions, the second-largest lottery game in the United States, shows that the actual extant choice pattern in the data is not generally rationalizable by any of these models, using commonly utilized functional forms employed in the literature over a wide range of parameter values. This is particularly intriguing, as the annual sales of Mega Millions tickets is in the range of a few billion dollars and therefore captures the choice behavior of millions of people, and there are only three Mega Million ticket options available. The data patterns are also not due to player unawareness of some of the available ticket options, nor a lag in lottery consumption adjustment, nor liquidity constraints binding on the more expensive ticket options. The paper argues that this apparent anomaly is due to the insufficiency of the (state space) payoff distribution for modeling the decision making process for this game. The paper proposes that the major driver of the choice behavior is a difference in the generation of ex post feedback on foregone choices, depending on which ticket option is purchased. Another factor likely impacting choice behavior is minimal winner regret. A model is proposed accounting for both of these phenomena, and this model is able to more closely align with the revealed preferences in the sales data.

The central motivation for the paper is the introduction of a third Mega Millions ticket option in October 2017, called Just the Jackpot. Just the Jackpot gives the consumer the best value of any ticket option for winning the jackpot. Sales of this product started quite low at about 2% of Mega Millions sales, and then proceeded to fall below 0.5% by the end of 2017 and remain below that percentage through the writing of this paper. The sales data is anomalous for a few reasons. First, lottery sales data from many games over decades demonstrates that jackpot size is the primary driver of sales for jackpot games, with revenues and profits often growing exponentially as jackpots get excessively large. Given that Just the Jackpot gives the best value of any ticket option at winning the jackpot, it is surprising its sales are so low, especially at high jackpot levels. Second, over 90% of the sales are of the Standard option, even though the Standard option is not the expected value maximizer at any jackpot level. Third, evaluation of the choice framework through a decision theory lens suggests that Just the Jackpot should be a highly desirable product, and the Standard option the least desirable option, across wide parameter ranges for commonly used parameterizations over a variety of models and jackpot levels. Essentially, mainstream decision theory would likely have endorsed the creation of this product from a consumer welfare perspective, yet it has been nothing short of a disaster. To its credit, mainstream decision theory models suggests that introducing Just the Jackpot does not bring in many new players, and that the demand for Just the Jackpot would be almost entirely due to existing players opting for a more preferred ticket type. Interestingly, this paper shows that such behavior is not revenue or profit maximizing from Mega Million's perspective, and therefore questions the decision to create such a product in the first place. This study shows that there is indeed still more to be learned about the factors individuals take into consideration when making choices over risk, and provides some policy implications relating to lottery product design.

Rationalizing lottery consumption within a unified model consistent with other common behavior over risk dates back to the birth of decision theory. Attempting to incorporate participation in unfair lotteries within an Expected Utility (EU) framework proved to be a tricky proposition. An initial attempt at reconciling simultaneous (risk loving) lottery participation and (risk averse) insurance purchasing within an EU framework involved a utility function with multiple inflection points (Friedman and Savage, 1948). While this approach technically could explain the lottery-insurance paradox at some wealth levels, it opened up bigger cans of worms, including the predicted depopulation of the middle class due to risk loving behavior (for a comprehensive critique of the Friedman-Savage approach, see Hirshleifer, 1966). An alternative approach was to remove lottery and gambling participation from evaluation by the utility of wealth function, by classifying such activities as *pleasure-oriented* gambling, discernible from *wealth-oriented* gambling by its repetitive, small stake nature (Hirshleifer, 1966). *Pleasure-oriented* gambling qualifies as a consumption good and therefore not subject to consideration under EU, whereas *wealth-oriented* gambling would be. There are various rationales consistent with the *pleasure-oriented* classification, including: the (short-lived) right to dream or fantasize about potential winnings (Clotfelter and Cook, 1990); contributing to socially-desirable causes that are funded by proceeds from such activities (Clotfelter and Cook, 1990); an escape from the routine, mundane and predictable nature of modern industrial life (Bloch, 1951); a mechanism for releasing tensions and registering non-disruptive protests against an inequitable capitalistic system (Devereux, 1949; Frey, 1984); a way to establish social cohesion and maintain friendships (Guillen et al., 2012). In spite of these reasons, it is unlikely that individuals partake in games of chance solely or even mostly for non-pecuniary purposes. About half of California lottery players polled stated that they played the lottery for the money more so than the fun, the share of which moved inversely with income (Los Angeles Times, 1986). More recent evidence further validates this, as a poll of over 1,000 US adults estimated that '21% of Americans, and 38% of those with incomes below $25,000, think that winning the lottery represents the most practical way for them to accumulate several hundred thousand dollars' (Consumer Federation of America, 2006). There is even evidence that lottery players place value on the means of lottery risk resolution, for instance, with preferences over self-selection of winning numbers for lottery entries (see Simon 2008 for a comprehensive discussion of number selection behavior in lotteries).

As the evidence against EU as a sufficient framework to model decision behavior over risk began to pile up, a number of models emerged as modifications or wholesale alternatives to EU. These models sought to maintain much of the normative appeal of EU while allowing behavior that was becoming increasingly viewed as common and even rational. Some of the more widely utilized and referenced models include: prospect theory (Kahneman and Tversky, 1979); anticipated utility (Quiggin, 1982); cumulative prospect theory (Tversky and Kahneman, 1992); disappointment aversion (Gul, 1991); regret theory (Loomes and Sugden, 1982; Bell 1982); salience theory (Bordalo et al. 2012). These models are able to handle the lottery-insurance paradox fully within their frameworks, in spite of differing axiomatic foundations and psychological motivations. These models maintain the sufficiency of the (state space) payoff distribution for decision making, with no need to attribute lottery or gambling play to consumption in any way, and without explicitly modeling any of the non-pecuniary motives mentioned above. However, these models are unable to explain the Mega Millions choice patterns present in the sales data, as this paper demonstrates.

The rest of the paper proceeds as follows. Section 2 introduces Mega Millions and the sales data. Section 3 reviews the decision theoretic models and their predictions regarding the Mega Millions ticketing options. Section 4 addresses non-decision theoretic explanations of the lack of interest in Just the Jackpot, namely player unawareness of its existence, slow adjustment of sales, and liquidity constraints due to its relative costliness. Section 5 proposes behavioral mechanisms that could be impacting Mega Millions choice behavior. Section 6 incorporates these mechanisms into a model that is able to explain Mega Millions choice behavior. Section 7 analyzes a few interesting counterfactuals with implications on lottery design. Section 8 concludes.

2 Mega Millions Data

Mega Millions is one of the two major interstate lottery games in the United States, the other being Powerball. To put the size of the US lottery industry into perspective, aggregate lottery revenue across states and games in fiscal year 2018 was $85.6 billion (NASPL, 2020). This is larger than the combined gambling revenues of $41.7 billion of commercial casinos (AGA, 2019) and $33.7 billion of Indian tribal gaming (NIGC, 2019), a total of $75.4 billion. Aggregate lottery spending in the United States outpaces that of sports tickets, books, video games, movie box office tickets, and music - combined (CNNMoney, 2015). Powerball and Mega Millions are the two largest selling lottery games in the United States. In fiscal year 2018, Powerball recorded $5.2 billion in sales, while Mega Millions recorded $3.2 billion, combining for about 10% of aggregate lottery sales in the United States.

A Mega Millions entry requires the player to select five numbers between 1 and 70 without replacement and a sixth ball from 1 to 25. Players may choose their own numbers or opt for a random assignment of numbers. Drawings for each game are held twice a week, during which a large drum spits out five numbers from the larger range and a second drum spits out the sixth number from the smaller range. If a player's ticket has partial number matches with the numbers drawn, that ticket is eligible for certain prize amounts based on the number of balls matched. If a ticket matches all six balls (order independent), that ticket is eligible to collect the jackpot amount, albeit in installments over 30 years. Jackpot winners also have the option to cash out the jackpot in a single immediate payment, for an amount that is less than the advertised jackpot. If multiple tickets match all six balls within a drawing, the jackpot is split among the winners. All other lower prize amounts are given as advertised in a single payment, regardless of the number of winners at that prize tier.[1] If there is no ticket that matches all six numbers in a drawing, the jackpot rolls over into the next drawing and increases by an amount determined by the projected and actual sales for that drawing. The current iteration of Mega Millions features prizes ranging from $2 to the jackpot, which starts at $40 million. The odds of winning $2 are 1 in 37, the odds of winning the jackpot are 1 in 302,575,350, and the overall odds of winning any prize are 1 in 24 (for the full prize-odds matrix, see the Appendix).[2]

There are three ticket types offered by the Mega Millions lottery game. It will be convenient to categorize Mega Millions prizes in two groups: all the prizes besides the jackpot are grouped into the lower tier (L), and the jackpot (J) is its own category. This is a natural classification as only the jackpot can roll over and is the only parimutuel prize.. The first is the Standard ticket, which increased from a price of $1 to $2 during the latest Mega Millions prize-odds changes in October 2017. This ticket amounts to a single entry into both the lower tier and the jackpot. The second is the Megaplier, which increased from a price of $2 to $3 during the October 2017 changes. The Megaplier ticket yields a single entry into the lower tier and the jackpot, just like the Standard. However, prospective lower tier prize amounts are at least doubled with a Megaplier ticket. During the drawing, a Megaplier value is displayed by a random number generator, in addition to the winning numbers selected from the drums. This Megaplier multiplier takes a value of 2, 3, 4 or 5 with probabilities of $1/3$, $2/5$, $1/5$ and $1/15$, respectively, amounting to a multiplier expected value of 3. So, if a ticket qualifies for a lower tier prize amount and the purchaser paid the extra dollar for the Megaplier option, the holder would be entitled to that prize amount multiplied by the Megaplier value. If the Megaplier option was not added, the holder would be entitled only to the prize amount. The Megaplier impacts all prize amounts except the jackpot. The third option is the Just the Jackpot ticket, which is a new option introduced during the October 2017 changes. It has no similar counterpart in Powerball, unlike the Megaplier. Just the Jackpot costs $3 and entitles the purchaser to two entries into the jackpot drawing only, the two entries are not eligible for any lower tier prizes. Figure 1 displays the probability tree diagram for each of the ticket types.

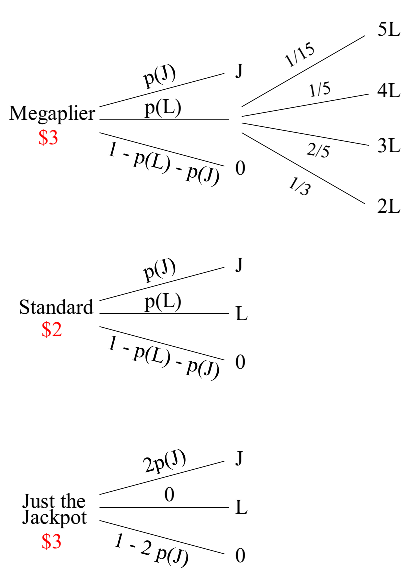
It is worthwhile to briefly consider the choice framework of the current version of the Mega Millions game. A Standard ticket costs $2, while the Megaplier and Just the Jackpot each cost $3. The expected value of the lower tier for a Standard ticket is $0.25, whereas it is $0.75 for the Megaplier lower tier due

[1]The exception is California, which determines payouts for non-jackpot prizes on a parimutuel basis.

[2]Mega Millions started as the Big Game in September 1996, selling tickets in six states: Georgia, Illinois, Maryland, Massachusetts, Michigan and Virginia. In May 2002, the Big Game turned into Mega Millions. In January 2010, a cross-selling agreement was made between the Mega Millions consortium and the Multi State Lottery Association (MUSL, which runs Powerball). Up to that point, a state could not sell both Mega Millions and Powerball products. This agreement allowed states to offer both. Over the years more and more states began offering Mega Millions. Currently, Mega Millions is available for purchase in 45 states, Washington DC and the US Virgin Islands. The most recent addition to the Mega Millions family was the state of Mississippi, which began selling Mega Millions tickets in January 2020 (for even more Mega Millions history see Mega Millions, 2020).

to the expected Megaplier multiplier of 3. The expected value of the minimum jackpot of $40 million, assuming no prize sharing, is $0.13, and a jackpot of $75 million yields about the same expected value of $0.25 as the lower tier prizes for a Standard ticket. From an expected valued maximization perspective of a single ticket with a single jackpot winner, the Standard ticket is best for jackpots below $378 million, and Just the Jackpot for jackpots in excess of that, with Megaplier never being the best option under any feasible jackpot. However, modeling the decision behavior based on the actual ticket costs introduces a cost effect between the Standard ticket and the other two options. For example, consider a half-Standard ticket, which is a Standard ticket with all of the win probabilities cut in half, and let the cost of this be $1. If this payoff distribution is valued much more by a player than a dollar, then the decision problem pitting the three ticket options with their costs against each other would be under-valuing the Standard ticket option, and vice versa. Modifying the decision problem by adding this half-Standard ticket to the Standard ticket would yield one and a half Standard tickets at a cost of $3. Comparing this with the other two options more appropriately captures the decision framework. Players are not limited to a single ticket purchase and can buy as many tickets as they want, and this cost neutral transformation better captures this reality. The choice problem is not so much which ticket option would a player prefer, but rather which payoff distribution a player would prefer at a given cost. The cost neutral approach will be the baseline approach for the model evaluations. Now, ranking tickets by expected value under the assumption of a single jackpot winner yields the Megaplier with the maximum expected value for jackpots below $223,870,250 and Just the Jackpot for jackpots above that: the cost neutral Standard option never maximizes the expected value at any feasible jackpot level.

Figure 1: Mega Millions Ticket Types



In 2018, the Mega Millions jackpot reached an enormous $1.6 billion, making it the largest lottery payout in US history to a single individual (Mega Millions, 2019). Figure 2 presents the Mega Millions sales data for this historic round, spanning from July 27 to October 23. A Mega Millions jackpot draw is a specific Tuesday or Friday on which the winning numbers are drawn, whereas a round consists of the set of draws beginning with the minimum jackpot up until the next jackpot reset. A key takeaway is the exponential climb in draw sales as the jackpot gets abnormally high. The sales data for the whole two years, including the historic round, is in Table A2 in the Appendix. The Total column represents the total Mega Millions sales for that draw of the three ticket types in DC and the 14 states that offer Just the Jackpot: total nationwide sales are approximately three times those amounts. The Standard, Megaplier and Just the Jackpot columns represent the percentage of sales in dollars (not tickets) of each ticket type. The percentage reported of the number of tickets would further elevate the Standard percentages over the other two options, as it costs one dollar less. There are a few things to note right off the bat. First, sales do not monotonically increase with jackpot at lower jackpot levels. This is due to the weekend effect: there is a drawing on Tuesday night and one on Friday night, the sales for Friday drawings will typically outdo those for the following Tuesday, even with the increase in jackpot. Second, as the jackpot rolls over and sales respond to the increased expected jackpots, the difference in successive jackpots increases. In this case, the difference start at $5 million and maxes out at $600 million.

Figure 2: Mega Millions Sales for $1.6 Billion Jackpot

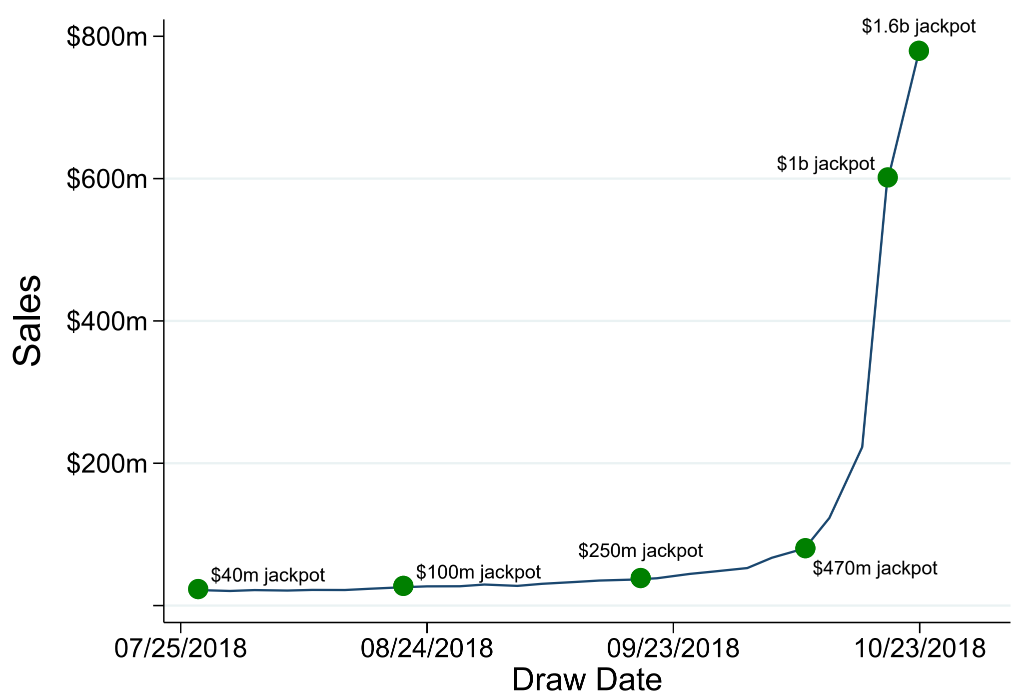
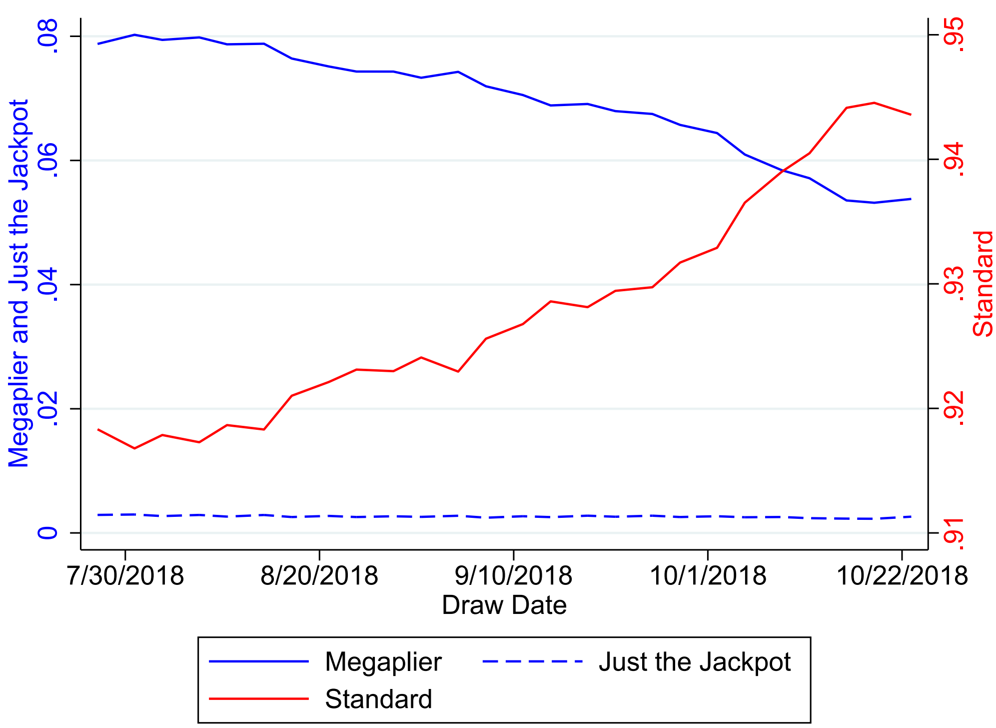


Figure 3 presents the sales percentages of each ticket type during the historic round, which are representative of the overall trends over the two years. The Standard ticket is selected in over 90% of transactions across all jackpot levels, the percentage increasing with jackpot size, in spite of it not being an expected value maximizer at any jackpot level under the cost neutral approach. The Megaplier seems to hold some niche appeal, with a range of 5% to 8% of sales, decreasing with jackpot size. This negative relationship between the Megaplier sales percentage and jackpot size is as expected: from an expected value viewpoint, Megaplier holds its biggest relative appeal at the minimum jackpot level. As the jackpot increases, the share of expected value going to the jackpot increases, implying a reduction in the relative appeal of Megaplier. The Just the Jackpot sales numbers are particularly intriguing. Once again, the sales and percentages reported are only for those states that offer Just the Jackpot. The percentage of sales never goes above a measly 0.3% for any of the draws. Even more bizarre is the flat or perhaps even slightly negative relationship between the Just the Jackpot sales percentage and the jackpot amount. A positive relationship is expected, since Just the Jackpot holds minimal relative appeal at the minimum jackpot amount, and increases in relative appeal with each successive rollover. The diminutive sales of Just the Jackpot tickets are perhaps surprising, given the clear impact jackpot size is having on aggregate sales in Figure 2, let alone all the evidence highlighting the impact of jackpot sizes on sales and profitability.[3] This product is nothing short of a flop, to the point where the state of Wisconsin discontinued Just the Jackpot on October 30, 2018, citing a "lack of interest" in the product, per an email exchange the author of this paper had with a Wisconsin lottery official. The next section introduces a number of the more popular decision models and demonstrates their collective inability to explain the sales data.

Figure 3: Mega Millions Sales Proportions for $1.6 Billion Jackpot



[3]The positive impact of jackpot size on lottery sales is well-established in the lottery literature (DeBoer, 1990; Cook and Clotfelter, 1993; Forrest, Simmons and Chesters, 2002). There is even evidence of the positive impact of jackpot size on lottery sales and a negative impact of expected value, but the high correlation between expected value and jackpot size may be affecting the estimates (Cook and Clotfelter, 1993). Lottery sales decreases in New York in the 1980s were attributed to increased participation resulting in fewer rollovers and smaller jackpots, with a recommendation to reduce the odds of winning to generate larger jackpots (DeBoer, 1990). Many of the innovations to jackpot lottery games over the years have come about with the intent of increasing jackpot sizes: increased ticket costs, larger minimum starting jackpots, larger target player pools, and decreased odds of winning the jackpot. The development of interstate lotteries was due to the belief that larger player pools with decreased jackpot odds would allow jackpots to grow in size, with a disproportional effect on revenue and profitability. Mega Millions ran with this notion by introducing its Just the Jackpot ticket option, made available in 14 states and Washington, DC since the most recent Mega Millions iteration in October 2017.

3 Decision Theory Model Predictions

This section evaluates the Mega Millions choice problem in a number of decision models. These models require one or more functional specifications to concretely predict choice behavior over risk. For example, Expected Utility requires the specification of a utility function over wealth. Additionally, estimation of these models generally proceed by assuming a certain family of functions. For instance, the power function x^α is often used as the functional family for utility over wealth in a number of models. The majority revealed preference in the data is for the Standard option across jackpot levels. A minority of players prefer the Megaplier, and Just the Jackpot participation is effectively negligible. For a model to be realistically considered to rationalize the data, it should be able to demonstrate the primary trends in the data using the common functional forms assumed in the literature across a sufficiently wide range of parameter values. Specifically, the parameter ranges should be consistent with those estimated in the experimental model estimation literature. Additionally, at least some of those parameter values should predict a preference for the Standard option over the range of jackpots observed in the data, modeling the behavior of a frequent Mega Millions player who buys at all jackpot levels. Perhaps there should be more parameter-jackpot combinations consistent with Standard preferences than either of the two other Mega Millions options. However, simply using the percentage of parameter combinations predicting each preference as an indicator of population preferences implicitly assumes uniformity of the population across parameter values. This assumption is extremely tenuous and is a weakness of the proposed approach. It turns out that none of the models tested are able to demonstrate even a single parameter combination that can predict Standard selection across the range of feasible Mega Millions jackpots, so that the distribution of the population over parameter values effectively becomes a moot point. Additionally, there should be a good number of parameter-jackpot combinations that predict not participating in Mega Millions at all, since most people do not play Mega Millions. Even restricting the population to existing lottery players suggests that participation predictions should not be too high, particularly at low jackpot levels, as Mega Millions accounted for less than 5% of US lottery sales in 2018. Such a restriction is reasonable, since some people may have moral or religious objections to lottery play and therefore the choice problem becomes trivial.

An important assumption is that the choice framework utilized in model evaluation is selecting one of the three ticket distributions, or a fourth outside option of not buying any and therefore having the cost of the ticket with probability 1. In reality, players may opt to play another lottery game or do anything else with those unspent funds, but the analysis forthcoming essentially subsumes this within the option of not playing and thereby having those funds. This assumption essentially just sets a threshold for Mega Millions play. For example, consider \$3 that can be spent on Mega Millions or in some other way. It is possible that there is an outside option with a cost of \$3 with a higher utility than simply having those \$3. This would just raise the required utility to play Mega Millions. So, in situations where modeling a utility of \$3 implies just barely choosing a Mega Millions option, raising that threshold could result in the model predicting no participation. It turns out that across models, situations in which Standard is preferred sit disproportionately close to the participation threshold relative to the two other ticket types, so that increasing the threshold would result in even worse predictions of Standard preferences. Also, model implications are determined purely using distributional and state space payoff information: non-monetary concerns like entertainment utility or preferences over how lotteries are resolved are excluded from the analysis. Even if such considerations exist, assuming these non-monetary considerations impact each of the Mega Millions options equally would not impact results.

3.1 Expected Utility

In order for Expected Utility to explain participation in unfair lotteries, convex utility is required, at least over certain ranges of wealth, as in Friedman and Savage (1948). Jackpot games like Mega Millions sometimes offer positive expected returns when jackpots get excessively large. Under the assumption of a single jackpot winner, the requisite break-even jackpot amounts are about \$530 million for the \$2 Standard purchase, \$685 million for the Megaplier purchase, and \$453 million for the Just the Jackpot purchase (two jackpot only entries). Expected value calculation and the decision modeling that follows use the advertised jackpot amount, not the cash value (one-time payout) of the jackpot. This latter amount becomes more relevant for the revenue estimations that happen in Section 7. The minimum starting jackpot is \$40 million, jackpot amounts above \$400 million are relatively uncommon. Factoring in the likelihood of multiple jackpot winners based on the number of tickets sold further increases these break-even thresholds. Therefore, the general case for Mega Millions participation would require a convex utility function to meet the individual rationality constraint.

One thing to consider before making functional and parametric assumptions is whether any of these ticket options dominate any other. No option FOSD any other: while the Megaplier FOSD the Standard option at the lower tiers, the higher probability of winning the jackpot in the cost neutral Standard option negates FOSD over the whole distribution; both the Standard and Megaplier FOSD over Just the Jackpot is nullified by the higher probability of winning the jackpot with a Just the Jackpot entry. There is some conditional dominance in the second order: both the Standard and Megaplier options SOSD Just the Jackpot at jackpot levels below approximately \$224 million, above which the added value of the better jackpot odds under Just the Jackpot negates SOSD; there is no cost neutral SOSD between the Standard and Megaplier ticket at any jackpot level. The SOSD results imply that at jackpot levels below \$224 million, which the majority of latent jackpots meet, a risk averse EU maximizer will not buy Just the Jackpot, irrespective of parametric considerations. While this result is consistent with the lack of take up of Just the Jackpot in the data, risk averse EU maximizers would not buy any of the options at jackpot levels below \$224 million, as the expected return for each is negative. So dominance results under EU cannot rationalize the data.

The next step is to consider functional forms of utility with the appropriate parametric value ranges. As Mega Millions tickets usually offer negative expected returns, convex utility is required for individual rationality to hold. As was mentioned earlier, Mega Millions can be simplified into a game with a probability of getting a lower tier prize (L) and a smaller probability of getting the jackpot (J), which rolls over into the next jackpot if no one wins. Without loss of generality, Mega Millions can be reduced to a lower tier and jackpot two branch lottery, and the expected utility of each ticket can be represented as:

1. Standard: $1.5p_L u(L) + 1.5p_J u(J)$
2. Megaplier:$p_L(\sum_{m=2}^{5} p_m u(m * L)) + p_J u(J)$
3. Just the Jackpot: $2p_J u(J)$

Here the u(0) is normalized to 0, and the subscript m refers to the four possible Megaplier values. Regarding the Megaplier, a convex u(.) and a mean Megaplier value of 3 imply that $(\sum_{m=2}^{5} p_m u(m*L)) > 3u(L)$. These conditions reduce to:

$$0.5p_J u(J) > 1.5p_L u(L) \tag{1}$$

$$0.5p_J u(J) < 1.5p_L u(L) \tag{2}$$

$$1.5p_L u(L) + 1.5p_J u(J) > u(3) \tag{3}$$

where (1) is the preference condition for Standard over Megaplier, (2) is the preference condition for Standard over Just the Jackpot, and (3) is the rationality constraint. It is immediately apparent that (1) and (2) cannot simultaneously hold, meaning that under the assumption of a convex utility function, the Standard option can never be the most preferred option under EU, irrespective of the rationality constraint or wealth level. Given the argued necessity of a convex utility function to capture lottery participation at most jackpot levels, the strong conclusion is that EU is unable to account for the preference patterns in the data.

3.2 Prospect Theory

The theory that has gained the most traction as an alternative to EU theory is Prospect Theory (Kahneman and Tversky, 1979), abbreviated to PT, and its refined version Cumulative Prospect Theory (Tversky and Kahneman, 1992), which will be addressed in the next section. Among its advantages is its innate ability to explain the insurance-lottery paradox via the introduction of a probability weighting function $w(p)$, which over-weights small probabilities and under-weights large probabilities. A preference for participation in an unfair lottery no longer necessitates a convex utility of wealth function, as sufficient probability over-weighting can overcome even heavily concave utility and lead to risk loving behavior. Under PT, the three cost neutral probability-weighted expected utility representations of the simplified Mega Millions lottery options are:

1. Standard: $w(1.5p_L)u(L) + w(1.5p_J)u(J)$
2. Megaplier: $w(p_L)(\sum_{m=2}^{5} w(p_m)u(m * L)) + w(p_J)u(J)$
3. Just the Jackpot: $w(2p_J)u(J)$

The difference in representation here in relation to EU is the incorporation of the possibly non-linear $w(p)$, allowing $\sum_n w(p_n) \neq 1$. A general feature of $w(.)$ is the over-weighting of small probabilities and the under-weighting of large ones. The probabilities of winning any Mega Millions prize are much smaller than the typical inflection point probabilities both theoretically postulated (Prelec, 1998) and experimentally estimated (Tversky and Kahneman, 1992; Camerer and Ho, 1994; Wu and Gonzalez, 1996). Therefore, subadditivity is expected to hold at the Mega Millions win probabilities under PT: $w(xp) < xw(p), x > 1$. The two preference conditions and rationality constraint for the Standard option to be selected are:

$$[w(1.5p_J) - w(p_J)]u(J) > [\delta w(p_L) - w(1.5p_L)]u(L) \tag{4}$$

$$w(1.5p_L)u(L) > [w(2p_J) - w(1.5p_J)]u(J) \tag{5}$$

$$w(1.5p_L)u(L) + w(1.5p_J)u(J) > \lambda u(3) \tag{6}$$

Simplifications beyond these conditions are not as readily made without further assumptions, unlike EU. Condition (4) is the preference condition for Standard over Megaplier, (5) is the preference condition for Standard over Just the Jackpot, and (6) is the rationality constraint, with $\lambda > 1$ capturing the loss aversion associated with paying \$3 to play the lottery. The presence of loss aversion in the Mega Millions PT representation depends on whether each of the three ticket evaluations include the payment of \$3, or if not playing is a separate fourth lottery that gives \$3. Loss aversion does not impact preference between Mega Millions tickets and only raises the threshold for Mega Millions participation. Loss aversion serves the same role as an outside option with a higher threshold. Therefore, loss aversion will not be included as a model parameter ($\lambda = 1$), or equivalently, not playing and keeping \$3 is an outside fourth option. Condition (4) takes advantage of the fact that $(\sum_{m=2}^{5} w(p_m)u(m * L)) = \delta u(L)$ for some real number $\delta > 1$. The value of δ is determined by both the shapes of $u(.)$ and $w(.)$. The Megaplier value probabilities are much closer to typical inflection points than the prize win probabilities, so that δ may be more heavily influenced by $u(.)$ and similar but larger in value to a δ for the same utility function under EU. This is because the Megaplier values of 4 and 5, which are above the mean of 3, have probabilities that fall in the generally understood over-weighting portion of $w(.)$, whereas the Megaplier probability values of 2 and 3 are close to the usual inflection point range. A consequence of the subadditivity is that while both the LHS of (4) and the RHS of (5) represent a change in probability of $0.5p_J$, the LHS of (4) is larger than the RHS of (5). This allows a wider range of permissible behavior relative to the EU constraints.

Given expected population variation of preferences, a parametric modeling of PT should demonstrate a preference for the Standard option over a substantial range of parameter values and jackpot levels. The most commonly used utility function in the parameter estimation literature of PT is the CRRA power function

$$u(x) = x^{\alpha}, x \geq 0, \alpha > 0 \tag{7}$$

(see Abdellaoui, 2000 for a list of studies using this formulation). The PT estimations will test two of the more referenced $w(.)$ in the literature: $p^{\gamma}/[p^{\gamma}+(1-p)^{\gamma}]^{\frac{1}{\gamma}}$ (Tversky and Kahneman, 1992) and the more theoretically-motivated $exp(-(-lnp)^{\gamma})$ (Prelec, 1998). Tversky and Kahneman (1992) generate experimental data and estimate median values of $\alpha = 0.88$, $\gamma = 0.61$ for gains. All lottery options are framed as gains, so the loss side estimates of parameter values can be ignored, as symmetry is not an implicit assumption of PT. These estimates are consistent with diminishing sensitivity and strong probability distortion. They can also serve as starting points for determining parameter ranges that support a rational preference for the Standard ticket over various jackpot amounts.

At these starting points, the Standard option is highly individually rational, with a net PT utility of about 50 at the minimum jackpot of \$40 million, increasing to a net PT utility of about 700 for a jackpot of \$1 billion. This jump in net PT utility with jackpot change highlights the fact that a minimum of 75% of the PT utility value of the Standard option at these median parameter values is due to the jackpot PT utility valuation, share increasing with jackpot size. While Standard is rational, it is not preferred at these parameter values: for jackpots up to about \$100 million, Megaplier is preferred, while Just the Jackpot is preferred for all larger jackpots. Model predictions were generated for jackpot levels of \$40 million, \$100 million, \$200 million, \$300 million, \$400 million, \$500 million, \$750 million and \$1 billion, with lower jackpot levels occurring in reality with more frequency than higher ones. Using the Tversky and Kahneman (1992) functional specifications, utility power function parameter α is allowed to run from 0.71 to 1.10. This range is chosen because it is the interquartile range of parameter values estimated for subjects modeled under PT in the experimental estimation of Abdellaoui, Bleichrodt and L'Haridon (2008). While an experimental setting may arguably not constitute a representative sample of Mega Millions players, it is also difficult to find a reason why lottery player preference estimates would necessarily fall outside of this range. Furthermore, results are robust to the extension of the range of permissible α to $[0.5, 1.5]$, not just here but in all the popular models tested in this paper. Probability distortion parameter γ from both the Tversky and Kahneman (1992) and Prelec (1998) specifications run from 0.50 to 0.99. A γ of 1 reduces to EU, a γ above 1 is distortion in the opposite direction than the literature finds evidence for.

Model predictions are presented in Table 1, including for EU for which no parameter-jackpot combination predicts a Standard preference. The total number of parameter-jackpot combinations in each PT specification is 16,000: 40 utility parameter values, 50 probability distortion parameter values, and 8 jackpots. The results reported in Table 1 are of the specification most favorable to a Standard preference, which happens to be the Prelec (1998) weighting function specification. It is generally more likely to predict lottery play given its relative steepness at low probability levels in comparison to the Tversky and Kahneman (1992) specification: 91.4% of the 16,000 parameter-jackpot tests predict playing some version of Mega Millions, much more than the 77.2% in the Tversky and Kahneman (1992) specification. Of these 16,000 combinations, only 104 predict a preference for the Standard option, about 0.7%. The number of jackpot tests for which Standard is preferred and rational is at most 2 of a possible 8 for any parameter pair, as indicated by the Max Jackpots Standard column in Table 1. No parameter pair predicts Standard preference at the minimum $40 million jackpot, and most of the predictions are for the rarer high jackpot amounts. Figure 4 gives a mapping of preferences over parameter values by jackpot amount for the Prelec PT specification. Overall, PT does very poorly in explaining the strong preference for Standard in the data. In fact, it actually predicts a strong preference for Just the Jackpot, contrary to the choice behavior in the data.

Table 1: Summary of Cost Neutral Model Predictions

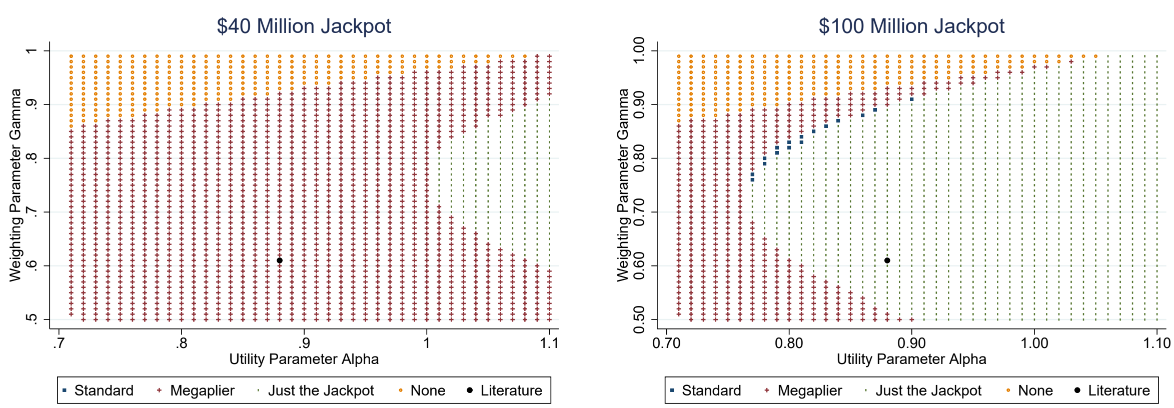
	Percent of Parameter-Jackpot Combinations				
Model	Standard	Megaplier	Just the Jackpot	None	Max Jackpots Standard
Expected Utility	0%	0%	20.3%	79.7%	0
Prospect Theory	0.7%	12.5%	78.2%	8.6%	2
Rank Dependent Utility	2.6%	8.3%	65.4%	23.7%	3
Disappointment Aversion	0%	1.1%	13.2%	85.7%	0
Regret Theory	0.1%	0.7%	75.1%	24.1%	1
Salience Theory	0%	8.8%	79.4%	11.8%	0

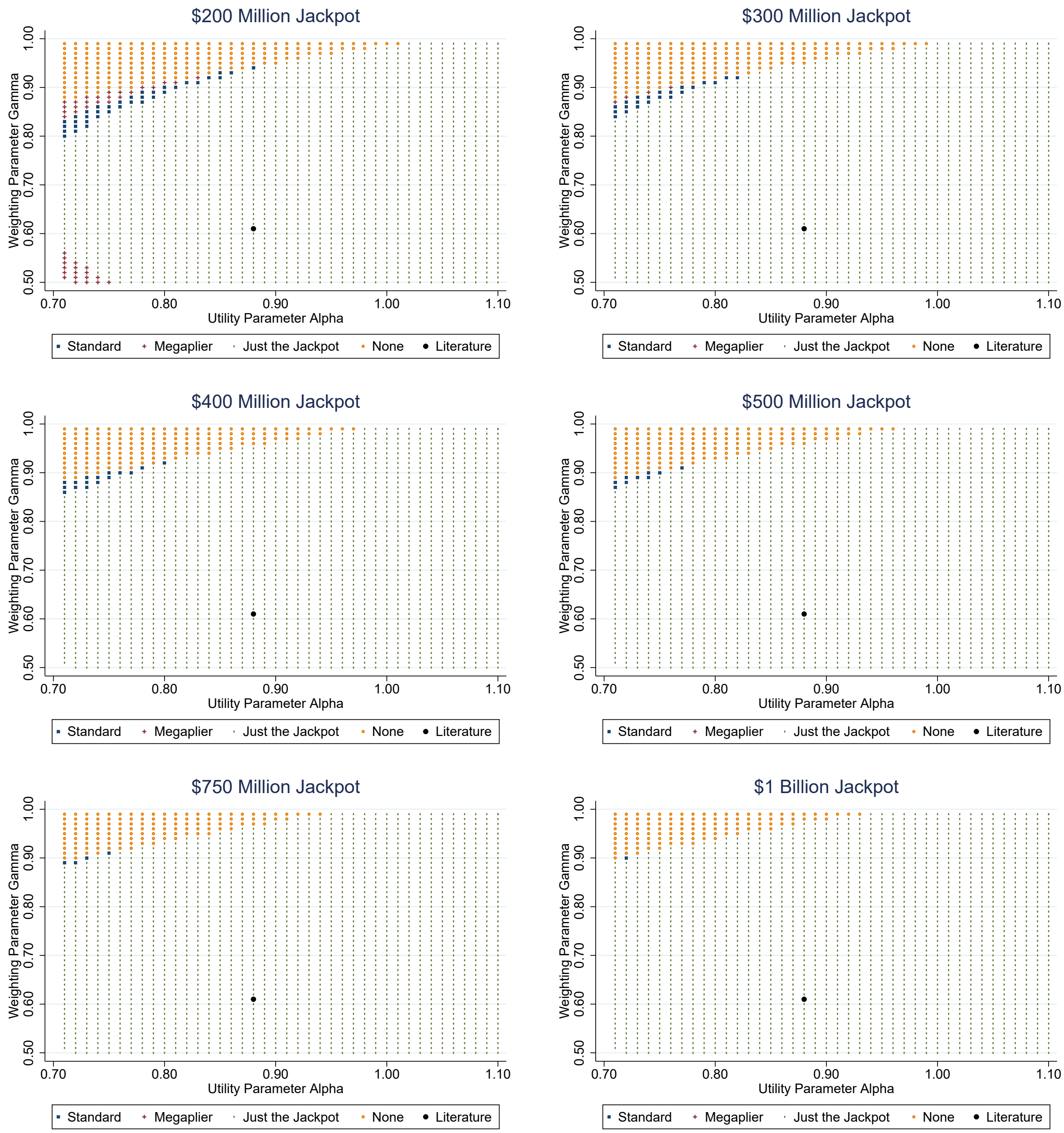
Table 1 shows the percentages of tested parameter-jackpot combinations that predict preferences for each of the four options, along with the maximum number of jackpots that any one parameter set predicts a Standard preference for. Prospect Theory uses the Prelec (1998) weighting function; Rank Dependent Utility is Cumulative Prospect Theory using the Tversky and Kahneman (1992) weighting function; Disappointment Aversion with a certainty equivalent less than 4 specification; Regret Theory under pairwise aggregation and full overlap scenario; Salience Theory under a no overlap scenario with $\theta = 1,000,000$.

A major driver in the preference relationship between Mega Millions options under PT is subadditivity. The cost neutral comparison boils down to getting bigger lower tier prizes and the jackpot with probability p with the Megaplier, a shot at both the lower tier and jackpot with a probability of $1.5p$

with the Standard, and two shots at only the jackpot with Just the Jackpot. The effective non-linearity between these probabilities strongly impacts preference under PT. However, the intuitive motivation of subadditivity in the case of Mega Millions participation is not so clear cut. Subadditivity implies that if buying multiple Mega Millions tickets in sequence, the value of the first ticket is larger than that of the second, the value of the second larger than that of the third, etc. This results in every player only buying a finite number of Mega Millions entries, because at some point the subadditivity will drive ticket valuations below the cost of participation under the assumption of approximate linearity of $u(.)$ over small wealth intervals (Rabin, 2000). At some point, all existing players will drop out, and the only way to maintain participation is to bring in new players, which does not seem sustainable. It is also not consistent with the existence of regular players who play such games on a frequent basis for extended periods of time. There needs to be some "resetting" of subadditivity at some point. Is it with every transaction? Every change in jackpot? Resetting after every draw seems more reasonable, as technically the lottery changes after every draw and the previously purchased tickets are not valid for future draws. However, that would allow a player buying multiple tickets over consecutive drawings to value an earlier ticket at a lower jackpot over a later ticket at a higher jackpot, violating FOSD, a violation which PT does allow, but does not seem reasonable in this case. A Mega Millions scenario perhaps most consistent with PT and subadditivity is a regular player who purchases a single ticket for every draw. Subadditivity coupled with the significant value added of a possibility of winning prizes, however small, ensure that only a single entry is purchased per draw. Subadditivity resetting with every draw allows this pattern to continue indefinitely. The Standard ticket is the cheapest option that gives entry into the Mega Millions, and the added perks of the Megaplier and Just the Jackpot do not offset the additional dollar cost. While this story may have descriptive appeal, it should be able to be validated within a parametric modeling of PT. Parametric estimations of PT utility comparisons for single purchases using both the Tversky and Kahneman (1992) and Prelec (1998) weighting functions result respectively in a paltry 2 and 22 parameter pairs predicting Standard preference, even worse than the cost neutral framework. The single purchase framework is discussed more thoroughly in Section 5 below.

Figure 4: Mapping of Prelec PT Preferences





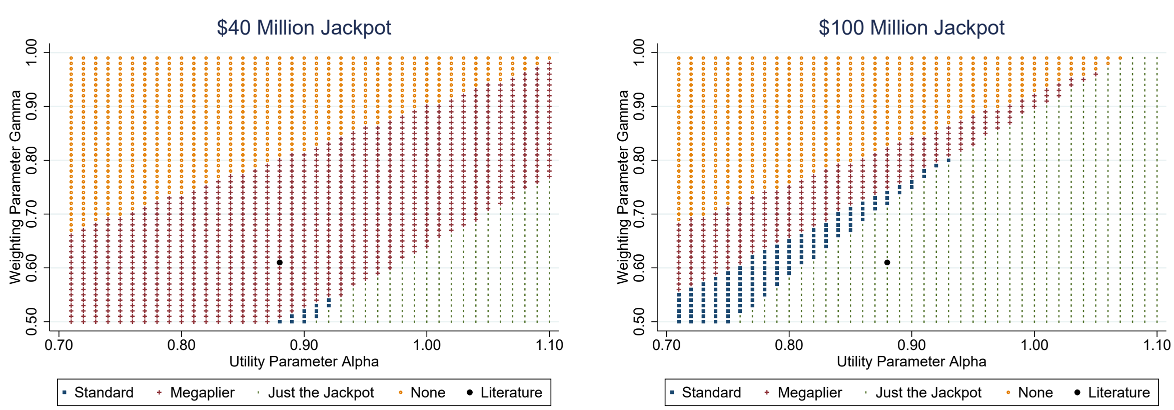
3.3 Rank Dependent Utility

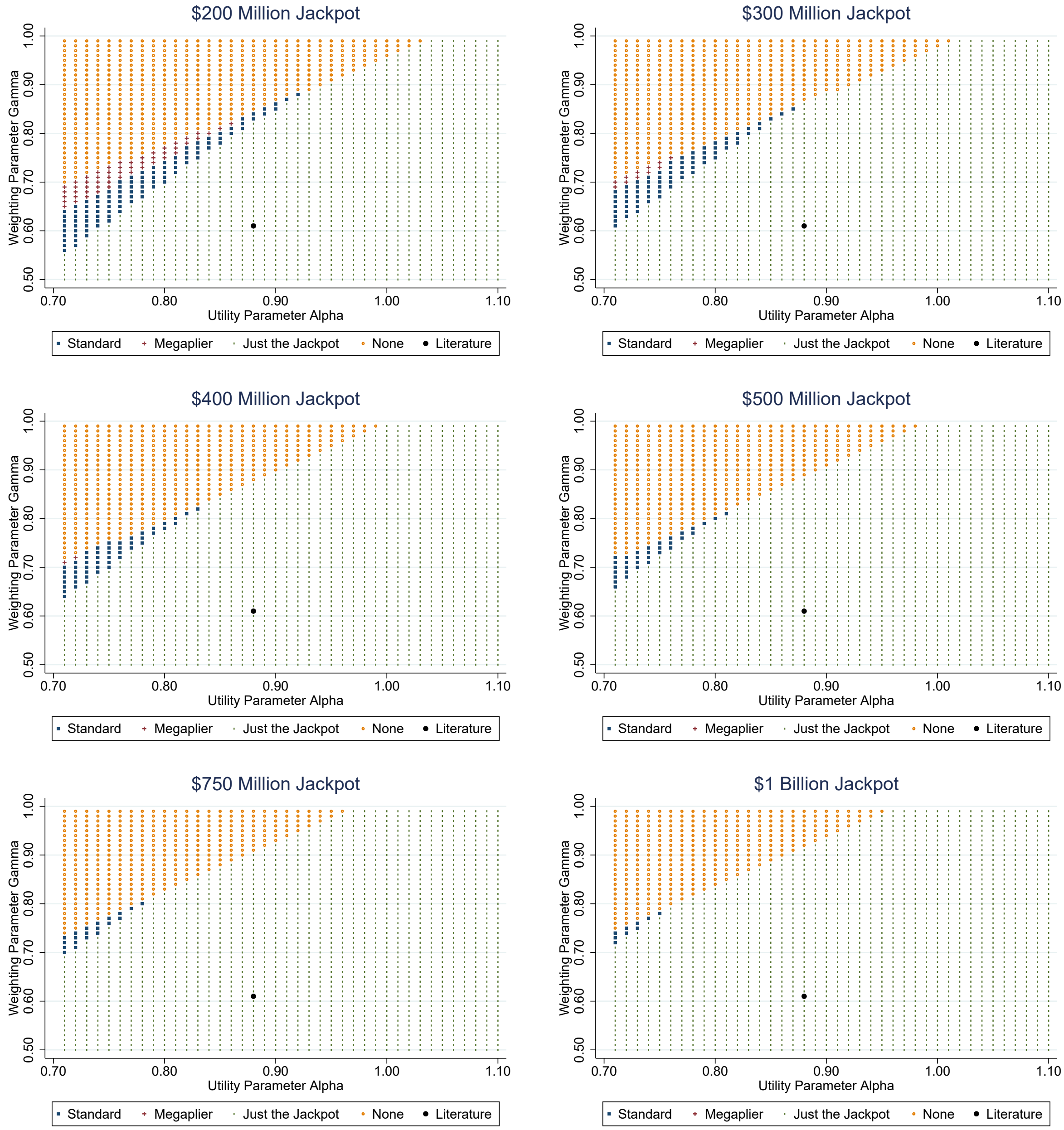
One criticism of PT is its allowance of FOSD violations. Quiggin (1982) introduced Anticipated Utility (AU) theory, which uses the cumulative probability distribution to determine the probability weights and eliminates FOSD violations. Outcomes are ordered by size, and weights are determined by the marginal impact of the probability. The implication is that $w(p)$ can change depending on where in the distribution it falls, with higher distortions occurring when it falls closer to 0 or 1. Using the marginal probability contribution ensures that $\sum w(p) = 1$. Kahneman and Tversky modified PT into Cumulative Prospect Theory (CPT, 1992) by incorporating Quiggin's idea of marginal probability contributions. Models that use the cumulative probability distributions to determine weights have been termed Rank Dependent Utility (RDU) models. While CPT incorporates more structure than

AU (loss aversion, separate weighting over gains and losses, reference point, etc.), in the case of Mega Millions, the only difference is how the weights are determined via the cumulative distribution. For example, consider a lottery with three outcomes $x_1 < x_2 < x_3$ with probabilities p_1, p_2 and p_3. Under AU, the weightsπ_1, π_2 and π_3 are $w(p_1), w(p_1 + p_2) - w(p_1), 1 - w(p_1 + p_2)$. Under CPT, the weights are $1 - w(p_3 + p_2), w(p_3 + p_2) - w(p_3), w(p_3)$. The preference and rationality constraints are the same as in (4), (5) and (6) from PT, except the weights are now the marginal weights derived from the cumulative probability distribution. Once again, the parametrization will employ the power function in (7) and the two weighting functions used in the PT modeling over the same ranges.

The two weighting functions and two rank dependent models yield four specifications. Once again, Table 1 only reports the specification most favorable to a Standard preference, which happens to be CPT using the Tversky and Kahneman (1992) weighting function. Under this specification, 12,215 parameter pairs are consistent with playing Mega Millions, of which 422 predict a preference for the Standard ticket, about 2.6%. The number of jackpot tests for which Standard is preferred and rational is 3 of a possible 8 for any parameter pair, one more than under PT modeling. Figure 5 gives a mapping of preferences over parameter values by jackpot amount for the Tversky Kahneman CPT specification. The preference for Standard is better distributed across jackpot levels here, although only 11 parameter pairs support a Standard preference at the most common jackpot amount of $40 million. The estimations were carried out applying the relevant probability distortion to the Megaplier probabilities. Forcing linear weighting over Megaplier probabilities does not substantially change the results of the predictions. The results of all four RDU specifications end up quite similar to those of PT: low preferences for Standard, high preferences for Just the Jackpot, and seemingly high rates of Mega Millions play. All of these run contrary to the choice behavior observed in the sales data.

Figure 5: Mapping of Tversky Kahneman CPT Preferences





3.4 Disappointment Aversion

Disappointment Aversion (DA) is an axiomatic model that swaps in an alternative to the independence axiom, with the intent of accommodating the Allais paradox while minimally straying from EU (Gul, 1991). Given a lottery p, there must exist a certainty equivalent for it. All outcomes larger than the certainty equivalent are considered elation outcomes, all outcomes below the certainty equivalent are considered disappointment outcomes (DA is a reference dependent model of sorts). Let a be the summed elation probabilities, q and r the normalized elation and disappointment sub-lotteries, so that $aq + (1-a)r = p$. The DA utility representation is

$$\left\{ [\gamma(a) \sum_{x \in q} u(x) * q(x)] + [(1-\gamma(a)) \sum_{x \in r} u(x) * r(x)] \quad \gamma(a) = \frac{a}{1+(1-a)\beta} \right. \tag{8}$$

with $\beta \in (-1, \infty)$. A negative β indicates an over-weighting of the elation outcomes and therefore elation loving, whereas a positive β indicates an over-weighting of the disappointment outcomes, hence disappointment averse, and $\beta = 0$ reduces to EU. The Allais paradox is consistent with disappointment averse preferences.

There has not been much work done on fitting individual decisions over risk using DA modeling. Abdellaoui and Bleichrodt (2007) propose an elicitation method for estimating DA parametrization using a power function as in (7) for utility. Estimation results from their experiment implementing the elicitation procedure are a median value for the power function parameter of $\alpha = 0.89$ for gains, and median DA parameter β ranging from about 0 to 3.5, depending on the probability of winning. They strongly reject the notion of a constant β for an individual, and that as the probability of winning increases, the higher β reflects the increased desire to avoid disappointment. When the probabilities of winning are low, the feeling of disappointment ex post will not be as intense compared to when the probability is higher, since the expectation is for a disappointing outcome (Abdellaoui and Bleichrodt, 2007). This result falls outside of the scope of the DA model as proposed by Gul (1991). The lowest win probability tested in Abdellaoui and Bleichrodt (2007) was 0.1, which gave a median β slightly below 0, indicative of elation loving. As the win probabilities for Mega Millions are exponentially smaller than 0.1, projecting the inverse relationship between win probability and elation loving onto the Mega Millions data should predict β estimates well in the elation loving range. Parameter values that are more likely to predict purchasing Mega Millions products will be lower values for β and higher α values for the power utility function.

A necessary prerequisite for DA modeling is determining certainty equivalents of the lotteries in question. As these may vary significantly for players of Mega Millions, DA modeling for Mega Millions executed here allows certainty equivalents to fall into two possible ranges that should cover the actual certainty equivalents for most players: certainty equivalents less than 4, and certainty equivalents between 4 and 10. This requires two separate estimations. Letting α to once again move between 0.71 and 1.10 in increments of 0.01 and β range from -0.9 to 4.0 in increments of tenths, with the same 8 jackpot levels, there are 16,000 possible parameter-jackpot combinations under each specification. There is a grand total of 0 parameter-jackpot combinations in which the Standard option is both rational and most preferred under either certainty equivalent specification. Results in Table 1 are for the certainty equivalent less than four specification, selected since the cost of tickets in the cost neutral framework is \$3. No mapping of parameters is given here since no tested parameter combination predicts Standard preference at any feasible jackpot level. The only DA parameter values that allow a rational preference for the Standard option are $\beta < -.95$, corresponding to absurd elation loving. Such extreme elation loving equates to a minimum of 30% weight going to elation outcomes under DA utility evaluation of the Standard option for certainty equivalents below 4 which have an actual probability of about 2% of occurring. In the case of certainty equivalents between 4 and 10, the weight is about 9% for outcomes with an objective probability of approximately 0.5%. These weights are somewhat incredulous, especially if they are to hold across decision problems. The conclusion must be that DA is not an appropriate model to account for the preferences for the Standard option displayed in the Mega Millions data.

3.5 Regret Theory

The models analyzed so far are structured so that lottery evaluation is independent of the set of lotteries available: evaluation of a lottery is wholly within-lottery. The remaining models require lottery evaluation to be contingent on the opportunity set, allowing for both within and between-lottery factors in lottery evaluation. Regret Theory (RT) was independently and simultaneously developed by Bell (1982) and Loomes and Sugden (1982). The environment for RT is in the mold of Savage (1954), in which the decision framework is choosing among acts that result in consequences depending on the state of the world that occurs. This native environment differs from those of the models discussed so far, which are choices between probability distributions over outcomes. The premise is that utility consists of an objective part, separate from the choice setting, as well as a part that depends on the other choices available. Specifically, an individual can make a choice or take a certain course of action between two available, and then some state of the world resolves, and the outcome associated with that action in that state of the world ensues. An individual could perhaps feel some elation or rejoice if that outcome is better than the outcome in the same state of the world under the other course of action, and perhaps some regret if the outcome observed is worse than what would have occurred under the other course of action. RT attempts to capture the anticipation of such feelings in the decision making process. Theories that allow for set-dependent lottery evaluation may have a better shot at explaining the Mega Millions choice behavior, due to the interdependence of the ticket types. With a given ticket in hand, a regret-influenced player may feel regret for playing at all if it is a losing ticket. In the case of the ticket winning a lower tier prize, such a player will feel no regret or maximal rejoice if the ticket is a Megaplier, a combination of regret and rejoice if it is Standard, and maximal regret if it is Just the Jackpot, even more so than not playing. If the ticket is a jackpot winner, there is no difference in outcome across ticket types, and rejoice is felt over not playing.

The method of action evaluation under RT is fundamentally between two actions only. Loomes and Sugden (1982) specify that

$$A_i \succeq A_k \Longleftrightarrow \sum_{j=1}^{n} p_j[c_{ij} - c_{kj} + R(c_{ij} - c_{kj}) - R(c_{kj} - c_{ij})] \geq 0 \tag{9}$$

where c_{ij} is the choiceless utility that the outcome of action A_i yields in state j, and $R(.)$ is a strictly increasing regret-rejoice function. This condition equates to the modified expected utility of A_i being greater than that of A_k. Defining a function $Q(\xi) = \xi + R(\xi) - R(-\xi)$, the evaluation condition simplifies to

$$A_i \succeq A_k \Longleftrightarrow \sum_{j=1}^{n} p_j[Q(c_{ij} - c_{kj})] \geq 0 \tag{10}$$

Loomes and Sugden (1982) demonstrate that a convex $Q(.)$ is consistent with typical EU violations like the common consequence effect, Allais paradox and the lottery-insurance paradox. One drawback of RT is that its extension to decisions between three or more actions is not straightforward. A keystone of the axiomatic foundation of RT is the relinquishing of transitivity and maintenance of the sure-thing principle. Therefore, as RT is built to compare two actions, invoking transitivity for decisions over three or more actions is not acceptable (Loomes and Sugden, 1982).

Bleichrodt et al. (2010) develop and perform the trade-off estimation procedure to experimentally estimate parametric forms of RT. The power function family in (7) was used to measure both choiceless utility c and the regret function $Q(.)$. Mean estimates of choiceless utility were effectively linear at 0.98 and 1.01 across two elicitation problems. However, there was significant variation at the individual level, with more subjects classified under concave utility than convex. Mean estimates of the power function parameter for $Q(.)$ yielded mean estimates of 1.73 and 1.89 across two elicitation problems.

There is some variance at the individual level, but the overwhelming majority of subjects display a convex $Q(.)$ under RT. The convexity estimated is consistent with the functional requirements in RT for many commonly observed choice behaviors. The relevant set of actions are purchasing any three of the Mega Millions ticket types, or not playing at all, so four actions. As in Bleichrodt et al. (2010), both choiceless utility c and regret function $Q(.)$ will take a power function representation as in (7). The power parameter in c will run from 0.71 to 1.10 in hundredths intervals as in previous models, and the power parameter for $Q(.)$ will run from 0.6 to 3.0 in tenths. The Mega Millions choice problem is not pairwise, so a RT extension is necessary, and two different ones will be employed. For pairwise mean utility, a simple average of the pairwise utilities is used to predict preference, where the option yielding the maximum average is preferred (Loomes and Sugden, 1982). In the case of state-wise mean utility, the input corresponds to $c_{ij} - mean(c_{-ij})$, where c_{-ij} refers to the outcomes of all actions besides A_i (Loomes and Sugden, 1987; Sugden 1993). There does not seem to be any compelling reason to weigh certain actions and outcomes more so than others.

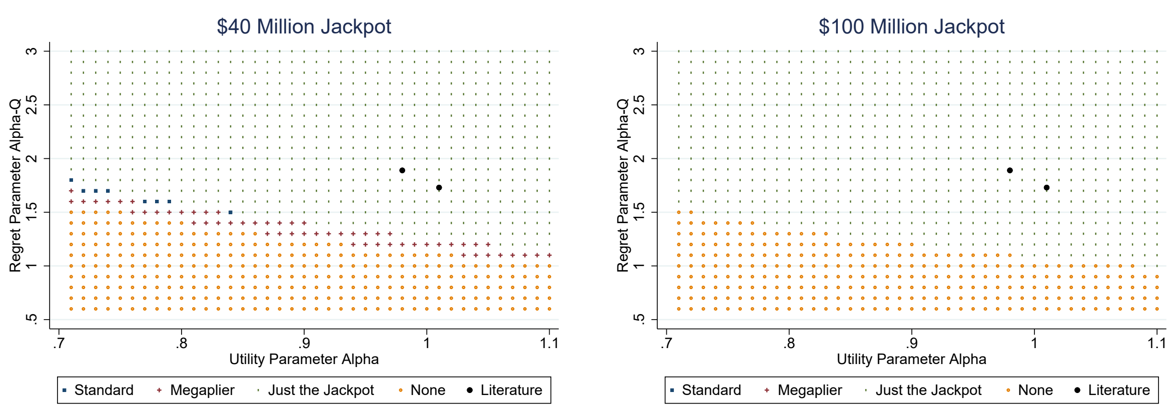
In order for RT to model a decision process, a matrix of state-contingent outcomes must be fully specified (Loomes and Sugden, 1982). With independence of the available lotteries assumed, this criterion is generally satisfied. However, the cost neutral framing of the decision problem may not satisfy independence. Consider the cost neutral pairwise choice between a Standard and Megaplier purchase. For any purchase amount, the Megaplier yields a smaller set of number combinations eligible for prizes, although winning combinations result in a higher payoff for lower tier prizes. Consider spending \$6 on either two Megapliers or three Standard entries, and assume the Megaplier value is 3, meaning lower tier prizes are multiplied by three. The number combinations on the first two tickets would be the same, regardless of Megaplier or Standard status. The third entry is what can cause a matrix uniqueness violation, as Table 2 displays. If the third entry is Ticket 3a, Ticket 3a can only win in states of the world that Tickets 1 and 2 do not. However, if the third entry is Ticket 3b, Ticket 3b and Ticket 1 have the same Mega Ball entry, entitling them both to the lowest possible prize if the Mega Ball value drawn is 25. Whether Ticket 3a or 3b is the third Standard ticket changes the set of possible outcomes pairs and breaks the outcome matrix uniqueness. One way to tackle this is to assume the case of Ticket 3a, of no overlapping numbers. With a given ticket, the probability of randomly drawing another ticket with no matching numbers is $(\frac{24}{25}) * \binom{65}{5}/\binom{70}{5} \approx 65\%$. Note that this probability will decrease greatly as the number of existing tickets required to not match a new ticket increases. The other option of considering Ticket 3b is more problematic in that there are other potential scenarios like Ticket 3b that result in further differing outcome matrices, such as sharing two numbers, three numbers, etc. Therefore, the polar overlap case of sharing all winning states for lower prize levels will be tested. While technically this scenario cannot manifest in reality, since matching the Mega Ball is a condition for some lower level prizes, it serves as a limiting case to test the RT model. Allowing the jackpot winning to overlap can only hurt the Standard option, since the jackpot is simply split among the winning tickets, so the jackpot entries from additional Standard entries will not overlap. Both the no overlap and full overlap cases are tested, as depending on the parameter values of c and $Q(.)$, one of these boundary cases will always make the Standard option as favorable as it can be under RT.

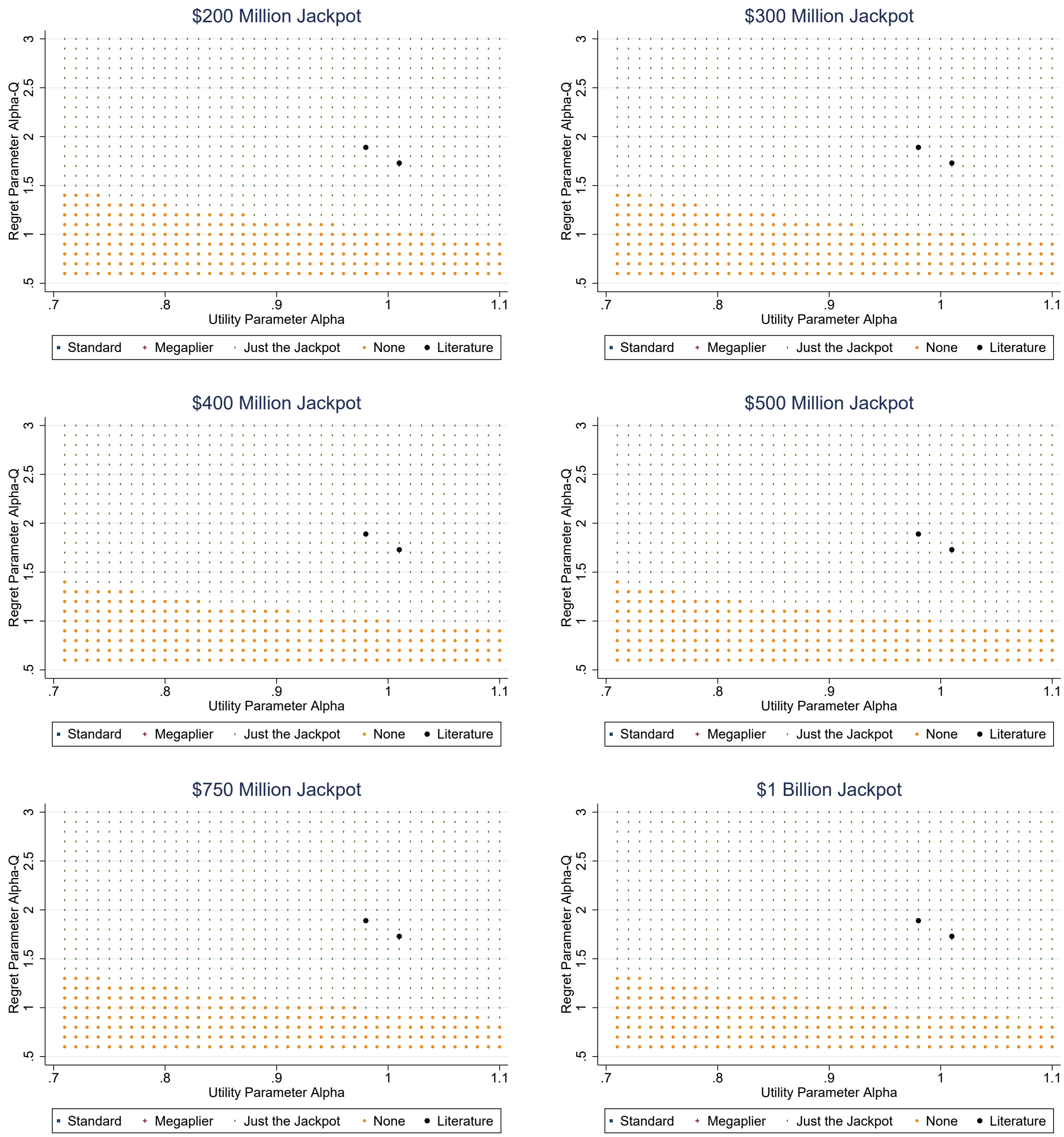
Table 2: Example of Outcome Matrix Differences

	Combination (#-#-#-#-#|Mega Ball)	Outcome Pairs with Ticket 1, Megaplier=3
Ticket 1	1-2-3-4-5|**25**	
Ticket 2	6-7-8-9-10|24	N/A
Ticket 3a	11-12-13-14-15|23	{(J,0);(0,J);(3L,0);(0,L);(6,0);(0,2);(0,0)}
Ticket 3b	16-17-18-19-20|**25**	{(J,2);(6,J);(3L,0);(0,L,);(3L,2);(6,L,);(6,2);(0,0)}

The total number of distinct parameter-jackpot combinations per specification comes out to 8,000: 40 parameter values for c, 25 parameter values for $Q(.)$, 8 jackpot levels. Note that there are also four specifications: 2 non-pairwise aggregation methods and 2 overlap scenarios. First, there is no parameter-jackpot combination that predicts a preference for the Standard option under the state-wise mean utility specification with both overlap scenarios. Under the pairwise mean utility specification, the choice of overlap scenario is essentially inconsequential to prediction. With no overlap of winning combinations, only a single parameter combination predicts Standard preference at a jackpot of $40 million, and none for all other jackpots. Under a full overlap scenario, 8 parameter combinations predict Standard preference at a jackpot of $40 million, and none for all other jackpots. The results of the pairwise mean specification under a full overlap scenario are presented in Table 1, as it is the least inconsistent with the actual choice behavior. Figure 6 gives a mapping of preferences over parameter values by jackpot amount for this specification. One potential critique is that the native problem of Mega Millions choice does not generate a unique outcome matrix, and therefore RT cannot make a prediction for this game. However, the polar cases of possible outcome matrices via overlap scenarios were both considered and result variation is negligible. Both are suggestive of a minimal preference for the Standard and Megaplier options, with preference behavior mostly being for Just the Jackpot or not participating at all.

Figure 6: Mapping of Pairwise RT Preferences under Full Overlap





3.6 Salience Theory

The most recent of the reviewed decision models in this paper is Salience Theory (ST), which allows the relative salience of payoffs within states of the world to impact the weight placed on those states of the world (Bordalo et al., 2012). The psychological motivation is that salient payoffs are over-weighted by the decision maker, and not so salient payoffs are under-weighted. ST is also set-dependent like RT. To apply ST to the Mega Millions problem, let's start with an example. Each of the ticket options gives some chance at the jackpot for \$3: Megaplier gives 1 entry, the Standard option 1.5 entries, and Just the Jackpot 2 entries. Since the native framework in ST is binary choice, just consider the Standard option and Just the Jackpot for now as the only two options available. The jackpot odds break down into 2 meaningful states of the world: the shared 1.5 entries, and the extra half entry of

Just the Jackpot. Since the outcome of the 1.5 shared entries is the same, the salience in this state is the minimum of zero. This would lead to an under-weighting of this state relative to its objective probability. The bonus half entry of Just the Jackpot is highly salient, as the payoff difference is a minimum of $40 million. This state would be over-weighted relative to its objective probability. This example highlights a major diversion between ST and PT/RDU: small probabilities only get over-weighted in ST if they are salient, whereas PT/RDU consistently over-weights small probabilities.

The notation used in Bordalo et al. (2012) is replicated here for model exposition. Consider a lottery L_i that has outcomes x_s^i across the states s$\in S$, and lottery L_j with outcomes x_s^{-i}. Let $\sigma(x_s^i, x_s^{-i})$ be the salience function determining the salience of a state. Like RT, the natural setting of ST is making binary comparisons. In the binary case, $\sigma(.)$ is symmetric, but not necessarily so when extended to non-pairwise decision making. They propose

$$\sigma(x_s^i, x_s^{-i}) = \frac{\mid x_s^i - x_s^{-i} \mid}{\mid x_s^i \mid + \mid x_s^{-i} \mid + \theta} \tag{11}$$

where $\theta > 0$. Salience increases with the difference in payoff between the two lotteries in a state, but decreases as the absolute average payoff deviates from 0. For example, consider L_A with payoffs {1,6} and L_B with payoffs {2,5}. The payoff difference between lotteries in each state is 1, but the salience in the first state is larger because those payoffs are closer to zero. States are then ranked by decreasing salience, so the most salient gets a rank of 1. The salience rank of state s for lottery L_i is denoted k_s^i. For a $\delta \in (0, 1]$ and objective state probabilities of π_s, the modified probability of state s is

$$\Big\{ \pi_s^i = \pi_s * \omega_s^i \quad \omega_s^i = {}^{\delta^{k_s^i}}/_{\sum_r \delta^{k_r^i} * \pi_r} \tag{12}$$

specified as such so that the modified probabilities sum to 1. The valuation of lottery L_i uses these modified weights: $V(L_i) = \sum_s \pi_s^i u(x_s^i)$. A state is over-weighted if and only if it is more salient than average, under-weighted otherwise. Notice that salience is determined independently of the objective state probabilities. Since the Mega Millions decision problem is non-pairwise, an extension of ST is required for proper evaluation. Bordalo et al. (2012) elaborate on a non-pairwise extension in their online appendix. They propose non-pairwise salience function $\hat{\sigma}(x_s^i, x_s^{-i}) = \sigma(x_s^i, f(x_s^{-i}))$, where $\sigma(.)$ is the salience function and $f(x_s^{-i}) = \frac{1}{N-1} \sum_{j \neq i} x_s^j$, a simple average. Once the composite alternative state outcomes are calculated, lottery valuation proceeds exactly as in the binary case. However, $\hat{\sigma}(.)$ is not necessarily symmetric, so valuations need to be calculated for each lottery under consideration. As in RT, calculations will be impacted depending on whether additional Standard tickets overlap in number combinations with existing counterfactual Megapliers. Therefore, both full overlap and no overlap scenarios will be tested.

While $\delta \in (0, 1]$ is the possible range for the distortion parameter, Bordalo et al. (2012) show that $\delta = 0.7$ is consistent with a number of observed EU violations. Konigsheim et al. (2019) experimentally estimate the distortion parameter δ and find that the Bordalo et al. (2012) assumptions of $\delta = 0.7$ and linear utility are reasonable and consistent with the results of their estimation, where δ is estimated to live in the range of 0.5 to 0.8 among subjects who behave according to ST. Bordalo et al. (2012) also show that a $\theta \geq 0.1$ is also consistent with observable EU violations that ST can account for; the choice of θ can impact the salience ranking within a lottery. Setting $\theta = 0.1$ in the Mega Millions choice problem yields some questionable salience rankings. For instance, from the perspective of Standard evaluation under a no overlap scenario, the state of the world in which the Standard ticket gives $4 and the other tickets give nothing is more salient than the state of the world in which Standard and Just the Jackpot win the jackpot while the Megaplier wins nothing. Recall salience ranking is independent of the probabilities of states of the world. The choice problems in Bordalo et al. (2012) that justify $\theta = 0.1$ had payoffs thousands of times smaller than jackpot levels. A selection of $\theta = 1,000,000$ seems to yield significantly more reasonable salience rankings. As a robustness check, estimations are run for both overlap scenarios using both $\theta = 0.1$ and $\theta = 1,000,000$. The results reported in Table 1 are for the no overlap scenario with $\theta = 1,000,000$. For the parametrization run here, non-linearity of utility will be permitted, and the power function in (7) will once again be the functional form employed. The power parameter α will run from 0.71 to 1.10 in increments of 0.01, distortion parameter δ will run from 0.5 to 0.99 in increments of 0.01, and the 8 jackpot levels used in previous estimations will also be tested. This amounts to a total of 16,000 distinct parameter-jackpot combinations per specification. Not a single one of these corresponds to a preference for the Standard option under any specification, and a majority of them correspond to a preference for Just the Jackpot. Therefore, no figure will be provided displaying the preference distribution across parameters. The extreme salience of the additional jackpot entries for Just the Jackpot overpowers the relative advantages of the Standard and Megaplier options under ST, rendering it inept at matching the choice behavior in the data.

4 Non-Decision Theoretic Explanations

There are a few reasons to conjecture that certain aspects of the real world implementation of Mega Millions outside of the simple choice problem may be constraining consumers, leading to consumption choices that would differ absent such constraints. It is plausible that since Just the Jackpot is relatively new, many consumers may not be aware of its existence, and therefore attention is limited to the Megaplier and Standard options. It may be that newly offered lottery products take time to acquire a strong sales base, and Just the Jackpot has not had enough time to do so. This could be an effect of consumer inattention, or just due to slow adjustment in consumption. There could also be some liquidity constraints at play, since both Megaplier and Just the Jackpot cost \$3 relative to the Standard price of \$2. Severely constrained consumers could opt for the cheaper Standard option even if they prefer one of the other distributions. This section takes a closer look at all of these possibilities.

4.1 Limited Attention

Since Just the Jackpot has only been around since October 2017, it is plausible that many players are simply not aware of its existence. If inattentiveness or limited attention (LA) applies in this case, it could be that for some consumers, C(Megaplier, Standard, Just the Jackpot) = Just the Jackpot, and C_{LA}(Megaplier, Standard, Just the Jackpot) = C(Megaplier, Standard) = Standard. This could account for the main data patterns, since the Standard option comprises over 90% of sales, and all of the models tested above find Just the Jackpot to be the most appealing Mega Millions product overall. At face value, testing for limited attention is not really feasible. The author's correspondence with numerous lottery officials in various states suggests that advertising of Just the Jackpot can vary significantly at the retailer level (like signs outside the retail location mentioning the new option, a retailer bringing up the product in conversation with customers, etc.), and acquiring retail-level data is quite costly, let alone the effective impossibility of determining Just the Jackpot advertising intensity at each retailer, which is not part of any compiled data set the author is aware of.

Fortunately, the lottery commission of Kansas created a natural test of limited attention by running statewide promotions for Just the Jackpot. On November 2, 2018, a complementary Just the Jackpot ticket was given to customers who purchased \$6 worth of Mega Millions in a single transaction, resulting in \$15,975 in promotional sales, or 5,325 Just the Jackpot giveaways. On May 3, 2019, a complementary Just the Jackpot ticket was awarded to players who purchased \$10 in Mega Millions, resulting in \$13,017 in promotional sales, or 4,339 Just the Jackpot ticket giveaways. If the promotion is successful, there may or may not be an increase in overall sales, but there should surely be an increase in the percentage of sales that are Just the Jackpot sales. This experiment also provides evidence that not all players are purchasing a single ticket per draw. For the November 2, 2018 promotion, about 20% of the total sales for that drawing qualified for the promotion, meaning at least that percentage of sales were due to multiple ticket purchases. About the same percentage of total sales qualified for the May 3, 2019 promotion, which had a higher threshold for qualification. While it is true that some players may have increased their ticket consumption from their norm to qualify for the promotion, it is also true that some players may have purchased more than the promotion-qualifying amount in Mega Millions expenditures.

The results of the test strongly reject the hypothesis of limited attention impacting Just the Jackpot sales. Consider breaking up the sample into dates prior to the first promotion, dates between promotions, and dates after the second promotion. The mean percentages of Just the Jackpot sales of total sales in each time period are 0.22%, 0.10% and 0.10%, respectively. The number of draws during these time periods are 105, 51 and 42, respectively. The 74 largest Just the Jackpot percentages belong to draws before either promotion, and only 2 of the 89 largest belong to either of the promotional periods. Statistically confirming the compelling results are Mann-Whitney U tests of significance levels better than 0.0001 on any reasonable dichotomies of Just the Jackpot sales, Just the Jackpot sales as a percentage of total sales, and Just the Jackpot as a percentage of the jackpot. The test results actually indicate a strong negative effect of the promotions on Just the Jackpot sales. This may be chalked up to a phenomenon known as jackpot fatigue (Matheson and Grote, 2004), in which overall sales drop the longer a game runs without having a restructuring of the odds-payoff matrix (assuming this more strongly affects Just the Jackpot sales relative to other Mega Millions ticket sales). It also could just be the result of a decreasing interest in the product with time. The jackpots for the draws in which promotions were run were \$52 million and \$252 million, both of which are within typical jackpot levels. Most of the players during these promotions would have been regular players and not jackpot frenzy players, who generally only play when jackpots get excessively high. Therefore, it must be concluded that for at least regular Mega Millions consumers, limited attention is not culpable for the dearth of Just the Jackpot sales.

4.2 Slow Sales Adjustment

There is a possibility that there is a significant lag in consumption adjustment after the introduction of a new lottery product. More precisely, the introduction of an additional ticket option within an existing lottery game may require a long timeline to acquire its stable sales range. Data consistent with this hypothesis would show an increase in sales percentage of a new lottery product with the passage of time, controlling for appropriate factors like jackpot size. In the case of Just the Jackpot, there should be some indication of an increased share in the Mega Millions sales percentage as time passes. A quick glance at Table A2 in the Appendix, which displays the time series of Mega Millions sales for states that offer Just the Jackpot since its inception on October 31, 2017 up through September 2019, seems to indicate the opposite. The Just the Jackpot sales percentage starts off at 2.08% on the first day it is available for purchases, but quickly drops below 1% in a week's time, then settles below 0.5%

about a month after that and further declining into the 0.2% to 0.4% range and never rebounding. This is in line with a consumption lag of no more than a month, and the adjustment runs opposite to what the hypothesis would require.

A linear regression of the Just the Jackpot sales percentage on time further validates this hypothesis rejection. Table 3 presents a couple of variations on this regression, where Time is a counting variable for each passing draw date, set equal to 1 for the first day Just the Jackpot was available for purchase. Also, since jackpots for a specific draw are due to a combination of the previous jackpot rolling over and the actual sales for that draw, using jackpot as a regressor in a sales regression introduces endogeneity: a lotto consumer likely takes into account the expected jackpot when deciding on lottery consumption, but that lottery consumption itself is impacting the jackpot size (Cook and Clotfelter, 1993). Therefore, two regressions are provided in Table 3: one regressing the sales percentage on time and the log of the rollover, which is the previous draw's jackpot amount, unless the previous jackpot was won, in which case the rollover is zero; the other using the log of the rollover and time as instruments for the log of the jackpot and regressing sales percentage on time and the predicted log jackpot. Both regressions produce essentially the same highly significant negative estimate of -0.000018. An additional puzzling result is the significant negative coefficients on Log Rollover and Log Jackpot, indicative of the trend highlighted above for the historic $1.6 billion Mega Millions run. As the jackpot increases, Just the Jackpot should increase in relative appeal, as the lower tier distribution remains unchanged. The regression results indicate the opposite, and this will be addressed in greater detail later on.

Table 3: Regressions of Just the Jackpot Sales Percentage

	(1)	(2)
Intercept	0.0062*** (.0005)	0.0218*** (.005)
Time	-0.000018*** (.000002)	-0.000018*** (.000002)
Log Rollover	-0.000087*** (.000024)	
Log Jackpot		-0.0009*** (.00026)

Time is a counting variable for passing draw dates. Log Rollover is the natural log of the rollover amount, which is equal to the previous draw's jackpot or zero, in case the previous jackpot was won. In this case, the natural log of the rollover of zero is also set equal to zero. Log Jackpot is the natural log of the jackpot amount. (1) presents linear regression results, whereas (2) presents two stage least squares estimates, in which Log Jackpot is instrumented for with Log Rollover and Time. Note that jackpot and rollover have a correlation coefficient of 0.97. Standard errors are in parentheses below the estimates. *** indicates significance at better than 1%.

4.3 Liquidity Constraints

Given that many lottery products costing a dollar or less, the Just the Jackpot price of $3 may be too steep for some consumers. Lotteries are often considered to be a poor man's tax, regressive in nature. Data bears out the regressivity, but only in so much as lower income tiers spend a higher percentage of their income on lottery products, as aggregate spending is relatively steady across income levels (Clotfelter and Cook, 1990; Kearney, 2005). Since lotteries are consumed even at the lowest income levels, where liquidity constraints are most likely to bind, it is plausible that some constrained consumers may opt to purchase the cheaper Standard ticket at the price of \$2, even if Just the Jackpot is the preferred option without such constraints.

There are two reasons up front why liquidity concerns are likely not the primary determinant of the data patterns. First, regressivity varies significantly within lotteries: instant games are highly regressive, draw games with jackpots that rollover are much less so, even somewhat progressive when jackpots become exceedingly large (Clotfelter and Cook, 1987). Mega Millions is precisely a draw game with a rollover jackpot that can get exceedingly large, much larger than the Maryland jackpots of a few million dollars analyzed in their study, and therefore Mega Millions players are on average wealthier than many other lottery game players. Second, from the limited attention test on the Kansas Mega Millions data, at least 20% of sales during typical jackpots are due to ticket purchases of at least \$6, meaning multiple tickets were purchased in a single transaction. It is likely that a larger percentage than that abounds during atypically high jackpot draws, in which jackpot frenzy buyers join the fray and the lottery becomes more progressive.

The most recent changes to the Mega Millions game on October 31, 2017 involved both a change to the payoff-odds matrix and an increase in the cost of the ticket options. Standard tickets doubled in cost from \$1 to \$2, and Megaplier increased 50% in cost, from \$2 to \$3. Assessing the impact on Megaplier sales of this cost increase to \$3 can provide some further insight regarding liquidity constraints on Just the Jackpot, which also costs \$3 but was newly introduced on October 31, 2017. For the two years prior to the change, median nationwide Megaplier sales by draw amounted to about \$2.14 million. After the change, the median Megaplier sales remarkably remained within just a few hundred dollars of the median for the two years before the change. This implied an approximate one-third reduction in Megaplier entries purchased after the cost change was instituted. So the cost change was essentially revenue-neutral for Megaplier sales. This is not true for the Standard ticket. Median Standard ticket revenue before the cost change was about \$16.7 million, whereas after the cost change it rose to \$25.4 million, over a 50% increase in median revenue, implying about a 25% reduction in median Standard entries. Therefore, overall median Mega Millions revenue grew after the increased cost, reduced odds and resultant increased jackpots of the October 31, 2017 restructuring.

Is this evidence of the presence of liquidity constraints for Mega Millions consumption? The most recent changes implemented to Mega Millions altered the expected return and effective price of both the Standard and Megaplier options. Effective price of a lottery entry can be thought of as the normalized expected loss, so the cost minus the expected value of the ticket, divided by the cost. Expected values vary as jackpots increase and with the probability of sharing the jackpot, so here the minimum jackpot is used in expected value determination, along with the simplifying assumption of no jackpot sharing. Prior to the changes, the effective price of the Standard ticket was 77 cents, and for the Megaplier it was 63 cents. After the 2017 changes, effective prices increased to 81 cents for the Standard ticket and 71 cents for the Megaplier. The Standard effective price increase was about $\frac{0.81}{0.77} - 1 \approx 5\%$, whereas for Megaplier it was about $\frac{0.71}{0.63} - 1 \approx 13\%$. The Megaplier became relatively more pricey after the changes than the Standard ticket did, and this by itself can rationalize the shifts in sales behavior witnessed with the 2017 changes. An increase in effective price should lead to a reduction in quantity demanded, which the data confirms for both ticket types. The higher relative price increase on Megaplier is more consistent with a net move from Megaplier to Standard, and not the other way around. While it is not possible to track individual purchasing behavior to confirm this with certainty, a higher percentage reduction in Megaplier purchases relative to Standard entries is consistent with this idea. The price increase on the Megaplier was high enough to diminish Megaplier sales to the point of revenue-neutrality, whereas the price increase on the Standard option was low enough so that revenue actually increased.

Notice that this rationalization of the data does not require a liquidity constraint story. It is conceivable that with the cost increase, constrained consumers who had previously purchased Megaplier under the old cost regime could buy Megaplier less frequently, switch to buying Standard tickets, or stop playing Mega Millions. While liquidity constraints are consistent with the observed reduction in Megaplier entries observed, the price increase channel described above can also explain it. The pricing motive may arguably better explain the shifts in consumption than liquidity constraints, since after the change in cost regime, the Standard cost doubled, whereas the Megaplier only increased in cost by 50%, both increasing by the same amount of $1. It is plausible that a dollar increase in cost will cause liquidity constraints to bind for more individuals the smaller the original cost is, so that Standard sales would be more impacted than Megaplier sales by binding constraints. The point estimates of price elasticity are indicative of this as well: a 5% price increase in the Standard option led to about a 25% reduction in quantity consumed, whereas a 13% price increase of the Megaplier resulted in only a 33% reduction in quantity consumed. More tightly binding liquidity constraints will strongly reduce consumption when prices increase, implying a higher price elasticity absent those constraints. The Standard price elasticity is approximately double that of the Megaplier, suggesting that if liquidity constraints are even present, they are likely impacting Standard consumption more than Megaplier consumption. Furthermore, recall that the motivation for looking at Megaplier consumption shifts with a cost increase was to see if low Just the Jackpot sales percentages were due to liquidity constraints. Recall that a minimum purchase of Megaplier and Just the Jackpot both costs $3. At best, liquidity constraint arguments are confounded by the price channel predicting the same directional movements in the data. Even with the increase in cost, Megaplier sales account for between 5.5% to 8.5% of Mega Millions dollars sales, 20 to 30 times more than the paltry average 0.3% of Just the Jackpot, which costs the same as the Megaplier. For liquidity constraint arguments to have bite, there should have been a much larger reduction in Megaplier sales after the cost increase. Therefore, it must be concluded that liquidity constraints are not the primary driver of low Just the Jackpot sales.

5 Behavioral Mechanisms

5.1 Feedback-Conditional Regret Theory

The various models analyzed share a general consensus that with jackpots relatively low, high concavity of utility and other model-specific parameter ranges, Megaplier or not playing at all are the preferred options. For larger jackpots, convex or even relatively linear utility and relevant model-specific parameter ranges, Just the Jackpot is the preferred option. Data consistent with any of these models should show a low percentage of Standard sales, with Megaplier and Just the Jackpot sales making up the majority of sales, and a decent level of non-participation. Furthermore, the non-decision model explanations of limited attention, slow sales adjustment and liquidity constraints have been ruled out. One hypothesis that would essentially bypass the entire conundrum is that lottery play is not rationalizable, especially given the inability of a wide range of models with varying rationality criteria to explain the Mega Millions data. One manifestations of this hypothesis is lottery and gambling play being classified as consumption goods (Hirshleifer, 1966), not subject to utility of wealth evaluations.

However, sensitivity of lottery sales to expected returns across a variety of games (Kearney 2005) and surveys reporting that lottery players mainly view the lottery as a means to acquire wealth counter this hypothesis. Additionally, the attempts of many models in the literature at explaining the so called lottery-insurance paradox is a concession in the literature that lottery consumption is primarily a wealth concern that can be rationalizable via preference modeling.

The remaining possibility is that Just the Jackpot is simply not appealing to Mega Millions consumers. Decision theory suggests that the poor appeal is not justifiable via the payoff distribution or even the choice set. Perhaps the mechanics of actually playing the Mega Millions lottery will provide some insight into the choice behavior. Once a decision to play is made, a player must either pick a set(s) of numbers or have one(s) randomly assigned. The player must also determine which of the three ticket options to select. Consider spending $3 on Mega Millions. If the player goes with the Megaplier, after the winning numbers are drawn, a player will feel no regret if the ticket is eligible for a prize. If the ticket wins a lower tier prize, that is more than would have been won had that ticket been a Standard or Just the Jackpot. In the case of winning the jackpot, there is no difference between ticket types. However, in the event of the Megaplier not winning a prize (a 96% probability), a player may regret not opting for one of the other ticket types. That original ticket would not have won regardless of the ticket type, but additional counterfactual entries may have been eligible for prizes. However, that ex post regret can only be felt in a probabilistic sense, since those additional tickets were never generated. Now consider originally opting for Just the Jackpot, which would give two different entries into only the jackpot portion. In the unlikely event of one of them winning the jackpot, no regret would be felt, and possibly some rejoice. This is because at any given cost, Just the Jackpot gives the most entries into the jackpot, and it may be that opting for another ticket type would not have generated the winning ticket. But in the nearly certain event of Just the Jackpot entries not being jackpot eligible, a partial ticket match may stir up regret, since if the ticket had been one of the other ticket types, it would have won a prize. This regret may be different from the regret engendered by the Megaplier, since in the case the Megaplier is not eligible for a prize, a player will not know with certainty the outcomes of the foregone Standard or Just the Jackpot entries. However, most models do not take such a distinction into account, including state and opportunity set-dependent models like Regret and Salience Theory. The ex post regret would be the same if additional voided entries were given for players who opted for the Megaplier or the Standard option, bringing the total of void and non-voided entries equal to the number of counterfactual Just the Jackpot entries.

There is precedence in the literature that this distinction in engendered regret ex post is contributing to the distaste for Just the Jackpot. Specifically, "feedback about what definitely would have occurred produces a greater potential for regret than pallid, abstract knowledge of what was statistically likely to occur" (Larrick, 1993). This description captures the regret difference here, since a Just the Jackpot holder has feedback about what definitely would have occurred if the Megaplier was chosen, whereas the Megaplier holder is missing definitive feedback for the foregone second Just the Jackpot ticket. The implication is that Megaplier buyers would prefer not being given the voided second Just the Jackpot ticket, and a similar extension can be made for Standard buyers with respect to the Just the Jackpot alternative. While Regret Theory and its ilk may be said to account for anticipated regret, this is not the same as ex post regret or post-decision regret, which refers to having knowledge of the outcomes of foregone options in at least some states of the world. A number of studies in the psychology literature have detected effects on decision making of the presence of ex post regret in varying settings: choice between two risky gambles (Zeelenberg at al., 1996); consumer purchase decisions (Inman and

Zeelenberg, 1998); an ultimatum game in which the proposer knows that he/she will be informed of the minimum acceptable offer after the proposal is made (Zeelenberg and Beatie, 1997); and most relevant to the problem at hand, influencing participation behavior and regret motivation in the Dutch postcode lottery, for which the entry is the player's postal code (Zeelenberg and Pieters, 2004). The Dutch postcode lottery has ex post regret built into it, since a player will know the outcome of the foregone option independent of participation. This is not true for lotteries in general: by entering, one will know whether one wins or loses, as well as the outcome of not entering (keeping the ticket cost); whereas by not entering, one will never know the outcome had they entered.

Given the evidence for ex post regret considerations in decision making, Humphrey (2004) attempts to add an ex post regret flavor to original Regret Theory (Loomes and Sugden, 1982; Loomes and Sugden, 1987). The specifics of the Humphrey (2004) modification to Regret Theory is provided in the Appendix. The main takeaway is that even incorporating differences in ex post feedback into Regret Theory does not yield a model capable of matching the strong preference for Standard in the data. The predictive power is comparable to that of the Rank Dependent Utility results listed in Table 1; detailed results of Feedback-Conditional Regret Theory (FCRT) are listed in Table A1 in the Appendix. These disappointing results do not necessarily mean that feedback consideration is not impacting choice behavior per se, as Regret Theory may also not be an appropriate base model to apply feedback considerations to. The structural interdependence of the Mega Millions ticket options engenders feedback variation based upon the option chosen, which at least descriptively seems to have the potential to impact choice behavior. Indeed, the model developed in the Section 6 does include a feedback parameter which contributes to its relatively strong predictive power.

5.2 Winner and Loser Regret

FCRT attempts to capture what is probably a major factor in the aversion to Just the Jackpot. The albeit unlikely possibility of purchasing a Just the Jackpot ticket that would have been eligible for a lower tier prize, particularly the million dollar prize level, may weigh heavily on some consumers. While FCRT is not able to rationalize the data, the variation in the elicitation of feedback on foregone actions ex post does seem to be a reasonable factor in the decision making process from a psychological and behavioral perspective. Returning to the mechanics of Mega Millions, any given number combination could be a Megaplier, Standard or Just the Jackpot entry. In the case of that ticket being jackpot eligible, ticket type is inconsequential. If it is not eligible for any prize, once again the type is inconsequential. However, if it is eligible for a lower prize, Megaplier gives the best possible outcome, Just the Jackpot gives the worst, and Standard is intermediary. In such a scenario, Standard and Just the Jackpot holders may experience ex post regret for not having opted for a Megaplier, with Just the Jackpot eliciting (perhaps disproportionately) more regret than the Standard option.

For example, consider the state of the world in which the ticket designated as a Megaplier would win \$5 million, in which case the Standard version of that ticket would win \$1 million, and Just the Jackpot would win nothing. For ease of exposition, assume linear utility of wealth and a state-specific regret magnitude, defined as the difference of the maximum attainable outcome in that state and the outcome of the chosen ticket type. In this state of the world, Megaplier yields no regret, Standard yields a regret of 4 million, and Just the Jackpot yields a regret of 5 million, so that the regret of a Standard designation is 80% of that of a Just the Jackpot designation. A linear RT framework also yields an 80% state-specific utility ratio of Standard foregoing Megaplier and Just the Jackpot foregoing Megaplier. A reduction of this ratio will generally result in the Standard option becoming relatively more favorable compared to the Megaplier without also making Just the Jackpot relatively more favorable.

There is precedence for thinking that this ratio should indeed be less than 80%, even with linear utility assumptions. In the million dollar state described above, Megaplier is the winner and Just the Jackpot the loser, while Standard falls in between the two. From a regret perspective, holding a Megaplier engenders no regret, while the other two may do so. Just the Jackpot can be said to engender loser regret, as it yields the worst possible state-specific outcome. Standard may be classified as engendering winner regret, as the holder walks away with $1 million but could have walked away with $5 million. Filiz-Ozbay and Ozbay (2007) theoretically propose and experimentally demonstrate that observed overbidding relative to the Risk Neutral Nash Equilibrium in first price auctions can be attributed to bidders anticipating loser regret. Loser regret would be felt ex post if a bidder loses an auction, is then told the winning bid, and realizes a higher but still rational bid could have won the auction. Their experiment also demonstrates that bidders do not strongly anticipate winner's regret. Winner's regret in a first price auction occurs if a bidder wins an auction and is told the second highest bid, and realizes a lower, more profitable bid would have still won the auction. Mean bid differences between treatment groups, as well as self-reported feelings of anticipated winner and loser regret, both suggest that only loser regret is significantly impacting bidding behavior in first price auctions. Engelbrecht-Wiggans and Katok (2008) run a similar test of winner and loser regret in first price auctions, but find significant effects of both winner and loser regret on bidding behavior. However, they note that winner regret becomes more prevalent in bidding behavior with successive auctions in a repeated experimental session, suggesting that winner regret is not as anticipated as loser regret and becomes more impactful with bidding experience.

The insights on winner and loser regret from the first price auction experiments are relevant to the Mega Millions choice problem. If the conclusion from Filiz-Ozbay and Ozbay (2007) carries over to the Mega Millions setting, the implication would be that players deciding on which ticket type to purchase would focus on the lower tier prize disparity between Megaplier and Just the Jackpot disproportionately more than that between Megaplier and Standard. Incorporating this into a model would improve the overall desirability of the Standard option. In order to determine where a given action's outcome in a certain state falls on the regret scale, the best and worst outcomes for the state must be known. A parameterized term that can capture the intensity of winner regret is

$$\gamma_{kj} = \begin{cases} \left(\dfrac{\max\limits_{A_i \in A} u(x_{ij}) - u(x_{kj})}{\max\limits_{A_i \in A} u(x_{ij}) - \min\limits_{A_i \in A} u(x_{ij})} \right)^{\rho} & \max\limits_{A_i \in A} u(x_{ij}) \neq \min\limits_{A_i \in A} u(x_{ij}) \\ 0 & \max\limits_{A_i \in A} u(x_{ij}) = \min\limits_{A_i \in A} u(x_{ij}) \end{cases} \tag{13}$$

where A represents the set of available actions, x_{kj} is the outcome of action k in state j, and $u(.)$ is the (choiceless) utility function, and $\rho > 0$ is a winner regret intensity parameter. The power functional form is used to allow for substantial variations in γ_{kj} depending on where x_{kj} falls within the range of state payoffs. The denominator of (13) is the maximum possible utility difference in state j, essentially the maximum regret. The numerator of (13) gives the action-specific regret in state j. If action k happens to be the best action in state j, then (13) reduces to 0, irrespective of ρ. If action k happens to be the worst action in state j, then (13) reduces to 1, irrespective of ρ. More generally, when there are only two actions in the choice set, (13) will always reduce to either 0 or 1. When there are only two possible outcomes in a certain state of the world, there can be only a max and a min outcome, and hence no potential for winner regret. Going back to the lower tier evaluation of

Megaplier, Standard and Just the Jackpot, (13) equals 0 for Megaplier evaluation, it equals 1 for Just the Jackpot evaluation, and it equals something in between 0 and 1 for Standard evaluation. If not playing at all means having the ticket cost in a cost neutral framework, for instance the minimum cost of \$3 that can buy any type, not playing also will fall between 0 and 1, albeit much closer to 1 as the lower prize tier being evaluated increases. The distinction between winner and loser regret is more ambiguous with this definition in comparison to the first price auction setting. An action yielding the worst possible outcome in a state will engender loser regret, but getting a marginally better than worst outcome can hardly be considered as shifting to winner regret. This is exemplified by not playing Mega Millions with a counterfactual \$5 million Megaplier: the player still has the \$3 ticket cost relative to nothing from Just the Jackpot, but the difference in regret is negligible. A more appropriate description is that the designation in (13) allows for shades of winner and loser regret.

5.3 Demand Quota

The argument up to this point has been for the cost neutral choice framework. While that framework is more appropriate from a purely decision modeling perspective, the fact of the matter is that single purchases dominate Mega Millions sales. The Kansas Lottery Commission promotion of a free Just the Jackpot entry with a \$6 Mega Millions purchase shows that only 20% of the sales in that period qualified for the promotion, and therefore more than 80% of the transactions did not qualify. The only multi-entry transaction that would not qualify for the promotion is a purchase of two Standard tickets. It is difficult to come up with any defensible reasons as to why most of the transactions would be of pairs of Standard tickets. Based on the data, it seems reasonable that there is a large drop off in the number of transactions after single ticket purchases.

It is important to identify possible reasons for the dominance of single purchases. It actually is not so important to nail down a specific reason, but to rule out certain ones that would constrain consumers to choose a sub-optimal option. One constraining reason is liquidity constraints, but the argument provided earlier shows that the high levels of Standard sales are likely not primarily due to liquidity constraints forcing constrained players to settle for their less preferred Standard option. A non-constraining reason is that players set some sort of demand quota, either per draw(s) or by jackpot amount. Some players may just make one purchase for every draw, or once a week, or once the jackpot goes over \$200 million. This could be due to a force of habit, or even to regulate potentially addictive consumption. It could also be to avoid regret of not playing. A single entry into either the Standard or Megaplier would shield a player from large amounts of regret in states of the world with big winnings, even with the lack of ex post feedback that comes by not playing. It is therefore worthwhile to consider a single purchase framework, which differs from the cost neutral framework only in how the Standard option is distributed. The single purchase framework puts back an additional dollar to the Standard option in every state of the world, and takes away the additional half Standard entry. A single purchase analysis of the decision models tested in Section 3 is provided in Table 4. The predictions are even more strongly aligned against the Standard option relative to the cost neutral framework, except for Salience Theory, which now has a few instances of Standard preference prediction. However, factoring in the winner regret into the decision modeling within a single purchase framework should favorably impact the Standard standing, since relative to the Megaplier and Just the Jackpot, it leaves an additional dollar in the consumer's pocket in the likely event of not being prize eligible. The next section will show results from the proposed model in both the cost neutral and single purchase frameworks.

Table 4: Summary of Single Purchase Model Predictions

	Percent of Parameter-Jackpot Combinations				
Model	Standard	Megaplier	Just the Jackpot	None	Max Jackpots Standard
Expected Utility	0%	0%	20.3%	79.7%	0
Prospect Theory	0.1%	12.8%	78.5%	8.6%	2
Rank Dependent Utility	0.2%	9.4%	66.7%	23.7%	2
Disappointment Aversion	0%	1.1%	13.2%	85.7%	0
Regret Theory	0.1%	0.8%	75.1%	24.0%	1
Salience Theory	0.1%	5.8%	79.5%	14.6%	2

Table 4 shows the percentages of tested parameter-jackpot combinations that predict preferences for each of the four options, along with the maximum number of jackpots that any one parameter set predicts a Standard preference for. Prospect Theory uses the Prelec (1998) weighting function; Rank Dependent Utility is Cumulative Prospect Theory using the Tversky and Kahneman (1992) weighting function; Disappointment Aversion with a certainty equivalent less than 4 specification; Regret Theory under pairwise aggregation and full overlap scenario; Salience Theory under a no overlap scenario with $\theta = 1,000,000$.

6 Data Rationalization

6.1 Model

The previous section argued for incorporation of feedback on foregone choices ex post and winner regret into a cost neutral or single purchase framework. A model incorporating these features would be more similar to models that incorporate between-lottery information, like Regret Theory and Salience Theory, than within-lottery only models like Expected Utility, Rank Dependent Utility, Disappointment Aversion, etc. Incorporating ex post feedback on foregone outcomes into a decision model requires information about other lotteries in the consideration set, in addition to the lottery being evaluated. The proposed winner regret coefficient requires knowing the span of outcomes over each state for evaluation purposes. A model of the Mega Millions game that considers these factors therefore implicitly holds that the choice evaluation process is a function of the available choices, which will allow for subsequent counterfactual analyses of reduced choice sets and removal of feedback and winner regret mechanisms from Mega Millions.

The process for developing a model to account for the Mega Millions choice data is behavioral, in the sense that psychological and behavioral motives are identified in the actual game, which then inform an appropriate model. The approach is not preference-based and therefore will not be presented with an axiomatic foundation. There may be no single "right" model given the approach, but the intent is to develop as simple and parsimonious a model as possible. Given the argued importance of regret in the Mega Millions game structure, some sort of regret model seems to be a good starting point for a model capturing the Mega Millions decision making process. There are a plethora of regret-based decision models that have developed over the past few decades: Minimax Regret (Savage, 1951); Regret Theory (Loomes and Sugden, 1982; Bell, 1982); Reference-Dependent Regret (Krahmer and Stone, 2005); Smooth Regret Aversion (Hayashi, 2008), to name a few. Some of these models are intended for pairwise choice scenarios with extensions to non-pairwise choice, like Regret Theory. Others, like Minimax Regret, fundamentally allow for non-pairwise choice settings. The Mega Millions choice setting has four options and would more naturally fit in a non-pairwise model.

Before introducing the model, the choice framework will be laid out. The framework is in the spirit of Savage (1954) and resembles the framework of most regret-based models, in particular Loomes and Sugden (1982). There are a finite number N states of the world, where each state of the world $j \in N$ corresponds to a unique vector of consequences $\overrightarrow{x_j}$ of finite length M. Each state j has an associated probability $p_j \in [0,1]$ with $\sum_j p_j = 1$. There are M actions to choose from, each action $A_i \in A$ being an N-tuple of consequences. x_{ij} refers to the consequence of choosing action A_i if state j realizes.

A decision maker is tasked with choosing a single action from the set of available actions. The decision maker is aware of not only the probability of every consequence in each action, but also the vector of consequences $\overrightarrow{x_j}$ for each possible state of the world. A decision maker behaves according to Regret Weighted Feedback Minimax (RWFM) if the choice satisfies

$$\phi(A) = \begin{cases} \underset{A}{\arg\min} \sum_j \left[p_j * \gamma_{kj} * \left(\underset{A}{\max} u(x_{ij}) - u(x_{kj})\right)\right] & x_{kj} \ Reveals \underset{A}{\max} x_{ij} \\ \underset{A}{\arg\min} \sum_j \left[p_j * \gamma_{kj} * \delta\left(\underset{A}{\max} u(x_{ij}) - u(x_{kj})\right)\right] & otherwise \end{cases} \tag{14}$$

where γ_{kj} is as specified in (13), $\delta < 1$ captures the difference in regret engendered ex post if choosing action A_k and state j realizing does not reveal $\underset{A}{\max} x_{ij}$. If under every action and every state there is always either no feedback or feedback, then the decision criteria reduces to just the first line of (14), removing the need for the feedback parameter δ. The criteria in (14) is a probability-weighted Minimax, with an additional winner regret weight γ_{kj} and possible scaling down of the regret term by δ if action A_k and state j realizing does not reveal $\underset{A}{\max} x_{ij}$.

The method to proceed with evaluating (14) is to start with any of the available actions in A, which will take the role of A_k. Then for each state of the world, the $\underset{A}{\max} x_{ij}$ must be identified, so that with a given utility function, $\underset{A}{\max} u(x_{ij}) - u(x_{kj})$ can be evaluated. It then must be determined if in state j action A_k is revealing of the $\underset{A}{\max} x_{ij}$. If so, the first case of (14) is used for evaluating A_k in state j. If not, the individual-specific δ is applied using the second case of (14). γ_{kj} can also be calculated by identifying the $\underset{A}{\min} x_{ij}$ and applying the individual-specific ρ. p_j is the probability of state j, which is not the same as and will generally be smaller than the probability of getting the consequence x_k under A_k, as x_k may realize in more than one state. The valuation of A_k can then be established by summing over all states. Proceed in the same manner for all $A_i \in A$, and the action that yields the minimum is the indicated choice of action.

Since this is a newly proposed model, there is not really a precedent on what range of values δ and ρ can practically take. The utility function will be parameterized with the power function as in (7), and power parameter α will range from 0.71 to 1.10 in increments of hundredths as in the other model estimations above. δ will be allowed to run from 0.1 to 1 in increments of tenths, and ρ will run from 0 to 5 in increments of 1. To give an idea of how the extreme values of these parameters impact evaluation, return to the Mega Millions choice problem, and assume linear utility for ease of exposition. Consider evaluating the Standard option in the state of the world in which the Megaplier wins \$5 million and the Standard wins \$1 million. Since opting for Standard in this state of the world reveals the counterfactual Megaplier of \$5 million and state max, the first case of (14) is used and δ falls out of consideration. Notice that $\gamma_{kj} = \left(\frac{5{,}000{,}000-1{,}000{,}000}{5{,}000{,}000-0}\right)^{\rho} = 0.8^{\rho}$. At the allowable minimum, $0.8^0 * (5,000,000 - 1,000,000) = 4,000,000$. At the allowable maximum, $0.8^5 * (5,000,000 - 1,000,000) = 1,310,720$, resulting in a difference in regret by a factor of more than three, so that the Standard option will become more favorable in this state of the world as ρ increases. Now consider evaluating Megaplier in the state of the world in which Just the Jackpot wins the \$40 million jackpot with the additional jackpot entry it gets relative to the Megaplier. Holding a jackpot-losing Megaplier ticket does not reveal the additional counterfactual Just the Jackpot winning entry, bringing δ into play. For a given γ_{kj}, the extreme values of δ result in regrets of $\gamma_{kj} * 1 * (40,000,000) = \gamma_{kj} * 40,000,000$ and $\gamma_{kj} * 0.1 * (40,000,000) = \gamma_{kj} * 4,000,000$, a difference by a factor of ten. So, highly discounting non-feedback (low δ) and diminished winner regret (high ρ) has the potential to strongly alter choice prediction.

A cost neutral analysis of the Mega Millions choice problem using RWFM will ensue, followed by a single purchase analysis. Since this is still a regret-based model, predictions will be dependent on the overlap specification of additional Standard entries. Once again, both extremes of full and no overlap are considered, in order to set bounds on predictive power. The total number of parameter-jackpot combinations considered is 19,200: 40 utility parameter values, 10 values of feedback parameter δ, six values of winner regret parameter ρ, tested at eight jackpots. Table 5 presents the results. In the no overlap scenario, 1,956 parameter-jackpot combinations are consistent with a Standard preference, a substantial 10.2% of the combinations in this overlap scenario. Furthermore, Standard is the Mega Millions option with the largest number of combinations predicting its selection, Just the Jackpot at 6.3% and Megaplier at 1.8%. The estimates do not change substantially under a full overlap scenario, with 10.7% of parameter-jackpot combinations indicative of Standard selection and 81.6% predicting non-participation. The insensitivity to the overlap specification is an additional plus of this model, since some sort of mixing between these two extremes is most practical.

There is one downside to the estimation results of the cost neutral RWFM, as no parameter combination predicts Mega Millions participation of any type at the minimum jackpot of \$40 million, although there are numerous combinations that consistently predict Standard selection across the other seven tested jackpots. It should be noted that allowing utility convexity to go just a few hundredths above 1.1 will yield some combinations predicting Mega Millions play and specifically Standard selection. However, it may be a bit concerning that in the concavity-convexity range that experimental studies identify as holding much of the density there is no predicted Mega Millions play. If players approach Mega Millions with a self-imposed demand quota of a single minimum purchase per draw, week, jackpot threshold, etc., the cost neutral approach is no longer appropriate. Under a single purchase approach, all options but Standard remain the same as in the cost neutral approach, but Standard loses its additional entries. So a Standard ticket differs from a Megaplier in that it keeps an additional dollar per Megaplier in exchange for reduced payouts in lower tier eligible states. Since there are no additional Standard entries modeled, there is no longer a need to consider variation in overlap scenarios. The estimation results of both the cost neutral and single purchase approach can be seen side to side in Table 5. Using the same parameter-jackpot combinations as in the cost neutral case, 4,994 of the 19,200 combinations indicate Standard selection, or 26.0%. A majority of combinations still predict non-participation at 64.7%. The primary benefit of this approach over the cost neutral one is that now 318 of the 2,400 parameter combinations (13.3%) tested at the \$40 million jackpot indicate Standard selection, with no other Mega Millions option favored at the \$40 million jackpot. The single purchase approach predicts greater Mega Millions participation at each jackpot level relative to the cost neutral approach.

Table 5: Cost Neutral vs Single Purchase RWFM Predictions

	Percent of Parameter-Jackpot Combinations				
Model	**Standard**	**Megaplier**	**Just the Jackpot**	**None**	**Max Jackpots Standard**
No Overlap	10.2%	1.8%	6.3%	81.7%	7
Full Overlap	10.7%	1.3%	6.4%	81.6%	7
Single Purchase	26.0%	2.6%	6.7%	64.7%	8

Table 5 shows the percentages of tested parameter-jackpot combinations that predict preferences for each of the four options, along with the maximum number of jackpots that any one parameter set predicts a Standard preference for. No overlap is the cost neutral specification for which additional Standard entries share no winning states of the world with Megaplier. Full overlap is the cost neutral specification for which additional Standard entries share all winning states of the world with Megaplier. Single purchase is the specification of a single $2 Standard purchase, $3 Megaplier purchase and $3 Just the Jackpot purchase.

From Table 5 alone it is not necessarily clear as to whether the cost neutral or single purchase approach is more appropriate to model the Mega Millions choice problem. It is possible that cost neutral better captures the problem for some players, while single purchase does better for others. It is even possible that players facing low jackpots restrict themselves to single purchases given the frequency of low jackpots, but switch to behavior more consistent with cost neutrality when jackpots become excessively high. Whatever the explanation may be, each RWFM specification differs with and does significantly better than all of the models analyzed in this paper in three key ways consistent with the sales data:

1. Standard is the Mega Millions ticket option most preferred over wide ranges of parameter combinations.
2. Standard preference is demonstrated for a large set of parameter values across the range of feasible jackpot values.
3. Not playing Mega Millions at all is the preference for most parameter-jackpot combinations.

As was noted earlier, there is effectively no precedent as to an appropriate range of parameter values capturing feedback and winner regret considerations in the population. A wide range of values were tested, perhaps excessively so, in order to see the full implications of the model. Therefore, results like a higher percentage of parameter-jackpot combinations predicting Just the Jackpot than Megaplier should be taken with a grain of salt. This does not imply that we should expect to see a similar pattern in the data, which we don't. This observation applies to a lesser degree to the common models tested above, since the parameter values tested in those cases are based on precedents established in the experimental literature. But the distribution of the population over the presumptive parameter values tested is most certainly non-uniform, so simply imposing the predicted percentages onto the population as expected behavior would be misleading. What the predictions can tell is if a product is expected to be preferred at all, and perhaps give some notion of the intensity of preference in the population. Literal interpretation of the magnitudes of the results in any model rests upon unrealistically strong assumptions.

In the pursuit of parsimony, RWFM can be constrained to remove either the feedback or winner regret parameter. The former is equivalent to setting $\delta = 1$ in (14), whereas the latter is equivalent to setting $\gamma_{kj} = 1$ for all k, j. Table 6 reports the parameter percentages with a no feedback restriction, and Table 7 reports the percentages for a no winner regret restriction. There are some trade offs relative to the unrestricted version of the model, results of which are in Table 5. Removing feedback distinctions renders the cost neutral framework inept at capturing the strong Standard preference in the data over the gamut of feasible jackpots, but the single purchase framework still is quite effective, although the Megaplier is now never predicted at any parameter-jackpot combination. Eliminating the winner regret parameter is marginally better for the cost neutral framework than removing the feedback parameter, but only predicts a Standard preference for a maximum of 4 of the 8 tested jackpots for any of the parameter values considered. Once winner regret is removed, the single purchase framework becomes about as effective in mimicking the data trends as the common models tested earlier. A case can be made that accounting for winner regret and dropping the feedback parameter from the model does well enough at predicting the general data patterns. Accepting this restriction de facto sets the single purchase framework as the appropriate modeling framework as well. Figure 7 displays the parameter distribution at each tested jackpot for the restricted RWFM single purchase model dropping the feedback parameter.

Table 6: RWFM Predictions with No Feedback

	Percent of Parameter-Jackpot Combinations				
Model	**Standard**	**Megaplier**	**Just the Jackpot**	**None**	**Max Jackpots Standard**
No Overlap	1.4%	0%	14.9%	83.7%	2
Full Overlap	1.1%	0%	15.7%	83.2%	2
Single Purchase	25.6%	0%	10.9%	63.5%	8

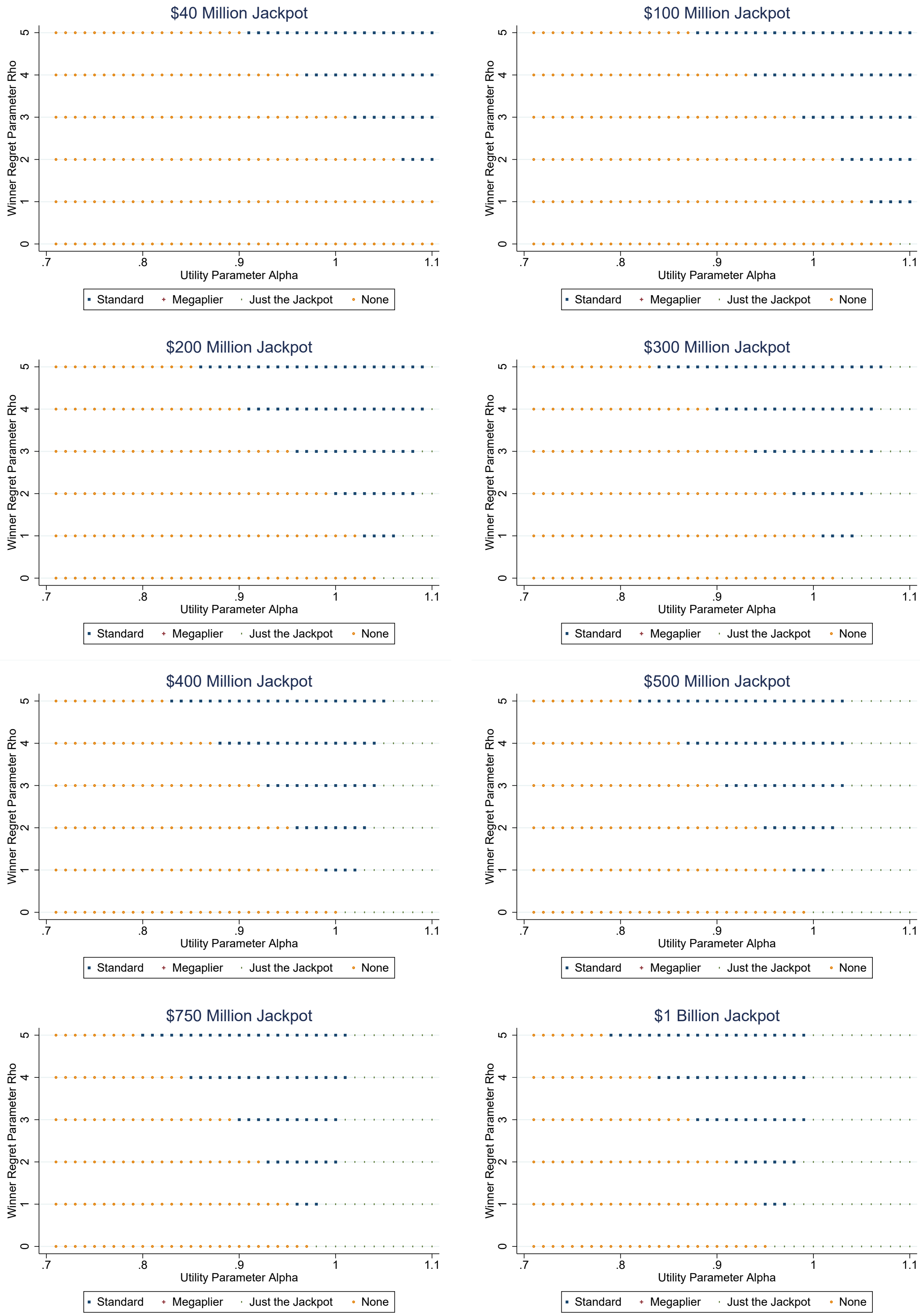
Table 6 shows the percentages of tested parameter-jackpot combinations that predict preferences for each of the four options, along with the maximum number of jackpots that any one parameter set predicts a Standard preference for, under a RWFM model without a feedback differentiation parameter. No overlap is the cost neutral specification for which additional Standard entries share no winning states of the world with Megaplier. Full overlap is the cost neutral specification for which additional Standard entries share all winning states of the world with Megaplier. Single purchase is the specification of a single \$2 Standard purchase, \$3 Megaplier purchase and \$3 Just the Jackpot purchase.

Table 7: RWFM Predictions with No Winner Regret

	Percent of Parameter-Jackpot Combinations				
Model	**Standard**	**Megaplier**	**Just the Jackpot**	**None**	**Max Jackpots Standard**
No Overlap	5.3%	4.8%	7.7%	82.2%	4
Full Overlap	6.0%	4.2%	7.7%	82.1%	4
Single Purchase	0.7%	7.7%	9.3%	82.3%	2

Table 7 shows the percentages of tested parameter-jackpot combinations that predict preferences for each of the four options, along with the maximum number of jackpots that any one parameter set predicts a Standard preference for, under a RWFM model without a winner regret parameter. No overlap is the cost neutral specification for which additional Standard entries share no winning states of the world with Megaplier. Full overlap is the cost neutral specification for which additional Standard entries share all winning states of the world with Megaplier. Single purchase is the specification of a single \$2 Standard purchase, \$3 Megaplier purchase and \$3 Just the Jackpot purchase.

Figure 7: Mapping of Single Purchase RWFM with No Feedback



6.2 Limited Attention, Revisited

Another, perhaps even more puzzling facet of the sales data is the flat or even downward trend of Just the Jackpot sales as a percentage of total sales as jackpots grow in size. This is not consistent with RWFM predictions in any of its specifications analyzed above. As jackpot size increases, not only does the number of parameter combinations predicting Just the Jackpot selection increase, but also the percentage of the combinations predicting Mega Millions play that are Just the Jackpot. The regressions in Table 3 confirm the inverse relationship between Just the Jackpot sales percentage and jackpot size. Note that as the jackpot size increases, Just the Jackpot must become relatively more appealing under any decision framework taking probabilities into account. As the jackpot increases in size, lower tier prizes and probabilities remain unchanged. Therefore, Just the Jackpot sales percentages should increase with jackpot size, but the data indicates the opposite.

The Kansas Lottery Commission field experiment demonstrated that limited attention is not responsible for driving low aggregate sales levels of Just the Jackpot tickets. The jackpot amounts for the draws the promotions were active were \$52 million and \$252 million, both typical jackpot amounts for Mega Millions. There are two general types of players that play games with rolling jackpots. There are the frequent players who play at some frequency across all jackpot levels, with intensity of play perhaps increasing with the jackpot size. Then there are the jackpot frenzy players, who generally abstain from playing until jackpots become unusually high. What the Kansas experiment established is that limited attention is not the cause of poor Just the Jackpot sales among frequent players. The argument proposed here is that extrapolating that conclusion to jackpot frenzy players is not appropriate. Frequent lottery consumers should be expected to have increased awareness of the products available in that market, much more so than infrequent consumers. There is also evidence that as jackpots get excessively high, regressivity of the lottery becomes more proportional or even progressive (Clotfelter and Cook, 1987), meaning that wealthier and higher income individuals make up a much higher percentage of the consumer base of jackpot games at higher jackpots amounts. The sales base changes with the jackpot level: exponential sales increases at unusually high jackpots are not primarily due to regular Mega Millions consumers scaling up their purchases many times over. The change in income distribution at low and high jackpot levels indicates that the large jackpots are driving the participation of the relatively wealthier players. It is reasonable to presume that these wealthier, higher income players derive much less marginal benefit from lower tier prize winnings than poorer players. Therefore, it seems likely that they would be willing to give up the chances for lower tier prizes for additional entries into the jackpot at a given desired expenditure amount. There is also an increase in the pooling of funds for lottery ticket purchases at higher jackpot levels by groups of individuals, like co-workers, friends or family. For such pools that have the intent to split winnings of any tickets purchased, it is not conceivable that any of the lower tier prizes, maybe save the \$1 million, can hold any substantial bearing in the decision to purchase a bunch of tickets. These reasons collectively point to limited attention reducing Just the Jackpot selection at higher jackpot levels. Since there has not been a similar experiment to the Kansas one conducted at an excessively high jackpot level, these reasons highlighting the difference in the player composition at the very least allow for limited attention to remain a viable hypothesis at higher jackpot levels. The author of this paper holds it to be the only plausible explanation, given no theoretical or psychological explanation for the Just the Jackpot sales percentage to decrease with jackpot size.

7 Counterfactual Analysis

7.1 No Feedback Just the Jackpot

The first counterfactual analyzed is how behavior would look if Just the Jackpot tickets did not give ex post feedback on the Standard and Megaplier options. There is a very simple and practical way to accomplish this: require Just the Jackpot entries to be the selection of a single number between 1 and 302,575,350. Such an entry would give no feedback on the foregone Standard or Megaplier options, in the same way that those options don't give feedback on the foregone additional jackpot entries. There is an assumption of indifference between the current selection process and the proposed one, and that players are fully aware of the jackpot odds under both processes. It is possible that the convoluted selection mechanism currently in use allows players to be unaware of the actual odds, although they are displayed clearly on the Mega Millions website. They are not advertised in any way though, so the proposed change to the selection mechanism becomes an advertisement of the jackpot odds, which may influence behavior. This issue is set to the side for the counterfactual analysis.

Enacting the mechanism change for Just the Jackpot only makes it no longer revealing of the outcomes of the foregone Megaplier and Standard tickets. In the model, this means Just the Jackpot evaluation now utilizes δ in all states. Table 8 reports the RWFM predictions for this restructured Mega Millions game. The predictions give further credence to the single purchase specification over the cost neutral ones, as they are no longer able to rationalize a Standard preference over a sufficiently large range of jackpots. For the most part, some of the players opting for Standard or Megaplier in the original game now opt for Just the Jackpot. In fact, comparing the percentages not playing between Tables 6 and 10 indicate that removing the feedback on Just the Jackpot does not increase the overall play rate. This begs the question of whether or not cannibalization by Just the Jackpot of other ticket types is desirable for Mega Millions.

Table 8: RWFM Predictions with No Feedback Just the Jackpot

	Percent of Parameter-Jackpot Combinations				
Model	**Standard**	**Megaplier**	**Just the Jackpot**	**None**	**Max Jackpots Standard**
No Overlap	0.5%	0%	17.9%	81.6%	2
Full Overlap	0.6%	0%	19.5%	79.9%	2
Single Purchase	17.9%	0%	14.3%	67.8%	8

Table 8 shows the percentages of tested parameter-jackpot combinations that predict preferences for each of the four options, along with the maximum number of jackpots that any one parameter set predicts a Standard preference for, under a RWFM model with a restructured Just the Jackpot option that provides no feedback on foregone Megaplier and Standard selections. No overlap is the cost neutral specification for which additional Standard entries share no winning states of the world with Megaplier. Full overlap is the cost neutral specification for which additional Standard entries share all winning states of the world with Megaplier. Single purchase is the specification of a single \$2 Standard purchase, \$3 Megaplier purchase and \$3 Just the Jackpot purchase.

7.2 Optimal Sales and Profits

RWFM is not unique in its prediction of Just the Jackpot primarily cannibalizing existing Mega Millions sales. Table 9 reports the differences in the overall Mega Millions play rates using the various models tested earlier, with and without Just the Jackpot as an option. In spite of the inability of these models to justify the choice behavior in the sales data, these models consistently predict negligible impacts of introducing Just the Jackpot on the extensive margin. Such an analysis could have been undertaken leading up to the creation of Just the Jackpot and would have indicated that the benefit of its introduction would be increased consumer welfare. However, Mega Millions and other lottery products are not designed with consumer welfare as the primary optimization objective. Many states have legislation explicitly stating the objective of lotteries to be revenue maximization for the state. Therefore, inducing players to switch from another Mega Millions option to Just the Jackpot should result in increases in revenue, in order to comply with state legislation.

Consider three counterfactual Mega Millions games, each of which only has one of the three ticketing options available: Standard Mega Millions, Megaplier Mega Millions, and Just the Jackpot Mega Millions. The current Mega Millions game ends up most resembling Standard Mega Millions, given the high percentage of Standard sales. The approach then is to construct hypothetical jackpot and sales progressions in each of the counterfactual games, using estimated jackpot and sales growth in the existing game, along with the existing profitability rate, which is relatively stable across draws. With the hypothetical sales and jackpot progressions in each game, the long run average sales and profitability per draw in each game can be estimated, and by that determine the sales of which ticket type are most in line with state revenue objectives. This procedure implicitly assumes that consumption patterns in these counterfactual games mimic that of the existing game: constant initial sales at the minimum jackpot across games, the same predicted sales growth as a function of the cash value of the jackpot, and the same target profitability rate in each game. These assumptions are quite strong, but if accepted allow for clear predictions about Mega Millions revenue maximization.

Table 9: Cost Neutral Play Rates

	Mega Millions Play Rate		
Model	**All Options**	**No Just the Jackpot**	**Difference**
Expected Utility	20.3%	18.8%	**1.5%**
Prospect Theory	91.4%	91.2%	**0.2%**
Rank Dependent Utility	76.3%	76.1%	**0.2%**
Disappointment Aversion	14.3%	12.9%%	**1.4%**
Regret Theory	75.9%	75.6%	**0.3%**
Salience Theory	88.2%	87.2%	**1.0%**

Table 9 shows the predicted play rates under a cost neutral framework for each of the common models, both with and without Just the Jackpot as an available option. Prospect Theory uses the Prelec (1998) weighting function; Rank Dependent Utility is Cumulative Prospect Theory using the Tversky and Kahneman (1992) weighting function; Disappointment Aversion with a certainty equivalent less than 4 specification; Regret Theory under pairwise aggregation and full overlap scenario; Salience Theory under a no overlap scenario with $\theta = 1,000,000$.

The sales data over the two year period shows a remarkably consistent net revenue rate of 52% of gross sales. As an aside, net revenue is not the same as profits, which would be net revenue less retailer commission and costs of running Mega Millions. Further assume the same retailer commission and additional cost structure across counterfactual game types, so net revenue optimization translates directly to profit maximization. Net revenue for a given draw is defined as gross sales since the last jackpot reset, less lower tier prize payouts since the last jackpot reset, less the cash value of the jackpot for that draw. For example, a minimum jackpot of $40 million may have a cash value of $25 million, depending on interest rates and other factors. If the sales for that draw were $20 million and lower tier payouts were \$2 million, potential net revenue would be $\$20,000,000 - \$2,000,000 - \$25,000,000 = -\$7,000,000$. Consider the jackpot rolling over to \$50 million with a cash value of \$30 million, with sales for that draw of \$20 million and lower tier payouts of \$2 million for that draw. Potential net revenue for that draw would be $(\$20,000,000 + \$20,000,000) - (\$2,000,000 + \$2,000,000) - \$30,000,000 = \$6,000,000$. Mega Millions essentially chooses the cash value of the jackpot for the coming round based on predicted sales and converts that into an advertised annuitized amount so that net revenue is at 52%, save for the first few draws of a given round of jackpots due to the relatively low sales compared to the jackpot size. This is true for all the jackpot rounds except the first one after the October 2017 changes, in which net revenue is lower and more volatile. There is an inherent endogeneity of jackpot and sales determination: sales are the sole influencer of jackpot size, while jackpot size will influence sales. Mega Millions announces a tentative jackpot for the next round at the end of the draw, which they determine based on sales projections. Mega Millions at times scales up the jackpot due to unexpectedly high sales, but does not reduce the jackpot if sales are abnormally low. Over the time range in the data, Mega Millions does reasonably well in setting jackpot amounts to keep expected net revenue stable at about 52%: the interquartile range over the two years of data for draws outside of the first few and the first round of jackpots is 51.8% to 52.3%. Note that expected net revenue in the data is calculated using expected and not actual lower tier payouts, and that it can be calculated for every draw, although it will only realize for a draw with a winning jackpot.

The sales data can be leveraged to establish a relationship between jackpots and sales to predict sales growth as a function of the natural log of the lagged cash values of the jackpot. A quadratic fit works much better than a linear fit at modeling the sales growth, highlighting convexity in growth as jackpots increase. In each of the counterfactual games the same minimum jackpot of \$40 million and initial gross sales for the first draw of \$22 million are assumed, which is approximately the average first draw sales across the time period. For the first few draws in the actual data, jackpots usually increase in increments of \$5 million, with sales holding relatively constant with the increasing jackpots. However, profitability increases significantly with each draw, settling at the 52% after about 4 draws. This sales and jackpot initiation are copied to the three counterfactual games. After the first few draws the lagged cash jackpots are used to estimate sales growth and therefore predicted sales for the next draw. Using these predicted sales, a cash value of the jackpot can be determined by setting the profitability rate to 52%. Profitability is determined differently in each version: Megaplier Mega Millions has high expected lower tier payoffs, so that jackpot growth is relatively low draw to draw; Just the Jackpot Mega Millions has no lower tier payoffs, so all the payout is packed into the jackpot, which grows fastest in this version; Standard Mega Millions takes a middle path with moderate lower tier payments and jackpot growth. Note that Megaplier Mega Millions gives the worst odds at the jackpot, while Just the Jackpot Mega Millions gives the best. The result is that the expected length of a jackpot round (23) is longest under Megaplier Mega Millions, and lowest (13) under Just the Jackpot Mega Millions.

The construction of hypothetical sales and jackpot progressions also allows for calculating the probability of reaching a specific draw number in each counterfactual game via the binomial distribution. It would be misleading to simply choose a probability level and compare the net revenues between games, since this would not be giving an appropriate notion of comparable revenue, due to variation in the number of draws to reach that probability level across games. What can be done is to normalize the revenues in each draw by dividing revenue in a draw by the number of draws up to that point in the round. Once that is done, the probabilities of reaching a certain draw in each game can be used to construct CDFs of net revenue in each game. The convexity of net revenue progressions allows net revenue per draw to be increasing monotonically in draws. Figure 8 displays the CDFs of net revenue in each game, and Figures 8a and 8b are magnifications of Figure 8 for ease of visibility. There is no FOSD of any distribution, but the Megaplier CDF is below and to the right of the CDF of Just the Jackpot for probabilities larger than about 15%. Standard is also dominated by Megaplier for most of the distribution, although there is some crossover at the top few percent. Median net revenue occurs at the draw number for which the probability of reaching that draw number falls below the 50% threshold, which occurs at different draw numbers in each game. Median Megaplier net revenue is about \$14.4 million, larger than median Standard net revenue of about \$13.7 million and median Just the Jackpot net revenue of \$13.4 million. These amounts can be seen visually by checking where the 50% probability line intersects each of the CDFs in Figure 8b. The same task is also undertaken for gross sales with similar patterns. Figure 9 displays the CDFs of gross sales for each counterfactual game, accompanied by Figures 9a and 9b as magnifications of Figure 9.

Figure 8: CDFs of Net Revenue

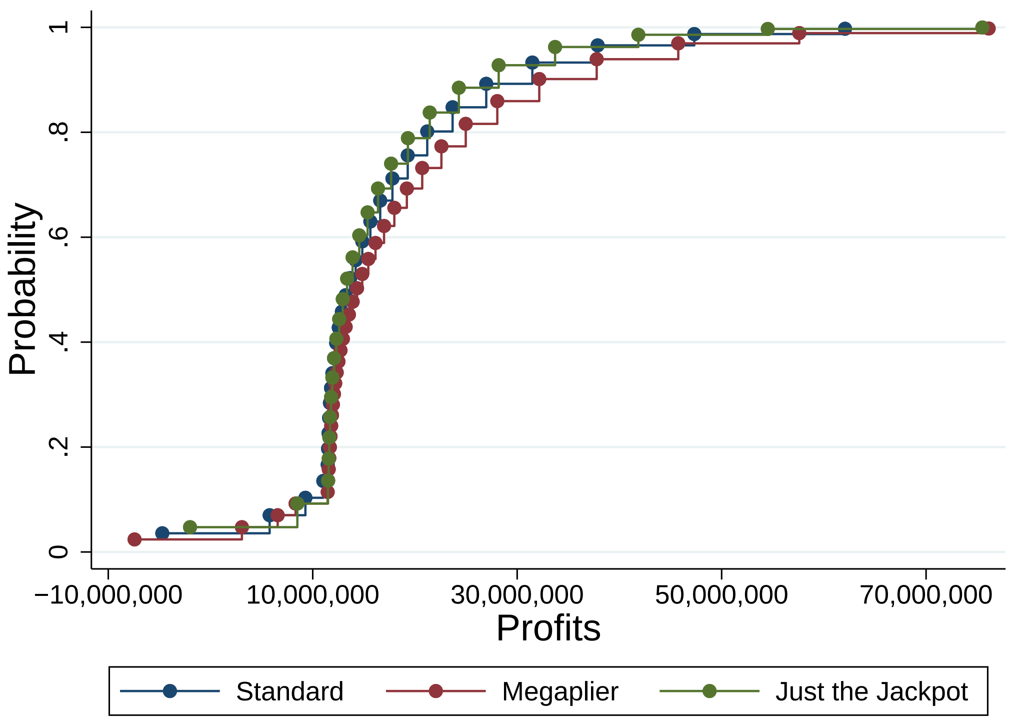


Figure 8a: Lower CDFs of Net Revenue

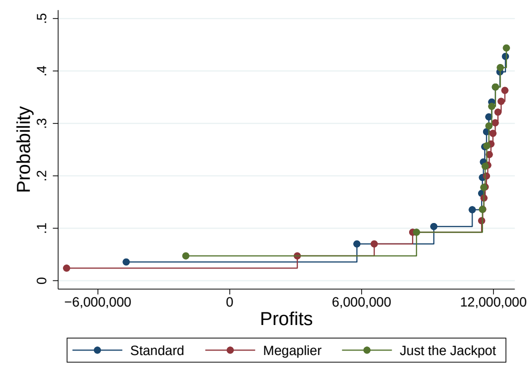


Figure 8b: Upper CDFs of Net Revenue

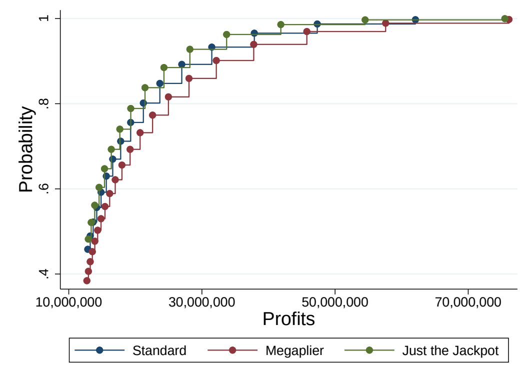


Figure 9: CDFs of Gross Sales

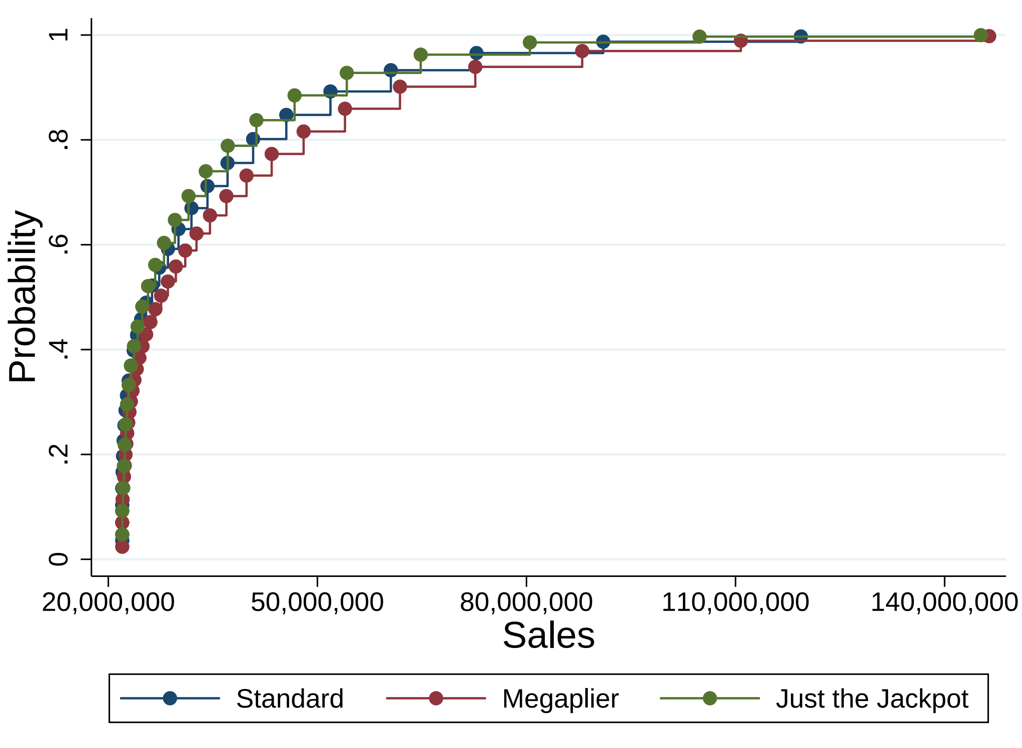


Figure 9a: Lower CDFs of Gross Sales

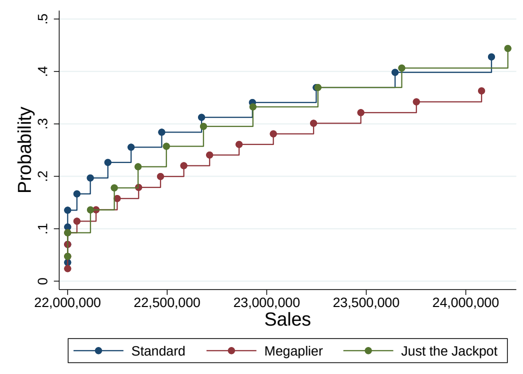
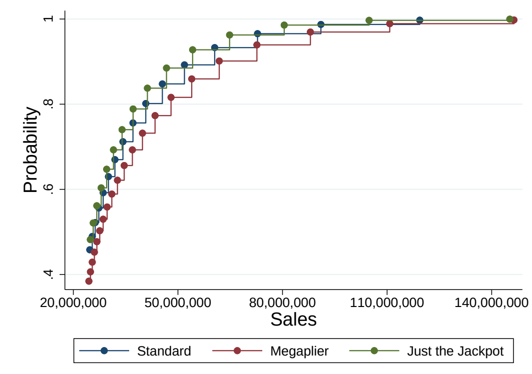


Figure 9b: Upper CDFs of Gross Sales



While the CDFs and median net revenue comparisons give some idea of product desirability from the point of view of revenue maximization for states, average net revenue is the most useful comparison, since it is something states could actually work into budget plans. The convexity of net revenue results in a right-skewed distribution, so that the median values do not give approximations to the means. Means are estimated by running through 1,000 concurrent simulations of each of the counterfactual games. This results in about 70% more draws in the Megaplier version than the Just the Jackpot version, and about 35% more than the Standard version. The 1,000 simulations result in 12,904 draws in the Just the Jackpot game. The mean net revenue and sales for the Just the Jackpot game can be estimated by dividing the total net revenue and sales by 12,904. The simulations result in 16,271 draws of the Standard game. One way to estimate the mean would be to proceed in the same way as Just the Jackpot mean determination, by dividing aggregates by the number of rounds. An alternative way is to keep the number of draws fixed between games instead of the number of rounds, so that time is constant across games. This requires finding the simulation number in the Standard and Megaplier versions for which the number of draws crosses 12,904, the number of draws in the 1,000 Just the Jackpot simulations. Only aggregate Standard and Megaplier revenues and sales for the simulations before the 12,904th draw is crossed and normalize. Results are robust to either mean calculation method. The mean sales and net revenue estimates using the fixed draw approach are presented in Table 10. The Megaplier version not only has the largest median sales and net revenues, but also has the highest mean ones as well. The Just the Jackpot version only yields approximately 85% the average sales and net revenues of Megaplier, while the Standard version yields about 90% of the Megaplier values. These results indicate that attempting to get players to switch from Megaplier and Standard selections to Just the Jackpot is not revenue maximizing and therefore not in line with the objectives explicitly outlined in the legislation of many states that offer lottery products. Just the Jackpot introduction would only be consistent with state objectives if it brought in players who were either not playing any lotteries or switching from an even less profitable lottery product outside Mega Millions. However, the mainstream preference-based decision models do not predict gains on the extensive margin by introducing Just the Jackpot into the existing Mega Millions game. The analysis places some serious question marks on the introduction of Just the Jackpot as a Mega Millions option.

Table 10: Mean Sales and Net Revenues

	Sales	Net Revenues	Sales % Megaplier	Net Revenues % Megaplier
Megaplier	$46,923,590	$24,346,334	100%	100%
Standard	$42,224,822	$21,890,201	90.0%	89.9%
Just the Jackpot	$39,938,215	$20,705,705	85.1%	85.0%

Table 10 shows the mean sales and net revenues for each of the hypothetical games based on 1,000 simulations of the prospective jackpot and sales progressions, as well as each of the game's means as a percentage of Megaplier means.

8 Conclusion

This paper has used sales data to analyze the underlying choice behavior in the Mega Millions lottery game since the introduction of the Just the Jackpot ticket option in October 2017. Just the Jackpot had some initial interest on its opening draw of about 2% of Mega Millions sales, but levels dropped to the 0.2% to 0.3% range within a few months. A variety of the more popular and accepted decision theoretic models all predicted higher levels of interest in the Just the Jackpot option then the data demonstrated, as well as low interest in the Standard option. Alternative explanations outside of the theoretical decision modeling framework, like limited attention, liquidity constraints and slow sales adjustments, were also ruled out. Further investigation pinned down the likely culprits to differences in ex post feedback on foregone outcomes between the various choices, along with low amounts of winner regret that having a Standard entry engenders in lower prize tier-eligible states of the world. The Regret Weighted Feedback Minimax model that captures these two behavioral tendencies is proposed and demonstrated to unequivocally outperform existing models in rationalizing the data from all perspectives. The inverse relationship between Just the Jackpot sales percentage and jackpot size in the data, which no reasonable model can account for, is argued to be due to limited attention of jackpot frenzy players. These players only participate when jackpots become abnormally high and may not have much experience with or be fully aware of the options available to play. If such players were made fully aware of the Just the Jackpot option, it is presumed that the expected positive relationship between jackpot size and Just the Jackpot sales percentage would be observed in the data. Counterfactual analysis suggests that introducing Just the Jackpot into the existing Mega Millions structure will not bring in new players, but rather cause some existing players to switch to Just the Jackpot from another ticket option. Under reasonable assumptions, such switching is shown to not be net revenue maximizing and therefore not in line with state legislative mandates. These results pose serious doubts relating to the introduction of Just the Jackpot in the first place, independent of the actual disinterest in the product demonstrated by players.

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Appendix

Mega Millions Prize-Odds Matrix[4]

Match	Prize*	Odds
○ ○ ○ ○ ○ + ●	Jackpot	1 in 302,575,350
○ ○ ○ ○ ○	$1,000,000	1 in 12,607,306
○ ○ ○ ○ + ●	$10,000	1 in 931,001
○ ○ ○ ○	$500	1 in 38,792
○ ○ ○ + ●	$200	1 in 14,547
○ ○ ○	$10	1 in 606
○ ○ + ●	$10	1 in 693
○ + ●	$4	1 in 89
●	$2	1 in 37
Overall chances of winning any prize:		1 in 24

[4]Taken from https://www.megamillions.com/How-to-Play.aspx on 09/17/20

Feedback-Conditional Regret Theory

Feedback-Conditional Regret Theory (FCRT) is an extension of Regret Theory proposed by Humphrey (2004) which allows for variation in feedback on outcomes of foregone choices to be included in decision modeling. The model maintains Regret Theory's native pairwise environment. The decision criteria is a generalization of (9)

$$A_i \succeq A_k \Longleftrightarrow \sum_{j=1}^{n} p_j[M(x_{ij}, x_{kj}) - M(x_{kj}, x_{ij})] \geq 0$$

For the purposes of the forthcoming analysis, (13) will be directly related to (9) by setting

$$M(x_{ij}, x_{kj}) = c_{ij} + R(c_{ij} - c_{kj})$$

where modified utility function $M(x_{ij}, x_{kj})$ can lose its symmetric character if the ex post regret status in state j differs between $A_i$and A_k. In the case that in state j, receiving x_{ij} fully reveals the state of the world and foregone outcome x_{kj}, $M(x_{ij}, x_{kj}) = m(x_{ij}, x_{kj})$. In the case that in state j, receiving x_{ij} does not fully reveal the state of the world and foregone outcome x_{kj}, $M(x_{ij}, x_{kj}) = \mu(x_{ij}, x_{kj})$. However, "the decision-maker has anticipated a state of the world under which they will receive x_{ij} and forego x_{kj}, but actually receiving x_{ij} does not reveal x_{kj} (as opposed to some other outcome, say, x_{kj*}) as the outcome of the foregone act" (Humphrey, 2004). Three conditions are imposed on $M(.,.)$ when $x_{ij} > x_{kj}$:

1. $m(x_{ij}, x_{kj}) > \mu(x_{ij}, x_{kj})$
2. $m(x_{kj}, x_{ij}) < \mu(x_{kj}, x_{ij})$
3. $\mu(x_{kj}, x_{ij}) - m(x_{kj}, x_{ij}) > m(x_{ij}, x_{kj}) - \mu(x_{ij}, x_{kj})$

The first two conditions highlight that an action fully revealing the state of the world and foregone outcome in that state has amplified utility when the outcome is larger than the foregone outcome relative to non-revelation, and diminished utility when the outcome is smaller than the foregone outcome relative to non-revelation. The third condition states that the magnitude of the difference in utilities between revelation and non-revelation is larger in the case of regret than rejoice. Humphrey (2004) does not provide a precise structural form or suggested parameterization to his FCRT. This paper will assume original RT, with a choiceless utility portion that is the same regardless of feedback, and an $R(.)$ function, where $R(.)$ is scaled up by a $\delta > 1$ if A_i will reveal the outcome of A_k in state j. With no feedback of the outcome of the foregone action ex post in state j, $R(c_{ij} - c_{kj})$ is used, and with feedback, $\delta * R(c_{ij} - c_{kj})$ is used. This functional choice of the relationship between $m(.,.)$ and $\mu(.,.)$ is chosen because of its simplicity and ease of interpretation, but is clearly not the only formulation consistent with the Humphrey (2004) stipulations. Notice that in order to fulfill the third condition on $M(.,.)$ using (7) to model $R(c_{ij} - c_{kj})$, $\beta > \alpha$ is necessary. To start, $\beta = \alpha$ will be forced so that the third condition is only weakly satisfied.

Notice that ex post feedback is not necessarily symmetric: A_i may reveal the outcome of A_k in state j, but A_k may or may not reveal the outcome of A_i in state j. For example, the actions of opting into a fair coin flip and not opting in display the asymmetry: flipping the coin reveals the foregone outcome of not playing in either resultant states of the world, whereas opting out does not reveal the foregone outcome, since the coin is never flipped. However, the choice could be modified so that the coin will be flipped and state of the world revealed, independent of choice of action. In that case, both actions would be fully revealing of the state of the world and outcomes of the foregone action. The implication is that in this modified game, rejoice would be larger in magnitude than in the original game for a favorable flip, and regret would also be larger in magnitude in the event of an unfavorable flip. It is also possible that an action may only reveal the outcomes of foregone actions in some but not all states of the world. Then, only in those states would $m(.,.)$ be used for utility evaluation, $\mu(.,.)$ would be used in all other states.

Mega Millions falls into this partial revelation framework. Not playing Mega Millions leaves one with the cost of the ticket with certainty, and there is no knowledge ex post of what the outcome of playing any ticket type would have been. Pairwise comparisons between not playing and each of the three ticket types results in utilizing $\mu(.,.)$ in every state when evaluating not playing, and utilizing $m(.,.)$ in every state when evaluating any ticket type. Pairwise comparisons between ticket types is different. At any given cost, Just the Jackpot gives the greatest number of distinct entries. For example, at a cost of \$6, Just the Jackpot gives four number combination entries, Standard gives three, and Megaplier gives two. So, consider evaluating the choice between Standard and Just the Jackpot. A choice of Just the Jackpot would be fully revealing of the counterfactual Standard in every resultant state: if one or more of those tickets are partial number matches, those would have been eligible for lower tier prizes had they been Standard. If none of them are partial matches, well then if they had been Standard tickets, they would not have been eligible. If one of them won the jackpot, it also would have won the jackpot had it been Standard. Now consider the perspective of choosing Standard. If any of those three tickets are lower tier eligible, that would indicate that they would be ineligible had they been Just the Jackpot. If any of them were jackpot winners, they also would be under Just the Jackpot designation. Now the asymmetry sets in. If none of the tickets are prize eligible, under Just the Jackpot designation a fourth ticket would have been generated, with two possibilities: that ticket wins the jackpot, or is a partial match or complete mismatch and wins nothing. Therefore, Standard is fully revealing of foregone outcomes of Just the Jackpot in some states, whereas Just the Jackpot is fully revealing of foregone Standard outcomes in every state. Revelation moves in increasing order respectively with not playing, Megaplier, Standard and Just the Jackpot. It may be that Just the Jackpot as the most revealing option is diminishing its appeal.

The framework of FCRT is the same as that of RT, so the same concerns and considerations highlighted in RT carry over to modeling FCRT. Two aggregation methods were used in RT modeling, pairwise and state-wise mean utilities. For FCRT, only pairwise mean utilities will be modeled. This is because with the factoring in of feedback into RT, calculating $c_{ij} - mean(c_{-ij})$ as the input to $R(.)$ becomes difficult to interpret. Relative to A_i in state j, some foregone actions may have their outcomes revealed whereas others may not. It becomes unclear as to whether or not $m(.,.)$ or $\mu(.,.)$ should be used in evaluation. Therefore this aggregation method is not modeled under FCRT, only pairwise is, in which averages are taken of pairwise utilities and the feedback parameter can be applied unequivocally. The two overlap scenarios from RT also carry over, and in this case there is no confounding that feedback provides in interpretation. Under FCRT, full overlap does impact the feedback channel relative to no overlap. With full overlap of lower tier prizes, the state-wise differentiation in feedback between Megaplier and Standard is effectively removed. Since Megaplier and Standard have the same sets of winning numbers under full overlap, Megaplier becomes fully revealing of foregone Standard outcomes in all states, whereas it does not in states in which a non-overlapping Standard ticket would be prize eligible.

The total number of tested parameter-jackpot combinations is 32,000 per overlap scenario: the choiceless utility power function parameter runs from .71 to 1.10 in increments of hundredths; $R(.)$ also takes the form of a power function, and its parameter runs from 0.6 to 3.0 in increments of tenths; feedback parameter δ runs from 2 to 5 in increments of one. Both boundary overlap scenarios are tested under pairwise mean aggregation for the same 8 jackpot levels tested in all other models. Only 8 parameter-jackpot combinations are consistent with a Standard preference under the no overlap scenario of a possible 32,000. Table A1 presents the test results of FCRT using the full overlap scenario, which finds 305 parameter-jackpot combinations indicative of a Standard preference. Still, no parameter combination predicts a Standard preference for the five highest of the eight jackpots under either overlap specification. FCRT does only marginally better than RT at capturing the choice behavior in the data, and cannot be said to be remotely consistent with it.

Table A1: Feedback-Conditional Regret Theory Predictions

	Percent of Parameter-Jackpot Combinations				
Model	Standard	Megaplier	Just the Jackpot	None	Max Jackpots Standard
FCRT	1.0%	0.9%	74.7%	23.4%	3
Regret-only FCRT	2.6%	0.7%	71.4%	25.3%	4

Table A1 shows the percentages of tested parameter-jackpot combinations that predict preferences for each of the four options, along with the maximum number of jackpots that any one parameter set predicts a Standard preference for. FCRT is Feedback-Conditional Regret Theory under the full overlap specification; Regret-only FCRT is Feedback-Conditional Regret Theory on regretful outcomes only under the full overlap specification.

As was noted above, the previous analysis set $\beta = \alpha$ and therefore only weakly satisfied condition three on $M(.,.)$, whereas Humphrey (2004) lists the condition in strict form. It is important to mention that this condition is essentially a behavioral assumption that regret is a more salient feeling than rejoice, although one might incline to agreement. The other extreme, which does satisfy condition three, is to assume that ex post feedback does not increase the feeling of rejoice, but only that of regret. This is consistent with the example given in Zeelenberg (1999), and amounts to

$$\begin{cases} m(x_{ij}, x_{kj}) = \mu(x_{ij}, x_{kj}) & x_{ij} > x_{kj} \\ m(x_{kj}, x_{ij}) = \delta * \mu(x_{kj}, x_{ij}) & x_{ij} > x_{kj}, \delta > 1 \end{cases}$$

This other extreme is also tested, with somewhat better results than the forced equivalency of rejoice and regret above. The same 64,000 parameter-jackpot-scenario combinations are run through, with no overlap yielding just 4 parameter-jackpot combinations at only the two lowest jackpots with a predicted Standard preference. The full overlap condition is much better, with 830 parameter-jackpot combinations consistent with a Standard preference, or 2.6% of the full overlap combinations. Results of this specification are presented in Table A1. These span seven of the eight jackpots, with high concentration at the lowest jackpots. Furthermore, no one set of parameters can explain a Standard preference for more than four jackpot levels. While the full overlap scenario fares somewhat decently relative to most of the models and scenarios tested, full overlap is again a boundary condition, with partial or no overlap much more realistic assumptions. Bolstering this conclusion is the fact that about 80% of Mega Millions tickets in Texas over the past ten years were randomly generated and not self-selected, and therefore no or partial overlap is unequivocally the better assumption for those 80%, let alone the likelihood of players purchasing multiple entries and self-selecting the same number combinations. One positive takeaway from full overlap FCRT is that this is the first model in which

Standard does not hold the fewest parameter-jackpot predictions, as it just outperforms the Megaplier. However, the conclusion is that FCRT is also unable to account for the observed Mega Millions choice behavior.

Table A2: Mega Millions Sales for States Offering Just the Jackpot

Date	Jackpot	Total	Standard	Megaplier	Just the Jackpot
10/31/2017	$40,000,000	$7,282,759	91.15%	6.76%	2.08%
11/3/2017	$48,000,000	$8,037,966	91.89%	6.94%	1.17%
11/7/2017	$59,000,000	$7,435,573	91.83%	7.17%	0.99%
11/10/2017	$71,000,000	$7,980,197	92.01%	7.14%	0.85%
11/14/2017	$82,000,000	$7,741,920	92.09%	7.09%	0.82%
11/17/2017	$95,000,000	$8,517,323	92.31%	6.97%	0.71%
11/21/2017	$106,000,000	$8,847,507	91.95%	7.40%	0.65%
11/24/2017	$119,000,000	$8,353,300	91.78%	7.62%	0.60%
11/28/2017	$132,000,000	$9,239,587	92.01%	7.41%	0.58%
12/1/2017	$145,000,000	$10,199,466	92.07%	7.41%	0.52%
12/5/2017	$160,000,000	$9,870,955	92.52%	6.94%	0.54%
12/8/2017	$176,000,000	$10,157,419	92.75%	6.76%	0.49%
12/12/2017	$191,000,000	$10,455,870	92.77%	6.74%	0.49%
12/15/2017	$208,000,000	$11,932,542	92.90%	6.65%	0.45%
12/19/2017	$223,000,000	$12,930,848	93.20%	6.37%	0.43%
12/22/2017	$253,000,000	$16,356,936	93.57%	6.05%	0.38%
12/26/2017	$277,000,000	$14,952,197	93.50%	6.11%	0.39%
12/29/2017	$306,000,000	$27,881,874	93.93%	5.73%	0.33%
1/2/2018	$361,000,000	$32,219,824	94.15%	5.53%	0.32%
1/5/2018	$450,000,000	$50,135,632	94.14%	5.56%	0.30%
1/9/2018	$40,000,000	$8,934,178	92.37%	7.19%	0.44%
1/12/2018	$45,000,000	$8,435,116	92.14%	7.47%	0.40%
1/16/2018	$50,000,000	$7,680,835	92.21%	7.38%	0.40%
1/19/2018	$55,000,000	$8,413,244	92.17%	7.45%	0.38%
1/23/2018	$63,000,000	$8,010,639	92.16%	7.44%	0.40%
1/26/2018	$76,000,000	$8,799,877	92.23%	7.41%	0.35%
1/30/2018	$89,000,000	$8,265,817	92.15%	7.47%	0.39%
2/2/2018	$104,000,000	$9,644,471	92.25%	7.40%	0.35%
2/6/2018	$120,000,000	$9,245,950	92.29%	7.34%	0.37%
2/9/2018	$136,000,000	$9,875,370	92.41%	7.27%	0.33%
2/13/2018	$153,000,000	$9,801,786	92.43%	7.22%	0.35%
2/16/2018	$168,000,000	$10,701,619	92.63%	7.05%	0.32%
2/20/2018	$185,000,000	$9,999,342	92.53%	7.12%	0.34%
2/23/2018	$204,000,000	$11,552,057	92.70%	6.99%	0.31%
2/27/2018	$222,000,000	$12,165,814	92.85%	6.83%	0.32%
3/2/2018	$243,000,000	$12,912,058	92.70%	6.99%	0.31%
3/6/2018	$265,000,000	$13,858,603	93.00%	6.69%	0.31%
3/9/2018	$290,000,000	$14,282,330	92.91%	6.80%	0.30%
3/13/2018	$318,000,000	$16,366,748	93.24%	6.47%	0.29%
3/16/2018	$345,000,000	$18,870,642	93.38%	6.35%	0.26%

Date	Jackpot	Total	Standard	Megaplier	Just the Jackpot
3/20/2018	$377,000,000	$19,415,088	93.31%	6.38%	0.31%
3/23/2018	$421,000,000	$21,290,674	93.29%	6.40%	0.31%
3/27/2018	$458,000,000	$24,879,498	93.52%	6.17%	0.31%
3/30/2018	$521,000,000	$51,350,316	94.18%	5.58%	0.25%
4/3/2018	$40,000,000	$8,664,295	91.87%	7.80%	0.33%
4/6/2018	$45,000,000	$8,741,042	91.90%	7.80%	0.30%
4/10/2018	$50,000,000	$8,136,523	91.82%	7.86%	0.32%
4/13/2018	$55,000,000	$9,039,829	92.02%	7.70%	0.28%
4/17/2018	$67,000,000	$8,228,002	91.90%	7.78%	0.32%
4/20/2018	$80,000,000	$9,019,060	92.05%	7.66%	0.29%
4/24/2018	$96,000,000	$8,643,616	92.08%	7.62%	0.31%
4/27/2018	$111,000,000	$9,946,819	92.35%	7.38%	0.27%
5/1/2018	$126,000,000	$9,798,001	92.23%	7.48%	0.29%
5/4/2018	$143,000,000	$10,506,463	92.35%	7.37%	0.28%
5/8/2018	$40,000,000	$7,498,599	91.79%	7.91%	0.30%
5/11/2018	$45,000,000	$7,960,610	91.94%	7.78%	0.28%
5/15/2018	$50,000,000	$7,356,060	91.74%	7.96%	0.30%
5/18/2018	$55,000,000	$8,085,046	91.96%	7.77%	0.28%
5/22/2018	$60,000,000	$7,416,149	91.86%	7.85%	0.29%
5/25/2018	$73,000,000	$8,378,826	92.07%	7.66%	0.27%
5/29/2018	$84,000,000	$7,456,218	91.89%	7.82%	0.30%
6/1/2018	$97,000,000	$9,107,809	92.12%	7.61%	0.27%
6/5/2018	$110,000,000	$9,291,018	92.22%	7.49%	0.28%
6/8/2018	$127,000,000	$9,955,497	92.44%	7.30%	0.26%
6/12/2018	$144,000,000	$9,784,722	92.45%	7.27%	0.28%
6/15/2018	$161,000,000	$10,495,370	92.56%	7.18%	0.26%
6/19/2018	$175,000,000	$10,214,179	92.54%	7.18%	0.28%
6/22/2018	$192,000,000	$11,325,558	92.60%	7.13%	0.27%
6/26/2018	$212,000,000	$12,108,530	92.66%	7.05%	0.29%
6/29/2018	$232,000,000	$13,578,444	92.71%	7.01%	0.28%
7/3/2018	$256,000,000	$14,206,276	92.63%	7.07%	0.30%
7/6/2018	$283,000,000	$14,250,322	92.69%	7.02%	0.29%
7/10/2018	$306,000,000	$16,881,618	93.01%	6.70%	0.30%
7/13/2018	$340,000,000	$20,728,668	93.17%	6.56%	0.27%
7/17/2018	$375,000,000	$21,488,624	93.30%	6.42%	0.28%
7/20/2018	$433,000,000	$34,642,408	93.88%	5.88%	0.24%
7/24/2018	$522,000,000	$60,161,128	94.29%	5.48%	0.23%
7/27/2018	$40,000,000	$8,131,869	91.83%	7.88%	0.29%
7/31/2018	$45,000,000	$7,665,313	91.68%	8.02%	0.30%
8/3/2018	$50,000,000	$8,271,744	91.79%	7.94%	0.27%
8/7/2018	$55,000,000	$7,805,630	91.73%	7.98%	0.29%
8/10/2018	$63,000,000	$8,363,887	91.87%	7.87%	0.26%

Date	Jackpot	Total	Standard	Megaplier	Just the Jackpot
8/14/2018	$75,000,000	$8,013,807	91.83%	7.88%	0.29%
8/17/2018	$88,000,000	$8,825,262	92.10%	7.64%	0.26%
8/21/2018	$102,000,000	$9,315,947	92.21%	7.51%	0.27%
8/24/2018	$118,000,000	$10,036,574	92.31%	7.43%	0.26%
8/28/2018	$134,000,000	$9,822,489	92.30%	7.43%	0.27%
8/31/2018	$152,000,000	$10,976,064	92.41%	7.33%	0.26%
9/4/2018	$167,000,000	$10,000,861	92.30%	7.43%	0.28%
9/7/2018	$187,000,000	$11,436,642	92.56%	7.19%	0.25%
9/11/2018	$207,000,000	$11,832,820	92.68%	7.05%	0.27%
9/14/2018	$227,000,000	$12,810,338	92.86%	6.89%	0.25%
9/18/2018	$252,000,000	$12,799,656	92.81%	6.91%	0.28%
9/21/2018	$275,000,000	$13,730,861	92.94%	6.79%	0.26%
9/25/2018	$303,000,000	$15,337,032	92.97%	6.75%	0.28%
9/28/2018	$336,000,000	$17,082,332	93.17%	6.57%	0.26%
10/2/2018	$367,000,000	$18,566,592	93.29%	6.44%	0.27%
10/5/2018	$420,000,000	$24,050,072	93.65%	6.10%	0.25%
10/9/2018	$470,000,000	$28,247,306	93.90%	5.84%	0.26%
10/12/2018	$548,000,000	$43,123,228	94.05%	5.71%	0.24%
10/16/2018	$667,000,000	$78,702,176	94.41%	5.36%	0.23%
10/19/2018	$1,000,000,000	$207,492,976	94.45%	5.32%	0.23%
10/23/2018	$1,600,000,000	$270,925,600	94.36%	5.38%	0.26%
10/26/2018	$40,000,000	$13,036,618	91.83%	7.87%	0.30%
10/30/2018	$45,000,000	$10,551,508	91.71%	8.02%	0.27%
11/2/2018	$52,000,000	$10,219,383	91.52%	8.08%	0.40%
11/6/2018	$70,000,000	$10,557,753	92.38%	7.38%	0.24%
11/9/2018	$90,000,000	$9,887,921	91.85%	7.92%	0.23%
11/13/2018	$106,000,000	$9,892,155	91.94%	7.81%	0.24%
11/16/2018	$122,000,000	$10,390,820	91.89%	7.89%	0.22%
11/20/2018	$139,000,000	$10,697,782	92.06%	7.71%	0.23%
11/23/2018	$155,000,000	$9,695,451	91.81%	7.97%	0.23%
11/27/2018	$172,000,000	$10,518,311	92.08%	7.70%	0.22%
11/30/2018	$190,000,000	$11,379,043	92.15%	7.64%	0.21%
12/4/2018	$208,000,000	$11,760,428	92.25%	7.52%	0.23%
12/7/2018	$226,000,000	$11,971,119	92.35%	7.43%	0.21%
12/11/2018	$245,000,000	$11,941,937	92.37%	7.41%	0.22%
12/14/2018	$262,000,000	$12,661,328	92.46%	7.32%	0.22%
12/18/2018	$284,000,000	$12,616,857	92.45%	7.32%	0.23%
12/21/2018	$305,000,000	$14,181,272	92.64%	7.14%	0.22%
12/25/2018	$321,000,000	$20,179,728	93.32%	6.48%	0.19%
12/28/2018	$370,000,000	$22,724,544	93.07%	6.74%	0.20%
1/1/2019	$425,000,000	$35,750,088	93.40%	6.40%	0.21%
1/4/2019	$40,000,000	$9,123,219	91.73%	8.05%	0.22%

Date	Jackpot	Total	Standard	Megaplier	Just the Jackpot
1/8/2019	$45,000,000	$8,562,782	91.66%	8.11%	0.23%
1/11/2019	$50,000,000	$8,886,587	91.62%	8.17%	0.21%
1/15/2019	$55,000,000	$8,388,774	91.62%	8.16%	0.22%
1/18/2019	$68,000,000	$9,132,415	91.80%	8.00%	0.20%
1/22/2019	$82,000,000	$8,193,440	91.56%	8.22%	0.22%
1/25/2019	$96,000,000	$9,172,900	91.78%	8.02%	0.20%
1/29/2019	$109,000,000	$9,492,464	92.00%	7.79%	0.21%
2/1/2019	$125,000,000	$10,061,010	91.86%	7.93%	0.21%
2/5/2019	$139,000,000	$10,369,302	92.03%	7.76%	0.21%
2/8/2019	$157,000,000	$10,528,767	92.07%	7.73%	0.20%
2/12/2019	$173,000,000	$10,018,091	91.92%	7.86%	0.22%
2/15/2019	$190,000,000	$11,229,604	92.22%	7.58%	0.20%
2/19/2019	$206,000,000	$11,180,536	92.22%	7.56%	0.22%
2/22/2019	$224,000,000	$12,400,790	92.39%	7.42%	0.20%
2/26/2019	$245,000,000	$12,685,180	92.45%	7.34%	0.21%
3/1/2019	$267,000,000	$14,024,939	92.39%	7.41%	0.21%
3/5/2019	$40,000,000	$8,244,622	91.61%	8.17%	0.22%
3/8/2019	$45,000,000	$8,944,821	91.81%	7.99%	0.20%
3/12/2019	$50,000,000	$8,670,861	91.74%	8.04%	0.21%
3/15/2019	$40,000,000	$9,088,265	91.94%	7.86%	0.20%
3/19/2019	$45,000,000	$9,401,534	92.02%	7.76%	0.22%
3/22/2019	$50,000,000	$10,410,177	92.12%	7.67%	0.21%
3/26/2019	$57,000,000	$10,683,040	92.09%	7.68%	0.23%
3/29/2019	$75,000,000	$10,570,955	92.05%	7.73%	0.22%
4/2/2019	$88,000,000	$9,339,371	91.75%	8.04%	0.21%
4/5/2019	$104,000,000	$10,642,903	91.96%	7.84%	0.20%
4/9/2019	$120,000,000	$10,459,799	92.06%	7.73%	0.21%
4/12/2019	$140,000,000	$11,029,933	92.15%	7.66%	0.19%
4/16/2019	$157,000,000	$10,935,413	92.13%	7.66%	0.21%
4/19/2019	$175,000,000	$11,457,466	92.20%	7.60%	0.19%
4/23/2019	$192,000,000	$11,350,473	92.19%	7.60%	0.21%
4/26/2019	$212,000,000	$12,572,596	92.34%	7.47%	0.20%
4/30/2019	$229,000,000	$12,952,414	92.36%	7.44%	0.21%
5/3/2019	$252,000,000	$14,050,305	92.37%	7.34%	0.29%
5/7/2019	$273,000,000	$13,644,412	92.48%	7.31%	0.21%
5/10/2019	$295,000,000	$14,407,691	92.57%	7.23%	0.20%
5/14/2019	$316,000,000	$15,046,406	92.67%	7.12%	0.20%
5/17/2019	$339,000,000	$15,925,653	92.74%	7.07%	0.19%
5/21/2019	$367,000,000	$16,168,593	92.72%	7.08%	0.21%
5/24/2019	$393,000,000	$17,346,194	92.73%	7.08%	0.19%
5/28/2019	$418,000,000	$18,204,460	92.88%	6.92%	0.20%
5/31/2019	$444,000,000	$23,855,912	93.08%	6.73%	0.19%

Date	Jackpot	Total	Standard	Megaplier	Just the Jackpot
6/4/2019	$475,000,000	$25,635,468	93.12%	6.68%	0.20%
6/7/2019	$530,000,000	$32,804,596	93.44%	6.36%	0.20%
6/11/2019	$40,000,000	$8,313,913	91.47%	8.32%	0.21%
6/14/2019	$45,000,000	$8,669,905	91.47%	8.32%	0.20%
6/18/2019	$50,000,000	$8,134,308	91.38%	8.41%	0.21%
6/21/2019	$55,000,000	$8,676,327	91.52%	8.30%	0.18%
6/25/2019	$60,000,000	$8,174,943	91.44%	8.36%	0.21%
6/28/2019	$71,000,000	$8,766,333	91.47%	8.34%	0.19%
7/2/2019	$83,000,000	$8,580,800	91.43%	8.37%	0.21%
7/5/2019	$95,000,000	$8,544,161	91.41%	8.39%	0.19%
7/9/2019	$107,000,000	$8,979,822	91.67%	8.13%	0.20%
7/12/2019	$121,000,000	$9,703,712	91.79%	8.02%	0.19%
7/16/2019	$137,000,000	$9,908,994	91.82%	7.97%	0.20%
7/19/2019	$154,000,000	$10,679,969	91.95%	7.86%	0.20%
7/23/2019	$168,000,000	$10,430,294	91.97%	7.83%	0.21%
7/26/2019	$40,000,000	$7,675,488	91.48%	8.32%	0.20%
7/30/2019	$45,000,000	$7,227,490	91.35%	8.44%	0.21%
8/2/2019	$50,000,000	$7,928,006	91.40%	8.41%	0.19%
8/6/2019	$55,000,000	$7,479,508	91.42%	8.37%	0.21%
8/9/2019	$60,000,000	$7,850,505	91.51%	8.31%	0.18%
8/13/2019	$65,000,000	$7,472,842	91.43%	8.36%	0.21%
8/16/2019	$70,000,000	$7,926,022	91.54%	8.27%	0.18%
8/20/2019	$79,000,000	$7,714,175	91.56%	8.23%	0.20%
8/23/2019	$90,000,000	$8,595,792	91.75%	8.06%	0.19%
8/27/2019	$103,000,000	$8,902,318	91.91%	7.89%	0.20%
8/30/2019	$113,000,000	$9,777,008	91.89%	7.91%	0.19%
9/3/2019	$127,000,000	$9,092,326	91.72%	8.06%	0.22%
9/6/2019	$139,000,000	$10,153,715	91.87%	7.94%	0.19%
9/10/2019	$154,000,000	$10,299,151	91.90%	7.89%	0.21%
9/13/2019	$172,000,000	$11,530,413	92.07%	7.73%	0.20%
9/17/2019	$192,000,000	$11,325,551	92.09%	7.70%	0.21%
9/20/2019	$211,000,000	$12,694,116	92.33%	7.47%	0.20%
9/24/2019	$227,000,000	$12,992,502	92.38%	7.40%	0.22%