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Economics 422

Practice Problems

QUESTION 1

Assume that the weight of cereal in a "10-ounce box" is $N(\mu, \sigma^2)$. To test $H_0 : \mu = 10.1$ against $H_1 : \mu > 10.1$, we take an $i.i.d.$ sample of size $n = 16$ and observe that

$$\overline{X} = \frac{1}{16} \sum_{i=1}^{16} X_i = 10.4$$

and

$$s^2 = \frac{1}{15} \sum_{i=1}^{16} \left(X_i - \overline{X} \right)^2 = 0.16.$$

Do we reject or fail to reject H_0 at the 5% significance level? Justify your answer with the necessary calculations.

QUESTION 2

Let $\mathbf{X}$ and $\mathbf{Y}$ be discrete random variables with joint probability distribution given by

(x, y)	$(0, 0)$	$(0, 1)$	$(0, 2)$	$(1, 1)$	$(1, 2)$	$(2, 2)$
$p(x, y)$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{1}{12}$

where $p(x, y)$ is equal to zero elsewhere. Find

(a) the marginal distribution of $\mathbf{X}$;

(b) the marginal distribution of $\mathbf{Y}$;

(c) $E(\mathbf{X})$

(d) $E(\mathbf{Y})$

(e) ρ_{XY}, i.e., the correlation coefficient of $\mathbf{X}$ and $\mathbf{Y}$.

QUESTION 3

Let X_1, X_2, ..., X_{64} be an $i.i.d.$ sample drawn from a normal distribution with mean μ and variance σ^2. Find the probability of the event

$$\left|\overline{X} - \mu\right| < 0.5\sigma,$$

where $\overline{X}$ is the sample average.

QUESTION 4

Let X_1, X_2, ..., X_n be an $i.i.d.$ sample drawn from a normal distribution with mean 0 and variance σ^2

(a) Is

$$\tilde{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}X_i^2$$

an unbiased estimator of σ^2? If so, prove it. If not, explain why not.

(b) If $n = 20$ and $\sigma^2 = 16$, find c such that $P\left(\tilde{\sigma}^2 > c\right) = 0.05$.

QUESTION 5

Suppose that a random sample of 200 twenty-year-old men is selected from a population and that these men's height and weight are recorded. A regression of weight on height yields

$$\widehat{Weight} = -101 + 3.2 \times Height,\ R^2 = 0.81,\ SER = 10.2,$$

where $Weight$ is measured in pounds and $Height$ is measured in inches.

(a) What is the regression's weight prediction for someone who is 64 inches tall?

(b) Suppose that a man has a late growth spurt and grows 2 inches over the course of a year. What is the regression's prediction for the increase in this man's weight?