Economics 422

Final Examination

This exam contains 5 questions. You should try to do all 5 questions. Please provide explanations for all your answers. Right answers with no explanation will receive no credit. On the other hand, wrong answers with thoughtful explanations may receive some credit. Partial credit will be given. **GOOD LUCK!**

QUESTION 1 (20 POINTS)

Suppose that you wish to conduct an empirical analysis of the relationship between years of work experience and earnings, and you believe that the relationship is nonlinear in variables. Hence, you estimate the following quadratic regression

$$\widehat{W}_{i} = 3.73 + 0.298E_{i} - 0.0061E_{i}^{2}$$

$$n = 526, R^{2} = 0.093,$$
(1)

where W_i and E_i denote, respectively, the wage and years of work experience of individual i.

- (a) Is the E_i^2 variable statistically significant at the 5% level? Show your calculations.
- (b) At what point does the (estimated) marginal effect of experience on wage (i.e., $\partial \widehat{W}/\partial E$) go from positive to negative? Show your calculations.
- (c) Do the estimation results imply that work experience has a diminishing effect on wage? Explain.

QUESTION 2 (20 POINTS)

Consider the following system of equations

$$z_i = \beta x_i + u_i,$$

 $y_i = \pi z_i + v_i, \quad i = 1, ..., n;$

where (y_i, x_i, z_i) are i.i.d.; $Cov(u_i, v_i) \neq 0$; $\pi \neq 0$; $\beta \neq 0$ and, with probability one, $E[u_i|x_i] = 0$ and $E[v_i|x_i] = 0$. In addition, assume that $E[y_i^4] < \infty$, $E[z_i^4] < \infty$, and $E[x_i^4] < \infty$.

- (a) Is the coefficient β identified? Explain.
- (b) Is the coefficient π identified? Explain
- (c) Explain how you would estimate the parameter(s) that you think is (are) identified in parts (a) and (b) above. Write down an explicit formula for any estimator you propose.

QUESTION 3 (20 POINTS)

Suppose you are interested in explaining chief executive officer (CEO) salary, and you estimate the following equation

$$\log (\widehat{salary_i}) = 4.59 + 0.257 \log (sales_i) + 0.11 roe_i + 0.158 D_i^F + 0.181 D_i^{CP} - 0.283 D_i^U
n = 209, R^2 = 0.357,$$

where $salary_i$ is the salary of the CEO at firm i, $sales_i$ denotes the annual sales of firm i, and roe_i is the return to equity of firm i. In addition,

$$\begin{split} D_i^F &= \left\{ \begin{array}{l} 1 & \text{if the } i^{th} \text{ firm is in the financial industry} \\ 0 & \text{if the } i^{th} \text{ firm is } \mathbf{not} \text{ in the financial industry} \end{array} \right., \\ D_i^{CP} &= \left\{ \begin{array}{l} 1 & \text{if the } i^{th} \text{ firm is in the consumer products industry} \\ 0 & \text{if the } i^{th} \text{ firm is } \mathbf{not} \text{ in the consumer products industry} \end{array} \right., \\ D_i^U &= \left\{ \begin{array}{l} 1 & \text{if the } i^{th} \text{ firm is in the utilities industry} \\ 0 & \text{if the } i^{th} \text{ firm is } \mathbf{not} \text{ in the utilities industry} \end{array} \right., \end{split}$$

The omitted industry is transportation.

- (a) Compute the (approximate) percentage difference in estimated salary between a CEO in the utilities industry vis-à-vis one in the transportation industry, holding sales and roe fixed. Is the difference statistically significant at the 5% significance level? Show your work.
- (b) What is the (approximate) percentage difference in estimated salary between a CEO in the consumer products industry vis-à-vis one in the financial industry? Write down an equation that would allow you to test whether this difference in statistically significant. (Do not actually implement such a test.) Show your work.

QUESTION 4 (20 POINTS)

Consider the following scenarios involving the specification of an instrumental variables regression.

(a) Suppose you have a model of the form

$$Y_i = \beta X_i + u_i,$$

$$X_i = \pi Z_i + v_i,$$

where (Y_i, X_i, Z_i) are i.i.d.; $Cov(u_i, v_i) \neq 0$; $\pi \neq 0$; and, with probability one, $E[v_i|Z_i] = 0$. In addition, assume that $E[Y_i^4] < \infty$, $E[Z_i^4] < \infty$, and $E[X_i^4] < \infty$. You are not sure, however, whether the condition $Cov(Z_i, u_i) = 0$ holds or not. You compute the J-statistic and find that

$$J = 0$$
.

Are you surprised by this result? Can you test the exogeneity assumption using the J-test in this case? Explain.

(b) Consider instead the model

$$Y_{i} = \beta_{0} + \beta_{1}X_{i} + \gamma W_{i} + u_{i},$$

$$X_{i} = \pi_{0} + \pi_{1}Z_{1i} + \pi_{2}Z_{2i} + \pi_{3}Z_{3i} + \pi_{4}Z_{4i} + \varphi W_{i} + v_{i},$$

where $(Y_i, X_i, W_i, Z_{1i}, Z_{2i}, Z_{3i}, Z_{4i})$ are i.i.d.; $Cov(u_i, v_i) \neq 0$; $\pi_k \neq 0$ (for k = 1, 2, 3, 4); and, with probability one, $E[v_i|W_i, Z_{1i}, Z_{2i}, Z_{3i}, Z_{4i}] = 0$. Also assume that $\varphi \neq 0$, and, with probability one, $E[u_i|W_i] = 0$. In addition, assume that $E[Y_i^4] < \infty$, $E[W_i^4] < \infty$, $E[Z_{ki}^4] < \infty$ (for k = 1, 2, 3, 4), and $E[X_i^4] < \infty$. You wish to test the exogeneity of your instruments. You compute the J-statistic and find that

$$J = 12.01$$
.

What would you conclude at the 5% significance level? Support your answer by providing the relevant details.

QUESTION 5 (20 POINTS)

Consider the panel data regression

$$Y_{it} = \alpha_i + \beta X_{it} + u_{it}, \ i = 1, ..., n; \ t = 1, 2.$$
 (3)

Suppose that n is much larger than 2. Define $\Delta Y_i = Y_{i2} - Y_{i1}$, $\Delta X_i = X_{i2} - X_{i1}$, and $\Delta u_i = u_{i2} - u_{i1}$ to be the "first-differences" of the variables Y_{it} , X_{it} , and u_{it} , respectively. Then, one way to estimate the parameter β is to take first-differences of the variables in equation (3) above to obtain the specification

$$\Delta Y_i = \beta \Delta X_i + \Delta u_i, \ i = 1, ..., n; \tag{4}$$

and then estimate (4) by ordinary least squares (OLS). Show that this procedure leads to exactly the same estimator for β as that which is obtained by running the so-called "entity-demeaned" OLS regression (in the terminology of Stock and Watson, 2011).