This exam contains 4 regular questions and 1 bonus question. The total number of points for the regular questions is 75. The total time allowed for the exam is 75 minutes. There is, of course, the opportunity to gain 20 extra points by completing the bonus question correctly. It is suggested that you allocate to each question the same number of minutes as the number of points that the question is worth; e.g., allocate 20 minutes to a question that is worth 20 points. You should provide explanations for your answers. Right answers with no explanation will receive no credit. On the other hand, wrong answers with thoughtful explanations may receive some credit. Partial credit will be given. GOOD LUCK!
QUESTION 1 (21 POINTS)

For a particular population, the mean $\mu_Y = 100$ and the variance $\sigma_Y^2 = 43$. Use the central limit theorem to answer the following questions:

(a) In a random sample of size $n = 100$, find $\Pr(\bar{Y} < 101)$.
(b) In a random sample of size $n = 64$, find $\Pr(101 < \bar{Y} < 103)$.
(c) In a random sample of size $n = 165$, find $\Pr(\bar{Y} > 98)$.

Here, $\bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i$ denotes the sample average.

QUESTION 2 (24 POINTS)

A survey of 1055 registered voters is conducted, and the voters are asked to choose between candidate A and candidate B. Let $p$ denote the fraction of voters in the population who prefer candidate A, and let $\hat{p}$ denote the fraction of voters in the sample who prefer Candidate A. Suppose that, in the survey, $\hat{p} = 0.54$.

(a) Test $H_0 : p = 0.5$ versus $H_1 : p \neq 0.5$ using a 5% significance level.
(b) Test $H_0 : p = 0.5$ versus $H_1 : p > 0.5$ using a 5% significance level.
(c) Construct a 95% confidence interval for $p$.

QUESTION 3 (10 POINTS)

Let $X_1, X_2, \ldots, X_{64}$ be an i.i.d. sample drawn from a normal distribution with mean $\mu$ and variance $\sigma^2$. Find the probability of the event $|\bar{X} - \mu| < 0.1 \sigma$, where $\bar{X}$ is the sample average.
QUESTION 4 (20 POINTS)

Suppose that a random sample of 200 twenty-year-old men is selected from a population and that these men’s height and weight are recorded. A regression of weight on height yields

\[ \hat{Weight} = -99.41 + 3.94 \times Height, \quad R^2 = 0.81, \quad SER = 10.2, \]

where Weight is measured in pounds and Height is measured in inches.

(a) What is the regression’s weight prediction for someone who is 70 inches tall?

(b) Suppose that a man has a late growth spurt and grows 1.5 inches over the course of a year. What is the regression’s prediction for the increase in this man’s weight?

BONUS QUESTION (20 POINTS)

Let \( Y_1, Y_2, \ldots, Y_n \) be a sequence of i.i.d. random variables with mean \( \mu_Y \) and variance \( \sigma^2_Y \) such that \( 0 < \sigma^2_Y < \infty \).

(a) Show that \( \bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i \) is an unbiased estimator of \( \mu_Y \).

(b) Is \( \bar{Y}^2 \) an unbiased estimator of \( \mu^2_Y \)? Justify your answer carefully.