Department of Economics
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Economics 422

Midterm Examination
Solution Sheet

Multiple-Choice Questions

1. c
2. b
3. a
4. b
5. d
6. a
7. b
8. a
9. d
10. b

Problems

11. (a) From the OLS formulae, we have

$$\begin{aligned}\widehat{\beta}_1 &= \frac{\sum_{i=1}^{110}\left(X_i-\overline{X}\right)\left(Y_i-\overline{Y}\right)}{\sum_{i=1}^{110}\left(X_i-\overline{X}\right)^2}\\ &= \frac{7625.9}{1248.9}\\ &\approx 6.1061\end{aligned}$$

Moreover,

$$\begin{aligned}\overline{Y} &= \frac{1}{110}\sum_{i=1}^{110}Y_i=\frac{17,375}{110}\approx 157.955,\\ \overline{X} &= \frac{1}{110}\sum_{i=1}^{110}X_i=\frac{7665.5}{110}\approx 69.686\end{aligned}$$

so that

$$\widehat{\beta}_0=\overline{Y}-\widehat{\beta}_1\overline{X}=157.955-(6.1061)(69.686)\approx -267.555$$

(b) Note first that

$$\begin{aligned}\widehat{Y}_i-\overline{Y} &= \widehat{\beta}_0+\widehat{\beta}_1X_i-\overline{Y}\\ &= \overline{Y}-\widehat{\beta}_1\overline{X}+\widehat{\beta}_1X_i-\overline{Y}\\ &= \widehat{\beta}_1\left(X_i-\overline{X}\right)\end{aligned}$$

It follows that

$$\begin{aligned}
R^2 &= \frac{\sum_{i=1}^{110}\left(\widehat{Y}_i - \overline{Y}\right)^2}{\sum_{i=1}^{110}\left(Y_i - \overline{Y}\right)^2} \\
&= \frac{\widehat{\beta}_1^2 \sum_{i=1}^{110}\left(X_i - \overline{X}\right)^2}{\sum_{i=1}^{110}\left(Y_i - \overline{Y}\right)^2} \\
&\approx (6.1061)^2 \frac{1248.9}{94,228.8} \\
&\approx 0.4942.
\end{aligned}$$

What this R^2 measure shows is that around 49% of the variance in Y has been explained by X.

12.
(a) Note that, since the data sample is $i.i.d.\,(0,4)$ under the null hypothesis,

$$\begin{aligned}
E\left[\overline{X}_n\right] &= 0 \\
Var\left(\overline{X}_n\right) &= \frac{4}{\sqrt{n}},
\end{aligned}$$

so that by the central limit theorem

$$Z_n = \frac{\overline{X}_n}{2/\sqrt{n}} = \frac{\sqrt{n}\overline{X}_n}{2} \overset{a}{\sim} N(0,1)$$

when the sample size n is large. This implies that

$$\sqrt{n}\overline{X}_n = 2Z_n \overset{a}{\sim} N(0,4).$$

(b) Note that

$$\begin{aligned}
& \Pr\left[-1.25 \leq \overline{X}_n - 1 \leq -0.5\right] \\
=\ & \Pr\left[-0.25 \leq \overline{X}_n \leq 0.5\right] \\
=\ & \Pr\left[\frac{\sqrt{100}}{2}(-0.25) \leq \frac{\sqrt{100}\overline{X}_n}{2} \leq \frac{\sqrt{100}}{2}0.5\right] \\
\approx\ & \Pr\left[-1.25 \leq Z \leq 2.5\right] \\
=\ & \Pr\left[Z \leq 2.5\right] - \Pr\left[Z \leq -1.25\right] \\
=\ & 0.9938 - 0.1056 \\
=\ & 0.8882.
\end{aligned}$$