Solutions to Econ 422 Problem Set 3

1a) Average wage is $957.95. Average IQ is 101.28. Sample standard deviation of IQ is 15.05 points.

1b) The predicted increase in wage for an increase in IQ of 15 points is (15 x 8.30) = $124.5. With an R² of 0.0955, IQ explains only 9.55% of the variation in wage.

2a) Average participation rate is 87.36%. Average match rate is 0.73 (a $1 contribution by the worker is matched by a 73¢ contribution by the firm).

2b) \( prate = 83.08 + 5.86 \text{mrate} \) where 83.08 is the intercept and 5.86 is the coefficient on mrate. There are 1534 observations in the sample. R² is 0.0747 (7.47% of the variation in prate is explained by the regression).

2c) Interpretation of the intercept: Predicted participation rate is 83.08% when the firm does not contribute anything to the worker’s plan (match rate = 0). Interpretation of the coefficient on mrate: Predicted prate increases by 5.86% for each unit increase (a $1 contribution by the worker is matched by a $1 contribution by the firm) in mrate.

2d) When mrate = 3.5, predicted prate is 103.59%. This is not a reasonable prediction as the participation rate cannot be greater than 100%. This illustrates that, especially when dependent variables are bounded, a simple regression model can give strange predictions for extreme values of the independent variable. (In the sample of 1534 firms, only 34 have mrate ≥ 3.5.)

2e) 7.47% of the variation in prate is explained by mrate. This is not much, and suggests that many other factors influence 401(k) plan participation rates.

3a) \( sleep = 3586.38 – 0.1507 \text{totwrk} \) where 3586.38 is the intercept and -0.1507 is the coefficient on totwrk (a one-minute increase in time spent in paid work decreases time spent in sleeping per week by 0.1507 minutes). There are 706 observations in the sample. R² = 0.1033. The intercept implies that the estimated amount of sleep per week for someone who does not work is 3586.4 minutes, or about 59.77 hours.

3b) If totwrk increases by 2 hours per week, sleep is estimated to fall by (120 x 0.1507) = 18.08 minutes. This is only a few minutes a night.

4a) 95% confidence interval for \( \beta_0 \): \( 43.2 \pm (t_{n-2} \times 10.2) = 43.2 \pm (2.048 \times 10.2) = (22.31, 64.09) \)

4b) \( t = \frac{61.2 - 55}{10.2} = 0.878 \). Under the null hypothesis, this t-statistic is distributed as \( t_{n-2} = t_{28} \). The two-sided 5% critical value for a t distribution with 28 degrees of freedom is about 2.048, since \( t = 0.878 < 2.048 \), we cannot reject \( \beta_1 = 55 \) at the 5% level.
4c) $t = \frac{\hat{\beta}_1 - \beta_1}{\sigma}$ = 0.878. Under the null hypothesis, this t-statistic is distributed as $t_{n-2} = t_{28}$. The one-sided 5% critical value for a t distribution with 28 degrees of freedom is about 1.701, since $t = 0.878 < 1.701$, we cannot reject $\beta_1 = 55$ at the 5% level.