1a) \[ C = M + F \]

\[ E(C) = E(M + F) = E(M) + E(F) = 40,000 + 45,000 = 85,000 \]

b) \[ \text{corr}(M, F) = \frac{\text{cov}(M, F)}{\sigma_M \sigma_F} \]

So that \[ \text{cov}(M, F) = \text{corr}(M, F) \cdot \sigma_M \sigma_F \]

\[ = 0.8 \times 12,000 \times 18,000 \]

\[ = 172,800,000 \]

c) \[ \sigma_C^2 = \sigma_M^2 + \sigma_F^2 + 2 \cdot \text{cov}(M, F) \]

\[ = (12,000)^2 + (18,000)^2 + 2 \cdot 172,800,000 \]

\[ = 813,600,000 \]

\[ \sigma_C = \sqrt{813,600,000} = 2,852.4 \]

d) i) \[ 85,000 / 1.3 = 65,385 \text{ Euros} \]

ii) Correlation is unit-free, and is unchanged with change in unit of measurement.

\[ \text{cov}(M, F) = 0.8 \times \frac{12,000}{1.3} \times \frac{18,000}{1.3} \]

\[ = 10,224,852.1 \text{ Euros} \]

iii) \[ 2,852.4 / 1.3 = 2,194.2 \text{ Euros} \]
2a) $E(Y) = 0 \times Pr(Y = 0) + 1 \times Pr(Y = 1)$
   $= 0 \times 0.05 + 1 \times 0.95$
   $= 0.95$

b) \text{Unemployment rate} = \frac{\# \text{ (unemployed)}}{\# \text{ (labor force)}}$
   $= Pr(Y = 0)$
   $= 0.05$
   $= 1 - 0.95$
   $= 1 - E(Y)$

c) \text{Calculate conditional probabilities first.} $
   \text{Pr}(Y = 0 | X = 0) = \frac{Pr(X = 0, Y = 0)}{Pr(X = 0)}$
   $= \frac{0.045}{0.754} = 0.0597$
   \text{Pr}(Y = 1 | X = 0) = \frac{Pr(X = 0, Y = 1)}{Pr(X = 0)}$
   $= \frac{0.709}{0.754} = 0.9403$
   \text{Pr}(Y = 0 | X = 1) = \frac{Pr(X = 1, Y = 0)}{Pr(X = 1)}$
   $= \frac{0.005}{0.246} = 0.0203$
   \text{Pr}(Y = 1 | X = 1) = \frac{Pr(X = 1, Y = 1)}{Pr(X = 1)}$
   $= \frac{0.241}{0.246} = 0.9797$
2 c) Continued

\[ E(Y \mid X = 1) = 0 \times Pr(Y = 0 \mid X = 1) + 1 \times Pr(Y = 1 \mid X = 1) \]
\[ = 0 \times 0.0203 + 1 \times 0.9797 \]
\[ = 0.9797 \]

\[ E(Y \mid X = 0) = 0 \times Pr(Y = 0 \mid X = 0) + 1 \times Pr(Y = 1 \mid X = 0) \]
\[ = 0 \times 0.0597 + 1 \times 0.9403 \]
\[ = 0.9403 \]

d) Use the solution to part (b)

Unemployment rate for college grads

\[ = 1 - E(Y \mid X = 1) = 1 - 0.9797 = 0.0203 \]

Unemployment rate for non-college grads

\[ = 1 - E(Y \mid X = 0) = 1 - 0.9403 = 0.0597 \]

e) Educational achievement and employment status are NOT independent because they do not satisfy that, for all values of \(X\) and \(Y\),

\[ Pr(Y = y \mid X = x) = Pr(Y = y) \]

For example,

\[ Pr(Y = 0 \mid X = 0) = 0.0597 \neq Pr(Y = 0) = 0.05 \]

3 a) Probability distribution function for \(Y\)

<table>
<thead>
<tr>
<th>Outcome (number of heads)</th>
<th>(Y = 0)</th>
<th>(Y = 1)</th>
<th>(Y = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>([T,T])</td>
<td>0.25</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>([T,H]) or ([H,T])</td>
<td>(\frac{1}{2} \times \frac{1}{2})</td>
<td>(\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2})</td>
<td>(\frac{1}{2} \times \frac{1}{2})</td>
</tr>
</tbody>
</table>
3 b) Cumulative distribution function for $Y$

<table>
<thead>
<tr>
<th>Outcome (number of heads)</th>
<th>$Y &lt; 0$</th>
<th>$0 \leq Y &lt; 1$</th>
<th>$1 \leq Y &lt; 2$</th>
<th>$Y \geq 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0</td>
<td>0.25</td>
<td>0.75</td>
<td>1.0</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
0.25 & + 0.50 & = 0.75 & + 0.25 & = 1.00
\end{align*}
\]

c) \[
E(Y) = 0 \times 0.25 + 1 \times 0.50 + 2 \times 0.25
\]
\[= 1\]

\[
\text{var}(Y) = E(Y^2) - [E(Y)]^2
\]

\[
E(Y^2) = (0^2 \times 0.25) + (1^2 \times 0.50) + (2^2 \times 0.25)
\]
\[= 1.50\]

So that \[
\text{var}(Y) = E(Y^2) - [E(Y)]^2
\]
\[= 1.50 - 1^2
\]
\[= 0.50\]

4 a) \[
E(X^3) = 0^3 \times (1 - p) + 1^3 \times p = p
\]

b) \[
E(X^k) = 0^k \times (1 - p) + 1^k \times p = p
\]

c) \[
E(X) = 0 \times (1 - p) + 1 \times p
\]
\[= p = 0.3\]

\[
\text{var}(X) = E(X^2) - [E(X)]^2
\]
\[= 0.3 - (0.3)^2
\]
\[= 0.21\]