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Econ 422 Problem Set # 1

1 a) $C = M + F$

$$E(C) = E(M+F) = E(M) + E(F) = \$40,000 + \$45,000$$
$$= \$85,000$$

b) $$\text{corr}(M,F) = \frac{\text{cov}(M,F)}{\sigma_M \sigma_F}$$

so that $\text{cov}(M,F) = \text{corr}(M,F)\,[\sigma_M \sigma_F]$

$$= 0.8 \times 12000 \times 18000$$
$$= \$172800000$$

c) $$\sigma_c^2 = \sigma_M^2 + \sigma_F^2 + 2\,\text{cov}(M,F)$$
$$= (12000)^2 + (18000)^2 + 2(172800000)$$
$$= \$813600000$$

$$\sigma_c = \sqrt{813600000} = \$28524$$

d) i) $85000 / 1.3 = 65385$ Euros

ii) Correlation is unit-free, and is unchanged with change in unit of measurement.

$$\text{cov}(M,F) = 0.8 \times \frac{12000}{1.3} \times \frac{18000}{1.3}$$
$$= 102248521 \text{ Euros}$$

iii) $28524 / 1.3 = 21942$ Euros

2 a) $E(Y) = 0 \times Pr(Y=0) + 1 \times Pr(Y=1)$

$= 0 \times 0.05 + 1 \times 0.95$

$= 0.95$ //

b) $\text{Unemployment rate} = \dfrac{\#(\text{unemployed})}{\#(\text{labor force})}$

$= Pr(Y=0)$

$= 0.05$

$= 1 - 0.95$

$= 1 - E(Y)$

c) Calculate conditional probabilities first:

$Pr(Y=0 \mid X=0) = \dfrac{Pr(X=0, Y=0)}{Pr(X=0)}$

$= \dfrac{0.045}{0.754} = 0.0597$

$Pr(Y=1 \mid X=0) = \dfrac{Pr(X=0, Y=1)}{Pr(X=0)}$

$= \dfrac{0.709}{0.754} = 0.9403$

$Pr(Y=0 \mid X=1) = \dfrac{Pr(X=1, Y=0)}{Pr(X=1)}$

$= \dfrac{0.005}{0.246} = 0.0203$

$Pr(Y=1 \mid X=1) = \dfrac{Pr(X=1, Y=1)}{Pr(X=1)}$

$= \dfrac{0.241}{0.246} = 0.9797$

2 c) Continued

$$E(Y \mid X=1) = 0 \times Pr(Y=0 \mid X=1) + 1 \times Pr(Y=1 \mid X=1)$$
$$= 0 \times 0.0203 + 1 \times 0.9797$$
$$= 0.9797$$

$$E(Y \mid X=0) = 0 \times Pr(Y=0 \mid X=0) + 1 \times Pr(Y=1 \mid X=0)$$
$$= 0 \times 0.0597 + 1 \times 0.9403$$
$$= 0.9403$$

d) Use the solution to part (b),

Unemployment rate for college grads

$= 1 - E(Y \mid X=1) = 1 - 0.9797 = 0.0203$

Unemployment rate for non-college grads

$= 1 - E(Y \mid X=0) = 1 - 0.9403 = 0.0597$

e) Educational achievement and employment status are NOT independent because they do not satisfy that, for all values of X and Y,

$$Pr(Y=y \mid X=x) = Pr(Y=y)$$

For example,

$$Pr(Y=0 \mid X=0) = 0.0597 \neq Pr(Y=0) = 0.05$$

3 a) Probability distribution function for Y

Outcome (number of heads)	$Y=0$ ← {T, T}	$Y=1$ ← {T, H} or {H, T}	$Y=2$ ← {H, H}
probability	0.25	0.50	0.25
	$\frac{1}{2} \times \frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2}$

3 b) Cumulative distribution function for Y

Outcome (number of heads)	$Y < 0$	$0 \leq Y < 1$	$1 \leq Y < 2$	$Y \geq 2$
Probability	0	0.25	0.75	1.0

$\uparrow$ $0.25 + 0.50$ $\qquad$ $\uparrow$ $0.25 + 0.50 + 0.25$

c) $E(Y) = 0 \times 0.25 + 1 \times 0.50 + 2 \times 0.25$

$= 1$ //

$var(Y) = E(Y^2) - [E(Y)]^2$

$E(Y^2) = (0^2 \times 0.25) + (1^2 \times 0.50) + (2^2 \times 0.25)$

$= 1.50$

So that $var(Y) = E(Y^2) - [E(Y)]^2$

$= 1.50 - 1^2$

$= 0.50$ //

4 a) $E(X^3) = 0^3 \times (1-p) + 1^3 \times p = p$

b) $E(X^k) = 0^k \times (1-p) + 1^k \times p = p$

c) $E(X) = 0 \times (1-p) + 1 \times p$

$= p = 0.3$ //

$var(X) = E(X^2) - [E(X)]^2$

$= 0.3 - (0.3)^2$

$= 0.21$ //