Has Financial Innovation Made the World Riskier? CDS, Regulatory Arbitrage and Systemic Risk\textsuperscript{1}

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Abstract

The paper analyzes the use of credit default swaps (CDS) for regulatory capital relief and its consequences for systemic risk. Equity capital acts as a buffer against losses, and reduces incentives for excessive risk taking. Basel capital regulation states that banks can lower capital requirements using CDS. When the cost of capital is too steep, CDS allows banks to invest in good projects, which would have been by-passed in the absence of CDS. However, CDS can also be used for regulatory arbitrage to lower capital requirements resulting in excessive risk taking. Furthermore, the bank and the insurer prefer high correlation in their returns and jointly shift risk to the regulator. CDS can be traded at a price higher than its fair value reflecting the value of capital relief. I also analyze how CDS can help banks expand balance sheets and fuel asset price bubbles.

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1 Introduction

Financial innovation has done many good over centuries. However, in the presence of adverse incentives and an inadequate regulatory framework it can make the world riskier (Rajan 2005, Korinek 2012). While in most cases the source of financial distress is the realization of a credit risk event, various financial instruments can have an effect that would amplify the initial shock. For example, new financial products can allow financial intermediaries to expand credit and increase their leverage to levels that can potentially prove to be inefficiently high. This can help building up of risks in the financial system, which, in turn, amplify the adverse effects of the initial shock leading to losses above and beyond the losses associated with the shock.¹ This paper looks at one particular case by focusing on the effects of the use of CDS to free up regulatory capital in bank’s balance sheets and its consequences for systemic risk.

The market for CDS has grown exponentially. According to the International Swaps and Derivatives Association the size of the CDS market grew from a notional value of around $918 billion at the end of 2001 to over $25 trillion at the end of 2010, reaching its peak of $62 trillion at the end of 2007 (see Table 1). While the size and the growth of the market is remarkable, CDS contracts are mostly confined to the financial industry. Bank for International Settlements data indicates that more than 95 percent of credit default swap transactions are between financial institutions.

--- Table 1 here ---

Equity capital has an important role in promoting financial stability (Gale, 2004). Equity capital acts as a buffer against losses, and, especially in the presence of limited liability and a safety net that is not adequately priced, it reduces the incentives to take excessive risk by increasing banks’ skin in the game. Hence, the regulators use minimum capital requirements as a major tool to promote financial stability.

¹There is a vast literature on amplification mechanisms such as information contagion (Chen, 1999), direct exposures (Allen and Gale, 2000b), resale externalities (Shleifer and Vishny, 1992 and Allen and Gale, 1994), or both in the form of the interaction between direct exposures, regulatory solvency constraints and marking to market of the assets (Cifuentes, Ferucci and Shin, 2005), to cite only a few.
Basel capital regulation suggests that CDS can be used as a credit risk mitigant, that is, when a bank acquires a CDS contract against a risky investment, the bank can hold less capital against such risky investments.\(^2\) Lowering regulatory capital requirements and freeing up capital on banks’ balance sheet is one of the most dominant and active motive for using CDS (Weistroffer (2009)). For example, AIG Financial Products was a large seller of CDS for relief from regulatory capital requirements. AIG has claimed in its public financial statements that 72% of the notional amounts of CDSs sold by AIG Financial Products as of December 2007 were used by European and other banks for capital relief.\(^3\)

The use of CDS to free up regulatory capital may undo the stabilizing effects of the capital requirements. For example, banks may spread their capital very thin, and even invest in highly risky projects that they would not have invested absent the mitigating effects of CDS on regulatory capital requirements. Furthermore, when there are states of the world where the probability of the bank and the insurer failing at the same time is high, in other words, when defaults are correlated, CDS only provides a false perception of safety. That is, the banking system enters such systemic risk states with highly risky projects in their balance sheet and inadequately low levels of capital, which only increases the costs of systemic failures. Gary Gensler, the chairman of the U.S. Commodity Futures Trading Commission, in his keynote address at Markit’s Outlook for OTC Derivatives Markets Conference on March 9, 2010 (Gensler, 2010) highlights this point:

“Credit default swaps also play a significant role in how banks manage their regulatory capital requirements. Under the Basel II capital accords, large banks and investment banks could significantly decrease their regulatory capital by relying on “credit risk mitigants,” including CDS positions on existing exposures. …The reliance on CDS, enabled by the Basel II

\(^2\)To illustrate simply, when a bank has an exposure of \(E\) and buys a guarantee of \(G\), risk-weighted assets in this case is calculated as \(RWA = (E - G)b + Gr\), where \(b\) and \(r\) denote the risk weights of the borrower and the guarantor, respectively. For example, when a bank lends to a corporate with a rating of BBB+ to BB- (or a corporate that is not rated) the risk weight would be \(b = 100\%\). For claims on corporates with a rating below BB- the risk weight would be \(b = 150\%\). However, when the bank acquires a CDS contract from a AAA to AA- rated institution the risk weight would be \(r = 20\%\). See BCBS (2004) for further details.

\(^3\)Approximately $379 billion (consisting of the corporate loans and prime residential mortgages) of the $527 billion in notional exposure of AIG Financial Products’s super senior CDS portfolio as of 31 December 2007 represents derivatives written for financial institutions, principally in Europe, for the purpose of providing regulatory capital relief rather than risk mitigation (source: AIG 2007 Form 10-K, p. 122). Also see ECB (2009) for a discussion.
capital accords, allowed many banks to lower regulatory capital requirements to what proved to be dangerously low levels. ...Bank capital regulation should be modified to make the use of CDS for capital reduction more restrictive.”

The paper builds a model where banks have access to some risky projects that pay a high return when they succeed. However, these projects can have an expected return lower than the expected return from a safe projects and in those cases should not be taken from a social welfare point of view. But, because of limited liability, when the bank finances itself with insured deposits that are not adequately priced, the bank has an incentive to invest in the risky projects and shift the risk to the deposit insurance fund. To prevent these risk shifting incentives regulators can require banks to hold equity capital so that the bank has enough skin in the game. Basel capital regulation suggests that when a bank acquires a CDS contract against a risky investment, the bank can hold less capital against such risky investments. However, this may undo the stabilizing effects of capital requirements, and the bank can go around the capital requirement and invest in highly risky projects, which they would not have invested under the capital requirements in the absence of CDS as a credit risk mitigant.

The first result from the benchmark model is that when CDS provides full insurance, that is, when there is no counterparty risk, and when the CDS is fairly priced the bank is not willing to acquire the CDS since the bank incurs the entire cost while a significant portion of the benefits accrue to the depositors (or indirectly to the deposit insurance fund).

When the failure of the risky project and the insurer are correlated, the likelihood of the credit risk and the counterparty risk materializing at the same time, that is, the probability of “double default” increases so that the CDS provides only partial insurance.\textsuperscript{4} Even though the individual probability of failure of the insurer can be low, that is, the insurer can be highly rated, if the returns of the bank and the insurer are highly correlated, when the bank experiences difficulty, it is exactly those times that the insurer will have difficulty in honoring the CDS contract. Hence, as correlation between the risky project and the insurer increases, CDS provides less insurance and its fair value decreases. That is, even though the probability

\textsuperscript{4}The counterparty risk associated with CDS can be reduced when CDS are cleared by central counterparties (CCPs), where the CCP would be placed between the counterparties of the bilateral transactions, and thus take on the counterparty credit risk of the bilateral trades. However, at the end of 2010, only less than 10% of CDS had been cleared with CCPs (Vause (2010)).
of success for the insurer in isolation can be high, when correlation between the risky project and the insurer is high, the insurance provided by the CDS can be limited. However, the CDS can still provide sufficient reduction in regulatory capital when the insurer and the bank are treated in isolation, that is, when the correlation risk is not taken into account appropriately. I show that when the risky project and the insurer are correlated, and, in turn, when the CDS is sufficiently cheap and allows for a sufficient reduction in bank’s capital requirement, the bank prefers to acquire the CDS and uses it as a risk mitigant to invest in highly risky projects. On the one hand the CDS is cheap compared to the underlying risk because of the counterparty risk associated with the insurer. On the other hand CDS contracts can be traded above their fair value, which reflects how much banks value regulatory arbitrage generated by exploiting the CDS to lower regulatory capital requirements.

When we consider the bank and the insurer jointly, we can show that the sum of the profits of the bank and the insurer is maximized when the correlation between the bank and the insurer is highest. Note that the deposit insurance fund guarantees the deposits of the bank. Hence, when the correlation between the bank and the insurer is high, the expected cost to the deposit insurance fund is high since the insurance by the CDS is minimal. Thus, high correlation allows the bank to acquire cheap CDS and minimizes the expected cost to the CDS seller, which would still achieve regulatory arbitrage and allow the bank and the insurer jointly shift the risk to the deposit insurance fund.

Especially during times of distress, the cost of capital can be prohibitively high so that banks may bypass good projects because of the capital requirements that become strictly binding. In those cases, CDS allows banks to lower their capital requirements and invest in the good projects that would have been bypassed otherwise. I show that when CDS is used for generating investment in good projects banks go for the high quality CDS, in contrast to the case, where banks use low quality CDS for regulatory arbitrage to invest in bad projects.

I analyze three extensions of the benchmark model. First, I look at how banks expand their balance sheet using CDS. Second, I show that the bank will be willing to pay a price higher than its fair-value for the risky project if the purchase is financed with insured deposits as in Allen and Gale (2000a), and that CDS helps banks increase their leverage fueling the asset price bubble. Third, I extend the benchmark model to allow for multiple banks that acquire CDS from the insurer.

The paper is related to the literature on financial innovation and systemic risk (Rajan,
The paper is also related to the vast literature on the role of the safety net and insured deposits in creating moral hazard, and the role of capital requirements in mitigating such moral hazard (Gale 2004). In addition, the paper is related to the papers on the relation between leverage and credit expansion (Adrian and Shin, 2010) and leverage and asset price bubbles (Allen and Gale, 2000a).

Allen and Gale (2000b) provide a model of contagion in the interbank market arising from banks’ cross holdings with each other. Zawadowski (2012) models a financial system where banks hedge risks using over-the-counter (OTC) contracts and show how an idiosyncratic failure can lead to a systemic run due to counterparty exposure. Heise and Kühn (2012) examine contagion through CDS in a network of corporates and financial institutions. They show that, by creating additional contagion channels, CDS can lead to greater instability in times of economic stress, more so when banks use CDS to expand their loan books.

Some recent papers analyze counterparty risk in credit derivatives (Thompson, 2010 and Biais, Heider and Hoerova, 2010). Bolton and Oehmke (2011) analyze the empty creditor problem and how CDS contracts can create incentives for excessive liquidations. There is also a growing literature on the reform of credit derivatives, their trading and regulation (Duffie and Zhu, 2011 and Duffie, Li and Lubke, 2010 and Acharya and Bisin, 2011). More closely, Parlour and Winton (2011) analyze loan sales and the use of CDS to lay off credit risk.

The focus of the paper differs significantly from these papers. This paper focuses on a specific question, namely the use of CDS contracts for regulatory arbitrage and its role in excessive risk taking in the context of credit risk mitigation role of CDS in the Basel regulatory framework. To the best of my knowledge, this is the first paper that analyzes that particular question.

The remainder of the paper is structured as follows. Section 2 present the benchmark model, the analysis and the main results. Section 3 looks at three extensions of the benchmark model. Section 4 presents discussion and evidence of the assumptions and results. Section 5 concludes and the proofs that are not in the text are contained in the Appendix.

2 Benchmark model

The model has two dates \( t \in \{0, 1\} \), a bank, depositors that finance the bank, an insurer that provides insurance to the bank through CDS, a regulator that sets capital requirements and a deposit insurance fund. The bank can invest in safe and risky projects. I assume that the safe project has a certain return of \( r > 1 \) at \( t = 1 \) for an investment of \$1 at \( t = 0 \). The risky project requires an investment of \$1 at \( t = 0 \) per unit of investment. The risky project succeeds with a probability of \( \alpha \), in which case it has a return of \( R > r \), and when it fails it has a return of 0, for simplicity. Unless stated otherwise, I have the following assumption about the probability of success \( \alpha \):

**Assumption 1** I assume that \( \frac{r - 1}{R - 1} < \alpha < \alpha^* = \frac{R + 1 - \sqrt{(R + 1)^2 - 4r}}{2} \), where \( \alpha^* < r/R \).

Note that for \( \alpha R < r \) the risky project has a lower expected return than the safe project. Thus, from a social welfare point of view, the bank should invest in the safe project, not in the risky project.

I assume that the bank has a measure one of depositors each endowed with \$1 at \( t = 0 \). Depositors have access to a storage technology, where they receive \$1 at date \( t + 1 \) per \$1 invested at date \( t \). I assume that deposits are fully insured by the deposit insurance fund and the bank does not pay any premium for insurance. Since deposits are insured the bank can borrow by offering a net return of 0.

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6 The assumption for \( \alpha < \alpha^* \) simplifies the analysis for capital requirements as discussed in Section 2.1 but is not necessary for the results.

7 This assumption simplifies the analysis. However, the results would go through as long as the bank pays something less than its fair value for deposit insurance, that is, when deposit insurance is not fairly priced. Many countries do not have risk-adjusted deposit premiums (Demirguc-Kunt, Karacali and Laeven, 2005). Furthermore, in the US, which has risk-adjusted premiums, the FDICIA and the Deposit Insurance Act of 1996 specified that if the FDIC’s deposit insurance fund (DIF) reserves exceed the 1.25 percent designated reserve ratio (DRR) of reserves to total insured deposits, the FDIC was prohibited from charging insurance premiums to banks in the lowest risk category. During the 1996-2006 period, DIF reserves were above 1.25 percent of insured deposits and, because the majority of banks were classified in the lowest risk category, these banks did not pay for deposit insurance. See Pennacchi (2009) and Acharya, Santos and Yorulmazer (2010) for further discussion.
Suppose, for now, that the bank finances itself through insured deposits and has no capital of its own. Due to limited liability, the expected profit of the bank from the risky project is

\[ \pi = \alpha(R - 1) > r - 1, \]

since \( \frac{r-1}{R-1} < \alpha \) by assumption 1. Hence, the bank will invest in the risky project even though the safe project has a higher expected return.\(^8\) This is the usual risk shifting argument due to limited liability and a safety net that is not fairly priced.

### 2.1 Capital requirements

As suggested by Gale (2004) capital can act as a buffer against losses and capital requirements can help correct adverse incentives and prevent banks from taking excessive risk. In this paper, the adverse incentives are due to limited liability of banks and deposit insurance that is not priced adequately, as already shown above.

Here, I introduce capital and capital requirements into the model. I show that when the regulator can subject the bank to capital requirements, where the bank is required to hold a minimum amount of capital against an investment in the risky project, it can prevent the bank from taking excessive risk.

Capital is a costly way of finance for the bank since deposits are insured and the bank does not pay the fair price for deposit insurance, which makes deposits a more attractive way of financing for the bank compared to capital.\(^9\)

When the bank holds \( k \) units of capital per unit of investment in the risky project and finances the rest \( (1 - k) \) through insured deposits, the bank’s expected profit is given as

\[ \pi(k) = \alpha(R - (1 - k)) - k, \]

\(^8\)I could allow for the bank to invest in both safe and risky projects. This makes the analysis more complicated without changing the results qualitatively. I chose to keep the analysis simple to convey the main message of the paper in a clear way.

\(^9\)The cost of capital can, for example, be due to asymmetric information as in Myers and Majluf (1984). The cost of capital can be modelled as follows. When the bank raises (holds) \( x \) units of capital, it incurs a cost of \( c(x) \), where the cost is weakly increasing and weakly convex in \( x \), that is, \( c' \geq 0 \) and \( c'' \geq 0 \), with \( c(0) = 0 \). However, throughout most of the paper, this is not necessary for the results. Hence, I assume \( c(x) = 0 \) unless stated otherwise. Introducing a positive cost for capital would only make the results stronger.
since it pays back \((1-k)\) to the depositors when the project succeeds. Note that \(\frac{\partial \pi}{\partial k} = \alpha - 1 < 0\), so that the bank’s expected profit from the risky project is decreasing in the amount of capital \(k\).

As suggested by Gale (2004) capital requirements aim at creating a buffer against losses and creating the right incentives for banks. The risky project fails with probability \((1 - \alpha)\), so that the probability of default is given as \(PD = 1 - \alpha\). In case of failure, the loss is the one unit of initial investment so that the loss given default is given as \(LGD = 1\). Hence, the expected loss from the risky investment is \(1 - \alpha\), and the capital requirement \(k_b = 1 - \alpha\) would serve the purpose of acting as a buffer against expected losses.

Furthermore, the capital requirement should prevent the bank from taking the risky project since for \(\alpha R < r\) the safe project has a higher return. Since the safe project is riskless, I assume that the bank is not required to hold any capital if it invests in the safe project. Using the expected profit the bank makes from the risky project given equation (2), we can show that when the subject to a capital requirement of

\[
 k_{IC} = \begin{cases} 
 \frac{\alpha(R-1) - (r-1)}{1-\alpha} & \text{for } \alpha < r/R \\
 0 & \text{for } \alpha \geq r/R 
\end{cases}
\]

it chooses to invest in the safe project since \(\pi(k_{IC}) < r - 1\) for \(\alpha < r/R\). Note that for \(\alpha \geq r/R\), the risky project has a higher expected return than the safe project, and \(k_{IC} = 0\) so that the bank is required to hold \(k^* = k_b = 1 - \alpha\). In that case, we obtain

\[
\pi(k^*) = \alpha R - 1 + \alpha(1 - \alpha),
\]

which is increasing in \(\alpha\). In addition, for \(\alpha = r/R\), the bank makes an expected profit of \(r - 1 + \frac{r(R-r)}{R^2} > r - 1\). This gives us the following Proposition.

**Proposition 1** Under the capital requirement \(k^* = \max\{k_b, k_{IC}\}\), the bank would take the risky investment if and only if \(\alpha \geq r/R\).

Furthermore, note that \(k_b\) is decreasing in \(\alpha\), whereas \(k_{IC}\) is increasing in \(\alpha\) for \(\alpha < r/R\). For \(\alpha < \alpha^*\), we obtain \(k_b > k_{IC}\) so that \(k^* = k_b = 1 - \alpha\). Figure 1 illustrates the capital requirements.
Hence, the minimum capital requirement $k^* = \max \{k_b, k_{IC}\}$ serves both purposes, that is, it acts as a buffer against expected losses and banks invest in risky project only when they have a higher expected return than the safe project. Thus, although deposit insurance and limited liability create incentives for the bank to take excessively risky projects, such incentives can be prevented by appropriately designed capital requirements.

2.2 CDS

Next, I introduce CDS into the model and analyze how the bank can use CDS to free up regulatory capital, which, in turn, can lead to regulatory arbitrage by allowing the bank to go around minimum capital requirements and invest in the excessively risky project.

CDS contract is such that the insurer agrees to pay the bank $1 when the project fails in exchange for a premium. Throughout I assume that the CDS is fairly priced. In the following I use the framework in Acharya and Yorulmazer (2007, 2008), where I look at the joint probabilities and the correlation between the returns of the bank and the insurer.

When the risky project (or the bank) has the high return $R$, the bank is successful and I denote that as state $S$, whereas when the risky project has the low return 0, it fails, which is denoted as state $F$, the state where the credit risk materializes. When the insurer does not have funds to honor the CDS contract, the insurer fails (state $F$), where counterparty risk materializes, otherwise the insurer is successful (state $S$). Suppose that the marginal probability of the insurer being successful is $\alpha$. I keep the marginal probability of the risky project being successful as $\alpha < \alpha^*$. I assume that $\frac{\gamma}{R} < \beta$. In that case, the probability of success for the insurer is higher than the probability of success for the risky project so that the insurer has a higher rating than the risky project, and the CDS can provide regulatory capital relief for the bank.\footnote{Recall that a risky project that has the probability of success $\beta$ (as the probability of success for the insurer) should be taken since $\beta R > 1$ and will actually be taken by the bank under the minimum capital requirement $k^*(\beta) = 1 - \beta$.}

We have the following joint probabilities for the risky project and the insurer:

- The probability that the project and the insurer are successful (state $SS$) is $a$.  

\begin{align*}
\text{(10)}
\end{align*}
• The probability that the project fails and the insurer is successful (state FS) is $p$.
• The probability that the project is successful and the insurer fails (state SF) is $b$.
• The probability that the project and the insurer fail (state FF) is $d$.

Note that state FF represents the double default state. The joint probabilities are presented in Table 2. As mentioned above, I keep the marginal probabilities of success and failure for the risky project and the insurer unchanged, so that $a = \beta - p$, $d = 1 - \alpha - p$ and $b = \alpha - \beta + p$. Note that $p \in [\beta - \alpha, 1 - \alpha]$.\(^{11}\)

--- Table 2 here ---

The important issue is that the credit rating of the insurer depends on the individual probability of failure $(1 - \beta)$, which is kept fixed throughout the following analysis. Hence, how much a bank can free up capital by acquiring a CDS from the insurer is independent of the joint probabilities as long as the marginal probability of failure of the insurer is fixed (at $(1 - \beta)$ in this case).

Since $\beta > r/R$, we obtain $\pi(k^*(\beta)) > 0$. However, as I will analyze below, the marginal probabilities are not sufficient and may not accurately represent the true counterparty risk, especially when the returns of the risky project and the insurer are correlated. Hence, when the capital requirement is based only on marginal probabilities of success, it can allow the bank to lower its capital requirement more than it should, leading to the possibility of regulatory arbitrage. Before I go into the analysis of regulatory capital requirements, and the role of correlation and regulatory arbitrage, I would like to present my first result that the bank never buys a CDS contract and invests in the risky project when the insurer always honors the contract and the CDS is fairly priced.

\(^{11}\)I assume that the agents know the true probabilities of default and the correlation between the bank and the insurer. Another important factor can be the miscalculation and underestimation of these risks, in particular the risk of systemic failure states, which would only make the results of the paper stronger. For a more detailed discussion, see section 4.
Since the CDS contract is fairly priced, the premium the bank pays is equal to the joint probability that the project fails and the insurer honors the CDS contract, that is, the fair premium is $p$.\(^{12}\)

If the bank invests in the risky project and buys a CDS at the fair price, its expected profit is given as

$$\pi^{CDS} = \alpha(R - (1 - k)) + p(1 - (1 - k)) - p - k,$$

$$= \alpha(R - (1 - k)) - p(1 - k) - k,$$

since with probability $p$ the project fails and the insurer pays the bank one unit out of which $1 - k$ goes to the depositors, and the bank pays the fair price $p$ to acquire the CDS.

Suppose that the CDS contract is always honored, that is, there is no counterparty risk so that $\beta = 1$ and $p = 1 - \alpha$. In that case, we obtain

$$\pi^{CDS} = \alpha R - 1 < r - 1.$$ \hspace{1cm} (6)

This results in the following proposition.\(^{13}\)

**Proposition 2** The bank never buys a CDS contract and invests in the risky project when the insurer always honors the CDS contract and the CDS is fairly priced.

First, note that the result is quite general and holds for any level of capital $k \in [0, 1]$ the bank holds. The reason for this result lies in the fact that the CDS without any counterparty risk that is fairly priced undoes the subsidy the bank receives from deposit insurance since the bank bears the entire cost of the CDS whereas some of the benefits accrue to the depositors (and indirectly to the deposit insurance fund).

\(^{12}\)Note that the conditional probability that the insurer honors the CDS contract when the risky project fails is $\frac{p}{1 - \alpha}$, which can also be interpreted as the quality of the CDS.

\(^{13}\)The results would not change when the CDS contract pays the full return $R$ in case of failure. In that case, if the bank invests in the risky project and buys a CDS, its expected payoff is given as

$$\pi^{CDS} = \alpha(R - (1 - k)) + p(R - (1 - k)) - pR - k - c(k) = \alpha(R - (1 - k)) - p(1 - k) - k - c(k),$$

which is equal to the expected profit with the CDS that pays 1 in case of failure.
2.2.1 Correlation and regulatory arbitrage

When the bank acquires a CDS contract from the insurer it is required to hold $k^*(\beta) = 1 - \beta$ units of capital. Hence, if the bank chooses the risky project and buys a CDS, we obtain

$$\pi^{CDS} = \alpha(R - \beta) - p\beta - (1 - \beta).$$

(7)

Note that $\pi^{CDS}$ is decreasing in $p$ and $\pi^{CDS} > 0$ if and only if

$$p < \frac{\alpha R - r}{\beta} + (1 - \alpha) = p^*.$$

(8)

which results in the following proposition.

**Proposition 3** *If the joint probability $p$ that the risky project fails and the insurer honors the CDS contract is low enough ($p < p^*$), the bank buys CDS and invests in the risky project.*

First, note that how much capital the bank needs to hold against the risky investment depends on $\beta$ when the bank acquires the CDS, whereas the true level of insurance provided by the CDS depends on the joint probability $p$, not the marginal probability $\beta$ of the insurer being successful. Furthermore, some of the benefits from the CDS accrue to the depositors (or the deposit insurance fund) while the bank pays the fair price for the CDS. Hence, the bank may not be willing to pay the fair price for a high-quality CDS since the bank mostly uses the CDS for regulatory arbitrage to free up regulatory capital. As a result, the bank is willing to acquire the CDS as long as it is not too expensive and it provides sufficient capital relief.

2.2.2 Incentive-compatible capital requirements

Note that the joint probability $p$ that the project fails and the insurer honors the CDS contract determines the extent of insurance coverage by the CDS. Hence, when the capital requirements depend only on marginal probabilities of success for the project and the insurer but not on the correlation between the outcomes of the project and the insurer, they can miss their main objective. In particular, such a requirement can lead to regulatory arbitrage by allowing the bank to invest in the risky project using low quality (hence cheap) CDS and to load up on tail risk.
Next, I derive the capital requirements that would prevent the bank from taking the risky investment, which I call “incentive-compatible capital requirements”. Not surprisingly, this capital requirement will depend on the joint probabilities, as well as the marginal probabilities for the returns of the project and the insurer. Furthermore, the capital required by the bank to hold will increase in the joint probability of failure \( d \) for the project and the insurer.

Recall that \( p = 1 - \alpha - d \) and when the bank chooses the risky project and buys a CDS its expected profit is given as

\[
\pi^{CDS} = \alpha(R - (1 - k)) - (1 - \alpha - d)(1 - k) - k.
\]  

(9)

Hence, when the bank is required to hold a minimum capital level of \( k^*(\alpha, d) \), where \( \pi^{CDS}(k^*(\alpha, d)) = r - 1 \), the bank does not invest in the risky project. This gives the following proposition.

**Proposition 4** When the bank is required to hold a minimum capital of \( k^*(\alpha, d) = \frac{\alpha R - r + d}{d} \), the bank would not take the risky investment. Furthermore, \( \frac{\partial k^*}{\partial d} > 0 \).

### 2.2.3 Is CDS cheap or expensive? Price of regulatory arbitrage

On the one hand, the price of the CDS is low compared to the underlying risk since \( p < 1 - \alpha \). This is due to the counterparty risk of the insurer and the partial, not full, insurance provided by the CDS.

On the other hand, the price of the CDS can be high compared to the level of insurance it provides. Note that for \( p < p^* \) the bank acquires the CDS at its fair price and still makes a positive profit \( \pi^{CDS} > 0 \) from the risky project. Figure 1 illustrates the price of regulatory arbitrage, which is simply the profit \( \pi^{CDS} \) the bank makes through risk-shifting using the CDS contract.

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In this case, the bank is willing to pay up to a price of \( \bar{p} = p + \pi^{CDS} \) for the CDS contract. Note that the insurer is willing to sell the CDS contract for a price greater than or equal to its fair price \( p \). Hence, for the price of the CDS contract between \( p \) and \( \bar{p} \), we would observe trade between the bank and the insurer. While the interaction between the bank and the insurer
can be modelled in many different ways, for example, when we look at the Nash bargaining solution between the bank and insurer we obtain the following proposition.\textsuperscript{14}

**Proposition 5** The Nash bargaining solution between the bank and the insurer would result in a price of $\frac{p+p_2}{2}$ for the CDS.

What is interesting is that the CDS contract can be sold at a price higher than its fair value $p$ due to the profits it helps generate for the bank through regulatory arbitrage and risk-shifting to the deposit insurance fund. I analyze the correlation between the returns of the bank and the insurer and the issue of joint risk-shifting by the bank and the insurer to the deposit insurance fund and its systemic risk consequences further in the next section.

2.2.4 Systemic risk and fiscal costs

Suppose that the CDS is traded at a price $p'$ between the bank and the insurer. In that case the profit for the bank and the insurer are given as follows, respectively:

$$\pi_B = \bar{p} - p' \text{ and } \pi_I = p' - p,$$

so that the sum of the profits of the bank and the insurer are

$$\pi_B + \pi_I = \bar{p} - p = \pi^{\text{CDS}}.$$

Note that the sum of the profits of the bank and the insurer is maximized when the correlation between the returns of the bank and the insurer are highest, that is, when $p$ is low.

The risky project is a negative NPV project and should not be taken in the first place from a social welfare point of view. As we have shown, the CDS contract can allow banks to go around the capital requirements that are designed to prevent the bank from taking the risky investment in the first place.

The analysis is more interesting when we consider what the bank and the insurer can do jointly when the high correlation between the bank and the insurer creates systemic risk and has fiscal costs. Note that the deposit insurance fund guarantees the deposits of the bank.

\textsuperscript{14}Kenyon and Green (2012) estimate that 20\% to 50\% of observed CDS spread could be due to capital relief provided by CDS in the Basel regulatory framework.
Hence, the deposit insurance fund is liable when the bank fails and the insurer cannot honor the CDS contract, an event that occurs with a probability of $1 - \alpha - p$. Hence, the expected amount of funds needed by the deposit insurance fund is $1 - \alpha - p$, which is decreasing in $p$. This gives us the following proposition.

**Proposition 6** The sum of the profits of the bank and the insurer, and, at the same time, the expected cost to the deposit insurance fund are decreasing in $p$.

When the returns of the bank and the insurer are highly correlated, that is, when $p$ is low, the CDS provides the minimum insurance, yet allows for the maximum amount of risk-shifting jointly by the bank and the insurer to the deposit insurance fund. High correlation allows the bank to acquire cheap CDS and minimizes the cost to the insurer, which would still achieve regulatory arbitrage and allow the bank and the insurer to jointly shift the risk to the deposit insurance fund.

### 2.3 Good CDS

So far, we focused on the use of CDS for regulatory arbitrage, in particular, its use to go around capital requirements to take bad projects. However, as argued before, financial innovation can do many good, and so can CDS even when they are used for freeing up regulatory capital. In particular, especially during periods of system-wide distress, the cost of capital can be prohibitively high and the banks can be capital constrained due to the regulatory requirements so that they may have to by-pass good projects. Hence, when used prudently CDS contracts can help finance good projects that would not have been financed absent CDS, which is in line with the Introduction that financial innovation can do many good.

In particular, suppose that when the bank raises $x$ units of capital, it incurs a cost of $c(x)$, with $c' \geq 0$, $c'' \geq 0$, and $c(0) = 0$. Furthermore, suppose that the same regulatory capital requirements are in place so that the bank is required to hold a capital of $k^*(\alpha) = 1 - \alpha$ for $\alpha \geq r/R$. Note that for $\alpha \geq r/R$ the risky project should be taken from a social welfare point of view. The bank’s expected profit from the risky project without the CDS is given as

$$\pi(k) = \alpha R - 1 + \alpha(1 - \alpha) - c(1 - \alpha).$$
Note that for \( \pi(k) < r - 1 \) the bank makes a negative expected profit from the project and will not take it, even though the project has a higher expected return than the safe project. However, when the bank acquires a CDS from an insurer, it can lower its capital requirement and can make a positive profit from the project. Hence, CDS, by freeing up regulatory capital, facilitates financing of good projects. In particular, when the bank acquires a CDS contract from the insurer it is required to hold \( k^*(\beta) = 1 - \beta < 1 - \alpha \) units of capital for \( \alpha < \beta \) and the bank’s expected profit can be written as

\[
\pi^{CDS} = \alpha(R - \beta) - p\beta - (1 - \beta) - c(1 - \beta).
\]

Note that the bank’s expected profit is again decreasing in \( p \) so that the bank would choose the lowest \( p \), say \( p = \beta - \alpha \). Even in that case, we can show that the CDS is welfare improving since it can facilitate invest in the good project. In particular, the bank’s expected profit can be written as

\[
\pi^{CDS} = \alpha R - 1 + \beta(1 - \beta) - c(1 - \beta).
\] (10)

For \( \beta = 1 \), the bank’s expected profit from the risky project is higher than the return from the safe project, that is, \( \alpha R - 1 > r - 1 \) since \( \alpha > r/R \). Furthermore, we obtain

\[
\frac{\partial \pi^{CDS}}{\partial \beta} = -2\beta + 1 + c'(1 - \beta),
\]

so that when the cost of capital increases steeply enough, that is, for \( c' > 1 \), the bank’s expected profit is increasing in \( \beta \). Hence, for \( c' > 1 \), there is a critical value of \( \beta \), denoted by \( \beta^* \), so that for \( \beta > \beta^* \), \( \pi^{CDS} \) is greater than the return \( r - 1 \) from the safe project. This gives the following formal proposition.

**Proposition 7** For \( c' > 1 \), bank’s profit given in equation (10) is increasing in \( \beta \). Furthermore, there is a critical value of \( \beta \), denoted by \( \beta^* \), where \( \pi^{CDS}(\beta^*) = r - 1 \) and the bank invests in the risky project using CDS with \( \beta > \beta^* \).

As argued in the Introduction, financial innovation can do many good, so can CDS, when used prudently. This illustrates a case where the cost of capital for the bank increases steeply and the CDS allows the bank to invest in good projects, which would have been by-passed without CDS. In contrast to the earlier result where the bank uses the CDS for regulatory arbitrage and, hence, prefers the lowest quality CDS that provides enough capital relief, in this case the bank would prefer the highest quality CDS.
3 Extensions

In this section, I look at three extensions of the benchmark model. In the first extension, I look at how CDS can allow banks to expand their balance sheet. In the second extension, I look at the effect of CDS on asset prices and how CDS can help fuel asset price bubbles. Lastly, I extend the model to allow for multiple banks that acquire CDS from the insurer.

3.1 Credit expansion

First, I allow the bank to choose the amount of investment in the risky project and show that the size of the investment in the risky project is decreasing in the amount of capital the bank holds, that is, there is a positive relation between leverage and credit expansion. In this extension, I keep the cost of capital $c$ as in section 2.3, which allows me to get a finite size for the bank’s investments in the risky project.

Let $I$ be the number of risky projects the bank invests in. Recall that for $\alpha < r/R$ the socially optimal level of the risky project zero. When the bank holds $k$ units of capital per unit of investment in the risky project, the bank’s expected profit is given as

$$\pi = I [\alpha(R - (1 - k))] - Ik - c(Ik).$$

Recall that the cost of capital $c$ is increasing and convex so that there is an interior solution for the amount of investment in the risky project that the bank will choose. Furthermore, for the same reason, we can show that as the bank holds more capital $k$ per unit of investment in the risky asset, it becomes less attractive for the bank to invest in the risky project, that is, $I$ is decreasing in $k$. This gives the following formal proposition.

**Proposition 8** The amount of investment $I$ in the risky project is given as:

$$c'(I_k) = \frac{\alpha(R - 1) - k(1 - \alpha) - (r - 1)}{k}.$$  

Furthermore, $I$ is decreasing in $k$, that is, $\frac{\partial I}{\partial k} < 0$.

Hence, as the bank holds less capital $k$, it invests more in the risky project and the bank’s balance sheet expands. However, as we showed in the benchmark model, the regulator can
subject the bank to a minimum regulatory capital requirement of $k^*(\alpha) = 1 - \alpha$ per unit of investment in the risky project, that is, the regulator can make the risky project expensive enough so that the bank chooses not to invest in the risky project at all.

Next, we conduct the same exercise in the presence of CDS that is fairly priced as in the benchmark case, keeping everything else the same. With the CDS, the bank needs to hold $k^*(\beta) = 1 - \beta$ per unit of investment in the risky project. In this case, the bank’s expected profit can be written as

$$\pi^{CDS} = I \left[ \alpha(R - \beta) - p\beta - (1 - \beta) \right] - c(I(1 - \beta)).$$

We obtain the following proposition.

**Proposition 9** With fairly-priced CDS contract, the amount of investment $I$ in the risky project is given as:

$$c'(I(1 - \beta)) = \frac{\alpha(R - \beta) - p\beta - (1 - \beta)}{1 - \beta}.\]$$

Furthermore, $I$ is decreasing in $p$, that is, $\frac{\partial I}{\partial p} < 0$.

As in the case without CDS, as the bank holds less capital $k$, it invests more in risky project. Note that the CDS is fairly priced as in the benchmark case and the amount of investment in the risky project is decreasing in $p$, that is, the bank invests more in the risky project when the CDS is cheaper, given that it provides enough capital relief, a result similar to the one characterized in Proposition 3 in the benchmark model.

### 3.2 Asset price bubbles

This analysis mostly follows Allen and Gale (2000a). Suppose the bank acquires the risky asset (once it is generated) rather than investing 1 unit to generate the investment itself. Next, I look at how much the bank is willing to pay for the asset and show that as the bank finances the acquisition with less capital (own money) and more insured deposits, it is willing to pay more for the asset.

When the bank pays the maximum price $\bar{q}$, its expected profit can be written as

$$\pi = \alpha(R - (\bar{q} - k)) - k,$$

(11)
where it finances the acquisition with \( k \) units of own funds (capital) and \((\tilde{q} - k)\) of insured deposits. Hence, the maximum price the bank is willing to pay is given as

\[
\tilde{q} = R - \frac{k(1 - \alpha) + (r - 1)}{\alpha}.
\] (12)

Note that \( \tilde{q} \) is decreasing in \( k \) so that as the bank finances itself with less capital, it is willing to pay more for the asset.

We can conduct a similar analysis by introducing the CDS contract, where the CDS contract promises to pay \( \tilde{q} \) when the return is low. In that case, the bank needs to hold \((1 - \beta)\) units of capital and we can write the bank’s expected profit as

\[
\pi^{CDS} = \alpha(R - (\tilde{q} - (1 - \beta))) - p(\tilde{q} - (1 - \beta)) - (1 - \beta).
\] (13)

We can obtain the maximum price the bank is willing to pay as follows

\[
\tilde{q} = \frac{\alpha R - (1 - \beta)(1 - \alpha - p) - (r - 1)}{\alpha + p}.
\] (14)

We can show that the bank is willing to pay a higher price for the asset when the price (therefore the quality) of the CDS is low (low \( p \)). This gives us the following proposition.

**Proposition 10** The maximum price the bank is willing to pay for the asset is decreasing in \( p \), that is, \( \frac{dq}{dp} < 0 \).

### 3.3 Multiple banks

In this section, I extend the model to allow for multiple banks that acquire CDS from the insurer. Suppose there are two banks, Bank A and Bank B. Both banks are ex-ante identical. Banks have the risky projects that they can invest in as in the benchmark case. For simplicity, I assume that the returns from each bank’s risky project are independent. Hence, we have the following joint probabilities for the banks’ returns:

- The probability that both banks are successful (state \( S_A S_B \)) is \( \alpha^2 \).
- The probability that one bank is successful and the other bank fails (state \( S_A F_B \) and \( F_A S_B \)) is \( \alpha(1 - \alpha) \).
The probability that both banks fail (state $F_AF_B$) is $(1 - \alpha)^2$.

The joint probabilities are presented in Table 3.

\[ \text{Table 3 here} \]

Next, I look at two different CDS contracts that differ in their quality. In particular, I denote the CDS contract that is honored only in individual failure states (state $S_AF_B$ and $F_AS_B$ but not state $F_AF_B$) as $CDS_1$, whereas I denote the CDS contract that is honored in all states as $CDS_2$. Note that the fair value of $CDS_2$ is $p = 1 - \alpha$. By proposition 3, the bank will not demand $CDS_2$.

On the other hand, the bank may demand $CDS_1$. The joint probabilities for an individual bank, Bank $j$ with $j = A, B$, and the insurer are given in Table 4.

\[ \text{Table 4 here} \]

When Bank $j$ fails, the insurer honors the CDS contract only if the other bank did not fail, an event that occurs with probability $\alpha(1 - \alpha)$, whereas the insurer cannot honor the CDS contract if both banks fail, an event that occurs with probability $(1 - \alpha)^2$. Hence, $CDS_1$ provides partial insurance since with probability $(1 - \alpha)^2$ the bank fails and the insurer cannot honor the CDS. Note that the probability that Bank $j$ fails and the insurer honors $CDS_1$ is $p = \alpha(1 - \alpha)$. From Proposition 3, for $\alpha(1 - \alpha) < p^*$, where $p^*$ is given in equation 8, the bank acquirers the CDS and invests in the risky project.

4 Discussion

In this section I provide a discussion of some of the related issues that are relevant and important for the issues addressed in the paper.

**Capital requirements for CDS sellers:** Banks that are subject Basel capital requirements have to treat any CDS they sell as if they have wholesale exposure to the credit risk of the
reference entity and need to hold regulatory capital accordingly. However, this was not the case for all institutions that sell CDS. Furthermore, the seller of the CDS need not be a regulated entity and is not required to hold reserves to pay off the claims of the CDS holders. While some of the CDS sellers that are outside the umbrella of the Basel capital requirements hold collateral and post margins, this was not necessarily the case as I explain in the case of AIG below. As a result the seller of the CDS may not have enough funds to back up the claims when they come due leading to double default. Hence, when the CDS seller does not set aside enough reserves ex ante, the CDS may provide a false sense of safety especially when the probability of systemic risk states are miscalculated. For example, AIG Financial Products’ many exposures were not initially collateralized, that is, no initial or variation margin were posted for the CDS contracts it sold. Rather, AIG Financial Products relied on AIG’s backing and high ratings. However, the credit rating triggers in the CDS contract lead to margin calls when market conditions deteriorated, which was one of the main reason for AIG’s near failure.

**Miscalculated or ignored risks:** In this paper I assumed that all agents know about the true probabilities. The results of the paper and the systemic risk consequences of using the CDS for regulatory arbitrage would be stronger if I allow for the probabilities of failure and more importantly for the correlation between the banks and the insurer to be miscalculated. For example, agents may miscalculate or rationally ignore the risk of an aggregate crisis (Gennaioli, Shleifer and Vishny, 2012), where both banks and the insurer fail, that is, the correlation risk being underestimated. This would only make the CDS cheaper ex ante and would strengthen the incentives for risk shifting using the CDS.

The regulatory rules through which CDS can help free up capital depends crucially on the credit risk of the CDS buyer and the counterparty risk associated with the CDS seller as estimated by their ratings. Some argue that the rating agencies misrated various institutions and financial products prior to the crisis. Jarrow (2009) shows the ratings of major financial institutions one month before their failure or near-failure. For example, AIG, a major CDS

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15This can result from mistakes in the calculation of the ratings, whereas some agency problems can also play a role. Rating agencies are paid by the institutions they rate, which can lead to rating shopping, where the institutions that are rated choose among agencies based on the ratings they give. This, in turn, can lead to inflated ratings. See Coval, Jurek and Stafford (2008), Jarrow and Xu (2010), Bolton, Freixas and Shapiro (2011), to cite a few.
seller, has a high enough rating that would provide the highest capital relief through the CDS contracts it sells.

**Asset sales and CDS for freeing up regulatory capital:** In addition to acquiring CDS contracts, banks can off-load credit risk and free up capital by selling the loans they originate (Parlour and Winton (2011)). Acharya and Schnabl (2011) document that commercial banks set up conduits to securitize assets while insuring the newly securitized assets using explicit guarantees. They argue that regulatory arbitrage and reducing capital requirements was the main motive behind setting up conduits, and the guarantees provided recourse to bank balance sheets for outside investors. As a result, conduits provided little risk transfer since losses from conduits remained with banks rather than outside investors.

In the paper, I looked at the bank’s decision to invest in highly risky and negative NPV projects. If the bank wants to sell this risky project, it would get a price of \( \alpha R < 1 \) and the bank would not be able to make a positive profit. Hence, the asset sale would not provide the same incentives as the CDS. However, the acquirer of the loans can be another entity under the umbrella of the deposit insurance guarantee as the originating bank so that the acquiring bank would be willing to pay a price higher than the fundamental value for the loan, as discussed in Section 3.2. Or, the acquirer of the loans can be an affiliated institution that is kept under the safety net through guarantees as argued in Acharya and Schnabl (2011). In those cases, even though assets are sold and the bank can hold less capital against them, they would still be under the umbrella of the safety net and asset sales would allow for similar adverse incentives as the CDS.

**Costs of crises:** The costs of bank failures, especially when the regulators feel compelled to bailout or restructure failed banks using public funds can be significant.\(^{16}\) The provision of immediate funds to pay off failed deposits entails fiscal costs for the regulator. The fiscal costs of providing funds with immediacy can be linked to a variety of sources, most notably, (i) distortionary effects of tax increases required to fund bailouts; and, (ii) the likely effect of

\(^{16}\)See, for example, the discussion on fiscal costs associated with banking collapses and bailouts in Calomiris (1998). Hoggarth, Reis and Saporta (2002) find that the cumulative output losses have amounted to 15-20% annual GDP in the banking crises of the past 25 years. Caprio and Klingebiel (1996) document that the estimated cost of bailouts were 16.8% for Spain, 6.4% for Sweden and 8% for Finland. Honohan and Klingebiel (2000) find that countries spent 12.8% of their GDP to clean up their banking systems whereas Claessens, Djankov and Klingebiel (1999) set the cost at 15-50% of GDP.
government deficits on the country’s exchange rate, manifested in the fact that banking crises and currency crises have often occurred as twins in many countries (especially, in emerging market countries).

To generate the needed funds, governments can also borrow domestically. However, its effectiveness depends on how well the public debt markets are established. In countries where a well-established government bond market does not exist, governments may not be able to generate the necessary funds domestically at a reasonable cost. Alternatively, countries can borrow internationally. While some countries may have easier access to international capital markets, it may not be so easy for others. For example, countries with weak economic health would have lower credit ratings and the market would attach higher probabilities for these countries to default on their debt. This would ultimately increase these countries’ borrowing costs and may force them to borrow at increasing rates with very short maturities, which may result in the increasing and convex fiscal costs. Ultimately, to generate the necessary funds, the governments have to introduce taxes. In countries where the tax base is narrow, large increases in tax rates have to be introduced, which can introduce a significant tax bill on (some parts of) the society.

5 Concluding remarks

In this paper, I analyzed the use of CDS to free up regulatory capital in bank’s balance sheets and its consequences for systemic risk. The paper showed that in the presence of adverse incentives and an inadequate regulatory framework, the use of CDS for regulatory capital relief can have adverse effects and lead to excessive risk taking. However, this does not mean that CDS (or financial innovation in general) is bad per se, rather, to generate its full benefit for the society, financial innovation needs to be accompanied by an adequate regulatory framework to set the right incentives. In particular, when CDS is used for generating investment in good projects, it is socially beneficial. Hence, the design and the regulation of credit derivatives, and in particular CDS, is an extremely important issue. To get a better understanding of the benefits and potential costs of financial innovation more research, both theoretical and empirical, is needed.

Correlation risk, that is, in this paper’s context, the risk that both the bank and the insurer fail at the same time is an extremely important issue and the main driver of the
results of the paper. While recent attempts in implementing regulation with a focus on the system as a whole (macroprudential) rather than only focusing on individual institutions in isolation (microprudential) are steps in the right direction, design and implementation of macroprudential regulation is a very important area for policy makers and for future research.

The theoretical results of the paper also pose important empirical questions for further research. In particular, empirical work on getting an estimate of how much capital banks freed up in their balance sheets prior to the crisis may shed some light on the systemic risk consequences of the practice of using CDS for capital relief purposes, and would help us analyze the optimal design of capital regulation and the risk mitigation role of CDS. Finally, while there are some recent empirical studies on CDS, as Stulz (2010) points there is a dearth of empirical studies on the social benefits and costs of credit default swaps and other derivatives and we need more serious empirical work on the topic.\footnote{This is potentially due to data availability problems. While recently DTCC made some data available on CDS, for the purposes of the issues raised in this paper, we would need data going back to pre-crisis, in particular, the build-up period leading to the crisis. Arora, Gandhi and Longstaff (2011) and Oehmke and Zawadowski (2012) are two recent empirical studies that use data for the period starting 2008.}

References


Arora, Navneet, Priyank Gandhi and Francis A. Longstaff (2011) Counterparty Credit Risk and the Credit Default Swap Market, Working Paper, UCLA.


**Proofs**

**Proof of Proposition 3:** Note that the minimum value $p$ can take is $\beta - \alpha$. We need to check that there is a range of parameter values the proposition holds, that is, $p^* > \beta - \alpha$. Note that $p^* > \beta - \alpha$ if and only if $\frac{\alpha R - r}{\beta} + 1 > \beta$, that is,

$$
\beta^2 - \beta + (r - \alpha R) < 0,
$$

which holds when

$$
\frac{1 - \sqrt{1 - 4(r - \alpha R)}}{2} < \beta < \frac{1 + \sqrt{1 - 4(r - \alpha R)}}{2}.
$$

**Proof of Proposition 4:** From $\pi^{CDS}(k^*(\alpha, d)) = 0$, we obtain

$$
k^* = \frac{\alpha R - r + d}{d},
$$

which gives us

$$
\frac{\partial k^*}{\partial d} = -\frac{\alpha R}{d^2} > 0.
$$
Proof of Proposition 5: Suppose that the CDS is traded at a price \( p_N \) between the bank and the insurer. Note that the threat point for the insurer and the bank is 0 and \( p_N \) has to be between \( p \) and \( \bar{p} \). In that case, the utility for the bank and the insurer, denoted by \( u_B \) and \( u_I \), respectively, are given as follows:

\[
 u_B = \bar{p} - p_N \quad \text{and} \quad u_I = p_N - p.
\]

The Nash bargaining solution maximizes the product of the utilities

\[
 U = u_B \times u_I = (\bar{p} - p_N)(p_N - p).
\]

We obtain

\[
 \frac{dU}{dp_N} = -(p_N - p) + (\bar{p} - p_N) = \bar{p} + p - 2p_N,
\]

so that \( p_N = \frac{\bar{p} + p}{2} \).

Proof of Proposition 8: Note that when the bank invests \( I \) units in the risky project, it needs to hold \( x = Ik \) units of capital. Taking \( k \) as given, the bank will choose \( I \) where

\[
 \frac{\partial \pi}{\partial I} = \alpha(R - (1 - k)) - k - kc'(Ik) = r - 1.
\]

Hence, the size of the investment in the risky project \( I \) is determined by

\[
 c'(Ik) = \frac{\alpha(R - (1 - k)) - k(1 - \alpha)}{k}.
\]

We can also show that \( I \) is decreasing in \( k \) using the implicit function theorem. By taking the derivative of the FOC in equation (15) with respect to \( k \), we obtain

\[
 \alpha - 1 - c'(Ik) - Ik c''(Ik) \frac{\partial I}{\partial k} = 0,
\]

so that

\[
 \frac{\partial I}{\partial k} = \frac{\alpha - 1 - c'(Ik)}{Ik c''(Ik)} < 0,
\]

since the denominator is positive and the numerator is negative. Hence, as the bank holds less capital \( k \), it invests more in the risky project.
Proof of Proposition 9: The bank will choose $I$ where
\[
\frac{\partial \pi}{\partial I} = \alpha(R - \beta) - p\beta - (1 - \beta) - (1 - \beta)c'(I(1 - \beta)) = r - 1.
\]
Hence, the size of the bank is determined by
\[
c'(I(1 - \beta)) = \frac{\alpha R - \beta(\alpha + p) - (r - 1)}{1 - \beta} - 1.
\]
We can show that $I$ is decreasing in $p$ using the implicit function theorem. By taking the derivative of the FOC with respect to $p$, we obtain
\[
-\beta - (1 - \beta)^2 c''(I(1 - \beta)) \frac{\partial I}{\partial p} = 0,
\]
so that
\[
\frac{\partial I}{\partial p} = -\frac{\beta}{(1 - \beta)^2 c''(I(1 - \beta))} < 0.
\]

Proof of Proposition 10: Note that
\[
\frac{dq}{dp} = -\frac{\alpha R - (1 - \beta)}{(\alpha + p)^2} < 0,
\]
(16)
since for the bank to take the investment and acquire the CDS we need $\alpha R - (1 - \beta) > 0$. 

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TABLES AND FIGURES

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Table 1: Notional amount of CDS outstanding (year-end, $ trillion) (source ISDA)

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<th>Insurer</th>
<th>$</th>
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<tr>
<td>S</td>
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<td>$b = \alpha - \beta + \rho$</td>
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<tr>
<td>F</td>
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Table 2: Joint probabilities of the bank and the insurer.

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<td>$\alpha^2$</td>
<td>$\alpha(1 - \alpha)$</td>
</tr>
<tr>
<td>$\alpha(1 - \alpha)$</td>
<td>$(1 - \alpha)^2$</td>
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Table 3: Joint probabilities of the two banks.

<table>
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</thead>
<tbody>
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<td>F_i</td>
<td>F_i</td>
</tr>
<tr>
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</tr>
</tbody>
</table>

Table 4: Joint probabilities of the bank and the insurer.
Figure 1: Capital requirements.

Figure 2: Price of regulatory arbitrage and the fair price of the CDS.