Contracting Frictions in Global Sourcing: Implications for Welfare*

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10 July 2020

Abstract

Contracting frictions affect firm sourcing decisions, but how much do such frictions — that firms encounter in their interactions with suppliers around the world — ultimately matter for trade patterns and country welfare? We develop a model of firm global sourcing in the presence of contracting frictions, that we embed into a quantitative general-equilibrium trade framework. At the micro level, each firm identifies a set of suppliers for its customized inputs, and these interactions are subject to a bilateral holdup problem, following the Grossman-Hart-Moore property-rights approach to the theory of the firm. For each input, the firm chooses the country to source from, as well as the organizational mode: whether to source from within or outside firm boundaries. These firm-level decisions aggregate into a gravity equation for bilateral trade flows by industry and organizational mode, and hence yield a structural expression for the intrafirm trade share. We derive a closed-form expression for welfare that nests the Arkolakis, Costinot and Rodriguez-Clare (AER 2012) formula, making clear how contracting frictions affect welfare relative to that benchmark. By associating key contracting-related parameters in the model to observables (specifically, input industry characteristics and source country rule of law), we are then able to estimate the model using U.S. data on intrafirm trade shares. We demonstrate through a series of counterfactual exercises how and how much such contracting frictions affect the pattern of trade and country welfare.

*For their valuable comments, we thank Andrés Rodríguez-Clare, Michael Sposi, Dan Trefler, and audiences at the SAET Annual Conference, Dartmouth, the World Bank, the Danish International Economics Workshop, Simon Fraser University, Tokyo University, Southern Methodist University, and the NBER ITI Summer Institute.

1 Introduction

The rise of global sourcing arrangements has been a defining feature of production, in the globalization wave that the world has ridden in recent decades. The declines in barriers to trade and communication have made it possible for firms to tap into the capabilities of suppliers located around the world, even for the sourcing of some highly specialized inputs. Not surprisingly, the input content of goods that ultimately derives from foreign sources has been increasing (Johnson and Noguera 2012; Koopman et al. 2014), giving rise to such monikers as "Made in the World" and "Global Value Chains" to describe this production phenomenon.

The question of what drives patterns of global sourcing – which countries firms choose to source particular inputs from – has naturally attracted the attention of researchers and policymakers. While a core line of work has focused on neoclassical explanations such as technology and factor endowments (Feenstra and Hanson 1996; Grossman and Rossi-Hansberg 2008), there is now also a keen recognition that institutional forces related to contracting frictions can weigh on these decisions. Put simply, variation across countries in institutional conditions pertaining to the enforcement of contracts can affect the decisions that firms make over where to source an input from. The existing evidence on institutions and trade flows points to such a link: Country rule of law, for example, is known to correlate positively with exports, particularly in contract-intensive industries in which production and input procurement are more exposed to holdup problems (Levchenko 2007; Nunn 2007; Ranjan and Lee 2007).[1]

The global sourcing problem of a firm that operates in the shadow of contracting frictions features a second active decision margin, namely over organizational mode. When the terms of a written contract are not fully enforceable by third-party courts, the relationship between the firm and any given supplier is exposed to opportunistic behavior, particularly when the inputs involved are highly customized. Should the firm then integrate a supplier to assume ownership and control, or instead source the input at arm's-length? We now have well-developed frameworks that analyze how this organizational mode decision can serve to offset (albeit partially) the inefficiencies caused by limits on contracting. A key insight from the Grossman-Hart-Moore approach to the theory of the firm is that this organizational mode choice would seek to assign greater residual rights of control to the party responsible for the more important input, in order to provide that party with better incentives to invest in relationship-specific effort. In the global sourcing context (e.g., Antràs and Helpman 2004, 2008), this logic would call for integration when firm headquarter services are particularly important, with the converse being true – favoring outsourcing – in industries that are more intensive in their use of supplier inputs.

The international trade data once again offer a lens through which researchers have tested these predictions on organizational mode in global sourcing. In particular, the share of trade flows that is observed to be between parties related by firm ownership – commonly termed the intrafirm trade share

[1] Nunn and Trefler (2014) includes a comprehensive survey of this literature, on the role of contracting and other institutional forces in driving country comparative advantage.

– has been used regularly as a measure of the propensity to integrate the sourcing of inputs from the industry in question. This intrafirm trade share has been shown to correlate positively with proxies for the importance of headquarter inputs, such as an industry's capital-intensity (e.g., Antràs 2003; Nunn and Trefler 2008, 2013; Corcos et al. 2013). The existing evidence therefore points robustly to the broad relevance of this key prediction stemming from the Grossman-Hart-Moore approach. That said, it remains an open question *how much* such contracting frictions matter for input sourcing patterns, and ultimately, country welfare.

We set out to make progress on this research question, to provide a fuller quantitative assessment of the implications of contracting frictions for global sourcing. Toward this goal, we build a firm-level model of input sourcing under incomplete contracts, that we then embed into a general equilibrium setting that is amenable to estimation and quantification. At the level of the firm, production of a final-good variety requires intermediate inputs from a discrete number of industries. Following Tintelnot (2017) and Antràs et al. (2017), we model the composite input in each of these industries as formed by a unit continuum of input varieties. The firm in turn makes an optimal decision over *sourcing mode* for each input variety, this being a joint decision over both the country to source from, as well as the organizational mode – integration or outsourcing – to adopt for the chosen supplier.[2] (In a world with J countries, there are thus $2J$ sourcing mode options for each input variety; this includes the possibility of sourcing from the home country.)

The incomplete contracting environment is modeled following the Grossman-Hart-Moore framework: For each input variety, both the firm and the chosen supplier are responsible for delivering relationship-specific task inputs. At the same time, both parties are prone to under-invest in the delivery of their respective task inputs relative to the efficient benchmark. The model incorporates parameters that encode the inherent degree of contractibility of headquarter (respectively, supplier) task inputs, as in the partial contractibility formulations of Acemoglu et al. (2007) and Antràs and Helpman (2008). Separately, the outcome of the ex-post renegotiation that takes place between the buyer and the supplier is described by a pair of bargaining share parameters, corresponding to the division of surplus that would take place under integration and outsourcing respectively. In line with the property-rights view of firm boundaries, we assume that integration confers the firm a better residual bargaining share relative to outsourcing, all else equal. Of note, the model is flexible in allowing for the above contractibility and bargaining share parameters to differ by the identity of the industry – reflecting intrinsic differences in input features – as well as by source country – reflecting variation in country institutions such as the rule of law. This is in addition to the model's ability to accommodate differences across industries in their underlying intensity in the use of headquarter services relative to supplier inputs.

Following Eaton and Kortum (2002) and Antràs et al. (2017), each firm obtains a separate set of draws for each input variety, which determine the unit production costs for supplier inputs that the firm would face under each sourcing mode. The continuum of input varieties allows us to define a

[2] While we acknowledge that there can be varying degrees of ownership that a firm might take in its supplier, we follow the literature in modeling this as a binary decision.

notion of sourcing shares, this being the share of input varieties for which a given sourcing mode is optimal from the perspective of the firm's overall payoff. We specify a productivity distribution for each set of $2J$ draws that builds in a correlation among the draws that emanate from the same country. This is intended to reflect common technological drivers for suppliers within a country (regardless of the organizational mode), and moreover will relax the Independence of Irrelevant Alternatives (IIA) property that would otherwise emerge with the more standard Fréchet distribution when comparing shares across any pair of sourcing modes. This setup delivers a closed-form expression for sourcing shares that has an intuitive nested logit form: The share of inputs obtained from country i under (say) integration is equal to the share sourced from country i, multiplied by the share sourced under integration conditional on having chosen country i. The expressions moreover show in a transparent manner how contracting frictions enter these sourcing shares, as terms that retard the effective state of technology that the firm has access to.

While the firm-level model builds in a lot of structure, it nevertheless aggregates neatly across all firms and exhibits the analytical tractability of a familiar class of quantitative trade models. Our framework therefore delivers an expression for welfare, and consequently too for welfare changes in response to shifts in fundamentals such as trade costs and contracting frictions. This welfare change expression nests the corresponding formulae from Arkolakis et al. (2012) and Costinot and Rodríguez-Clare (2014) in the limit case where all inputs are fully contractible. We are therefore able to evaluate counterfactuals where trade costs are reduced, or where contracting frictions are lowered, following the hat algebra approach in Dekle et al. (2008) and Caliendo and Parro (2015).

Our model facilitates a tight transition from the theory to estimation. We show how to derive an exact estimating equation for the intrafirm trade share, allowing us to attach a structural interpretation to this key variable which has featured in otherwise reduced-form regression-based empirical work on the correlates of firm boundary decisions.[3] This implies a relatively low set of data requirements to quantify the model: All that is needed to estimate the key contractibility and bargaining parameters is data on the intrafirm trade share suitably aggregated at the country-pair by industry level, together with a functional form specification for how these parameters map to observables, such as country rule of law and intrinsic industry characteristics.

We implement this via a (weighted) non-linear least squares (NLLS) estimator, that seeks to minimize the distance between the model-predicted and observed intrafirm trade shares for countries and NAICS 3-digit industries in the US data.[4] The estimation reveals that the key contractibility parameters vary with country rule of law, with a measure of industry product specificity from Rauch (1999), and with measures of industry input contractibility constructed from Nunn (2007), in rich but nevertheless intuitive ways.[5] We moreover obtain an estimate for the within-country productivity draw correlation

[3]See Antràs (2015) for a survey of this literature, that covers a broad range of data settings.

[4]In practice, we solve the NLLS problem with a mixture of heuristic and gradient-based optimization algorithms, and the computational load can be reasonably handled without the use of high-speed computer clusters.

[5]This follows the lead of several empirical papers, including Nunn and Trefler (2008, 2013), Bernard et al. (2010), and Defever and Toubal (2013), which have reported correlations between intrafirm trade shares, input characteristics related

parameter that points to its relevance in accounting for the variation exhibited by the intrafirm trade share.

With these estimates, we carry out several counterfactual exercises to shed light on the welfare implications of contracting frictions. In a first exercise, we eliminate contracting frictions; our model conveniently allows us to execute this counterfactual as the limit case where headquarter and supplier services embodied in all industries are fully contractible. We find clear benefits from such an improvement in contracting: all countries register welfare gains, with the mean increase across countries being 3.49%. In a second exercise, we consider the welfare consequences of reducing the firm's bargaining share under integration to that under outsourcing, in effect removing the distinction between these two organizational modes. We find that countries that initially exhibited high intrafirm import shares – implying a greater dependence on integration – record modest losses to country welfare. Third, we compare the gains from trade under current contracting conditions and a hypothetical world where all contracting frictions have been removed. Interestingly, we find that the move to a full contractibility world tends to magnify the gains from trade for those countries that had high initial levels of rule of law, as these countries became increasingly plugged in to sourcing opportunities abroad. Last but not least, we evaluate the consequences of an improvement in rule of law for a particular country of interest, China.

As should be clear, the work in this paper brings together two strands in the international trade literature. On the one hand, it builds on the firm-level models that have leveraged insights from the Grossman-Hart-Moore framework, to develop a deeper understanding of the interplay between location and ownership decisions in global sourcing.[6] On the other hand, it adds to a growing body of work that seeks to quantitatively assess various implications of global supply chains in a general equilibrium setting; see for example, Yi (2010), Antràs and de Gortari (2017), and Johnson and Moxnes (2019), though these preceding papers do not examine contracting frictions explicitly. Fally and Hillberry (2018) and Boehm (2018) are two exceptions that provide welfare assessments of contracting frictions in a multi-country general equilibrium setting. That said, contracting frictions in these papers are modeled on a Coasian (or transactions-cost based) approach, in which the firm-supplier relationship features a one-sided (rather than bilateral) holdup problem. On a more closely-related note, Boehm and Oberfield (2020) perform a rich exercise to quantify the extent to which misallocation in the sourcing of relationship-specific inputs has negatively impinged on the productivity of Indian firms.

The rest of the paper proceeds as follows. Section 2 sets up the firm-level decision problem over the global sourcing of its inputs. This provides the building block for the general equilibrium model

to specificity and contractibility, as well as country characteristics related to the rule of law. The broad survey of this line of work in Antràs (2015) suggests that it has not been easy to establish robust correlations across these variables. We seek to shed light on the relevance of these country and industry variables related to contracting conditions with a more structural approach.

[6]The firms in the model we develop in this paper need to interact with a large number of suppliers, but these sourcing decisions are made simultaneously. See Antràs and Chor (2013) and Alfaro et al. (2019) for property-rights based models of firm boundaries where the sequentiality of the sourcing decisions plays a key role in influencing organizational decisions in a long supply chain.

in Section 3. In Section 4, we show how we take the model to the data, in order to estimate its key parameters. Section 5 delves into the quantitative insights from a series of counterfactual exercises. We provide detailed derivations in a series of appendices.

2 The Model: Global Sourcing at the Firm Level

We develop a multi-country, multi-industry model of global sourcing, where each final-good producer (or "firm") sources a continuum of inputs (from "suppliers"). For each input variety, each firm faces a decision over: (i) which country to source it from; and (ii) under which organizational mode – integration or outsourcing – to conduct this sourcing. We set up the firm decision problem in this section; this lays the groundwork for Section 3, where we will aggregate across firms to obtain predictions on industry-level trade flows, the intrafirm trade share, as well as country welfare.

2.1 Basic Setup

We consider a world with $J > 1$ countries. Throughout the exposition, we use j to refer to the home country of a final-good producer, and i to refer to input source countries. The utility of the representative consumer in country j is given by:

$$U_j = \left(\int_{\omega} c_j(\omega)^{\rho} d\omega \right)^{\frac{1}{\rho}}, \ \rho, \in (0,1). \tag{1}$$

Here, $c_j(\omega)$ is the quantity consumed of the final-good differentiated variety indexed by ω, and the elasticity of substitution across each pair of these varieties is equal to $1/(1-\rho) > 1$. In any given country, each ω is produced by a distinct firm; the market over these varieties features monopolistic competition. We assume that final-good varieties are consumed in the country in which they are assembled (i.e., they are not traded across borders), but that the input varieties required to produce each final-good are sourced globally.[7]

We start by describing the production function, for the final-good firm in country j that produces variety ω. Let ϕ be the firm's core productivity; this is obtained as an iid draw from an underlying distribution with cdf $G_j(\phi)$, whose support is a connected subset of the positive real line. The production function for this firm with core productivity ϕ is given by:

$$y_j(\phi) = \phi \left(\prod_{k=1}^{K} \left(X_j^k(\phi) \right)^{\eta^k} \right)^{1-\alpha} L_j(\phi)^{\alpha}. \tag{2}$$

Production requires inputs from $K \geq 1$ distinct industries (indexed by k), where $X_j^k(\phi)$ denotes the composite input from industry $k \in \{1, \ldots, K\}$ that is used by the firm in question. These composite inputs are assembled in a Cobb-Douglas fashion, with the industry-k exponent being equal to $\eta^k \in (0,1)$ (and $\sum_{k=1}^{K} \eta^k = 1$). These industry inputs are combined with an amount $L_j(\phi)$ of assembly labor, with $\alpha \in (0,1)$ being the labor share.

[7] Since final-good varieties are tied to the country in which they are consumed, one should more precisely add a j subscript to ω. We suppress this to save on notation.

Each industry-k composite input is itself an aggregate over a continuum of industry-k input varieties, denoted by $\tilde{x}_j^k(\phi; l)$, where $l \in [0,1]$. In particular:

$$X_j^k(\phi) = \left(\int_{l=0}^{1} \tilde{x}_j^k(\phi; l)^{\rho^k} dl \right)^{\frac{1}{\rho^k}}, \tag{3}$$

where $\rho^k \in (0,1)$ governs the degree of substitutability of input varieties in industry k; the relevant elasticity of substitution is equal to $1/(1-\rho^k) > 1$. The measure of these input varieties is normalized to 1, with $l \in [0,1]$ indexing these input varieties.[8] The above specification of a continuum of input varieties follows closely Tintelnot (2017) and Antràs, Fort and Tintelnot (2017), and will facilitate the derivation of expressions for input sourcing shares that are smooth functions of the underlying model parameters. We assume that $0 < \rho < \rho^k < 1$ for all industries k, so that input varieties from the same industry are closer substitutes than are final-good varieties.

Each input variety l in the right-hand side of (3) is in turn a Cobb-Douglas combination of headquarter services, $h_j^k(\phi; l)$, and supplier services, $x_j^k(\phi; l)$:

$$\tilde{x}_j^k(\phi; l) = \left[h_j^k(\phi; l) \right]^{\alpha^k} \left[x_j^k(\phi; l) \right]^{1-\alpha^k}. \tag{4}$$

The parameter $\alpha^k \in (0,1)$ reflects the headquarter-intensity of industry-k input varieties. As the variable names suggest, the input services embodied in $h_j^k(\phi; l)$ are provided by the firm headquarters (e.g., design or managerial inputs), while $x_j^k(\phi; l)$ is obtained from the unique supplier whom the firm has contracted with (e.g., parts and components). In the incomplete contracting environment we consider, both headquarter and supplier services are relationship-specific in the sense that they are each customized as inputs for the particular final-good variety ω. Thus, $h_j^k(\phi; l)$ and $x_j^k(\phi; l)$ have a diminished value if one were to attempt to use either of these in the production of other final-good varieties. This opens up the production process to a bilateral holdup problem, as in Grossman and Hart (1986) and Hart and Moore (1990).

We further accommodate the possibility that headquarter and supplier services might differ across industries in their degree of contractibility, depending on the inherent features of these inputs. We adopt the formulation from Acemoglu et al. (2007) and Antràs and Helpman (2008), by specifying that $h_j^k(\phi; l)$ and $x_j^k(\phi; l)$ are each composed of a unit continuum of "tasks", indexed by ι_h and ι_x respectively, with:

$$h_j^k(\phi; l) = \exp\left\{ \int_{\iota_h=0}^{\mu_{hij}^k} \log h_j^k(\iota_h; \phi, l) d\iota_h + \int_{\iota_h=\mu_{hij}^k}^{1} \log h_j^k(\iota_h; \phi, l) d\iota_h \right\}, \text{ and} \tag{5}$$

$$x_j^k(\phi; l) = \exp\left\{ \int_{\iota_x=0}^{\mu_{xij}^k} \log x_j^k(\iota_x; \phi, l) d\iota_x + \int_{\iota_x=\mu_{xij}^k}^{1} \log x_j^k(\iota_x; \phi, l) d\iota_x \right\}. \tag{6}$$

For $x_j^k(\phi; l)$, all supplier tasks in the full measure $\iota_x \in [0,1]$ are performed by the unique supplier that the firm has selected for that input variety l. Among these, the tasks in the range $[0, \mu_{xij}^k]$ are fully

[8] To be more precise, l should carry with it a superscript k to identify the industry to which the variety l belongs, but we omit this to avoid cluttering the notation.

contractible in the sense that the supplier effort levels can be fully specified in the ex-ante contract and enforced ex-post by an independent third-party (such as a court of law). The remaining tasks in $(\mu^k_{xij}, 1]$ are noncontractible, and the supplier has full discretion over how much effort to exert on each of these tasks. Likewise for $h^k_j(\phi; l)$, all headquarter tasks indexed on the unit interval are provided by the firm; tasks in the range $[0, \mu^k_{hij}]$ are fully contractible, while tasks in $(\mu^k_{hij}, 1]$ are noncontractible. The parameters μ^k_{hij} and μ^k_{xij} thus capture the contractibility of headquarter and supplier services respectively. In particular, when $\mu^k_{hij} = \mu^k_{xij} = 1$, all inputs become fully contractible; as we will see, the expressions our model delivers in that special case correspond to the first-best world with no contracting frictions. Note that μ^k_{hij} and μ^k_{xij} depend on the identity of the industry k, reflecting the inherent characteristics of inputs from that industry; these contractibility parameters could also depend on the identities of the source country i and destination country j, reflecting for example the role of institutions in each country that are relevant for the enforcement of commercial contracts.[9] Moving forward, to simplify the notation, we will use ι to refer to either ι_h or ι_x, though it should be understood that the headquarter and supplier tasks in (5) and (6) are distinct tasks performed by different agents.

This concludes our description of the structure of the production function. At its upper-most layer, the final-good variety is put together using assembly labor and composite inputs from each of K industries. Each composite input is a CES aggregate of input varieties, that are themselves composed of headquarter and supplier services, which are in turn composed of contractible and noncontractible tasks. The output of the final-good firm is then sold domestically. The CES utility function from (1) implies that the quantity demanded, $q_j(\phi)$, of a final-good variety (assembled by a firm with core productivity ϕ) is a familiar isoelastic function of its price, $p_j(\phi)$, namely:

$$q_j(\phi) = A_j p_j(\phi)^{-\frac{1}{1-\rho}}. \tag{7}$$

The aggregate demand shifter, A_j, is a function of the total income I_j in country j. This is given explicitly by:

$$A_j = I_j P_j^{\frac{\rho}{1-\rho}}, \tag{8}$$

where the ideal price index P_j for final-good varieties in country j is:

$$P_j = \left(N_j \int_\phi p_j(\phi)^{-\frac{\rho}{1-\rho}} dG_j(\phi) \right)^{-\frac{1-\rho}{\rho}}. \tag{9}$$

Here, N_j is the exogenous and fixed mass of firms in country j; there are thus N_j firms that sport each level of core productivity ϕ. As is well known, the constant elasticity demand structure implies that the revenue of the firm, $R_j(\phi)$, is a concave power function of the quantity demanded:

$$R_j(\phi) = A_j^{1-\rho} q_j(\phi)^\rho. \tag{10}$$

[9] To be fully formal, the subscripts of $h^k_j(\phi; l)$ and $x^k_j(\phi; l)$ should feature a further i index; we have omitted this to keep the notation neater. Note that the identity of the source country i from which each input variety l is obtained will be an endogenous decision of the firm.

2.2 Input Sourcing Environment

We turn now to set up the firm's sourcing decision, to secure the input varieties l from each industry k – the $\tilde{x}_j^k(\phi; l)$ in (4) – that are required for production of the final good. Each input variety is assumed to be entirely customized to the purposes of the final-good producer and thus has zero outside value. The sourcing decision over each input variety entails a choice over both location and organizational mode: The input variety can be obtained from any of the J countries in the world, and it can be procured either from a supplier that is integrated within the ownership boundaries of the firm or via an outsourcing arrangement. There are thus $2J$ possible sourcing options or modes for each input variety.

We assume that there is a large pool of identical potential suppliers for each input variety within each country. As a reminder, the supplier that is ultimately picked for an input variety l will perform the entire unit measure of tasks indexed by ι_x in (6) – both the contractible and noncontractible tasks – that are associated with l. At the same time, the firm is responsible for performing the entire measure of headquarter tasks indexed by ι_h in (5) for that input variety. We furthermore assume that suppliers can make ex-ante transfers to the final-good firm. Competition among potential suppliers then means that the firm will – in equilibrium – be able to extract all ex-post rents that would accrue to the chosen supplier through an ex-ante transfer (which acts as a participation fee). The decisions that the firm makes over organizational mode – whether or not to integrate the supplier of a given input variety l – are therefore not influenced by ex-post rent-extraction motives; rather, the organizational mode selected is that which best balances the incentives of the two parties as they undertake their relationship-specific investments in their respective noncontractible tasks ($h_j^k(\iota; \phi, l)$ for all $\iota \in (\mu_{hij}^k, 1]$ and $x_j^k(\iota; \phi, l)$ for all $\iota \in (\mu_{xij}^k, 1]$).

Contracting and Bargaining: The environment is one of incomplete contracts. While the firm and each input supplier sign a contract prior to the commencement of production, the only binding and enforceable terms in the contract are: (i) the ownership arrangement, namely whether the relationship is integrated within firm boundaries, or whether the supplier remains at arm's length; and (ii) the contractible tasks levels, $h_j^k(\iota; \phi, l)$ for all $\iota \in [0, \mu_{hij}^k]$, and $x_j^k(\iota; \phi, l)$ for all $\iota \in [0, \mu_{xij}^k]$. Any other terms – such as over transactions prices, or investments in the non-contractible tasks – are intrinsically unverifiable and hence unenforceable by a third-party.

The actual division of the surplus generated by the firm-supplier relationship is instead determined in ex-post renegotiation, for which we adopt a Nash bargaining protocol. The payoffs of each party from this bargaining process depend on whether integration or outsourcing is chosen as the organizational mode. Let β_{ijV}^k denote the (generalized) Nash bargaining share that accrues to a country-j firm if it integrates an industry-k input supplier from country i, and let β_{ijO}^k denote the corresponding bargaining share if outsourcing is adopted instead. The natural assumption here is that $0 < \beta_{ijO}^k < \beta_{ijV}^k < 1$, reflecting the greater residual rights of control that the firm would have over the input – in the event, say, of a breakdown in the bilateral relationship – by virtue of its ownership position (Grossman and Hart 1986). Note that β_{ijV}^k and β_{ijO}^k feature a k superscript: in practice, the inherent properties of the input could affect the relative bargaining position of the firm vis-à-vis the supplier. For example, in industries that feature relatively homogeneous products (Rauch 1999), it is presumably easier for the firm to recover pecuniary value from outside markets, should the need arise to exercise its control rights. A lower degree of industry-k specificity would thus be associated with a stronger firm bargaining position under integration, β^k_{ijV}, relative to under outsourcing, β^k_{ijO} (e.g., Antràs 2015, Eppinger and Kukharskyy 2020). The bargaining shares are moreover allowed to depend on i and j, so that both source and destination country institutions can matter for bargaining outcomes. In countries with stronger rule of law, for example, these institutional protections help to preserve the value of inputs over which the firm has control rights, by restraining the supplier from taking unilateral actions to destroy the inputs in the event of a dispute.

Timing: The timing of the sourcing and production process is as follows. For simplicity, we refer to the tuple (i, χ) as the "sourcing mode", under which the firm obtains the input in question from a supplier in country $i \in \{1, \dots, J\}$ while adopting organizational mode $\chi \in \{V, O\}$.

- Prior to any contracting or production, the firm observes the full set of draws that will govern its marginal costs for sourcing each input variety l (from each industry k) under each of the possible sourcing modes (i, χ). In other words, for each input variety l, the firm obtains $2J$ draws corresponding to each possible (i, χ), much as in Eaton and Kortum (2002) and Tintelnot (2017); we will specify the underlying distribution that governs these draws in equation (15) below. It is worth reiterating that these stochastic draws are associated with the final-good producer. In other words, each particular draw is akin to a technological capability that the firm can transfer to whichever supplier in country i it chooses to contract with for input variety l under χ as the organizational mode.

- Having observed its draws, the firm then chooses the optimal sourcing mode (i, χ) for each input variety l. The firm posts contracts in country i that specify: (i) an ex-ante participation fee; (ii) the sourcing mode (i, χ) over l; and (iii) the investment levels for the contractible tasks, $h^k_j(\iota; \phi, l)$ for $\iota \in [0, \mu^k_{hij}]$ and $x^k_j(\iota; \phi, l)$ for $\iota \in [0, \mu^k_{xij}]$.

- Suppliers apply for these posted contracts, and the firm picks a supplier for each input variety l. The firm is indifferent with regard to the actual identity of the supplier, since the ex-ante transfer will allow the firm to extract all of the supplier's ex-post surplus.

- The supplier of l discretionarily chooses how much to invest in providing the noncontractible supplier tasks, i.e., $x^k_j(\iota; \phi, l)$ for $\iota \in (\mu^k_{xij}, 1]$. The firm simultaneously chooses how much to invest in the noncontractible headquarter tasks, i.e., $h^k_j(\iota; \phi, l)$ for $\iota \in (\mu^k_{hij}, 1]$. At the same time, both the supplier and the firm make their contractible investments – $x^k_j(\iota; \phi, l)$ for $\iota \in [0, \mu^k_{xij}]$ and $h^k_j(\iota; \phi, l)$ for $\iota \in [0, \mu^k_{hij}]$ – as prescribed in the initial contract. Upon delivery of the headquarter and supplier tasks, the firm then bargains bilaterally with each supplier over the incremental revenue contributed by the input variety l in question. This determines the respective parties' payoffs from this sourcing relationship.

- Each input variety l is put together from the task inputs following (4). Each composite input is in turn put together from the $\tilde{x}_j^k(\phi;l)$'s following (3). The composite inputs $X_j^k(\phi)$ are then combined with assembly labor $L_j(\phi)$ based on (2) to obtain the final-good variety.

We take the stand that the firm engages in simultaneous bilateral bargaining with all of its input suppliers. In particular, in the bilateral bargaining with a given input supplier l from industry k, the effort levels of all other suppliers, i.e., $x_j^k(\phi;l')$ for all $l' \neq l$, and $x_j^{k'}(\phi;l')$ for all $k' \neq k$ and all $l' \in [0,1]$, as well as the headquarter services channelled toward these other input varieties, are taken as given. The firm and this input supplier l thus bargain over the surplus – or more precisely, over the incremental contribution to the firm's overall revenue – that this supplier generates by being a participant in this sourcing relationship (i.e., from delivering the supplier services $x_j^k(\phi;l)$).

We calculate this incremental revenue following the method in Acemoglu, Antràs and Helpman (2007). This involves first deriving the revenue contributed by any single supplier when the unit measure of input varieties is divided equally across $\mathcal{L}$ identical suppliers, and then obtaining the limit expression as $\mathcal{L} \longrightarrow \infty$. As we show in Appendix B.1, this yields the following expression for the incremental revenue associated with each input variety $l \in [0,1]$:

$$r_j^k(\phi;l) = (1-\alpha)\frac{\rho\eta^k}{\rho^k}R_j(\phi)\left(\frac{\tilde{x}_j^k(\phi;l)}{X_j^k(\phi)}\right)^{\rho^k}. \tag{11}$$

Equation (11) has an intuitive interpretation, namely that the bilateral interaction with the supplier l contributes a fraction $(1-\alpha)\frac{\rho\eta^k}{\rho^k}(\tilde{x}_j^k(\phi;l)/X_j^k(\phi))^{\rho^k}$ of the final-good producer's revenue, $R_j(\phi)$. This fraction is increasing in the importance of input variety l in the composite industry-k input, $\tilde{x}_j^k(\phi;l)/X_j^k(\phi)$; at the same time, it decreases the easier it is to substitute for l with other industry-k input varieties (i.e., the higher is ρ^k).[10]

2.3 Input Task Decisions

We now spell out the decision problems of the firm and the supplier of l in their bilateral interaction, and solve for their investment levels in the input tasks – i.e., the $h_j^k(\iota;\phi,l)$'s and $x_j^k(\iota;\phi,l)$'s – via backward induction. Accordingly, we first solve for each party's investments in noncontractible tasks, given the levels of contractible task input and the sourcing mode that have been written into the ex-ante contract. This will then allow us to derive the levels of contractible task input that the firm will post

[10]Note that summing up the incremental revenue in (11) across all input varieties l from all industries k, we have:

$$\sum_{k=1}^{K}(1-\alpha)\frac{\rho\eta^k}{\rho^k}R_j(\phi)\int_{l=0}^{1}\left(\frac{\tilde{x}_j^k(\phi;l)}{X_j^k(\phi)}\right)^{\rho^k}dl = \left(\sum_{k=1}^{K}(1-\alpha)\frac{\rho\eta^k}{\rho^k}\right)R_j(\phi).$$

The parameter restriction $\rho < \rho^k$ that was introduced earlier is sufficient to ensure that: $\sum_{k=1}^{K}(1-\alpha)\frac{\rho\eta^k}{\rho^k} < (1-\alpha)\sum_{k=1}^{K}\eta^k < 1$, so that the payments to input suppliers do not fully exhaust the revenue of the firm.

in the ex-ante contract, that fully anticipate the investment levels that both parties will select for their respective noncontractible task inputs.

Noncontractible input tasks: Taking the sourcing mode (i, χ) as given, the firm chooses $h_j^k(\iota; \phi, l)$ for all $\iota \in (\mu_{hij}^k, 1]$ in order to maximize:

$$\beta_{ij\chi}^k r_j^k(\phi; \ell) - s_j \int_{\mu_{hij}^k}^1 h_j^k(\iota; \phi, \ell) d\iota, \tag{12}$$

this being the share $\beta_{ij\chi}^k$ of the incremental revenue that the firm obtains from its interaction with supplier l, net of the cost of providing these noncontractible headquarter tasks. Here, s_j is the cost that the firm bears per unit of $h_j^k(\iota; \phi, l)$; this cost is incurred in country-j human capital (or skilled labor), which we can interpret more broadly as a factor of production that is used exclusively for headquarter tasks.

On the other hand, the supplier in question receives the remaining share, $1 - \beta_{ij\chi}^k$, of the incremental revenue. This supplier chooses the noncontractible task levels, $x_j^k(\iota; \phi, l)$ for all $\iota \in (\mu_{xij}^k, 1]$, in order to maximize:

$$(1 - \beta_{ij\chi}^k) r_j^k(\phi; \ell) - c_{ij\chi}^k(\phi; l) \int_{\mu_{xi}^k}^1 x_j^k(\iota; \phi, l) d\iota. \tag{13}$$

Here, $c_{ij\chi}^k(\phi; l)$ is the unit cost incurred for these supplier tasks under sourcing mode (i, χ), which we describe in more detail below. The factor of production used by suppliers is country-i labor, with associated wage equal to w_i. Although individual firms and input suppliers take w_i and s_j as given, these factor prices will be pinned down endogenously in general equilibrium. Note that both (12) and (13) are solved taking as given the pre-specified investment levels in the contractible tasks for l, as well as the investment levels in all tasks – $h_j^k(\iota; \phi, l')$ and $x_j^k(\iota; \phi, l')$ – for all other input varieties $l' \neq l$.

Supplier cost structure: The unit cost of a supplier under sourcing mode (i, χ) is given by:

$$c_{ij\chi}^k(\phi; l) = \frac{d_{ij}^k w_i}{Z_{ij\chi}^k(\phi; l)}. \tag{14}$$

Here, $d_{ij}^k \geq 1$ is the iceberg trade cost of shipping the input from its source country i to the destination country j, so the supplier has to produce d_{ij}^k units of the input variety in order for one unit to be delivered to the firm; note that we set $d_{jj}^k = 1$. As stated above, w_i is the unit cost of labor in the country from which the input variety l is being sourced.

The labor productivity of the supplier is given by the $Z_{ij\chi}^k(\phi; l)$ term in (14). (Note that this is not to be confused with the core productivity ϕ of the final-good producer.) For each input variety l (from industry k), the firm receives $2J$ productivity draws, one for each of the $2J$ possible sourcing modes (i, χ), where $i \in \{1, \ldots, J\}$ and $\chi \in \{V, O\}$. Each set of $2J$ productivity terms is independently drawn for each l from an underlying productivity distribution. We specify this to be a "nested-Fréchet" distribution with cumulative distribution function (cdf):

$$Pr\left(Z^k_{1jV} \leq z^k_{1jV}, Z^k_{1jO} \leq z^k_{1jO}, \ldots, Z^k_{JjO} \leq z^k_{JjO}\right) = \exp\left\{-\sum_{i=1}^{J} T^k_i \left(\left(z^k_{ijV}\right)^{-\frac{\theta^k}{1-\lambda_i}} + \left(z^k_{ijO}\right)^{-\frac{\theta^k}{1-\lambda_i}}\right)^{1-\lambda_i}\right\}, \tag{15}$$

where $T^k_i > 0$, $\theta^k > 1$ and $0 < \lambda_i < 1$ for each source country i. The above extends the more conventional Fréchet distribution from Eaton and Kortum (2002), to allow for a correlation structure among the realized productivity draws from a given "nest". The distribution in (15) features J nests corresponding to each source country, with two productivity draws obtained – for the two possible organizational modes $\chi = V$ and $\chi = O$ – from within each nest. It is worth stressing that while the $2J$ draws for a given input variety l exhibit a within-nest correlation, the productivity draws across any pair of input varieties l and $l' \neq l$ are independent. As in Eaton and Kortum (2002), the T^k_i's are scale parameters, with a larger T^k_i associated with a higher mean productivity level for draws for source country i. In turn, $\theta^k > 1$ is the shape parameter, which is inversely related to the dispersion of the productivity draws.

The λ_i parameters in (15) regulate the strength of the correlation across the productivity draws obtained from the country-i nest, with a higher λ_i corresponding to a stronger correlation. The limit case $\lambda_i = 1$ implies an identical productivity under both integration and outsourcing from country i. On the other hand, if $\lambda_i = 0$ for all countries i, (15) reduces to the special case where the $2J$ draws are each from independent Fréchet distributions with cdf: $\exp\left\{-T^k_i(z^k_{ij\chi})^{-\theta^k}\right\}$. In the application to firm sourcing decisions that we are putting this model to work to, it is relevant to allow for this correlation in productivity draws within source countries. If the draws were instead from independent Fréchet distributions as in Eaton and Kortum (2002), the sourcing probabilities would feature the Independence of Irrelevant Alternative (IIA) property, whereby the probability that a given sourcing mode is optimal relative to another would not depend on the sourcing probability of any third options. The correlation parameter breaks this IIA property across nests: With the nested Fréchet distribution, the share of firms that outsource from China relative to the share that outsource from Vietnam would no longer be independent of the share that obtains the input under integration from China, due to the positive correlation that has been introduced between the productivity draws for sourcing from China under the two organizational modes.[11]

As we will see, this nested Fréchet distribution retains sufficient analytical tractability to yield predictions on aggregate trade flows and welfare, even while it enriches the sourcing problem at the micro level for the firm. In our quantitative work, we will moreover obtain estimates for the correlation parameter that point to its relevance for explaining the data on intrafirm trade shares.

[11] The joint cdf in (15) is the Fréchet analogue of the nested logit error structure that is often adopted to relax the IIA property inherent in multinomial logit models; in fact, the joint cdf of the $\ln Z^k_{ij\chi}(\phi; l)$'s would be precisely a nested logit distribution. Other papers that work with Fréchet distributions with within-nest correlations include: Lagakos and Waugh (2013), Ramondo and Rodríguez-Clare (2013), Arkolakis et al. (2018), Brandt et al. (2019), among others. See also Lind and Ramondo (2018) for a more general treatment of correlated productivity draws in quantitative trade models.

The solutions to the firm's problem in (12) and the supplier's problem in (13), together with the specification of supplier costs in (14), yield expressions for the noncontractible task investments undertaken by the respective parties. Note that these expressions – for $h^k_j(\iota;\phi,l)$ for $\iota \in (\mu^k_{hij},1]$ and $x^k_j(\iota;\phi,l)$ for $\iota \in (\mu^k_{xij},1]$ – are functions of the terms of the ex-ante contract, including the contractible task levels and the sourcing mode that have been specified there.

Contractible input tasks: We now back up to the ex-ante stage of the bilateral interaction, and solve for the contractible task levels, $h^k_j(\iota;\phi,l)$ for $\iota \in [0,\mu^k_{hij}]$ and $x^k_j(\iota;\phi,l)$ for $\iota \in [0,\mu^k_{xij}]$, that will be written into the original contract. Recall that the supplier makes an ex-ante transfer when it enters the contractual relationship. Competition among the large pool of potential suppliers implies that the transfer will be equal to the full amount of the ex-post supplier payoff in (13), so that the firm appropriates the entire surplus generated from its interaction with the supplier for l it ultimately selects. Bearing this in mind, the firm thus sets the contractible task levels in the initial contract to maximize:

$$F^k_{ij}(\phi;l) = r^k_j(\phi;l) - s_j \int_0^1 h^k_j(\iota;\phi,l)d\iota - c^k_{ij\chi}(\phi;l)\int_0^1 x^k_j(\iota;\phi,l)d\iota. \tag{16}$$

To be clear, $F^k_{ij}(\phi;l)$ is the contribution to the firm's overall profits that arises from the bilateral sourcing relationship for this input variety l, after taking into account the ex-ante transfer that the supplier would make to secure its participation in this production and sourcing process. Note that we will substitute in the expressions for the noncontractible tasks investments that we have solved for from (12) and (13) into the $F^k_{ij}(\phi;l)$ maximand in (16), before taking the first-order conditions for the contractible task investments, i.e., $h^k_j(\iota;\phi,l)$ for $\iota \in [0,\mu^k_{hij}]$ and $x^k_j(\iota;\phi,l)$ for $\iota \in [0,\mu^k_{xij}]$.

Solution for input task levels: Given the symmetry across tasks in (5) and (6), the investment levels across all noncontractible headquarter tasks will be equal; we therefore define: $h^k_{nj}(\phi;l) = h^k_j(\iota;\phi,l)$ for all $\iota \in (\mu^k_{hij},1]$ to be the optimal investment level in each of these noncontractible headquarter tasks.[12] Likewise, we define $x^k_{nj}(\phi;l) = x^k_j(\iota;\phi,l)$ for all $\iota \in (\mu^k_{xij},1]$ to be the common investment level that will be chosen by the supplier across all noncontractible tasks; as well as $h^k_{cj}(\phi;l) = h^k_j(\iota;\phi,l)$ for all $\iota \in [0,\mu^k_{hij}]$ and $x^k_{cj}(\phi;l) = x^k_j(\iota;\phi,l)$ for all $\iota \in [0,\mu^k_{xij}]$ to be the investment levels for contractible

[12] More formally, $h^k_{nj}(\phi;l)$ depends not only on the identity of the input variety l, but also on the sourcing mode (i,χ) that is specified for l. To streamline the notation, we have not made this dependence on (i,χ) explicit in the arguments of $h^k_{nj}(\phi;l)$, nor analogously in the arguments of $x^k_{nj}(\phi;l)$, $h^k_{cj}(\phi;l)$, and $x^k_{cj}(\phi;l)$.

headquarter and supplier tasks respectively. As solved for in Appendix B.1, we have:

$$h_{cj}^{k}(\phi;l) = \frac{\alpha^k}{s_j}\frac{\rho^k}{1-\rho^k}\left(\Xi_{ij\chi}^{k}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\left(Z_{ij\chi}^{k}(\phi;l)\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}, \tag{17}$$

$$x_{cj}^{k}(\phi;l) = \frac{1-\alpha^k}{d_{ij}^k w_i}\frac{\rho^k}{1-\rho^k}\left(\Xi_{ij\chi}^{k}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\left(Z_{ij\chi}^{k}(\phi;l)\right)^{\frac{1-\rho^k\alpha^k}{1-\rho^k}}, \tag{18}$$

$$h_{nj}^{k}(\phi;l) = \frac{\alpha^k\beta_{ij\chi}^k}{s_j}\frac{\rho^k}{1-\rho^k}\left(\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\right)\left(\Xi_{ij\chi}^{k}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\left(Z_{ij\chi}^{k}(\phi;l)\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}, \text{ and} \tag{19}$$

$$x_{nj}^{k}(\phi;l) = \frac{\left(1-\alpha^k\right)\left(1-\beta_{ij\chi}^k\right)}{d_{ij}^k w_i}\frac{\rho^k}{1-\rho^k}\left(\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\right)\left(\Xi_{ij\chi}^{k}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\left(Z_{ij\chi}^{k}(\phi;l)\right)^{\frac{1-\rho^k\alpha^k}{1-\rho^k}}, \tag{20}$$

where in the interest of keeping the notation compact, we have collected various terms in $\Xi_{ij\chi}^k$, $\zeta_{ij\chi}^k$, and ζ_{ij}^k. These are defined as:

$$\Xi_{ij\chi}^{k} = \left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X_j^k(\phi))^{\rho^k}}\right)^{\frac{1}{\rho^k(1-\alpha^k)}} \times \left(\frac{1-\rho^k}{\rho^k}\right)^{\frac{1-\rho^k}{\rho^k(1-\alpha^k)}}\left(\frac{\alpha^k}{s_j}\right)^{\frac{\alpha^k}{1-\alpha^k}}\left(\frac{1-\alpha^k}{d_{ij}^k w_i}\right)$$
$$\times\left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{\frac{\zeta_{ij}^k}{\rho^k(1-\alpha^k)}}\left(\beta_{ij\chi}^k\right)^{\frac{\alpha^k(1-\mu_{hij}^k)}{1-\alpha^k}}\left(1-\beta_{ij\chi}^k\right)^{1-\mu_{xij}^k}, \tag{21}$$

$$\zeta_{ij\chi}^k = 1-\rho^k\alpha^k\left(1-\mu_{hij}^k\right)\beta_{ij\chi}^k - \rho^k\left(1-\alpha^k\right)\left(1-\mu_{xij}^k\right)\left(1-\beta_{ij\chi}^k\right), \text{ and} \tag{22}$$

$$\zeta_{ij}^k = 1-\rho^k\alpha^k\left(1-\mu_{hij}^k\right) - \rho^k\left(1-\alpha^k\right)\left(1-\mu_{xij}^k\right). \tag{23}$$

The optimal task levels in (17)-(20) are functions of the factor prices, w_i and s_j, as well as of the underlying parameters that govern the production function (e.g., ρ^k and α^k), the contractibility of headquarter and supplier services (μ_{hij}^k and μ_{xij}^k), and the bargaining process ($\beta_{ij\chi}^k$). It is straightforward to see from (17) and (18) that the ratio of the investment in contractible headquarter to supplier tasks, $h_{cj}^k(\phi;l)/x_{cj}^k(\phi;l)$ is: (i) increasing in the headquarter-intensity of industry k, α^k; but is (ii) decreasing in the relative unit cost $s_j/c_{ij\chi}^k(\phi;l)$ of headquarter relative to supplier tasks. From (19) and (20), these properties hold too for $h_{nj}^k(\phi;l)/x_{nj}^k(\phi;l)$; in addition, $h_{nj}^k(\phi;l)/x_{nj}^k(\phi;l)$ increases with $\beta_{ij\chi}^k$, as a larger Nash bargaining share accruing to the firm will raise its incentives to invest in noncontractible headquarter services.

One can moreover show that the task investment levels in (17)-(20) are all increasing in μ_{hij}^k and μ_{xij}^k.[13] In fact, when $\mu_{hij}^k = \mu_{xij}^k = 1$, the expressions for $h_{cj}^k(\phi;l)$, $x_{cj}^k(\phi;l)$, $h_{nj}^k(\phi;l)$, and $x_{nj}^k(\phi;l)$ reduce to the respective first-best effort levels that would be chosen by the firm in the absence of any contracting frictions. (Note in particular that $\zeta_{ij}^k/\zeta_{ij\chi}^k \leq 1$, with equality if and only if $\mu_{hij}^k = \mu_{xij}^k = 1$.) This points to the usefulness of this formulation of partial input contractibility that we have adopted

[13]This follows from the fact that $\zeta_{ij}^k/\zeta_{ij\chi}^k$ and $\Xi_{ij\chi}^k$ are increasing in both μ_{hij}^k and μ_{xij}^k. This can be established for $\zeta_{ij}^k/\zeta_{ij\chi}^k$ by direct differentiation of the definitions in (22) and (23). As for $\Xi_{ij\chi}^k$, this property follows as a consequence of Lemma 1 in Section 2.4. Note too that $h_{nj}^k(\phi;l)/h_{cj}^k(\phi;l)$ and $x_{nj}^k(\phi;l)/x_{cj}^k(\phi;l)$ are both increasing in μ_{hij}^k and μ_{xij}^k; reductions in contracting friction are thus associated with a greater propensity to invest in noncontractible relative to contractible tasks.

from Antràs and Helpman (2008): We will later simulate reductions in contracting frictions by increasing the μ_{hij}^k and μ_{xij}^k parameters, with the limit case $\mu_{hij}^k = \mu_{xij}^k = 1$ corresponding to an environment with full contractibility.

2.4 Sourcing mode choice

We are now in a position to characterize the optimal sourcing mode for the procurement of input varieties. Substituting the input task levels from (17)-(20) into equation (16), we show in Appendix B.1 that the firm's payoff from its bilateral interaction with the supplier of input variety l can be re-expressed as:

$$F_{ij}^k(\phi; l) = \left(\Xi_{ij\chi}^k Z_{ij\chi}^k(\phi; l)\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}. \tag{24}$$

It follows that the firm's optimal sourcing mode for this input variety is given by:

$$\arg\max_{(i,\chi)} \ \Xi_{ij\chi}^k Z_{ij\chi}^k(\phi; l). \tag{25}$$

For any individual input variety l, this choice naturally depends on the particular realization of the draw $Z_{ij\chi}^k(\phi; l)$. We can nevertheless make use of the properties of the nested Fréchet distribution to compute the probability, $\pi_{ij\chi}^k$, that (i, χ) will be the optimal sourcing mode for this input variety. By the law of large numbers, $\pi_{ij\chi}^k$ will then also be equal to the share of input varieties on the unit interval for which (i, χ) is the sourcing mode that maximizes $F_{ij}^k(\phi; l)$. In Appendix B.1, we prove that:

$$\pi_{ij\chi}^k = \pi_{ij}^k \pi_{\chi|ij}^k. \tag{26}$$

The sourcing probability $\pi_{ij\chi}^k$ can therefore be decomposed conveniently into the product of: (i) π_{ij}^k, the probability that the input variety will be sourced from country i; and (ii) $\pi_{\chi|ij}^k$, the probability that the organizational mode will be χ, conditional on selecting i as the source country.[14] In turn, π_{ij}^k and $\pi_{\chi|ij}^k$ are given explicitly by:

$$\pi_{ij}^k = \frac{T_i^k (d_{ij}^k w_i)^{-\theta^k} (B_{ij}^k)^{\theta^k}}{\sum_{i'=1}^J T_{i'}^k (d_{i'j}^k w_{i'})^{-\theta^k} (B_{i'j}^k)^{\theta^k}}, \text{ and} \tag{27}$$

$$\pi_{\chi|ij}^k = \frac{(B_{ij\chi}^k)^{\frac{\theta^k}{1-\lambda_i}}}{(B_{ijV}^k)^{\frac{\theta^k}{1-\lambda_i}} + (B_{ijO}^k)^{\frac{\theta^k}{1-\lambda_i}}}, \tag{28}$$

where we have grouped together the terms that depend on the contractibility and bargaining parameters (i.e., μ_{hij}^k, μ_{xij}^k, and $\beta_{ij\chi}^k$) under:

$$B_{ij\chi}^k = \left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{\frac{\zeta_{ij}^k}{\rho^k(1-\alpha^k)}} \left(\beta_{ij\chi}^k\right)^{\frac{\alpha^k\left(1-\mu_{hij}^k\right)}{(1-\alpha^k)}} \left(1-\beta_{ij\chi}^k\right)^{\left(1-\mu_{xij}^k\right)}, \text{ and} \tag{29}$$

$$B_{ij}^k = \left(\frac{1}{2}\left[(B_{ijV}^k)^{\frac{\theta^k}{1-\lambda_i}} + (B_{ijO}^k)^{\frac{\theta^k}{1-\lambda_i}}\right]\right)^{\frac{1-\lambda_i}{\theta^k}}. \tag{30}$$

[14] This decomposition is analogous to that which we see for choice probabilities in nested logit models of discrete choice.

Note that B^k_{ij} is the generalized power mean of B^k_{ijV} and B^k_{ijO} with exponent equal to $\frac{\theta^k}{1-\lambda_i}$.

The expression for π^k_{ij} in (27) should resonate with readers familiar with Eaton and Kortum (2002). This probability of sourcing from country i (regardless of organizational mode) is identical to that in the baseline Eaton-Kortum framework, except that the technology parameter T^k_i has been replaced in the above with T^k_i multiplied by $(B^k_{ij})^{\theta^k}$. The following lemma will help us to better interpret this $(B^k_{ij})^{\theta^k}$ term (see Appendix B.1 for the proof):

Lemma 1 *B^k_{ijV}, B^k_{ijO} and B^k_{ij} are increasing in both μ^k_{hij} and μ^k_{xij}. Moreover, B^k_{ijV}, B^k_{ijO}, $B^k_{ij} \leq 1$ with equality if and only if $\mu^k_{hij} = 1$ and $\mu^k_{xij} = 1$.*

Since $(B^k_{ij})^{\theta^k} \leq 1$, this multiplicative term in (27) thus captures how contracting frictions in effect retard the state of technology T^k_i available to country-j firms when they seek to access country-i suppliers for industry-k inputs. This is an interpretation of $(B^k_{ij})^{\theta^k}$ that we will refer to with some regularity. Moving forward, it will also be convenient to define the denominator of π^k_{ij} in (27) as:

$$\Phi^k_j \equiv \sum_{i'=1}^{J} T^k_{i'} (d^k_{i'j} w_{i'})^{-\theta^k} (B^k_{i'j})^{\theta^k}. \tag{31}$$

This Φ^k_j summarizes how state of technology, prevailing labor costs, trade frictions, as well as contracting frictions across all potential source countries. Following Antràs, Fort and Tintelnot (2017), we will refer to Φ^k_j as the *sourcing capability* of country-j firms in procuring inputs from industry k.

Several implications that should be familiar from Eaton and Kortum (2002) follow as a consequence of equation (27) for the π^k_{ij}'s. The relative probability that a firm will source from country i as opposed to country i' is:

$$\frac{T^k_i}{T^k_{i'}} \frac{(d^k_{ij} w_i)^{-\theta^k} (B^k_{ij})^{\theta^k}}{(d^k_{i'j} w_{i'})^{-\theta^k} (B^k_{i'j})^{\theta^k}}.$$

Not surprisingly, a country-j firm will be more likely to source from country i if the state of technology there, T^k_i, is high relative to that in country i', $T^k_{i'}$. Moreover, sourcing is more likely to take place with country i if labor costs are lower, shipping to country j is less costly, or if contracting frictions are less severe, in country i relative to country i'.

Turning to $\pi^k_{\chi|ij}$ in (28), observe that this sourcing probability by organizational mode depends only on the bargaining shares (the $\beta^k_{ij\chi}$'s), production parameters (θ^k, λ_i, ρ^k, and α^k), and contracting parameters (μ^k_{hij} and μ^k_{xij}). After the identity of the source country has been conditioned out, the relative probability of sourcing under mode χ does not depend on the state of technology (T^k_i), trade costs (d^k_{ij}), or labor costs (w_i).

Conditional on sourcing from country i, the relative probability of sourcing under integration versus outsourcing is given by:

$$\left(\frac{B^k_{ijV}}{B^k_{ijO}}\right)^{\frac{\theta^k}{1-\lambda_i}} = \left[\frac{\left(\zeta^k_{ijV}\right)^{\frac{\zeta^k_{ij}}{\rho^k(1-\alpha^k)}} \left(\beta^k_{ijV}\right)^{\frac{\alpha^k\left(1-\mu^k_{hij}\right)}{(1-\alpha^k)}} \left(1-\beta^k_{ijV}\right)^{\left(1-\mu^k_{xij}\right)}}{\left(\zeta^k_{ijO}\right)^{\frac{\zeta^k_{ij}}{\rho^k(1-\alpha^k)}} \left(\beta^k_{ijO}\right)^{\frac{\alpha^k\left(1-\mu^k_{hij}\right)}{(1-\alpha^k)}} \left(1-\beta^k_{ijO}\right)^{\left(1-\mu^k_{xij}\right)}}\right]^{\frac{\theta^k}{1-\lambda_i}}. \tag{32}$$

This relative probability of adopting integration is a non-monotonic function of the bargaining share parameters, β_{ijV}^k and β_{ijO}^k. This reflects the inherent tradeoff that emerges should say the firm's bargaining share under integration rise: On the one hand, this allows the firm to retain a greater share of the bilateral surplus, making integration more attractive. However, a higher β_{ijV}^k would also disincentivize supplier task investments, which would dampen the firm's payoff under integration; in the limit as $\beta_{ijV}^k \longrightarrow 1$, this latter effect is so strong that the probability that integration would be optimal tends to 0.

The effects of the contractibility parameters μ_{hij}^k and μ_{xij}^k on (32) can also be pinned down. An increase in supplier contractibility μ_{xij}^k allows the firm to spell out the investment levels required of the supplier for a larger subset of supplier tasks. This reduces the need to adopt outsourcing as an organizational mode to incentivize the actions of the supplier, raising (32) and hence the sourcing share under integration $\pi_{V|ij}^k$. An increase in headquarter contractibility μ_{hij}^k has the opposite effect, as in Antràs and Helpman (2008). When headquarter services are highly contractible, the firm would in fact find it optimal to commit to procuring a greater share of the supplier inputs via outsourcing, so as not to overly deter supplier effort.

Separately, a larger θ^k or λ_i would magnify the effects of the contracting friction terms, B_{ijV} and B_{ijO}, in (32). This is a key feature that comes from our specification of the nested Fréchet productivity draws: A larger θ^k or a higher λ_i will lead to stochastic productivity draws that exhibit a smaller variance and a higher correlation across organizational modes (within country i). This would amplify the role of non-stochastic forces, in particular B_{ijV} and B_{ijO}, in determining the relative share of inputs that are integrated as opposed to outsourced.

In principle, if firm-level data were available on sourcing patterns by country and organizational mode for a rich enough set of inputs, one could pursue a quantification strategy that seeks to fit this data to the model-implied firm-level sourcing shares in (27) and (28). In the absence of such detailed data however, we instead derive predictions from aggregating these sourcing decisions across suppliers and firms, in order to map these model-based expressions to data on trade flows, and more specifically to country-by-industry intrafirm trade shares. This is the approach which we develop below.

A quick subsection on the technical issue of the gap between observed trade flows and sourcing probabilities. Sum up the model in "layers".

3 Aggregate Implications: Welfare and Trade Flows

In this section, we aggregate over the choices made by the final-good producers in a given country, to derive implications for country welfare and bilateral trade flows under each organizational mode. (The details of these derivations are documented in Appendix B.2.)

3.1 Steps towards Aggregation

Composite industry-k input: We first derive an expression for $X_j^k(\phi)$, after taking into account the optimal sourcing choices that a given firm would make over the full unit measure of industry-k input varieties. This is a key intermediate step, since the task levels in (17)-(20) are functions of $\Xi_{ij\chi}^k$, which from the definition in (21) in turn depends on $X_j^k(\phi)$. In Appendix B.2, we show that:

$$X_j^k(\phi) = (1-\alpha)\rho\eta^k R_j(\phi)\left(\frac{\alpha^k}{s_j}\right)^{\alpha^k}\left(1-\alpha^k\right)^{1-\alpha^k}\left(\Phi_j^k\right)^{\frac{1-\alpha^k}{\theta^k}}\left(\bar{\Gamma}^k\right)^{\frac{1-\rho^k}{\rho^k}}\left(\Upsilon_j^k\right)^{-\frac{1-\rho^k}{\rho^k}}. \tag{33}$$

where $\bar{\Gamma}^k \equiv \Gamma\left(1-\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}\right)$ is a constant, with $\Gamma(\cdot)$ being the Gamma function.[15] In turn, Υ_j^k is given by:

$$\Upsilon_j^k = \left(\sum_{i=1}^{J}\sum_{\chi\in\{V,O\}}\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\pi_{ij}^k\pi_{\chi|ij}^k\right)^{-1}. \tag{34}$$

The composite input $X_j^k(\phi)$ is thus a linear function of the final-good producer's revenue, $R_j(\phi)$, and is increasing with the sourcing capability at the disposal of industry-k firms in country j, Φ_j^k.

Contracting frictions exert a negative effect on this composite input through two channels. First, more severe contracting frictions lower the firm's sourcing capability, Φ_j^k. Recall in particular from (31) that this sourcing capability is increasing in each B_{ij}^k, and in turn in each of the μ_{hij}^k and μ_{xij}^k input contractibility parameters (see Lemma 1); the sourcing capability is thus at its highest possible value when there is full contractibility of headquarter and supplier inputs from all source countries ($\mu_{hij}^k = \mu_{xij}^k = 1$). Second, the contractibility parameters also affect Υ_j^k. Since $\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k} \leq 1$, we have from (34) that: $(\Upsilon_j^k)^{-(1-\rho^k)/\rho^k} \leq (\sum_{i=1}^J\sum_{\chi\in\{V,O\}}\pi_{ij}^k\pi_{\chi|ij}^k)^{-(1-\rho^k)/\rho^k} = 1$. Applying this to (33), one can see that the $(\Upsilon_j^k)^{-(1-\rho^k)/\rho^k}$ term tends to reduce $X_j^k(\phi)$, with equality if and only if $\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k} = 1$ for all sourcing modes (i,χ), namely when $\mu_{hij}^k = \mu_{xij}^k = 1$ for all source countries i. The $(\Upsilon_j^k)^{-(1-\rho^k)/\rho^k}$ term therefore captures the effect of contracting frictions on the composite input – due to less than first-best levels of investment in headquarter and supplier tasks – holding the sourcing capability constant.

Payoff function of the final firm: We next work out an expression for the full payoff of the firm, $F_j(\phi)$. Bearing in mind the ex-ante transfers from suppliers, the firm fully internalizes in its payoff function all payments to labor that each supplier makes.[16] $F_j(\phi)$ will therefore be equal to the firm's revenue from selling the final-good, $R(\phi)$, less all factor costs incurred. The latter comprises: (i) the headquarter task services, $s_j\int_0^1 h_j^k(\iota;\phi,l)d\iota$, for all input varieties l; (ii) the supplier tasks $c_{ij\chi}^k(\phi;l)\int_0^1 x_j^k(\iota;\phi,\lambda)d\iota$, over all l; and (iii) the labor employed in final assembly, $w_jL_j(\phi)$. With some

[15] In order for $\bar{\Gamma}^k$ to be well-defined, we require the parameter restriction: $1-\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k} > 0$. This ensures that the dispersion of the nested Fréchet productivity draws is not too large, so that a power function of these draws that will be relevant for evaluating $X_j^k(\phi)$ has a finite expectation.

[16] Each supplier's payoff will be zero after deducting the ex-ante transfer and its payments to labor.

extensive algebra, one can show that:

$$F_j(\phi) = R_j(\phi) - \sum_{k=1}^{K} \int_{l=0}^{1} \left[s_j \int_0^1 h_j^k(\iota;\phi,l)d\iota + c_{ij\chi}^k(\phi,l) \int_0^1 x_j^k(\iota;\phi,l)d\iota \right] dl - w_j L_j(\phi)$$
$$= R_j(\phi)\bar{\Upsilon}_j - w_j L_j(\phi) \tag{35}$$

where $\bar{\Upsilon}_j$ is defined as:

$$\bar{\Upsilon}_j = 1 - (1-\alpha) \sum_{k=1}^{K} \frac{\rho\eta^k}{\rho^k} \left(1 - (1-\rho^k)\Upsilon_j^k\right). \tag{36}$$

It is straightforward to establish that: $\Upsilon_j^k \leq 1/(1-\rho^k)$, for all industries k.[17] (36) then implies that $\bar{\Upsilon}_j \leq 1$, which is consistent with the interpretation that $\bar{\Upsilon}_j$ is a revenue share, this being the share of $R_j(\phi)$ that accrues to the firm after summing over the payoffs, $F_{ij}^k(\phi;l)$ in (16), from its bilateral interactions with all suppliers of relationship-specific inputs.

Labor Demand: We can now pin down the amount $L_j(\phi)$ of labor that the firm employs in the final assembly of the good. Taking the first-order condition of (35) with respect to $L_j(\phi)$ yields:

$$L_j(\phi) = \frac{\alpha\rho}{w_j}\bar{\Upsilon}_j R_j(\phi). \tag{37}$$

The amount of assembly labor employed by the final-good producer is thus also a linear function of its revenue. It is straightforward to see that $\bar{\Upsilon}_j$, and hence also $L_j(\phi)$, is increasing in the assembly labor share α in the final-good production function (2). Moreover, the larger is $\bar{\Upsilon}_j$, the greater is $L_j(\phi)$, so that the ability to retain a larger share of revenue vis-á-vis suppliers encourages the firm to raise its labor demand in final-assembly.

Output: Combining the expressions for the composite input from (33) and assembly labor from (37) into the production function in (2), the quantity of the final-good produced is:

$$q_j(\phi) = A_j \phi^{\frac{1}{1-\rho}} \left[\rho \frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{(w_j)^\alpha} (\bar{\Upsilon}_j)^\alpha \right]^{\frac{1}{1-\rho}}$$
$$\times \prod_{k=1}^{K} \left[\left(\frac{\alpha^k}{s_j}\right)^{\alpha^k} \left(1-\alpha^k\right)^{1-\alpha^k} \eta^k \bar{\Gamma}^{\frac{1-\rho^k}{\rho^k}} \left(\Phi_j^k\right)^{\frac{1-\alpha^k}{\theta^k}} \left(\Upsilon_j^k\right)^{-\frac{1-\rho^k}{\rho^k}} \right]^{\eta^k \frac{1-\alpha}{1-\rho}}. \tag{38}$$

3.2 Welfare

We are now ready to evaluate an expression for welfare based on the utility function in (1). From each individual firm's perspective, the quantity of the final-good variety it produces – and which is then consumed in the home economy – is given entirely by (38). From an aggregate perspective however, the level of market demand A_j is endogenous. From (8), we have:

$$A_j = I_j P_j^{\frac{\rho}{1-\rho}} = I_j \left(N_j \int_\phi \left(\frac{q_j(\phi)}{A_j} \right)^\rho dG_j(\phi) \right)^{-1},$$

[17] Using the definitions of $\zeta_{ij\chi}^k$ in (22) and ζ_{ij}^k in (23), one can show that $\zeta_{ij}^k/\zeta_{ij\chi}^k \geq 1-\rho^k$. The fact that $\Upsilon_j^k \leq 1/(1-\rho^k)$ then follows from the definition of Υ_j^k in (34).

which implies:

$$A_j = (I_j)^{\frac{1}{1-\rho}} \left(N_j \int_\phi q_j(\phi)^\rho dG_j(\phi) \right)^{-\frac{1}{1-\rho}}. \tag{39}$$

Recall here that N_j is the fixed measure of final-good producers in the economy. The CES consumption aggregate over final-good varieties appears in (39); to derive an expression for this, we substitute (39) into (38), integrate over all varieties, and simplify to obtain:

$$\begin{aligned} \left(\int_\phi q_j(\phi)^\rho dG_j(\phi) \right)^{\frac{1}{\rho}} &= \left(\frac{\rho I_j}{N_j} \right) \left(\frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{(w_j)^\alpha} (\bar{\Upsilon}_j)^\alpha \right) \bar{\phi} \\ & \times \prod_{k=1}^{K} \left[\left(\frac{\alpha^k}{s_j} \right)^{\alpha^k} \left(1 - \alpha^k \right)^{1-\alpha^k} \eta^k \bar{\Gamma}^{\frac{1-\rho^k}{\rho^k}} \left(\Phi_j^k \right)^{\frac{1-\alpha^k}{\theta^k}} \left(\Upsilon_j^k \right)^{-\frac{1-\rho^k}{\rho^k}} \right]^{\eta^k (1-\alpha)} \end{aligned} \tag{40}$$

In this last expression, $\bar{\phi} = \left(\int_\phi \phi^{\frac{\rho}{1-\rho}} dG_j(\phi) \right)^{\frac{1-\rho}{\rho}}$ is an average productivity level, evaluated over the underlying distribution $G_j(\phi)$ of core firm productivities.

Aggregate income, I_j, is in turn pinned down as follows. There are two primary factors of production in this economy: (i) human capital, that is employed in the provision of headquarter services; and (ii) labor, that is used in supplier inputs and in final-good assembly. Let $\bar{H}_j$ and $\bar{L}_j$ denote respectively the country-j endowments of these two factors. To account for firm profits, we specify that these are rebated to country-j households, via a domestic asset market. Therefore, aggregate income in country j is:

$$I_j = w_j \bar{L}_j + s_j \bar{H}_j + N_j \int_\phi \bar{\Upsilon}_j R(\phi) dG_j(\phi) - N_j \int_\phi w_j L_j(\phi) dG_j(\phi).$$

Using the expression for $L_j(\phi)$ from (37), together with the fact that revenues aggregated over all firms, $N_j \int_\phi R(\phi) dG_j(\phi)$, will be equal to I_j, we have:

$$I_j = \frac{w_j \bar{L}_j + s_j \bar{H}_j}{1 - (1 - \alpha\rho)\bar{\Upsilon}_j}. \tag{41}$$

Note that to simplify matters, we have assumed that there is no international trade in assets; in the quantitative implementation of the model, we will treat any observed trade imbalances to be fixed as given in the data, following Dekle et al. (2008). (Although we have not explicitly written down the trade deficits in the equations above as a component of aggregate income, these have been incorporated and accounted for in the quantitative work.)

Since $q_j(\phi) = c_j(\phi)$ for all varieties, and $U_j = \left(\int_\phi N_j q_j(\phi)^\rho dG_j(\phi) \right)^{\frac{1}{\rho}}$, we now use (40) to obtain the following closed-form expression for welfare:

$$\begin{aligned} U_j &= (N_j)^{\frac{1-\rho}{\rho}} \rho I_j \frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{(w_j)^\alpha} (\bar{\Upsilon}_j)^\alpha \bar{\phi} \\ & \times \prod_{k=1}^{K} \left[\left(\frac{\alpha^k}{s_j} \right)^{\alpha^k} \left(\frac{1-\alpha^k}{w_j} \right)^{1-\alpha^k} \eta^k \left(\frac{\bar{\Gamma}}{\Upsilon_j^k} \right)^{\frac{1-\rho^k}{\rho^k}} \left(\frac{T_j^k}{\pi_{jj}^k} \right)^{\frac{1-\alpha^k}{\theta^k}} (B_{jj}^k)^{1-\alpha^k} \right]^{\eta^k (1-\alpha)}. \end{aligned} \tag{42}$$

Note that we have applied a familiar substitution – $\Phi_j^k = T_j^k (w_j)^{-\theta^k} (B_{jj}^k)^{\theta^k} / \pi_{jj}^k$ when setting $i = j$ in (27) – in order to express the sourcing capability as a function of the domestic sourcing share, π_{jj}. We therefore have a compact expression for consumer welfare in country j in terms of: (i) demand parameters (ρ); (ii) technological parameters and variables (ρ^k, θ^k, λ_j, η^k, α, α^k, T_j^k); (iii) the share of input varieties purchased from domestic suppliers (π_{jj}^k); (iv) terms related to contracting frictions (B_{jj}^k, Υ_j^k, $\bar{\Upsilon}_j$); and (v) terms that depend on endogenous factor prices (w_j, s_j, I_j).

We denote the proportional change in a variable X by: $\hat{X} \equiv X'/X$, using the standard notation in the literature. The hat algebra analogue of (42) for welfare changes in response to an underlying shock to either trade costs or contracting frictions is given by:

$$\widehat{U_j} = \widehat{I}_j \left(\widehat{w_j}\right)^{-\alpha} \left(\prod_{k=1}^{K} \left[\left(\widehat{w_j}\right)^{-(1-\alpha^k)} \left(\widehat{s_j}\right)^{-\alpha^k} \right]^{\eta^k (1-\alpha)} \right) \left(\widehat{\bar{\Upsilon}_j}\right)^{\alpha}$$
$$\times \prod_{k=1}^{K} \left[\left(\widehat{\Upsilon_j^k}\right)^{-\frac{1-\rho^k}{\rho^k}} \left(\widehat{\pi_{jj}^k}\right)^{-\frac{1-\alpha^k}{\theta^k}} \left(\widehat{B_{jj}^k}\right)^{1-\alpha^k} \right]^{\eta^k (1-\alpha)} \tag{43}$$

The expression in (43) shares features with the gains-from-trade formulae that have been commonly derived in the literature. Looking first at the terms in the square brackets, one should quickly spot the familiar role of the change in the domestic-sourcing share, $\widehat{\pi_{jj}^k}$, which is inversely related to welfare change.[18] In the current setup with contracting frictions, the gains-from-trade formula is augmented by several terms. First, the $\widehat{B_{jj}^k}$ term captures the direct impact of contracting frictions on the sourcing capability of home-country firms.[19] From (43), an improvement in contracting conditions – such as an increase in μ_{hij}^k or μ_{xij}^k for inputs from a particular source country i – would be reflected in an increase in B_{jj}^k. This has a direct positive impact on country j's sourcing capability, which filters through to country welfare.

Second, the $\widehat{\Upsilon_j^k}$ term reflects how contracting frictions affect input task investment levels, holding the sourcing capability constant; this interpretation follows from our earlier discussion of the effect of contracting frictions on the industry composite input, $X_j^k(\phi)$. Since an improvement in contractibility is associated with an increase in $(\Upsilon_j^k)^{-(1-\rho^k)/\rho^k}$, and hence an increase in the amount of composite input in (33), we see that a positive shift in $(\Upsilon_j^k)^{-(1-\rho^k)/\rho^k}$ generates an improvement in welfare in (43).

Third, the $\widehat{\bar{\Upsilon}_j}$ term in (43) reflects how the contractibility of inputs affects the share of revenues that accrues to the firm. While an increase in μ_{hij}^k or μ_{xij}^k raises the amount of composite inputs $X_j^k(\phi)$ that the firm obtains, this also increases payments for these inputs as a share of firm revenues, so that $\bar{\Upsilon}_j$ decreases. This in turn tends to decrease the firm's choice of assembly labor $L_j(\phi)$, as seen in (37), which has a dampening effect on welfare as captured by the $\left(\bar{\Upsilon}_j\right)^{\alpha}$ term in the first line of (43). Last but

[18]Note that this domestic-sourcing share, π_{jj}, strictly refers to the share of input varieties that are sourced by the firm from the home country. Unlike in Eaton and Kortum (2002), this share is not equal to the trade share by value that is sourced domestically, given that the nature of the distribution of prices differs when these are determined by a bargaining process shaped by contracting frictions rather than in a competitive market. In Appendix C.5, we are careful to establish the model-consistent mapping between the trade share by fraction of inputs and the trade share by value of the inputs. This will be important for a correct execution of the welfare counterfactuals in Section 5.

[19]Recall in particular that B_{jj}^k enters the welfare expression in (42) precisely because of the substitution for Φ_j^k that was performed there to introduce the domestic-sourcing share into the welfare formula.

not least, (43) contains terms in $\widehat{w_j}$, $\widehat{s_j}$ and $\widehat{I_j}$. These reflect endogenous responses of the factor prices, and hence aggregate income. The magnitudes of these responses will naturally be pinned down by factor market-clearing conditions in general equilibrium (see Section 3.4 below).[20] These last two observations – on the role of $\widehat{\bar{\Upsilon}_j}$ and on shifts in factor prices – speak to the presence of subtle mechanisms within the model that need to be taken into account when assessing the overall impact of an improvement in contracting conditions on welfare. These offsetting effects can be strong, for example, if assembly labor were very important in the production function (i.e., α is high), or if shifts in sourcing patterns were to result in a strong decrease in demand for domestic factors (i.e., $\hat{w}_j$ and/or $\hat{s}_j$ are less than 1).

A quick corollary is that in the special case of a world with only one factor of production (i.e., both headquarter services and supplier inputs are made with one type of labor), and in the absence of contracting frictions, the welfare gains formula would collapse to:

$$\widehat{U_j} = \prod_{k=1}^{K} (\widehat{\pi_{jj}^k})^{-\frac{\eta^k(1-\alpha^k)}{\theta^k}(1-\alpha)}. \tag{44}$$

Our model with contracting frictions thus nests the gains-from-trade formula for a class of models with multiple industries, as surveyed in Costinot and Rodriguez-Clare (2014).[21]

3.3 Trade flows

We next tease out implications for trade flows. To map our model to trade data, we need to take a stand on what constitutes the value of trade flows that are recorded by the customs authority. We take the position here that the trade flows reported at customs are valuing the items being shipped at cost. This would be consistent with the underlying modeling assumption that there is a large pool of potential input suppliers in each country, who compete away any rents (markups) that suppliers might otherwise earn.[22] Under this approach, the value of the industry-k inputs that a country-j firm with core productivity ϕ would import under sourcing mode (i, χ) is:

$$t_{ij\chi}^k(\phi) = \int_{l \in \Omega_{ij\chi}^k} c_{ij\chi}^k \left(\mu_{xij}^k x_{cj}^k(\phi; l) + (1 - \mu_{xij}^k) x_{nj}^k(\phi; l) \right) \, dl, \tag{45}$$

[20]Ordinarily, in a model with perfect competition, I_j would be proportional to w_j and hence the corresponding $\hat{w}_j$ term would be equal to 1. However, with the present setup, total income and hence total expenditure in the home country are equal to the sum of payments to factors and the total surplus from the bargaining processes that accrues to home-country firms. The latter payoff to firms responds to shocks to contracting conditions, and hence I_j as a whole shifts too with such shocks.

[21]Recall that in the absence of contracting frictions, $B_{jj}^k = 1$, $\Upsilon_j^k = 1$ and $\bar{\Upsilon}_j = 1 - \rho(1 - \alpha)$.

[22]As an alternative, we have also considered a specification where the value of trade flows observed is equal to the spot payments made by the country-j firm to suppliers in country i, at the point of shipment across international borders. This is equal to the share $1 - \beta_{ij\chi}^k$ of the incremental revenue generated from the bilateral relationship between the firm-supplier pair. For a given firm with productivity ϕ, its total purchases of industry-k inputs under sourcing mode (i, χ) would then be equal to:

$$t_{ij\chi}^{k,alt}(\phi) = \int_{l \in \Omega_{ij\chi}^k} \left(1 - \beta_{ij\chi}^k\right) r_j^k(\phi; l) \, dl.$$

The estimation results from this specification yield qualitatively similar insights, and are available on request.

where $\Omega^k_{ij\chi}$ is the set of industry-k input varieties $l \in [0,1]$ for which (i,χ) is the lowest-cost sourcing mode.

The value of bilateral trade observed by industry and sourcing mode can then be obtained by aggregating (45) over all firms. As we show in Appendix B.2, this yields:

$$t^k_{ij\chi} = \frac{(1-\alpha)\rho\eta^k}{\rho^k}\frac{\Upsilon^k_j}{\Phi^k_j} I_j \rho^k (1-\alpha^k) T^k_i (w_i)^{-\theta^k} \left(B^k_{ij}\right)^{-\frac{\theta^k \lambda_i}{1-\lambda_i}} \left(d^k_{ij}\right)^{-\theta^k}$$
$$\times \left(\mu^k_{xij} + (1-\mu^k_{xij})(1-\beta^k_{ij\chi})\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\right)\frac{1}{2}\left(B^k_{ij\chi}\right)^{\frac{\theta^k}{1-\lambda_i}}. \tag{46}$$

Trade flows aggregated at the country-industry level, $t^k_{ij\chi}$, can therefore be written as a function of: (i) terms specific to the destination-country (j) and industry (k), including variables related to destination-country demand (e.g., I_j); (ii) terms specific to the source-country (i) and industry (k), including variables related to supply conditions in i (e.g., T^k_i); (iii) trade and contracting frictions – d^k_{ij} and B^k_{ij} – that vary by country pair (i-j) and industry (k); and (iv) terms that vary by country pair (i-j), industry (k), and organizational mode (χ). Thus, (46) delivers a gravity equation for bilateral trade flows by industry and sourcing mode, with θ^k as the distance elasticity. Note that the terms in (iv) – that appear on the second line of equation (46) – will be of key interest to us, these being functions of the bargaining shares, the contractibility parameters, and other deep model parameters. These terms will feature prominently as drivers of the intrafirm trade share, i.e., the share (by value) of trade flows that are brought in under integration rather than under outsourcing, $t^k_{ijV}/(t^k_{ijV} + t^k_{ijO})$.

3.4 Closing the Model

The model as set up thus far treats factor costs in each country as given. We now close the model in general equilibrium, by presenting the factor market clearing conditions that pin down w_j and s_j in each country. (Recall that while firms earn profits, this gets rebated to households/workers through a domestic asset market, and is accounted for in I_j.)

The labor supply in country j is given exogenously by $\bar{L}_j$. On the other hand, there are two sources of labor demand for these workers. First, workers are employed by country-j firms to assemble final-good varieties; the total amount of labor employed for this is: $N_j \int_\phi L_j(\phi)\, dG_j(\phi)$. Second, country-$j$ input suppliers also employ labor to produce input varieties that are then sourced by firms around the world (including by country-j's own final-good producers). The total amount of labor employed for this latter purpose in country j is obtained by summing over the labor used to provide inputs to final-good producers in country $m = \{1, \ldots, J\}$ under sourcing mode (j,χ), where $\chi = \{V, O\}$. Accounting for both sources of labor demand, one arrives at the following labor market-clearing condition (see Appendix B.2 for a derivation):

$$w_j \bar{L}_j = \rho\alpha\bar{\Upsilon}_j I_j \tag{47}$$
$$+\rho(1-\alpha)\sum_{k=1}^{K}\left(1-\alpha^k\right)\eta^k \sum_{m=1}^{J} I_m \Upsilon^k_m \sum_{\chi\in\{V,O\}} \pi^k_{jm}\pi^k_{\chi|jm}\left(\mu^k_{xjm} + \left(1-\mu^k_{xjm}\right)\beta^k_{jm\chi}\frac{\zeta^k_{jm}}{\zeta^k_{jm\chi}}\right)$$

Separately, the endowment $\bar{H}_j$ of human capital is used only to perform headquarter services in country j. After aggregating the demand for human capital across all firms in country j, this second factor market-clearing condition is given by:

$$s_j \bar{H}_j = (1-\alpha)\,\rho I_j \sum_{k=1}^{K} \alpha^k \eta^k \Upsilon_j^k \sum_{i=1}^{J} \sum_{\chi=V,O} \pi_{ij}^k \pi_{\chi|ij}^k \left(\mu_{hij}^k + \left(1-\mu_{hij}^k\right) \beta_{ij\chi}^k \frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k} \right). \tag{48}$$

4 Transition to Empirics

4.1 Estimation Strategy

We now discuss how to take the implied industry trade flow expressions by sourcing mode to the data and back out the key parameters in our model. Our objective is to show how the model can be quantified and its key parameters estimated, using data on U.S. intrafirm trade shares made publicly available by the U.S. Census Bureau. Recall that the model-based expression for trade flows is given by $t_{ij\chi}^k$ in (46). We have, in particular, that $i = \text{US}$ or $j = \text{US}$, i.e., either the importing country or the exporting country, is always the U.S. in the dataset that we use.

To map this into an empirical specification, we assume that the actual trade flows, denoted by $\tilde{t}_{ij\chi}^k$, are observed with noise. In particular, suppose that: $\tilde{t}_{ij\chi}^k = t_{ij\chi}^k \epsilon_{ij\chi}^k$, where $\epsilon_{ij\chi}^k$ is an i.i.d. Poisson noise term with unit mean. We can now write observed trade flows as:

$$\tilde{t}_{ij\chi}^k = a_{ij}^k \cdot a_{ij\chi}^k \cdot \epsilon_{ij\chi}^k, \tag{49}$$

where:

$$a_{ij}^k = (1-\alpha)\rho\eta^k \frac{\Upsilon_j^k}{\Phi_j^k} I_j \left(1-\alpha^k\right) T_i^k (w_i)^{-\theta^k} \left(B_{ij}^k\right)^{-\frac{\theta^k \lambda_i}{1-\lambda_i}} \left(d_{ij}^k\right)^{-\theta^k} \frac{1}{2} \left(\frac{1}{\zeta_{ij}^k}\right)^{\frac{\zeta_{ij}^k}{\rho^k(1-\alpha^k)} \frac{\theta^k}{1-\lambda_i}}, \text{ and} \tag{50}$$

$$\begin{aligned} a_{ij\chi}^k = {} & \left(\zeta_{ij\chi}^k\right)^{\frac{\zeta_{ij}^k}{\rho^k(1-\alpha^k)} \frac{\theta^k}{1-\lambda_i}} \left(1-\beta_{ij\chi}^k\right)^{\frac{\theta^k}{1-\lambda_i}\left(1-\mu_{xij}^k\right)} \left(\beta_{ij\chi}^k\right)^{\left(1-\mu_{hij}^k\right)\frac{\alpha^k}{1-\alpha^k}\frac{\theta^k}{1-\lambda_i}} \\ & \times \left(\mu_{xij}^k + (1-\mu_{xij}^k)(1-\beta_{ij\chi}^k) \frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k} \right). \end{aligned} \tag{51}$$

The above formulation implies that the observed trade flow in mode χ is Poisson-distributed with $a_{ij\chi}^k \cdot a_{ij}^k$ as its mean parameter. As is well-known, the specification of a Poisson noise term is consistent with the presence of zeros in the trade flow data (Santos Silva and Tenreyro 2006). In addition, it facilitates the derivation of a model-consistent moment condition that we can directly estimate, as we explain below.

Conditional on the summation of two independent Poisson random variables, $\tilde{t}_{ijV}^k + \tilde{t}_{ijO}^k = \tilde{t}_{ij}^k$, the distribution of $\tilde{t}_{ijV}^k$ is a binomial distribution where $\tilde{t}_{ij}^k$ is the number of the trials and $a_{ijV}^k a_{ij}^k / \sum_{\chi=\{V,O\}} a_{ij\chi}^k a_{ij}^k$ is the success probability. Applying this property of the binomial distribution, it is straightforward to see that the observed intrafirm trade share, $\tilde{t}_{ijV}^k / \tilde{t}_{ij}^k$, conditional on $\tilde{t}_{ij}^k$, follows a binomial distribution where the number of trials is equal to 1 (also known as a Bernoulli distribution) and the success probability is $a^k_{ijV} a^k_{ij} / \sum_{\chi=\{V,O\}} a^k_{ij\chi} a^k_{ij}$. Therefore:

$$E\left[\frac{\tilde{t}^k_{ijV}}{\tilde{t}^k_{ij}} \middle| \tilde{t}^k_{ij}\right] = \frac{a^k_{ijV} a^k_{ij}}{\sum_{\chi=\{V,O\}} a^k_{ij\chi} a^k_{ij}} = \frac{a^k_{ijV}}{\sum_{\chi=\{V,O\}} a^k_{ij\chi}}. \tag{52}$$

In words, if the trade flows are observed up to a Poisson error, then the intrafirm trade share conditional on total trade flows obeys a Bernoulli distribution with mean $a^k_{ijV} / \sum_{\chi=\{V,O\}} a^k_{ij\chi}$. Equation (52) thus delivers a moment condition in which the left-hand side can be taken directly from data, while the right-hand side is a model-based structural expression for the predicted intrafirm trade share.[23]

We provide in Appendix C.2 an alternative justification for the moment condition in (52). There, we derive the expression for the quasi-maximum likelihood estimator of the a^k_{ij}'s, treating these as country-pair-by-industry fixed effects in a Poisson Pseudo-Maximum Likelihood (PPML) estimation of (49). If we were then to substitute the implied quasi-maximum likelihood estimator for the a^k_{ij}'s back into (49), a quick rearrangement would deliver the same moment condition as in (52).[24]

Our interest is in recovering the fundamental parameters to allow us to quantify the welfare change expression in (43). We proceed by taking a stance on functional form for the $\beta^k_{ij\chi}$, μ^k_{hij}, and μ^k_{xij} in terms of observable characteristics related to contracting institutions in countries i and j, as well as the properties of products from industry k. These can then be substituted in equation (52), and in principle, a non-linear estimator can then be used to recover estimates for: (i) the deep parameters of the model, i.e., θ^k, ρ^k and λ_i; as well as (ii) parameters related to the mapping of the $\beta^k_{ij\chi}$, μ^k_{hij}, and μ^k_{xij} to observable characteristics. This would then allows us to provide quantitative content to the $\widehat{B^k_{jj}}$, $\widehat{\Upsilon^k_j}$, and $\widehat{\widehat{\Upsilon}_j}$ terms, since they are themselves functions of the $\beta^k_{ij\chi}$, μ^k_{hij}, and μ^k_{xij}, and other deep model parameters.

Since $\mu^k_{hij}, \mu^k_{xij} \in [0, 1]$, we adopt a logistic function specification for the two measures of contractibility:

$$\mu^k_{hij} = \frac{e^{\mathbf{h}(i,j,k)}}{1 + e^{\mathbf{h}(i,j,k)}} = \frac{1}{1 + e^{-\mathbf{h}(i,j,k)}}, \tag{53}$$

$$\mu^k_{xij} = \frac{e^{\mathbf{x}(i,j,k)}}{1 + e^{\mathbf{x}(i,j,k)}} = \frac{1}{1 + e^{-\mathbf{x}(i,j,k)}}, \tag{54}$$

where $\mathbf{h}(i, j, k)$ and $\mathbf{x}(i, j, k)$ are polynomial functions of country-i, j, and industry-k characteristics that we will specify below. We simplify the firm's bargaining share under outsourcing to:

$$\beta^k_{ijO} = \beta_O, \tag{55}$$

and the firm's bargaining share under the alternative organizational mode of integration to:

$$\beta^k_{ijV} = (1 - \delta^k_{ij})\beta_O + \delta^k_{ij}, \tag{56}$$

[23] The derivation of this moment condition is akin to that in Eaton, Kortum and Sotelo (2013), in that the moment condition is not exact, but rather holds in expectation after accounting for the distribution of the multiplicative error term associated with observed trade flows (the $\tilde{t}^k_{ij\chi}$'s).

[24] As we show in Appendix C.2, this is because the implied quasi-maximum likelihood estimator of the a^k_{ij}'s delivers predicted trade flows within each country-pair-by-industry bin that are equal in value to the actual trade flows observed in that bin. This is closely related to the observation in Fally (2015), that the PPML delivers estimates that satisfy an adding-up constraint in gravity equations.

where $\delta_{ij}^k \in [0,1]$ is a parameter capturing the share of incremental revenues that the firm would be able to recover from the supplier in the event of a breakdown in the relationship. The δ_{ij}^k parameter captures the residual rights of control in the Grossman and Hart (1986) approach to the theory of the firm: the firm is assured of at least a share δ_{ij}^k of the surplus, and is further assumed to obtain a share β_O of the remaining $1-\delta_{ij}^k$ of the surplus. We in turn relate δ_{ij}^k to country and industry observables via an analogous logistic function:

$$\delta_{ij}^k = \frac{e^{\mathbf{d}(i,j,k)}}{1+e^{\mathbf{d}(i,j,k)}} = \frac{1}{1+e^{-\mathbf{d}(i,j,k)}}. \tag{57}$$

Similarly, we model $\alpha^k \in [0,1]$, the intensity of headquarter services in inputs from industry-k, as a function of physical capital intensity, $\ln(K/L)$, in industry k:

$$\alpha^k = \frac{e^{\mathbf{a}(k)}}{1+e^{\mathbf{a}(k)}} = \frac{1}{1+e^{-\mathbf{a}(k)}}, \tag{58}$$

This follows the lead of Antràs (2003), who pointed to the relevance of capital intensity as a proxy for the relative importance of headquarter services in such buyer-supplier interactions in incomplete contracting environments.

In our baseline specification, we assume that:[25]

$$\begin{aligned}
\mathbf{x}(i,j,k) &= \gamma_1 + \gamma_2\,(\text{Supplier Contractibility}_k) + \gamma_3\,(\text{Supplier Contractibility}_k)^2 \\
&\quad + \gamma_4 \text{ROL}_i + \gamma_5\,(\text{ROL}_i)^2 + \gamma_6 \text{ROL}_j + \gamma_7\,(\text{ROL}_j)^2 \\
&\quad + \gamma_8\,(\text{Supplier Contractibility}_k) \times \text{ROL}_i + \gamma_9\,(\text{Supplier Contractibility}_k) \times \text{ROL}_j, \text{ and} \\
\mathbf{h}(i,j,k) &= \gamma_{11} + \gamma_{12}\,(\text{HQ Contractibility}_k) + \gamma_{13}\,(\text{HQ Contractibility}_k)^2 \\
&\quad + \gamma_{14} \text{ROL}_i + \gamma_{15}\,(\text{ROL}_i)^2 + \gamma_{16} \text{ROL}_j + \gamma_{17}\,(\text{ROL}_j)^2 \\
&\quad + \gamma_{18}\,(\text{HQ Contractibility}_k) \times \text{ROL}_i + \gamma_{19}\,(\text{HQ Contractibility}_k) \times \text{ROL}_j, \text{ and} \\
\mathbf{d}(i,j,k) &= \gamma_{21} + \gamma_{22}\,(\text{Specificity}_k) + \gamma_{23}\,(\text{Specificity}_k)^2 + \gamma_{24} \text{ROL}_i + \gamma_{25}\,(\text{ROL}_i)^2 \\
&\quad + \gamma_{26} \text{ROL}_j + \gamma_{27}\,(\text{ROL}_j)^2 + \gamma_{28}\,(\text{Specificity}_k) \times \text{ROL}_i + \gamma_{29}\,(\text{Specificity}_k) \times \text{ROL}_j, \text{ and} \\
\mathbf{a}(k) &= \gamma_{31} + \gamma_{32} \ln(K/L)_k + \gamma_{33}\,(\ln(K/L)_k)^2 .
\end{aligned}$$

Here, "Supplier Contractibility$_k$" and "HQ Contractibility$_k$" are the measures of the contractibility of supplier and headquarter input tasks respectively in industry k. We construct these drawing on the ideas in Nunn (2007), who proposed a measure of the overall contractibility of production in industry k that is the share of inputs (by value) classified by Rauch (1999) as traded on an open exchange or reference-priced. We further designate inputs that are from NAICS 6-digit codes that feature an above-median capital-labor ratio to be headquarter inputs, with the view here being that the firm headquarters

[25] The functions $\mathbf{x}(i,j,k)$, $\mathbf{h}(i,j,k)$, and $\mathbf{d}(i,j,k)$ are full second-order polynomials of an industry-level variable and the ROL of the exporting and the importing countries, except for the omission of $\text{ROL}_i \times \text{ROL}_j$. The interaction between the two ROL measures cannot be included in the regression because, as explained later in this section, we are using the US Census Bureau's Related Party Database, in which the US will be either the importer or the exporter. Including $\text{ROL}_i \times \text{ROL}_j$ will lead to colinearity issue with either ROL_i or ROL_j.

is more likely to be responsible for the provision of capital-intensive inputs; the remaining below-median capital-intensity inputs are designated as supplier inputs. We then compute the share of inputs (by value) used in industry k that are traded on an open exchange or reference-priced separately for each of these subsets of inputs, to arrive at the "HQ Contractibility$_k$" and "Supplier Contractibility$_k$" measures. We include these in respectively the $\mathbf{h}(i,j,k)$ and the $\mathbf{x}(i,j,k)$ functions that capture the μ_{hij}^k and μ_{xij}^k contractibility parameters in our model. Separately, "Specificity$_k$" is the Rauch (1999) measure of the extent to which industry-k is comprised of differentiated products. We include this in the $\mathbf{d}(i,j,k)$; conceptually, it would be easier for products with a *lower* specificity to fetch resale value on an open market, and so we would expect a lower Specificity$_k$ to be associated with a higher δ_{ij}^k. The variable "ROL$_i$" is country i's Rule-of-Law index from the World Governance Indicators. We include this in the $\mathbf{h}(i,j,k)$, $\mathbf{x}(i,j,k)$ and $\mathbf{d}(i,j,k)$ functions, to allow a role for source- and destination-country institutions to affect these contracting and bargaining parameters. The variable $(K/L)_k$ is the capital-to-labor ratio from the NBER-CES Manufacturing Productivity Database.

Stress generality of this functional form.

A quick subsection on identifying variation. Does the expression for the wedge between sourcing probabilities and trade shares matter for identification.

This is key to understanding what pins down the order of magnitude of our results. Strengthen the interpretation of the counterfactuals.

In addition to the $\gamma_{(\cdot)}$ parameters, we also need to take a stand on ρ^k, λ_i, and θ^k. To reduce the dimension of the parameter space, we back out ρ^k from the estimates of demand elasticity from Soderbery (2015). We estimate the nested Freéchet correlation parameter, λ_i, by income group of the exporting country, following the classification of the World Bank. As explained in the next section, three income groups exist in our sample: the lower-middle (λ_1), upper-middle (λ_2), and high income countries (λ_3). We estimate θ^k for each industry.

In the end, we perform the estimation of (52) via weighted non-linear least squares (NLLS). The vector of parameters to be estimated is: $\Theta = \{\gamma_{(\cdot)}, \beta_O, \lambda_i, \theta^k\}$. Let $\mathbf{X}_{ij}^k$ denote the full vector of country and industry observables that enter into the $\mathbf{h}(i,j,k)$, $\mathbf{x}(i,j,k)$, $\mathbf{d}(i,j,k)$ and $\mathbf{a}(k)$ functions. The weighting matrix is the volume of trade flow in each $\{i,j,k,\chi\}$ cell to alleviate the concern that the cells with lower trade volumes are imprecisely measured. The moment condition is therefore:

$$m(\Theta) = \mathbb{E}\left[\frac{\tilde{t}_{ij\chi}^k}{\sum_{\chi\in\{V,O\}}\tilde{t}_{ij\chi}^k} - \frac{a_{ij\chi}^k}{\sum_{\chi\in\{V,O\}}a_{ij\chi}^k}\Big|\mathbf{X}_{ij}^k\right] = 0, \tag{59}$$

and the NLLS estimator is formally defined as:

$$\Theta^* = \text{argmin}_\Theta \left(m(\Theta)\right)^T \cdot \Omega \cdot \left(m(\Theta)\right),$$

where Ω is the weighting matrix with $\tilde{t}_{ij\chi}^k$ in the diagonal.

4.2 Data and Sample

We briefly discuss the data and sample issues here, while referring the readers to Appendix C.1 for details. We define industry k as a NAICS 3-digit level industry. Our specificity data come from Rauch (1999), which uses the SIC classification of industry. We bridge the industry's classification from SIC to NAICS using the Feenstra-Romalis-Schott import data to construct concordance weights. We compute both the supplier and headquarter contractibility by using the 1997 U.S. Input-Output Tables and the Rauch-specificities of the input industries. Lastly, the capital intensity comes from the NBER-CES dataset as the log of the average real capital stock per worker.

At the country dimension, we include the largest 50 trading partners of the U.S. and exclude Iraq, Saudi Arabia, Venezuela, and Hong Kong from our sample. These excluded countries are either oil-exporting countries or serve as trading gateways for other countries. This sample exclusion leaves us with 47 countries, including the United States. The Rule of Law of a country comes from the World Governance Indicators, and the income group classification comes from the World Bank. Table A.1 in the appendix provides the details of the countries included in each sample.

Lastly, the intrafirm trade share data come from the U.S. Census Bureau's Related Party Database. We use both the U.S. imports and exports data. The sample restrictions above leave us with 1,926 observations, coming from 21 industries and 47 countries. Out of these observations, 960 are U.S. import trade flows, and the remaining 966 are exports.

4.3 Estimation Algorithm

We solve the NLLS as a constrained minimization problem subject to: $\theta^k > 1$, $\lambda_i \in (0, 1)$, and $\beta_O \in (0, 1)$. In practice, we solve the minimization problem using a mixture of heuristic and gradient-based algorithms. Without taking a stand on the initial guess of the solution, we start the solution algorithm with Particle Swarm Optimization (PSO). Doing so allows us to search the entire parameter space, thus reducing the possibility of finding a local optimum. We switch to a gradient-based algorithm, Levenberg-Marquardt, after 1000 randomized calls using the PSO, with the optimal solution from the PSO as the initial guess. Compared to the PSO, a gradient-based method reduces computational load and guarantees local convergence. With a reasonably good initial guess, switching to a gradient-based method improves the efficiency of the estimation. Lastly, the standard errors, clustered at the industry-level, are computed via Gauss-Newton Regressions (GNR). We refer to the readers for more details of the estimation algorithm in Appendix C.4.

4.4 Estimation Results

Table 1 and Table 2 in the main text and Table A.2 in the Appendix report the point estimates and the standard errors of the elements of the Θ parameter vector. We first illustrate the goodness of fit between the intrafirm trade shares predicted by these estimates and the corresponding shares that are directly in the data. Figure 1 provides reassurance that the model provides a reasonable fit to the data

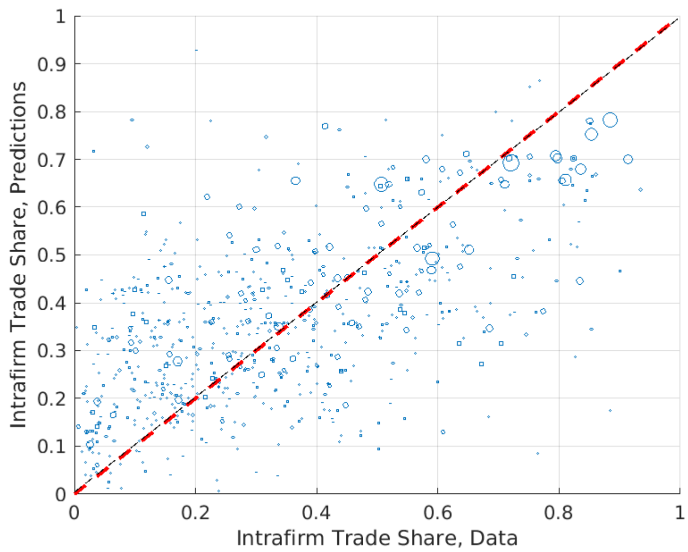


Figure 1: Intrafirm Trade Share, Data v.s. Prediction

Note: the figure presents the predicted intrafirm trade share against the data. Each point represents an observation at the country-pair-industry level. The size of the circle indicates the weight of the observation, which is the observed trade flow, $\tilde{t}^k_{ij}$. The red dashed line is the 45-degree line. The plot is based on the estimated $\gamma_{(\cdot)}$ as reported in Table 1 and A.2.

about the 45 degree line, in spite of the large variation in values assumed by the intrafirm trade share between 0 and 1.

With the non-linearity of the $\mathbf{h}(\cdot)$, $\mathbf{x}(\cdot)$, and $\mathbf{d}(\cdot)$ functions, it is easier to illustrate their behavior by focusing on the surface plots in Figure 2 and Figure 3, rather than on directly interpreting the point estimates of $\gamma_{(\cdot)}$ in Table 1 and Appendix Table A.2. In Figure 2, we plot the μ^k_{xij}, μ^k_{hij}, and δ^k_{ij} functions against the rule of law of the exporting and the importing countries, for a particular industry k. For each variable we present two industries, one at the 10th percentile of the industry characteristic, and the other at the 90th percentile. For the μ^k_{xij} and μ^k_{hij} surfaces, the relevant industry characteristic is the NAICS supplier- and HQ-contractibility, respectively, and for δ^k_{ij}, it is the NAICS specificity of the industry. Similarly, Figure 3 plots the surfaces of these functions across the importer ROL and the industry characteristic dimension, conditional on low and high levels of exporter ROL.

Across the entire sample, the average of μ^k_{xij} and μ^k_{hij} are 0.59 and 0.41, which through the literal lens of the model would mean that 59 and 41 percent of the supplier and HQ tasks respectively are contractible in the baseline estimation. As shown in Figure 2, both μ^k_{xij} and μ^k_{hij} span a wide range, and they positively correlate with the rule of law of the importing country. By contrast, μ^k_{xij} and μ^k_{hij} are less responsive to differences in the rule of law across exporting countries. Note that conditional on the identity of the industry, as the ROL of the importing country improves, the μ^k_{xij} and μ^k_{hij} surfaces increase at different rates depending on the ROL of the exporting country. For example, the μ^k_{xij} surface starts to increase at lower values of importer ROL when the exporting country has low ROL in Panel (a), as compared to the case in Panel (b) with a high ROL in the exporting country. The μ^k_{hij} surface

name	est.	se	95% CI	Δ F
γ_{32}	0.414	0.009	[0.40, 0.43]	-
γ_{33}	-0.134	0.003	[-0.14, -0.13]	-
β_O	0.678	0.007	[0.66, 0.69]	-
λ_{01}	0.921	0.001	[0.92, 0.92]	-
λ_{02}	0.884	0.002	[0.88, 0.89]	-
λ_{03}	0.782	0.004	[0.77, 0.79]	-
F-val	56.58	-	-	-

Table 1: Estimation Results

Note: this table reports the point estimates of the parameters in the NLLS. The second column is the point estimate; the third column is the standard error (clustered by country-pair), and the last column the 95% confidence interval. The other γ parameters are reported in Table A.2 in the Appendix.

exhibits a similar pattern.

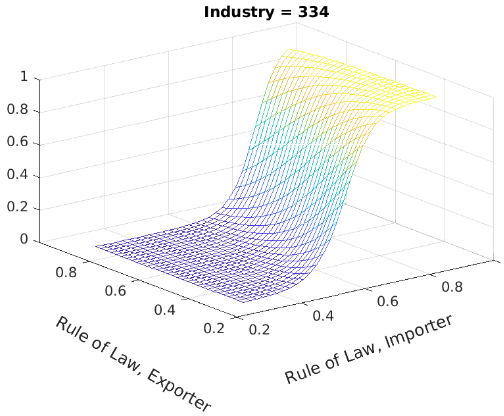
Figure 3 highlights how the surfaces vary across industries. The estimated μ^k_{hij} mildly increases with the NAICS HQ-contractibility, especially when the ROL of the importing country is high. On the other hand, the relationship between μ^k_{xij} and the supplier contractibility is non-monotonic: it first increases with the supplier contractibility before declining.

The average δ^k_{ij}, the share of incremental revenue that a firm would be able to recover in the event of relationship breakdown, is 0.39 across the entire sample. The firm's ability to recuperate its investments significantly drops as the NAICS-specificity of the inputs increases, as highlighted in both Figure 2 and 3. At the 10th percentile of input specificity, the average δ^k_{ij} can be as high as 58 percent, while at the 90th percentile, it drops to 31 percent. This pattern is expected as the market value for highly specific inputs should be lower than the non-specific ones. Moreover, the firms are better protected if the importing country has a higher ROL as well. For example, in Panel (e) of Figure 2, the average δ^k_{ij} is 38 percent when Russia is the importer, and it more than doubles to 82 percent if the importer is Finland with a significantly higher ROL. On a related note, our point estimate of β_O, the bargaining share under outsourcing, is 0.678, as reported in Table 1. Together with the estimated δ^k_{ij}, the implied firm bargaining share under integration β^k_{ijV} averages out at 0.804. Following the pattern of δ^k_{ij}, the firm's bargaining power under vertical integration is also higher in the importing countries with better institutions and industries with lower input specificity.

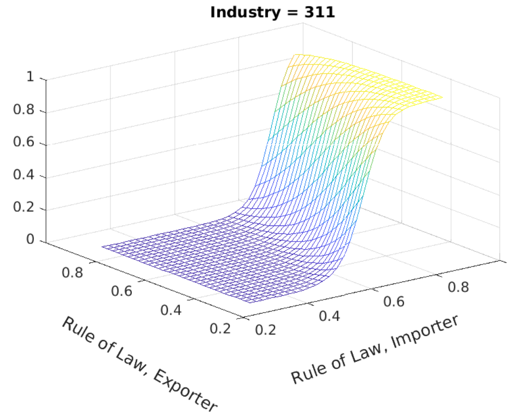
The last three $\gamma_{(\cdot)}$ estimates governing the shape of α^k are reported in Table 1, and the predicted values of α^k by industry are reported in the first column of Table 2. Similar to Antràs (2003), industries with higher capital intensity place a higher weight on headquarter inputs in the production function. The estimated α^k varies between 0.494 and 0.707, reflecting the variation in the underlying capital intensity across industries. In the estimation, we have fixed γ_{31}, the constant term in $\mathbf{a}(k)$ function to 0.619, so that the industry with the average level of capital intensity in our sample will have an $\alpha^k = 0.65$.[26]

Lastly, we describe our estimates of θ^k and λ_i. We first note that our estimation strategy only

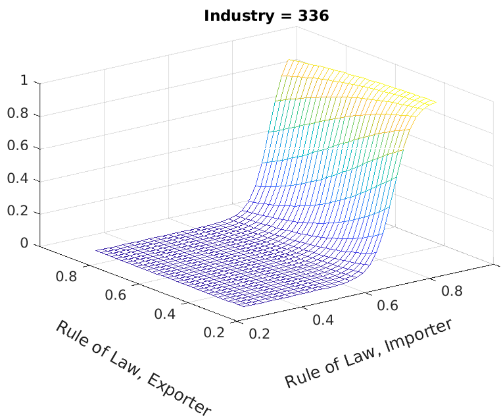
[26] We normalize the average level of capital intensity across industries to 0.



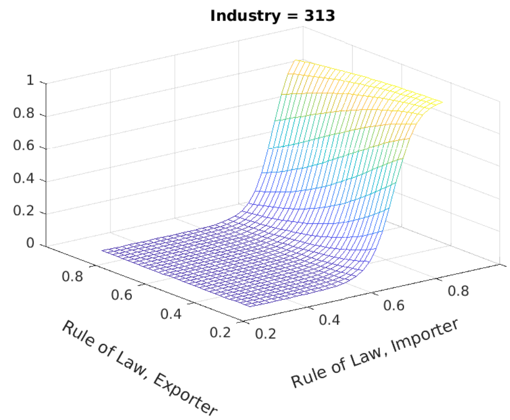
(a) μ^k_{xij}, Low Supplier Contractibility



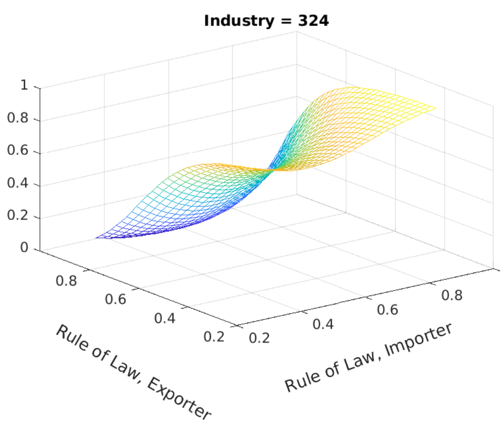
(b) μ^k_{xij}, High Supplier Contractibility



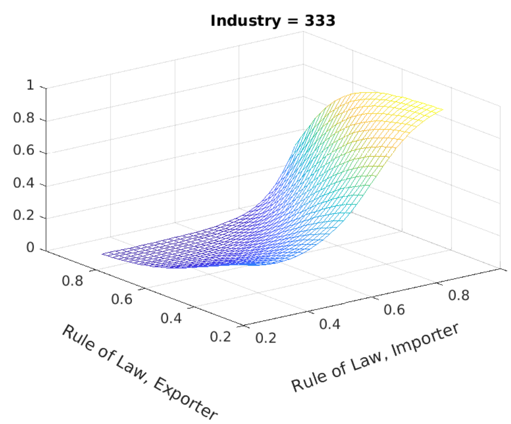
(c) μ^k_{hij}, Low HQ Contractibility



(d) μ^k_{hij}, High HQ Contractibility



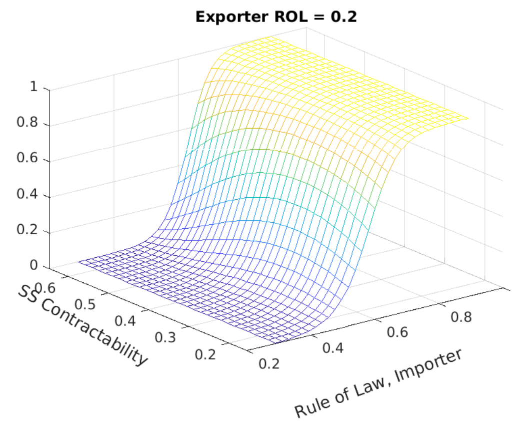
(e) δ^k_{ij}, Low Specificity



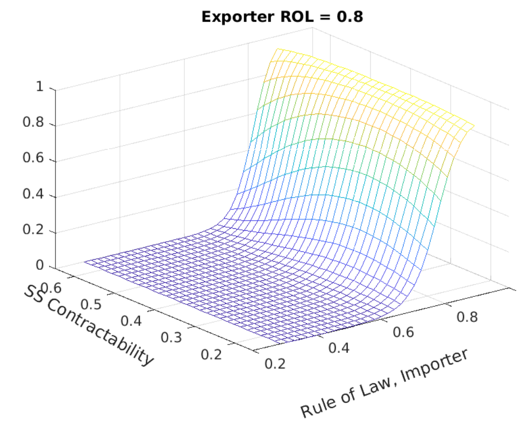
(f) δ^k_{ij}, High Specificity

Figure 2: Estimated Surfaces

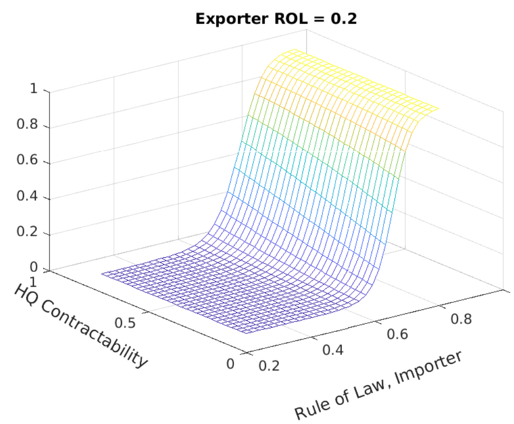
Note: the figures present the surface plot of the μ^k_{xij}, μ^k_{hij}, and δ^k_{ij} functions as specified in equation (54), (53), and (57), respectively. The plots are based on the estimated $\gamma_{(\cdot)}$ as reported in Table 1.



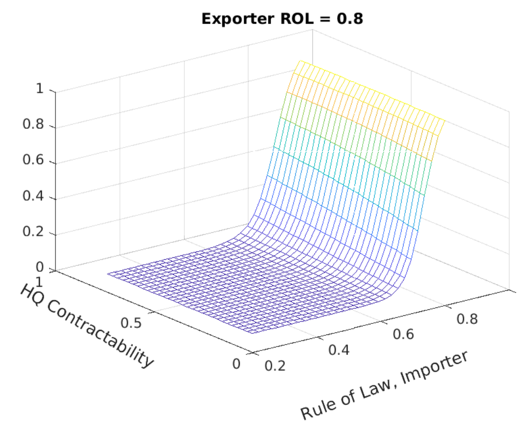
(a) μ^k_{xij}, Low Exporter ROL



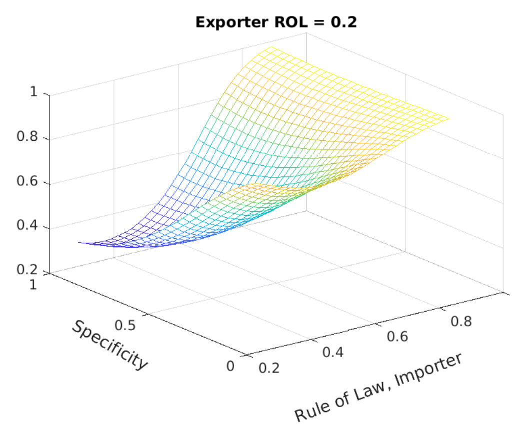
(b) μ^k_{xij}, High Exporter ROL



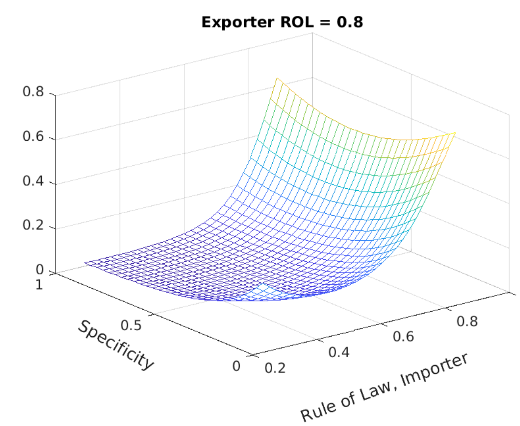
(c) μ^k_{hij}, Low Exporter ROL



(d) μ^k_{hij}, High Exporter ROL



(e) δ^k_{ij}, Low Exporter ROL



(f) δ^k_{ij}, High Exporter ROL

Figure 3: Estimated Surfaces, Selected Exporters

Note: the figures present the surface plot of the μ^k_{xij}, μ^k_{hij}, and δ^k_{ij} functions as specified in equation (54), (53), and (57), respectively. The plots are based on the estimated $\gamma_{(\cdot)}$ as reported in Table 1.

ID	NAICS3	Desc	α^k	θ^k	ρ^k (Soderbery)	$1 - \frac{(1-\alpha^k)\rho^k}{\theta^k(1-\rho^k)}$
1	311	Food Manufacturing ...	0.646	4.000	0.886	0.315
2	312	Beverage and Tobacco Prod ...	0.701	16.223	0.788	0.932
3	313	Textile Mills ...	0.659	3.805	0.821	0.590
4	314	Textile Product Mills ...	0.539	8.432	0.768	0.820
5	315	Apparel Manufacturing ...	0.521	9.478	0.852	0.708
6	316	Leather and Allied Produc ...	0.555	12.714	0.781	0.875
7	321	Wood Product Manufacturin ...	0.560	8.858	0.777	0.827
8	322	Paper Manufacturing ...	0.698	8.641	0.580	0.952
9	323	Printing and Related Supp ...	0.587	16.020	0.688	0.943
10	324	Petroleum and Coal Produc ...	0.707	11.193	0.881	0.806
11	325	Chemical Manufacturing ...	0.708	21.514	0.771	0.954
12	326	Plastics and Rubber Produ ...	0.626	13.045	0.879	0.791
13	327	Nonmetallic Mineral Produ ...	0.656	39.243	0.738	0.975
14	331	Primary Metal Manufacturi ...	0.700	36.932	0.891	0.934
15	332	Fabricated Metal Product ...	0.607	10.690	0.708	0.911
16	333	Machinery Manufacturing ...	0.639	18.377	0.841	0.897
17	334	Computer and Electronic P ...	0.687	15.771	0.728	0.947
18	335	Electrical Equipment Appl ...	0.625	1.835	0.632	0.650
19	336	Transportation Equipment ...	0.664	23.198	0.749	0.957
20	337	Furniture and Related Pro ...	0.494	9.246	0.297	0.977
21	339	Miscellaneous Manufacturi ...	0.575	7.802	0.714	0.864
-	-	Mean	0.626	14.144	0.751	0.839

Table 2: Estimation Results, Industry Level Results

Note: this table reports the estimated results at the industry level. α^k is the predicted values based on the estimates of γ reported in Table 1. θ^k is the shape parameter of the productivity distribution. The third column reports the implied ρ^k using the demand elasticities reported in Soderbery (2015). The last column reports the theoretical restriction on the parameter values in order to ensure that the inputs to the $\Gamma(\cdot)$ function are positive in equation (B.14). We do not impose the restriction at the estimation stage, and only check the condition after the estimation, as reported here.

identifies θ^k and λ_i up to a scaling factor. This is because in the expression for $a^k_{ij\chi}$'s in equation (51), the two terms only show up together as the ratio $\frac{\theta^k}{1-\lambda_i}$. For this reason, we exogenously fix θ^1 prior to the estimation, and identify all the other θ^k's and λ_i's relative to θ^1. In practice, we set $\theta^1 = 4$, implying a trade elasticity of 4 in the food manufacturing industry.

The estimated θ^k varies greatly across industries. It is lower in complex industries with differentiated products, such as "Electrical Equipment" ($\theta^k = 1.835$) and "Textile Mills" ($\theta^k = 3.805$). On the other end of the spectrum, higher θ^k are often observed in the primary industries with relatively homogeneous outputs, such as "Non-Metallic Mineral Products" ($\theta^k = 39.243$), and "Primary Metal Manufacturing" ($\theta^k = 36.932$). Recall that θ^k is the shape parameter of the Fréchet distribution, and it is inversely related to the dispersion of the productivity draws across the sourcing modes. In light of this, our estimates suggest that the productivity draws of the primary and homogeneous industries are distributed more evenly relative to the productivity draws of more complex industries. The average of θ^k across all industries is 14.14.

The correlation parameter, λ_i, is estimated to be 0.921 in lower-middle-income countries, and 0.884 and 0.782 in upper-middle-income and high-income countries, respectively. While the differences between the lower- and upper-middle-income countries are not statistically significant, the estimated λ_i in the high-income countries is smaller than that of lower-middle-income countries. The higher λ_i among lower-income countries suggests that these productivity draws are more homogeneous than in high-income countries. Regardless of the relative magnitudes, the three estimated λ_i are high, which suggests that the productivity draws of suppliers across the two modes of organization are highly correlated within each country.

5 Quantitative Implications

In this section, we study the quantitative implications of contracting frictions. Our starting point is the baseline parameterization presented in the previous section. Following the "hat-algebra" approach in Dekle, Eaton, and Kortum (2008), we directly compute the percentage deviation between the counterfactual and the baseline model without solving for the equilibrium in levels. In particular, we use x' to denote the counterfactual value, and $\widehat{x} \equiv x'/x$ to denote the percentage difference between the counterfactual and the baseline value of variable x. Appendix C.6 provides the details on the hat-algebra simulations.

In addition to the estimated parameters in the previous section, we also need data on initial trade flows to compute the sourcing probability, π^k_{ij}, in the baseline equilibrium for a full matrix of countries. For this purpose, we turn to the Inter-Country Input-Output Tables (ICIO) from the OECD. Out of the 47 countries in our estimation sample, only 41 are included in the ICIO tables. We add the Rest of the World (ROW) to the counterfactual sample, bringing the final number of countries in the quantitative exercises to 42. Table A.1 in the Appendix provides the list of the countries, and Appendix C.1 discusses details of how the ICIO data were processed. There is a subtle complication here in that the trade shares we compute from the ICIO data do not map exactly to the sourcing shares π_{ij}^k in the model; this is because prices and quantities in our model are the outcome of a bargaining process, unlike the competitive markets in Eaton and Kortum (2002). In Appendix C.5, we derive a model-consistent correction factor that allows us to infer the π_{ij}^k's from the ICIO trade shares.

To be clear, our approach here will take the parameter estimates of the model that have been obtained using solely the U.S. intrafirm trade share data, and apply these to the global system of 42 countries constructed from the ICIO. While this assumes that the U.S.-based parameter estimates provide an accurate description of the sourcing problem faced by final-good producers in other countries, we should note that our quantification strategy is general enough that intrafirm trade data from other countries (should these become available) can be readily included in the vector of data moments as part of the estimation.

In addition to the parameters estimated from Section 4, we also need the following parameters to simulate the model. The demand elasticity of the final good, ρ, is set to 0.75. The share of labor in the final assembly, α, equals to 0.66. Lastly, η^k reflects the weight of industry k in the production of the final good. We computed η_k as $\frac{\text{VADD}_k}{\sum_{k'=1}^{K} \text{VADD}_{k'}}$, where VADD_k is the value-added of industry k from the NBER-CES dataset.

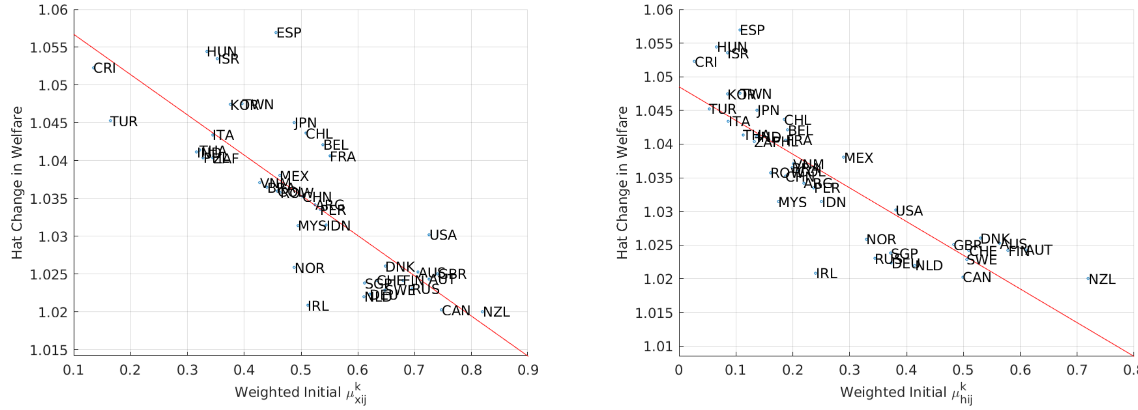
In the rest of the section, we present four sets of counterfactual exercises. In the first two exercises, we: (i) remove all the contracting frictions in the world by setting all the μ_{hij}^k's and μ_{xij}^k's to 1, and (ii) remove the possibility of vertical integration by setting $\delta_{ij}^k = 0$. In the third exercise, we study the interaction between the gains from trade and contracting frictions. In the last exercise, we focus on the implication of improving the rule of law in one specific country – China – towards the world's frontier.

5.1 Improving μ_{hij}^k and μ_{xij}^k to 1

In the first exercise, we increase both μ_{hij}^k and μ_{xij}^k to 1 for all i,j, and k, entirely removing the contracting frictions in headquarter services and supplier inputs. In the baseline estimation, the average μ_{xij}^k is 0.59, and the average μ_{hij}^k is 0.41, so the counterfactual exercise roughly doubles the level of contractibility. The welfare in all the countries increases after the removal of contracting frictions. The average improvement is around 3.49 percent, and the median is 3.55 percent. As expected, countries with lower initial values of μ_{xij}^k and μ_{hij}^k in the baseline equilibrium see more substantial gains in welfare. We compute the initial μ_{xij}^k for a given country as the weighted average of all the μ_{xij}^k where the country is either an importer or an exporter. The weight is the trade flow in the baseline equilibrium. As shown in Figure 4, while Turkey, a country with a poor contracting environment, experiences a 4.5 percent increase, the welfare in Canada only increases by 2.0 percent.

5.2 Decreasing δ_{ij}^k to 0

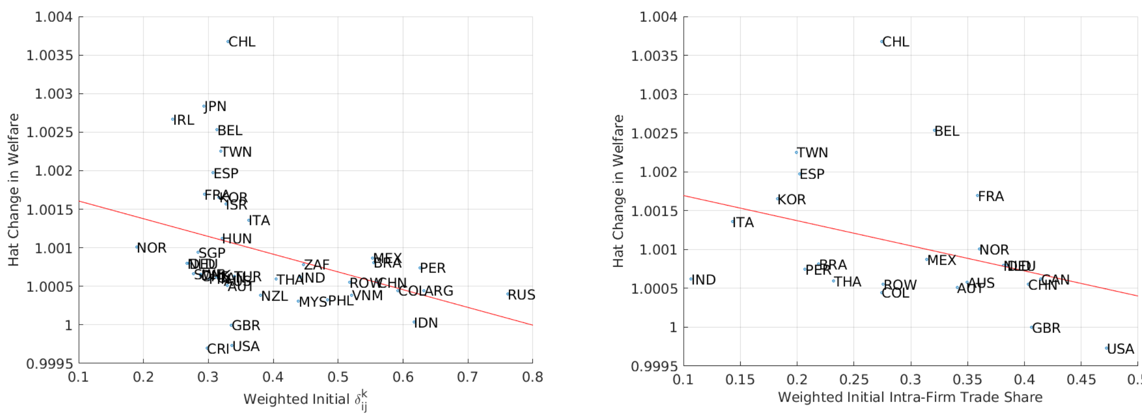
In the second exercise, we decrease δ_{ij}^k to 0, implying $\beta_{ijV}^k = \beta_{ijO}^k$ for all i, j, k. In this counter-factual world, vertical integration is no longer a distinct mode of sourcing, and all the firms will be effectively sourcing from suppliers at arms-length.



(a) Initial μ^k_{xij} (b) Initial μ^k_{hij}

Figure 4: Changes in Welfare v.s. Initial Levels of Contractibility

Note: the figures plot the hat changes in welfare against the initial levels of μ^k_{xij} and μ^k_{hij} in each country. The counterfactual equilibrium is the one in which all the μ^k_{xij} and μ^k_{hij} are set to 1.



(a) δ^k_{ij} (b) Intrafirm Trade Share

Figure 5: Changes in Welfare v.s. Initial Levels of δ^k_{ij} and Intrafirm Trade Share

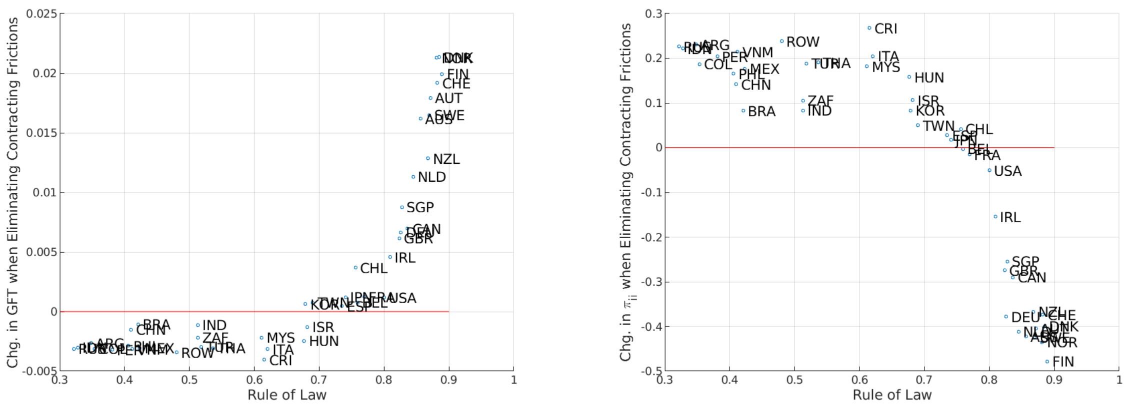
Note: the figures plot the hat changes in welfare against the initial levels of δ^k_{xij} and intrafirm trade share in each country. The counterfactual equilibrium is the one in which all the δ^k_{ij} are set to 0.

Address the literature on quantitative models of MP, including Garetto (2013), Ramondo and Rodriguez-Clare (2013), Ramondo (2014), Alvariez (2019), Arkolakis et al. (2018).

The average welfare change is positive but small at 0.09 percent, the median change is 0.06 percent, and a small number of countries lose from setting $\delta^k_{ij} = 0$. Across the countries, the changes in welfare are negatively correlated with the initial levels of δ^k_{ij} and intrafirm trade share, as shown in the two panels in Figure 5. Thus, countries that initially rely more on vertical integration as an organizational mode stand to lose if this mode of sourcing is no longer available. For example, in Panel (b) of Figure 5, the country with the highest initial level of intrafirm trade share, the U.S., also suffers the most extensive welfare loss of 0.03 percent. In contrast, the countries with initial intrafirm trade shares close to zero slightly benefit from such a change.

5.3 Gains from Trade

In the third exercise, we study the interaction between contracting frictions and the gains from trade. Specifically, we compare the gains from trade in a model with and without contracting frictions to understand the interaction. We interpret the model based on the estimated $\gamma_{(\cdot)}$ as the baseline case with contracting frictions, and the model with $\mu^k_{xij} = \mu^k_{hij} = 1$, for all i, j, k as the counterfactual scenario void of contracting frictions. Figure 6 summarizes the results.



(a) Gains from Trade

(b) Domestic Absorption Ratio

Figure 6: Changes in Gains from Trade and Domestic Absorption Ratio against Rule of Law

Note: this figure plots the changes in gains from trade and the domestic absorption ratio when removing the contracting frictions against the ROL of a country. Removing contracting frictions means setting $\mu^k_{xij} = \mu^k_{hij} = 1$ for all i, j, k.

Countries with higher ROL gain more from trade once the contracting frictions are eliminated, while countries with lower ROL see smaller gains from trade. Panel (a) of Figure 6 plots the changes in gains from trade once the contracting frictions are eliminated against the initial ROL of the country. In high-ROL countries such as Denmark and Finland, removing the contracting frictions increases the gains from trade by around 2 percentage points. At the same time, countries with lower ROL such as Russia and Argentina see their gains from trade decline by approximately 0.3 percentage points.

The positive correlation between ROL and the changes in gains from trade is rooted in the fact that low-ROL countries have lower μ^k_{xij} and μ^k_{hij} in the baseline model. Setting all the μ^k_{xij} and μ^k_{hij} to 1 improves the contracting environment in the domestic market relatively more as compared to the international markets in these countries. As a result, as rule of law improves, the final-good producers in low-ROL countries see a relative increase in sourcing from their domestic suppliers relative to foreign suppliers; this leads to a higher domestic absorption ratio, as shown in Panel (b) of the same figure. Subsequently, the potential welfare gains from removing trade frictions in these low-ROL countries declines. In high-ROL countries, the opposite forces prevail. Improving contractibility only marginally improves the contracting environments in the domestic market in a high-ROL country. Still, it significantly boosts the contracting environment with foreign suppliers, leading to a lower domestic sourcing ratio and higher gains from trade.

5.4 Improving ROL in China

In the last exercise, we turn our attention to China. We improve the ROL in China from its baseline value, which ranked 36th out of the 42 countries, to the world frontier, the level of Finland. The improvement in ROL is substantial, as the magnitude is equivalent to 2.5 standard deviations in our sample. Higher ROL directly improves μ^k_{xij}, μ^k_{hij}, and δ^k_{ij}. As suggested by Figure 2, the improvements are more pronounced when China is the importer.

Welfare Across Countries: All countries around the world benefit from higher ROL in China, and the average welfare gain, excluding China, is 0.21 percent, with a median of 0.14 percent. As shown in the first two panels of Figure 4, countries that see larger increases in their trade volumes with China enjoy higher welfare gains. Upon closer look, the countries that gained the most also tended to be engaged initially in trade with China in industries that feature low contractibility values. To highlight this point, we correlate the country welfare gains against the weighted average value of μ^k_{xij} (respectively, μ^k_{hij}) computed as follows:

$$\bar{\mu}_{xi} = \frac{\sum_{k=1}^{21} \mu^k_{xi,\text{CHN}} t^k_{i,\text{CHN}} + \sum_{k=1}^{21} \mu^k_{x\text{CHN},i} t^k_{\text{CHN},i}}{\sum_{k=1}^{21} t^k_{i,\text{CHN}} + \sum_{k=1}^{21} t^k_{\text{CHN},i}}.$$

We illustrate these correlations in the remaining two panels of Figure **??**. As shown in these panels, the welfare gains are negatively correlated with the initial level of contractibility of the country's trade flow with China. In other words, countries whose trade with China was initially subject to weaker contracting conditions stand to benefit more, as they enjoy more considerable relative changes in the contractibility of their trade flows with China. Lastly, China itself reaps the largest welfare gain at 3.1 percent. The increase in China is roughly the same magnitude as in the first exercise (3.5 percent) in which we remove the contracting frictions in all the countries.

6 Conclusion

In this paper, we have developed a global sourcing model in the presence of contracting frictions, in which the interaction between a firm and each input supplier is subject to a bilateral holdup problem. The firm's decision problem over organizational mode – whether to integrate the supplier under its ownership and control, or whether to maintain an arm's-length relationship – is modeled closely on the Grossman-Hart Moore approach to the theory of the firm, in which the assignment of residual rights of control plays a central role. These firm-level decision problems have often been analyzed in partial equilibrium in the literature. We show in this paper how to provide a modeling bridge between this rich firm-level structure and a class of quantitative trade models. Doing so allows us to shed light on the macro consequences of these contracting frictions in global sourcing.

Based on this framework, we derive a closed-form expression for welfare, describing how contracting frictions affect welfare. Moreover, aggregating up from the firm-level decisions leads to a gravity equation for bilateral trade flows by industry and organizational mode, and hence yields a structural expression for the intrafirm trade share. We estimate the model using publicly available data, and use a series of counterfactual exercises to highlight the relationship between contracting frictions, trade patterns, and country welfare. The framework developed in this paper is rich and flexible, but much scope remains for future work. The model as it stands features a relatively short production chain, and it would be interesting to extend it in future work to explore the welfare implications of contracting frictions in global value chains, in which production and sourcing exhibit a more meaningful sequentiality. Another potentially fruitful extension would be to incorporate a richer exporting decision for firms, to speak to the role of countries as export platforms.

Allude to potential for estimation with firm-level data. firm-level work.

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A Additional Tables and Figures

ARG	DEU	IND	PER
AUS	DNK	IRL	PHL
AUT	DOM*	ISR	ROW†
BEL	DZA*	ITA	RUS
BGD*	ESP	JPN	SGP
BRA	FIN	KOR	SWE
CAN	FRA	MEX	THA
CHE	GBR	MYS	TUR
CHL	GTM*	NLD	TWN
CHN	HND*	NOR	USA
COL	HUN	NZL	VNM
CRI	IDN	PAK*	ZAF

Table A.1: Countries in the quantitative exercise

Note: the table lists all the countries in the structural estimation and the counterfactual exercises. The six countries with a * are only in the structural estimation. They are absent in the counterfactual simulations because they are missing in the ICIO data. The two countries with a † are only in the counterfactual analysis. USA is missing in the estimation because we use the US Related Party Trade data, where the importer is fixed to be the US. "ROW" stands for "rest-of-the-world". All the other 42 countries show up in both exercises.

name	est.	se	95% CI	Δ F
γ_1	-7.154	0.987	[-9.09, -5.22]	-
γ_2	4.856	0.286	[4.30, 5.42]	-
γ_3	-18.223	0.160	[-18.54, -17.91]	-
γ_4	-9.843	0.296	[-10.42, -9.26]	-
γ_5	2.624	0.245	[2.14, 3.10]	-
γ_6	6.853	2.485	[1.98, 11.72]	-
γ_7	11.132	1.588	[8.02, 14.24]	-
γ_8	-3.744	0.067	[-3.88, -3.61]	-
γ_9	12.949	0.296	[12.37, 13.53]	-
γ_{11}	-17.156	2.106	[-21.28, -13.03]	-
γ_{12}	8.760	0.877	[7.04, 10.48]	-
γ_{13}	-0.527	0.035	[-0.60, -0.46]	-
γ_{14}	-6.376	0.232	[-6.83, -5.92]	-
γ_{15}	1.661	0.210	[1.25, 2.07]	-
γ_{16}	11.580	4.696	[2.38, 20.78]	-
γ_{17}	16.951	2.693	[11.67, 22.23]	-
γ_{18}	0.229	0.050	[0.13, 0.33]	-
γ_{19}	-9.446	1.093	[-11.59, -7.30]	-
γ_{21}	6.394	0.066	[6.26, 6.52]	-
γ_{22}	-7.824	0.056	[-7.93, -7.72]	-
γ_{23}	2.746	0.051	[2.65, 2.85]	-
γ_{24}	1.930	0.271	[1.40, 2.46]	-
γ_{25}	-7.522	0.242	[-8.00, -7.05]	-
γ_{26}	-17.544	0.140	[-17.82, -17.27]	-
γ_{27}	17.369	0.153	[17.07, 17.67]	-
γ_{28}	0.224	0.038	[0.15, 0.30]	-
γ_{29}	4.742	0.044	[4.66, 4.83]	-
γ_{31}	0.619	-	-	-

Table A.2: Countries in the quantitative exercise

Note: the table lists the estimated $\gamma_{(.)}$ from the NLLS. γ_{31} is normalized to match the average α^k across the industries.

B Model Appendix (ONLINE ONLY)

B.1 Proofs from Section 2

Derivation of incremental revenue contribution of each input variety: We calculate the incremental contribution to firm revenue that is attributable to each input variety, following the heuristic in Acemoglu, Antràs and Helpman (2007). Suppose that the number of input varieties in industry k is finite and equal to $\mathcal{L}$. Each input variety is of measure $\epsilon = 1/\mathcal{L}$, so that the input varieties continue to span the unit interval. With a slight abuse of notation, we use l to refer to input varieties, where $0 \leq l \leq \mathcal{L}$. We seek to compute the incremental contribution to firm revenue arising from the successful delivery of $\tilde{x}_j^k(\phi; l)$ for the input variety l; for this computation, the assembly labor input of the firm, $L_j(\phi)$, and the headquarter and supplier services embodied in all other input varieties – those in industry k other than l, as well as all input varieties in all other industries outside of k – are taken as given.

When the services embodied in input variety l – as captured by $\tilde{x}_j^k(\phi; l)$ – are included in the production process, the value of firm revenue is equal to:

$$\widetilde{r}_{\text{IN},j}^k(\phi; l) = A_j^{1-\rho} \phi^\rho L_j(\phi)^{\alpha\rho} \left[\prod_{k' \neq k} \left(X_j^{k'}(\phi) \right)^{\eta^{k'}(1-\alpha)\rho} \right] \times \left[\left(\sum_{l' \neq l} \tilde{x}_j^k(\phi; l')^{\rho^k} \epsilon' \right) + \left[\left(h_j^k(\phi; l) \right)^{\alpha^k} \left(x_j^k(\phi; l) \right)^{1-\alpha^k} \right]^{\rho^k} \epsilon \right]^{\frac{\eta^k (1-\alpha)\rho}{\rho^k}}. \quad \text{(B.1)}$$

On the other hand, the value of firm revenue without input variety l is:

$$\widetilde{r}_{\text{OUT},j}^k(\phi; l) = A_j^{1-\rho} \phi^\rho L_j(\phi)^{\alpha\rho} \left[\prod_{k' \neq k} \left(X_j^{k'}(\phi) \right)^{\eta^{k'}(1-\alpha)\rho} \right] \left[\sum_{l' \neq l} \tilde{x}_j^k(\phi; l')^{\rho^k} \epsilon' \right]^{\frac{\eta^k (1-\alpha)\rho}{\rho^k}}. \quad \text{(B.2)}$$

Combining (B.1) and (B.2), the incremental revenue generated by input variety l is:

$$\begin{aligned}
\widetilde{r}(l; \epsilon) &= \widetilde{r}_{\text{IN}} - \widetilde{r}_{\text{OUT}} \\
&= A_j^{1-\rho} \phi^\rho L_j(\phi)^{\alpha\rho} \left[\prod_{k' \neq k} \left(X_j^{k'}(\phi) \right)^{\eta^{k'}(1-\alpha)\rho} \right] \times \\
&\left\{ \left[\left(\sum_{l' \neq l} \tilde{x}_j^k(\phi; l')^{\rho^k} \epsilon' \right) + \left[\left(h_j^k(\phi; l) \right)^{\alpha^k} \left(x_j^k(\phi; l) \right)^{1-\alpha^k} \right]^{\rho^k} \epsilon \right]^{\frac{\eta^k (1-\alpha)\rho}{\rho^k}} - \left[\left(\sum_{l' \neq l} \tilde{x}_j^k(\phi; l')^{\rho^k} \epsilon' \right) \right]^{\frac{\eta^k (1-\alpha)\rho}{\rho^k}} \right\}.
\end{aligned}$$

We approximate the above term in the curly braces via a first-order Taylor expansion about $\epsilon = 0$.

Bearing in mind that $\widetilde{r}(l;0)=0$, we have:

$$\begin{aligned}\frac{\widetilde{r}(l;\epsilon)}{\epsilon} &\approx \frac{\widetilde{r}(l;0)}{\epsilon}+\left.\frac{\partial\widetilde{r}(l;\epsilon)}{\partial\epsilon}\right|_{\epsilon=0}\\
&= A_j^{1-\rho}\phi^\rho L_j(\phi)^{\alpha\rho}\left[\prod_{k'\neq k}\left(X_j^{k'}(\phi)\right)^{\eta^{k'}(1-\alpha)\rho}\right]\\
&\times\left[\left(\sum_{l'\neq l}\tilde{x}_j^k(\phi;l')^{\rho^k}\epsilon'\right)+\left[\left(h_j^k(\phi;l)\right)^{\alpha^k}\left(x_j^k(\phi;l)\right)^{1-\alpha^k}\right]^{\rho^k}\epsilon\right]^{\frac{\eta^k(1-\alpha)\rho}{\rho^k}-1}\\
&\times\left(\frac{\eta^k(1-\alpha)\rho}{\rho^k}\right)\left[\left(h_j^k(\phi;l)\right)^{\alpha^k}\left(x_j^k(\phi;l)\right)^{1-\alpha^k}\right]^{\rho^k}.\end{aligned}$$

As $\mathcal{L}\to\infty$, we have $\epsilon=1/\mathcal{L}\to 0$, and the summation term in the above equation becomes a Riemann integral. The incremental revenue contribution from input variety l is thus:

$$\begin{aligned}r_j^k(\phi;l)=\lim_{\mathcal{L}\to\infty}\frac{\widetilde{r}(\epsilon)}{\epsilon} &= A_j^{1-\rho}\phi^\rho L_j(\phi)^{\alpha\rho}\left[\prod_{k'\neq k}\left(X_j^{k'}(\phi)\right)^{\eta^{k'}(1-\alpha)\rho}\right]\times\\
&\left[\int_{l'=0}^1\tilde{x}_j^k(\phi;l')^{\rho^k}dl'\right]^{\frac{\eta^k(1-\alpha)\rho}{\rho^k}-1}\left(\frac{\eta^k(1-\alpha)\rho}{\rho^k}\right)\left[\left(h_j^k(\phi;l)\right)^{\alpha^k}\left(x_j^k(\phi;l)\right)^{1-\alpha^k}\right]^{\rho^k}\\
&= A_j^{1-\rho}\phi^\rho L_j(\phi)^{\alpha\rho}\left[\prod_{k'=1}^K\left(X_j^{k'}(\phi)\right)^{\eta^{k'}(1-\alpha)\rho}\right]\times\\
&\left(X_j^k(\phi)\right)^{-\rho^k}\left(\frac{\eta^k(1-\alpha)\rho}{\rho^k}\right)\left[\left(h_j^k(\phi;l)\right)^{\alpha^k}\left(x_j^k(\phi;l)\right)^{1-\alpha^k}\right]^{\rho^k}\\
&= (1-\alpha)\frac{\rho\eta^k}{\rho^k}R_j(\phi)\left(\frac{\tilde{x}_j^k(\phi;l)}{X_j^k(\phi)}\right)^{\rho^k}.\end{aligned}$$

This yields the expression reported earlier in equation (11) in the main text.

Noncontractible tasks: We solve first for the supplier's choice of $x_j^k(\iota;\phi,l)$ for all input tasks $\iota\in(\mu_{xij}^k,1]$, that would maximize the supplier's payoff given in (13). Note that in this first step, the supplier takes the investment levels specified for all contractible tasks $\iota\in[0,\mu_{xij}^k]$ as given, these being enforceable terms from the ex-ante contract; the supplier also takes the investment levels for all headquarter service tasks, $h_j^k(\iota;\phi,l)$ as given.

For $\iota\in(\mu_{xij}^k,1]$, the first-order condition with respect to $x_j^k(\iota;\phi,l)$ simplifies to:

$$\begin{aligned}x_j^k(\iota;\phi,l)=\left(1-\beta_{ij\chi}^k\right)(1-\alpha)\frac{\rho\eta^k R_j(\phi)}{\left(X_j^k(\phi)\right)^{\rho^k}}&\left[\left(h_j^k(\phi;l)\right)^{\alpha^k}\left(\exp\left[\int_0^{\mu_{xij}^k}\log x_j^k(\iota';\phi,l)d\iota'\right]\right)^{1-\alpha^k}\right]^{\rho^k}\\
\times\left(1-\alpha^k\right)\left(\exp\left[\int_{\mu_{xij}^k}^1\log x_j^k(\iota';\phi,l)d\iota'\right]\right)^{(1-\alpha^k)\rho^k}&\frac{1}{c_{ij\chi}^k(\phi;l)}.\end{aligned}$$

Inspecting this last equation, note that the right-hand side does not depend on the specific identity of the noncontractible task input ι that is being considered. It follows that the supplier for input variety l will choose an identical investment level for each of the noncontractible supplier tasks, so that $x_j^k(\iota;\phi,l) = x_{nj}^k(\phi,l)$ for all $\iota \in (\mu_{xij}^k, 1]$. This follows in particular from the inherent symmetry across noncontractible supplier tasks in the production function in (6). We now substitute this observation back into the first-order condition in the above equation. After some algebraic simplification, this yields:

$$\left(x_{nj}^k(\phi;l)\right)^{1-\rho^k(1-\alpha^k)(1-\mu_{xij}^k)} = \frac{(1-\alpha)\rho\eta^k R_j(\phi)}{\left(X_j^k(\phi)\right)^{\rho^k}} \left(\frac{\left(1-\alpha^k\right)\left(1-\beta_{ij\chi}^k\right)}{c_{ij\chi}^k(\phi;l)}\right) \times \left(\exp\left[\int_0^{\mu_{xij}^k} \log x_j^k(\iota';\phi,l)d\iota'\right]\right)^{\rho^k\left(1-\alpha^k\right)} \left[h_j^k(\phi;l)\right]^{\rho^k\alpha^k}, \tag{B.3}$$

which is an expression for the supplier's investment level in her noncontractible tasks ($\iota \in (\mu_{xij}^k, 1]$), conditional on the headquarter task inputs and the pre-specified investments for the contractible supplier tasks ($\iota \in [0, \mu_{xij}^k]$).

Turning to the firm's problem of choosing $h_j^k(\iota;\phi,l)$ for all $\iota \in (\mu_{hij}^k, 1]$, the corresponding first-order condition based on the objective function in (12) is:

$$h_j^k(\iota;\phi,l) = \beta_{ij\chi}^k \frac{(1-\alpha)\rho\eta^k R_j(\phi)}{\left(X_j^k\right)^{\rho^k}} \left[\left(x_j^k(\phi;l)\right)^{1-\alpha^k} \left(\exp\left[\int_0^{\mu_{hij}^k} \log h_j^k(\iota';\phi,l)d\iota'\right]\right)^{\alpha^k}\right]^{\rho^k} \times \alpha^k \left(\exp\left[\int_{\mu_{hij}^k}^1 \log h_j^k(\iota';\phi,l)d\iota'\right]\right)^{\alpha^k\rho^k} \frac{1}{s_j}.$$

As with the noncontractible supplier tasks, the investment level that the firm makes in each of its noncontractible headquarter tasks is identical. In other words, $h_j^k(\iota;\phi,l) = h_{nj}^k(\phi,l)$ for all $\iota \in (\mu_{hij}^k, 1]$. (Once again, this is a consequence of the symmetry across noncontractible headquarter tasks in the production function in (5).) We substitute this property into the above first-order condition, and simplify to obtain:

$$\left(h_{nj}^k(\phi;l)\right)^{1-\rho^k\alpha^k(1-\mu_{hij}^k)} = \frac{(1-\alpha)\rho\eta^k R_j(\phi)}{\left(X_j^k(\phi)\right)^{\rho^k}} \left(\frac{\alpha^k\beta_{ij\chi}^k}{s_j}\right) \times \left(\exp\left[\int_0^{\mu_{hij}^k} \log h_j^k(\iota';\phi,l)d\iota'\right]\right)^{\rho^k\alpha^k} \left[x_j^k(\phi;l)\right]^{\rho^k(1-\alpha^k)}. \tag{B.4}$$

This is an expression for the firm's investment level in each noncontractible headquarter task ($\iota \in (\mu_{hij}^k, 1]$), conditional on the supplier task inputs and the pre-specified investments for the contractible headquarter tasks ($\iota \in [0, \mu_{hij}^k]$).

We now simplify (B.4) and (B.3), in order to express $h_{nj}^k(\phi,l)$ and $x_{nj}^k(\phi,l)$ as functions just of contractible task investment levels. Dividing B.4) by (B.3), one obtains:

$$\frac{h^k_{nj}(\phi;l)}{x^k_{nj}(\phi;l)} = \frac{\alpha^k}{1-\alpha^k}\frac{\beta^k_{ij\chi}}{1-\beta^k_{ij\chi}}\frac{c^k_{ij\chi}(\phi;l)}{s_j}.$$

We then substitute the implied expression for $h^k_{nj}(\phi;l)$ from the above ratio back into equation (B.3). After simplifying, this yields:

$$\left(x^k_{nj}(\phi;l)\right)^{\zeta^k_{ij}} = \frac{(1-\alpha)\rho\eta^k R_j(\phi)}{\left(X^k_j(\phi)\right)^{\rho^k}} \times \left(\exp\left[\int_0^{\mu^k_{hij}} \log h^k_j(\iota;\phi,l)d\iota\right]\right)^{\rho^k\alpha^k} \left(\exp\left[\int_0^{\mu^k_{xij}} \log x^k_j(\iota;\phi,l)d\iota\right]\right)^{\rho^k\left(1-\alpha^k\right)}$$
$$\times \left(\frac{\alpha^k\beta^k_{ij\chi}}{s_j}\right)^{\rho^k\alpha^k\left(1-\mu^k_{hij}\right)} \left(\frac{\left(1-\alpha^k\right)\left(1-\beta^k_{ij\chi}\right)}{c^k_{ij\chi}(\phi;l)}\right)^{1-\rho^k\alpha^k\left(1-\mu^k_{hij}\right)}, \tag{B.5}$$

where:

$$\zeta^k_{ij} = 1-\rho^k\alpha^k\left(1-\mu^k_{hij}\right)-\rho^k\left(1-\alpha^k\right)\left(1-\mu^k_{xij}\right)$$

as defined in equation (23) in the main text. Performing an analogous substitution into equation (B.4), it is straightforward to show that for the noncontractible headquarter tasks, we have:

$$\left(h^k_{nj}(\phi;l)\right)^{\zeta^k_{ij}} = \frac{(1-\alpha)\rho\eta^k R_j(\phi)}{\left(X^k_j(\phi)\right)^{\rho^k}} \times \left(\exp\left[\int_0^{\mu^k_{hij}} \log h^k_j(\iota;\phi,l)d\iota\right]\right)^{\rho^k\alpha^k} \left(\exp\left[\int_0^{\mu^k_{xij}} \log x^k_j(\iota;\phi,l)d\iota\right]\right)^{\rho^k\left(1-\alpha^k\right)}$$
$$\times \left(\frac{\alpha^k\beta^k_{ij\chi}}{s_j}\right)^{1-\rho^k(1-\alpha^k)\left(1-\mu^k_{xij}\right)} \left(\frac{\left(1-\alpha^k\right)\left(1-\beta^k_{ij\chi}\right)}{c^k_{ij\chi}(\phi;l)}\right)^{\rho^k(1-\alpha^k)\left(1-\mu^k_{xij}\right)} \tag{B.6}$$

To sum up, (B.6) and (B.5) are the noncontractible task investment levels for headquarter and supplier tasks, expressed as a function of the contractible task investment levels.

Contractible input tasks: Working backwards, we are now in a position to solve for the investment levels in contractible task inputs that would be spelled out by the firm in the initial contract. These contractible input levels are chosen to maximize the joint surplus from the bilateral interaction with the supplier for input variety l; this is bearing in mind that the firm has the ability to extract all of the supplier surplus through the ex-ante participation fee when the contract is initially posted. The payoff to the firm from this bilateral interaction, $F^k_{ij}(\phi;l)$, is given by (16) in the main text. We substitute into (16): (i) the expression for $h^k_{nj}(\phi;l)$ from (B.6), in place of $h^k_j(\iota;\phi,l)$ for all $\iota \in [0,\mu^k_{hij}]$; and (ii) the expression for $x^k_{nj}(\phi;l)$ from (B.5), in place of $x^k_j(\iota;\phi,l)$ for all $\iota \in [0,\mu^k_{xij}]$. After some algebra,

$F_{ij}^{k}(\phi;l)$ can be re-written as:

$$\frac{1}{\rho^k}\zeta_{ij\chi}^k\left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X_j^k(\phi))^{\rho^k}}\right)^{\frac{1}{\zeta_{ij}^k}}\times\left(\exp\left[\int_0^{\mu_{hij}^k}\log h_j^k(\iota;\phi,l)d\iota\right]\right)^{\frac{\rho^k\alpha^k}{\zeta_{ij}^k}}\left(\exp\left[\int_0^{\mu_{xij}^k}\log x_j^k(\iota;\phi,l)d\iota\right]\right)^{\frac{\rho^k\left(1-\alpha^k\right)}{\zeta_{ij}^k}}$$

$$\times\left(\frac{\alpha^k\beta_{ij\chi}^k}{s_j}\right)^{\frac{\rho^k\alpha^k(1-\mu_{hij}^k)}{\zeta_{ij}^k}}\left(\frac{\left(1-\alpha^k\right)\left(1-\beta_{ij\chi}^k\right)}{c_{ij\chi}^k(\phi;l)}\right)^{\frac{\rho^k\left(1-\alpha^k\right)\left(1-\mu_{xij}^k\right)}{\zeta_{ij}^k}}$$

$$-s_j\int_0^{\mu_{hij}^k}h_j^k(\iota;\phi,l)d\iota-c_{ij\chi}^k(\phi;l)\int_0^{\mu_{xij}^k}x_j^k(\iota;\phi,l)d\iota. \tag{B.7}$$

Recall that we defined $\zeta_{ij\chi}^k$ in the main text in equation (22) as:

$$\zeta_{ij\chi}^k=1-\rho^k\alpha^k\left(1-\mu_{hij}^k\right)\beta_{ij\chi}^k-\rho^k\left(1-\alpha^k\right)\left(1-\mu_{xij}^k\right)\left(1-\beta_{ij\chi}^k\right).$$

Using the expression for $F_{ij}^k(\phi;l)$ from (B.7), we now take the first-order condition with respect to $x_j^k(\iota;\phi,l)$ for $\iota\in[0,\mu_{xij}^k]$:

$$x_j^k(\iota;\phi,l)=(1-\alpha^k)\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X_j^k(\phi))^{\rho^k}}\right)^{\frac{1}{\zeta_{ij}^k}}$$

$$\times\left(\exp\left[\int_0^{\mu_{hij}^k}\log h_j^k(\iota';\phi,l)d\iota'\right]\right)^{\frac{\rho^k\alpha^k}{\zeta_{ij}^k}}\left(\exp\left[\int_0^{\mu_{xij}^k}\log x_j^k(\iota';\phi,l)d\iota'\right]\right)^{\frac{\rho^k\left(1-\alpha^k\right)}{\zeta_{ij}^k}}$$

$$\times\left(\frac{\alpha^k\beta_{ij\chi}^k}{s_j}\right)^{\frac{\rho^k\alpha^k(1-\mu_{hij}^k)}{\zeta_{ij}^k}}\left(\frac{\left(1-\alpha^k\right)\left(1-\beta_{ij\chi}^k\right)}{c_{ij\chi}^k(\phi;l)}\right)^{\frac{\rho^k\left(1-\alpha^k\right)\left(1-\mu_{xij}^k\right)}{\zeta_{ij}^k}}\frac{1}{c_{ij\chi}^k(\phi;l)}.$$

Inspecting the right-hand side of this previous equation, we find once again that the headquarter task investment levels will be equal for all $\iota\in[0,\mu_{xij}^k]$. In other words, we have: $x_j^k(\iota;\phi,l)=x_{cj}^k(\phi,l)$ for all $\iota\in[0,\mu_{xij}^k]$. Substituting this property back into the above first-order condition and simplifying, we can solve for $x_{cj}^k(\phi,l)$ as:

$$\left(x_{cj}^k(\phi,l)\right)^{\frac{1-\rho^k\alpha^k(1-\mu_{hij}^k)}{\zeta_{ij}^k}}=(1-\alpha^k)\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X_j^k(\phi))^{\rho^k}}\right)^{\frac{1}{\zeta_{ij}^k}}\times\left(\exp\left[\int_0^{\mu_{hij}^k}\log h_j^k(\iota;\phi,l)d\iota\right]\right)^{\frac{\rho^k\alpha^k}{\zeta_{ij}^k}}$$

$$\times\left(\frac{\alpha^k\beta_{ij\chi}^k}{s_j}\right)^{\frac{\rho^k\alpha^k(1-\mu_{hij}^k)}{\zeta_{ij}^k}}\left(\frac{\left(1-\alpha^k\right)\left(1-\beta_{ij\chi}^k\right)}{c_{ij\chi}^k(\phi;l)}\right)^{\frac{\rho^k\left(1-\alpha^k\right)\left(1-\mu_{xij}^k\right)}{\zeta_{ij}^k}}\frac{1}{c_{ij\chi}^k(\phi;l)}. \tag{B.8}$$

Next, we use the expression $F_{ij}^k(\phi;l)$ from (B.7), and take the first-order condition for the contractible headquarter tasks, i.e., $h_j^k(\iota;\phi,l)$ for all $\iota \in [0,\mu_{hij}^k]$. This yields:

$$\begin{aligned} h_j^k(\iota;\phi,l) = \alpha^k \frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X_j^k(\phi))^{\rho^k}}\right)^{\frac{1}{\zeta_{ij}^k}} \\ \times\left(\exp\left[\int_0^{\mu_{hij}^k}\log h_j^k(\iota;\phi,l)d\iota\right]\right)^{\frac{\rho^k\alpha^k}{\zeta_{ij}^k}}\left(\exp\left[\int_0^{\mu_{xij}^k}\log x_j^k(\iota;\phi,l)d\iota\right]\right)^{\frac{\rho^k\left(1-\alpha^k\right)}{\zeta_{ij}^k}} \\ \times\left(\frac{\alpha^k\beta_{ij\chi}^k}{s_j}\right)^{\frac{\rho^k\alpha^k(1-\mu_{hij}^k)}{\zeta_{ij}^k}}\left(\frac{\left(1-\alpha^k\right)\left(1-\beta_{ij\chi}^k\right)}{c_{ij\chi}^k(\phi;l)}\right)^{\frac{\rho^k\left(1-\alpha^k\right)\left(1-\mu_{xij}^k\right)}{\zeta_{ij}^k}}\frac{1}{s_j}, \end{aligned}$$

from which we conclude again that the investment levels that are specified for all contractible headquarter tasks are identical, i.e., $h_{cj}^k(\phi,l) = h_j^k(\iota;\phi,l)$ for all $\iota \in [0,\mu_{hij}^k]$ due to the symmetry across contractible headquarter tasks. We substitute this property back into the above first-order condition to obtain:

$$\begin{aligned} \left(h_{cj}^k(\phi,l)\right)^{\frac{1-\rho^k(1-\alpha^k)(1-\mu_{xij}^k)}{\zeta_{ij}^k}} &= \alpha^k \frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X_j^k(\phi))^{\rho^k}}\right)^{\frac{1}{\zeta_{ij}^k}} \times\left(\exp\left[\int_0^{\mu_{xij}^k}\log x_j^k(\iota;\phi,l)d\iota\right]\right)^{\frac{\rho^k(1-\alpha^k)}{\zeta_{ij}^k}} \\ &\quad\times\left(\frac{\alpha^k\beta_{ij\chi}^k}{s_j}\right)^{\frac{\rho^k\alpha^k(1-\mu_{hij}^k)}{\zeta_{ij}^k}}\left(\frac{\left(1-\alpha^k\right)\left(1-\beta_{ij\chi}^k\right)}{c_{ij\chi}^k(\phi;l)}\right)^{\frac{\rho^k\left(1-\alpha^k\right)\left(1-\mu_{xij}^k\right)}{\zeta_{ij}^k}}\frac{1}{s_j}. \end{aligned} \tag{B.9}$$

To solve out fully for $h_{cj}^k(\phi,l)$ and $x_{cj}^k(\phi,l)$, we divide (B.9) by (B.8):

$$\frac{h_{cj}^k(\phi;l)}{x_{cj}^k(\phi;l)} = \frac{\alpha^k}{1-\alpha^k}\frac{c_{ij\chi}^k(\phi;l)}{s_j}.$$

We take the expression for $h_{cj}^k(\phi;l)$ as a function of $x_{cj}^k(\phi;l)$ implied by the above ratio, and substitute this back into the first-order conditions in (B.9) and (B.8). After some algebra, this yields:

$$\begin{aligned} \left(x_{cj}^k(\phi,l)\right)^{1-\rho^k} &= \left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X_j^k(\phi))^{\rho^k}}\right)\left(\frac{\alpha^k}{s_j}\right)^{\rho^k\alpha^k}\left(\frac{1-\alpha^k}{c_{ij\chi}^k(\phi;l)}\right)^{1-\rho^k\alpha^k} \\ &\quad\times\left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{\zeta_{ij}^k}\left(\beta_{ij\chi}^k\right)^{\rho^k\alpha^k(1-\mu_{hij}^k)}\left(1-\beta_{ij\chi}^k\right)^{\rho^k\left(1-\alpha^k\right)\left(1-\mu_{xij}^k\right)}, \text{ and} \end{aligned} \tag{B.10}$$

$$\begin{aligned} \left(h_{cj}^k(\phi,l)\right)^{1-\rho^k} &= \left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X_j^k(\phi))^{\rho^k}}\right)\left(\frac{\alpha^k}{s_j}\right)^{1-\rho^k(1-\alpha^k)}\left(\frac{1-\alpha^k}{c_{ij\chi}^k(\phi;l)}\right)^{\rho^k(1-\alpha^k)} \\ &\quad\times\left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{\zeta_{ij}^k}\left(\beta_{ij\chi}^k\right)^{\rho^k\alpha^k(1-\mu_{hij}^k)}\left(1-\beta_{ij\chi}^k\right)^{\rho^k\left(1-\alpha^k\right)\left(1-\mu_{xij}^k\right)}. \end{aligned} \tag{B.11}$$

It is straightforward to see that the expressions we have derived for $h_{cj}^k(\phi,l)$ in (B.11) and for

$x^k_{cj}(\phi, l)$ in (B.10) correspond to those reported in the main paper in equations (17) and (18), bearing in mind the definition for $\Xi^k_{ij\chi}$ introduced in (21). We next substitute from (B.11) and (B.10) into (B.6) and (B.5); after some simplification, this leads to the expressions for $h^k_{nj}(\phi, l)$ in (19) and $x^k_{nj}(\phi, l)$ in (20) from the main text.

We can now use these expressions for $h^k_{cj}(\phi, l)$, $x^k_{cj}(\phi, l)$, $h^k_{nj}(\phi, l)$, and $x^k_{nj}(\phi, l)$, and substitute these back into (B.7). The contribution to the firm's payoff that comes from its interaction with the supplier of input variety l can then be re-expressed as:

$$F^k_{ij}(\phi; l) = \left(\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X^k_j(\phi))^{\rho^k}}\right)^{\frac{1}{1-\rho^k}} \times \frac{1-\rho^k}{\rho^k} \left(\frac{\alpha^k}{s_j}\right)^{\frac{\rho^k\alpha^k}{1-\rho^k}} \left(\frac{1-\alpha^k}{c^k_{ij\chi}(\phi; l)}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}$$
$$\times \left(\frac{\zeta^k_{ij\chi}}{\zeta^k_{ij}}\right)^{\frac{\zeta^k_{ij}}{1-\rho^k}} \left(\beta^k_{ij\chi}\right)^{\frac{\rho^k\alpha^k(1-\mu^k_{hij})}{1-\rho^k}} \left(1-\beta^k_{ij\chi}\right)^{\frac{\rho^k(1-\alpha^k)(1-\mu^k_{xij})}{1-\rho^k}},$$

which is equation (24) in the main text. This expresses the firm's payoff in terms of the fundamental parameters of the model and factor prices.

Sourcing shares: Let the set of possible sourcing modes be Ω, i.e., $\Omega = \{(i', \chi') : i' \in \{1, \ldots, N\}, \chi' \in \{V, O\}\}$. The share of inputs for which (i, χ) is the sourcing mode, $\pi^k_{ij\chi}$, is evaluated explicitly as:

$$\begin{aligned} \pi^k_{ij\chi} &= Pr\left(Z^k_{i'j\chi'} \leq \frac{\Xi^k_{ij\chi}}{\Xi^k_{i'j\chi'}} Z^k_{ij\chi},\ \forall\ (i,\chi) \in \Omega\right) \\ &= \int_{z=0}^{\infty} Pr\left(Z^k_{ij\chi} = z\ \&\ Z^k_{i'j\chi'} \leq \frac{\Xi^k_{ij\chi}}{\Xi^k_{i'j\chi'}} z,\ \forall\ (i,\chi) \neq (i', \chi')\right)\ dz. \end{aligned} \tag{B.12}$$

We can evaluate the above probability by differentiating the joint distribution on the right-hand side of (15) with respect to $z^k_{ij\chi}$, setting $z^k_{ij\chi} = z$ and $z^k_{i'j\chi'} = \frac{\Xi^k_{ij\chi}}{\Xi^k_{i'j\chi'}} z$ for all $(i', \chi') \neq (i, \chi)$, and then integrating over all possible values of z. It follows that the probability in (B.12) is equal to:

$$
\begin{aligned}
& \int_{z=0}^{\infty} \exp\left\{-\sum_{i'=1}^{J} T_{i'}^{k}\left(\left(\frac{\Xi_{ij\chi}^{k}}{\Xi_{i'jV}^{k}}z\right)^{-\frac{\theta^k}{1-\lambda_{i'}}}+\left(\frac{\Xi_{ij\chi}^{k}}{\Xi_{i'jO}^{k}}z\right)^{-\frac{\theta^k}{1-\lambda_{i'}}}\right)^{1-\lambda_{i'}}\right\} \\
& \quad \times\left\{T_{i}^{k}\left(\left(\frac{\Xi_{ij\chi}^{k}}{\Xi_{ijV}^{k}}z\right)^{-\frac{\theta^k}{1-\lambda_{i}}}+\left(\frac{\Xi_{ij\chi}^{k}}{\Xi_{ijO}^{k}}z\right)^{-\frac{\theta^k}{1-\lambda_{i}}}\right)^{-\lambda_{i}}\right\} \times \theta^k z^{-\frac{\theta^k}{1-\lambda_i}-1}\, dz \\
= & \int_{z=0}^{\infty} \exp\left\{-\left(\Xi_{ij\chi}^{k}z\right)^{-\theta^k}\sum_{i'=1}^{J} T_{i'}^{k}\left(\left(\Xi_{i'jV}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i'}}}+\left(\Xi_{i'jO}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i'}}}\right)^{1-\lambda_{i'}}\right\} \\
& \quad \times\left\{T_{i}^{k}\left(\Xi_{ij\chi}^{k}\right)^{\frac{\theta^k\lambda_i}{1-\lambda_i}}\left(\left(\Xi_{ijV}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i}}}+\left(\Xi_{ijO}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i}}}\right)^{-\lambda_{i}}\right\} \times \theta^k z^{-\theta^k-1}\, dz \\
= & \frac{T_{i}^{k}\left(\left(\Xi_{ijV}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i}}}+\left(\Xi_{ijO}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i}}}\right)^{1-\lambda_{i}}}{\sum_{i'=1}^{J} T_{i'}^{k}\left(\left(\Xi_{i'jV}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i'}}}+\left(\Xi_{i'jO}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i'}}}\right)^{1-\lambda_{i'}}} \times \frac{\left(\Xi_{ij\chi}^{k}\right)^{\frac{\theta^k}{1-\lambda_i}}}{\left(\Xi_{ijV}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i}}}+\left(\Xi_{ijO}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i}}}} \\
& \quad \times\left[\exp\left\{-\left(\Xi_{ij\chi}^{k}z\right)^{-\theta^k}\sum_{i'=1}^{J} T_{i'}^{k}\left(\left(\Xi_{i'jV}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i'}}}+\left(\Xi_{i'jO}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i'}}}\right)^{1-\lambda_{i'}}\right\}\right]_{z=0}^{\infty}
\end{aligned}
$$

It is straightforward to see that the last term in square brackets above is equal to 1. It follows that $\pi_{ij\chi}^{k} = \pi_{ij} \times \pi_{\chi|ij}$, where:

$$
\begin{aligned}
\pi_{ij}^{k} &= \frac{T_{i}^{k}\left(\left(\Xi_{ijV}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i}}}+\left(\Xi_{ijO}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i}}}\right)^{1-\lambda_{i}}}{\sum_{i'=1}^{J} T_{i'}^{k}\left(\left(\Xi_{i'jV}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i'}}}+\left(\Xi_{i'jO}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i'}}}\right)^{1-\lambda_{i'}}}, \text{ and} \\
\pi_{\chi|ij}^{k} &= \frac{\left(\Xi_{ij\chi}^{k}\right)^{\frac{\theta^k}{1-\lambda_i}}}{\left(\Xi_{ijV}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i}}}+\left(\Xi_{ijO}^{k}\right)^{\frac{\theta^k}{1-\lambda_{i}}}}.
\end{aligned}
$$

Recalling the definitions of $\Xi_{ij\chi}^{k}$ from (21), $B_{ij\chi}^{k}$ from (29), and B_{ij}^{k} from (30), the above expressions for π_{ij}^{k} and $\pi_{\chi|ij}^{k}$ simplify to those reported in (27) and (28) respectively in the main paper.

Properties of $B_{ij\chi}^{k}$ (Lemma 1) We show here that $B_{ij\chi}^{k}$ as defined in (29) is increasing in the contractibility of headquarter and supplier tasks. To see this, note that the sign of the derivative of

$B^k_{ij\chi}$ with respect to μ^k_{hij} is given by the sign of:

$$\rho^k(1-\alpha^k)\frac{d}{d\mu^k_{hij}}\ln B^k_{ij\chi} = \frac{d}{d\mu^k_{hij}}\left(\zeta^k_{ij}\ln\left(\frac{\zeta^k_{ij\chi}}{\zeta^k_{ij}}\right) + \rho^k\alpha^k(1-\mu^k_{hij})\ln\beta^k_{ij\chi} + \rho^k(1-\alpha^k)(1-\mu^k_{xij})\ln\left(1-\beta^k_{ij\chi}\right)\right)$$
$$= \rho^k\alpha^k\left[\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\beta^k_{ij\chi} - \ln\left(\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\beta^k_{ij\chi}\right) - 1\right].$$

Since $\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\beta^k_{ij\chi} \in [0,1]$, it thus suffices to study the behavior of the function: $x - \ln x - 1$ in the range $x \in [0,1]$. It is straightforward to show that at $x = 0$, this function takes a value that tends to $+\infty$. But as x increases towards 1, $x - \ln x - 1$ decreases until it reaches a value of 0 at $x = 1$. It follows that $x - \ln x - 1 \geq 0$ for $x \in [0,1]$ with equality if and only if $x = 1$. It follows that $B^k_{ij\chi}$ is increasing in μ^k_{hij}. The argument showing that $B^k_{ij\chi}$ is also increasing in μ^k_{xij} (holding all else constant) is analogous.

B.2 Proofs from Section 3

Composite industry-k input: From the definition of $X^k_j(\phi)^{\rho^k}$, we have:

$$X^k_j(\phi)^{\rho^k} = \mathbb{E}_l\left[\tilde{x}^k_j(\phi;l)^{\rho^k}\right]$$
$$= \sum_{i=1}^{J}\sum_{\chi\in\{V,O\}}\left(\frac{\alpha^k\left(\beta^k_{ij\chi}\right)^{1-\mu^k_{hij}}}{s_j}\right)^{\rho^k\alpha^k}\left(\frac{(1-\alpha^k)\left(1-\beta^k_{ij\chi}\right)^{1-\mu^k_{xij}}}{d^k_{ij}w_i}\right)^{\rho^k(1-\alpha^k)}\left(\frac{\rho^k}{1-\rho^k}\right)^{\rho^k}$$
$$\times\left(\frac{\zeta^k_{ij\chi}}{\zeta^k_{ij}}\right)^{\zeta^k_{ij}-1}\left(\Xi^k_{ij\chi}\right)^{\frac{(\rho^k)^2(1-\alpha^k)}{1-\rho^k}}\mathbb{E}_l\left[\tilde{Z}^k_{ij\chi}(\phi;l)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\right]. \quad \text{(B.13)}$$

Note that for the above, we have substituted in the definitions from (5) and (6) into (4), and further made use of the expressions for the optimal task investment levels in (17)-(20). In this last line of (B.13, $\tilde{Z}^k_{ij\chi}(\phi;l)$ is the realized supplier productivity level associated with the optimal sourcing mode for input variety l, i.e., the (i,χ) that solves (25).

For convenience, define: $\bar{z}^k_{ij\chi} \equiv \mathbb{E}_l[\tilde{Z}_{ij\chi}(\phi;l)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}]$, since this expression will appear repeatedly in the computations below. We have:

$$\bar{z}^k_{ij\chi} = \int_{z=0}^{\infty} z^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\exp\left\{-\left(\Xi^k_{ij\chi}z\right)^{-\theta^k}\sum_{i'=1}^{J}T^k_{i'}\left(\left(\Xi^k_{i'jV}\right)^{\frac{\theta^k}{1-\lambda_{i'}}}+\left(\Xi^k_{i'jO}\right)^{\frac{\theta^k}{1-\lambda_{i'}}}\right)^{1-\lambda_{i'}}\right\}$$
$$\times\left\{T^k_i\left(\Xi^k_{ij\chi}\right)^{\frac{\theta^k\lambda_i}{1-\lambda_i}}\left(\left(\Xi^k_{ijV}\right)^{\frac{\theta^k}{1-\lambda_i}}+\left(\Xi^k_{ijO}\right)^{\frac{\theta^k}{1-\lambda_i}}\right)^{-\lambda_i}\right\}\times\theta^k z^{-\theta^k-1}\,dz.$$

We evaluate the above integral by performing the change of variables: $y = \left(\Xi^k_{ij\chi}z\right)^{-\theta^k}v^k_j$, where

$\upsilon_j^k \equiv \sum_{i'=1}^{J} T_{i'}^k \left(\left(\Xi_{i'jV}^k\right)^{\frac{\theta^k}{1-\lambda_{i'}}} + \left(\Xi_{i'jO}^k\right)^{\frac{\theta^k}{1-\lambda_{i'}}} \right)^{1-\lambda_{i'}}$. This yields:

$$
\begin{aligned}
\bar{z}_{ij\chi}^k &= \int_{z=0}^{\infty} \exp\{-y\} \times \left\{ T_i^k \left(\Xi_{ij\chi}^k\right)^{\frac{\theta^k \lambda_i}{1-\lambda_i}} \left(\left(\Xi_{ijV}^k\right)^{\frac{\theta^k}{1-\lambda_i}} + \left(\Xi_{ijO}^k\right)^{\frac{\theta^k}{1-\lambda_i}} \right)^{-\lambda_i} \right\} \times \theta^k z^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k} - \theta^k - 1} dz \\
&= \int_{y=\infty}^{0} \exp\{-y\} \times \left\{ T_i^k \left(\Xi_{ij\chi}^k\right)^{\frac{\theta^k \lambda_i}{1-\lambda_i}} \left(\left(\Xi_{ijV}^k\right)^{\frac{\theta^k}{1-\lambda_i}} + \left(\Xi_{ijO}^k\right)^{\frac{\theta^k}{1-\lambda_i}} \right)^{-\lambda_i} \right\} \\
&\qquad \times \theta^k \left[\left(\frac{y}{\upsilon_j^k}\right)^{-\frac{1}{\theta^k}} \frac{1}{\Xi_{ij\chi}^k} \right]^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k} - \theta^k - 1} d\left[\left(\frac{y}{\upsilon_j^k}\right)^{-\frac{1}{\theta^k}} \frac{1}{\Xi_{ij\chi}^k} \right] \\
&= \int_{y=0}^{\infty} \exp\{-y\} \times \left\{ T_i^k \left(\Xi_{ij\chi}^k\right)^{\frac{\theta^k \lambda_i}{1-\lambda_i}} \left(\left(\Xi_{ijV}^k\right)^{\frac{\theta^k}{1-\lambda_i}} + \left(\Xi_{ijO}^k\right)^{\frac{\theta^k}{1-\lambda_i}} \right)^{-\lambda_i} \right\} \\
&\qquad \times \left(\frac{y}{\upsilon_j^k}\right)^{-\frac{\rho^k(1-\alpha^k)}{\theta^k(1-\rho^k)}} \left(\frac{1}{\Xi_{ij\chi}^k}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k} - \theta^k} \frac{1}{\upsilon_j^k}\, dy \\
&= \left(\frac{1}{\Xi_{ij\chi}^k}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k} - \theta^k} \left(\frac{1}{\upsilon_j^k}\right)^{-\frac{\rho^k(1-\alpha^k)}{\theta^k(1-\rho^k)} + 1} \times \left\{ T_i^k \left(\Xi_{ij\chi}^k\right)^{\frac{\theta^k \lambda_i}{1-\lambda_i}} \left(\left(\Xi_{ijV}^k\right)^{\frac{\theta^k}{1-\lambda_i}} + \left(\Xi_{ijO}^k\right)^{\frac{\theta^k}{1-\lambda_i}} \right)^{-\lambda_i} \right\} \\
&\qquad \times \int_{y=0}^{\infty} \exp\{-y\}\, y^{-\frac{\rho^k(1-\alpha^k)}{\theta^k(1-\rho^k)}}\, dy \\
&= \left(\Xi_{ij\chi}^k\right)^{\frac{\theta^k}{1-\lambda_i} - \frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \times \frac{T_i^k \left(\left(\Xi_{ijV}^k\right)^{\frac{\theta^k}{1-\lambda_i}} + \left(\Xi_{ijO}^k\right)^{\frac{\theta^k}{1-\lambda_i}} \right)^{-\lambda_i}}{\left(\upsilon_j^k\right)^{-\frac{\rho^k(1-\alpha^k)}{\theta^k(1-\rho^k)} + 1}} \times \Gamma\left(1 - \frac{\rho^k(1-\alpha^k)}{\theta^k(1-\rho^k)}\right) \\
&= \Gamma\left(1 - \frac{\rho^k(1-\alpha^k)}{\theta^k(1-\rho^k)}\right) \left(\pi_{ij}^k\right)^{1 - \frac{1}{\theta^k} \frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \left(\pi_{\chi|ij}^k\right)^{1 - \frac{1-\lambda_i}{\theta^k} \frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \left(T_i^k\right)^{\frac{1}{\theta^k} \frac{\rho^k(1-\alpha^k)}{1-\rho^k}},
\end{aligned} \tag{B.14}
$$

where $\Gamma(\cdot)$ denotes the Gamma function, and we assume $1 - \frac{1}{\theta^k} \frac{\rho^k(1-\alpha^k)}{1-\rho^k} > 0$ in order for this function to be well-defined.

Separately, using the expressions for π_{ij}^k from (27) and $\pi_{\chi|ij}^k$ from (28), one can show that:

$$
\begin{aligned}
&\left(\pi_{ij}^k\right)^{-\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\left(\pi_{\chi|ij}^k\right)^{-\frac{1-\lambda_i}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\left(T_i^k(d_{ij}^k w_i)^{-\theta^k}\right)^{\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\\
&=\left(\frac{T_i^k(d_{ij}^k w_i)^{-\theta^k}\left((B_{ijV}^k)^{\frac{\theta^k}{1-\lambda_i}}+(B_{ijO}^k)^{\frac{\theta^k}{1-\lambda_i}}\right)^{1-\lambda_i}}{\Phi_j^k}\right)^{-\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\left(\frac{(B_{ij\chi}^k)^{\frac{\theta^k}{1-\lambda_i}}}{(B_{ijV}^k)^{\frac{\theta^k}{1-\lambda_i}}+(B_{ijO}^k)^{\frac{\theta^k}{1-\lambda_i}}}\right)^{-\frac{1-\lambda_i}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\\
&\quad\times\left(T_i^k(d_{ij}^k w_i)^{-\theta^k}\right)^{\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\\
&=\left(T_i^k(d_{ij}^k w_i)^{-\theta^k}\right)^{-\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\left((B_{ijV}^k)^{\frac{\theta^k}{1-\lambda_i}}+(B_{ijO}^k)^{\frac{\theta^k}{1-\lambda_i}}\right)^{-\frac{1-\lambda_i}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\left(\Phi_j^k\right)^{\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\\
&\quad\times\left(\frac{(B_{ij\chi}^k)^{\frac{\theta^k}{1-\lambda_i}}}{(B_{ijV}^k)^{\frac{\theta^k}{1-\lambda_i}}+(B_{ijO}^k)^{\frac{\theta^k}{1-\lambda_i}}}\right)^{-\frac{1-\lambda_i}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\left(T_i^k(d_{ij}^k w_i)^{-\theta^k}\right)^{\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\\
&=\left(\Phi_j^k\right)^{\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\left(B_{ij\chi}^k\right)^{-\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}.
\end{aligned}
$$

Combining the above equation with (B.14), it quickly follows that:

$$
\bar{z}_{ij\chi}^k=\bar{\Gamma}^k\times\pi_{ij}^k\pi_{\chi|ij}^k\left(\Phi_j^k\right)^{\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\frac{\left(d_{ij}^k w_i\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}}{\left(B_{ij\chi}^k\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}},
$$

where we define: $\bar{\Gamma}^k\equiv\Gamma\left(1-\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}\right)$.

Substituting this last expression for $\bar{z}_{ij\chi}^k$ into (B.13) and simplifying, one arrives at the expression for the $X_j^k(\phi)$ reported in the main text in equation (33).

Payoff function of the final firm. As an intermediate step, it will be useful to derive a simpler expression for $\frac{\rho^k}{1-\rho^k}\left(\Xi^k_{ij\chi}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \bar{z}^k_{ij\chi}$ that we will be using later on:

$$
\begin{aligned}
&\frac{\rho^k}{1-\rho^k}\left(\Xi^k_{ij\chi}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \bar{z}^k_{ij\chi} \\
&= \left[\frac{(1-\alpha)\rho\eta^k R_j(\phi)}{(X^k_j(\phi))^{\rho^k}}\right]^{\frac{1}{1-\rho^k}} \left(\frac{\alpha^k(\beta^k_{ij\chi})^{1-\mu^k_{hij}}}{s_j}\right)^{\frac{\rho^k\alpha^k}{1-\rho^k}} \left(\frac{(1-\alpha^k)(1-\beta^k_{ij\chi})^{1-\mu^k_{xij}}}{d^k_{ij}w_i}\right)^{\frac{\rho^k\left(1-\alpha^k\right)}{1-\rho^k}} \\
&\quad \times \frac{\rho^k}{1-\rho^k} \times \frac{1-\rho^k}{\rho^k} \times \left(\frac{\zeta^k_{ij\chi}}{\zeta^k_{ij}}\right)^{\frac{\zeta^k_j}{1-\rho^k}} \times \bar{\Gamma}^k \times \pi^k_{ij}\pi^k_{\chi|ij}\left(\Phi^k_j\right)^{\frac{1}{\theta^k}\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \frac{\left(d^k_{ij}w_i\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}}{\left(B^k_{ij\chi}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}} \\
&= \left(\frac{\zeta^k_{ij\chi}}{\zeta^k_{ij}}\right)^{\frac{\zeta^k_j}{1-\rho^k}} \Big[(1-\alpha)\rho\eta^k R_j(\phi)\Big] \Upsilon^k_j \pi^k_{ij}\pi^k_{\chi|ij} \frac{\left(1-\beta^k_{ij\chi}\right)^{\frac{\rho^k\left(1-\alpha^k\right)}{1-\rho^k}(1-\mu^k_{xij})} \left(\beta^k_{ij\chi}\right)^{\frac{\rho^k\alpha^k}{1-\rho^k}(1-\mu^k_{hij})}}{\left(B^k_{ij\chi}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}} \\
&= \Big[(1-\alpha)\rho\eta^k R_j(\phi)\Big] \Upsilon^k_j \pi^k_{ij}\pi^k_{\chi|ij}. \qquad \text{(B.15)}
\end{aligned}
$$

Turning to the overall payoff function for the firm:

$$
\begin{aligned}
F_j(\phi) &= R_j(\phi) - \sum_{k=1}^{K}\int_{l=0}^{1}\left[s_j\int_0^1 h^k_j(\iota;\phi;l)d\iota + c^k_{ij\chi}(\phi,l)\int_0^1 x^k_j(\iota;\phi,l)d\iota\right]dl - w_jL_j(\phi) \\
&= R_j(\phi) - \sum_{k=1}^{K}\int_{l=0}^{1} s_j\left[\mu^k_{hij}h^k_{cj}(\phi;l) + \left(1-\mu^k_{hij}\right)h^k_{nj}(\phi;l)\right]dl \\
&\quad - \sum_{k=1}^{K}\int_{l=0}^{1} c^k_{ij\chi}(\phi,l)\left[\mu^k_{xij}x^k_{cj}(\phi;l) + \left(1-\mu^k_{xij}\right)x^k_{nj}(\phi;l)\right]dl - w_jL_j(\phi) \qquad \text{(B.16)}
\end{aligned}
$$

Using the solution of $h^k_{cj}(\phi;l)$ and $h^k_{nj}(\phi;l)$ in equation (17) and (19), the total costs of headquarter services can be written as:

$$
\begin{aligned}
&\sum_{k=1}^{K}\int_{l=0}^{1} s_j\left[\mu^k_{hij}h^k_{cj}(\phi;l) + \left(1-\mu^k_{hij}\right)h^k_{nj}(\phi;l)\right]dl \\
&= \sum_{k=1}^{K}\sum_{i=1}^{J}\sum_{\chi=V,O}\int_{l\in\Omega^k_{ij\chi}} s_j\left[\frac{\rho^k}{1-\rho^k}\frac{\alpha^k}{s_j}\left(\Xi^k_{ij\chi}Z^k_{ij\chi}(\phi;l)\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\right]\left[\mu^k_{hij} + \left(1-\mu^k_{hij}\right)\beta^k_{ij\chi}\left(\frac{\zeta^k_{ij\chi}}{\zeta^k_{ij}}\right)^{-1}\right]dl \\
&= \sum_{k=1}^{K}\sum_{i=1}^{J}\sum_{\chi=V,O}\alpha^k\left[\mu^k_{hij} + \left(1-\mu^k_{hij}\right)\beta^k_{ij\chi}\left(\frac{\zeta^k_{ij\chi}}{\zeta^k_{ij}}\right)^{-1}\right]\frac{\rho^k}{1-\rho^k}\left(\Xi^k_{ij\chi}\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\bar{z}^k_{ij\chi}
\end{aligned}
$$

Substituting in the expression from equation (B.15), we have:

$$\begin{aligned}
&\sum_{k=1}^{K}\int_{l=0}^{1} s_j \left[\mu_{hij}^k h_{cj}^k(\phi;l) + \left(1-\mu_{hij}^k\right) h_{nj}^k(\phi;l)\right] dl \\
&= \sum_{k=1}^{K}\sum_{i=1}^{J}\sum_{\chi=V,O} \alpha^k \left[\mu_{hij}^k + \left(1-\mu_{hij}^k\right)\beta_{ij\chi}^k \left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{-1}\right] \left[(1-\alpha)\rho\eta^k R_j(\phi)\right] \Upsilon_j^k \pi_{ij}^k \pi_{\chi|ij}^k \\
&= (1-\alpha) R_j(\phi) \sum_{k=1}^{K} \rho\eta^k \Upsilon_j^k \sum_{i=1}^{J}\sum_{\chi=V,O} \alpha^k \left[\mu_{hij}^k + \left(1-\mu_{hij}^k\right)\beta_{ij\chi}^k \left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{-1}\right] \pi_{ij}^k \pi_{\chi|ij}^k \qquad \text{(B.17)}
\end{aligned}$$

Similarly, the total investment undertaken by the intermediate goods suppliers is:

$$\begin{aligned}
&\sum_{k=1}^{K}\int_{l=0}^{1} c_{ij\chi}^k(\phi,l) \left[\mu_{xij}^k x_{cj}^k(\phi;l) + \left(1-\mu_{xij}^k\right) x_{nj}^k(\phi;l)\right] dl \\
&= \sum_{k=1}^{K}\sum_{i=1}^{J}\sum_{\chi=V,O} \int_{l\in\Omega_{ij\chi}^k} c_{ij\chi}^k(\phi,l) \left[\frac{\rho^k}{1-\rho^k}\frac{1-\alpha^k}{c_{ij\chi}^k(\phi,l)} \left(\Xi_{ij\chi}^k Z_{ij\chi}^k(\phi;l)\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\right] \\
&\times \left[\mu_{xij}^k + \left(1-\mu_{xij}^k\right)\left(1-\beta_{ij\chi}^k\right)\left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{-1}\right] dl \\
&= \sum_{k=1}^{K}\sum_{i=1}^{J}\sum_{\chi=V,O} (1-\alpha^k) \left[\mu_{xij}^k + \left(1-\mu_{xij}^k\right)\left(1-\beta_{ij\chi}^k\right)\left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{-1}\right] \frac{\rho^k}{1-\rho^k}\left(\Xi_{ij\chi}^k\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \bar{z}_{ij\chi}^k \\
&= (1-\alpha) R_j(\phi) \sum_{k=1}^{K} \rho\eta^k \Upsilon_j^k \sum_{i=1}^{J}\sum_{\chi=V,O} (1-\alpha^k) \left[\mu_{xij}^k + \left(1-\mu_{xij}^k\right)\left(1-\beta_{ij\chi}^k\right)\left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{-1}\right] \pi_{ij}^k \pi_{\chi|ij}^k
\end{aligned}$$

Combining the above two expressions, the costs incurred on all relationship-specific inputs is:

$$
\begin{aligned}
&\sum_{k=1}^{K} \int_{l=0}^{1} \left[s_j \int_0^1 h_j^k(\iota;\phi;l)d\iota + c_{ij\chi}^k(\phi,l) \int_0^1 x_j^k(\iota;\phi,l)d\iota \right] dl \\
&= (1-\alpha)\, R_j(\phi) \sum_{k=1}^{K} \left(\rho\eta^k\right) \Upsilon_j^k \sum_{i=1}^{J} \sum_{\chi=V,O} \pi_{ij}^k \pi_{\chi|ij}^k \\
&\quad \times \left\{ \alpha^k \left[\mu_{hij}^k + \left(1-\mu_{hij}^k\right) \beta_{ij\chi}^k \left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{-1} \right] + \left(1-\alpha^k\right) \left[\mu_{xij}^k + \left(1-\mu_{xij}^k\right)\left(1-\beta_{ij\chi}^k\right) \left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{-1} \right] \right\} \\
&= (1-\alpha)\, R_j(\phi) \sum_{k=1}^{K} \left(\rho\eta^k\right) \Upsilon_j^k \sum_{i=1}^{J} \sum_{\chi=V,O} \pi_{ij}^k \pi_{\chi|ij}^k \\
&\quad \times \left\{ \alpha^k \mu_{hij}^k + \left(1-\alpha^k\right)\mu_{xij}^k + \left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{-1} \left[\alpha^k \left(1-\mu_{hij}^k\right)\beta_{ij\chi}^k + \left(1-\alpha^k\right)\left(1-\mu_{xij}^k\right)\left(1-\beta_{ij\chi}^k\right) \right] \right\} \\
&= (1-\alpha)\, R_j(\phi) \sum_{k=1}^{K} \left(\rho\eta^k\right) \Upsilon_j^k \sum_{i=1}^{J} \sum_{\chi=V,O} \pi_{ij}^k \pi_{\chi|ij}^k \left[\frac{\zeta_{ij}^k - (1-\rho^k)}{\rho^k} + \left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{-1} \frac{1-\zeta_{ij\chi}^k}{\rho^k} \right] \\
&= (1-\alpha)\, R_j(\phi) \sum_{k=1}^{K} \left(\frac{\rho\eta^k}{\rho^k}\right) \Upsilon_j^k \sum_{i=1}^{J} \sum_{\chi=V,O} \left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{-1} \pi_{ij}^k \pi_{\chi|ij}^k \left[\zeta_{ij\chi}^k - \left(1-\rho^k\right)\left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right) + \left(1-\zeta_{ij\chi}^k\right) \right] \\
&= (1-\alpha)\, R_j(\phi) \sum_{k=1}^{K} \left(\frac{\rho\eta^k}{\rho^k}\right) \Upsilon_j^k \sum_{i=1}^{J} \sum_{\chi=V,O} \left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{-1} \pi_{ij}^k \pi_{\chi|ij}^k \left[1 - \left(1-\rho^k\right)\left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right) \right] \\
&= (1-\alpha)\, R_j(\phi) \sum_{k=1}^{K} \frac{\rho\eta^k}{\rho^k} \left(1 - (1-\rho^k)\Upsilon_j^k\right)
\end{aligned}
$$

The third-to-last step used the definition of ζ_{ij}^k and $\zeta_{ij\chi}^k$ in equation (23) and (22). At the same time, the last step made use of the definition of Υ_j^k from (34), and the fact that the same overall sourcing shares $\sum_{i=1}^{J}\sum_{\chi=V,O} \pi_{ij}^k \pi_{\chi|ij}^k$ is equal to 1 for any given industry k. From the definitions of ζ_{ij}^k and $\zeta_{ij\chi}^k$, one can further show that:

$$
\begin{aligned}
\zeta_{ij}^k - (1-\rho^k)\zeta_{ij\chi}^k &= (1-\rho^k)(1-\zeta_{ij\chi}^k) + \rho^k\alpha^k\mu_{hij}^k + \rho^k(1-\alpha^k)\mu_{xij}^k \\
&> 0.
\end{aligned}
$$

It follows that $\left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{-1} > (1-\rho^k)$, and hence that $\Upsilon_j^k = \left(\sum_{i=1}^{J}\sum_{\chi\in\{V,O\}} \left(\frac{\zeta_{ij\chi}^k}{\zeta_{ij}^k}\right)^{-1} \pi_{ij}^k\pi_{\chi|ij}^k\right)^{-1} < \frac{1}{1-\rho^k}$. This means that: $1-(1-\rho^k)\Upsilon_j^k \in [0,1]$. Under the further parameter assumptions that $0 < \rho < \rho^k < 1$ and $0 < \alpha, \eta^k < 1$, we have that the total costs paid for relationship-specific inputs, $(1-\alpha)\sum_{k=1}^{K} \frac{\rho\eta^k}{\rho^k}\left(1-(1-\rho^k)\Upsilon_j^k\right) \in [0,1]$.

Labor Demand. We take the first-order condition of (35) with respect to $L_j(\phi)$:

$$\begin{aligned} \frac{\partial F_j(\phi)}{\partial L_j(\phi)} &= 0 \\ \Rightarrow \quad \bar{\Upsilon}_j \frac{\partial R_j(\phi)}{\partial q_j(\phi)} \frac{\partial q_j(\phi)}{\partial L_j(\phi)} &= w_j, \end{aligned}$$

where $\bar{\Upsilon}_j$ is the share of final revenues, $R_j(\phi)$, that accrues to the final-good producer in its payoff function. It follows that:

$$\begin{aligned} \bar{\Upsilon}_j \rho A_j^{1-\rho} q_j(\phi)^{\rho-1} \left\{ \phi \left[\prod_{k=1}^{K} \left(X_j^k(\phi) \right)^{\eta^k} \right]^{1-\alpha} \alpha L_j(\phi)^{\alpha-1} \right\} &= w_j \\ \alpha\rho\bar{\Upsilon}_j \frac{R_j(\phi)}{L_j(\phi)} &= w_j. \end{aligned}$$

This yields equation (37) in the main text for $L_j(\phi)$.

Output. Substituting the expression for $X_j^k(\phi)$ from (33) and $L_j(\phi)$ from (37) into the production function (2), and using the fact that $R_j(\phi) = A_j^{1-\rho} q_j(\phi)^\rho$, we have:

$$\begin{aligned} q_j(\phi) &= \phi \left(\prod_{k=1}^{K} \left(X_j^k(\phi) \right)^{\eta^k} \right)^{1-\alpha} (L_j(\phi))^\alpha \\ &= \phi \prod_{k=1}^{K} \left[\left(\frac{\alpha^k}{s_j} \right)^{\alpha^k} \left(1-\alpha^k \right)^{1-\alpha^k} \left[(1-\alpha)\rho\eta^k R_j(\phi) \right] \left(\bar{\Gamma}^k \right)^{\frac{1-\rho^k}{\rho^k}} \left(\Phi_j^k \right)^{\frac{1-\alpha^k}{\theta^k}} \left(\Upsilon_j^k \right)^{-\frac{1-\rho^k}{\rho^k}} \right]^{\eta^k(1-\alpha)} \\ &\quad \times \left(\frac{\alpha\rho}{w_j} \bar{\Upsilon}_j R_j(\phi) \right)^\alpha \\ \frac{q_j(\rho)}{R_j(\phi)} &= \phi \left(\rho \frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{(w_j)^\alpha} \left(\bar{\Upsilon}_j \right)^\alpha \right) \prod_{k=1}^{K} \left[\left(\frac{\alpha^k}{s_j} \right)^{\alpha^k} \left(1-\alpha^k \right)^{1-\alpha^k} \eta^k \bar{\Gamma}^{\frac{1-\rho^k}{\rho^k}} \left(\Phi_j^k \right)^{\frac{1-\alpha^k}{\theta^k}} \left(\Upsilon_j^k \right)^{-\frac{1-\rho^k}{\rho^k}} \right]^{\eta^k(1-\alpha)} \end{aligned}$$

This corresponds to equation (38) in the main paper.

Trade Flows. Using the expression for $x_{cj}^k(\phi; l)$ and $x_{nj}^k(\phi; l)$ from (18) and (20) respectively, and substituting these into (45), we obtain:

$$\begin{aligned} t_{ij\chi}^k(\phi) &= \int_{l \in \Omega_{ij\chi}^k} (1-\alpha^k) \left(\mu_{xij}^k + (1-\mu_{xij}^k)(1-\beta_{ij\chi}^k) \frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k} \right) \frac{\rho^k}{1-\rho^k} \left(\Xi_{ij\chi}^k Z_{ij\chi}^k(\phi; l) \right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} dl \\ &= (1-\alpha^k) \left(\mu_{xij}^k + (1-\mu_{xij}^k)(1-\beta_{ij\chi}^k) \frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k} \right) \frac{\rho^k}{1-\rho^k} \left(\Xi_{ij\chi}^k \right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \bar{z}_{ij\chi}^k. \end{aligned}$$

We plug in the expression for $\left(\Xi_{ij\chi}^k\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \bar{z}_{ij\chi}^k$ from equation (B.15) in the above to get:

$$t_{ij\chi}^k(\phi) = (1-\alpha^k)\left(\mu_{xij}^k + (1-\mu_{xij}^k)(1-\beta_{ij\chi}^k)\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\right)\left[(1-\alpha)\rho\eta^k R_j(\phi)\right]\Upsilon_j^k \pi_{ij}^k \pi_{\chi|ij}^k.$$

It follows that the expression for trade flows observed by industry and sourcing mode, after aggregating over all firms is:

$$t_{ij\chi}^k = \frac{(1-\alpha)\rho\eta^k}{\rho^k}\Upsilon_j^k \rho^k(1-\alpha^k)\left(\mu_{xij}^k + (1-\mu_{xij}^k)(1-\beta_{ij\chi}^k)\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\right)\pi_{ij}^k \pi_{\chi|ij}^k N_j \int_\phi R_j(\phi) dG_j(\phi).$$

Bearing in mind that: $I_j = N_j \int_\phi R_j(\phi) dG_j(\phi)$, this yields the aggregate trade flow expression reported in (46) in the main paper.

Factor Market-Clearing. First, the amount of labor that is employed in assembly labor is given by:

$$N_j \int_\phi L_j(\phi)\ dG_j(\phi) = \frac{\alpha\rho}{w_j}\bar{\Upsilon}_j N_j \int_\phi R_j(\phi)\ dG_j(\phi) = \frac{\alpha\rho}{w_j}\bar{\Upsilon}_j I_j,$$

where we have used the expression for $L_j(\phi)$ from (37). Second, country-j input suppliers also employ labor to produce input varieties that are then sourced by firms around the world (including by country-j's own final-good producers). The total amount of labor employed for this latter purpose in country j is obtained by summing over the labor used to provide inputs to final-good producers in country $m = \{1, \ldots, J\}$ under sourcing mode (j, χ), where $\chi = \{V, O\}$. Let $\Omega_{jm\chi}^k$ be the set of input varieties $l \in [0,1]$ from industry k for which sourcing mode (j, χ) is optimal for a final-good firm headquartered in country m. The demand for labor by input suppliers in country j is then given specifically by:

$$\begin{aligned}
&\sum_{k=1}^K \sum_{m=1}^J \sum_{\chi\in\{V,O\}} N_m \int_\phi \int_{l\in\Omega_{jm\chi}^k} \frac{d_{jm}^k \left[\mu_{xjm}^k x_{cm}^k(\phi;l) + (1-\mu_{xjm}^k) x_{nm}^k(\phi;l)\right]}{Z_{jm\chi}^k(\phi;l)}\ dl\ dG_m(\phi) \\
&= \sum_{k=1}^K \sum_{m=1}^J \sum_{\chi\in\{V,O\}} N_m \int_\phi \int_{l\in\Omega_{jm\chi}^k} \frac{d_{jm}^k}{Z_{jm\chi}^k(\phi;l)} \left[\frac{\rho^k}{1-\rho^k}\frac{1-\alpha^k}{c_{jm\chi}^k(\phi,l)}\left(\Xi_{jm\chi}^k Z_{jm\chi}^k(\phi;l)\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}}\right] \\
&\quad \times \left[\mu_{xjm}^k + (1-\mu_{xjm}^k)(1-\beta_{jm\chi}^k)\frac{\zeta_{jm}^k}{\zeta_{jm\chi}^k}\right]\ dl\ dG_m(\phi) \\
&= \sum_{k=1}^K \sum_{m=1}^J \sum_{\chi\in\{V,O\}} N_m \int_\phi \int_{l\in\Omega_{jm\chi}^k} \frac{1-\alpha^k}{w_j}\frac{\rho^k}{1-\rho^k}\left(\Xi_{jm\chi}^k Z_{jm\chi}^k(\phi;l)\right)^{\frac{\rho^k(1-\alpha^k)}{1-\rho^k}} \\
&\quad \times \left[\mu_{xjm}^k + (1-\mu_{xjm}^k)(1-\beta_{jm\chi}^k)\frac{\zeta_{jm}^k}{\zeta_{jm\chi}^k}\right]\ dl\ dG_m(\phi)
\end{aligned}$$

Using the substitution implied by (B.15), we have after some simplification:

$$\sum_{k=1}^{K}\sum_{m=1}^{J}\sum_{\chi\in\{V,O\}} N_m \int_{\phi}\int_{l\in\Omega^k_{jm\chi}} \frac{d^k_{jm}\left[\mu^k_{xjm}x^k_{cm}(\phi;l)+(1-\mu^k_{xjm})x^k_{nm}(\phi;l)\right]}{Z^k_{jm\chi}(\phi;l)}\, dl\ dG_m(\phi)$$
$$=\sum_{k=1}^{K}\sum_{m=1}^{J}\sum_{\chi\in\{V,O\}} N_m \int_{\phi}\frac{1-\alpha^k}{w_j}\left[(1-\alpha)\rho\eta^k R_m(\phi)\right]\Upsilon^k_m\pi^k_{jm}\pi^k_{\chi|jm}$$
$$\times\left[\mu^k_{xjm}+(1-\mu^k_{xjm})(1-\beta^k_{jm\chi})\frac{\zeta^k_{jm}}{\zeta^k_{jm\chi}}\right]dG_m(\phi)$$
$$=\sum_{k=1}^{K}\sum_{m=1}^{J}\sum_{\chi\in\{V,O\}}(1-\alpha)\rho\eta^k I_m\Upsilon^k_m\pi^k_{jm}\pi^k_{\chi|jm}\frac{1-\alpha^k}{w_j}\left[\mu^k_{xjm}+(1-\mu^k_{xjm})(1-\beta^k_{jm\chi})\frac{\zeta^k_{jm}}{\zeta^k_{jm\chi}}\right]$$

Summing up the above two components of labor demand in country j, and equating this to the endowment of labor, this gives us the labor market-clearing condition spelled out in (47).

We use similar steps as in the above case of supplier labor to derive the demand for capital in country j, after aggregating over all firms:

$$\sum_{k=1}^{K}N_j\int_{\phi}\int_{l=0}^{1}\left[\mu^k_{hij}h^k_{cj}(\phi;l)+\left(1-\mu^k_{hij}\right)h^k_{nj}(\phi;l)\right]dl\ dG_j(\phi)$$
$$=\sum_{k=1}^{K}(1-\alpha)\rho\eta^k I_j\Upsilon^k_j\frac{\alpha^k}{s_j}\sum_{i=1}^{J}\sum_{\chi=V,O}\pi^k_{ij}\pi^k_{\chi|ij}\left(\mu^k_{hij}+\left(1-\mu^k_{hij}\right)\beta^k_{ij\chi}\frac{\zeta^k_{ij}}{\zeta^k_{ij\chi}}\right)$$

Equating this to the endowment of capital yields the capital market-clearing condition in (48).

C Quantitative Work Appendix (ONLINE ONLY)

C.1 Data notes

Intrafirm Trade Share We use data from the U.S. Census Bureau's Related Party Database to construct our measure of the intrafirm trade share. This is constructed as the value of related party imports and exports in a NAICS 6-digit industry, expressed as a share of the sum of the value of related and non-related party imports and exports in that same industry code, respectively. For each industry, we compute the intrafirm import share for each year between 2001 and 2005; then, we take the simple average across years to smooth out the idiosyncratic noise from any given year.

Capital Intensity We use a measure of physical capital intensity in our estimation of the functional form for α^k. This is computed from the NBER-CES dataset as the log of the average real capital stock per worker for each NAICS1997 6-digit industry, computed as the average of annual values between 2001 and 2005. For 3-digit industries, we sum up the real capital stock and employment at the three-digit level before computing the log capital stock per worker each year and taking the average across the five years.

Specificity We use the measures in Rauch (1999). We first merge the Rauch measure at the SIC level with Feenstra-Romalis-Schott import data by HS-NAICS-SIC. Then, for each NAICS 6-digit, we compute the share of these codes that are classified as differentiated, reference-priced, or exchange-traded. If missing, we successively replace with the share of codes that are classified as such at the next higher level of aggregation, until the 3 digit level.

Import Demand Elasticities We infer the import demand elasticities in two steps. In the first step, we use Soderbery(2015)'s estimates at the 10-digit HS level. Elasticities at higher aggregation levels are computed as the weighted average of constituent HS10 elasticities, using HS10 import volumes as weights from Feenstra-Romalis-Schott (FRS) import data from 1989-2006. We do this successfully till HS2.

In the second step, we compute NAICS6 elasticities as a weighted average of constituent HS codes, using concordance weights from HS to NAICS provided by FRS. We fill in the missing NAICS6 elasticities with a weighted average of constituent HS codes at successively higher aggregation levels. This gives NAICS6 elasticities for all manufacturing industries. We then take a weighted average to get NAICS5, 4, 3-digit elasticities.

Contractibility Our overall strategy is to construct the contractibility of each input and then infer the HQ and the supplier contractibility from these measures. In the first step, we obtain a NAICS-IO concordance from the 1997 US Input-Output Tables and construct direct requirements coefficients for each NAICS output industry in manufacturing. For a given direct requirements coefficient, for each NAICS input, if multiple IO input industries map to it, we divide the direct requirements by the number of these IO input industries (i.e., equal apportionment). Then, we compute for each NAICS 6-digit output industry the weighted-average of specificity values of the NAICS inputs, using the direct requirements coefficients as weights. Analogously we calculate this for higher levels of NAICS aggregation. If missing at the NAICS 6-digit level, we successively replace with the contractibility value at the next higher level of aggregation, until the 3 digit level.

We further designate inputs from NAICS 6-digit codes that feature an above-median capital-labor ratio to be headquarter inputs. The remaining below-median capital-intensity inputs are designated as supplier inputs. We then compute the share of inputs (by value) used in industry k that are traded on an open exchange or reference-priced separately for each of these subsets of inputs, to arrive at the "HQ Contractibility$_k$" and "Supplier Contractibility$_k$" measures.

Trade Flows Our trade flow data come from the ICIO Tables from the OECD. We use the data from the year 2005 in the 2018 version of the dataset. The dataset classifies industries at the ISIC Rev 4, which we map into NAICS3 by matching the industry descriptions. If an ISIC code maps into multiple NAICS3 codes, we split the ICIO trade flows into the NAICS trade flows using the value-added from NBER-CES dataset as weights. For example, one ICIO code, 10T12 (Food, Beverage, and Tobacco), maps into NAICS 331 (Food) and 332 (Beverage and Tobacco). In the NBER-CES database, NAICS 311 is roughly three times larger than 312 by value-added, and therefore we assign 75 percent of the trade flows in the ICIO code to NAICS 311 and the rest 25 percent to 312.

C.2 (Quasi-)Maximum Likelihood Estimation: PPML

In the main text we have estimated the intra-firm trade share using NLLS by showing that if the trade flows are observed with a Poisson error term, then the intra-firm trade share follows a Bernoulli distribution. In this appendix, we show that the expression of trade flows, $t^k_{ij\chi}$, also lends itself to (quasi-)maximum likelihood estimation. To do this, we assume that the error term $\epsilon^k_{ij\chi}$ follows a Poisson distribution with unit mean. Given this, the conditional expectation of $\tilde{t}^k_{ij\chi}$ is:

$$E[\tilde{t}^k_{ij\chi}|a^k_{ij\chi}, a^k_{ij}] = \exp\left\{\log a^k_{ij\chi} + \log a^k_{ij}\right\}.$$

Bearing in mind the Poisson distributional assumption on the error term, the probability mass function of $\tilde{t}^k_{ij\chi}$ conditional on $a^k_{ij\chi}$ and a^k_{ij} is:

$$\begin{aligned}\Pr(\tilde{t}^k_{ij\chi}|a^k_{ij\chi}, a^k_{ij}) &= \frac{\exp\left\{\tilde{t}^k_{ij\chi}(\log a^k_{ij\chi} + \log a^k_{ij})\right\} \times \exp\left\{-\exp\{\log a^k_{ij\chi} + \log a^k_{ij}\}\right\}}{\tilde{t}^k_{ij\chi}!} \\ &= \frac{1}{\tilde{t}^k_{ij\chi}!}\exp\left\{\tilde{t}^k_{ij\chi}\log a^k_{ij\chi}\right\} \times \exp\left\{-a^k_{ij}a^k_{ij\chi} + \tilde{t}^k_{ij\chi}\log a^k_{ij}\right\}\end{aligned}$$

Technically, $\tilde{t}^k_{ij\chi}!$ is only well-defined if the observed trade flow is an integer. This is why the procedure here should be viewed more as a quasi-maximum likelihood, rather than a true maximum likelihood. Moving forward, we omit this term in the denominator, and replace the "=" sign by a "$\propto$" sign, to denote equality up to a positive multiplicative constant. As the observations are independently distributed, it suffices to focus on observations within an i-k cell, in order to obtain an estimate for a^k_{ij}. To do this, observe that the conditional joint probability for the observations within an i-k cell is:

$$\begin{aligned}\Pr\left(\tilde{t}^k_{ijV}, \tilde{t}^k_{ijO}|a^k_{ij\chi}, a^k_{ij}\right) &\propto \prod_{\chi\in\{V,O\}}\exp\left\{\tilde{t}^k_{ij\chi}\log a^k_{ij\chi}\right\} \times \exp\left\{-a^k_{ij}a^k_{ij\chi} + \tilde{t}^k_{ij\chi}\log a^k_{ij}\right\} \\ &= \left(\prod_{\chi\in\{V,O\}}\exp\left\{\tilde{t}^k_{ij\chi}\log a^k_{ij\chi}\right\}\right)\exp\left\{-a^k_{ij}\sum_{\chi\in\{V,O\}}a^k_{ij\chi} + \log a^k_{ij}\sum_{\chi\in\{V,O\}}\tilde{t}^k_{ij\chi}\right\}.\end{aligned}$$

The first-order condition of the above with respect to a^k_{ij} implies that the quasi-maximum likelihood estimator satisfies:

$$\begin{aligned}a^k_{ij}\sum_{\chi\in\{V,O\}}a^k_{ij\chi} &= \sum_{\chi\in\{V,O\}}\tilde{t}^k_{ij\chi} \\ \Rightarrow \quad a^k_{ij} &= \frac{\sum_{\chi\in\{V,O\}}\tilde{t}^k_{ij\chi}}{\sum_{\chi\in\{V,O\}}a^k_{ij\chi}}\end{aligned}$$

Substituting this quasi-maximum likelihood estimator of a^k_{ij} back into equation (49), we have:

$$\begin{aligned}\tilde{t}^k_{ij\chi} &= a^k_{ij\chi}\left(\frac{\sum_{\chi\in\{V,O\}}\tilde{t}^k_{ij\chi}}{\sum_{\chi\in\{V,O\}}a^k_{ij\chi}}\right)\varepsilon^k_{ij\chi} \\ \Rightarrow \quad \frac{\tilde{t}^k_{ij\chi}}{\sum_{\chi\in\{V,O\}}\tilde{t}^k_{ij\chi}} &= \frac{a^k_{ij\chi}}{\sum_{\chi\in\{V,O\}}a^k_{ij\chi}}\varepsilon^k_{ij\chi} \qquad \text{(C.1)}\end{aligned}$$

The left-hand side of this last equation is the share of trade from country i in industry k that is sourced through mode χ; when $\chi = V$, this is simply the intrafirm import share for the source-country by industry cell in question. In other words, if we limit ourselves to the sub-sample of observations where $\chi = V$, what (C.1) provides is a structural equation of the intrafirm import share, the latter being a commonly-used dependent variable in the empirical trade literature on the determinants of firm boundaries. This is explained by terms on the right-hand side that are a function of the $a^k_{ij\chi}$'s, as given by equation (51), as well as by the error term that is assumed to be distributed Poisson. An implication of (C.1) is that it provides a justification for regression specifications in which the intrafirm trade share is the dependent variable, and a Pseudo-Poisson Maximum Likelihood (PPML) estimator is used.

C.3 Numerical Implementation

Particle Swarm Optimization (PSO) and Levenberg-Marquardt (LM) We use a combination of PSO and LM to solve the minimization problem in the NLLS. To implement the PSO, we use a population size of 500, roughly 10 times the number of parameters to be estimated. We use randomized neighborhoods with size of 50 to enhance the performance of the algorithm. The PSO converges if the optimal point hits a cumulative stall limit of 10. We call the PSO routines 1000 times with randomized seeds, and use the best point estimate from these 1000 calls from as the starting point to the LM solver. The Levenberg-Marquardt solver comes from the MINPACK library in the public domain. We use `1.0E-4` as the tolerance level to stop the LM algorithm.

Upon convergence, we use Gauss-Newton Regression (GNR) to compute the standard errors. The Jacobin matrix in GNR was numerically approximated with a relative step-size of `1.0E-8`. Using smaller step sizes yields similar results. We cluster the standard errors at the industry level in the GNR to estimate the clustered standard errors.

C.4 Technical details on estimation

Computing the values of $a^k_{ij\chi}$ is numerically unstable because $\frac{\theta^k}{1-\lambda_i}$ can take on large values, which in turn implies that $\left(1-\beta^k_{ij\chi}\right)^{\frac{\theta^k}{1-\lambda_i}+1}$ or $\left(\beta^k_{ij\chi}\right)^{\frac{\alpha^k}{1-\alpha^k}\frac{\theta^k}{1-\lambda_i}}$ can be numerically identical to zero. This problem is made worse in the case when $\beta^k_{ij\chi}$ is close to 0 or 1. To reliably simulate the intrafirm trade share, we first re-write it as:

$$E\left[\frac{\tilde{t}^k_{ijV}}{\tilde{t}^k_{ij}}\middle|\tilde{t}^k_{ij}\right] = \left(1+\frac{a^k_{ijO}}{a^k_{ijV}}\right)^{-1},$$

and then compute the log of the ratio $\frac{a^k_{ijO}}{a^k_{ijV}}$ as:

$$\log\left(\frac{a^k_{ijO}}{a^k_{ijV}}\right) = \left[\left(1-\mu^k_{hij}\right)\frac{\alpha^k}{1-\alpha^k}\frac{\theta^k}{1-\lambda_i}\right]\log\left(\frac{\beta^k_{ijO}}{\beta^k_{ijV}}\right) + \left[\left(1-\mu^k_{xij}\right)\frac{\theta^k}{1-\lambda_i}+1\right]\log\left(\frac{1-\beta^k_{ijO}}{1-\beta^k_{ijV}}\right)$$
$$+\left[\frac{\zeta^k_{ij}}{\rho^k(1-\alpha^k)}\frac{\theta^k}{1-\lambda_i}-1\right]\log\left(\frac{\zeta^k_{ijO}}{\zeta^k_{ijV}}\right) + \log\left(\frac{\left(1-\mu^k_{xij}\right)\zeta^k_{ij}+\mu^k_{xij}\frac{\zeta^k_{ijO}}{1-\beta^k_{ijO}}}{\left(1-\mu^k_{xij}\right)\zeta^k_{ij}+\mu^k_{xij}\frac{\zeta^k_{ijV}}{1-\beta^k_{ijV}}}\right). \quad \text{(C.2)}$$

C.5 Technical details on mapping from sourcing probabilities to trade shares

The sourcing probability π_{ij}^k in (28) is equal to the share of input varieties from industry k that will be sourced by country j from origin-country i. In Eaton and Kortum (2002), π_{ij}^k would also be the share of trade by value in industry k that is imported by country j from origin-country i; this follows from the property that the distribution of prices that country j conditional on country i being the lowest-cost source is independent of the identity of the origin-country i. In our current setting, prices are not determined in a competitive setting, as the interaction between a final-good firm and its suppliers is instead mediated by a bargaining process. We therefore need an additional step to map the sourcing probability π_{ij}^k to the corresponding trade share by value.

Recall that the expression for trade flows from country i to j in industry k under organizational mode χ is given in equation (46) in the main text. Summing across organizational modes, we have:

$$\begin{aligned} t_{ij}^k &= \sum_{\chi=\{V,O\}} t_{ij\chi}^k \\ &= \frac{(1-\alpha)\rho\eta^k}{\rho^k}\frac{\Upsilon_j^k}{\Phi_j^k} I_j \rho^k (1-\alpha^k) T_i^k (w_i)^{-\theta^k} \left(B_{ij}^k\right)^{-\frac{\theta^k\lambda_i}{1-\lambda_i}} \left(d_{ij}^k\right)^{-\theta^k} \\ &\quad \times \sum_{\chi\in\{V,O\}} \left(\mu_{xij}^k + (1-\mu_{xij}^k)(1-\beta_{ij\chi}^k)\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\right) \frac{1}{2} \left(B_{ij\chi}^k\right)^{\frac{\theta^k}{1-\lambda_i}}. \end{aligned}$$

Denote the correction term as σ_{ij}^k:

$$\sigma_{ij}^k = \left(B_{ij}^k\right)^{-\frac{\theta^k}{1-\lambda_i}} \sum_{\chi\in\{V,O\}} \left(\mu_{xij}^k + (1-\mu_{xij}^k)(1-\beta_{ij\chi}^k)\frac{\zeta_{ij}^k}{\zeta_{ij\chi}^k}\right) \frac{1}{2} \left(B_{ij\chi}^k\right)^{\frac{\theta^k}{1-\lambda_i}}.$$

Applying this correction term to the observed bilateral trade flows, $\tilde{t}_{ij}^k$, in the data allows us to recover the model-implied sourcing probabilities, π_{ij}, since:

$$\frac{\tilde{t}_{ij}^k/\sigma_{ij}^k}{\sum_{i'=1}^J \tilde{t}_{i'j}^k/\sigma_{i'j}^k} = \frac{T_i^k\left(w_i d_{ij}^k\right)^{-\theta^k}\left(B_{ij}^k\right)^{-\frac{\theta^k\lambda_i}{1-\lambda_i}}\left(B_{ij}^k\right)^{\frac{\theta^k}{1-\lambda_i}}}{\sum_{i'=1}^J T_{i'}^k\left(w_{i'} d_{i'j}^k\right)^{-\theta^k}\left(B_{i'j}^k\right)^{-\frac{\theta^k\lambda_{i'}}{1-\lambda_{i'}}}\left(B_{i'j}^k\right)^{\frac{\theta^k}{1-\lambda_{i'}}}} = \frac{T_i^k\left(w_i d_{ij}^k\right)^{-\theta^k}\left(B_{ij}^k\right)^{\theta^k}}{\sum_{i'=1}^J T_{i'}^k\left(w_{i'} d_{i'j}^k\right)^{-\theta^k}\left(B_{i'j}^k\right)^{\theta^k}} = \pi_{ij}^k.$$

After we have estimated the model, the σ_{ij}^k correction term can be evaluated exactly in the baseline equilibrium. When we perform the hat-algebra counterfactuals, where we need the initial values of π_{ij}^k as key sufficient statistics to evaluate general equilibrium responses, we now first apply the correction terms in order to back out π_{ij}^k, instead of reading this off directly from the entries of the inter-country Input-Output table. Note that the σ_{ij}^k correction term is equal to 1 in the special case of full contractibility ($\mu_{hij}^k = \mu_{xij}^k = 1$).

C.6 Technical details on computing counterfactuals

Given counterfactual values of μ_{hij}^k, μ_{xij}^k, and $\beta_{ij\chi}^k$, the system of equations to solve the model in changes is as follows:

$$\left(\zeta_{ij}^{k}\right)' = 1-\rho^{k}+\rho^{k}\alpha^{k}\left(\mu_{hij}^{k}\right)'+\rho^{k}\left(1-\alpha^{k}\right)\left(\mu_{xij}^{k}\right)' \tag{C.3}$$

$$\left(\zeta_{ij\chi}^{k}\right)' = 1-\rho^{k}\alpha^{k}\left[1-\left(\mu_{hij}^{k}\right)'\right]\left(\beta_{ij\chi}^{k}\right)'-\rho^{k}\left(1-\alpha^{k}\right)\left(1-\left(\mu_{xij}^{k}\right)'\right)\left(1-\left(\beta_{ij\chi}^{k}\right)'\right). \tag{C.4}$$

$$\left(B_{ij\chi}^{k}\right)' = \left[1-\left(\beta_{ij\chi}^{k}\right)'\right]^{1-\left(\mu_{xij}^{k}\right)'}\left[\left(\beta_{ij\chi}^{k}\right)'\right]^{\left(1-\left(\mu_{hij}^{k}\right)'\right)\frac{\alpha^{k}}{1-\alpha^{k}}}\left[\frac{\left(\zeta_{ij\chi}^{k}\right)'}{\left(\zeta_{ij}^{k}\right)'}\right]^{\frac{\left(\zeta_{ij}^{k}\right)'}{\rho^{k}\left(1-\alpha^{k}\right)}} \tag{C.5}$$

$$\left(B_{ij}^{k}\right)' = \left(\frac{1}{2}\left[\left(\left(B_{ijV}^{k}\right)'\right)^{\frac{\theta^{k}}{1-\lambda_{i}}}+\left(\left(B_{ijO}^{k}\right)'\right)^{\frac{\theta^{k}}{1-\lambda_{i}}}\right]\right)^{\frac{1-\lambda_{i}}{\theta^{k}}}. \tag{C.6}$$

$$\left(\pi_{\chi|ij}^{k}\right)' = \frac{\left((B_{ij\chi}^{k})'\right)^{\frac{\theta^{k}}{1-\lambda_{i}}}}{\left((B_{ijV}^{k})'\right)^{\frac{\theta^{k}}{1-\lambda_{i}}}+\left((B_{ijO}^{k})'\right)^{\frac{\theta^{k}}{1-\lambda_{i}}}}. \tag{C.7}$$

$$\widehat{\pi_{ij}^{k}} = \frac{\left(\widehat{d_{ij}^{k}}\widehat{w_{i}}\right)^{-\theta^{k}}\left(\widehat{B_{ij}^{k}}\right)^{\theta^{k}}}{\widehat{\Phi_{j}^{k}}} \tag{C.8}$$

$$\widehat{\Phi_{j}^{k}} \equiv \sum_{i=1}^{J}\pi_{ij}^{k}(\widehat{d_{ij}^{k}}\widehat{w_{i}})^{-\theta^{k}}\left(\widehat{B_{ij}^{k}}\right)^{\theta^{k}} \tag{C.9}$$

$$\left(\upsilon_{ij\chi}^{k}\right)' = \frac{\left(\pi_{ij}^{k}\right)'\left(\pi_{\chi|ij}^{k}\right)'}{\frac{\left(\zeta_{ij\chi}^{k}\right)'}{\left(\zeta_{ij}^{k}\right)'}} \tag{C.10}$$

$$\left(\Upsilon_{j}^{k}\right)' = \left\{\sum_{i=1}^{J}\sum_{\chi\in\{V,O\}}\frac{\left(\pi_{ij}^{k}\right)'\left(\pi_{\chi|ij}^{k}\right)'}{\frac{\left(\zeta_{ij\chi}^{k}\right)'}{\left(\zeta_{ij}^{k}\right)'}}\right\}^{-1} = \left\{\sum_{i=1}^{J}\sum_{\chi=\{V,O\}}\left(\upsilon_{ij\chi}^{k}\right)'\right\}^{-1} \tag{C.11}$$

$$\left(\bar{\Upsilon}_j\right)' = 1-(1-\alpha)\sum_{k=1}^{K}\frac{\rho\eta^k}{\rho^k}\left[1-(1-\rho^k)\left(\Upsilon_j^k\right)'\right]. \tag{C.12}$$

$$\left(I_j\right)' = \frac{\widehat{w_j}w_j\bar{L}_j+\widehat{s}_j s_j\bar{H}_j}{1-(1-\alpha\rho)\left(\bar{\Upsilon}_j\right)'} \tag{C.13}$$

$$\widehat{w_j}w_j\bar{L}_j = \rho\alpha\left(\bar{\Upsilon}_j\right)'\left(I_j\right)'+\rho\left(1-\alpha\right) \\ \times\sum_{k=1}^{K}\left(1-\alpha^k\right)\eta^k\sum_{m=1}^{J}\left(I_m\right)'\left(\Upsilon_m^k\right)'\sum_{\chi\in\{V,O\}}\left(\upsilon_{jm\chi}^k\right)'\left[\frac{\left(\mu_{xjm}^k\right)'\left(\zeta_{jm\chi}^k\right)'}{\left(\zeta_{jm}^k\right)'}+\left(1-\left(\mu_{xjm}^k\right)'\right)\left[1-\left(\beta_{jm\chi}^k\right)'\right]\right] \tag{C.14}$$

$$\widehat{s_j^h}s_j^h\bar{H}_j = \rho(1-\alpha)\left(I_j\right)'\sum_{k=1}^{K}\alpha^k\eta^k\left(\Upsilon_j^k\right)'\sum_{i=1}^{J}\sum_{\chi=V,O}\left(\upsilon_{ij\chi}^k\right)'\left[\frac{\left(\mu_{hij}^k\right)'\left(\zeta_{ij\chi}^k\right)'}{\left(\zeta_{ij}^k\right)'}+\left(\beta_{ij\chi}^k\right)'\left(1-\left(\mu_{hij}^k\right)'\right)\right] \tag{C.15}$$

To solve the above system of equations, we need data on: π_{ij}^k, $w_m\bar{L}_m$, and $s_j^h\bar{H}$. The exogenous changes are $\widehat{d_{ij}^k}$ and $\widehat{\beta_{ij\chi}^k}$, and the endogenous unknowns that we need to solve for are $\widehat{w_m}$, $\widehat{s_j^h}$, $\widehat{I_j}$, $\widehat{\pi_{ij}^k}$, $\widehat{\pi_{\chi|ij}^k}$, $\widehat{\Upsilon_j^k}$, $\widehat{\bar{\Upsilon}_j^k}$, and $\widehat{\Phi_j^k}$.

The algorithm:

1. Given $(\mu_{hij}^k)'$, $(\mu_{hij}^k)'$ and $\left(\beta_{ij\chi}^k\right)'$, use equation (C.6) to solve for $(B_{ij\chi}^k)'$.
2. Use equation (C.7) and $(B_{ij\chi}^k)'$ to get $\left(\pi_{\chi|ij}^k\right)'$ and $\widehat{\pi_{\chi|ij}^k}$.
3. Guess a vector of $\widehat{w}_j$ and $\widehat{s_j}$.
4. Conditional on the guessed $\widehat{w}_j$ and $\widehat{s_j}$, use equation (C.9) to solve for $\widehat{\Phi_j^k}$.
5. Use $\widehat{\Phi_j^k}$ and equation (C.8) to solve for $\widehat{\pi_{ij}^k}$ and $\left(\pi_{ij}^k\right)'$.
6. With $\left(\pi_{ij}^k\right)'$, we can use equation (C.11) and (C.12) to get $\left(\Upsilon_m^k\right)'$ and $\left(\bar{\Upsilon}_m\right)'$.
7. With $\left(\bar{\Upsilon}_j\right)'$, use equation (C.13) to solve for $(I_j)'$.
8. With all the above information, invert equation (C.14) to get a new $\widetilde{w_j}$ Similarly, we can update the price of capital, $\widetilde{s_j}$ by inverting equation (C.15):
9. Update $(\widehat{w_j}, \widehat{s_j})$ with $(\widetilde{w_j}, \widetilde{s_j})$, and iterate from step 3 until convergence.

A note on computing $\widehat{B_{ij}^k}$. As $\theta^k/(1-\lambda_i)$ might take on very large values, it is numerically impossible to compute the $\left(B_{ij\chi}^k\right)^{\frac{\theta}{1-\lambda_i}}$ terms in levels. As a result, the B_{ij}^k terms in levels are also impossible to compute using equation (C.6). To get $\widehat{B_{ij}^k}$, we first infer $\left(\widehat{B_{ij\chi}^k}\right)^{\frac{\theta}{1-\lambda_i}}$ from $\frac{\theta^k}{1-\lambda_i}\log(B_{ij\chi}^k)$ terms:

$$\exp^{\frac{\theta^k}{1-\lambda_i}\log((B_{ij\chi}^k)')-\frac{\theta^k}{1-\lambda_i}\log((B_{ij\chi}^k))} = \frac{\left(\left(B_{ij\chi}^k\right)'\right)^{\theta^k/(1-\lambda_i)}}{\left(\left(B_{ij\chi}^k\right)\right)^{\theta^k/(1-\lambda_i)}} = \left(\widehat{B_{ij\chi}^k}\right)^{\frac{\theta^k}{1-\lambda_i}}$$

With $\left(\widehat{B_{ij\chi}^k}\right)^{\frac{\theta}{1-\lambda_i}}$ terms, we proceed to compute $\widehat{B_{ij}^k}$ as a linear combination of them:

$$\begin{aligned}
\widehat{B_{ij}^k} = \frac{\left(B_{ij}^k\right)'}{\left(B_{ij}^k\right)} &= \left[\frac{\left(\left(B_{ijV}^k\right)'\right)^{\frac{\theta^k}{1-\lambda_i}} + \left(\left(B_{ijO}^k\right)'\right)^{\frac{\theta^k}{1-\lambda_i}}}{\left(\left(B_{ijV}^k\right)\right)^{\frac{\theta^k}{1-\lambda_i}} + \left(\left(B_{ijO}^k\right)\right)^{\frac{\theta^k}{1-\lambda_i}}}\right]^{\frac{1-\lambda_i}{\theta^k}} \\
&= \left[\frac{\left(\left(B_{ijV}^k\right)'\right)^{\frac{\theta^k}{1-\lambda_i}}}{\left(B_{ijV}^k\right)^{\frac{\theta^k}{1-\lambda_i}}} \frac{\left(B_{ijV}^k\right)^{\frac{\theta^k}{1-\lambda_i}}}{\left(B_{ijV}^k\right)^{\frac{\theta^k}{1-\lambda_i}} + \left(B_{ijO}^k\right)^{\frac{\theta^k}{1-\lambda_i}}} + \frac{\left(\left(B_{ijO}^k\right)'\right)^{\frac{\theta^k}{1-\lambda_i}}}{\left(B_{ijO}^k\right)^{\frac{\theta^k}{1-\lambda_i}}} \frac{\left(B_{ijO}^k\right)^{\frac{\theta^k}{1-\lambda_i}}}{\left(B_{ijV}^k\right)^{\frac{\theta^k}{1-\lambda_i}} + \left(B_{ijO}^k\right)^{\frac{\theta^k}{1-\lambda_i}}}\right]^{\frac{1-\lambda_i}{\theta^k}} \\
&= \left[\left(\widehat{B_{ijV}^k}\right)^{\frac{\theta^k}{1-\lambda_i}} \pi_{ijV}^k + \left(\widehat{B_{ijO}^k}\right)^{\frac{\theta^k}{1-\lambda_i}} \pi_{ijO}^k\right]^{\frac{1-\lambda_i}{\theta^k}}.
\end{aligned}$$

The above shortcut works for most of the cases. However, in several cases with large variations, the $\left(\widehat{B_{ij\chi}^k}\right)$ deviates significantly from 1. As a result, it is impossible to numerically represent $\left(\widehat{B_{ij\chi}^k}\right)^{\frac{\theta}{1-\lambda_i}}$ as floating point numbers either. For example, given an estimate of $\frac{\theta^k}{1-\lambda_i}$ close to 1500 and $\widehat{B_{ij\chi}^k}$ at around 0.5, $\left(\widehat{B_{ij\chi}^k}\right)^{\frac{\theta}{1-\lambda_i}}$ is numerically zero. In these cases, we compute the B_{ij}^k and $\left(B_{ij}^k\right)'$ terms directly as:

$$B_{ij}^k = \left(\frac{1}{2}\right)^{\frac{1-\lambda_i}{\theta^k}} B_{ijV}^k \left[1 + \left(\frac{B_{ijO}^k}{B_{ijV}^k}\right)^{\frac{\theta^k}{1-\lambda_i}}\right]^{\frac{1-\lambda_i}{\theta^k}}. \tag{C.16}$$

In the above expression, the only term that is unstable to compute is $\left(\frac{B_{ijO}^k}{B_{ijV}^k}\right)^{\frac{\theta^k}{1-\lambda_i}}$. If $\left(\frac{B_{ijO}^k}{B_{ijV}^k}\right) > 1$ and $\frac{\theta^k}{1-\lambda_i}$ is large, the term might exceed the upper limit of floating point numbers and thus return infinity.

In this case, we directly approximate B_{ij}^k as:

$$B_{ij}^k = \left(\frac{1}{2}\right)^{\frac{1-\lambda_i}{\theta^k}} B_{ijO}^k \left[1 + \left(\frac{B_{ijV}^k}{B_{ijO}^k}\right)^{\frac{\theta^k}{1-\lambda_i}}\right]^{\frac{1-\lambda_i}{\theta^k}} \approx \left(\frac{1}{2}\right)^{\frac{1-\lambda_i}{\theta^k}} B_{ijO}^k.$$

To see this, note that as $\left(\frac{B_{ijO}^k}{B_{ijV}^k}\right)^{\frac{\theta^k}{1-\lambda_i}} \to \infty$, $\left(\frac{B_{ijV}^k}{B_{ijO}^k}\right)^{\frac{\theta^k}{1-\lambda_i}} \to 0$. Lastly, as (C.16) yields the exact results in non-approximating cases, and the results are identical to the linear-combination case, we use (C.16) for all the counterfactual simulations for simplicity.

A note on equilibrium factor payments in hat-algebra The model implies that in equilibrium the ratio between $w_j\bar{L}_j$ and $s_j\bar{H}_j$ is a function of model parameters. When we collect the data vectors $\{w_j\bar{L}_j, s_j^h\bar{H}_j\}$, there is no guarantee that the equilibrium relationships are satisfied by these data vectors. In other words, in the actual data, the factor payment ratios will not be in equilibrium in our model. This poses a difficulty in applying the hat-algebra, as the market clearing conditions will push $\{\hat{w}, \hat{c}\}$ towards the equilibrium values, not the counterfactual values.

The solution is that we only collect data on $w_j\bar{L}_j$, and use the equilibrium condition to directly solve for $s_j^h\bar{H}_j$, and then solve the hat-system. Intuitively, we can justify this as taking a broader view of what might constitute the factor that goes into producing headquarter services, as this might not map neatly to skilled labor in the data.