Sovereign Default, Private Investment, and Economic Growth∗

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Abstract

The literature on sovereign debt explains the ability of a country to borrow through various costs of a sovereign default. Existing models typically capture part of these costs as transitory drops in output of low persistence around the default. However, I show empirically that the costs of defaults seem long-lived. Even ten years after a default, GDP is roughly six percentage points lower than it would have been without a default. Output losses thus seem to be highly persistent or even permanent. Based on this observation, I develop a small open economy model that incorporates government borrowing from abroad and endogenous growth through the adoption of new varieties of intermediate goods. The model captures important stylized facts associated with sovereign borrowing, the business cycle in emerging economies and sovereign default. In the model, a sovereign default triggers a persistent loss in GDP relative to trend through a temporary reduction in technology adoption and investment. This persistence of the GDP losses adds to the cost of a default. Numerical experiments show that the long-run cost channel increases the average debt-to-GDP ratio by between 33 and 100 percent while keeping the default frequency constant or reducing it.

JEL Codes: E32, E44, F34, F43

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1 Introduction

According to the data provided in the World Development Indicators (2013), in 2010, 129 countries had positive public long-term external debt.\(^1\) The median country of those 129 nations was holding debt of this type that was worth 20 percent of its GDP. The literature on sovereign debt commonly explains the ability of a country to borrow from abroad through various costs of a sovereign default.\(^2\) Understanding the sources of these costs is not only important in any study that examines the access of a nation to foreign capital markets, it also helps us to forecast debt crisis and to design effective interventions that can be employed by third parties in a crisis. Quantitative papers that analyze sovereign defaults on external debt typically model the cost as short-lived output or consumption losses that are triggered by the default.\(^3\)

However, this paper shows empirically that the costs of defaults seem long-lived. For instance, I estimate that the level of GDP ten years after a default is roughly six percentage points lower than what we would have expected if no default had occurred. In light of the empirical finding, I build a novel default model that is capable of endogenously generating persistent output losses after a default to study the role of long-run GDP losses as a new default cost channel. The extra cost has quantitatively sizable effects on the decision to default and the amount of resources the country examined in the model is able to borrow.

If the persistent GDP loss is partially triggered by the default, this adds to the potential costs of not repaying external debt. Figure 1 illustrates this point. In both parts of this stylized graph the solid line indicates the development of GDP if no default occurs and the broken line the path of GDP if a default actually takes place. In the left panel GDP returns back to the blue line after some time, while in the right plot this does not occur. The cost of a default in terms of forgone GDP is the area between the two graphs and we see that the total loss is higher when GDP losses persist longer. We can therefore conclude that it is important to understand how long the output losses last on average.

I base my empirical result on regressions using a country panel data set that was constructed from the database of the World Development Indicators (2013) and covers over 60 defaults between 1970

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\(^1\)Public long-term external debt means debt that has a maturity of more than one year and that is owed to nonresidents by the government of an economy and repayable in foreign currency, goods, or services.

\(^2\)As discussed in Panizza, Sturzenegger, and Zettelmeyer (2009), the enforcement institutions for sovereign debt are much weaker than the ones for corporate debt. Therefore, they do not suffice as an explanation for the ability of a government to borrow substantial amounts from abroad. If a nation is suffering some sort of cost from failing to service its debt, the desire to avoid this cost can help to explain why a nation repays its foreign obligations most of the time.

\(^3\)See, for example, the discussions in Borensztein and Panizza (2009), Panizza, Sturzenegger, and Zettelmeyer (2009) or Stähler (2013).
and 2007. My results suggest that GDP either recovers slowly or never returns to its pre-crisis trend after a sovereign default event. In the year of a default, GDP is, on average, three percentage points lower. Losses accumulate over the following years until they reach six percentage points after ten years.

While standard models of sovereign default either impose short-lived output losses (see, for example, Arellano (2008), Chatterjee and Eyigungor (2012)) or derive them from frictions that interact with the default (see for example Mendoza and Yue (2012)), none of these models capture the long-lasting effects of defaulting. However, if we accept that the output losses around a default are partially the result of the government’s decision to not repay its debt, as most of the quantitative models of sovereign default assume, then the empirical results drive us to the conclusion that the persistently lower GDP, even a decade later, is also, in part, a result of the default. In the current paper, I achieve this transmission by modeling the medium and long-run dynamics of the economy as the adoption of new technologies through entrepreneurs. To motivate this choice, I show that a sovereign default in the 1970s or 1980s predicts a significant reduction in the imports of computers up to five years after the default using a panel of countries. This finding suggests a slowdown in the adoption of computer-based innovations into the production process of the defaulting countries at a time when the usage of computers in production started to spread rapidly throughout the world.

The model environment is a small open economy. The production of the final good involves the usage of capital, labor and a continuum of intermediate goods. The number of intermediate good varieties grows over time, as new goods get adopted by entrepreneurs who sell them to the household. This leads to endogenous growth in the model, which follows the idea of Romer (1990). The amount
of spending on technology adoption is determined by a cost-benefit comparison. Shocks that influence the gains from intermediate good adoption therefore lead to fluctuations in the growth rate of the economy. Through this channel, temporary disturbances can lead to long-lasting movements in GDP, an idea put forward in Fatas (2000b) and Comin and Gertler (2006).

The model also incorporates a government that uses taxation and external borrowing to finance government expenditures. The authorities lack the ability to commit to debt repayment and are impatient. The government defaults on its debt whenever the cost of honoring the promises made to lenders exceeds the cost of a default. A default triggers a loss of access to external finance for the government and a rise in import costs through higher working capital costs. The latter channel is consistent with the findings described by Gopinath and Neiman (2012) and Zymek (2012) about trade adjustments during crisis and the differential impact of defaults on firms according to their need for external finance, and is initially studied by Mendoza and Yue (2012). The increase in the cost of producing intermediate goods leads to a recession, as the intermediate goods producers pass on their costs to the final good producer, thereby reducing the usage of such goods in production. As the demand for intermediate goods is expected to be low in the near future, the profits from adoption are expected to be low and adoption falls because of a substitution effect. In addition, households try to smooth consumption during the crisis and this makes them less willing to provide funds. This income effect further enhances the drop in adoption and also in capital investment. Therefore, the short-lived recession triggers a long lasting reduction in the level of production relative to the level of output if no default occurred. As a result, the default permanently reduces the level of output in the economy.

Section 4 and 5 present the quantitative analysis of the model. The model is solved using global numerical methods that are based on a projection approach that takes into account the nature of the dynamic game between household, firms and government, and the nonlinearities induced by the default decision. The model is calibrated to match a set of moments that are deemed to be typical for recent defaulters. The model qualitatively and quantitatively replicates the stylized facts of the business cycle as well as of the dynamics around a default. This observation adds to the evidence on the long-run costs of sovereign default, as it shows that a standard model of economic long-run development placed into a sovereign default model can generate persistent output losses triggered by a default while showing a reasonable quantitative performance. My model implies that a default leads to a reduction in the growth of the number of intermediate varieties of 0.9 percentage points annualized, leading to a long-run reduction in growth of the same magnitude.
In the remainder of Section 5, I demonstrate that the adverse effect of a default on growth raises the cost of a default. I do this by performing a sequence of experiments. First, I show that a subsidy that eliminates the negative effect of a default on profits of intermediate goods producers increases the default frequency by 22 percent, while reducing the average debt-to-GDP ratio by 15 percent. Both demonstrate a sizable reduction in the cost of a default, as a lower cost of defaulting makes failing to service the external debt more attractive for the government. This increased likelihood of a default, in turn, raises borrowing costs ex-ante, which leads to a reduction in the average debt-to-GDP ratio. The subsidy does not fully remove the cost of a default generated by reduced adoption and it is not clear how to eliminate the cost from the model without affecting the government’s trade-off along other dimensions. Therefore, I modify the model so that technology adoption becomes a simpler process, thus allowing me to eliminate the effect of a default on adoption in an easy manner. Comparing the versions of the model with and without the effect of a default on adoption, I see that without this cost channel the default frequency more than doubles, while the debt-to-GDP ratio is more than 30 percent lower. Finally, I construct a simple endowment economy that is similar to the ones commonly employed in the literature. I find that adding a permanent output loss of the size found in my calibrated model to the endowment process leads to an increase in the average debt-to-GDP ratio by a hundred percent while keeping the default frequency roughly constant. All the experiments indicate the substantial extra cost of a default that the growth channel can generate.

1.1 Related Literature

A couple of recent empirical papers documented persistent output losses following deep recessions. For instance, the paper by Cerra and Saxena (2008) shows that large economic or political crisis tend to lead to long-lasting GDP losses. Focusing on sovereign default crisis, De Paoli, Hoggarth, and Saporta (2009) and Blyde, Daude, and Fernández-Arias (2010) use a set of defaulting countries to demonstrate that GDP and productivity are lower after a default, for at least some years, by separately extrapolating the trend of GDP/productivity before the default for each country under consideration. Furceri and Zdzienicka (2012) use a cross-country panel approach to demonstrate that defaults lead, on average, to persistent GDP losses for at least eight years after a default. However, beside past growth rates, their paper does not control for initial conditions or other types of crisis at the time of a default. In my paper, I demonstrate that GDP is lower after a default, even when controlling for the conditions, such as the presence of banking and currency crisis, at the onset of the
default. Looking at further evidence from other data sources allows me to document the presence of long-run losses after a default over longer time spans.

The model developed in this paper employs the ideas of the medium-term fluctuation models based on Comin and Gertler (2006) that builds on Romer (1990) and other growth models to show how short-run fluctuations can induce longer lasting fluctuation in the economy. Queraltó (2013) is the first to combine the idea of with other frictions to study persistent productivity losses after financial crisis in emerging markets. While Guerron-Quintana and Jinnai (2013) apply a similar model to analyze the U.S. Great Recession, Ates and Saffie (2014) combine the idea with micro level firm data to study how the selection of entrants influences productivity growth during a financial crises in Chile. All these papers, in addition to my own, model short-run phenomena and growth over longer horizons as interrelated phenomena and provide evidence that this approach can help to explain a set of empirical observations. A similar point is also made in Fatas (2000a) and Fatas (2000b), both of which document a strong cross-country correlation between long-term growth rates and the persistence of output fluctuations. He further shows that modeling growth as an endogenous, return-driven process can explain this observation, pointing out that a unit root process would have trouble doing the same. I use elements from this literature to model the output losses observed after a default and study for the first time in the literature how the presence of this mechanism influences the decisions that a government makes in terms of debt accumulation and default.

Aguiar and Gopinath (2007) propose that emerging market economies are more affected by a non-stationary component of total factor productivity (TFP) than developed economies, which helps to explain certain differences in the business cycle between the two groups of countries. Based on this idea, Aguiar and Gopinath (2006) have studied the role such trend shocks can play in models of sovereign default. The latter could provide an alternative story of the correlation I observe. My paper differs in that I model the movements in the trend as the endogenous response to stationary shocks and policy changes. My approach is supported by some of the existing research described above and it can also help to explain why some countries seem to have a larger non-stationary component than others, as this can be viewed as the outcome of a more crisis-prone environment.

Finally, my paper contributes to the growing literature on the sources and effects of interest rate fluctuations on the business cycle of emerging market economies like Neumeyer and Perri (2005), Uribe and Yue (2006) and Fernández-Villaverde, Guerron-Quintana, Rubio-Ramírez, and Uribe (2011). In particular it contributes to the literature on quantitative sovereign default models, for example Arellano (2008), Chatterjee and Eyigungor (2012) and Mendoza and Yue (2012), and surveyed in
Within this body of work, the modeling of intertemporal links in production in a default model connects my paper closest to the work of Bai and Zhang (2012), Gordon and Guerron-Quintana (2013), Roldán-Peña (2012) and Park (2012), all of which introduce capital into the basic sovereign default model. Bai and Zhang (2012) uses the setup to study international risk sharing in the presence of default risk while the remaining researchers model capital as a government-controlled asset and study its effect on default dynamics and sovereign interest spreads. The latter three all find that, as the government has full control over the capital stock, more capital allows for more borrowing and reduces interest spreads on government bonds. In all four papers, capital also serves as a second means of insurance against fluctuations beside debt, reducing consumption volatility. Although production in sovereign default models has been previously studied, it is typically assumed to be a static production setting (Mendoza and Yue (2012), Cuadra, Sanchez, and Sapriza (2010)). My model also allows for capital accumulation. I add to the sovereign default literature by studying the role of long-run GDP losses for the decision to default, endogenizing those losses through endogenous growth and, from a modeling perspective, by allowing the investment and adoption decisions to be decided by the household and not the government itself. The latter is important as it reduces the effect of an impatient government on the accumulation of private sector assets.

The rest of the paper is organized as follows. In Section 2, I present evidence for the observation that a sovereign default typically coincides with subsequent output losses. In Section 3, I introduce the model, which I calibrate in Section 4. Section 5 discusses the quantitative findings. The last section concludes the paper.

2 Default and Growth: Empirical Evidence

In this section, I study the relationship between sovereign defaults and the subsequent development of GDP. I construct a panel data set and estimate the average dynamics of GDP in the aftermath of a debt crisis. After describing the data used, I describe and discuss the results from the baseline estimation. The next subsection summarizes the results from some variations of the regression specification to show the robustness of the key results. Finally, I perform a similar exercise for computer imports into defaulting countries in the 1980s as a means of providing some suggestive evidence that reduced technology adoption plays a role in propagating the effects of a default.
2.1 Data

The estimation is based on an unbalanced yearly panel of countries over the period 1970-2007. The time period is based on the time covered by the database used to determine the default dates. I consider all countries contained in the World Development Indicators (2013) database that is provided by the World Bank. This leads to a set of 214 countries. I use data for real GDP, government consumption, investment, credit to the private sector, inflation, population, exchange rates and openness to trade from this source. The World Development Indicators (2013) is a World Bank broad collection of development indicators that is compiled from international sources. I drop all country-year pairs if observations are not available, which leaves me with 164 countries in the sample. Table 1 provides summary statistics of the data.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Observations</th>
<th>Mean</th>
<th>Std.Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP (in 2005$)</td>
<td>7204</td>
<td>1.87e+11</td>
<td>8.14e+11</td>
</tr>
<tr>
<td>Government Expenditure (% of GDP)</td>
<td>6377</td>
<td>16.32007</td>
<td>6.864598</td>
</tr>
<tr>
<td>Openness</td>
<td>6677</td>
<td>80.87904</td>
<td>49.91241</td>
</tr>
<tr>
<td>Exchange Rate (in local currency per dollar)</td>
<td>7223</td>
<td>931101</td>
<td>7.91e+07</td>
</tr>
<tr>
<td>Domestic credit to private sector (% of GDP)</td>
<td>6346</td>
<td>60.50408</td>
<td>438.4617</td>
</tr>
<tr>
<td>Population</td>
<td>9313</td>
<td>2.50e+07</td>
<td>1.03e+08</td>
</tr>
<tr>
<td>Gross Fixed Capital Formation (% of GDP)</td>
<td>6193</td>
<td>22.09522</td>
<td>8.323847</td>
</tr>
<tr>
<td>Inflation (based on GDP deflator)</td>
<td>6585</td>
<td>43.46217</td>
<td>499.726</td>
</tr>
</tbody>
</table>

Table 1: Descriptive Statistics

In addition, I need to date sovereign defaults on external debt from private lenders. Unfortunately, this is not a trivial exercise and there are considerable differences between sources that try to date defaults over the same time period. It might, for instance, be unclear if the announcement of a default or the first missed payment constitutes the starting date of a default. Similarly, unilaterally induced debt exchanges by the debtor might be viewed as a default. As a final example, one can think of a country that defaulted and later renegotiated a settlement with its creditors. It then defaults on the first payment under this settlement a few weeks later. It is not clear if this should actually be counted as two defaults or as one default, as the country effectively failed to settle its first default. For the purposes of the estimation, I will use the database that was prepared by Laeven and Valencia (2008) to date defaults. However, I will perform robustness exercises by also using the
dating of defaults presented in the work of Borensztein and Panizza (2009). The former captures 63 episodes of sovereign default based on other sources: Beim and Calomiris (2001), World Bank (2002), and Sturzenegger and Zettelmeyer (2006), and not closer described IMF Staff reports. Borensztein and Panizza (2009) instead use the default dates from Standard & Poor’s database and counted for the time period of the estimation 129 default dates. In addition, I also use Laeven and Valencia (2008) to date banking and currency crisis.

2.2 Estimation

I estimate the following model to capture the dynamics of a default:

\[ y_{i,t+k} - y_{i,t-1} = \alpha_i^k + \beta_k D_{i,t} + \rho^k X_{i,t} + \epsilon_{i,t}. \]  

(1)

Here, \(i\) is the country index, \(t\) the time index. Logged real GDP in country \(i\) in year \(t\) is denoted \(y_{i,t}\). I include a country-specific dummy or fixed effect, depending on the details of the estimation, which I denote by \(\alpha_i^k\). Dummies that take the value 1 if a country defaults in period \(t\) are denoted by \(D_{i,t}\). The regression error is \(\epsilon_{i,t}\). Finally, \(X_{i,t}\) contains a set of country specific-controls. In the baseline estimation, they are logged GDP growth in \(t-1\), logged GDP in \(t-1\), logged GDP growth in \(t-2\) and the gross fixed capital formation to GDP ratio, openness, real exchange rate growth, GDP based inflation, logged population growth and logged credit growth in period \(t\). In addition, dummies for banking and currency crisis in period \(t\) are added.\(^4\)

The main parameter of interest are the values of the \(\beta_k\)'s, which describe by how much we should expect logged GDP \(k\) periods later to be lower if there is a default in period \(t\). They summarize all the dynamics after a default, including the effects of changes in other variables like investment and government expenditure on GDP. This approach is based on the idea of local linear projections discussed in Jordá (2005). The parameter \(k\), which governs the number of years after period \(t\), is varied between 0 and 10.

I estimate equation 1 using ordinary least squares with standard errors clustered by countries with \(\alpha_i^k\) being a country dummy. When I perform robustness checks I also look at country specific fixed effects. In that case I will take first differences of Equation 1, use lags of \(X_{i,t}\) as instruments, and use the generalized method of moments in order to allow for a more general error structure following

\(^4\)I experimented with additional controls like the lagged level of logged GDP, further lags of GDP growth or the capital stock. The latter was constructed using the perpetual inventory method. Adding them did not change the qualitative results and had very little effect on the quantitative ones.
the approach discussed in Roodman (2006).

### 2.3 Results

This section reports the results from estimating Equation 1 with ordinary least squares using country dummies. I start by reporting the results of the estimation for \( k = 0, 1, 10 \) in Table 2 below.

<table>
<thead>
<tr>
<th>Variable</th>
<th>( k=0 )</th>
<th>( k=1 )</th>
<th>( k=10 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default (_{i,t})</td>
<td>-.0352***</td>
<td>-.0503***</td>
<td>-.0663***</td>
</tr>
<tr>
<td>( \Delta y_{i,t-1} )</td>
<td>.1329***</td>
<td>.1337*</td>
<td>-.0901</td>
</tr>
<tr>
<td>( \Delta y_{i,t-2} )</td>
<td>-.0415</td>
<td>-.0523</td>
<td>-.2961***</td>
</tr>
<tr>
<td>( y_{i,t-1} )</td>
<td>-.0173***</td>
<td>-.038097***</td>
<td>-.1835***</td>
</tr>
<tr>
<td>Bank Crisis (_{i,t})</td>
<td>-.0079*</td>
<td>-.0328***</td>
<td>-.0242*</td>
</tr>
<tr>
<td>Currency Crisis (_{i,t})</td>
<td>-.0256***</td>
<td>-.0276***</td>
<td>.0011</td>
</tr>
<tr>
<td>Gross Fixed Capital Formation (_{i,t})</td>
<td>.0010***</td>
<td>.0014***</td>
<td>.0010</td>
</tr>
<tr>
<td>Inflation (_{i,t})</td>
<td>-.00001</td>
<td>-.00003</td>
<td>-.0001**</td>
</tr>
<tr>
<td>( \Delta \log(\text{Population}_{i,t}) )</td>
<td>.5032*</td>
<td>1.0113*</td>
<td>-.5508</td>
</tr>
<tr>
<td>Government Expenditure (_{i,t})</td>
<td>-.0021***</td>
<td>-.0032***</td>
<td>-.0030</td>
</tr>
<tr>
<td>Openness (_{i,t})</td>
<td>.0004***</td>
<td>.0009***</td>
<td>.0017***</td>
</tr>
<tr>
<td>( \Delta \text{Exchange Rate}_{i,t} )</td>
<td>-.00001**</td>
<td>-.00002**</td>
<td>-.00001</td>
</tr>
<tr>
<td>( \Delta \text{Credit}_{i,t} )</td>
<td>.0377***</td>
<td>.05016***</td>
<td>.02530</td>
</tr>
<tr>
<td>Country Dummies</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.2969</td>
<td>0.3802</td>
<td>0.7276</td>
</tr>
<tr>
<td>Number of observations</td>
<td>3742</td>
<td>3741</td>
<td>3026</td>
</tr>
</tbody>
</table>

Table 2: Regression Results

Note: Results from the OLS estimation of equation 1 for \( k=0,1,10 \). Country dummies are not shown. A * indicates significance at 90 percent, ** indicates significance at 95 percent and *** significance at 99 percent. \( \Delta \) indicates first differences.

The results displayed are reassuring. The \( R^2 \) is 0.3 or higher within the typical range for this type of regression (see, for example, Borensztein and Panizza (2009), Yeyati and Panizza (2011)). The controls enter with the expected signs and many of them are significant. For instance, higher capital formation and larger credit supply lead to higher GDP growth, while a higher government expenditure share and banking crisis have a negative effect at all time horizons. Most importantly for the topic of this study is that a default leads to a fall in forecasted GDP for all three time horizons.

To summarize the results from all 11 regressions, I plot \( \beta^k \)'s for \( k \) from 0 to 10 together with a 95 percent confidence interval for each coefficient in Figure 2. This captures the expected effect on the GDP dynamics after a default.

As we can see, GDP is expected to be below trend for ten years after a default. It initially falls by roughly 3.5 percent at the point estimate. This is consistent with other findings summarized, for
example, in Panizza, Sturzenegger, and Zettelmeyer (2009). The losses then accumulate over time and reach over six percent after ten years. Both the point estimates and the confidence intervals lie below the line of a zero effect and one cannot reject the hypothesis of no recovery of GDP back to a fictitious trend without default. This could still mean that a very persistent shock hit the economy and led both to the default and a long-lasting reduction of GDP. However, if we are willing to accept that a default generates an output loss as a large fraction of the literature assumes, then the results here clearly suggest that we should expect them to have a long-lasting impact. Another aspect to keep in mind when interpreting the impulse responses shown, is that the dynamics after the default also reflect the propagation of lower growth rates to future periods in general. A regression controlling for lagged GDP growth applied to periods after the default, might therefore find a weaker or no effect of a default on GDP growth without contradicting my results.

In the end, the main message the reader should take away from this picture is that, on average, GDP falls substantially around a sovereign default and that years later we can still see the effects of these losses in a country’s GDP.
2.4 Robustness

I perform a set of robustness exercises by modifying the empirical model. As for the baseline estimation, I summarize the results by showing the implied impulse responses of GDP after a default. They are displayed in Figure 3.

![Figure 3: Impulse Response Function: Robustness](image)

Note: The solid blue line depicts the point estimate for $\beta^k$ in each period from regression 1 with the appropriate modification. The broken lines are the 95% confidence intervals based on clustered or Windmeijer standard errors. “Time Dummies” adds year dummies. “Fixed Effects” is estimated using an Arrellano-Bond estimator and allowing for year dummies. “Alternative Dates” uses Borensztein and Panizza (2009) to date defaults. “Restricted Sample” drops countries that did not default between 1970 and 2007. “Time Dummies” estimates the model using ordinary least squares. In this case confidence intervals are based on clustered standard errors.

I start by adding year dummies to the regression. Sovereign defaults are not uniformly distributed across time. Out of the 63 episodes identified by Laeven and Valencia (2008) between 1970 and 2007, only six took place between 1990 and 1999, while there are 42 defaults between 1980 and 1989. It is therefore possible that the lower growth rates after a default reflect lower growth rates around the globe in the eighties. The results graphed under “Time Dummies” in Figure 3 indicate that the results are robust to the addition of time dummies. The point estimates and the confidence intervals shift up compared to the ones in the baseline, but the results are reasonably similar.\(^5\) In particular, we still expect a default to coincide with persistent output losses according to the results in all the periods with at least 90 percent confidence.

\(^5\)The shift up should be expected as the year dummies capture some of the default effect in periods in which many countries violated their debt contracts.
Next, I estimate the model in first differences using GMM and instrumenting regressors with lagged variables in order to control for potential country fixed effects, as explained above. This is done using the approach and code described by Roodman (2006). Standard errors use the correction derived by Windmeijer (2005) and are clustered. While I drop the country dummies, I keep the year dummy variables for the estimation. Results are shown under “Fixed Effects” in Figure 3. If we compare the point estimates to the ones obtained by ordinary least squares, we see that the estimated GDP losses are smaller. However, we also see that they are still statistically and economically significant across all of the ten years with a loss after ten years of 5.3 percent. Even in Period 6 and 9, we cannot reject, at a 90 percent significance level, that GDP is lower after the default took place.

As an additional exercise, I replace the sovereign default dummies utilized by Laeven and Valencia (2008) with those employed by Borensztein and Panizza (2009). Default dates are not the same between different studies because of the difficulties related to dating them and deciding if a default actually occurred. The estimated IRF is shown in the plot under “Alternative Dates” in Figure 3. It is reassuring that the results are similar. The estimated initial effect is weaker, suggesting that the selected default episodes in Borensztein and Panizza (2009) coincide with weaker recessions on average. After that, the dynamics look relatively close and eventually reach a slightly higher loss of 7.1 percent.

Next, I restrict the set of countries in the regression to the ones that defaulted at least once in the sample, using again the default dummies based on Laeven and Valencia (2008). In the baseline regression, I include countries that never defaulted during the years covered. They help me identify the effects of not defaulting given the observables. But it is not clear how informative, for example, the development of the United States is for the growth pattern of an emerging market economy like Indonesia. While I control for country effects, it is possible that nations that defaulted during the years 1970 to 2007 differ in other ways from the ones that never defaulted. I therefore restrict the sample. Results are shown in Figure 3 under “Restricted Sample”. Focusing on the point estimate, the losses in the later periods are slightly smaller. Confidence intervals widen as the number of observations shrink considerably. Nevertheless, the losses are always significant at a 90 percent confidence level and most of the time also at a 95 percent level.\(^6\)

Since the time period the panel spans is relative short, I perform another set of robustness exercises by using the GDP per capita data and the dates of sovereign default on external debt used in Reinhart

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\(^6\)I also performed the last two exercises using time dummies and allowing for country fixed effects at the same time. The main results stayed robust to the combination of those elements.
and Rogoff (2009). The data goes much further back in time and covers the whole 20th century for many countries and even the time since 1820 for others. I only consider year country pairs in which the country was independent. As the controls used before are not available for this exercise, I estimate the simpler model

\[ y_{i,t+k} - y_{i,t-1} = \alpha_i^k + \beta_i^k D_{i,t} + \rho_1^k y_{i,t-1} + \rho_2^k (y_{i,t-1} - y_{i,t-2}) + \gamma_t + \epsilon_{i,t} \]

using ordinary least squares allowing for country specific dummies and time dummies \((\gamma_t)\). \(y_{i,t}\) now denotes logged GDP per capita in country \(i\) in year \(t\) and \(D_{i,t}\) is still a default dummy. The results are shown in the upper half of figure 4. While the initial drop in GDP is smaller compared to the short sample, the main results are not affected. On average, we still see a significant and strong loss in GDP over the ten years following the default.

![Figure 4: Impulse Response Function: Longer Series](image)

Note: The solid blue line depicts the point estimate for \(\beta_i^k\) in each period from regression 1 with the appropriate modification. The broken lines are the 95% confidence intervals based on clustered standard errors.

The Latin American Debt Crisis is viewed as a particularly deep economic crisis in the countries affected (not only in Latin America). Similarly, the effects in the longer sample for the period of the great depression and the two world wars might be different for the defaulting countries due to the global nature of those crisis. It is possible that the particular group of defaults during these events strongly affects the results, both qualitatively and quantitatively, as they constitute a large fraction
of all the sovereign defaults in the sample. As a final robustness exercise, I therefore recreate the last estimation, excluding the years 1914-1918, 1928-1945 and 1980-1989. The results are shown in the lower part of Figure 4. The confidence intervals widen and, in the first periods, some of the point estimates are only significantly different from zero at a 90 percent threshold. However, the results remain robust. GDP is still expected to be over seven percent lower after ten years if a default occurred.

I conclude from the previous estimates, that after a default, one should expect GDP to be statistically and economically significantly lower for at least ten years than one would have expected without a default. While no causal claim can be made, the evidence at least does not lead us to reject the idea that a default causes highly persistent or even permanent output losses. Reassured by this observation, I will study the interaction of sovereign default and GDP growth further in the next sections.

2.5 Computer Adoption

The empirical results shown so far establish a correlation between sovereign defaults and the subsequent development of GDP. They do not provide any implication as to the sources of this association. In the subsequent parts of the paper, I am going to develop and analyze a model that generates the connection through the decisions of private agents on how much to invest in the development of new goods and the adoption of new technologies. The underlying basic model has been used to successfully analyze various questions in the business cycle literature, as discussed in the literature review. Nevertheless, I want to provide some suggestive evidence on the negative link between the adoption of new technologies and default.

In general, it is not easy to measure the diffusion of new technologies across countries that are affected by a sovereign default over short horizons. However, the period of the 1980s with its large number of debt crisis, provides us with a good opportunity. Focusing on the 1970s and 1980s, Caselli and Coleman (2001) produced a case study in which they examined the determinants of the diffusion of computer technology around the globe. According to the authors, two things made this exercise feasible. Firstly, the large-scale usage of computers in production had just commenced during that period. Secondly, computer technology is embodied in a machine. In order to track the diffusion it therefore suffices to look at the value of computers set up in a given country. As many countries did not call a sizable computer industry their own, Caselli and Coleman (2001) made the case that
tracking imports should form a good proxy for the increase in usage. Here, I am going to follow their idea by looking at the role defaults might play for the dynamics of computer imports, a connection that Caselli and Coleman (2001) did not consider.

I take data on computer imports in US dollars for the years from 1970 to 1990 provided by Feenstra, Lipsey, Deng, Ma, and Mo (2005), divide them by the population size of the respective country and adjust them using the U.S. National Income and Product Accounts deflator for equipment. This variable, real computer imports per capita, serves as my proxy for the degree of additional computer adoption in the respective year and country. Let us denote this variable in country \( i \) in year \( t \) by \( I_{i,t} \).

I estimate the following model to capture the effects of a default on the adoption speed in different years:

\[
\log\left(\sum_{j=0}^{k} I_{i,t+j}\right) = \alpha_k^i + \beta^k D_{i,t} + \rho^k X_{i,t} + \epsilon_{i,t}.
\]  

Here, \( i \) is again the country index, \( t \) the time index. I include a country-specific dummy, which I denote by \( \alpha_k^i \). Dummies that take the value 1 if a country defaults in period \( t \) are denoted by \( D_{i,t} \). The regression error is \( \epsilon_{i,t} \). Finally, \( X_{i,t} \) contains a set of country specific controls. They are logged GDP per capita in \( t \) and the growth rate in \( t \) and \( t-1 \), real computer imports per worker and its growth rate of real computer imports per worker in \( t-1 \). I also include gross capital formation relative to GDP, openness, primary school completion rate and credit to the private sector relative to GDP in year \( t-1 \). In addition, dummies for banking and currency crisis in period \( t \), country dummies and year dummies were added.

My choice of controls is determined by variables that Caselli and Coleman (2001) found to be important predictors of computer imports to the extent my data allows. I also include lagged GDP and its growth and lagged computer imports to control for the economic situation before the default occurred. I estimate Equation 2 using ordinary least squares with standard errors clustered by countries with \( \alpha_k^i \). Table 2.5 reports the results of the estimation for \( k = 0, 1, \ldots, 5 \) below. I am particularly interested in the coefficient for the default dummy, as it tells us the extent to which the total computer imports were reduced after the default and therefore provides a proxy for the slowdown in computer adoption.

Looking at the estimation results we see a good fit of the estimated equation. Most of the controls

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7While the results are robust to modifications of the specification I tried, I decided not to report those for brevity. In this regressions I focus on the first five years after a default as I follow Caselli and Coleman (2001) in using the time period from 1970 to 1990. Most defaults in the sample occur after 1980; as such, that attrition would rapidly shrink the sample if I were to increase the horizon further.
Table 3: Regression Results Computer Adoption

<table>
<thead>
<tr>
<th>Variable</th>
<th>k=0</th>
<th>k=1</th>
<th>k=2</th>
<th>k=3</th>
<th>k=4</th>
<th>k=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default(_{i,t})</td>
<td>-.533**</td>
<td>-.493**</td>
<td>-.430*</td>
<td>-.210**</td>
<td>-.205***</td>
<td>-.217***</td>
</tr>
<tr>
<td>(I_{t-1})</td>
<td>.346***</td>
<td>.247***</td>
<td>.163***</td>
<td>.134**</td>
<td>.130**</td>
<td>.118**</td>
</tr>
<tr>
<td>logged GDP per capita(_{i,t})</td>
<td>1.101***</td>
<td>1.299***</td>
<td>1.462***</td>
<td>1.481***</td>
<td>1.372***</td>
<td>1.275***</td>
</tr>
<tr>
<td>(\Delta I_{t-1})</td>
<td>-.074</td>
<td>-.078*</td>
<td>-.064*</td>
<td>-.032</td>
<td>-.029</td>
<td>-.027</td>
</tr>
<tr>
<td>(\Delta\text{logged GDP per capita}_{i,t})</td>
<td>-.951</td>
<td>-.270</td>
<td>.289</td>
<td>.035</td>
<td>.367</td>
<td>.267</td>
</tr>
<tr>
<td>(\Delta\text{logged GDP per capita}_{i,t-1})</td>
<td>.177</td>
<td>.322</td>
<td>-.297</td>
<td>-.206</td>
<td>-.071</td>
<td>.100</td>
</tr>
<tr>
<td>Bank Crisis(_{i,t})</td>
<td>.641</td>
<td>.176*</td>
<td>.168**</td>
<td>.104</td>
<td>.095</td>
<td>.089</td>
</tr>
<tr>
<td>Currency Crisis(_{i,t})</td>
<td>-.117</td>
<td>-.123</td>
<td>-.127</td>
<td>-.102</td>
<td>-.100</td>
<td>-.197</td>
</tr>
<tr>
<td>Gross Fixed Capital Formation(_{i,t})</td>
<td>.0212**</td>
<td>.018*</td>
<td>.018*</td>
<td>.013*</td>
<td>.008</td>
<td>.004</td>
</tr>
<tr>
<td>Openness(_{i,t})</td>
<td>.004</td>
<td>.005</td>
<td>.007**</td>
<td>.008***</td>
<td>.008***</td>
<td>.007**</td>
</tr>
<tr>
<td>Primary Education(_{i,t})</td>
<td>.005</td>
<td>.001</td>
<td>-.0001</td>
<td>-.001</td>
<td>-.002</td>
<td>-.002</td>
</tr>
<tr>
<td>Credit(_{i,t})</td>
<td>0.0002</td>
<td>.001</td>
<td>.0001</td>
<td>-.0009</td>
<td>-.0005</td>
<td>-.0007**</td>
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<td>Country Dummies</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>Time Dummies</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.907</td>
<td>0.945</td>
<td>0.956</td>
<td>0.967</td>
<td>0.973</td>
<td>0.975</td>
</tr>
<tr>
<td>Number of observations</td>
<td>642</td>
<td>629</td>
<td>617</td>
<td>608</td>
<td>600</td>
<td>555</td>
</tr>
</tbody>
</table>

Note: Results from the OLS estimation of equation 1 for k=0-5. Country dummies are not shown. A * indicates significance at 90 percent, ** indicates significance at 95 percent and *** significance at 99 percent. \(\Delta\) indicates first differences.

enter insignificantly, which might partially be a reflection of the inclusion of country and time dummies.\(^8\) The level of development, as measured by GDP per capita, is the main predictor of computer imports in the regression. This is not surprising as, for example, a higher level of development should make adoption of new technologies more likely. The other strong predictor in the regression results is past imports. This suggests, for example, that countries that started using the technology in the past keep adding to their stock of machines using computer components. In the past they might have established links to the producers, thus reducing the cost of importing more machinery.

In the period of a default and the following years, the effect of a default on computer imports is significant and sizable. In each year following the default the reduction of accumulated computer imports is more than 20 percent.\(^9\) In total, the results displayed support the idea of reduced adoption of computer technology after a default, as the fall in computer imports is sizable. As the defaulting countries in the time period covered lie outside the developed world, we can assume that imports are their main way of receiving machines that incorporate the new technology. I view these results

\(^8\)Caselli and Coleman (2001) acknowledge that country dummies would pick up a lot of the variation in their data.

\(^9\)Depending on the exact specification the default dummy for k=0 sometimes became statistically insignificant with a p-value between 0.1 and 0.2 while staying economically sizable. This might be the result of extra variation in variables across countries during a default period introduced by the timing of the default within the year. For the remaining k’s the statistical significance was robust to changes in the exact specification.
therefore as suggestive evidence for an effect of a sovereign default on technology adoption.

3 Model

In the last section we saw that we should expect persistent GDP losses after a default. In the remainder of the paper I study how persistent output losses affect a government’s decision as to whether they should default on, or honor, its debt. I did not establish that part of the GDP losses are, in fact, caused by the default. However, I will show that the common assumption of temporary output losses around a default, paired with a fairly standard endogenous growth model can generate such persistent losses as a response to the default. Endogenous decisions by the private sector of the economy will map the short-lived downturn into persistent losses in growth, as households respond to the drop in current economic activity by reducing investment and technology adoption. As I will explain later, this will be the result of income and substitution effects. In this section I will introduce the model and present the quantitative analysis in the following sections. I will describe two models. I start with a simple model. It allows me to qualitatively illustrate how growth is affected by a default and how endogenous growth can enlarge the cost of a default to the government. After this, I progress to the quantitative model, which I use to study the interaction between the government, domestic households and foreign lenders. The latter model combines the sovereign default model produced by Arellano (2008) with the model of technology adoption developed by Romer (1990). It also allows for capital investment, labor supply choices and long-run debt.

3.1 A Simple Model (to be updated)

Time in the economy is discrete with an infinite horizon. However, to keep things simple, I assume that all agents make choices with intertemporal effects only in the first period. The economy is populated by a representative family consisting of a government, a final good producer, and a continuum of entrepreneurs. The family likes consumption \((c_t)\). Its preference over consumption streams are given by

\[
\sum_{t=0}^{\infty} \beta^t (c_t)^{1-\sigma} \frac{1}{1-\sigma}.
\]

Final output is product from a variety of intermediate goods. Each intermediate good is produced by an entrepreneur, who is a monopolist for the production of his variety. The intermediate goods
are product one for one from units of the final good. Beside this usage, the final good can also be used as a consumption good and, in the first period, to pay for the cost of new entry. Denoting the quantity of the final good by $Y_t$, $m_{it}$ the quantity of intermediate good $i$ used, and $N_t$ the number of intermediate goods, final output in period $t$ is given by

$$Y_t = A_t \int_0^{N_t} m_{it}^\alpha di,$$

where $\alpha \in (0, 1)$ and $A_t$ is the level of productivity. The latter is 1, if the country is not punished for a default, and $D < 1$ otherwise. The final good is product by a price taking final good producer, who pays his profits to the family.

The government has to raise lump sum taxes $T$ in form of the final good each period to repay some debt, that was acquired before the start of the economy. It can decide to default in period 0. In this case the family does not have repay the debt, however it is being punished for a stochastic length of time. After period 0 the punishment ends with probability $p$. When making the decision the government maximizes the utility of the family.

In period 0 the measure of varieties is 1. Entrepreneurs can use resources to set up new varieties. Using $S$ units of the final good results in $S$ addition varieties. So in all the following periods the measure of varieties is $1 + S$. The optimal number of new varieties is determined by comparing the cost of forgone consumption to the household with the discounted stream of profits the entrepreneur makes. Given the market structure and functional forms the entrepreneur will charge a price $\frac{1}{\alpha}$. The demand for each variety in equilibrium can be shown to be $m_t = (\frac{1}{A_t^{\alpha}})^{\frac{1}{\alpha}}$. Profits are therefore $\pi_t = (\frac{1}{\alpha} - 1)m_t$. As the goods market clearing condition of the model is $Y_t = c_t + \int_0^{N_t} m_{it} di + S_t + T_t$ the condition determining new varieties if the country is not in default is then

$$(m^\alpha - m - S - T)^{-\sigma} = \frac{\beta}{1 - \beta}((1 + S)(m^\alpha - m) - T)^{-\sigma}(\frac{1}{\alpha} - 1)m.$$

If the country defaults $S$ is determined by

$$(Dm^\alpha - m - S)^{-\sigma} = \frac{\beta(1 - p)}{1 - (1 - p)\beta}((1 + S)(Dm^\alpha - m))^{-\sigma}(\frac{1}{\alpha} - 1)m + \frac{1}{1 - (1 - p)\beta}((1 + S)(m^\alpha - m))^{-\sigma}(\frac{1}{\alpha} - 1)m.$$

One can show for $T$ close to zero and $p$ close to 1, that $S$ is smaller in default than if the country keeps repaying. The reason is a wealth effect, as the reduced production in period 0 leads to a gain of front loading consumption. In addition, there are two opposing effects if $p$ is positive. The first is
a wealth effect from reduced production in the punishment states. This effect tends to increase $S$. However, it is counteracted by lower profits of the intermediate good producers. These effects will also play out in the bigger model to be presented next.

Finally, the reduction of $S$ in default is more expensive to the family and the government, than the entrepreneurs consider when they decide on $S$. The reason is that the marginal gain in total output from a new variety is $m^2 - m$, which is larger than $\frac{1}{\alpha} m - m$, the profit that the entrepreneurs use in their calculation. This force will tend to make the government more reluctant to default than in the case when $S$ is unaffected by the default, at least when the reduction in $S$ is small.

3.2 The Full Model

The model essentially describes a small open economy. There are seven types of agents: The household, final good producers, intermediate goods producers, importers, entrepreneurs, the government and foreign lenders. The representative household works, consumes and invests in capital and shares of intermediate good producers. He supplies labor and capital services in a competitive market to a representative final good producer. The final good producer generates the final good using labor services, capital services and intermediate goods. He sells his product to the household, the government and intermediate good producers in competitive markets. A continuum of firms produce intermediate goods by combining the final good and imported inputs, selling their variety as monopolists.\(^{10}\) During each period, new intermediate goods producers are set up by entrepreneurs and sold immediately to the household. Importers buy a set of different foreign goods from abroad and combine them to generate the imported input. Some of the imported foreign goods are subject to a working capital constraint. The government collects taxes and engages in spending on government consumption. It trades bonds with foreign lenders and decides whether it will default on its obligations or service them. A default excludes both the government and the importers from financial markets for a stochastically determined amount of time.

The timing in a given period is as follows. First, every agent observes the realization of the TFP shock and potentially the reentry shock. Then the government makes its choices. After observing the choices of the government, the lenders, the firms and the household make their choices. This means all agents’ plans are functions of the policies of the government. This is important, as the government lacks the ability to commit. Therefore all choices should not only be thought of as a

\(^{10}\) I will refer to this process as (technology) adoption, introduction of new technologies and introduction of intermediate goods interchangeably.
function of the history of shocks and previous aggregates, but also, and especially, of the choices the
government made and is making in the current period.

The rest of this section explains the elements of the model in detail. Before doing so, it is helpful to
quickly preview the effects of various parts of the model. Working capital and imports allow the model
to generate a recession as a response to a default.\footnote{The idea for this part of the setup is taken from Mendoza and Yue (2012).} Monopoly rents from the sale of intermediate
goods generate procyclical incentives to introduce new varieties, the source of endogenous growth
in the model. A default will reduce expected profits, leading to temporary lower growth. Capital
accumulation will also contribute to growth in the model but is mainly introduced for the quantitative
performance, as will be explained later. Finally, the separation of government and household is
necessary to be able to generate a stronger desire of the government to borrow in the form of a lower
discount factor while simultaneously not distorting the capital and adoption decisions.

\subsection{Households}

There is a representative household who likes consumption ($c_t$) and dislikes supplying labor ($l_t$). His
preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t_{HH} \frac{(c_t - N_tG(l_t))^{1-\sigma}}{1 - \sigma}.$$  

Here, and in the rest of the paper, $E_t$ denotes the conditional expectations operator in period $t$. $\beta$ is
the discount factor, $\sigma$ the parameter governing the risk aversion. $G$ maps the disutility from working
into consumption units and is given by $G(l_t) = \omega_1 l_t^{\omega_2}$. $N_t$ is the number of varieties in the whole
economy in period $t$.\footnote{I assume this form of disutility originally introduced in Greenwood, Hercowitz, and Huffman (1988) here as, for
e.g. Correia, Neves, and Rebelo (1995) and Neumeyer and Perri (2005) have shown that it helps generating the
right comovements in small open economy models when it comes to hours worked. $G$ is multiplied by $N_t$ to ensure
that labor supply shows no trend along as the economy grows. One way to justify this assumption is to assume $G(l_t)$
measures forgone home production as in Benhabib, Rogerson, and Wright (1991), and the productivity of the home
sector grows at the same rate as the rest of the economy.}

In order to transfer resources across time, the household can invest in capital ($i_t$) and buy and sell
shares in intermediate goods producers ($s_t^i$) at a price $P_t^i$. He can also use his income to buy units of
the consumption good. The household’s income consists of multiple sources. For each unit of labor
supplies he earns a wage of $w_t$. Each unit of capital ($k_t$) generates capital services which earns a
payment of $r_t$. He receives profit income $\pi_t^i$ from the $i^{th}$ of the intermediate goods producers. He is
taxed lump sum by the government ($T_t$) and profits from the final good producer and the importers.
The resulting budget constraint in sequential form is given by

\[
c_t + i_t + \int_0^{N_t} s_t^i P_t^i di + T_t = w_t l_t + r_t k_t + \int_0^{N_{t+1}} s_{t+1}^i (\pi_t^i + P_t^i) di + \Pi_t;
\]

\[k_t, s_t^i \geq 0.
\]

The laws of motion for capital and varieties are given by

\[k_{t+1} = (1 - \delta_k) k_t + \phi_k \left( \frac{i_t}{k_t} \right) k_t;
\]

\[\delta_k \text{ are the depreciation rates of capital. } \phi_k \text{ is an adjustment functions that capture the cost of adjusting the capital stock. I use the functional form } \phi_k(.) = a_1 k_t^{\eta} + a_2,
\]

as in Jermann (1998).

The household maximizes his utility by choosing investment, share holdings, labor supply and consumption subject to the budget constraint and the law of motions taking prices and government policies as given:

\[\max_{(l_t)_{t=0}^\infty, (c_t)_{t=0}^\infty, (s_t^i)_{t=0}^\infty, (k_t)_{t=0}^\infty, (\pi_t^i)_{t=0}^\infty} \mathbb{E}_0 \sum_{t=0}^\infty \beta_t \mathbb{H} \left( c_t - N_t G(l_t) \right)^{1-\sigma}.
\]

s.t. \(c_t + i_t + \int_0^{N_t} s_t^i P_t^i di + T_t = w_t l_t + r_t k_t + \int_0^{N_{t+1}} s_{t+1}^i (\pi_t^i + P_t^i) di + \Pi_t, k_t, s_t^i \geq 0
\]

\[k_{t+1} = (1 - \delta_k) k_t + \phi_k \left( \frac{i_t}{k_t} \right) k_t;
\]

His optimal choices are therefore characterized by the constraints and the following first-order conditions:

\[N_t G'(l_t) = \frac{w_t}{1 + \tau_t}
\]

\[\frac{u'(c_t)}{\phi_k'(\frac{c_t}{k_t})} = \beta_t H \mathbb{E}_t u'(c_{t+1}) \left( r_t + \frac{1}{\phi_k'(\frac{i_t}{k_t})} (1 - \delta_k) - \frac{i_{t+1}}{k_{t+1}} + \phi_k'(\frac{i_{t+1}}{k_{t+1}}) \right)
\]

\[u'(c_t) P_t^i = \beta_t H \mathbb{E}_t u'(c_{t+1}) (P_{t+1}^i + \pi_{t+1}^i)
\]

The first one governs the intratemporal tradeoff between labor and consumption. The second one is the Euler equation for capital, while the last one is the Euler equation for varieties.

\[^{13}\text{In equilibrium these are zero.}\]
3.2.2 Final Good Producer

The final good is produced and sold by a representative firm. Its production technology is given by

\[ Y_t = a_t(K_t^\alpha L_t^{1-\alpha})^{1-\xi} M_t^\xi. \]

\( L_t \) and \( K_t \) are labor and capital services rented by the firm in competitive markets. \( a_t \) is a stationary neutral technology shock that follows an AR(1) process in logs:

\[ \log(a_t) = \rho_a \log(a_{t-1}) + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_a) \text{ and } \rho_a \in [0, 1). \]

\( \mathcal{N}(0, \sigma_a) \) denotes a normal distribution with mean zero and standard deviation \( \sigma_a \). \( M_t \) is a composite of all the varieties of intermediate goods the final good producer purchased. It is defined by a Dixit-Stiglitz aggregator:

\[ M_t = \left[ \int_0^{N_t} \frac{1}{m_{i,t}} \, di \right]^{1/\nu}. \]

\( m_{i,t} \) is the quantity of intermediate good \( i \in [0, N_t] \) purchased by the final good producer at the price \( p_{i,t} \). The final good producer operates in competitive markets for the final good and capital and labor services. He faces monopolistic price setters for each variety of the intermediate good which means he also takes the prices for the varieties as given. As he makes no intertemporal decisions, he solves a sequence of optimization problems of the form

\[
\max_{Y_t, K_t, L_t, (m_{i,t})_{i \in [0,N_t]}} \quad Y_t - r_t K_t - w_t L_t - \int_0^{N_t} p_{i,t} m_{i,t} \, di
\]

s.t.

\[ Y_t = a_t(K_t^\alpha L_t^{1-\alpha})^{1-\xi} M_t^\xi \]

\[ M_t = \left[ \int_0^{N_t} \frac{1}{m_{i,t}} \, di \right]^{1/\nu}. \]

His optimal choices are therefore characterized by the constraints and the following first-order conditions:

\[ w_t = (1 - \alpha)(1 - \xi) \frac{Y_t}{L_t} \]

\[ r_t = \alpha(1 - \xi) \frac{Y_t}{K_t} \]
\[ p^i_t = a_t(K^i_t L^i_t)^{1-a} \xi \nu \left[ \int_0^{N_t} m^i_t d\xi \right]^{\nu-1} \frac{1}{\nu} m^{i-1}_t. \] (3)

3.2.3 Intermediate Goods Producers

Intermediate goods producers are monopolistic price setters for their variety of the intermediate good facing a demand function \( D_{it}(\cdot) \). They have to supply the goods demanded at the price they chose. The producers create intermediate goods using final and foreign goods in the production technology

\[ m^i_t = (\theta_I(x^h_{i,t})^{\nu_I} + (1 - \theta_I)(x^f_{i,t})^{\nu_I})^{\frac{1}{\nu_I}}, \]

which they buy in competitive markets. The latter good has a price \( P^f_t \) in period \( t \). As he makes no intertemporal decisions the producer solves a sequence of optimization problems of the form

\[
\max_{\pi^i_t, x^f_{i,t}, x^h_{i,t}} D_{it}(p^*_i)\pi^i_t - x^h_{i,t} - P^f_t x^f_{i,t}
\]

s.t. \( D_{it}(p^*_i) = (\theta_I(x^h_{i,t})^{\nu_I} + (1 - \theta_I)(x^f_{i,t})^{\nu_I})^{\frac{1}{\nu_I}} \)

\[ \pi^i_t = D_{it}(p^*_i)\pi^i_t - x^h_{i,t} - P^f_t x^f_{i,t}. \]

For later reference, I state the cost of producing a unit of good \( i \), which is obtained by solving the corresponding cost minimization problem:

\[ P^c_t = \frac{1 + P^f_t (\frac{P^f_t \theta_I^{\nu_I}}{1 - \theta_I})^{\frac{1}{\nu_I}}}{[\theta_I + (1 - \theta_I)(\frac{P^f_t \theta_I^{\nu_I}}{1 - \theta_I})^{\frac{1}{\nu_I}}]^{\frac{1}{\nu_I}}}. \]

The optimal choice of the price is then characterized by the following first-order conditions:

\[
(D^i_t)'(p^*_i)\pi^i_t + D^i_t(p^*_i) = P^c_t (D^i_t)'(p^*_i).
\]

while the demand for the two inputs is characterized by

\[
x^h_{i,t} = \frac{D^i_t(p^*_i)}{[\theta_I + (1 - \theta_I)(\frac{P^f_t \theta_I^{\nu_I}}{1 - \theta_I})^{\frac{1}{\nu_I}}]^{\frac{1}{\nu_I}}} x^h_{i,t},
\]

\[
x^f_{i,t} = (\frac{P^f_t \theta_I}{1 - \theta_I})^{\frac{1}{\nu_I}} x^h_{i,t}.
\]

\[^{14}\text{In equilibrium the demand function is given by Equation 3.}\]
At the end of each period, each intermediate good producer has a probability of dying, which is given by \( \delta_n \).

### 3.2.4 Importers

Foreign goods are produced by the representative importer from a continuum \([0, 1]\) of foreign intermediate goods according to the technology

\[
X_f^t = \left( \int_0^1 (X_{j,t})^{\nu_f} \, dj \right)^{\frac{1}{\nu_f}}.
\]

Each unit of each variety of the good costs \( p_m \). \( \nu_f \) is assumed to be between 0 and 1 making the intermediate foreign goods imperfect substitutes. A fraction of \( \theta_f \) of varieties of those goods is subject to a working capital constraint. If the country is not in default, the importer has to pay for the good in advance facing an interest rate of \( 1 + r_f \). These goods are not traded if the country is in default.

This setup follows Mendoza and Yue (2012).

If the country is not in default, the importer is therefore solving the problem

\[
\max_{(X_{j,t}, X_f^t)_{j \in [0, 1]}} \quad P_f^t X_f^t - p_m \left[ (1 + r_f) \int_{\theta_f}^1 X_{j,t} \, dj + \int_{\theta_f}^1 \nu_f \, X_{j,t} \, dj \right]
\]

s.t. \( X_f^t = \left( \int_0^1 (X_{j,t})^{\nu_f} \, dj \right)^{\frac{1}{\nu_f}} \).

If the country is in default, the importer is solving

\[
\max_{(X_{j,t}, X_f^t)_{j \in [0, 1]}} \quad P_f^t X_f^t - p_m \left[ \int_{\theta_f}^1 X_{j,t} \, dj \right]
\]

s.t. \( X_f^t = \left( \int_{\theta_f}^1 (X_{j,t})^{\nu_f} \, dj \right)^{\frac{1}{\nu_f}} \).

Solving the first problem, one can show that

\[
P_f^t = p_m \left[ (1 - \theta_f) + \theta_f (1 + r_f)^{\nu_f^{-1}} \right]^{1 - \frac{1}{\nu_f}}.^{15}
\]

\(^{15}\)While there are also some conditions related to the demand for the two types of import, I am not showing them here as they are not used in computation.
In the case of a default, one instead finds

\[ P^f_t = p_m [1 - \theta_f]^{1 - \frac{1}{\eta f}}. \]

### 3.2.5 Entrepreneurs

Each period there is a continuum of risk-neutral entrepreneurs who live for a single period. They can borrow resources to invest into the adoption of new technologies. If entrepreneurs spend enough resources, they can set up a new intermediate goods producers that they can then sell to the household to cover their costs and consume their profits. Entrepreneurs are not allowed to die in debt. There is no constraint on entry into adoption. Following Comin and Gertler (2006) and Kung and Schmid (2012) I assume that the cost of adopting a new variety is given by

\[ 1 \alpha n \left( \frac{S_t}{N_t} \right)^{\eta n}, \]

where \( S_t \) is aggregate spending on adoption in period \( t \). It is assumed, following the previously mentioned papers, that entrepreneurs do not take the effect of their choices on present or future costs of adoption into account, so that the dependence of adoption works as a congestion externality. Free entry then implies that the number of new varieties is determined by

\[ \frac{1}{\alpha n} \left( \frac{S_t}{N_t} \right)^{\eta n} = P^i_t, \]

where \( P^i_t \) is the price of generic new variety.\(^{16}\)

This formulation implies the following aggregate law of motion for the measure of varieties

\[ N_{t+1} = (1 - \delta_n) N_t + a^n \left( \frac{S_t}{N_t} \right)^{1 - \eta n} N_t. \]

### 3.2.6 Private Equilibrium

I first define a private equilibrium of the model as an equilibrium of the economy taken government policies as given. This simplifies the description of the government’s problem. Later, I define an equilibrium of the whole economy.

A **Private Equilibrium** of the economy given choices of the government for government spending, debt, transfers, taxes and default \((g_t, b_t, T_t, d_t)_{t=0}^{\infty}\) are sequences (depending on realizations of the

\(^{16}\)Here, I implicitly impose equality of the price of each variety, which holds in equilibrium. Instead of introducing short-lived entrepreneurs, I could replace the representative agent with a continuum of identical households, who all compete in a free-entry market for the introduction of new goods. The results would be the same.
stochastic process and the choices of the government) of quantities

$$(i_t, l_t, c_t, k_{t+1}, (s^i_{t+1})_{i\in[0,N_{t+1}]}, (\pi^i_t)_{i\in[0,N_{t+1}]}, Y_t, K_t, L_t, (m^i_t)_{i\in[0,N_t]}, (x^f_{l,t})_{i\in[0,N_t]}, (x^h_{l,t})_{i\in[0,N_t]}, (X_{j,t}, X^f_t)_{j\in[0,1]}, S_t, N_t, (D^i_t)_{i\in[0,N_{t+1}])}_{t=0}^{17}$$

and prices

$$(p^*_t, (P^i_t)_{i\in[0,N_{t+1}]}, w_t, r_t, (p^i_t)_{i\in[0,N_{t+1}]}, P^f_t, P^c_t)_{t=0}^{\infty}$$
such that

- Given the prices, free entry decisions for entrepreneurs and government policies

  $$((i_t)_{t=0}^{\infty}, (l_t)_{t=0}^{\infty}, (c_t)_{t=0}^{\infty}, (k_{t+1})_{t=0}^{\infty}, ((s^i_{t+1})_{i\in[0,N_{t+1}])}_{t=0}^{\infty})$$
solves the household’s problem;

- Given the prices, $(\pi^i_t)_{i\in[0,N_{t+1}]})_{t=0}^{\infty}$ and government policies $(Y_t, K_t, L_t, (m^i_t)_{i\in[0,N_t])}$ solves the final good producer’s problem for all $t \in \{0, 1, 2, 3, \ldots\}$;

- Given the prices and government policies $(\pi^i_t, x^f_{l,t}, x^h_{l,t}, p^*_t)$ solves the intermediate goods producer i’s problem for all $t \in \{0, 1, 2, 3, \ldots\}$;

- Given the prices and government policies $(X_{j,t}, X^f_t)_{j\in[0,1]}$ solves the importer’s problem for all $t \in \{0, 1, 2, 3, \ldots\}$;

- Given the prices and government policies $S_t$ fulfills the free entry condition;

- Markets clear for all $t$:

  1. Good market: $Y_t = c_t + i_t + \int_0^{N_t} x^h_{l,t} di + p_m (1 + r^f) \int_0^{\theta_f} X_{j,t} dj + p_m \int_0^{1} X_{j,t} dj + T_t$;

  2. Labor market: $l_t = L_t$;

  3. Shares markets: $s^i_t = 1$ for all $i$;

  4. Intermediate goods markets: $m^i_t = D^i_t (p^*_t)$ for all $i$;

  5. Import market: $X^f_t = \int_0^{N_t} x^f_{l,t} di$;

- $(S_t, N_{t+1})_{t=0}^{\infty}$ fulfills $N_{t+1} = (1 - \delta_t) N_t + a^n \left( \frac{s_t}{N_t} \right)^{1-\eta^n} N_t$;

- Price predictions in intermediate goods markets are consistent: $p^i_t = p^*_t$. 

---

$^{17}$ $D^i_t$ a function of the price choice of the intermediate goods producer during period $t$. 
3.2.7 Government

For simplicity, the government of our small open economy is modeled as an infinitely lived agent. The government has preferences over government consumption \( (g_t) \), private consumption and labor. These are represented by the following specification:

\[
E_0 \sum_{t=0}^{\infty} \beta^t G \left[ \chi \left( \frac{(g_t)^{1-\sigma}}{1-\sigma} + (1-\chi) \left( c_t - N_t G(l_t) \right)^{1-\sigma} \right) \right].
\]

\( c_t \) and \( l_t \) are chosen by the household and the government directly controls \( g_t \). The government can raise lump sum taxes. In each period, when the country is not excluded from the financial market, the government can decide to default on the bonds and fail to repay them. If not in default, the government can borrow in international financial markets with a long-term bond. Suppose the government has borrowed \( B_t \). A fraction \( \lambda \) of the bonds comes due and is repaid. The government has to repay the coupon \( z \) on the remainder and rolls them over to the next period. It can, in addition, borrow additional resources or repay debt. This happens based on a market price schedule \( q_t \), which depends on the state and the choices of the small open economy. This setup for the long-run debt follows Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012) and Chatterjee and Eyigungor (2012). Those authors also show that bonds with a duration of more than one period improve the quantitative performance of default models. If the government decides to default, it does not pay the due amount of the debt or the coupon, and its debt is set to zero. However, the country is excluded from international financial markets during this period and only reenters it in any of the following periods with probability \( \phi \). I denote the choice to default by \( d_t = 1 \) and the choice not to default by \( d_t = 0 \). If a country is excluded, I restrict \( d_t \) to be 1. The budget constraint of the government in default is given by

\[
g_t = T_t.
\]

If the government is not in default, its budget constraint is

\[
g_t + b_t = T_t + b_t(\lambda + (1 - \lambda)z) + q_t[b_{t+1} - (1 - \lambda)b_t].
\]

The government initially lacks the ability to commit to a sequence of policies. This imposes that

18A previous version of this paper analyzed the case of proportional taxes on consumption. The main results are robust to this change. I chose to use lump sum taxes to simplify the exposition. A consumption tax introduces distortions into the household’s decisions. This adds distortion smoothing as a further motive of optimal policy. It also complicates the intuition for the determinants of adoption and investment, as the timing of the distortions is one of those determinants.
in every period, after any history, its choices must maximize its utility from this point on. The
government maximization problem is therefore given by
\[
\max_{(g_t)_{t=0}^{\infty}, (T_t)_{t=0}^{\infty}, (c_t)_{t=0}^{\infty}, (b_t)_{t=0}^{\infty}, (d_t)_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t^T \left[ \chi \left( \frac{(g_t)^{1-\sigma}}{1-\sigma} \right) + (1 - \chi) \left( \frac{(c_t - N_t G(l_t))^{1-\sigma}}{1-\sigma} \right) \right]
\]
subject to
- \( g_t = T_t \), if the country is in default;
- \( g_t + b_t = T_t + b_t(\lambda + (1 - \lambda)z) + q_t[b_{t+1} - (1 - \lambda)b_t] \), if the country is not in default;
- Given \((g_t)_{t=0}^{\infty}, (T_t)_{t=0}^{\infty}, (b_t)_{t=0}^{\infty}, (d_t)_{t=0}^{\infty}\), there exists a private equilibrium with \((c_t)_{t=0}^{\infty}, (l_t)_{t=0}^{\infty}\) as optimal choice of the household;
- \( \max_{(g_t)_{t=T}^{\infty}, (T_t)_{t=T}^{\infty}, (b_t)_{t=T}^{\infty}, (d_t)_{t=T}^{\infty}} \mathbb{E}_T \sum_{t=T}^{\infty} \beta_t^T \left[ \chi \left( \frac{(g_t)^{1-\sigma}}{1-\sigma} \right) + (1 - \chi) \left( \frac{(c_t - N_t G(l_t))^{1-\sigma}}{1-\sigma} \right) \right] \) for all \( T \).

The last constraint states that the government has to maximize at every point in time, after any
realization of shocks up to that point and given its own and others’ previous choices. While the
two latter constraints look rather complicated, they are easy to handle in the Markov equilibrium
I will employ. The constraint that the government maximizes after every history on the path of
the economy will be directly incorporated into a value function representation without the need to
impose any extra constraints.

### 3.2.8 Foreign Lenders

There is a continuum of foreign lenders. It is assumed that lenders are risk neutral and are able
to borrow and lend at a rate \( 1 + r_f \). They are financially unconstrained and the market for sovereign
debt is competitive. Bonds are therefore priced so that a zero profit condition is met and lender’s
expected returns equal the interest rate they are facing. The price of a bond therefore has to fulfill
the following equation:
\[
q_t = \mathbb{E}_t[\left(1 - d_{t+1}\right) \lambda + (1 - \lambda)z + q_{t+1}] / (1 + r_f).
\]

### 3.2.9 Equilibrium

An Equilibrium of the economy are sequences of choices of the government \((g_t, b_t, T_t, d_t)_{t=0}^{\infty}\), of price
schedules for debt \((q_t)_{t=0}^{\infty}\), of quantities \((i_t, l_t, c_t, k_{t+1}, s_{t+1}^j)_{t \in \{0, N_t + 1\}}, (\pi_t^i)_{t \in \{0, N_t + 1\}}, Y_t, K_t, L_t, (m_t^j)_{t \in \{0, N_t\}}, \)
The profits of any intermediate goods producer are therefore

\[ (x_{i,t}^f)_{i \in [0,N_t]}, (x_{i,t}^h)_{i \in [0,N_t]}, (X_{j,t}^f, X_{j,t}^l)_{j \in [0,1]}, S_t, N_t, (D_t^i)_{i \in [0,N_{t+1}]) \}_{t=0}^{\infty} \]

and prices \((p_t^*, P_t^i)_{i \in [0,N_{t+1}]}\), \(w_t, r_t, (p_t^*)_{i \in [0,N_{t+1}]}\), \(P_t^f, P_t^c\) such that

1. Given \((g_t, b_t, T_t, d_t)_{i \in [0,N_t]}\), \((i_t, t, c_t, k_{t+1}, (s_{t+1}^i)_{i \in [0,N_{t+1}]}), (\pi_t^i)_{i \in [0,N_{t+1}]}\), \(Y_t, K_t, L_t, (m_t^i)_{i \in [0,N_t]}\),
   \((x_{i,t}^f)_{i \in [0,N_t]}\), \((x_{i,t}^h)_{i \in [0,N_t]}\), \((X_{j,t}^f, X_{j,t}^l)_{j \in [0,1]}\), \(S_t, N_t, (D_t^i)_{i \in [0,N_{t+1}]}\) \(t=0\)
   and \((p_t^*, P_t^i)_{i \in [0,N_{t+1}]}\), \(w_t, r_t, (p_t^*)_{i \in [0,N_{t+1}]}\), \(P_t^f, P_t^c\) \(t=0\) form a private equilibrium;

2. \((g_t, b_t, T_t, c_t, l_t, d_t)_{i \in [0,N_t]}\) solves the government problem;

3. \(q_t\) is a bounded solution to \(q_t = \mathbb{E}_t[(1 - d_{t+1})^{\lambda(1-\lambda)(z+g_{t+1})}]\) \(t=0\).

### 3.3 Further Characterization and Discussion

Before calibrating the model and discussing quantitative results, in the next sections I briefly illustrate how the model can generate output losses after a default. I do this by further characterizing an equilibrium. It is easy to check that given the form of the aggregator, any intermediate goods producer will set its price to a constant markup over his cost. Therefore, it is the case that \(p_t^i = \nu P_t^c\). \(P_t^c\) still denotes the cost of inputs into the production of intermediate goods. It is also possible now to derive the quantity of each intermediate good in equilibrium:

\[ m_t^i = \left( \frac{\xi}{\nu P_t^c} (a_t K_t^\alpha L_t^{1-\alpha})^{1-\xi} N_t^{\nu \xi - 1} \right)^{\frac{1}{\nu}}. \]

The profits of any intermediate goods producer are therefore

\[ \pi_t^i = P_t^c (\nu - 1) \left( \frac{\xi}{\nu P_t^c} (a_t K_t^\alpha L_t^{1-\alpha})^{1-\xi} N_t^{\nu \xi - 1} \right)^{\frac{1}{\nu}} \]

and GDP is given by

\[ Y_t = \left( \int_0^{N_t} x_{i,t}^h di + p_m (1 + r) \int_{0}^{\theta_f} X_{j,t}^f dj + p_m \int_{\theta_f}^{1} X_{j,t}^l dj \right) \]

\[ = \left( \frac{\xi}{\nu P_t^c} \right)^{\frac{1}{\nu}} a_t K_t^\alpha L_t^{1-\alpha} N_t^{\frac{\nu - \xi}{\nu \xi - 1}} - P_t^c N_t \left( \frac{\xi}{\nu P_t^c} (a_t K_t^\alpha L_t^{1-\alpha}) N_t^{\nu \xi - 1} \right)^{\frac{1}{\nu}} \]

\(^{19}\) \(D_t^i\) a function of the price choice of the intermediate good producer during period \(t\).

\(^{20}\) The assumption of boundedness is common in the literature and rules out bubbles. It implies \(q_t \in [0, \frac{\lambda(1-\lambda)}{\lambda + \rho}]\) for all \(t\).
The last equation is declining in $P^t_{c}$. It illustrates how in this setting developed by Mendoza and Yue (2012) a default has a temporary effect on output. A default increases the cost of imports. Therefore, the price $P^t_{c}$, which reflects the cost of imported goods, rises and GDP falls. In essence, the mechanism is isomorphic to a drop in TFP of a fixed size. We also see that the economy is growing with $N_t$. Depending on the sum of the exponents of capital and varieties, the economy might not be growing with $N_t$ but with some power of it. In the calibration I will impose constraints that lead to an economy that grows at rate $\frac{N_t+\delta}{N_t}$ and so I will also focus on this case here.

In order to understand how variations in $\pi^t_{c}$ and GDP feed back into the growth rate, we next look at the determination of $S_t$. We start from the free entry condition:

$$\frac{1}{a^n} \left( \frac{S_t}{N_t} \right)^{\eta^n} = P^t_{i}.$$

Standard asset pricing results imply

$$P^t_{i} = \mathbb{E}_t \beta^H \left( \frac{c_{t+1} - N_{t+1}G(l_{t+1})}{c_{t} - N_{t}G(l_{t})} \right)^{-\sigma} \left[ \pi^t_{i+1} + (1 - \delta_n) P^t_{i+1} \right].$$

Combining the two we arrive at

$$(c_{t} - N_{t}G(l_{t}))^{-\sigma} \frac{1}{a^n} \left( \frac{S_t}{N_t} \right)^{\eta^n} = \mathbb{E}_t \beta^H \left( c_{t+1} - N_{t+1}G(l_{t+1}) \right)^{-\sigma} \left[ \pi^t_{i+1} + (1 - \delta_n) \frac{1}{a^n} \left( \frac{S_{t+1}}{N_{t+1}} \right)^{\eta^n} \right].$$

This Euler equation captures the main determinants of the amount of technology adoption. It is a rather complicated object in the full version of the model, but thinking back to the two-period model is helpful at this point. In equilibrium in the default state, $c_t - N_tG(l_t)$ will be lower, as will be $\pi^t_{i}$. The former is generating an income effect, while the latter is generating a substitution effect. We therefore see, that the same forces that are at work in the two-period model are also acting in the full model to generate a reduction in growth. As in the two-period model, there is a positive probability of a recovery, so $c_{t+1} - N_{t+1}G(l_{t+1})$ might revert upward. Therefore, we can expect the right-hand side to shrink more relative to the left-hand side. Both this, and the larger $(c_{t} - N_{t}G(l_{t}))^{-\sigma}$, induce $S$ to fall. The growth rate falls. Later, I will show that this occurs in the calibrated version of the model. This will also show that the crowding in effect of potentially lower taxation after default will not drive the result in the other direction.
4 Model Solution and Calibration

In this section I discuss how I solve the quantitative model numerically, select an equilibrium and calibrate the model.

4.1 Model Solution

As the model is rather complex, I will have to solve it numerically. I will focus on the Markov equilibria in the natural state variables \((z_t, k_t, N_t, b_t)\). To generate a stationary problem, I transform the model by normalizing all growing variables in Period \(t\) by dividing by \(N_t\). This adjustment also allows me to drop \(N_t\) as a state. In order to solve the model, the problem of the government is transformed into a recursive representation. The model is solved numerically using a mixture of value and policy function iteration. The TFP shocks are approximated using the method described in Rouwenhorst (1995), while policy and value function are approximated using linear and Chebychev interpolation (see, for example, Judd (1998)). In each iteration, I first solve the household’s and firms’ optimal choices for all the possible government choices today using a policy function iteration step that utilizes the Euler equations. Then I find the optimal policy of the government given those household and firm choices using a value function iteration step given the choices of the other agents. Using the derived default decisions, the debt price function is updated. After convergence is achieved, it is verified that the choices of the household and the firms are indeed a maximizer. Details about the computation are contained in Appendix B. I also discuss some of the other methods I used to compute equilibria to check the robustness of my approach.

The model is an infinite horizon game between multiple players. As such, it is prone to have multiple equilibria. In order to ensure a meaningful comparison between different policies or different calibrations, I use an equilibrium selection rule. Following, for example, Hatchondo and Martinez (2009), I select a unique equilibrium by solving the finite horizon problem and increasing the horizon until value functions and policy functions for the first and second period are sufficiently close. I then use the policy and value functions in the first period as equilibrium objects for the infinite horizon model. Putting it differently, I start from the solution to the static version of my model and iterate backward until convergence. While this tends to increase the computation time compared to starting from a “good” starting guess, like the solution found for a different calibration, it is necessary to employ equilibria consistently between computations.
4.2 Calibration

A model period is assumed to be a quarter. In order to calibrate the model, I start by fixing some parameters to conventional values in the literature. The remaining ones are set to match empirical observations of a set of recent defaulters.\footnote{The countries are Argentina, Ecuador, Indonesia, Mexico, Peru, Philippines, Russia, South Africa and Thailand. They are chosen as quarterly data for many aggregate series is available for them from the \textit{International Financial Statistics (2013)}.} I will compare the model behavior along business cycle dimensions to the moments of those countries in the next section. I set the risk aversion of the household and the government ($\sigma$) to 2. The discount factor of the household $\beta_{HH}$ is set to 0.99 and the interest rate faced by the foreign lenders $(1 + r^f)$ is chosen to be 1.01, both implying a yearly interest rate of four percent. Following Mendoza (1991) and Correira, Neves, and Rebelo (1995), I set the curvature parameter in the disutility of working ($\omega_2$) to 1.455.

I set the persistence of the TFP shock $\rho_a$ to 0.95, a standard value in the literature. Data that is provided by the United Nations and other sources (for example United Nations (1994-2013) and Lim (1996)) point to a small labor share for the countries considered. But, as discussed in Gollin (2002), low labor shares in the data submitted, for example, to the United Nations, for many countries are the result of the fact that the income of most self-employed and family workers is treated as capital income. He further shows, that for countries with sufficient data to make adjustments, the obtained labor shares tend to be close to the range of values assumed for the United States. I therefore set the labor share $1 - \alpha$ to be 0.7. The share of intermediate goods ($\xi$) is set to 0.45. This share is determined by taking the average over the share of intermediate goods in total output for the countries I considered. I obtain those shares by using data from United Nations (1994-2013). In order to have a balanced growth path in the model without shocks,\footnote{The existence of such a path is needed to be able to transform the model into a stationary environment for the computation.} it has to be true that $\alpha + \frac{\nu^f - \xi}{1 - \xi} = 1$. This implies that the elasticity of varieties has to be 1.856. I use the evidence from Mendoza and Yue (2012) to set elasticity estimates for the substitutability of goods given a lack of the necessary data for all the countries. This results in $\nu^f = 0.59$, $\theta_f = 0.62$ and $\nu^i = 0.65$. I set the depreciation rate of capital ($\delta_k$) to 0.02, a standard value. As I have no solid evidence on the destruction rate of good varieties ($\delta_n$), I follow Bilbiie, Ghironi, and Melitz (2012) and set the parameter to 0.025. This implies an annual destruction rate of ten percent. Following Comin and Gertler (2006) and Kung and Schmid (2012) I set exponent of the variety adjustment function $\eta^a$ to 0.83.

The price of the imported good ($p_m$) is normalized to 1. The reentry probability into financial
markets ($\phi$) is set to 0.083 following Mendoza and Yue (2012). The parameters in the long-run debt rule are set to $z = 0.03$ and $\lambda = 0.05$ following Chatterjee and Eyigungor (2012).

Seven parameters remain to be determined: $a^n$ in the variety adjustment function, $\eta^k$ in the capital adjustment cost function, $a^k_1$ and $a^k_2$ are set conditional on this value as explained in for example Van Binsbergen, Fernández-Villaverde, Koijen, and Rubío-Ramírez (2012). $\beta_G$, the volatility of TFP $\sigma_a$, the fraction of imported varieties subject to the working capital constraint $\theta_f$, the scale parameter of the disutility of work $\omega_1$, and the weight of government consumption in the governments utility function $\chi$. These parameters are set in order to match the following moments: the median of the standard deviation of investment (0.1048) and output (0.0358) for my recent group of defaulters after applying the Hodrick Prescott filter with a parameter of 1600, the average quarterly growth rate of these countries of 0.88 percent, the average government expenditure to GDP ratio of 10.76 percent, a default frequency of three times a century, a labor supply level of 1 and an amount of working capital of a fraction of 0.024 of GDP. The last number is based on the World Development Indicators (2013). I take the short-term external debt-to-GDP ratio and use it as a proxy for the amount of working capital. Short-term debt contains both claims on the private as well as on the government sector and there is no information on how it splits between the two groups. I therefore assume that the fractions are the same as for total external debt and only allocate that fraction of the short run external debt to the above target. Table 4 summarizes the calibration.

5 Results

This section discusses the quantitative performance of the model and studies the role of growth losses as part of the cost of default. I start by discussing some policy functions to demonstrate how debt, capital, shocks and default state influence the decisions taken by the government and the private sector. I then compare simulation results from the model with moments of emerging market economies. The next subsection performs an event study around defaults with the model and sets them in relation to default events in the data. Finally, I look at the role of growth as a default cost channel.

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$^{23}a^k_1$ and $a^k_2$ are set conditional on this value as explained in for example Van Binsbergen, Fernández-Villaverde, Koijen, and Rubío-Ramírez (2012).

$^{24}$The data is taken from International Financial Statistics (2013).
Table 4: Calibration

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor of the Household</td>
<td>$\beta_{HH}$</td>
<td>0.99</td>
</tr>
<tr>
<td>Risk Aversion of Household &amp; Government</td>
<td>$\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>Scale of Disutility of Labor</td>
<td>$\omega_1$</td>
<td>0.35</td>
</tr>
<tr>
<td>Curvature of Disutility of Labor</td>
<td>$\omega_2$</td>
<td>1.455</td>
</tr>
<tr>
<td>Discount Factor of Government</td>
<td>$\beta_G$</td>
<td>0.91387</td>
</tr>
<tr>
<td>Utility Weight of Government Consumption</td>
<td>$\chi$</td>
<td>0.07</td>
</tr>
<tr>
<td>Interest Rate of Lender</td>
<td>$r_f$</td>
<td>1.01</td>
</tr>
<tr>
<td>Capital Share in Production Function</td>
<td>$\alpha$</td>
<td>0.3</td>
</tr>
<tr>
<td>Intermediate Goods Exponent in Production Function</td>
<td>$\xi$</td>
<td>0.45</td>
</tr>
<tr>
<td>Elasticity of Varieties Aggregator</td>
<td>$\nu$</td>
<td>1.8556</td>
</tr>
<tr>
<td>Slope of Capital Adjustment Function</td>
<td>$a_k^1$</td>
<td>0.4887</td>
</tr>
<tr>
<td>Level of Capital Adjustment Function</td>
<td>$a_k^2$</td>
<td>-0.012</td>
</tr>
<tr>
<td>Slope of Variety Adjustment Function</td>
<td>$a^n$</td>
<td>0.1475</td>
</tr>
<tr>
<td>Exponent of Capital Adjustment Function</td>
<td>$\eta^k$</td>
<td>0.3</td>
</tr>
<tr>
<td>Exponent of Variety Adjustment Function</td>
<td>$\eta^n$</td>
<td>0.83</td>
</tr>
<tr>
<td>Depreciation Rate of Capital</td>
<td>$\delta_k$</td>
<td>0.02</td>
</tr>
<tr>
<td>Depreciation Rate of Varieties</td>
<td>$\delta_n$</td>
<td>0.025</td>
</tr>
<tr>
<td>Elasticity of Foreign Intermediate Goods Aggregator</td>
<td>$\nu^f$</td>
<td>0.59</td>
</tr>
<tr>
<td>Elasticity of Intermediate Goods Inputs Aggregator</td>
<td>$\nu^I$</td>
<td>0.65</td>
</tr>
<tr>
<td>Domestic Share of Final Good</td>
<td>$\theta$</td>
<td>0.62</td>
</tr>
<tr>
<td>Fraction of Goods Affected by Working Capital</td>
<td>$\theta_f$</td>
<td>0.39</td>
</tr>
<tr>
<td>Price of Foreign Good</td>
<td>$p_m$</td>
<td>1</td>
</tr>
<tr>
<td>Persistence of Technology Shock</td>
<td>$\rho_a$</td>
<td>0.95</td>
</tr>
<tr>
<td>Standard Deviation of Technology Shock</td>
<td>$\sigma_a$</td>
<td>0.0121</td>
</tr>
<tr>
<td>Reentry Probability</td>
<td>$\phi$</td>
<td>0.083</td>
</tr>
<tr>
<td>Duration of Debt</td>
<td>$\lambda$</td>
<td>0.05</td>
</tr>
<tr>
<td>Coupon of Debt</td>
<td>$z$</td>
<td>0.03</td>
</tr>
</tbody>
</table>

5.1 Policy Functions

To better understand the outcomes of the model and to validate the intuition developed in past sections, it is instructive to look at the policy functions conditional on equilibrium taxation and default decisions. I start with the policy function for growth in varieties from this to the next period. In Figure 5, I show how the growth rate of new goods varies with the amount of debt the government is holding relative to the level of development captured by the measure of varieties in the given period. Three lines are depicted on each of the plots and these show the growth rate for different levels of capital. The two plots are for different levels of TFP.

The adoption of new goods and therefore growth is increasing in capital and TFP as they increase the amount of resources the private sector and the government commands. Here, capital has a weaker
Figure 5: Growth Policy Function

Note: The TFP shocks are 2 standard deviations below and above the mean. Capital is shown at its average and two standard deviations above and below. The functions are adjusted to show the growth rate conditional on the no default state if no default occurs at this debt level and the growth rate conditional on default otherwise.

effect than changes in TFP. In the picture, the policy function is shown at the average capital stock and two standard deviations above and below. Growth varies by around 0.2 percentage points. For the TFP shock we see a change of a whole percentage point, when the current realization of the shock moves from two standard deviations below to two standard deviations above the mean.25 A simple explanation for this is that the capital-to-varieties ratio is not very volatile in the model and this therefore results in only small variations in output.

Let us now look at the policy function as a function of debt. Initially the growth rate reacts very little to the debt stock. In some calibrations, it was locally, even weakly, increasing. This is the case because of two income effects: one in the current period and one in the future. The government keeps taxation low and instead uses borrowing to finance its consumption and debt services. As such, households have more disposable income in the current period and are more willing to provide resources for projects that allow them to shift their income to the future. This desire is enhanced as they expect taxation to rise in the future, when the government needs to increase lump sum taxes. For an intermediate range of debt values, growth falls more steeply with the stock of borrowing, as the government is also taxing the household strongly today. In addition, three other effects interact.

25For clarity purposes, standard deviations refer here to the standard deviation of the ergodic distribution.
here. Firstly, the expected reduction of profits in case of a default in the future makes spending on new varieties less profitable. Secondly, two income effects in case of a default compete with each other. On the one hand, a default reduces the debt and tax burden in the future. On the other hand, the default triggers a recession and corresponding output losses. The total effect is a reduction in spending on adoption. Spending on the accumulation of new goods falls drastically at the precise level of debt, at which point the government defaults and is flat thereafter in the policy function, as in default the policy function is independent of debt.

In Figure 6, parts of the policy function for capital are shown for an initial capital stock relative to the measure of varieties. The picture tells the same story as the one for the growth of varieties; as such, I am not going to comment on it in detail.

Figure 6: Capital Policy Function

Note: The TFP shocks are 2 standard deviations below and above the mean. Capital is shown at its average and two standard deviations above and below. The functions are adjusted to show the growth rate conditional on the no default state if no default occurs at this debt level and the growth rate conditional on default otherwise. The functions are adjusted to show the capital stock relative to \( N_t \) tomorrow conditional on the no default state if no default occurs at this debt level and the capital stock relative to \( N_t \) tomorrow conditional on default otherwise.

Finally, we look at the policy function for debt. This is depicted in Figure 7 and constructed in the same way as the previous two.

We can see that the amount of borrowing increases both in capital and the level of TFP. This is the case as the government is more likely to default in bad times when resources are more limited and a larger fraction of current income would be needed to service current debt. As such, the price of debt is higher and the government is therefore more likely to take out loans. Again, variations in

\[^{26}\text{decided not to allow the household to borrow from abroad because of some conceptual issues and to keep the problem numerically simple. If the household would be allowed to save abroad, it should be the case that the income effect, together with the expected lower return on saving at home, would drive the household to invest his income abroad, leading to an even steeper fall of the curve.}\]
TFP lead to stronger responses than variations in capital. We also see that the level of indebtedness increases quickly for low levels of debt as a function of past borrowing and then levels off. For low levels of debt, the government is able to keep taxation low to foster private consumption and investment and, at the same time, can spend substantially on government consumption by means of external borrowing. As the probability of a default in the near future rises, it becomes less attractive for the government to borrow, as the foreign lenders are paying lower prices for the bonds and the government does not like the prospect of a default. The government therefore reduces the rate at which it is increasing its debt stock. When the debt level increases further, the effect reverses and the future debt level rises in the level of current debt. A default in the near future is now likely. Instead of reducing its borrowing, the government gambles in the hope that a good realization of the shocks tomorrow will keep the debt level sustainable. In case of a bad realization it defaults, as the cost of servicing the debt exceeds the benefits, and the debt level drops to zero. Finally, it should be noted that, as the economy grows, the debt-to-varieties ratio actually mildly declines in this region. Therefore, if no default occurs, the relative indebtedness of the country actually falls.

The final figures in this subsection show the price of debt as a function of TFP and debt-to-varieties ratio chosen for the next period for two levels of the capital stock next period. The higher one is two standard deviations above the average level, the other one two standard deviations below.

We see that higher TFP leads to higher prices holding borrowing fixed as a default is less likely.
Finally, a higher capital stock increases the price and reduces the likelihood of a default for the same reason as TFP, but the effect is much weaker.

5.2 Business Cycle Moments

I am now going to briefly assess the calibrated model by comparing moments from the model generated data with the same moments for a set of emerging market economies that went through a default in the time covered by the data available to me. I use seasonally adjusted quarterly data for Argentina, Ecuador, Indonesia, Mexico, Peru, Philippines, Russia, South Africa and Thailand from the International Financial Statistics (2013), the same data used for example by Gordon and Guerron-Quintana (2013). Data for the yearly external debt-to-GDP ratio is taken from World De-
Data counterparts from the model are generated by simulating the model 100 times for 500 periods and discarding the first 250 periods. Table 5 contains the results. The selected moments are commonly looked at in the literature.

Let us first have a look at the data moments. A few things are noteworthy. As it is well documented in the literature, private consumption is more volatile than output for emerging economies. This observation also holds for this sample. To an even more extreme degree, the same is true for public consumption. Finally, the debt-to-GDP ratio is, for most countries, around 30 percent.

The model reproduces some of the moments rather well, some a bit more poorly. I am able to match the median output volatility. Private consumption is less volatile than output. While it is within the bounds of the data counterparts, it is at the lower end. There are two reasons for this outcome. Capital adjustment costs are relatively low, so the household can smooth consumption by

Footnotes:
27 For South Africa and Russia the series start only in 1992.
28 See, for example, Neumeyer and Perri (2005).
### Table 5: Model and Data Moments

<table>
<thead>
<tr>
<th>Country</th>
<th>$100 \times \sigma(y_t)$</th>
<th>$\frac{\sigma(c_t)}{\sigma(y_t)}$</th>
<th>$\frac{\sigma(g_t)}{\sigma(y_t)}$</th>
<th>$100 \times E(\Delta y_t - 1)$</th>
<th>$\rho(y_t)$</th>
<th>$E\left(\frac{Debt}{GDP}\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>4.84</td>
<td>1.22</td>
<td>2.61</td>
<td>1.48</td>
<td>0.88</td>
<td>0.77</td>
</tr>
<tr>
<td>Ecuador</td>
<td>2.14</td>
<td>2.11</td>
<td>3.87</td>
<td>5.58</td>
<td>0.81</td>
<td>0.61</td>
</tr>
<tr>
<td>Indonesia</td>
<td>5.83</td>
<td>1.53</td>
<td>1.68</td>
<td>1.56</td>
<td>0.98</td>
<td>0.77</td>
</tr>
<tr>
<td>Mexico</td>
<td>2.84</td>
<td>1.38</td>
<td>2.88</td>
<td>3.31</td>
<td>0.58</td>
<td>0.52</td>
</tr>
<tr>
<td>Peru</td>
<td>5.57</td>
<td>1.03</td>
<td>2.00</td>
<td>2.14</td>
<td>0.82</td>
<td>0.63</td>
</tr>
<tr>
<td>Philippines</td>
<td>3.22</td>
<td>0.77</td>
<td>3.51</td>
<td>2.39</td>
<td>0.74</td>
<td>0.48</td>
</tr>
<tr>
<td>Russia</td>
<td>3.68</td>
<td>1.07</td>
<td>2.65</td>
<td>1.96</td>
<td>0.88</td>
<td>0.47</td>
</tr>
<tr>
<td>South Africa</td>
<td>1.82</td>
<td>1.47</td>
<td>3.55</td>
<td>2.22</td>
<td>0.62</td>
<td>0.72</td>
</tr>
<tr>
<td>Thailand</td>
<td>3.87</td>
<td>0.96</td>
<td>3.13</td>
<td>1.13</td>
<td>0.92</td>
<td>0.64</td>
</tr>
<tr>
<td>Model</td>
<td>3.68</td>
<td>0.96</td>
<td>2.94</td>
<td>0.85</td>
<td>0.88</td>
<td>0.75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$100 \times E(\text{Spread})$</th>
<th>Frequency Default</th>
<th>$100 \times \sigma(\text{Spread})$</th>
<th>Working Capital</th>
<th>$\frac{\sigma(l_t)}{\sigma(y_t)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>7.83</td>
<td>3 in 100 years</td>
<td>3.33</td>
<td>2.40% of GDP</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Note: All series with the exception of growth, spreads and the debt to GDP ratio are logged and hp-filtered with parameter 1600. $\sigma$ indicates the standard deviation, $\rho$ the autocorrelation, $E$ the average, $\Delta$ a growth rate. Spreads and debt to GDP ratios are annualized. Moments for the countries are based on International Financial Statistics (2013) and World Development Indicators (2013) - see text. $y$ denotes output, $c$ private consumption, $g$ public consumption, $i$ investment. $l$ labor.

adjusting investment. I could therefore improve the fit of consumption while reducing the volatility of investment by increasing those costs. The other reason is due to the absence of other frictions and shocks that are often included into small open economy models and generate more volatile consumption. Government consumption is even less volatile. This is the case as the GHH cost term makes private consumption more volatile. This leads the household to increase private consumption more strongly in good times to counteract the rise in the disutility. Looking at the data, it is clear that the model would need stronger frictions to match public consumption. This would require a more involved setup than modeling preferences about the two consumption types as symmetric.

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29 The problem is common to real business cycle setups in a default setting. See, for example, the discussion in Roldán-Peña (2012) or Gordon and Guerron-Quintana (2013).

30 In a previous version, I allowed for a higher adjustment cost for capital to allow for more volatile consumption of both forms. While the main message of this section was robust to this change, that version of the calibrated model did a worse job of matching the default dynamics in the next section. This either suggests that the countries listed in this subsection are not representative of the average country that defaulted between 1970 and 2007 or that I would
The model matches investment well and generates the same average growth rate as that depicted in the data. The average debt-to-GDP ratio is lower than that portrayed in the data. This is not uncommon in the literature. The autocorrelation of output growth matches the data counterpart well although it was not targeted. This is reassuring as it shows that introducing endogenous growth does not make the growth rate too volatile. While I lack data on spreads on government bonds going far enough back, the numbers I obtained for the mean spread and the volatility of the spread are in the range of normal emerging economies, as can be seen, for example, in the work of Roldán-Peña (2012). Working capital matches its constructed data counterpart. The model also delivers the targeted default frequency. Labor volatility is within the bounds for emerging markets in Neumeyer and Perri (2005). In total, the fit seems reasonably good.

5.3 Default Dynamics

Here, I want to see how the model-generated time series compares relative to their counterpart in the data in the event of a default. I take the series for logged real GDP, logged real gross capital formation, logged real private consumption and logged real government consumption in the ten years before and after a default. To make them comparable to each other, I subtract a linear trend that grows with the average growth rate of the series in the ten years before the default for each country and normalize the value of each series in the period before the default to be zero for all countries as well. The means across countries for the four series are shown in Figure 10, together with one standard deviation error bands.

Let us focus first on the mean of each series. We see a sizable drop in all four series in the years around the default. None of the series returns to its level right before the default. As the series are detrended, this implies that the all four variables stay below the pre-crisis trend. While GDP, private consumption and investment seem to stabilize, government consumption keeps falling for the entire time period. This default coincides with a severe depression in the average country. We also see wide error bands, indicating a substantial heterogeneity in the development of the economy after the sovereign debt crisis year. This is partially the result of not controlling for any other factors, like a banking crisis, that might have occurred in some of the countries that had a default.

In order to compare the data to the model, I simulate the model 100 times for 500 periods and need frictions that work outside a default to obtain a better fit.

31See the discussion in Chatterjee and Eyigungor (2012). They manage to get a high average debt-to-GDP ratio in their model by using long run debt and a state dependent exogenous default cost process.
discard the first 250 periods. I then select all default events from the remaining series elements, i.e. I record the series for consumption, investment and GDP between 40 quarters before and after the default. I aggregate the quarterly data from the model to years by randomly selecting the default quarter to be one of the four quarters in a year. Finally, I detrend the resulting yearly data using the average growth rate in the 10 years before a default and normalize the value of each series in the period of the default to zero. Figure 11 compares the mean of the events and one standard deviation confidence intervals to the mean of the data.

The means of the model and the data are reasonably close for GDP and private consumption before, and especially after, the default. The model does not generate the transition of the series towards the default in a sufficiently smooth manner. Investment is rising more on average in the data in the decade before a default and then falls strongly right before the default. The model is not fully able to reproduce the gradual slowdown before the default. It takes typically a sequence of negative TFP innovations to trigger a default in the model, but even so it cannot generate the pattern in the data. In the default period, another negative TFP shock hits and the government’s decision to stop servicing its debt triggers a further sharp decline in economic activity.

After the default, the investment series are still close and the data mean lies within the error
bands. However, it is also clear that the model underpredicts the fall in investment. Gross capital formation seems in total to be more volatile around a default. One explanation for this could be that, in reality, pre-default booms coincided with lending booms for the private sector. Allowing the household to borrow from abroad and benefit from low interest rates might make investment more volatile. Finally, the model fails to predict the magnitude of the drop in government consumption, which keeps falling in the years after the default and leaves the one standard deviation error bands in year eight after the default. The data suggests a longer lasting and stronger fiscal adjustment after a country defaulted. The model also underpredicts government spending before the default. It has a very simplistic view of the government’s spending and taxation decisions, which might be reflected in this failure to match the long-run behavior. Despite this caveat, it seems that the model is doing a decent job of capturing the average default dynamics. Additional standard elements, like private borrowing during a pre-default boom, may help the model to achieve a closer fit - for example, with investment - but would substantially complicate the numerical solution. For my exercise, it is especially reassuring that the model is able to capture the post-default trends well.
5.4 Growth and the Cost of Default

In this subsection I want to quantify the effect of incorporating persistent output losses on the decisions about how much to borrow and when to default. The model gives me a natural environment to do this as it allows me to separate changes in the growth rate caused by the decision of the government to default from other sources of fluctuation in GDP. In my setting, the other sources of variation in technology adoption and capital accumulation are TFP shocks, the other decisions that the government makes and changes in expectation over those. While ultimately all decisions are driven by TFP shocks, I am still able to separate, for example, the direct effect of those shocks from the response of the government by comparing policy functions. In fact, by comparing the policy functions for the growth of varieties in the default period, both for the case of a default and no default, I can see that the decision of the government to default leads to a drop in growth of varieties and therefore long-run growth by 0.218 percentage points on average in the quarter of the default alone.\textsuperscript{32}

My measures of the cost of a default are the yearly default frequency and the average quarterly debt-to- GDP ratio when the country is not in default. Holding everything else fixed, a higher cost of default implies that a country has a better ability to borrow, as lenders know that the country is less likely to default. Both numbers have to be used together, as the government in the model reacts to changes in the environment. The higher ability to borrow might lead the government to borrow more which, in turn, might lead to higher default rates, as it becomes more expensive to service the larger amount of debt. Looking at both numbers together prevents us from being fooled by potentially modest changes in one of them.

It might seem natural to study the role of endogenous growth by simply removing this element from the model, setting the economy to grow at an exogenous fixed rate that is equal to the one in the main model and comparing the predictions of the two versions of the model. However, this approach would lead to misleading results. Endogenous growth does not only change the effects of a default on the economy, it also changes the effects of TFP shocks. As technology adoption is positively correlated with higher realizations of the TFP process, an increase in TFP does not only mean that output rises now and in the future because of the positive serial correlation in TFP, it also implies that the economy will grow faster as the introduction of new varieties rises. This means

\textsuperscript{32}While the initial total drop in GDP is larger, TFP, labor supply and capital eventually recover, resulting in a period of increased GDP growth. In contrast, the loss in intermediate good varieties reduces the level of GDP permanently.
that the government is richer in the future after a positive innovation. Foreign borrowing allows the government to transfer resources from the future to the present and, as the government wants to do this because of its low discount factor, not being in default becomes more valuable and the likelihood of a default falls. A default is typically triggered by a sequence of negative TFP shocks, so the same logic implies that access to foreign lending becomes less attractive and that the probability of a default rises. It is therefore natural to expect that a model with constant growth rate produces less defaults and a lower debt-to-GDP ratio.\textsuperscript{33}

Instead, I will perform a sequence of simple experiments and comparisons. In the first exercise I introduce a subsidy to the profits of intermediate goods producers in the default state. The subsidy is set such that each intermediate goods producer makes the same profits as if there was no default.\textsuperscript{34} For simplicity, I assume the subsidy is paid by the household through lump sum taxes. As he receives the payment back through profits, his budget set is unchanged. Therefore, the subsidy only eliminates parts of the substitution effect by mitigating the expected reduction of profits in the default state. It leads to mildly higher growth (0.03 percentage points per quarter on average over all states) both in the default state and also when no default occurred, as entrepreneurs and households are forward looking and rational. However, the subsidy affects growth in the default states more strongly, reducing the cost of a default by reducing the gap between adoption in the default and no-default states. The debt-to-GDP ratio, when not in default, falls mildly from 0.48 to 0.42 and the frequency of defaults increases by 0.66 percentage points, showing that defaults become less costly thanks to the subsidy. This also demonstrates that the presence of the long-run cost channel does matter, as the frequency of defaults increases substantially. However, it should be noted that this simple experiment does not totally eliminate the cost due to variation in growth. Both substitution and income effects are still present and they reduce adoption in the case of a default. The income effect works through the reduced income because of the recession, while the substitution effect is triggered by lower expected innovation in the future in response to future income effects. Both increase the long-run losses after a default. In fact, the subsidy only reduces the growth loss by roughly 30 percent, as can seen by comparing the policy functions.

It is not straight forward to remove the remaining income and substitution effects without affecting other parts of the model, like capital accumulation or other costs of default. In addition, the interaction between policy choices and the decisions made in the private sector might also be affected.

\textsuperscript{33} Numerical experiments support this expectation.

\textsuperscript{34} As shown in section 3, debt and taxes do not influence the profits in my model.
Therefore, instead of trying to perform a comparison based on the original model, I modify it slightly. The second exercise consists of replacing the endogenous growth component in the model by a simple rule that was calibrated to match the process in the model in a non-structural way. I assume that

\[ N_{t+1} = (A_1 + A_2a_t + A_3d_t)N_t. \]

This process then allows me to turn off the effect of a default and compare the results without changing other things. It should be noted that, while the private sector is still allowed to adjust other choices in response to the process for varieties growth and the government choices, the adoption decision does not react to the choices of the government. This change in the response to the government’s policies will also lead to different choices by the government. I estimate the process by fitting it to the data generated from the baseline using a regression. \( A_3 \) is adjusted to -0.0022, the reduction found by comparing policy functions at the default point. The resulting process is \( N_{t+1} = (1.0088 + 0.1122a_t - 0.0022d_t)N_t. \) In order to turn off the default effect, I replace \( d_t \) with 0.

Solving the model with a response of \( N_{t+1} \) to a default and simulating it leads to a debt-to-GDP ratio of 0.5054 and a yearly default frequency of 1.22 percent. The corresponding numbers for the model without a default effect are 0.3526 and 3.41 percent. Both the amount of borrowing and the default frequency are lower in the model with the extra cost channel. The quantitative effects are sizable. The default frequency more than doubles, while the country has to service only two thirds of the debt on average when the effect of a default on the growth rate is removed. As the process was fit to the original model and although it is obviously misspecified, it demonstrates the strong increase in the cost of default induced by the long-run losses.

To connect my work back to most of the models commonly used in the literature, as my last experiment I solve a simple, calibrated endowment economy with and without a permanent GDP loss. I again use my estimate from the policy function comparison to pin down the size of this loss. In the endowment economy I keep the preferences of the government fixed, but replace the private production sector with a stochastic endowment process.

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35 As a reminder, \( a_t \) is logged TFP, while \( d_t \) is a default dummy.
36 The regression implied -0.0050, which increases the effects found here. The regression is likely to have attributed things like the effect of a falling capital stock to the default, so performing this adjustment places the resulting numbers closer to the effect in the main model.
37 Replacing \( d_t \) by the average of it over the simulated ergodic distribution did not change the results in any quantitative meaningful way.
The government solves the following problem in the endowment economy:\footnote{The separation between private and government consumption in the endowment economy does not matter. It would be possible, without changing the results, to aggregate them.}

\[
\max_{(g_t)_{t=0}^\infty, (c_t)_{t=0}^\infty, (b_t)_{t=0}^\infty, (d_t)_{t=0}^\infty} \mathbb{E}_0 \sum_{t=0}^\infty \beta^t G_t \left[ \chi (q_t)^{1-\sigma} + (1 - \chi) (c_t)^{1-\sigma} \right]
\]

subject to \( g_t + c_t = \gamma E_t \), if the country is in default

subject to \( g_t + c_t + b_t = \gamma E_t + b_t (\lambda + (1 - \lambda) z) + q_t [b_{t+1} - (1 - \lambda) b_t] \), if the country is not in default.

Here, \( E_t \) is the stochastic endowment process to be defined below. The price function of the bond will be determined by foreign lenders, as explained in Section 3. The same holds true for the rules that determine reentry after a default.

It remains to discuss how \( E_t \) is structured and calibrated. I assume that \( E_t \) can be written as the product of two components. The first component is denoted \( E^1_t \) and is assumed to be a stationary process. I assume it follows an autoregressive process of order one and use the estimates from Chatterjee and Eyigungor (2012) for Argentina to calibrate it. The persistence parameter is set to 0.93 and the standard deviation of the innovations to 0.0272. The cost of a default \( \gamma \) is set to two percent, the value used in Aguiar and Gopinath (2006). The second component of the stochastic process is \( E^2_t \) which is assumed to grow over time. Based on the previous results, it is \( E^2_{t+1} = 1.0088 E^2_t \) if the economy is not in default and \( E^2_{t+1} = 1.0066 E^2_t \) otherwise. Beside the long-run loss in growth, the calibration is quite standard.

I solve the model numerically. The quarterly debt-to-GDP ratio is 0.39 when I simulate the model, while the countries yearly default frequency is 3.06 percent. If I remove the long-run cost by setting \( E^2_{t+1} = 1.0088 E^2_t \) also in the default case, those numbers become 0.18 and 3.42 percent. The permanent losses in endowment lead to a much higher cost of default. Therefore, the country in the model gains a better access to external lending, which he uses to borrow substantially more, increasing the default probability back towards its old level. This experiment shows that introducing the long-run output losses implied by the main model can also enhance the borrowing capacity of an endowment model considerably through the same mechanism.

All the experiments indicate an important role for persistent output losses as an extra cost channel of default. The effects are quantitatively sizable. Even just weakening the channel by introducing a subsidy leads to an increase in the probability of a default of 22 percent and a 15 percent reduction in debt. The economic intuition is simple. A lower growth rate makes the country poorer, as GDP will
be lower in the future. Government and households suffer as government and private consumption also have to fall. A default becomes attractive if this channel is present and the country is able to accumulate a higher amount of debt.

6 Conclusion

At least since Eaton and Gersovitz (1981) economists have tried to understand which channels make it costly for a government to default and therefore allow the decision makers of a country to borrow large amounts from abroad. A common assumption made in quantitative models is that a nation suffers a deep recession as a consequence of the default. Either the recession is directly modeled as a temporary loss of output and consumption, or the temporary loss is generated through some mechanism that links the default to the production process. These models struggle to produce a sufficiently high debt-to-GDP ratio while at the same time generating a plausible default frequency and a plausible behavior of the aggregates.

My paper starts by questioning whether output and consumption losses are actually short-lived after a default. If the costs are not short-lived, a default might be more costly than what is implicitly assumed in many models. I first show empirically that a default is, on average, associated with long-lasting GDP losses. I document that we should not only expect GDP to be three percent lower in the year of a default than otherwise forecasted, but also that GDP is on average roughly six percent lower ten years after the default.

The regression results do not allow me to determine if the growth losses are the cause of the default or are themselves the result of the default. I therefore combine a prototypical sovereign default model along the lines of Mendoza and Yue (2012) with a standard endogenous growth model in order to study the role of persistent output losses in such models. I show that the model delivers a reasonable fit of important emerging market business cycle facts and the dynamics around a default. I also use it as a device to measure the role of growth in generating an extra cost. I find that, independent of the details of the way I eliminate the interaction of default and growth, a model without the channel can sustain substantially less debt and default is therefore more frequent. Finally, my model provides me with a way of measuring the contribution of the shock preceding the default and the decision to default itself in generating the reduction in growth. By comparing the policy functions, I find that the default decision leads to a 0.22 percentage point drop in the level of GDP in the first quarter after the default alone.
My work therefore shows that longer lasting effects can play an important role in analyzing defaults. However, at least two things require further study. First, my results involve a particular model that is stylized for reasons of tractability in many dimensions and studies a particular mechanism. Secondly, the estimated average effect masks a lot of heterogeneity. Both observations point to the need for future quantitative investigations into the exact channels that lead to a persistent reduction in the GDP level after a default or, more generally, after severe crisis. To address the first point, (micro-)data should be used to study the extent to which movements in the innovation, technology adoption and firm entry and exit margin play an important role during and after a default, or if, for example, a shrinking government sector is the main cause of the persistent drop in overall GDP. The recent sovereign debt crisis in Europe might provide new insights into this question. In terms of the heterogeneity of outcomes, it is a first-order concern to understand what influences the outcomes of a default. If persistent losses could, for example, be reduced or avoided by a quick settlement with the owners of the defaulted debt, or if third party intervention either reduced or enhanced the effects of a default, then it would be of high policy relevance to understand the determinants. I therefore view the results of this paper as a first step toward forming a solid understanding of the interaction between sovereign defaults and economic growth.
References


Appendix

A Data Description

For the estimation produced in Section 2 data is taken from World Development Indicators (2013) and the database introduced in Laeven and Valencia (2008). To construct dummies for currency crisis, banking crisis and sovereign defaults I take the dates from “Table 2. Timing of Financial Crises” in Laeven and Valencia (2008). For the alternative default dates, dates are taken from Borensztein and Panizza (2009) for the years from 1970 to 2007. All other variables are constructed based on the data contained in the World Development Indicators (2013). In detail I download the following series for the years 1969 to 2011:

- GDP (constant 2000 US$),
- Inflation, GDP deflator (annual %),
- Population, Gross capital formation (% of GDP),
- Gross fixed capital formation (% of GDP),
- Exports of goods and services (% of GDP),
- Domestic credit to private sector (% of GDP),
- Gross domestic savings (% of GDP),
- Foreign direct investment, net lows (% of GDP),
- Primary completion rate, total (% of relevant age group),
- External debt stocks, public and publicly guaranteed (PPG) (DOD, current US$),
- GDP (current US$),
- Official exchange rate (LCU per US$, period average).

Series in the regressions are then constructed from this data. Series for capital are constructed using the perpetual inventory method using a depreciation rate of eight percent.

The GDP per capita and the dates on the default on external debt for the robustness exercises on longer horizons are taken from the website underlying Reinhart and Rogoff (2009):
Data on computer imports is taken from Feehra, Lipsey, Deng, Ma, and Mo (2005).
Quarterly data on GDP, private consumption, government consumption, investment, imports and exports is taken from International Financial Statistics (2013).
B Model Solution

The algorithm consists of the following steps:

0. Create grids for debt, capital stock, taxes and productivity shocks within the approximation set \([b, \bar{b}] \times [k, \bar{k}] \times [T, \bar{T}] \times [\zeta, \bar{\zeta}]\) using Chebyshev nodes for debt, capital and taxes, and the grid corresponding to the Rouwenhorst discretization for the productivity shocks. To form an initial guess for value functions, policy functions and the price of debt, solve the static problem of a one period economy. Denote the value functions for the government by \(V\), the policy functions of the government by \(P_g\), the policy function of the household and the firms by \(P_h\) and the price schedule by \(q\).

1. Given a guess for the functions tomorrow, find the optimal choices of household and firms today using the resource constraints and Euler equations in all points of the grid including taxes and treating the grid point for debt as the choice of the government for tomorrow. The solutions are found by using the Nelder Mead algorithm (see Judd (1998)). Expectations are computed using the transition probabilities implied by the Rouwenhorst (1995) discretization. Choices outside of the grid are evaluated using Chebyshev interpolation. Denote the resulting solution by \(P'_h\).

2. Given a guess for the functions tomorrow, the price schedule and the solution of the household and firms problems find the optimal choices of the government and update the value function in all points of the grid excluding taxes and treating the grid point for debt as the level at the start of the period. Expectations are computed using the transition probabilities implied by the Rouwenhorst (1995) discretization. Choices outside of the grid were evaluated using Chebyshev interpolation. To perform the optimization I first performed a grid search and used the solution to it as an initial guess for the Nelder Mead algorithm.

3. Use the new policy functions for capital, debt, default and taxes to update the debt price schedule.

4. If the difference between value functions in two consecutive iterations is smaller than a given precision level stop. Otherwise return to 1 and use \(V', P'_g, P'_h, q'\) as new guesses.

5. Solve the problem of the household and firms using value functions given the government choices to check for the validity of the Euler equation approach.
I tried variations of this approach by trying spline interpolation or linear finite elements. (For more on different computation methods for solving models of this type see for example Judd (1998), Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006) or Hatchondo, Martinez, and Sapriza (2010)). If grids are set fine enough results are robust to the solution procedure. The same is true for varying the approximation method for the productivity shocks when I use the method suggested by Tauchen (1986) or using a combination of interpolation over the shock levels and Gauss-Hermite integration (see Judd (1998)). I also tried an approach similar to Chatterjee and Eyigungor (2012) and Gordon and Guerrero-Quintana (2013) using i.i.d. preference shocks. The current approach minimizes the computation time, while giving sufficiently precise solutions.