Wealth and Volatility*

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Abstract

We document that in the US over the course of the past century there has been a systematic negative relation between household wealth and business cycle volatility. We develop a micro-founded dynamic equilibrium model that contains elements of a traditional Keynesian framework in which economic fluctuations can be driven by sunspot-type fluctuations in household optimism or pessimism. The novel feature is that the scope for equilibrium fluctuations due to “sunspots” depends on the (exogenously given) value of household wealth. When wealth is high consumers demand is not sensitive to unemployment expectations and the economy is robust to confidence crisis. When wealth is low, consumers’ demand is sensitive to unemployment expectations, the economy becomes vulnerable to confidence-driven fluctuations and is, in general, more volatile. In this case, there is a potential role for public policies to stabilize demand. We conclude by providing micro-economic evidence on the importance of wealth in affecting the sensitivity of demand to aggregate conditions. We show that during the Great Recession consumers with low wealth, ceteris paribus, cut expenditures more sharply.

Keywords: Business cycles, Aggregate Demand, Multiple Equilibria

JEL classification codes: E12, E21

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1 Introduction

Figure 1 documents the relationship between household wealth and aggregate volatility in the United States. Volatility is measured as the standard deviation of the quarterly real GDP growth rate over a 10 years period (where the x axis in the graph denotes the end of that period), while wealth is measured as the average deviations from trend for net worth of households and non profit organization (from the Flow of Funds) over the same 10 years windows. The figure reveals that periods when average net worth is high relative to trend, reflecting high prices for housing and/or stocks, tend to be periods of low volatility in aggregate output (and hence employment and consumption). Conversely, periods in which net worth is below trend tend to be periods of high macroeconomic volatility. For example during periods ending in late 70s and early 1980s wealth is at its historical low and volatility is at its historical high, while periods ending during the late 1990s and early 2000s wealth is at its highest and volatility is at its lowest.

Figure 2 presents another piece of evidence that relates economic volatility and wealth, showing that the three periods over the past century in which the US economy has been more unstable, namely the 30s, the late 70s early 80s and the late 00s are all characterized by household net worth below trend.\footnote{Household net worth is from the Flow of Funds from 1945 onward and has been calculated using historical sources on stock values and housing prices for earlier periods, see the data appendix for more details}

In this paper we argue that (exogenously given) fluctuations in wealth are an important factor in determining the volatility of business cycles. We make the argument by developing a micro-founded dynamic equilibrium model that contains elements of a traditional Keynesian framework in which economic fluctuations are driven by fluctuations in household optimism or pessimism. The novel feature is that the scope for equilibrium fluctuations due to “animal spirits” depends crucially on the value of wealth in the economy. When wealth is high the economy has a unique equilibrium and behaves neoclassically. When wealth is low, the economy becomes vulnerable to additional confidence-driven fluctuations and hence is, in general, more volatile. In this case, there is a potential role for public policies to stabilize demand.

Our model economy deviates from a standard representative agent economy in two ways: the first is that we introduce frictional labor markets, with allows equilibrium unemployment, the second is that we introduce uninsurable unemployment risk, which makes household demand sensitive to expectations about economic conditions. The economy is populated by a large number of identical households, each of which comprises many members. Competitive firms operate a linear technology.
Note: Standard deviation of GDP growth are computed over 40 quarters rolling windows. Observations for net worth are average over the same windows.

Figure 1: Wealth and volatility

Figure 2: Household net worth in US in the long run
that use employed workers to produce a perishable consumption good. Households enjoy utility from non-durable consumption and from durable housing, which is in fixed supply. In each period, given current economic conditions and expectations, households submit consumption orders to firms. Given orders, firms hire the workers that are necessary to fulfill the orders. Then jobs are allocated by random matching in a decentralized frictional labor market. The key friction we introduce is that household cannot affect the probability of finding a job by asking a lower wage. The decentralized nature of the labor market is important because it means that equilibrium output will be determined by desired consumption demand, rather than by desired labor supply, and because it allows for the possibility of equilibrium unemployment.

Agents must finance their consumption orders using wage income, home equity, or non-collateralized borrowing. Unemployment expectations matters not only because the expected unemployment rate determines expected income, but also because if house prices are low the household anticipates that members who do not find jobs will have to use expensive credit. The fact that consumption for each member is committed prior to realization of the member’s idiosyncratic employment status increases the precautionary motive associated with perceived unemployment risk.

Our first result is to show that this environment allows for multiple equilibria in which households can collectively co-ordinate on a range of expectations about unemployment, each of which turns out to be self-fulfilling. In particular, there is a range of values for house prices in which the model has two steady states (with the same house price in each). In the optimistic steady state, households expect low unemployment, are therefore not too concerned about credit costs for unemployed workers, and set consumption demand high. Facing high demand, firms employ a large fraction of workers, and the expectation of low unemployment is rationalized. In the pessimistic steady state, households expect high unemployment. Because they do not want to commit to high consumption given high idiosyncratic employment risk and costly credit, they set consumption demand low. Facing low demand, firms hire few workers, and unemployment is in fact high, as expected.

Precautionary saving in housing offers a way for households to self-insure against unemployment risk. The less wealth a household has, the more reliant the household will be on costly credit in the event of unemployment. Thus the lower is household wealth, the more sensitive is consumption demand to the expected unemployment rate. This increased sensitivity of demand to expectations increases the range of unemployment rates that can be supported in a rational expectations equilibrium. To see this consider the extreme case in which households have no wealth. Then, if they are maximally pessimistic and expect 100% unemployment they will set consumption to zero, and
100% unemployment can occur in equilibrium. But if households have positive wealth they will choose positive consumption even if they expect 100% unemployment. Firms must then hire a positive fraction of workers to fill these orders, and so 100% unemployment cannot be an equilibrium. Thus the maximum magnitude of sun-spot driven equilibrium fluctuations will vary inversely with the level of house prices.

Home values in the model are endogenous equilibrium objects, and reflect both the fundamental flow utility from home ownership and the liquidity value of being able to finance consumption out of home equity in the event of unemployment. Because this liquidity value is tied to the level of unemployment, house prices themselves are indeterminate, and like the unemployment rate, can potentially fluctuate in response to changes in expectations. However, for most of our analysis we will explore what dynamics for unemployment are possible for alternative constant values for equilibrium house prices. We do this because in the data a large share of house price volatility is at lower frequencies than typical business cycle fluctuations.

We further limit the set of values for house prices we consider by introducing as an additional model element a fringe group of households that does not face unemployment risk. The presence of this group establishes a lower bound on housing demand and thus on equilibrium house prices. If this lower bound is sufficiently high, unemployed workers can finance consumption entirely out of home equity, and full employment is the only possible equilibrium.

We use the model to offer an interpretation of recent macroeconomic history. The Great Moderation was a time in which US house and stock prices were very high by historical standards. High household wealth levels in this period meant that the economy was robust in the sense that it was not subject to large recessions induced by declines in confidence. However, the sharp declines in house and stock prices between mid 2007 and mid 2009 left the economy fragile, and vulnerable to a confidence-driven recession. One possible “trigger” for a collective loss of confidence was the collapse of Lehman Brothers in the Fall of 2008. We show that following a transitory sunspot-induced recession, equilibrium dynamics imply a slow recovery, even though fundamentals remain unchanged. Of course, fluctuations in consumer confidence are only one source of business cycles, and over a longer history economic cycles in the United States likely have a number of causes above and beyond fluctuations in animal spirits.

The model has policy implications. We evaluate two specific policies. The first is a lump-sum unemployment benefit, financed by a proportional income tax. This policy makes unemployment less painful, and thereby reduces the sensitivity of demand to the expected unemployment rate.
A sufficiently generous benefit rules out sunspot-driven fluctuations and ensures full employment. The second policy we consider is government consumption financed by lump-sum taxation, in the spirit of the 2009 stimulus plan. This policy also makes aggregate (private plus public) demand less sensitive to expectations, and thereby rules out equilibria with very high unemployment rates. However, taxation also reduces asset values, which increases the sensitivity of demand to perceived unemployment risk. The two effects offset each-other and the end result is that fiscal policy can be very ineffective.

1.1 Related Literature

From a theoretical viewpoint, our paper is related to recent work by Guerrieri and Lorenzoni (2009), Chamley (2010), and Farmer (2010). In both our model and Farmer’s output is demand-determined, and asset prices play a critical role in determining demand. One difference is that agents in our model face idiosyncratic risk and thus have a precautionary motive for saving, which is central to delivering a connection between the level of asset prices and the volatility of output. Guerrieri and Lorenzoni develop a model in which risk-averse agents trade in a decentralized fashion and face idiosyncratic risk. Their model recessions feature an increase in precautionary saving, as do ours, but in their model an endogenous increase in precaution amplifies a fundamental aggregate productivity shock, while in ours it is a self-fulfilling prophecy.

Other recent models that argue for multiple equilibria as a source of aggregate fluctuations include Kaplan and Menzio (2013) where multiplicity is driven by a shopping externality, and Martin and Ventura (2012), where there are multiple possible prices of collateral.

A challenge in constructing models in which demand-side factors play an important role in that many forces that tend to reduce desired consumption demand (e.g. lower asset values, greater idiosyncratic risk) also tend to increase desired household labor supply. Hall (2005), Michaillat (2010) and Shimer (2012) have developed models with decentralized labor markets in which they assume that the real wage does not fall (much) in response to a negative productivity shock, leading to a large fall in vacancy posting and a surge in involuntary unemployment. These models lack clear microfoundations for wage formation. On the other hand, they offer a natural resolution to the longstanding discrepancy between small microeconomic estimates for labor supply elasticities, and the large macro elasticities implicit in the large movements in aggregate hours over the business cycle (see, for example, Chetty et al., 2011). The resolution is simply that large falls in aggregate hours during recessions do not reflect an unwillingness to supply labor, but instead indicate that
wages are “stuck” at too-high levels. In our model, the household is assumed to first choose a reservation wage, and then the firm only gets to decide whether or not to accept or reject a potential match. Workers then always extract all the surplus from a firm-worker match and, given a linear technology, firms are always indifferent about how many workers to hire. The level of demand then effectively selects a particular employment rate.

We are not the first to argue for a link between asset values and volatility, but our mechanism reverses the usual direction of causation. Others (see Lettau et al. 2008) have pointed out that higher aggregate risk should drive up the risk premium on risky assets relative to safe assets. Lower prices for risky assets like housing and equity then just reflect higher expected future returns on these assets. In our model, asset prices are the primitive, and the level of asset prices determines the possible range of equilibrium output fluctuations, i.e. macroeconomic volatility.

2 Model

There are two goods in the economy: a perishable consumption good, produced by a continuum of identical competitive firms using labor, and an durable asset, which is in fixed supply and which we label housing. There are two types of households in the model, and a continuum of identical households of each type. These types share common preferences, but differ with respect to the risk they face: income for the first type is risky, while income for the second is not. The only role of the riskless type of household is to establish a floor for asset prices.

Each household of the first “risky” type contains a continuum of measure one of individuals, while the second “riskless” type is measure zero. The measure of firms is equal to the measure of risky households. Thus we can envision a representative firm interacting with mass one members of a representative risky household. The price of the consumption good is normalized to one in each period. The quantity of housing is normalized to one. The economy is closed.

Let $s_t$ denote the current state of the economy, and $s^t$ denote the history up to date $t$. In each period, households of the risky type send out members to buy consumption and to look for jobs. Employment opportunities are randomly allocated across household members, but assets must be allocated across members before labor market outcomes are realized. It is therefore optimal to give each member an equal fraction $h(s^{t-1})$ of the assets the household carries in the period. The household can give its members consumption and savings instructions that are contingent on their labor market outcomes. The fraction $1 - u(s^t)$ of household members who find a job are paid a
wage \( w(s^t) \) and use wage income and asset holdings to pay for consumption \( c^w(s^t) \). The fraction \( u(s^t) \) who are unemployed can only use wealth and (potentially) unemployment benefits to pay for consumption \( c^u(s^t) \).

At the end of the period the household regroups and pools resources, which determines the quantity of the asset carried into the next period \( h(s^t) \). This model of the household is a simple way to introduce idiosyncratic risk and a precautionary motive, without having to keep track of the cross-sectional distribution of wealth.

At the start of each period \( t \) households observe \( s_t \), update \( s_t \), and assign probabilities to future sequences \( \{ s_\tau \}_{\tau = t+1}^\infty \). We assume that all households form the same expectations.

Preferences for a household are given by

\[
\sum_{t=0}^\infty \beta^t \sum_{s^t} \pi(s^t) \left\{ \left[ 1 - u(s^t) \right] \log c^w(s^t) + u(s^t) \log c^u(s^t) + \phi h(s^{t-1}) \right\} ,
\]

where \( \beta \) is the discount factor, \( \pi(s^t) \) is the probability of history \( s^t \) as of date 0, and \( \phi \) is a parameter determining the utility from housing.

The household budget constraints for a risky household have the form:

\[
\left[ 1 - u(s^t) \right] c^w(s^t) + u(s^t)c^u(s^t) + p(s^t) \left[ h(s^t) - h(s^{t-1}) \right] \leq \left[ 1 - u(s^t) \right] \left[ w(s^t) - T(s^t) \right] + u(s^t) b
\]

\[
c^u(s^t) \leq p(s^t)h(s^{t-1}) + b \quad (2)
\]

\[
c^u(s^t) \leq p(s^t)h(s^{t-1}) + w(s^t) - T(s^t) \quad (3)
\]

\[
c^w(s^t), c^u(s^t), h(s^t) \geq 0
\]

The left hand side of eq. (1) captures total household consumption and the cost of net asset purchases. The first term on the right hand side is earnings for workers \( w(s^t) \) less payroll taxes \( T(s^t) \), while the second is unemployment benefits \( b \) for the fraction \( u(s^t) \) of members who do not find a job. Note that \( h(s^{t-1}) \) was effectively chosen in the previous period. In the current period, given aggregate variables \( u(s^t), w(s^t) \) and \( p(s^t) \), the choices for \( c^w(s^t) \) and \( c^u(s^t) \) implicitly define the quantity of wealth carried into the next period \( h(s^t) \). Equation (2) is the constraint that limits consumption of unemployment members to wealth plus unemployment benefits. Equation (3) is the analogous constraint for workers.

The budget constraint for the riskless household is identical, except that unemployment and transfers for this type are equal to zero.
The government balances its budget period by period, using payroll taxes on workers to finance unemployment benefits and (possibly) government spending $G$:

\[
[1 - u(s^t)] T(s^t) = u(s^t) b + G.
\]

2.1 Household’s problem

Consider the problem for the type that faces unemployment risk. Let $\mu(s^t)$ denote the multiplier on (1) and let $\lambda(s^t)$ denote the multiplier on (2). We conjecture and later verify that the other constraints do not bind in equilibrium.

The first order conditions for $h(s^t)$, $c^w(s^t)$ and $c^u(s^t)$ are respectively

\[
p(s^t)\mu(s^t) = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \left[ p(s^{t+1})\mu(s^{t+1}) + p(s^{t+1})\lambda(s^{t+1}) \right] + \beta \phi
\]

\[
\frac{1}{c^w(s^t)} = \mu(s^t)
\]

\[
\frac{u(s^t)}{c^u(s^t)} = \mu(s^t) u(s^t) + \lambda(s^t)
\]

Combining these gives

\[
\frac{p(s^t)}{c^w(s^t)} = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \left[ (1 - u(s^{t+1})) \frac{p(s^{t+1})}{c^w(s^{t+1})} + u(s^{t+1}) \frac{p(s^{t+1})}{c^u(s^{t+1})} \right] + \beta \phi
\]

where

\[
c^u(s^{t+1}) = \begin{cases} c^w(s^{t+1}) & \text{if } c^w(s^{t+1}) \leq p(s^{t+1}) h(s^t) + b \\ p(s^{t+1}) h(s^t) + b & \text{if } c^w(s^{t+1}) > p(s^{t+1}) h(s^t) + b \end{cases}
\]

This inter-temporal condition can alternatively be written

\[
\frac{p(s^t)}{c^w(s^t)} = \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \left[ \frac{p(s^{t+1})}{c^w(s^{t+1})} \left( 1 + \frac{u(s^{t+1}) \max \left\{ c^w(s^{t+1}) - \left[ \frac{p(s^{t+1}) h(s^t) + b}{p(s^{t+1}) h(s^t) + b} \right], 0 \right\}}{c^u(s^{t+1}) h(s^t) + b} \right) \right] + \beta \phi
\]

This first order condition can be interpreted as follows. The utility cost of buying an additional unit of housing is the price times the marginal utility of consumption for a worker. The return is the discounted utility flow $\beta \phi$ plus the next period price times next period marginal utility for workers plus an additional term that regulates the liquidity value of additional wealth in the next period. This liquidity value is proportional to the unemployment rate – which captures the number
of household members who will value extra liquidity – times the difference in consumption for employed versus unemployed workers – which captures the value of being to allocate consumption more evenly across household members. When either the unemployment rate is zero, or when the borrowing constraint is non-binding for unemployed workers – so that employed and unemployed members enjoy equal consumption – this liquidity term drops out, and the inter-temporal first-order condition takes the usual representative agent form. Conversely, when there is a positive probability of unemployment at \( t+1 \) and when workers consume more than the unemployed, there will be a precautionary motive to save that will be larger the higher are expected unemployment rates. Further, and most importantly, the precautionary motive to save will be stronger the lower are expected house prices, since lower asset prices will imply a higher marginal utility of consumption for unemployed household members.

The analogous first order condition for the type that does not face unemployment risk is

\[
\frac{p(s^t)}{\hat{c}^w(s^t)} \geq \beta \sum_{s^{t+1}} \pi(s^{t+1}|s^t) \left[ \frac{p(s^{t+1})}{\hat{c}^w(s^{t+1})} \right] + \beta \phi
\]

where hats denote allocations for this type. The inequality here reflects the fact that, given the preferences we will assume below, the type facing no unemployment risk will be at a corner in equilibrium, with zero housing.

2.2 Production and Labor Markets

We now describe how workers and firms meet, how production takes place, and how the equilibrium unemployment rate is determined.

Households observe the aggregate state \( s_t \) and the household head then gives its members a reservation wage \( w^*(s^t) \) specifying what wages to accept, along with contingent consumption instructions, \( c^w(s^t) \) and \( c^u(s^t) \). Each of the measure one of potential workers from a particular household are then randomly matched with firms across the economy, so each firm ends up matched with measure one of potential workers.

Each period consists of a continuous unit interval of time. Production at each representative firm takes place smoothly through this interval, using workers sequentially, one worker at a time. The production technology is linear: hiring measure \( n(s^t) \) workers produces \( y(s^t) \) units of output:

\[
y(s^t) = n(s^t)
\]
Potential workers matched with a particular firm are allocated a random position in the production queue indexed by \( i \in [0, 1] \). Each firm takes as given the reservation wage \( w^*(s^t) \) and decides whether or not to hire each successive worker in the queue. The optimal strategy for the firm is to employ a worker as long as the reservation wage \( w^*(s^t) \) is less than or equal to the worker’s marginal product (which is equal to one), and as long as the firm has not already produced sufficient output to satisfy anticipated demand \( d(s^t) \).

Understanding the firms’ incentives, a representative household head will optimally assign its members a reservation wage \( w^*(s^t) = 1 \). A lower reservation wage does not increase the probability that a given household member will find a job, while a higher reservation wage would guarantee non-employment.

Firms will hire workers and produce continuously until they have produced \( d(s^t) \) units of output. Thus, those workers with queue index \( i \leq d(s^t) \) end up employed, while those with \( i > d(s^t) \) are unemployed. The unemployment rate is demand determined: \( u(s^t) = 1 - d(s^t) \).

Once production is completed, at the instant of time that is fraction \( d(s^t) \) into the period, all output is immediately sold in a competitive centralized market at a price normalized to one. Firms have no interest in hiring workers with queue positions \( i > d(s^t) \) because they anticipate no additional demand within the period, because workers cannot consume the output of the firm at which they produce, and because output cannot be stored and carried into the next period.

Note that households make consumption decisions and firms make production plans given the same information set and identical expectations.

### 2.3 Equilibrium

A symmetric equilibrium in this model is a pair of policy parameters \((G, b)\), a process for the state \( s_t \) (which in some examples will be a sunspot) and associated decision rules and prices \( n(s^t), u(s^t), d(s^t), c^w(s^t), c^a(s^t), h(s^t), p(s^t), T(s^t) \) that satisfy, for all \( t \) and for all \( s_t \):

1. \( w(s^t) = w^*(s^t) = 1 \)

2. \( n(s^t) = 1 - u(s^t) \)
3. \( h(s^t) = 1 \) \hspace{1cm} (7)

4. \( d(s^t) = [1 - u(s^t)]c_u(s^t) + u(s^t)c_u(s^t) + G = 1 - u(s^t) \) \hspace{1cm} (8)

5. \[1 - u(s^t)]T(s^t) = u(s^t)b + G. \hspace{1cm} (9)

6. \( c_u(s^t) = \min\{c_u(s^t), p(s^t)h(s^t - 1) + b\} \) \hspace{1cm} (10)

7. \[\frac{p(s^{t+1})}{c_u(s^{t+1})} = \beta \sum_{s_{t+1}} \pi(s_{t+1}|s^t) \left[ \frac{p(s^{t+1})}{c_u(s^{t+1})} \left( 1 + \frac{u(s^{t+1})\max\{c_u(s^{t+1}) - \left[ p(s^{t+1})h(s^t) + b \right], 0\}}{p(s^{t+1})h(s^t) + b} \right) \right] + \beta \phi \] \hspace{1cm} (11)

8. \[\frac{p(s^t)}{1 - T(s^t)} \geq \beta \sum_{s_{t+1}} \pi(s_{t+1}|s^t) \frac{p(s^{t+1})}{1 - T(s^{t+1})} + \beta \phi \] \hspace{1cm} (12)

### 2.4 Discussion

In this environment, demand \( d(s^t) \) determines employment \( n(s^t) \) and unemployment \( u(s^t) = 1 - n(s^t) \). If orders fall short of potential output, i.e., if \( d(s^t) < 1 \), then labor supply will exceed labor demand, in the sense that all measure 1 of workers in each household are willing to work at any positive wage, while employment is determined by labor demand \( n(s^t) = d(s^t) < 1 \). However, no single atomistic household has an incentive to choose a lower reservation wage, because a lower wage will not increase the probability of its members forming successful matches. Thus unemployment does not exert downward pressure on wages, breaking the standard Walrasian adjustment process that ultimately equates labor demand and labor supply in models with frictionless labor markets.

Equation (12) indicates that the presence of the riskless type whose consumption is \( \check{c}(s^t) = 1 - T(s^t) \) puts a floor under house prices.\(^2\)

\(^2\)Note that with \( h(s^t) = 1 \) the inter-temporal first order condition for the risky household type (equation 11) would be identical if preferences were given by \( u(c, h, \phi) = \log c + \phi \log h \). Thus it is sufficient to assume linearity in preferences for the riskless type.
3 Steady States

Most of the analysis that follows focuses on a simple version of the model in which the government plays no role, so that \( b = G = T(s') = 0 \). We will return to consider various policy interventions in Section 7.

Steady states are constant values \((c^w, c^u, u, p)\) that satisfy equations (11) (the risky household’s FOC), (8) (goods market clearing), (10) (the budget constraint for unemployed members), and (12) (the pricing floor established by the riskless type). These equations are, respectively

\[
\frac{p}{c^w} = \beta \frac{p}{c^w} \left( 1 + \frac{u \max\{c^w - p, 0\}}{p} \right) + \beta \phi \\
(1 - u) c^w + u c^u = 1 - u \\
c^u = \min\{c^w, p\} \\
p \geq p \equiv \frac{\beta}{1 - \beta} \phi
\]

Let \( p_F(u) \) denote the fundamental price of housing: the price households would be willing to pay in steady state if there was perfect risk sharing within the household, so that \( c^w = c^u \):

\[
p_F(u) = \frac{\beta \phi}{1 - \beta} (1 - u) \leq p
\]

with strict inequality for \( u > 0 \).

**Proposition 1:** Any steady state with positive unemployment must feature limited risk sharing: \( u > 0 \implies c^w > c^u \).

**Proof:** See Appendix

The logic for this result is that in any steady state with positive unemployment, expected individual consumption is less than one. If each household member consumed expected individual consumption, the price of housing would equal \( p_F(u) \), which is below the price the riskless household (whose expected consumption is higher) is willing to pay, namely \( p = p_F(0) \). It therefore follows that in steady states with positive unemployment housing must have additional value as a source of liquidity for the risky household type. This in turn implies that in steady state unemployed agents must be consuming less than employed households – the term labeled “liquidity value of wealth” in eq. (4) must be positive – so that the additional liquidity associated with housing wealth is priced.
Proposition 2: Let $\tilde{\phi} \equiv \frac{1}{2} \sqrt{\left(\frac{4}{\beta} - 3\right)} - \frac{1}{2}$. If $\phi \geq \tilde{\phi}$, then the only possible steady state is $p = p_0, u = 0$. If $\phi < \tilde{\phi}$ then there exist a continuum of steady states in which the unemployment rate ranges from $u = 0$ to $u = u^+$ where

$$u^+ \equiv 1 - \frac{\beta}{1 - \beta} \phi(1 + \phi)$$

(14)

For each unemployment rate $u \in [0, u^+]$ the corresponding steady state house price is given by

$$p(u) = \frac{\beta (u + \phi)}{(1 - \beta) + \frac{\beta u(1 + \phi)}{1 - u}}$$

(15)

Proof: See Appendix

Note that the function $p(u)$ is concave and $p(0) = p(u^+) = p_0$. The following corollary follows:

Corollary: There is a range of values for $p \geq p_0$ such that for any $p$ in this range there are two distinct steady state values for $u$. For $p > p_0$, both of these steady states features positive unemployment.

We define the liquidity value of housing, given a steady state unemployment rate $u$, as the equilibrium price (eq. 15) minus the fundamental component (eq. 13):

$$p_L(u) = \frac{\beta (u + \phi)}{(1 - \beta) + \frac{\beta u(1 + \phi)}{1 - u}} - \frac{\beta \phi}{1 - \beta} (1 - u)$$

(16)

At $u = 0$, the liquidity value for housing is increasing in the unemployment rate, given $\phi \leq \tilde{\phi}$. The liquidity value shrinks to zero as $u \to 1$.

The nature of steady states, and the decomposition of housing value into fundamental and liquidity components, are best understood graphically. For the purposes of plotting numerical examples, we need to parameterize the model.

3.1 Parameterization

The model has only two parameters, $\beta$ and $\phi$. Thinking of a period length as a year, we set $\beta = (1 + 0.05)^{-1}$.

We then set our baseline value for $\phi$ so that the floor on house prices, $p_0 = \beta \phi / (1 - \phi)$, is equal to 0.75, which implies $\phi = 0.0375$. This choice has two important features. First, $\phi < \tilde{\phi}$, guaranteeing,
Figure 1: Steady state price $p(u)$.

Figure 3: Steady states for $\beta = (1 + 0.05)^{-1}$, $\phi = 0.0375$.

by virtue of Proposition 2, that the model will exhibit a continuum of steady states. The highest possible steady state unemployment rate $u^+ = 0.222$. Second, given $p = 0.75$, in steady states with unemployment, unemployed workers will consume $c^u = 0.75$, which is 25 percent less than full employment consumption. This is consistent with estimates of the size of the consumption loss for households who experience a job loss (Chodorow-Reich and Karabarbounis, 2014).

3.2 Understanding Steady State Multiplicity

Figure 3 shows steady state equilibria for our baseline parameterization. Specifically the hump-shaped solid black line plots $p(u)$ from eq. (15) which corresponds to the steady state price that the risky household is willing to pay as a function of the unemployment rate. Each point on this line is a steady state. The green horizontal line shows $p_r$: the lower bound on house prices established by the riskless household.

Suppose we start in the steady state with $p = p_r$ and $u = 0$, and consider how the steady state price $p(u)$ changes in response to a marginal increase in unemployment. On the one hand, higher unemployment reduces expected income, reducing fundamental housing demand and the fundamental component of the price $p_F(u)$. On the other hand, increasing unemployment raises
the liquidity value of housing, $p_L(u)$, since the household has a stronger incentive to accumulate housing as an asset that members can use to consumption smooth through unemployment spells. The marginal liquidity value is initially strong, because there is a large gap between the consumption levels of employed and unemployed workers. This means that a marginal increase in unemployment (starting from $u = 0$) translates into an increase in the steady state asset price. But for high enough unemployment rates, the marginal negative impact on fundamental value comes to dominate, so that the steady state price becomes a declining function of $u$. Thus there is a second equilibrium at $p = \overline{p}$ with $u = u^+$.

For each $p \geq \overline{p}$ in the range of the $p(u)$ function there are two steady states, one with low and one with high unemployment. In the low unemployment steady state, wealth is low relative to per capita consumption, but the household does not want to increase saving because there is low unemployment risk – and thus a modest precautionary motive to save. In the high unemployment equilibrium, unemployment risk is high, but the household does not want to increase saving further because wealth is already high relative to consumption. Thus, in the low unemployment equilibrium the fundamental share of house value is higher (and the liquidity share lower) than in the high unemployment equilibrium. Note that for $p > \overline{p}$, if steady states exist, the two steady state unemployment rates are closer together the larger is $p$. There are no steady states with $u > u^+$, because such high unemployment rates would imply values for $p$ below the lower bound $\overline{p}$ established by the marginal riskless household.

Figure 4 contrasts the baseline low $\phi$ parameterization described above to an alternative in which $\phi$ is larger and equal to $\tilde{\phi}$.

The top pair of red lines correspond to the case $\phi = \tilde{\phi}$, so that the taste for housing is high, while the bottom pair of black lines correspond to the baseline parameterization shown in the previous plot. In both cases, the solid lines depict the set of steady states, while the dashed lines show the respective price floors $\underline{p}$. Because the taste for housing is relatively strong with $\phi = \tilde{\phi}$, house prices are high, and as a consequence unemployed household members can afford similar consumption to employed household members. Thus the liquidity value of housing is relatively low, and the fundamental component is the primary determinant of house value. As a consequence the $p(u)$ curve is always (weakly) declining in $u$, and the equilibrium is therefore unique: $u = 0$ is the only steady state satisfying both $p = p(u)$ and $p \geq \overline{p}$. Zero is the only unemployment rate at which the steady state price $p_F(0)$ weakly exceeds the floor $\underline{p}$ established by the marginal riskless household.

Note that without the riskless type, there would be a continuum of steady states with unemploy-
ment rates between zero and one, with each unemployment rate corresponding to a different steady state asset price as given by eq. (13) (see Farmer 2010). The presence of the riskless type puts a floor on the asset price, which in turn establishes a floor for steady state consumption demand and output.

To summarize our steady state analysis, with strong demand for housing (high $\phi$), the fundamental component of house prices is large, which translates into a weak precautionary motive and a relatively small liquidity component to house values. This in turn implies a unique full employment steady state. With weak demand for housing (low $\phi$), house values are lower but are initially increasing in the unemployment rate, reflecting a high value for additional liquidity. This implies the existence of two possible steady state unemployment rates for the same price level.

4 Dynamics

We now introduce dynamics. Our focus will primarily be on constructing equilibria in which the unemployment changes over time, while asset prices are constant, so that $p(s') = p \geq p$. We start by considering unemployment dynamics in the perfect foresight case. We then show that one can introduce sunspots in the model, and thereby generate “confidence-driven” fluctuations in economic
activity. The ultimate goals of this section are two-fold. First, to characterize some general features of fluctuations driven by non-fundamental changes in expectations. Second, to show that the model can be used to interpret the time path for the unemployment rate in the US over the course of the Great Recession.

We will maintain – for the sake of simplifying the exposition – the assumptions $b = G = T(s^t) = 0$.

### 4.1 Deterministic Dynamics

Imposing $p(s^t) = p$ and the equilibrium market-clearing condition condition $h(s^t) = 1$, the perfect-foresight version of the inter-temporal FOC for the risky household type (eq. 11) is

$$\frac{p}{c^w_t} = \beta \cdot \frac{p}{c^w_{t+1}} \left( 1 + \frac{u_{t+1} \max\{c^w_{t+1} - p, 0\}}{p} \right) + \beta \phi$$

where the consumption of the representative worker and the unemployment rate are linked via the resource constraint:

$$(1 - u_t) c^w_t + u_t \min\{c^w_t, p\} = 1 - u_t.$$  

These two equations can be used to plot the implied dynamics for the unemployment rate. To do so, we use the same parameter values as before ($\phi = 0.0375$) and set $p = p = 0.75$.

Figure 5 plots the change in the unemployment rate $u_{t+1} - u_t$ against the unemployment rate $u_t$. The two points at which the change in the unemployment rate is zero correspond to the two steady state unemployment rates at $p = 0.75$. Denote these rates $u_L$ and $u_H$. The figure indicates that for any initial unemployment rate below $u_H$, unemployment will gradually converge, over time, to $u_L$. Thus the low unemployment steady state is locally dynamically stable: if unemployment starts out below $u_L$, unemployment will rise, while if it starts above $u_L$ (but below $u_H$) unemployment will fall. The fact that this steady state is dynamically stable will later allow us to introduce sunspot shocks that generate fluctuations in the neighborhood of $u_L$.

The high unemployment steady state is not stable. If unemployment starts above $u_H$, it will increase towards maximum unemployment, in expected terms. Note that any such paths are not equilibria, because in the limit they imply that households will end up with zero income and consumption, which cannot be optimal given positive wealth equal to $p$. 


4.2 The Great Recession

We now show that our model can generate dynamics for house prices and unemployment that are qualitatively similar to those experienced by the United States over the course of the Great Recession. Figure 6 shows time-paths for the unemployment rate and for house prices between the first quarter of 2005 and the first quarter of 2014. The house price series plotted is the Case-Shiller U.S. National Home Price Index, deflated by the GDP deflator, and relative to a 2% trend growth rate for the real price.\(^3\) Between the start of 2007 and the end of 2008 house prices fell by 30% relative to trend.\(^4\) The rise in the unemployment rate was concentrated in the second half of 2008,

\(^3\)This is the average growth rate for real GDP per capita between 1947 and 2007. It is also close to the average growth rate for real house prices between 1975 and 2006 (see Figure 1 in Davis and Heathcote, 2007).

\(^4\)This fall in house prices drove the ratio of per capita home equity to per capita annual disposable income from a post-War high of 1.61 in the first quarter of 2006 to 1.19 by the fourth quarter of 2007, and to a post-War low of 0.70 by the end of 2011.
and the first half of 2009. Thus the fall in house prices began well before the most severe portion of the recession.

![House Prices and Unemployment: Data](image.png)

**Figure 6: House Prices and Unemployment: Data**

The sequence of model events that generates the times series plotted in Figure 7 is as follows. Initially, the fundamental demand for housing is strong, so that there is a unique full employment steady state. Then, between 2007 and 2008, there is a permanent unanticipated decline in the taste for housing, which reduces $\phi$ from $\phi = \tilde{\phi}$ to $\phi = 0.0375$ (our baseline value). Agents initially believe that this will simply lead to a permanent change in house prices to the new implied new value for $p$, but that the unemployment rate will remain at zero. Given these beliefs there is no immediate change household demand – notwithstanding the decline in household wealth – and thus no change in the unemployment rate.

With $\phi$ now below $\tilde{\phi}$, however, the model now has multiple steady states, and the zero unemployment steady state is locally stable. We assume that in 2009 the economy is hit by an unanticipated shock to the expected path for the unemployment rate from 2009 onwards such that
that the unemployment rate jumps immediately to 10 percent. Households cut back consumption – thereby rationalizing the surge in unemployment – because they now expect persistently high unemployment and therefore have a strong precautionary motive to save. From this point onwards, the economy is hit by no further fundamental shocks to preferences or to expectations, so households enjoy perfect foresight over the evolution of the unemployment rate. The economy converges towards the low unemployment steady state according to the dynamics described in Figure 5.

Although the model we have developed is very simple, it can replicate some key features of the Great Recession: (i) a decline in asset values that precedes the decline in real economic activity, (ii) a rapid contraction, and (iii) a slow recovery. In the next section we will argue that there are good economic reasons for why confidence-driven recessions will typically be persistent in nature. A key part of the intuition is that a rapid expected recovery would imply a strong inter-temporal motive to borrow and spend in the near term, and therefore make it difficult to engineer a demand-driven recession in the first place.
Note that the model simulation just described relies on two zero probability shocks: one to preferences, and one to expectations. We now move to construct sunspot equilibria in which agents take as given a positive probability of switching between boom and recession states.

4.3 Two State Sunspot Equilibria

We now construct equilibria with sunspots. We start with perhaps the simplest possible equilibrium of this type: a two-state Markov sunspot equilibrium, in which asset prices are constant, and in which the unemployment rate bounces between zero and a positive value, with symmetric Markov transition probabilities. Let \( L \) and \( H \) denote the zero and positive unemployment states. Let \( \lambda \) denote the probability of that the state in the next period is \( L \) (\( H \)) given that it is \( L \) (\( H \)) in the present period. This is now a three parameter model, where the parameters are \( \beta \), \( \phi \) and \( \lambda \).

The unemployment rates in the two states are \( u(L) = 0 \) and \( u(H) > 0 \). Assuming that unemployed workers are constrained, \( c^u(H) = p \). From the resource constraint, consumption of workers is then given by

\[
\begin{align*}
c^w(L) & = 1 \\
c^w(H) & = 1 - \frac{u(H)p}{[1 - u(H)]}
\end{align*}
\]

The inter-temporal first-order conditions in the zero and positive unemployment rate states are (again assuming that unemployed workers are constrained)

\[
\begin{align*}
\frac{p}{c^w(L)} & = \beta (1 - \lambda)p \left( [1 - u(H)] \frac{1}{c^w(H)} + u(H) \frac{1}{p} \right) + \beta \lambda \frac{p}{c^w(L)} + \beta \phi \\
\frac{p}{c^w(H)} & = \beta \lambda p \left( [1 - u(H)] \frac{1}{c^w(H)} + u(H) \frac{1}{p} \right) + \beta (1 - \lambda) \frac{p}{c^w(L)} + \beta \phi
\end{align*}
\]

Existence of Equilibria with Sunspots: We first ask, when does a sunspot equilibrium of the type described exist? An equilibrium of the type described exists if and only if there is a solution \( \{u(H), c^w(H), c^w(L), p\} \) to the previous four equations that satisfies (i) \( u(H) \in (0, 1] \), (ii) \( p \geq p \), and (iii) \( c^w(H) > p \). We now provide a partition of the parameter space into a region in which a solution with these properties exists, and a region in which there is no such solution.

**Proposition 3:** A two state sunspot equilibrium of the type described exists if and only if \( \lambda \geq \Lambda \) where

\[
\Lambda = 1 - \frac{(2 + \rho)p^2 - p + 1}{p^2 - p + 1}
\]
and where \( p = \frac{\beta \phi}{1-\beta} \) is the floor on house prices established by the riskless household type.

**Proof:** See Appendix

From this proposition, it is immediate that the larger is \( \phi \), the larger is \( \lambda \) and thus the more persistent changes in the unemployment rate must be in any sunspot driven model of fluctuations. Consider some special cases.

- When \( \phi = 0 \), so that housing has no fundamental value, \( \lambda = 0.5 \), and aggregate fluctuations can be generated by a sunspot process that is iid over time.
- For \( \phi > 0 \), \( \lambda > \frac{1}{2} \) and thus confidence-driven fluctuations must be persistent. At the baseline parameter values \( \lambda = 0.8635 \), so the expected duration of the unemployment state must be at least \( 1/(1 - 0.8635) = 7.3 \) years.
- As \( \phi \to \tilde{\phi} \), \( \lambda \to 1 \) and thus regimes of zero or positive unemployment must be expected to be near permanent.
- For \( \phi \geq \tilde{\phi} \) there are no sunspot equilibria of this type. Thus, confidence-driven fluctuations are only possible if the taste for housing, and hence the fundamental component of housing value, is sufficiently low.

To summarize, sunspot-driven fluctuations can only arise when two conditions are satisfied: (i) asset values must be sufficiently low (precisely, \( \phi \) must be sufficiently low), and (ii) fluctuations must be sufficiently persistent in expected terms (\( \lambda \) must be sufficiently high).

The logic for the first result is straightforward. For households to be willing to pay the same price for housing in the positive unemployment / low consumption state as in the zero unemployment / high consumption state, it must be that housing has sufficient additional liquidity value in the positive unemployment state. Housing only has significant liquidity value when the fundamental component of housing value is low.

The logic for why confidence-driven recessions must be persistent is as follows. The reason households reduce spending when the sunspot shock flips the economy into the positive unemployment state is that they anticipate a high likelihood that the unemployment rate will be high in the next period, and thus they have a strong precautionary motive to save today. Iterating forward, expecting that the unemployment rate will be high in the next period (and thus that consumption
will be low) only makes sense if the unemployment rate is likely to be high two periods into the future.\footnote{Put differently, suppose one were to try to construct sunspot driven cycles in which the sunspot process were iid. Households would then have no differential precautionary motive to save in the two states, but they would have a strong inter-temporal motive to use wealth to support consumption in the high unemployment state. But then this consumption would translate into additional demand and employment, and the conjectured equilibrium would unravel.}

**Persistence and Volatility:** We now turn to a second question. For values for $\phi < \tilde{\phi}$ (and thus $\Lambda < 1$) and for $\lambda \in [\Lambda, 1]$, what is the relationship between $\lambda$ on the one hand, and the unemployment rate in the recession state $u(H)$ on the other?

Figure 8 shows $u(H)$ for a set of alternative model parameterizations in which we vary the persistence parameter $\lambda$. In each case, $\beta$ and $\phi$ are fixed at their baseline values. Corresponding to each different $\lambda$ there is a $\lambda$-specific constant house price $p$.  

---

**Figure 8:** Larger Recessions are More Persistent
The key takeaway from the figure is that, in this class of equilibria, more persistent fluctuations and larger fluctuations go hand in hand. As $\lambda \to 1$, the unemployment rate in the recession state $u(H)$ converges to zero, and fluctuations vanish. As we increase $\lambda$, unemployment in the recession state rises. In the limit $\lambda \to 1$ the expected duration of either state becomes infinite, and the unemployment rate in the recession state converges to highest possible value that can arise in steady state, namely $u^+$.\(^6\)

Why are more persistent fluctuations also larger in magnitude? At a basic level, standard permanent income logic suggests that if the recession state is expected to last a long time, households will reduce spending sharply when the economy enters the recession state. More precisely, the strength of the household’s inter-temporal savings motive is closely connected to expected income growth, which in turn depends (negatively) on both the persistence of the sunspot process, and (positively) on the level difference between output in the normal and recession states. The more persistent are shocks, the larger must be this level difference in order to maintain similar expected income growth in the recession state and to thereby make these fluctuations consistent with optimal inter-temporal consumption choices.

**Wealth and Volatility:** We next construct a slightly different set of sunspot equilibria. We again focus on two-state Markov equilibria, in which asset prices are constant. However, instead of focusing on equilibria in which $u(L) = 0$ and exploring the effects of varying $\lambda$, we instead fix $\lambda$ and explore how the unemployment rates $u(L)$ and $u(H)$ vary with different choices for $p$. The goal is to explore how macroeconomic volatility – the difference between $u(H)$ and $u(L)$ – varies with wealth (conditional on a parameterization in which this sort of sunspot equilibrium exists.)

Recall that $p$ is an endogenous equilibrium object in the model. But there exist sunspot equilibria with different constant values for $p$, just as there exist steady states with different constant values for $p$ (see Figure 3). In fact, one can roughly think of the exercise here as constructing equilibria in which the economy bounces between the two steady states.\(^7\)

In Figure 9 we fix $\lambda = 0.99$, and plot $u(H)$ (in red) and $u(L)$ (in blue) against $p$.\(^8\) The key point is that the larger is $p$, the smaller is the gap between the unemployment rates in the two states. Thus higher asset prices imply less macroeconomic volatility, consistent with our characterization of US macroeconomic history in Section 1. A second finding is that higher asset prices also translate

\(^6\)In both the limit $\lambda \to 1$ and the limit $\lambda \to 1$, the equilibrium house price $p$ converges to the lower bound $p$. The steady state price is a hump-shaped function of $\lambda$ for intermediate values for $\lambda$.

\(^7\)As $\lambda \to 1$ this analogy is exact.

\(^8\)The dashed lines show the same objects for a lower $\lambda = 0.98$. 

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into lower average unemployment rates over the cycle: in the equilibrium with maximum volatility \( (p \approx 0.78) \) the average unemployment rate is 9 percent, while in the equilibrium with least volatility \( (p \approx 0.84) \) the average unemployment rate is 6 percent. Thus, the model suggests that high levels of unemployment should go hand in hand with high volatility of unemployment.

One way to understand these theoretical predictions is that in order to support a price considerably above the zero unemployment fundamental \( (\bar{p} = 0.75) \) the fundamental and liquidity components of home value must both be large in each state. A large liquidity component in the low unemployment state implies a relatively high value for \( u(L) \), while a high fundamental component in the high unemployment state implies a relatively low value for \( u(H) \).

![Figure 9: Wealth and Volatility](image)

Note that in this class of sunspot equilibria, for a fixed \( \lambda \), the equilibrium with the highest constant value for asset prices \( \bar{p} \) implies a constant positive unemployment rate. This \( (\bar{p}, u) \) combination picks out one of the steady states illustrated in Figure (3). For each different value for \( \lambda \), the corresponding highest price equilibrium picks out a different steady state \( (\bar{p}, u) \) combination.
Recall that for $\phi \geq \tilde{\phi}$ the highest price in any steady state corresponds to zero unemployment and is equal to $p$. Combining these observations, it follows that $\phi \geq \tilde{\phi}$ rules out the existence of any sunspot equilibria of this type; in any such equilibrium we would have $p \leq \tilde{p}$ (by definition) and $\tilde{p} < p$ (by virtue of Proposition 2) thereby contradicting the requirement $p \geq \tilde{p}$.

5 Review: Asset Prices and Volatility

Recall that the starting point for this paper is the strong positive empirical correlation between the level of US household wealth and US macroeconomic volatility. We now collect together the various theoretical results we have established that relate to the relationship between asset prices and output volatility.

First, if the taste for housing is sufficiently strong ($\phi \geq \tilde{\phi}$) the model has a unique steady state with full employment. Furthermore there do not exist sunspot equilibria in which the unemployment rate fluctuates over time.

Second, if the taste for housing is weak ($\phi < \tilde{\phi}$) so that the fundamental component of home value is small, then the economy has multiple steady states. In particular, there is a range of values for $p \geq p$ such for any $p$ in that range the model has two steady state unemployment rates. The low unemployment steady state is locally stable, introducing the possible of sunspot driven fluctuations, in which changes in expectations unrelated to fundamentals translate into macroeconomic volatility.

Third, when a low fundamental component to home value makes possible fluctuations driven by self-fulfilling changes in expectations, the theory imposes restrictions on the nature of those fluctuations. In particular, fluctuations must be expected to be persistent in order for changes in the expected unemployment rate to be self-fulfilling. The larger is the fundamental component of house value – and thus the less sensitive is demand to expectations – the more persistent must be changes in the equilibrium unemployment rate. Thus the theory raises an interesting contrast between fundamentals-driven versus confidence-driven fluctuations: the persistence of a fundamentals-driven recession will be tightly linked to the persistence of the underlying fundamental shock, while a confidence-driven recession will necessarily be persistent.

Fourth, when sunspot driven fluctuations are possible, the model can be used to investigate the conditions under which equilibrium fluctuations are large in magnitude. We found, first, that fluctuations tend to be larger in magnitude the more persistent is the underlying sunspot process. We will later argue that this implication of the theory can perhaps shed light on why the US
experienced such slow recoveries from the Great Depression and the Great Recession – the two deepest recessions in the last century. Our second result is that, holding fixed structural parameters (including the persistence of the sunspot process) fluctuations are larger the lower is the level of asset prices.

Taking these observations in combination indicates that the empirical evidence we presented earlier on the link between the level of wealth and the volatility of macro aggregates can be interpreted on two levels: (i) when asset values are sufficiently high, confidence-driven fluctuations cannot arise, and (ii) when asset values are lower, the amplitude of confidence-driven fluctuations is larger the lower is the level of asset prices.

6 Microeconomic evidence

As discussed above, when wealth is low, demand (and hence output) in our model can fall in response to a negative shock to expectations. The mechanism is that when wealth is low, an increase in perceived unemployment risk generates a large precautionary fall in consumption, thereby making the expectation of higher unemployment self-fulfilling. If the decline in aggregate consumption during the Great Recession was in fact largely driven by an increase in precautionary savings, then low wealth households – for whom the marginal precautionary motive should be especially large – should have reduced consumption disproportionately sharply as unemployment risk rose. In the next section we will use micro data from the Consumer Expenditure (CE) Survey and the Panel Study of Income Dynamics (PSID) to verify that lower wealth households did in fact exhibit systematically larger spending declines.

9In the model we have laid out, all households are identical. To develop a theoretical link between household wealth on the one hand and the consumption response to a change in unemployment risk on the other we considered the following thought experiment. Take the baseline calibration of the model, and consider the two steady states at $p = p$, one of which has $u = 0$, and the other of which has $u = u^+$. Now consider dropping a single atomistic household with arbitrary initial wealth $h_0$ into each of the two steady states. For different values for $h_0$ we compute the optimal values for initial consumption in each steady state. In the steady state with zero unemployment, the optimal choice is $c_w^* = 1$ irrespective of the initial value for $h_0$ (this is an implication of the quasi-linear preferences we assumed). In the steady state with positive unemployment, the optimal initial consumption of both workers and unemployed agents is increasing in $h_0$: consumption of borrowing-constrained unemployed workers is mechanically increasing in $h_0$, while consumption of employed workers is increasing in $h_0$ because more initial wealth reduces the precautionary motive to save (see eq. 4). Thus, if we imagine the economy transiting unexpectedly from the full employment steady state to the positive unemployment steady state, the decline in household consumption will be larger the smaller is initial household wealth.
6.1 Consumer Expenditure Survey

The CE Survey contains information on household-level wealth, expenditures and income. Households in the CE survey are interviewed for a maximum of four consecutive quarters. Households report consumption expenditures in all four interviews, report income information in the first and last interview, and report wealth information in the last interview only. We use CE data from the first quarter of 2005 to the first quarter of 2011 and we select households for which we have four consumption expenditure observations, two income observations and one wealth observation.

The goal is to compare changes in consumption expenditures during the course of the Great Recession for wealth rich versus poor households, controlling for potentially different income trends across the groups. It is important that we measure changes in expenditures using the same households, rather than comparing the average consumption of different sections of the wealth distribution at different points of time, as changes in moments of the cross-sectional distribution might reflect changes in composition of the cross-sections and not actual changes in expenditures. To measure actual expenditure changes we exploit the limited panel dimension of the CE survey.

In particular, we first rank households in each survey quarter by net worth relative to average quarterly consumption expenditure, which we use as a proxy for permanent income. Net worth here includes net financial wealth plus housing wealth net of all mortgages (including home equity loans). We then construct two wealth groups in each month of the survey, using the median as the break-point. We compute aggregate nine-month consumption (income) growth for each group as the difference between aggregate 4th interview consumption (income) less aggregate 1st interview consumption (income). Note that we are using the exact same set of households to measure consumption and income growth. Finally we use a standard weighting scheme to convert these observations on monthly growth rates to a quarterly frequency, assuming constant growth between interview dates.

Before comparing changes in expenditures across groups, we compare aggregate consumption in our CE sample to a conceptually-similar definition of consumption from the National Income and Product Accounts (NIPA). In particular we take aggregate NIPA consumption expenditure minus the following categories: housing and utilities, health care, and financial services. Figure 10 compares the CE and NIPA series, both of which are real, per-capita measures. The timing and depth of the Great Recession are broadly similar across the two measures, although the extent of the recovery in expenditures is much weaker in the CE than in NIPA.

\footnote{We exclude the categories for which the matching of the CE with NIPA is particularly poor}
Table 1 reports some characteristics of the two groups. First the table indicates that with respect to demographics the wealth poor group appears to be composed by younger, less educated and poorer (in terms of income) households. The table also shows that the differences in average net worth across the different groups are large: the median per capita household wealth in the poorest group is close to 0 while the corresponding value for the rich group quartile exceeds 60000 2005 dollars.

Most important though, for our purposes, is the comparison of income and consumption growth for the groups. The table shows that although the wealth poor experience a slightly stronger income growth than the wealth rich, they experience a substantially stronger reduction in consumption expenditures (-5.6% v/s -3.1%). We view this as prima-facie evidence that the level of wealth is a major factor, independent from income, in determining demand response in a turbulent time.
Table 1. Characteristics of two wealth groups, 2005.1-2011.1

<table>
<thead>
<tr>
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<th>Wealth Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0-50</td>
</tr>
<tr>
<td>Sample size</td>
<td>8864</td>
</tr>
<tr>
<td>Average age of head</td>
<td>41.4</td>
</tr>
<tr>
<td>Percent of heads with college</td>
<td>25.7</td>
</tr>
<tr>
<td>Average household size</td>
<td>2.9</td>
</tr>
<tr>
<td>Per capita real net wealth (2005$)</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1498</td>
</tr>
<tr>
<td>Median</td>
<td>238</td>
</tr>
<tr>
<td>Average per capita after tax income (2005$)</td>
<td></td>
</tr>
<tr>
<td>Level</td>
<td>22117</td>
</tr>
<tr>
<td>Growth rate</td>
<td>-0.3%</td>
</tr>
<tr>
<td>Average per capita consumption expenditures (2005$)</td>
<td></td>
</tr>
<tr>
<td>Level</td>
<td>9353</td>
</tr>
<tr>
<td>Growth rate</td>
<td>-5.6%</td>
</tr>
</tbody>
</table>

We analyze the issue in more detail as figure 11 plots (annualized) growth rates of per capita real consumption for each of our four wealth groups from the 4th quarter of 2005 through the 1th quarter of 2011 while figure 12 plots the change in consumption rates (i.e. consumption expenditure over after tax total income) for the two groups. These 2 figures tell a clear story. The first suggests that the the wealth-poor group reduced consumption sooner and most periods more sharply than the wealth rich. The second suggests that the relative reduction in consumption expenditures of the wealth poor throughout the recession is not just driven by a poor relative income growth by the wealth poor but rather by an increasing saving (reduced consumption rate). Our theory suggests that this increase in relative saving rate of the wealth poor is the result of the strengthening of their precautionary motive to save, in response to higher unemployment risk.

6.2 Panel Study of Income Dynamics

The PSID is a panel of US households, selected to be representative of the US population, collected at a bi-annual frequency. Most importantly for our purposes starting in 2004 the PSID report, for every household in the panel information on income, wealth and comprehensive consumption expenditures. Our panel includes all households which have at least one member aged between 22 and 60, which report yearly consumption expenditures of at least $1000, and which are in the panel continuously from 2004 to 2010. In table 2 we report the statistics of interest for the Great recession period. In particular in 2006 and in 2008 we divide all households in the PSID sample in two groups, those with wealth to consumption ratio above the median and those with wealth to consumption ratio below it. The table shows that the differences in consumption behavior between
Figure 11: Growth in consumption expenditures for rich and poor

Figure 12: Changes in consumption rates for rich and poor
the two groups are even more starker than in the CE. Over the period 2006-2008 the low wealth group reduced its consumption from 68% of its disposable income to 52%. By comparison the high wealth group only reduced from 46% to 43%.

<table>
<thead>
<tr>
<th></th>
<th>Low Wealth</th>
<th>High Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disposable Income</td>
<td>36600</td>
<td>+15%</td>
</tr>
<tr>
<td>Consumption</td>
<td>24800</td>
<td>-13%</td>
</tr>
<tr>
<td>Consumption Ratio</td>
<td>68%</td>
<td>-16%</td>
</tr>
<tr>
<td></td>
<td>2008</td>
<td>2008-2010</td>
</tr>
<tr>
<td>Disposable Income</td>
<td>41200</td>
<td>+2%</td>
</tr>
<tr>
<td>Consumption</td>
<td>22600</td>
<td>+3%</td>
</tr>
<tr>
<td>Consumption Ratio</td>
<td>55%</td>
<td>+1%</td>
</tr>
</tbody>
</table>

7 Policy

In the model we outlined in Section 2 agents derive no disutility from working, and enjoy strictly increasing utility from consumption. Thus equilibria with positive unemployed are inefficient. The economy can get stuck in those equilibria because agents cannot get together and agree to maintain consumption demand at the full employment level of output. Government policy can potentially address this co-ordination failure.

We consider two policies.

The first is for the government to directly purchase goods, and to finance the purchases by taxing workers. The government budget constraint is

\[ [1 - u(s^t)]T(s^t) = G \]

where \( G \) denotes non-valued government purchases, and \( T(s^t) \) is the tax per worker.

The second is for the government to tax workers, and transfer the revenues to unemployed workers in the form of a benefit \( b \). The government budget constraint is then

\[ [1 - u(s^t)]T(s^t) = u(s^t)b \]

7.1 Tax and Spend

Given a credible government commitment to a fixed amount of purchases \( G \), only the private portion of aggregate demand is now potentially sensitive to expectations. One might therefore guess that
this policy should make aggregate demand less sensitive to the expected unemployment rate, and therefore help to rule out equilibria with positive unemployment. However, the problem with this policy is that the tax on workers required to finance expenditure reduces private consumption, and therefore reduces the fundamental component of housing values. This reduces risk-sharing within the household, and makes desired saving more sensitive to perceived unemployment risk. The net effect is that the set of steady state unemployment rates ends up being completely insensitive to $G$.

**Proposition 4:** The set of steady state unemployment rates is equal to $[0, u^+]$ for any $G \in [0, 1 - u^+]$.

**Proof:** See Appendix

Figure 13 illustrates this result, showing the set of steady states for two different values for government purchases: $G = 0$, and $G = 0.1$. With the higher value for $G$, the floor on house prices established by the riskless household is reduced, since workers in the riskless household now pay taxes. Moreover, the floor is now declining in the steady state unemployment rate $u$, since per worker taxes $T = G/(1 - u)$ are increasing in $u$. The plot offers a concrete visual verification of Proposition 4: the maximum steady state unemployment rate is $u^+$ for both values for $G$. We conclude that the tax and spend policy is not an effective way to rule out expectations driven recession because the beneficial spending component of the policy is likely to be undermined by the negative impact of the associated taxation on asset prices.$^{11}$

### 7.2 Unemployment Benefits

Now consider a policy of taxing workers to provide unemployment benefits. This policy is effective because it directly addresses the lack of within household risk-sharing that makes demand very sensitive to perceived unemployment risk. Once unemployment benefits are introduced, the gap between consumption of workers and the unemployed is reduced, and the precautionary motive to save in the face of perceived unemployment risk therefore declines. As a result, the liquidity component to house prices is smaller, and high unemployment rates are no longer consistent with house prices that exceed the floor $p$. Figure 14 shows the set of steady states for $b = 0$ and $b = 0.1$.

$^{11}$The government could reduce the set of steady state unemployment rates by committing to extremely high levels of government purchases. Since private consumption cannot be negative, $u \leq 1 - G$. Thus setting $G = 1$ would guarantee zero unemployment. Note, however, that such a policy would imply zero private consumption.
Figure 13: Tax and Spend Policy

Figure 14: Unemployment Insurance Policy
In fact, if
\[ b \geq b = \frac{1}{2\beta - 2} \left( \beta - \sqrt{4\beta - 3\beta^2 + 2\beta\phi} \right) \]
then there is a unique steady state, with full employment.\(^{12}\) Given our baseline parameter values, the critical threshold for benefits is \( b = 0.204 \). Note that a policy of setting \( b \geq b \) is costless in equilibrium, since the mere anticipation of unemployment benefits is enough to prevent agents from co-ordinating on an equilibrium with positive unemployment, and with full employment no taxes need be collected. In that sense the policy is analogous to bank deposit insurance: the presence of sufficiently generous deposit insurance (unemployment insurance) rules out the equilibrium in which agents run on the bank (increase precautionary saving), and on the equilibrium path no tax revenue need be collected. Thus, in the context of this simple model, unemployment benefits are an attractive policy instrument. We recognize, however, that this simple model does not capture the potential costs of an unemployment insurance scheme in terms of disincentives to supply labor.\(^{13}\)

8 Conclusions

[TO BE ADDED]

References


\(^{12}\)This expression can be derived following a similar approach to the derivation of the threshold \( \hat{\phi} \) for the existence of multiple steady states absent unemployment benefits (see Proposition 2).

\(^{13}\)In addition, a more complete analysis of unemployment insurance would ideally micro-found why private unemployment insurance markets do not exist. We have simply assumed that the government (but not households) can access a technology that allows it reshuffle resources between unemployment household members and employed household members within the period.


9 Appendix

Proof of Proposition 1

Suppose, contrary to the claim, that $u > 0$ and $c^u = c^w$. Then the price that solves the inter-temporal FOC would be

$$p = p_F(u) = \frac{\beta \phi}{1-\beta}(1-u) < p$$  \hspace{1cm} (17)

where $p_F(u)$ is the “fundamental” steady state price given $u$. But $p < p$ contradicts $p \geq p$ which must hold in any steady state.

Proof of Proposition 2

From the pricing equation for the riskless type, $p \geq p = \frac{\beta \phi}{1-\beta}$. Thus there can be no steady states with $p < p$.

From the previous proposition, if there is an equilibrium with $u > 0$, then unemployed agents are constrained. Thus $c^u = p$ and $c^w = 1 - \frac{u}{1-u}p$.

The first order condition for the risky household is therefore

$$\frac{p}{1-\frac{u}{1-u}p} = \beta \frac{p}{1-\frac{u}{1-u}p} \left[ 1 + \frac{u \left(1 - \frac{u}{1-u}p - p\right)}{p} \right] + \beta \phi$$

which simplifies to deliver the following equilibrium relation between steady state $u$ and $p$:

$$p(u) = \frac{\beta (u + \phi)}{(1-\beta) + \frac{\beta u (1+\phi)}{1-u}}$$  \hspace{1cm} (18)

Note that at $u = 0$, this relation implies $p = p$, so that is always a steady state.

Using the expression above, we can explore how $p$ varies with $u$.

$$\frac{\partial p}{\partial u} \propto - \left( 2u + \beta - 2u\beta + \beta \phi + 2u^2\beta + \beta \phi^2 - u^2 + u^2\beta \phi - 1 \right)$$

Now it is immediate that at $u = 0$

$$\left( \frac{\partial p}{\partial u} \right)_{u=0} = \begin{cases} > 0 & \text{for } \phi < \bar{\phi} \\ 0 & \text{at } \phi = \bar{\phi} \\ < 0 & \text{for } \phi > \bar{\phi} \end{cases}$$

Now turn to the second derivative:

$$\frac{\partial^2 p}{\partial u^2} \propto u + \beta - 2u\beta - u\beta \phi - 1$$
It is immediate that $\beta > 0.5$ is a sufficient condition for the second derivative to be negative, and thus for $p$ to be a concave function of $u$.

Combining these two results, it follows that for $\beta > 0.5$ and $\phi \geq \tilde{\phi}$, the value for $p$ that satisfies the risky type’s first-order condition is decreasing in $u$. Thus $p \leq p^{*}$.

But then for $\phi \geq \tilde{\phi}$, we have $p \geq p^{*}$ (from the FOC for the riskless type) and $p \leq p^{*}$ (from the FOC for the risky type). It follows that $p = p^{*}$ and $u = 0$ is the only steady state.

If $\phi < \tilde{\phi}$ then $\frac{\partial p}{\partial u} > 0$ at $u = 0$. Since $p$ is a continuous and concave function of $u$, and since $p = 0$ at $u = 1$ there must be a second steady state at $p = p^{*}$ with $u > 0$.

This unemployment rate is given by

$$u^+ = 1 - \frac{\beta}{1 - \beta} \phi(1 + \phi)$$

(19)

By similar reasoning, there is a range of values for $p > p^{*}$ such that given $\phi < \tilde{\phi}$ there are two steady states with positive unemployment.

Note that the uniqueness result with $\phi \geq \tilde{\phi}$ hinges on the presence of the riskless household type. Without this type, there would be a continuum of steady states with unemployment rates between zero and one, with each unemployment rate corresponding to a different steady state asset price as given by eq. 17 (see Farmer 2010). The presence of the riskless type puts a floor on the asset price, which effectively establishes a floor for steady state consumption demand and output.

**Proof of Proposition 3**

[TO BE COMPLETED]

**Proof of Proposition 4**

The steady state equilibrium condition that must be satisfied for the household facing unemployment risk is

$$\frac{p}{1 - \frac{u}{1-u}p - T(u)} = \beta p \left( \frac{(1 - u)}{1 - \frac{u}{1-u}p - T(u)} + \frac{u}{p} \right) + \beta \phi$$

where equilibrium consumption values for workers and unemployed members are

$$c^w = 1 - \frac{u}{1-u}p - T(u)$$

$$c^u = p$$

and where taxes paid per worker are $T(u) = G/(1-u)$. 38
Let \( p(u) \) denote the floor on house prices established by the riskless household type.

\[
p(u) = \frac{\beta \phi (1 - T(u))}{1 - \beta}.
\]

Recall that at \( u = 0 \), the house price is equal to \( p(0) \). Let \( u^\# \) denote the maximum possible steady state unemployment rate. At \( u = u^\# \) the price is again equal to the floor established by the riskless household type: \( p = p(u^\#) \). This implies the following relationship at \( u = u^\# \) between the per-worker tax required to finance \( G \) and the house price

\[
T(u^\#) = 1 - \frac{(1 - \beta)p(u^\#)}{\beta \phi}.
\]

Substituting this into the first-order condition for the household facing risk gives

\[
\frac{p(u^\#)}{(1 - \beta)p(u^\#)} - \frac{u}{1 - u} p(u^\#) = \beta p(u^\#) \left( \frac{1 - u^\#}{(1 - \beta)p(u^\#)} - \frac{u}{1 - u} p(u^\#) + \frac{u^\#}{p(u^\#)} \right) + \beta \phi
\]

which simplifies to the following quadratic equation:

\[
-u^\# \left( \beta \phi^2 + \beta \phi + u^\# + \beta - u^\# \beta - 1 \right) = 0
\]

One solution to this equation is \( u^\# = 0 \) – the minimum possible steady state unemployment rate. The other solution is

\[
u^\# = 1 - \frac{\beta}{1 - \beta} \phi (1 + \phi) = u^+.
\]