Investment Hangover and the Great Recession*

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Abstract

We present a model of investment hangover motivated by the Great Recession. In our model, overbuilding of residential capital requires a reallocation of productive resources to nonresidential sectors, which is facilitated by a reduction in the real interest rate. If the fall in the interest rate is limited by the zero lower bound and nominal rigidities, then the economy enters a liquidity trap with limited reallocation and low output. The drop in output reduces nonresidential investment through a mechanism similar to the acceleration principle of investment. The burst in nonresidential investment is followed by an investment boom due to low interest rates during the liquidity trap. The boom induces a partial and asymmetric recovery in which the residential sector is left behind, consistent with the broad trends of the Great Recession. In view of aggregate demand externalities, welfare can be improved by ex-post policies that slow down the decumulation of residential capital, as well as ex-ante policies that restrict the accumulation of capital.

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1 Introduction

Since 2008, the US economy has been going through the worst macroeconomic slump since the Great Depression. Real GDP per capita declined from more than $49,000 in 2007 (in 2009 dollars) to less than $47,000 in 2009, and surpassed its pre-recession level only in 2013. The civilian employment ratio, which stood at about 63% in January 2008, fell below 58% by January 2010, and remains below 59% in June 2014.

Recent macroeconomic views emphasize the burst of the housing bubble—and its effects on financial institutions, firms, and households—as the main culprit for these developments. The collapse of home prices arguably affected the economy through at least two principal channels. First, it triggered the financial crisis, which led financial institutions that suffered losses related to the housing market to cut back their lending to firms and households (Brunnermeier (2008), Gertler and Kiyotaki (2010)). Second, the reduction in home prices also generated a household deleveraging crisis, in which homeowners that suffered leveraged losses from their housing equity cut back their consumption so as to reduce their outstanding leverage (Guerrieri and Lorenzoni (2011), Eggertsson and Krugman (2012), Mian and Sufi (2014)). Both crises reduced aggregate demand, plunging the economy into a Keynesian recession. The recession was exacerbated by the zero lower bound on the nominal interest rate, also known as the liquidity trap, which restricted the ability of monetary policy to counter these demand shocks (Hall (2011), Christiano, Eichenbaum, Trabandt (2014)).

A growing body of empirical evidence shows that these views are at least partially correct: the financial and the household crises both appear to have played a part in the Great Recession\[1\] But these views also face a challenge in explaining the

\[1\]Several recent papers, such as Campello, Graham, and Harvey (2010) and Chodorow-Reich (2014), provide some evidence that financial crisis affected firms’ investment before 2010. Mian, Rao, Sufi (2013) and Mian and Sufi (2012) provide evidence that household deleveraging reduced
nature of the recovery after the Great Recession. As Figure illustrates, the recovery has been quite asymmetric across components of aggregate private spending. Non-residential investment and consumption—measured as a fraction of output—reached or exceeded their pre-recession levels by 2013, while residential investment remained depressed. One explanation for this pattern is that households are unable to buy houses due to ongoing deleveraging. But the left panel of Figure casts doubt on this explanation: sales of durable consumption goods such as cars—which should also be affected by household deleveraging—rebounded strongly in recent years while residential investment has lagged behind.

In this paper, we supplement the two accounts of the Great Recession with a third channel, which we refer to as the investment hangover, which could help explain the asymmetric recovery. Our key observation is that the housing bubble was an investment bubble as much as an asset price bubble. Overbuilding during the bubble years created excess supply of housing capital by 2007, especially certain types of capital such as owner occupied housing. Between 1996 and 2006, the share of US households living in their own homes rose from about 65% to about 69%. The homeownership rate fell back below 65% in 2014, suggesting that the housing capital might have been in excess for many years after 2007. The excess housing capital lowers residential investment and slows down economic activity.

Our argument so far is similar to the Austrian theory of the business cycle, in which recessions are times at which excess capital built during boom years is liquidated (Hayek (1931)). The Hayekian view, however, faces a challenge in explaining how low investment in the liquidating sector reduces aggregate output and employment. As noted by Krugman (1998), the economy has a natural adjustment mechanism household consumption and employment between 2007 and 2009.
Figure 1: The plots illustrate different components of aggregate demand as a fraction of GDP between 1999 and 2004. The data is quarterly and reported as the seasonally adjusted annual rate. Source: St. Louis Fed.
that facilitates the reallocation of labor (and other productive resources) from the liquidating sector to other sectors. As the interest rate falls during the liquidation phase due to low aggregate demand, other sectors expand and keep employment from falling. This reallocation process can be associated with some increase in frictional unemployment. But it is unclear in the Austrian theory how employment can fall in both the liquidating and the nonliquidating sectors, which seems to be the case for major recessions such as the Great Recession. To fit that evidence, an additional—Keynesian—aggregate demand mechanism is needed.

Accordingly, we depart from the Hayekian view by emphasizing that, during the Great Recession, the aggregate reallocation mechanism was undermined by the zero lower bound constraint on monetary policy. If the initial overbuilding is sufficiently large, then the interest rate hits a lower bound and the economy enters a liquidity trap. As this happens, low investment in the residential sector cannot be countered by the expansion of other sectors. Instead, low investment reduces aggregate demand and output, contributing to the Keynesian slump.

We also illustrate how overbuilding of residential capital can initially reduce nonresidential investment. The Keynesian slump reduces the return to nonresidential capital such as business equipment. We show that this can generate an initial reduction in nonresidential investment and capital, despite the low interest rate and the low cost of capital. The nonresidential investment response in turn aggravates the recession as emphasized by the previous literature on the acceleration principle of investment (see Samuelson (1939)). As the economy liquidates the excess residential capital, nonresidential investment gradually recovers in anticipation of a recovery in output. In fact, the initial burst in nonresidential investment is followed by an even greater boom due to low interest rates. It follows that, from the lens of our model,
the recession can be roughly divided into two phases. In the first phase, both types of investment decline, generating a severe slump. In the second phase, nonresidential investment increases, generating a partial recovery, but residential investment remains low. Hence, the residential sector is left behind in the recovery, as in Figure 1.

We finally investigate the implications of our analysis for policies directed towards controlling investment. A naive intuition would suggest that the planner should not interfere with the decumulation of residential capital, since the problems originate in this sector. We find that this intuition is incorrect: A constrained planner also decumulates the residential capital, but does so more slowly compared to an unregulated competitive equilibrium. Intuitively, the planner recognizes that raising residential investment during the liquidity trap stimulates aggregate demand and output. Thus, she optimally postpones some of the decumulation to future periods that do not feature a demand shortage.

Another naive intuition would suggest that, ex-ante, before the economy enters the liquidity trap, the planner should restrict the accumulation of residential capital. We find that this intuition is largely correct. Perhaps surprisingly, however, our analysis also reveals that a constrained planner restricts nonresidential investment, even though the problems are driven by residential investment. Intuitively, the planner recognizes that bringing less capital (of either type) into the liquidity trap leaves more slack for future investment. Thus, she restricts capital accumulation ex-ante so as to increase investment and aggregate demand ex-post, thereby internalizing aggregate demand externalities.

Our paper is part of a large macroeconomic literature that attempts to identify the mechanisms of the Great Recession.\footnote{In addition to the above mentioned papers, see Gertler and Karadi (2011), Jermann and Quadrini} Two features differentiate our analysis from
recent accounts that also emphasize demand shocks and the liquidity trap. First, moti-
vated by the asymmetric recovery depicted in Figure 1, we emphasize overbuilding of residential capital as a key driving factor of the recession. In contrast, Eggertsson and Krugman (2012) emphasize a consumption shock due to household deleveraging, and Christiano, Eichenbaum, Trabandt (2014) emphasize a nonresidential investment shock due to firms' financial frictions (as well as a consumption shock). Second, we illustrate how the problems in the residential investment sector can spread to nonresidential investment even without financial shocks. We do not claim that financial shocks were unimportant during the Great Recession. Rather, our point is that overbuilding was also an important contributing factor, with implications that confound financial shocks, especially earlier in the recession. These confounding effects should be taken into account by empirical analyses of the Great Recession.

Our paper is also related to a macroeconomic literature that investigates the role of reallocation shocks relative to aggregate shocks in generating unemployment fluctuations (see Lilien (1982), Abraham and Katz (1986), Blanchard and Diamond (1989), Davis and Haltiwanger (1990)). Our paper shows how reallocation shocks can *endogenously* turn into aggregate shocks. In our model, expanding sectors are constrained due to nominal rigidities and constrained monetary policy, which restricts reallocation and triggers a Keynesian recession. Caballero and Hammour (1996) alternatively emphasize a supply-side channel by which reallocation is restricted because the expanding sectors are constrained due to a hold-up problem.

As we have noted, our paper makes contact with the Austrian (or Hayekian) theory (2012), He and Krishnamurthy (2014), Midrigan and Philippon (2011) for quantitative dynamic macroeconomic models that emphasize either banks', firms', or households' financial frictions during the Great Recession. There is also a vast theoretical literature that analyzes the amplification mechanisms that could have exacerbated the financial crisis (see Brunnermeier, Eisenbach, Sannikov (2013) for a survey). Another theoretical literature investigates the liquidity trap and its policy implications (see Korinek and Simsek (2014) and the references therein).
of the business cycle. As DeLong (1990) discusses, liquidationist views along these lines were quite popular before and during the Great Depression, but were relegated to the sidelines with the Keynesian revolution in macroeconomics. Our paper illustrates how Hayekian and Keynesian mechanisms can come together to generate a recession. The Hayekian mechanism finds another modern formulation in the recent literature on news-driven business cycles. A strand of this literature argues that positive news about future productivity can generate investment booms, occasionally followed by liquidations if the news is not realized (see Beaudry and Portier (2013) for a review). This literature typically generates business cycles from supply side considerations (see, for instance, Beaudry and Portier (2004), Jaimovich and Rebelo (2009)), whereas we emphasize demand shortages as the key mechanism by which liquidations trigger a recession.

In recent work, Beaudry, Galizia, Portier (BGP, 2014) also investigate channels by which overbuilding can induce a recession driven by demand shortages. Their paper is complementary to ours in the sense that they use different ingredients and emphasize a different mechanism. In BGP, aggregate demand affects employment due to a matching friction in the labor market, whereas we obtain demand effects through nominal rigidities. In addition, BGP emphasize how overbuilding increases the ( uninsurable) unemployment risk, which exacerbates the recession due to households’ precautionary savings motive. In contrast, we emphasize how overbuilding reduces the return to other (nonresidential) types of capital, which exacerbates the recession due to the endogenous investment response. We also apply our model to explain the asymmetric recovery from the Great Recession.

Our paper is also related to the literature on the acceleration principle of investment (see Clark (1917)). This principle posits that the target level of capital is
proportional to output, so that investment is driven by changes in output. Samuelson’s (1939) famous multiplier-accelerator analysis shows that this principle can also aggravate business cycles driven by demand shocks. The acceleration principle fell out of fashion partly because it relies on mechanical relations between investment and output, without considering changes in the cost of capital (see Caballero (1999)). In our model, a version of the acceleration principle emerges endogenously from agents’ optimizing behavior. Intuitively, the liquidity trap keeps the cost of capital constant, resuscitating the acceleration principle and some of its macroeconomic implications.

The rest of the paper is organized as follows. Section 2 describes the basic environment, defines the equilibrium, and establishes the properties of equilibrium that facilitate subsequent analysis. The remaining sections characterize the dynamic equilibrium starting with excess residential capital. Section 3 presents our main result that excessive overbuilding induces a recession, and establishes conditions under which this outcome is more likely. Section 4 investigates the nonresidential investment response and discusses the relationship of our model with the acceleration principle. This section also discusses how our model can be extended to capture the nonmonotonic response of consumption in addition to investment. Section 5 discusses the welfare implications of our analysis and Section 6 concludes.

2 Basic environment and equilibrium

The economy is set in infinite discrete time \( t \in \{0, 1, \ldots\} \) with a single consumption good, and three factors of production: residential capital, \( h_t \), nonresidential capital, \( k_t \), and labor, \( l_t \). For brevity, we also refer to nonresidential capital as “capital.” Each unit of residential capital produces one unit of housing services. Capital and labor
are combined to produce the consumption good as we will describe below.

One unit of the consumption good can be converted into one unit of residential or nonresidential capital without any adjustment costs. Thus, the two types of capital evolve according to

\[ h_{t+1} = h_t (1 - \delta^h) + i_t^h \quad \text{and} \quad k_{t+1} = k_t (1 - \delta^k) + i_t^k. \]  

Here, \( i_t^h \) (resp. \( i_t^k \)) denote residential (resp. nonresidential) investment, and \( \delta^h \) (resp. \( \delta^k \)) denotes the depreciation rate for residential (resp. nonresidential) capital.

As we will see, absent shocks, the economy will be at a neoclassical steady state in which the two types of capital are kept at fixed levels denoted by \( h^* \) and \( k^* \). We will analyze situations in which the economy starts with excess residential capital, \( h_0 > h^* \) (see Eq. (19) below). This assumption can be thought of as capturing an unmodeled overbuilding episode that took place before the start of our model. We are agnostic about the reason for overbuilding, which could be driven, among other things, by optimistic beliefs (or news) in the past that were ultimately corrected. Our focus will be on understanding how the economy decumulates the overbuilt capital.

**Households** There is a representative household that makes consumption, labor supply, as well as residential investment decisions. For the baseline model, we assume the household has the following preferences,

\[ U (\hat{c}_t, l_t, h_t) = u (\hat{c}_t - v (l_t)) + u^h 1 [h_t \geq h^*]. \]  

Here, the function \( u (\cdot) \) is strictly increasing and concave. The expression \( 1 [h_t \geq h^*] \) is equal to 1 if \( h_t \geq h^* \) and zero otherwise, and \( u^h \) is a large constant. This specification
of preferences entails two simplifying assumptions. First, given housing capital \( h_t \), the household will choose

\[ h_{t+1} = h^*, \]  
which implies \( \dot{i}^h_t = h^* - h_t (1 - \delta^h) \).  

(3)

In particular, starting with some \( h_0 > h^* \), the household decumulates its residential capital as quickly as possible so as to reach and stay at the level \( h^* \). This assumption considerably simplifies the residential investment part of the model, and enables us to focus on the effect of overbuilding on the rest of the equilibrium allocations.

The second simplification in (2) is the functional form \( u(\hat{c}_t - v(l_t)) \), which implies that the household’s labor supply decision does not depend on its consumption (see Greenwood, Hercowitz and Huffman (1988)). Specifically, households’ optimal labor solves the static optimization problem,

\[ e_t = \max_{l_t} w_t l_t - v(l_t). \]  

(4)

Here, \( e_t \) denotes households’ net labor income, that is, labor income net of labor costs. We also define \( c_t = \hat{c}_t - v(l_t) \) as net consumption, which summarizes the household welfare within the period. Households’ consumption and saving problem can be written in terms of net variables as:

\[ \max_{\{c_t, a_{t+1}\}_t} \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{s.t.} \quad c_t + a_{t+1} + \dot{i}^h_t = e_t + a_t (1 + r_t) + \Pi_t, \]  

(5)

for each \( t \). Here, \( a_t \) denotes households’ financial assets and \( r_{t+1} \) denotes the real interest rate between dates \( t \) and \( t + 1 \). Households also receive profits \( \Pi_t \) from firms that will be described below.
The optimal household behavior in this model is summarized by Eq. (3) and problems (4) and (5). We also assume $u(\cdot)$ and $v(\cdot)$ satisfy the standard regularity conditions.

**Investment firms, the interest rate, and the liquidity trap** The capital stock of the economy is managed by a competitive investment sector. This sector issues financial assets to households, invests in capital, and rents the capital to the production firms that will be described below. The optimality conditions for the sector imply that the interest rate (equivalently, the cost of capital) satisfies,

$$r_{t+1} = R_{t+1} - \delta^k,$$

where $R_{t+1}$ denotes the rental rate of capital. The capital market clearing condition is $a_t = k_t$.

Our key ingredient is that the nominal interest rate is bounded from below, that is, $r_{t+1}^n \geq 0$. This constraint emerges because there is cash in circulation that provides households with transaction services. If the nominal interest rate fell below zero, then individuals would switch to hoarding cash instead of holding financial assets. Therefore, monetary policy cannot lower the nominal interest rate below zero. The situation in which the nominal interest rate is at its lower bound is known as the *liquidity trap*.

The constraint on the nominal interest rate might not affect the real allocations by itself. However, we also assume that nominal prices are completely sticky (as we formalize below) which ensures that the nominal and the real interest rates are the same, and thus the real interest rate is also bounded,

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3To simplify the notation and the exposition, however, we consider the limit in which the transaction value of cash approaches zero (as described in Woodford (2003)).
\[ r_{t+1}^n = r_{t+1} \geq 0 \text{ for each } t. \] (7)

**Production firms and output**  We introduce the nominal price rigidities with the standard New Keynesian model. Specifically, there are two types of production firms. A competitive final good sector uses intermediate varieties \( \nu \in [0, 1] \) to produce the final output according to the Dixit-Stiglitz technology,

\[
y_t = \left( \int_0^1 y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} \, d\nu \right)^{-\frac{\varepsilon}{\varepsilon-1}} \text{ where } \varepsilon > 1.
\] (8)

In turn, a unit mass of monopolistic firms labeled by \( \nu \in [0, 1] \) each produce the variety according to,

\[
y_t(\nu) = F(k_t(\nu), l_t(\nu)), \tag{9}
\]

where \( F(\cdot) \) is a neoclassical production function that satisfies the standard regularity conditions.

We make the extreme assumption that each monopolist has a preset and constant nominal price, \( P_t(\nu) = P \) for each \( \nu \). This also implies that monopolists are symmetric: they face the same real price (equal to one) and they choose the same level of inputs and outputs subject to an aggregate demand constraint. In particular, the representative monopolist’s problem can be written as:

\[
\Pi_t = \max_{k_t, l_t} F(k_t, l_t) - w_t l_t - R_t k_t \text{ s.t. } F(k_t, l_t) \leq y_t.
\] (10)

In the equilibria we will analyze, the monopolist’s marginal cost will be below its price so that it will find it optimal to meet all of its demand, that is, \( F(k_t, l_t) = y_t \).
Efficient benchmark and the monetary policy  It follows that the outcomes in this model are ultimately determined by the aggregate demand for the final good, \( y_t = \hat{c}_t + i^h_t + i^k_t \). This in turn depends on monetary policy, which controls the nominal and the real interest rate. Since the price level is fixed, we assume that the monetary policy focuses on replicating the efficient benchmark subject to the lower bound constraint in (7). \(^4\)

To formalize this policy, we first characterize the efficient benchmark. The appendix shows that the efficient level of employment and output are respectively given by,

\[
\begin{align*}
    l^*_t &= \arg \max_{l_t} F(k_t, l_t) - v(l_t) \quad \text{and} \quad y^*_t = F(k_t, l^*_t).
\end{align*}
\]

That is, the efficient benchmark maximizes the output net of labor costs given the capital stock and the technology in (8) and (9). We also define \( r^*_t \) recursively as the interest rate that obtains at time \( t \) when employment and output are at their efficient levels, and the monetary policy follows the rule in (11) at all future dates. Finally, we assume the monetary policy sets the interest rate,

\[
r^*_t + 1 = r^*_t = \max \left( 0, r^*_t \right) \text{ for each } t.
\]

It can be seen that this policy is constrained efficient in our environment as long as the monetary policy does not have commitment power.

Definition 1. The equilibrium is a path of allocations, \( \{ l_t, k_t, l_t, \hat{c}_t, c_t, i^h_t, i^k_t, y_t \} \_t \), and real prices and profits, \( \{ w_t, R_t, r^*_t, \Pi_t \} \_t \), such that the household allocations satisfy

\(^4\)In particular, the monetary policy does not replicate the “frictionless” equilibrium that would obtain if monopolists could reset their prices at every period. Instead, the monetary policy also corrects for the distortions that would stem from the monopoly pricing. Ideally, these distortions should be corrected by other policies, e.g., monopoly subsidies, and the monetary policy should focus on replicating the frictionless benchmark. We ignore this distinction in the baseline model, so as to simplify the notation.
and solve problems (4) and (5), a competitive final good sector produces according to (8), the intermediate good monopolists solve (10) for given fixed goods prices, the interest rate is set according to (12), and all markets clear.

3 Investment hangover and the Keynesian recession

We next turn to the characterization of equilibrium. We start by establishing some properties of the equilibrium within a period. We then analyze the dynamic equilibrium starting with excess residential capital stock and establish our main result.

Throughout, we find it useful to work with the variable,

\[ Y_t = y_t - v(l_t) + (1 - \delta^k) k_t, \]

which corresponds to output net of labor costs and depreciation costs, while also including the capital stock \( k_t \). We refer to \( Y_t \) as net output (with a slight abuse of terminology). The resource constraint of the economy can then be written as,

\[ c_t + k_{t+1} + i_t^h = Y_t. \]  

We also define the efficient (or supply determined) level of net output as,

\[ S(k_t) = \max_{l_t} F(k_t, l_t) - v(l_t) + k_t (1 - \delta^h). \]  

Note that this is the level of \( Y_t \) that obtains when employment and output are at their efficient levels [cf. (11)]. The feasibility constraints of this economy can then be
written as,

\[ Y_t \leq S(k_t) \text{ and } Y_t \geq (1 - \delta^k) k_t. \] (15)

The following lemma characterizes the possibilities for output, employment, and factor returns within a period.

**Lemma 1.** (i) If \( r_{t+1} > 0 \), then \( Y_t = S(k_t), l_t = l_t^*, \) and \( 1 + R_t - \delta^k = S'(k_t) \).

(ii) If \( r_{t+1} = 0 \), then \( Y_t \) satisfies (15) and (13). The labor supply is the unique solution to

\[ Y_t = F(k_t, l_t) - v(l_t) + (1 - \delta^k) k_t, \text{ over the range } l_t \in [0, l_t^*]. \] (16)

The gross return to capital satisfies \( 1 + R_t - \delta^k = s(k_t, Y_t) \leq S'(k_t) \), where the function \( s(k_t, Y_t) \) is strictly decreasing in \( k_t \) and strictly increasing in \( Y_t \).

The first part describes the case in which the interest rate is positive and the monetary policy in (12) replicates the efficient outcomes. The second part describes the liquidity trap scenario in which the interest rate is at its lower bound. In this case, net output satisfies the feasibility constraint in (13) but it is otherwise unrestricted. The actual level of net output is determined by the net aggregate demand at date \( t \) as illustrated by the resource constraint (13). Given \( Y_t \), the level of employment is found as the solution to (16) and satisfies \( l_t \leq l_t^* \).

The lemma also characterizes the gross return to capital, \( 1 + R_t - \delta^k \), which will be equated to the gross interest rate \( 1 + r_t \) in equilibrium [cf. Eq. (6)]. Towards an intuition for these results, consider the optimality conditions for problem (10), which
characterizes the monopolists’ factor demands,

\[(1 - \tau_t) F_k(k_t, l_t) = R_t \text{ and } (1 - \tau_t) F_l(k_t, l_t) = w_t. \tag{17}\]

Here, \(\tau_t \geq 0\) denotes the Lagrange multiplier on the demand constraint in (10), which is also known as the labor wedge. If the interest rate is positive, then employment is at its efficient level and the labor wedge is zero, \(\tau_t = 0\). In this case, the demand constraint effectively does not bind and capital (as well as labor) earns its marginal contribution to the efficient level of output, as captured by the first part of the lemma.

If instead the interest rate is zero, then employment is below its efficient level and the labor wedge is positive, \(\tau_t > 0\). In this case, the demand shortage lowers capital’s (as well as labor’s) rental rate relative to the efficient benchmark. The second part of the lemma shows further that the return to capital in this case can be written as a function of the capital stock and net output. Higher \(Y_t\) increases the return due to higher demand (and thus, lower labor wedge), while higher \(k_t\) reduces it due to diminishing returns.

Combining Lemma \([\square]\) with the lower bound in \([\square]\) also implies that the capital stock (and thus, nonresidential investment) is bounded from above, that is,

\[k_{t+1} \leq \bar{k} \text{ for each } t, \text{ where } S'(\bar{k}) = 1. \tag{18}\]

Here, the upper bound \(\bar{k}\) is the level of capital that delivers a gross return of one (or a net return of zero). Intuitively, investing beyond this level would not be profitable given the lower bound to the cost of capital in \([\text{7}]\). This bound on investment will play a central role in the subsequent analysis. It is also useful to define the steady state level of capital \(k^*\) as the solution to \(S'(k^*) = 1/\beta\).
3.1 Investment hangover

We next characterize the dynamic equilibrium under the assumption that the economy starts with too much residential capital,

\[ h_0 = (1 + b_0) h^*, \text{ where } b_0 > 0. \]  

(19)

This assumption can be thought of as capturing an unmodeled investment bubble that took place before the start of our model. The parameter \( b_0 \) measures the degree of overbuilding as a fraction of the steady-state stock of residential capital \( h^* \). Our main result, which we present in this section, shows that this type of overbuilding can induce a recession.

By Eq. (3), the residential investment level at date 0 is given by,

\[ i_0^h = h^* - (1 - \delta^h) h_0 = (\delta^h - b_0 (1 - \delta^h)) h^*. \]  

(20)

Note that the residential investment is below the level required to maintain the target residential capital \( i_0^h < \delta^h h^* \). Hence, overbuilding represents a negative shock to the residential investment demand relative to a steady state. The equilibrium depends on how the remaining components of aggregate demand—nonresidential investment and consumption—respond to this shock.

To characterize this response, we solve the equilibrium backwards. Suppose the economy reaches date 1 with \( h_1 = h^* \) and some capital level \( k_1 \leq \bar{k} \). Consider the continuation equilibrium. Residential investment is given by \( i_t^h = \delta^h h^* \). Since there are no further demand shocks, the equilibrium does not feature a liquidity trap, that is, \( r_{t+1} > 0 \) for each \( t \geq 1 \). Labor and net output are then at their efficient levels, respectively given by \( l_t^\ell \) and \( S(k_t) \). The equilibrium path \( \{c_t, k_{t+1}\}_{t=1}^{\infty} \) is characterized
as the solution to the neoclassical system

\[ c_t + k_{t+1} + \delta^h h^* = S(k_t) \tag{21} \]
\[ u'(c_t) = \beta S'(k_t) u'(c_{t+1}) , \]

along with a transversality condition. For the rest of the analysis, we make the following assumption, which ensures that the economy is able to afford the required residential investment at the initial period as well as the steady state.

**Assumption 1.** \( \min (S(k_0), S(k^*)) > k^* + \delta^h h^* \), and

This assumption ensure there is a unique solution to the system in (21) that converges to a steady state \((c^*, k^*)\) characterized in the appendix. The initial consumption can be written as \( c_0 = C(k_T) \), where \( C(\cdot) \) is an increasing function.

Next consider the equilibrium at date 0. The key observation is that both non-residential investment and consumption are bounded from above due to the lower bound on the interest rate. Recall from Eq. (18) that capital (and thus, nonresidential investment) is bounded from above, that is, \( k_1 \leq \bar{k} \). Consumption is similarly bounded. Combining the inequality \( c_1 \leq C(\bar{k}) \) with the lower bound on the interest rate implies

\[ c_0 \leq \bar{c}_0, \text{ where } u'(\bar{c}_0) = \beta u'(C(\bar{k})) . \tag{22} \]

Intuitively, investment and consumption can only be stimulated so much without violating the interest rate bound.

Combining the bounds in (18) and (22) with the demand shock in (20), the ag-
Aggregate demand (and output) at date 0 is also bounded from above, that is,

\[ Y_0 \leq \bar{Y}_0 \equiv \bar{k} + \bar{\tau}_0 + (\delta^h - b_0 (1 - \delta^h)) h^*. \]  

(23)

The equilibrium depends on the comparison between the maximum demand and the efficient level, i.e., whether \( \bar{Y}_0 < S(k_0) \). This in turn depends on whether the amount of overbuilding \( b_0 \) exceeds a threshold level,

\[ \bar{b}_0 \equiv \frac{\delta^h h^* + \bar{k} + \bar{\tau}_0 - S(k_0)}{(1 - \delta^h) h^*}. \]  

(24)

**Proposition 1** (Overbuilding and the Liquidity Trap). *Consider the model with \( b_0 > 0 \) (and thus \( h_0 > h^* \)). Suppose Assumption 1 holds.*

(i) Suppose \( b_0 \leq \bar{b}_0 \). Then, the date 0 equilibrium features

\[ r_1 \geq 0, Y_0 = S(k_0) \text{ and } l_0 = l_0^*. \]

(ii) Suppose \( b_0 > \bar{b}_0 \). Then, the date 0 equilibrium features a liquidity trap with

\[ r_1 = 0, k_1 = \bar{k}, Y_0 = \bar{Y}_0 < S(k_0) \text{ and } l_0 < l_0^*. \]

Moreover, output \( Y_0 \) and labor supply \( l_0 \) are decreasing in the amount of overbuilding \( b_0 \).

In either case, starting date 1, the economy converges to the steady state \((k^*, c^*)\) according to the system in (21).

Part (i) describes the equilibrium for the case in which the initial overbuilding is not too large. In this case, the economy does not fall into a liquidity trap. Residential disinvestment is offset by a reduction in the interest rate and an increase in nonresi-
Figure 2: Date 0 equilibrium variables as a function of the initial overbuilding $b_0$ (measured as a fraction of the target residential capital stock, $h^*$).

Differential investment and consumption, leaving the output and employment determined by productivity. The left part of the panels in Figure 2 (the range corresponding to $b_0 \leq \bar{b}_0$) illustrate this outcome. This is the Austrian case.

Part (ii) of Proposition 1, our main result, characterizes the case in which the initial overbuilding is sufficiently large. In this case, the demand shock associated with residential disinvestment is large enough to plunge the economy into a liquidity trap. The lower bound on the interest rate prevents the nonresidential investment and consumption sectors from expanding sufficiently to pick up the slack aggregate demand. As a consequence, the initial shock translates into a Keynesian recession with low output and employment. Figure 2 illustrates this result. A greater initial residential shock—driven by greater overbuilding—triggers a deeper recession. This
is the Keynesian case of our model.

### 3.2 Comparative statics of the liquidity trap

We next investigate the conditions under which a given amount of overbuilding $b_0$ triggers a liquidity trap. As illustrated by Eq. (24), factors that reduce aggregate demand at date $0$, such as a higher discount factor $\beta$ (that lowers $\bar{c}_0$), increase the incidence of the liquidity trap in our setting. More generally, other frictions that reduce aggregate demand during the decumulation phase, such as household deleveraging or the financial crisis, are also complementary to our mechanism.

Perhaps less obviously, Eq. (24) illustrates that a higher initial level of nonresidential capital stock $k_0$ also increases the incidence of a liquidity trap. A higher $k_0$ affects the equilibrium at date $0$ through two main channels. First, it increases output $F(k_0, l_0)$ for any given amount of labor, which makes it more likely that aggregate demand will fall short of the efficient level. Second, a higher $k_0$ also reduces nonresidential investment at date $0$, which in turn lowers aggregate demand. Hence, overbuilding of the two types of capital is complementary in terms of triggering a liquidity trap.

A distinguishing feature of residential capital is its high durability relative to other types of capital. A natural question is whether high durability is conducive to triggering a liquidity trap in our setting. Our model so far is not well suited to address this question, since changing the depreciation rate $\delta^h$ creates general equilibrium effects that are orthogonal to the question (for instance, it changes the steady-state composition of output between consumption, residential, and nonresidential investment).

To isolate the effect of durability, consider a slight variant of the model in which there are two types of residential capital denoted by $h^d$ and $h^n$, each of which has a
target level $h^*/2$. The key difference is their depreciation rate, which are respectively given by $\delta^d$ and $\delta^n$, with $\delta^d < \delta^n$. Thus, type $d$ (durable) residential capital has a lower depreciation rate than type $n$ (nondurable) residential capital. Suppose 
\( \left( \delta^d + \delta^n \right)/2 = \delta^h \) so that the average depreciation rate is the same as before. Let $h^d_0 = (1 + b^d_0) (h^*/2)$ and $h^n_0 = (1 + b^n_0) (h^*/2)$, so that $b^d_0$ and $b^n_0$ capture the overbuilding in respectively durable and nondurable capital relative to their target levels. Suppose also that $(b^d_0 + b^n_0)/2 = b_0$ so that the total amount of overbuilding is the same as before. The case with symmetric overbuilding, $b^d_0 = b^n_0 = b_0$, results in the same equilibrium as in the earlier model. Our next result investigates the effect of overbuilding one type of capital more than the other.

**Proposition 2 (Role of Durability).** Consider the model with two types of residential capital with different depreciation rates. Given the average overbuilding $b_0 = (b^d_0 + b^n_0)/2$, the incidence of a liquidity trap $1[l_0 < l^*_0]$ is increasing in overbuilding of the more durable residential capital $b^d_0$.

To provide an intuition, consider the aggregate demand at date 0, which can be written as

$$ \bar{Y}_0 = \bar{k} + \bar{c}_0 + \delta^h h^* - b^d_0 \left( 1 - \delta^d \right) \frac{h^*}{2} - b^n_0 \left( 1 - \delta^n \right) \frac{h^*}{2}. \quad (25) $$

Note that $1 - \delta^d > 1 - \delta^n$, and thus, overbuilding of the durable residential capital (relative to the nondurable capital) induces a greater reduction in aggregate demand at date 0. Intuitively, depreciation helps to "erase" the overbuilt capital naturally, thereby inducing a smaller reduction investment. When the capital is more durable, there is less natural erasing. This in turn leads to lower investment and aggregate demand, and makes a liquidity trap more likely. This result suggests that overbuilding
is more of a concern when it hits durable capital such as residential investment, structures, or infrastructure (e.g., railroads), as opposed to less durable capital such as equipment or machinery.

### 3.3 Aftermath of the recession

We next investigate the equilibrium behavior in the aftermath of the liquidity trap. Figure 3 plots the full dynamic equilibrium in the original model with single residential capital (and in the liquidity trap scenario). The initial shock generates a temporary recession, followed by a neoclassical adjustment after the recession.

The interest rate gradually increases during the aftermath of the recession, and might remain below its steady-state level for several periods. This is because the
Figure 4: The evolution of net return to capital over time, starting with $b_0 > \bar{b}_0$. The economy accumulates capital during the liquidity trap thanks to low interest rates. The economy decumulates this capital only gradually over time, which leaves the rate of return low after the recession. These low rates are reminiscent of the secular stagnation hypothesis, which was recently revived by Summers (2013). According to this hypothesis, the economy could permanently remain depressed with low interest rates due to a chronic demand shortage (see Eggertsson and Mehrota (2014) for a formalization). In our model, the economy eventually recovers. But the low rates in the aftermath suggest that the economy remains fragile to another demand shock, even though it does not feature secular stagnation.

Figure 3 illustrates further that, while there is a recession at date 0, several components of aggregate demand—especially nonresidential investment—actually expand. The recession is confined to the residential investment sector in which the shock originates. This feature is inconsistent with facts in major recessions, such as the Great Recession, in which all components of aggregate demand decline simultaneously. To resolve this puzzle, we next analyze the investment and consumption responses in more detail.
4 Investment response and the acceleration principle

This section investigates a slight variant of the model in which the liquidity trap persists over multiple periods. We show how the overbuilding of residential capital can induce an initial burst in nonresidential investment followed by a boom. We also discuss the relationship of this effect to the acceleration principle of investment. We finally discuss how our model can further be extended to induce an initial reduction in consumption, and how the extended model can explain the asymmetric recovery from the Great Recession depicted in Figure 1.

The analysis is motivated by Figure 4, which illustrates the evolution of the net return to capital $R_t - \delta^k$ corresponding to the equilibrium plotted in Figure 3. The near-zero return during the recovery phase reflects the high level of capital (and low interest rates). The figure illustrates that the net return at date 0 is even lower—in fact, in negative territory. Intuitively, the recession at date 0 lowers not only the output but also factor returns, including the return to capital (see Lemma 1). This suggests that, if nonresidential investment could respond to the shock during period 0, it could also fall.

To investigate this possibility, we modify the model so that the residential disinvestment is spread over many periods. One way to ensure this is to assume that there is a lower bound on housing investment at every period.

**Assumption 2.** $i^h_t \geq i^h$ for each $t$, for some $i^h < \delta^h h^*$. 

For instance, the special case $i^h = 0$ captures the idea that housing investment is irreversible. More generally, the lower bound provides a tractable model of adjustment costs. To simplify the exposition, we also assume that the initial overbuilding $b_0$ (and
thus \( h_0 = h^* (1 + b_0) \) is such that the economy adjusts to the target level in exactly 
\( T \geq 1 \) periods.

**Assumption 3.** \( \delta h^* = (\delta h_0) (1 - \delta h)^T + i^h \left( 1 - (1 - \delta h)^T \right) \) for an integer \( T \geq 1 \).

With these assumptions, the residential investment path is given by

\[
\dot{i}_t^h = \begin{cases} 
  i^h < \delta^h h^* & \text{if } t \in \{0, \ldots, T - 1\} \\
  \delta^h h^* & \text{if } t \geq T
\end{cases}.
\]  

(26)

For future reference, note that the parameter \( i^h \) also provides an (inverse) measure of the severity of the residential investment shock.

As before, we characterize the equilibrium backwards. The economy reaches date \( T \) with residential capital \( h_T = h^* \) and some \( k_T \leq \overline{k} \). The continuation equilibrium is characterized by the same conditions as before (see Eq. \((21)\)). In particular, consumption is given by \( c_T = C (k_T) \), where recall that \( C(\cdot) \) is an increasing function.

Next consider the equilibrium during the decumulation phase, \( t \in \{0, \ldots, T - 1\} \).

We conjecture that—under appropriate assumptions—there is an equilibrium that features a liquidity trap at all of these dates, that is, \( r_{t+1} = 0 \) for each \( t \in \{0, \ldots, T - 1\} \).

In this equilibrium, the economy reaches date \( T \) with the maximum level of capital, \( k_T = \overline{k} \). Consumption is also equal to its maximum level, that is, \( c_t = \overline{c}_t \) for each \( t \), where

\[
u'(\overline{c}_t) = \beta u'(\overline{c}_{t+1}) \text{ for each } t \in \{0, 1, \ldots, T - 1\}.
\]

We still need to characterize the path of the capital stock \( \{k_t\}_{t=1}^{T-1} \) during the decumulation phase.

To this end, consider the investment decision at some date \( t - 1 \), which determines
the capital stock at date $t$. The gross return from this investment is given by $1 + R_t - \delta^k$, which is determined by the function $s(k_t, Y_t)$ (cf. Lemma 1). Since $r_t = 0$, the gross cost of investment is given by $1 + r_t = 1$. The economy invests at date $t - 1$ up to the point at which the gross benefit is equal to the gross cost, which gives a break-even condition

$$s(k_t, Y_t) = 1 \text{ for each } t \in \{1, \ldots, T - 1\}. \tag{27}$$

Recall that the gross return function $s(\cdot)$ is decreasing in the capital stock $k_t$ and increasing in net output $Y_t$. Hence, Eq. (27) says that, if the (expected) output at date $t$ is large, then the economy invests more at date $t - 1$ and ends up with greater capital stock at date $t$.

The level of output is in turn determined by the aggregate demand at date $t$:

$$Y_t = c_t + k_{t+1} + i^h \text{ for each } t \in \{0, \ldots, T - 1\}. \tag{28}$$

Eqs. (27) and (28) represent a difference equation that can be solved backwards starting with $k_T = \bar{k}$. The resulting path corresponds to an equilibrium as long as $S(k_0) > Y_0$, so that there is a liquidity trap in the first period as we have conjectured. The next result establishes that this is the case if the shock is sufficiently severe, as captured by low $i^h$, and characterizes the behavior of nonresidential capital in equilibrium.\footnote{If the condition $i^h < i^l$ is violated, then there is an alternative equilibrium in which there is a partial liquidity trap at dates $t \in \{T_b - 1, \ldots, T - 1\}$ for some $T_b \geq 2$. We omit the characterization of these equilibria for brevity.} The result requires Assumption 4, which is a regularity condition on shocks and parameters that ensures an interior liquidity trap equilibrium at date 0 with positive output. This assumption is satisfied for all of our numerical simulations and is relegated to the appendix for brevity.
Proposition 3 (Nonresidential Investment Response). Consider the model with the adjustment length $T \geq 1$. Suppose Assumptions 1-3 and Assumption 4 in the appendix hold.

(i) There exists $i_1^h$ such that if $i^h < i_1^h$, then there is a unique equilibrium path $\{k_t, Y_{t-1}\}_{t=1}^T$, which solves Eqs. \(27\) and \(28\) along with $k_T = \bar{k}$. The equilibrium features a liquidity trap at each date $t \in \{0, \ldots, T-1\}$ with $r_{t+1} = 0$ and $Y_t < S(k_t)$.

(ii) There exists $i_2^h$ such that, if $i^h < i_2^h$, then the nonresidential capital declines at date 1, and then increases before date $T$:

$$k_0 > k_1 \text{ and } k_1 < k_T = \bar{k}.$$

The main result of this section is the second part, which establishes conditions under which the nonresidential capital (and investment) follow a non-monotone path during the recession: falling initially, but eventually increasing.

To understand the drop in investment, note that a negative shock to residential investment reduces aggregate demand and output. This in turn lowers nonresidential investment as captured by the break-even condition \(27\). When the shock is sufficiently severe, the aggregate demand at date 1 is sufficiently low that capital declines. Intuitively, the economy is optimally responding to the low return to capital depicted in Figure 4.

In later periods, aggregate demand and output gradually increase in anticipation of the eventual recovery. As this happens, the low interest rate—or the low cost of capital—becomes the dominant factor for nonresidential investment. Consequently, the economy starts reaccumulating capital, and in fact—exits the liquidity trap with the maximum level of capital $\bar{k}$ as in the earlier model.

Figure 5 illustrates the dynamic evolution of the equilibrium variables for the case
Figure 5: The evolution of equilibrium variables over time, given the length of decumulation $T = 2$. 
\( T = 2 \). The parameters are chosen so that the figure can be compared to Figure 3 after replacing a single period with two periods. The lower panels on the left illustrate the non-monotonic response of capital and investment identified in Proposition 3. The figure illustrates that the recession can be roughly divided into two phases. In the first phase, captured by date 0, both types of investment fall. This induces a particularly severe recession with low output and employment. In the second phase, captured by date 1 in the figure (and dates \( t \in \{1, ..., T - 1\} \) more generally), residential investment remains low whereas the nonresidential investment gradually recovers and eventually booms. The nonresidential investment response also raises aggregate demand and creates a partial recovery in output and employment.

4.1 Relationship to the acceleration principle

Our analysis of nonresidential investment is related to the accelerator theory of investment (see Clark (1917)). To illustrate this, let us linearize Eq. (27) around \((k, Y) \simeq (\bar{k}, S(\bar{k}))\), to obtain the approximation

\[
k_t \simeq \alpha + \beta E_{t-1}[Y_t] \text{ for each } t \in \{1, ..., T - 1\},
\]

where \( \beta = -s_Y/s_k > 0, \alpha = \bar{k} - \beta S(\bar{k}) \), and \( E_{t-1}[Y_t] = Y_t \). We introduce the (redundant) expectations operator to contrast our rational expectations approach with the previous literature. Taking the first differences of Eq. (29), and assuming that the depreciation rate is small, \( \delta^k \simeq 0 \), we further obtain

\[
i_t^k \simeq k_{t+1} - k_t \simeq \beta (Y_{t+1} - Y_t) \text{ for each } t \in \{0, ..., T - 2\}.
\]
Our model thus implies a version of the acceleration principle, which says that investment is proportional to changes in output (see Eckaus (1953) for a review). Note, however, that the relationship in (29) is mechanically assumed in the accelerator literature, whereas Eq. (27) emerges in our setting from the optimal investment behavior of firms.

Intuitively, the liquidity trap ensures that the interest rate and the cost of capital is constant. Consequently, the return on capital becomes the main determinant of investment. In our model (and in many settings), the return on capital is increasing in output, which yields a positive relationship between capital and output. In his review of the accelerator theory, Caballero (1999) notes: “the absence of prices (the cost of capital, in particular) from the right-hand side of the flexible accelerator equation has earned it disrespect despite its empirical success.” In our analysis, the liquidity trap keeps the cost of capital constant, reviving the acceleration principle.

Our model has several distinct features as compared to the accelerator theory. First, our acceleration principle applies only temporarily during the liquidity trap. From time $T$ onwards, investment is driven by neoclassical forces [cf. (21)]. Second, our acceleration principle captures a nonlinear relationship [cf. Eq. (27)], whereas the accelerator theory often uses a linear form as in (29). Third, our investing firms hold rational expectations, whereas the macroeconomic applications of the accelerator theory often use Eq. (29) with adaptive expectations, for instance, $E_{t-1} [Y_t] = Y_{t-1}$. We show that, even with rational expectations, the acceleration principle exacerbates the earlier phase of the recession similar to Samuelson (1939). However, our model does not feature the periodic oscillations of output emphasized in Samuelson (1939) or Metzler (1941), which seem to be driven by adaptive expectations.
4.2 Consumption response and the asymmetric recovery

While our model can account for the decline in investment in the earlier part of the recession, in view of the accelerator mechanism, it cannot generate a similar behavior for consumption. As Figure 5 illustrates, (net) consumption expands during the recession due to the Euler equation. However, the Euler equation—and the permanent income hypothesis that it implies—cannot fully capture the behavior of consumption in response to income changes in the data. After reviewing the vast empirical literature on this topic, Jappelli and Pistaferri (2010) note “there is by now considerable evidence that consumption appears to respond to anticipated income increases, over and above by what is implied by standard models of consumption smoothing.”

To make consumption more responsive to income, Appendix A.1 extends the model by introducing additional households that have high marginal propensities to consume (MPC) out of income. The main result shows that, if there are sufficiently many high-MPC households, then aggregate consumption initially declines. Intuitively, the low output earlier in the recession lowers all households’ incomes, which in turn reduces consumption via the high-MPC households. As output increases later in the recession, so does consumption. Hence, consumption also responds non-monotonically to overbuilding.

The model with high-MPC households can explain the asymmetric recovery from the Great Recession. Figure 6 in the appendix, which is the analogue of Figure 5, illustrates that the recession is naturally divided into two phases. In the first phase, all components of aggregate demand—including consumption—simultaneously fall,

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6 Actual consumption, $c_t = \hat{c}_t + v(l_t)$, might fall in view of the reduction in employment, $l_t$. We do not emphasize this result since it is mainly driven by the GHH functional form for the preferences, which we adopted for expositional simplicity.
triggering a deep recession. In the second phase, nonresidential investment booms, which also increases output, employment, as well as consumption. Hence, the second phase of the recession in our model represents a partial and asymmetric recovery in which the residential sector is left behind, similar to the aftermath of the Great Recession (see Figure 1).

The appendix also shows that the model with high-MPC households features a Keynesian income multiplier, in addition to the investment accelerator, with two implications. First, the recession is more severe than in the baseline model, because the decline of consumption exacerbates the reduction in aggregate demand and output. Second, the accelerator mechanism is more pronounced in the sense that investment decreases more early in the recession, while also increasing more later in the recession (as the economy continues to exit the trap with the maximum amount of capital, $k^*$. Intuitively, a more severe recession reduces the return to capital further, which translates into lower investment early in the recession. This in turn exacerbates the initial decline in output as well as consumption. In this sense, the multiplier and the accelerator mechanisms reinforce one another.

5 Policy implications

We next investigate the welfare implications of our analysis. We first discuss ex-post policies by which the government can improve welfare once the overbuilding is realized and the economy is in a recession. We then consider ex-ante “macroprudential” policies which the government can implement (prior to date 0) so as to mitigate the damage during the overbuilding episode. Since investment plays a central role in our analysis, we focus on policy interventions directed towards controlling investment.
5.1 Ex-post policies: Stimulating investment

Since our model features a liquidity trap, several policies that have been discussed in the literature are also relevant in this context. In particular, welfare can be improved with unconventional monetary policies as in Eggertsson and Woodford (2003), or unconventional tax policies as in Correira et al. (2013). Once we modify the model appropriately to include government spending, welfare can also be improved by increasing government spending during the recession as in Werning (2012) and Christiano et al. (2011). The common denominator of these policies is that they stimulate aggregate demand and output in an environment in which the monetary policy is constrained and output is below its efficient level (and thus, the opportunity cost of production is low).

A natural question in our setting concerns the optimal government policy regarding residential investment. In fact, since overbuilding is associated with residential capital, a naive intuition might suggest that the planner should not interfere with the decumulation of this type of capital. Our next result shows that this intuition is incorrect. To state the result consider the baseline model described in Section 3 in which the competitive equilibrium would decumulate the excess capital in a single period. Consider a constrained planner that can choose the next period’s residential capital, $h_{t+1}$, at every date (without commitment). The remaining allocations are determined in a competitive equilibrium as before. We next characterize the constrained efficient allocations.

Proposition 4 (Slowing Down Disinvestment). Consider the baseline model with $b_0 > \bar{b}_0$. There exists an integer $T \geq 1$ such that the constrained efficient allocation features $h_0 > h_1 > \ldots > h_{T+1} = h^*$, and $Y_t = S(k_t)$ for each $t$.

That is, a constrained planner optimally decumulates the residential capital in $T + \ldots$
1 \geq 2\) periods, while avoiding a recession at all dates. Consequently, the competitive equilibrium allocation that decumulates the excess residential capital in a single period while featuring a recession at date 0 is not constrained efficient.

To understand the result, note that choosing \(h_1 > h^*\) raises the aggregate demand at date 0, or equivalently, mitigates the demand shock at this date (see Eq. (20)). Note, however, that the demand shock does not disappear. Instead, it is relegated to future dates since the economy still needs to decumulate the excess residential capital, \(h_1\). Proposition 4 shows that the future decumulation can be carried out in a way that the economy avoids a recession at all dates.

Intuitively, at every date, the economy has positive yet limited capacity to absorb negative demand shocks (as illustrated for date 0 by Figure 2). Hence, a sufficiently large demand shock concentrated in a single period plunges the economy into a liquidity trap. However, if this demand shock is chopped into pieces and spread over time, then it can be absorbed by an increase in the remaining components of demand. Thus, slowing down the decumulation of residential capital improves welfare, as formalized by Proposition 4.

5.2 Ex-ante policies: Reducing investment

We next analyze whether the planner can improve welfare via ex-ante interventions. To this end, consider the baseline model with an ex-ante period, date \(-1\). Suppose also that the economy is in one of two states at date 0, denoted by \(s \in \{H, L\}\). State \(L\) is a low-demand state in which the target level of housing capital is \(h^*\) as before (and the planner has no tools for ex-post intervention). State \(H\) is a high-demand state in which the target level of housing capital is \(\lambda h^*\) for some \(\lambda > 1\). Let \(\pi\) denote the ex-ante probability of state \(L\). The households start with some \(h_{-1}, k_{-1}\), and
choose $h_0, k_0$ at date $-1$. We first characterize the competitive equilibrium, and then investigate the constrained efficiency of this equilibrium.

Given the preferences (2), households will choose,

$$h_0 = \lambda h^*,$$

which also implies

$$i_{-1} = \lambda h^* - (1 - \delta^h) h_{-1}.$$

Intuitively, since $u^h$ is assumed to be very large, the opportunity cost of not consuming housing in state $H$ of date 0 is also very large. Consequently, households invest in housing just enough to meet the target in the high-demand state. The assumption that $u^h$ is very large is unrealistic, but it helps us to keep the model tractable.

The equilibrium starting state $H$ of date 0 is straightforward. It solves the neo-classical system in (21) after replacing $h^*$ with $\lambda h^*$. The gross return to capital is given by $1 + R_0^H - \delta^k = S'(k_0)$.

The equilibrium starting state $L$ of date 0 is the same as before. We assume that $\lambda > 1 + \bar{b}_0$, so that the equilibrium features a liquidity trap and a recession at date 0 (cf. Proposition 1). The gross return to capital is given by $1 + R_0^L - \delta^k = s(k_0, Y_0^L)$.

Next consider the date 0 allocations. It can be seen that the equilibrium level of $k_0$ is characterized by the first order conditions,

$$u'(c_{-1}) = \beta \left( (1 - \pi) S'(k_0) u'(c_{0}^H) + \pi s(k_0, Y_0^L) u'(c_{0}^L) \right).$$

Decentralized households recognize that the return to capital in state $L$ will be low. However, they might still choose a high level of $k_0$, especially if they put a relatively low probability $\pi$ to the low-demand state.

\footnote{That said, the analysis also illustrates how households might choose a high level of $h_0$ so as to trigger the investment hangover that we analyzed in the earlier sections—especially if households put relatively low probability $\pi$ to the low-demand state.}
Next consider a constrained planner that can fully determine households’ date \(-1\) allocations, including the choice of \(k_{0,\text{pl}}, h_{0,\text{pl}}\) at date \(-1\), but cannot interfere with equilibrium allocations starting date 0. Like households, the planner also chooses \(h_{0,\text{pl}} = \lambda h^*\) in view of the large opportunity cost of not consuming housing in state \(H\). However, the planner’s choice of \(k_{0,\text{pl}}\) is potentially different.

To characterize \(k_{0,\text{pl}}\), let \(V^H(k_{0,\text{pl}})\) and \(V^L(k_{0,\text{pl}})\) denote the welfare of the representative household in equilibrium in respectively states \(H\) and \(L\) at date 0. It can be seen that,
\[
\frac{dV^H(k_{0,\text{pl}})}{dk_{0,\text{pl}}} = S'(k_{0,\text{pl}}) u'(c^H_0).
\]
Intuitively, since state \(H\) features a neoclassical equilibrium, another unit of capital increases households’ resources in equilibrium by exactly \(S'(k_{0,\text{pl}})\) units. In contrast,
\[
\frac{dV^L(k_{0,\text{pl}})}{dk_{0,\text{pl}}} = 0.
\]
This follows from Eq. (22), which illustrates that the initial level of capital does not affect households’ net consumption in equilibrium, \(c_{0,L} = c_0\). Intuitively, a greater initial capital (just like greater residential capital) reduces investment and aggregate demand. Since the economy is demand constrained, the reduction in demand reduces output and employment. Hence, raising the level of capital at date 0 is associated with negative aggregate demand externalities. In our model, these externalities are sufficiently strong that net consumption (or net output) does not depend on the initial capital stock.

It follows that the planner’s choice of capital at date \(-1\) is characterized by,
\[
u'(c_{-1,\text{pl}}) = \beta (1 - \pi) S'(k_{0,\text{pl}}) u'(c^H_{0,\text{pl}}).
\] (31)
Combining Eqs. (30) and (31) implies that the planner chooses a lower level of capital compared to an unregulated equilibrium, \( k_{0,pl} < k_0 \). Intuitively, even though the private return to capital in state \( L \) is lower than the efficient benchmark, the social return to capital is even lower than this level because the planner internalizes the aggregate demand externalities.

Note that, in our stylized model, the constrained planner does not interfere with households’ ex-ante residential investment, \( h_0 = h_{0,pl} = \lambda h^* \). This is because of our extreme assumptions that ensure residential investment is at a corner solution. In alternative and less extreme formulations of our model, the planner would reduce households’ ex-ante residential investment, because bringing less residential capital to date 0 is also associated with positive aggregate demand externalities.

Let us summarize the insights from our welfare analyses in this section. Ex-post, once the economy is in the liquidity trap, welfare can be improved by policies that slow down the decumulation of residential capital. Ex-ante, before the economy enters the liquidity trap, welfare can be improved by policies that restrict the accumulation of capital. The optimal ex-post and ex-ante policies share the broad principle that they attempt to intertemporally substitutes investment from periods that feature efficient outcomes to periods (or states) that feature deficient demand.

6 Conclusion

We have presented a model of investment hangover in the Great Recession that combines both Austrian and Keynesian features. On the Austrian side, the recession is precipitated by overbuilding in the residential sector, which necessitates a reallocation of resources to other sectors. The reallocation problem is exacerbated by the
durability of residential capital, which prevents depreciation from naturally erasing the overbuilt capital. On the Keynesian side, a lower bound on interest rates slows down reallocation and creates an aggregate demand shortage. The demand shortage can also reduce investment (as well as consumption) in sectors that are not overbuilt, leading to a severe recession. Eventually, nonresidential investment recovers, but the slump in the residential sector continues for a long time. The broad trends of the Great Recession on GDP, residential investment, and other types of investment (as well as consumption) are consistent with the predictions of this model. We also investigated the policy implications of our analysis. In view of aggregate demand externalities, welfare can be improved by ex-post policies that slow down the decumulation of residential capital, as well as ex-ante policies that restrict the accumulation of capital.

Although we have focused on the Great Recession, the model is more widely applicable. Perhaps the most straightforward extension is to overbuilding in sectors other than housing. In the 1930s, when both Hayek and Keynes wrote, speculative overbuilding was seen as a critical impetus to recessions, but the focus was more on railroads and perhaps industrial plant than on housing. In our model, such extensions would require only a relabeling of variables.

Less obvious is the extension to other forms of restrictions on interest rates, such as currency unions, which also slow down the Austrian reallocation of resources from the overbuilt sector to others. In the recent European context, such restrictions may have played a critical role, and generated Keynesian aggregate demand effects along the lines suggested by our model. We leave an elaboration of these mechanisms to future work.
References


A Appendix: Omitted extensions

A.1 Consumption response and the Keynesian multiplier

The baseline model features a representative household whose consumption satisfies the Euler equation. However, the Euler equation cannot fully capture the behavior of consumption in response to income changes in the data (see Japeli and Pistaferri, 2010). We next modify the model by introducing constrained agents that have high MPCs out of income. We show that, unlike the baseline model, this version of the model can account for the drop in consumption earlier in the recession. The model also features a Keynesian income multiplier, which exacerbates the recession and reinforces the investment accelerator mechanism.

Suppose, in addition to the representative household analyzed earlier, there is an additional mass \( l^{tr} \) of households which we refer to as income-trackers. These agents are excluded from financial markets so that they consume all of their income, that is, their MPC is equal to 1 (for simplicity). Each income-tracker inelastically supplies 1 unit of labor in a competitive market for a wage level \( w^{tr}_t \), which provides her only source of income. Consequently, total consumption is now given by \( c_t + w^{tr}_t l^{tr} \), where \( c_t \) is the consumption of the representative household and \( w^{tr}_t l^{tr} \) denotes the consumption of income-trackers.

The aggregate production function can generally be written as \( \tilde{F}(k_t, l_t, l^{tr}) \), where \( l_t \) is the labor supply by the representative household and \( l^{tr} \) is the total labor supply by income-trackers. To simplify the analysis, we focus on the special case

\[
\tilde{F}(k_t, l_t, l^{tr}) = F(k_t, l_t) + \eta^{tr} l^{tr},
\]

where \( F \) is a neoclassical production function and \( \eta^{tr} > 0 \) is a scalar. We continue to use the notation \( Y_t = F(k_t, l_t) \) to refer to the output excluding the supply of income-trackers. Total output is given by \( Y_t + \eta^{tr} l^{tr} \). The rest of the model is the same as in the previous section.

In view of these assumptions, the economy is subject to the resource constraint,

\[
c_t + k_{t+1} + i^h_t + w^{tr}_t l^{tr} = Y_t + \eta^{tr} l^{tr} \leq S(k_t) + \eta^{tr} l^{tr}. \tag{A.1}
\]

In Eq. \((A.1)\), the equality says that total demand equals total output, whereas the
inequality says that total output is below the maximum total supply. Lemma 2 in Appendix B characterizes the income-trackers’ wage level as

$$w_{tr}^t = \psi (k_t, Y_t) \eta^{tr}. \quad (A.2)$$

Here, $\psi (k_t, Y_t) \in [0, 1]$ is a measure of efficient resource utilization (more specifically, $\psi = 1 - \tau$ where $\tau$ is the labor wedge). Absent a liquidity trap, $\psi = 1$ and output is at its efficient level, in which case the income-trackers also earn their marginal product $\eta^{tr}$. In a liquidity trap, $\psi \leq 1$ and output is below its efficient level due to the demand shortage. In this case, the income-trackers’ wage is also below their marginal product, $w_{tr}^t \leq \eta^{tr}$. Moreover, their wage is increasing in $Y_t$, since greater demand increases factor returns.

Combining Eqs. (A.1) and (A.2) implies

$$Y_t = c_t + k_{t+1} + i^b_t + (\psi (k_t, Y_t) - 1) \eta^{tr} l^{tr} \quad \text{for each } t \in \{0, 1, \ldots, T - 1\}. \quad (A.3)$$

This expression illustrates a Keynesian cross as well as a Keynesian income multiplier in our setting. The equilibrium obtains when the actual and demanded net outputs are equal, as in a typical the Keynesian cross. Moreover, total demand depends on net output $Y_t$ through income-trackers’ consumption, illustrating the multiplier. Consider, for instance, a shock to aggregate demand that lowers net output. This lowers income-trackers’ income and their consumption, which in turn induces a second round reduction in aggregate demand and net output, and so on.

Next consider a residential investment shock that lasts $T$ periods as in the previous section. We conjecture an equilibrium with a liquidity trap for all dates $t \in \{0, 1, \ldots, T - 1\}$. As before, the optimality of investment implies the break-even condition (27). Eqs. (A.3) and (27) can then be solved backwards starting with $k_T = \bar{k}$. The next result establishes conditions under which the solution exists and corresponds to an equilibrium, and characterizes the behavior of consumption in equilibrium.

**Proposition 5 (Consumption Response).** Consider the model with mass $l^{tr}$ of income-trackers and the adjustment length $T \geq 1$. Suppose Assumptions 1-3 and Assumption 4' in the appendix hold.

(i) There exists $i^1_f$ such that if $i^b < i^1_f$, then there is an equilibrium path
Figure 6: The dynamic equilibrium with income-trackers (light bars) compared to the equilibrium without income-trackers (dark bars).

\{k_t, Y_{t-1}\}_{t=1}^T, \text{ which solves Eqs. (27) and (A.3) along with } k_T = \overline{k}. \text{ Any equilibrium features a liquidity trap at each date } t \in \{0, \ldots, T - 1\} \text{ with } r_{t+1} = 0 \text{ and } Y_t < S(k_t).

(ii) There exists \( l_1^{tr} \) such that if \( l^{tr} > l_1^{tr} \), then total consumption at date 0 (in any equilibrium) is below its steady-state level, that is

\[
    c_0 + w_0^{tr} l^{tr} < c^* + \eta^{tr} l^{tr}.
\]

The main result of this section is the second part, which establishes conditions under which overbuilding also lowers total consumption at date 0 in any equilibrium.\footnote{The equilibrium is unique in all of our numerical simulations. However, there could in principle be multiple equilibria because Eq. (A.3) represents an intersection of two increasing curves in \( Y_t \).}

When the economy is in a liquidity trap, output falls due to the demand shortage, which lowers income-trackers’ consumption. With sufficiently many income-trackers, this also reduces total consumption in contrast to the baseline model. The light
bars in Figure 6 illustrate this result by plotting the dynamic equilibrium using the same parameters as before except for the new parameters $\eta^{tr}, l^{tr} > 0$. Consumption declines early in the recession, and it recovers later in the recession due to the recovery in output. Hence, this version of the model can generate a nonmonotonic response in consumption, similar to the nonmonotonic response of investment identified in Proposition 3.

It follows that this version of the model can explain the asymmetric recovery from the Great Recession depicted in Figure 1. In the first phase of the recession, consumption as well as nonresidential investment simultaneously fall, triggering a deep recession. In the second phase, the boom in nonresidential investment increases output, which also increases consumption. Hence, the second phase is a partial and asymmetric recovery in which the residential sector is left behind, as in the aftermath of the Great Recession.

Figure 6 also contrasts this equilibrium with the earlier equilibrium without income-trackers, which is plotted with dark bars. Note that the equilibrium in this section features a greater drop in output and employment, as well as a greater labor wedge. Intuitively, the Keynesian income multiplier aggravates the recession. Perhaps less obviously, the equilibrium also features a more severe drop in investment at date 0, followed by a stronger recovery at date 1. Intuitively, a more severe recession implies a lower return to capital, which in turn lowers investment at date 0. Put differently, the Keynesian income multiplier exacerbates the investment accelerator mechanism. The decline in investment at date 0 further lowers net output and consumption, aggravating the Keynesian income multiplier. In this sense, the multiplier and the accelerator mechanisms reinforce each other.

B Appendix: Omitted proofs

B.1 Proofs for the baseline model

This section presents the proofs of the results for the baseline model and its variants analyzed in Sections 3 and 4.

Proof of Lemma 1. First consider the case $r_{t+1} > 0$. In this case, the monetary implements the efficient allocation with $l_t = l_t^r$ and $Y_t = S(k_t)$. In addition, the first
order conditions for problems (11) and (4) further imply,

\[ F_l(k_t, l_t^*) = v'(l_t^*) = w_t. \]

Combining this with Eq. (17) implies that the labor wedge is zero, \( \tau_t = 0 \). Combining Eq. (17) with the first order condition for problem (14) then imply the gross return to capital is given by \( F_k(k_t, l_t^*) + 1 - \delta^k = S'(k_t) \), completing the proof for the first part.

Next consider the case \( r_{t+1} = 0 \). In this case, Eq. (17) implies \( F_l(k_t, l_t) = v'(l_t) \).

This in turn implies that \( l_t \in [0, l_t^*] \). By feasibility, net output satisfies

\[ Y_t = c_t + k_t + i^h = F(k_t, l_t) - v(l_t) + (1 - \delta^k) k_t. \]

This expression is strictly increasing in \( l_t \) over the range \([0, l_t^*]\). The minimum and the maximum are respectively given by \((1 - \delta^k) k_t \) and \( S(k_t) \), establishing the constraints (13). Moreover, given \( Y_t \) that satisfies these resource constraints, there is a unique solution to problem (16), which we denote by \( L(k_t, Y_t) \). Combining this with Eq. (17), we further obtain the labor wedge as,

\[ 1 - \tau_t = \frac{v'(l_t)}{F_l(k_t, l_t)} = \frac{v'(L(k_t, Y_t))}{F_l(k_t, L(k_t, Y_t))}. \]

Plugging this into Eq. (17) for capital, we obtain the gross return to capital as,

\[ 1 + R_t - \delta^k = v'(L(k_t, Y_t)) \frac{F_k(k_t, L(k_t, Y_t))}{F_l(k_t, L(k_t, Y_t))} F_k(k_t, L(k_t, Y_t)) + 1 - \delta^k = s(k_t, Y_t), \]

where the last equality defines the function \( s(\cdot) \). Note that \( s(k_t, Y_t) \leq S'(k_t) \) since the labor wedge is nonnegative. It can also be checked that \( s_Y > 0 \) and \( s_k < 0 \), completing the proof.

Proof of Proposition 1. For each \( r_0 \geq 0 \), define the function \( K_1(r_0) \) as the solution to

\[ S'(K_1(r_0)) = 1 + r_0. \]

Note that \( K_1(r_0) \) is decreasing in the interest rate, with \( K_1(0) = \overline{k} \) and \( \lim_{r_0 \to \infty} K_1(r_0) = 0 \). Similarly, define the function \( C_0(r_0) \) as the solution to the
Euler equation
\[ u'(C_0(r_0)) = \beta (1 + r_0) u'(K_1(r_0)). \]

Note that \( C_0(r_0) \) is decreasing in the interest rate, with \( C_0(0) = \tau_0 \) and \( \lim_{r_0 \to \infty} C_0(r_0) = 0 \). Finally, define the aggregate demand function
\[ Y_0(r_0) = C_0(r_0) + K_1(r_0) + \ell^h. \]

Note that \( Y_0(r_0) \) is also decreasing in the interest rate, with \( Y_0(0) = \bar{Y}_0 \) and \( \lim_{r_0 \to \infty} Y_0(r_0) = \ell^h < \delta^h \ell^* \).

Next consider the time 0 equilibrium for the case \( b_0 \leq \bar{b}_0 \), which implies \( S(k_0) \leq \bar{Y}_0 \). Assumption 1 implies \( S(k_0) \geq k_0 + \delta^h \ell^* > \ell^h \). It follows that there is a unique equilibrium interest rate \( r_0 \in [0, \infty) \) such that \( Y_0(r_0) = S(k_0) \). The equilibrium consumption and investment are determined by \( c_0 = C_0(r_0) \) and \( K_1(r_0) = k_1 \), and the equilibrium output and labor supply satisfy \( Y_0 = S(k_0) \) and \( l_0 = l_0^* \).

Next consider the date 0 equilibrium for the case \( b_0 > \bar{b}_0 \). In this case, \( Y_0(r_0) < S(k_0) \) for each \( r_0 \geq 0 \). Thus, the unique equilibrium features \( r_0 = 0 \) and \( Y_0 = \bar{Y}_0 < S(k_0) \). Consumption and investment are given by \( c_0 = \tau_0 \) and \( k_1 = \bar{k}_1 \). Labor supply \( l_0 \) is determined as the unique solution to (16) over the range \( l_0 \in (0, l_0^*) \). Finally, Eq. (23) implies the equilibrium output, \( Y_0 = \bar{Y}_0 \), is declining in the initial overbuilding \( b_0 \).

In either case, it can also be checked that the economy reaches time 1 with capital stock \( k_1 \geq \min(k_0, k^*) \). Under Assumption 1, the system in (21) corresponds to a standard neoclassical model. Using the standard steps, there is a unique equilibrium path \( \{c_t, k_{t+1}\}_{t=1}^\infty \), which converges to the steady state \((c^*, k^*)\) characterized by the equations
\[ \beta S'(k^*) = 1 \quad \text{and} \quad c^* = S(k^*) - k^* - \delta^h \ell^*, \]
completing the proof.

\( \square \)

**Proof of Proposition 2.** Note that the recession is triggered if \( \bar{Y}_0 < S(k_0) \), where \( \bar{Y}_0 \) is given by Eq. (25). Since \( 1 - \delta^h \ell^* > 1 - \delta^h \ell^n \), increasing \( b_0^\ell \) (while keeping \( b_0 = (b_0^\ell + b_0^n)/2 \) constant) reduces \( \bar{Y}_0 \), proving the result. \( \square \)
To prove Proposition 3, we also make the following assumption.

**Assumption 4.** (i) \( i^h \in [-\bar{c}_T, S(\bar{k}) - \bar{k} - \bar{c}_0) \) and (ii) \( s(k_0, \bar{c}_0 - \bar{c}_T + \bar{k}) < 1 \).

Part (i) ensures that \( i^h \) is not too low to induce zero aggregate demand in any period, but also not too high so that a liquidity trap at date 0 is possible. Part (ii) ensures that the worst possible shock \( i^h = -\bar{c}_T \) is sufficient to induce a liquidity trap at date 0.

**Proof of Proposition 3.** We first claim that the solution to Eq. (27) can be written as \( k_t = K(Y_t), \) where \( K(\cdot) \) is an increasing function over \((0, S(\bar{k}))\). To this end, consider some \( Y \in (0, S(\bar{k})) \). Let \( \tilde{k} < \bar{k} \) denote the unique capital level such that \( Y = S(\tilde{k}) \). Note that

\[
s(\tilde{k}, Y) = S'(\tilde{k}) > 1 \text{ and } s(\bar{k}, Y) < S'(\bar{k}) = 1,
\]

where the latter inequality follows from Lemma 1 since \( Y < S(\bar{k}) \). Since \( s_k < 0 \), there exists a unique \( K(Y) \in (\tilde{k}, \bar{k}) \) such that \( s(K(Y), Y) = 1 \). Thus, the function \( K(\cdot) \) is well defined. Note also that \( K(\cdot) \) is continuous and strictly increasing. Note also that \( \lim_{Y \to 0} K(Y) = 0 \).

Given the function \( K(\cdot) \), the path of capital can be written as the solution to the system,

\[
k_t = K_t(Y_t), \text{ where } Y_t = \bar{c}_t + k_{t+1} + i^h \quad (B.1)
\]

for each \( t \in \{1, .., T - 1\} \), starting with \( k_T = \bar{k} \). To solve this system by induction, consider some \( k_{t+1} \in (0, \bar{k}] \). Consider the corresponding aggregate demand \( Y_t \). Part (i) of Assumption 4 implies \( Y_t > 0 \) (using \( \bar{c}_t \geq \bar{c}_T \) and \( Y_t < S(\bar{k}) \) (using \( \bar{c}_t \leq \bar{c}_0 \) and \( k_{t+1} \leq \bar{k} \)). We thus have \( Y_t \in (0, S(\bar{k})) \). Since \( K(\cdot) \) is a strictly increasing function, there is a unique solution to \((B.1)\) which also satisfies \( k_t \in (0, \bar{k}] \). By induction, we obtain a unique path for capital \( \{k_t\}_{t=1}^{T-1} \). Combining the path of capital with Eq. (28) also implies a unique path of output \( \{Y_{t-1}\}_{t=1}^{T-1} \). Since \( k_t < \bar{k} \) and \( s(k_t, Y_t) = 1 \), we also have \( Y_t < S(k_t) \) for each \( t \in \{1, .., T - 1\} \).

It remains to show that there is a liquidity trap also at date 0 with \( Y_0 < S(k_0) \), verifying our conjecture. We first claim this is the case for the worst allowed shock, \( i^h = -\bar{c}_T \). We then establish it also for any shock below a threshold level.

Consider the worst allowed shock \( i^h = -\bar{c}_T \). Note that \( K(Y_0) \in (0, \bar{k}) \) is well
defined, and describes the capital level at date 0 that would generate a gross return of 1 given the demand \( Y_0 \). The demand at date 0 is in turn given by

\[
Y_0 = \bar{c}_0 - \bar{c}_T + k_1 \leq \bar{c}_0 - \bar{c}_T + \bar{k}.
\]

Combining this with Part (ii) of Assumption 4, we obtain \( s(k_0, Y_0) < 1 \). This implies \( K(Y_0) < k_0 \), which further implies

\[
Y_0 < S(K(Y_0)) < S(k_0).
\]

Here, the first inequality from the definition of \( K(Y_0) \) and the second inequality follows since \( K(Y_0) < k_0 \). We thus have \( Y_0 < S(k_0) \), proving the claim that the worst allowed shock induces a liquidity trap.

Next note from Eq. \((\text{B.1})\) that, for any \( k_{t+1} \), the implied \( k_t \) is strictly increasing in \( i^h \). Consequently, \( k_1 \) and \( Y_0 \) are also strictly increasing in \( i^h \). Since \( i^h = -\bar{c}_T \) induces \( Y_0 < S(k_0) \), there exists \( i^h_1 > -\bar{c}_T \) such that \( Y_0 = S(k_0) \). It follows that there is a liquidity trap at date 0 with \( Y_0 < S(k_0) \) whenever \( i^h < i^h_1 \), proving the first part.

Similarly, we claim that the worst allowed shock \( i^h = -\bar{c}_T \) induces \( k_1 < k_0 \). To see this, consider the aggregate demand at date 1 given by

\[
Y_1 = \bar{c}_1 - \bar{c}_T + k_2 \leq \bar{c}_0 - \bar{c}_T + \bar{k}.
\]

Combining this with Part (ii) of Assumption 4, we obtain \( s(k_0, Y_1) < 1 \). This in turn implies \( k_1 = K(Y_1) < k_0 \), proving the claim. Since \( k_1 \) is strictly increasing in \( i^h \), there exists \( i^h_2 > -\bar{c}_T \) such that \( k_1 = k_0 \). It follows that \( k_1 < k_0 \) whenever \( i^h < i^h_2 \), completing the proof.

### B.2 Proofs for the extension with income-trackers

**Lemma 2.** The income-trackers’ wage level is given by Eq. \((\text{A.2})\) for some function \( \psi(k_t, Y_t) \), which has the following properties:

(i) \( \psi(k_t, Y_t) = 1 - \tau_t = \frac{\psi'(l_t)}{F(l_t)} \),

(ii) \( \psi(k_t, Y_t) = 1 \) if \( r_{t+1} > 0 \) and \( \psi(k_t, Y_t) \in [0, 1] \) if \( r_{t+1} = 0 \),

(iii) \( \psi(k_t, Y_t) \) is strictly decreasing in \( k_t \), and strictly increasing in \( Y_t \).
Proof. As in the proof of Lemma 1 let \( L(k, Y) \) denote the labor supply corresponding to capital level \( k \leq \overline{k} \) and output \( Y \in \left[ (1 - \delta^k) k, S(k) \right] \). Next consider the analogue of Problem (10) that also includes firms’ demand for hand-to-mouth labor. The firm’s optimization in this case implies

\[
 w^{tr}(k_t, Y_t) = (1 - \tau_t) \eta^{tr},
\]

where \( \tau_t \geq 0 \) is the Lagrange multiplier on the demand constraint. As before, the same problem also implies that \( \tau_t \) is equal to the labor wedge, that is:

\[
 1 - \tau_t = \frac{i'(L(k_t, Y_t))}{F_i(k_t, L(k_t, Y_t))} \equiv \psi(k_t, Y_t).
\]

Here, the last line defines the function \( \psi(k_t, Y_t) \). Combining these expressions proves the first part. Recall that the labor wedge satisfies \( \tau_t = 0 \) if \( r_{t+1} = 0 \), and \( \tau_t \in [0, 1] \) if \( r_{t+1} > 0 \), proving the second part. It can also be checked that \( \psi_k < 0 \) and \( \psi_Y > 0 \), completing the proof.

To prove Proposition 5, we strengthen Assumption 4 as follows.

**Assumption 4**. (i) \( i^h \in \left[ - (\overline{c}_T - \eta^{tr} \ell^{tr}), S(\overline{k}) - \overline{k} - \overline{c}_0 \right] \), (ii) \( s(k_0, \overline{c}_0 - \overline{c}_T + \overline{k}) < 1 \).

**Proof of Proposition 5**. Let \( K(Y) \) denote the function defined in the proof of Proposition 3 that describes the break-even capital level \( k_t = K(Y_t) \) given aggregate demand \( Y_t \). Eqs. (27) and (A.3) can then be written as

\[
 Y_t = f(Y_t) \equiv \overline{c}_t + k_{t+1} + i^h + \psi(K(Y_t), Y_t) - 1) \eta^{tr} \ell^{tr}, \tag{B.2}
\]

for each \( t \geq 1 \). The output at date 0 is separately characterized as the solution to Eq. (A.3) with the initial \( k_0 \) (as opposed to \( K(Y_0) \)).

We next claim that, given \( k_{t+1} \in (0, \overline{k}] \), there exists a solution to (B.2) over the range \( Y_t \in (0, S(\overline{k})) \). To see this, note that

\[
 \lim_{Y_t \to 0} f(Y_t) > \overline{c}_T + i^h - \eta^{tr} \ell^{tr} \geq 0,
\]

where the first inequality uses \( \overline{c}_t \geq \overline{c}_T, k_{t+1} > 0 \) and \( \psi \geq 0 \), and the second inequality
uses Part (i) of Assumption 4\textsuperscript{tr}. Next note that

\[ f(S(\bar{k})) \leq \bar{c}_0 + \bar{k} + \bar{\epsilon}^h < S(\bar{k}) , \]

where the first inequality uses \( \bar{c} \leq \bar{c}_0, k_{t+1} \leq \bar{k} \) and \( \psi \leq 1 \), and the second inequality reuses Part (i) of Assumption 4\textsuperscript{tr}. Combining the last two inequalities implies the existence of a solution \( Y_t \in (0, S(\bar{k})) \). This also implies a capital stock \( k_t = K(Y_t) \in (0, \bar{k}) \). Applying the same argument recursively, we obtain the path \( \{k_t, Y_t\}_{t=1}^{T-1} \). By the same argument, there exists \( Y_0 \) that solves Eq. \textsuperscript{(A.3)} with the initial \( k_0 \). Note that the solution satisfies \( Y_t < S(k_t) \) for each \( t \in \{1, ..., T - 1\} \) as in the proof of Proposition \textsuperscript{3}. Note that there could be multiple solutions to Eq. \textsuperscript{(B.2)} [and Eq. \textsuperscript{(A.3)} for date 0], which could generate multiple equilibria. We establish the desired results for the “best” equilibrium that has the highest capital and net output, which also implies the results for any other equilibrium. To this end, let \( Y_t^b \) denote the supremum over all \( Y_t \)'s that solve Eq. \textsuperscript{(B.2)} [and Eq. \textsuperscript{(A.3)} for date 0] given \( k_{t+1}^b \). Then let \( k_t^b = K(Y_t^b) \). By induction, we obtain a particular solution to Eq. \textsuperscript{(B.2)} [and Eq. \textsuperscript{(A.3)} for date 0]. It is easy to show that this is the “best” solution in the sense that \( k_t^b \geq k_t \) and \( Y_t^b \geq Y_t \) for each \( t \) for any other solution.

We next claim that, given the worst allowed shock \( \bar{\epsilon}^h = -(\bar{c}_T + \bar{\epsilon}^r l^r) \), the best solution results in a liquidity trap at date 0 with \( Y_0^b < S(k_0) \). To see this, note that the aggregate demand at date 0 satisfies

\[ Y_0^b = \bar{c}_0 - \bar{c}_T + k_1^b + (\psi - 1) \eta^r l^r \leq \bar{c}_0 - \bar{c}_T + \bar{k} . \]

Combining this with Part (ii) of Assumption 4\textsuperscript{tr}, we obtain \( s(k_0, Y_0^b) < 1 \). As in the proof of Proposition \textsuperscript{3}, this implies \( K(Y_0^b) < k_0 \), which in turn implies \( Y_0^b < S(k_0) \). Using \( Y_0 \leq Y_0^b \), this further implies that any solution features a liquidity trap at date 0 with \( Y_0 < S(k_0) \), proving the first part.

To prove the second part, first note that \( Y_t^b < S(k_t^b) \) also implies \( \psi_t(k_t^b, Y_t^b) < 1 \) for each \( t \in \{0, ..., T - 1\} \). Eqs. \textsuperscript{(B.2)} and \textsuperscript{(A.3)} then imply that \( Y_t^b \) is strictly decreasing in \( l^r \) for each \( t \in \{0, ..., T - 1\} \). Next note that the required inequality can be rewritten as

\[ Y_0^b = \bar{c}_0 - \bar{c}_T + k_1^b + (\psi - 1) \eta^r l^r < \bar{c}_0 - \bar{c}_T + \bar{k} . \]
\[ c_0 - c^* < (1 - \psi(k_0, Y_0)) \eta^{tr} l^{tr} \]  
(B.3)

Since \( Y_0^b \) is strictly decreasing in \( l^{tr} \), so is the expression \( \psi(k_0, Y_0^b) \). Thus, there exists \( l_1^{tr} \) such that (B.3) holds for the “best” equilibrium \( \{ k_{i+1}^b, Y_{i+1}^b \}_{t=0}^{T-1} \) if and only if \( l^{tr} > l_1^{tr} \). Note also that any other equilibrium features \( Y_0 \leq Y_0^b \), and thus \( \psi(k_0, Y_0) \leq \psi(k_0, Y_0^b) \). It follows that, if \( l^{tr} > l_1^{tr} \), then the inequality in (B.3) holds for any equilibrium, completing the proof. \( \square \)

### B.3 Proofs for the welfare analysis

**Proof of Proposition 4.** We first prove by induction that the planner avoids the recession at all dates, so that \( Y_t = S(k_t) \) and the return to capital is given by \( S'(k_t) \). Suppose this is true for all dates \( t + 1 \). Then, the constrained planning allocation given some \( k_t \leq \bar{k} \) and \( h_t \geq h^* \) at date \( t \) can be written as,

\[
V_t(k_t, h_t) = \max_{c_t, k_{t+1}, h_{t+1}} u(c_t) + \beta V_{t+1}(k_{t+1}, h_{t+1})  
\text{s.t.} \ c_t + k_{t+1} + h_{t+1} - h_t (1 - \delta^h_t) = Y_t \leq S(k_t),  
\text{and} \ u'(c_t) \geq \beta u'(C_{t+1}(k_{t+1}, h_{t+1})) S'(k_{t+1}),  
\text{and} \ S'(k_{t+1}) \geq 1,  
\text{and} \ h_{t+1} \geq h^*.  
\]  
(B.4)

Here, \( C_{t+1}(\cdot) \) denotes the policy function at date \( t + 1 \). The second line captures the feasibility constraint at date \( t \). The third line captures households’ Euler equation (which we wrote as an inequality to emphasize the sign of the corresponding Lagrange multiplier). The fourth line captures firms’ optimal investment condition along with the zero lower bound constraint. These two lines also use the conjecture that the feasibility constraint holds as equality starting date \( t + 1 \). The last line captures the optimality condition that the planner would never let the residential capital decline below \( h^* \) (in view of the preferences in (2)).

Let \( \lambda_t \) denote the Lagrange multiplier for the budget constraints. The first order conditions for \( h_{t+1} \) implies \( \lambda_t \geq \beta \lambda_{t+1} (1 - \delta^h) \) (since raising \( h_{t+1} \) weakly relaxes the constraints in the third and the fifth lines). Note also that \( \lambda \) is strictly positive in the long run since the economy converges to a neoclassical steady-state (more specifically,
\( \lim_{t \to \infty} \lambda(t) = \lambda^* > 0 \). Combining these observations implies that \( \lambda > 0 \) at all times. This in turn implies the feasibility constraint holds as equality also at date \( t \), that is, \( Y_t = S(k_t) \), proving the conjecture by induction.

Using the conjecture, problem \( (B.5) \) can further be written as,

\[
V(k, h) = \max_{c, k_+, h_+} u(c) + \beta V(k_+ + h_+),
\]

s.t. \( c + k_+ + h_+ = S(k) + h(1 - \delta^h) \),

and \( u'(c) \geq \beta S'(k_+) u'(c_+) \),

and \( S'(k_+) \geq 1 \) and \( h_+ \geq h^* \).

Here, we also dropped the time subscripts to simplify the notation. Note also that the value function does not depend on time. Using the first order conditions, it can be seen that if \( S'(k_+) > 1 \) (equivalently, \( k_+ < \bar{k} \)), then \( h_+ = h^* \). Thus, at least one of the constraints in the last line hold as equality at all times. Using this observation, the solution is characterized by the difference equation system,

\[
c + k_+ + h_+ = S(k) + h(1 - \delta^h)
\]

\[
u'(c) = \beta S'(k_+) u'(c_+)
\]

along with the constraints,

\( k_+ \leq \bar{k}, h_+ \geq h^* \) and \( (k_+ - \bar{k})(h_+ - h^*) = 0 \).

To characterize the solution to this system, first consider the special case \( k_0 = \bar{k} \) and \( h_0 = h^* \). In this case, as described in the main text, the unique solution to the difference equation coincides with the neoclassical adjustment path to the steady state. Let \( \tau_0 \) denote the solution to this neoclassical system. Consider the sequence \( \{\bar{c}_{-T}, ..., \bar{c}_{-1}, \bar{c}_0\} \) defined as the solution to the Euler equations,

\[
u'(\bar{c}_{-T}) = \beta u'(\bar{c}_{-T}) = ... \beta^T u'(\bar{c}_0).
\]

Note that \( \bar{c}_{-T} > ... > \bar{c}_0 \). Define a corresponding sequence of \( \{\bar{h}_{-T}, ..., \bar{h}_{-1}, \bar{h}_0\} \) as the
unique solution to,

\[ \tau_{-t} + \bar{k} + \bar{h}_{-(t-1)} = S(\bar{k}) + \bar{h}_{-t} (1 - \delta^h) \quad \text{for each } t \in \{0, 1, \ldots, T\}, \quad (B.10) \]

with the convention that \( \bar{h}_1 = h^* \). In words, \( \bar{h}_{-t} \) corresponds to the level of residential capital that satisfies the feasibility constraint in \((B.8)\) as equality when \( k = k_+ = \bar{k} \) and consumption is determined according to the Euler equation. Note also that \( h_0 = \bar{h}_0 = (1 + b_0) \) is equal to the threshold level of residential capital characterized in the main text, above which the competitive equilibrium features a liquidity trap [cf. Eq. \((24)\)].

We next claim that starting with \((\bar{h}_{-T}, \bar{k})\), the solution to the difference equation is given by the sequence \( \{\bar{h}_{-t}, \bar{k}_{-t} = \bar{k}, \tau_{-t}\}_{t=T}^0 \) for the first \( T + 1 \) dates followed by a neoclassical adjustment path to the steady-state. That is, starting with \((\bar{h}_{0}, \bar{k})\), the planning solution (just like the competitive equilibrium) decumulates the excess residential capital in a single period. Starting with \((\bar{h}_{-1}, \bar{k})\), the planning solution (unlike the competitive equilibrium) decumulates the excess residential capital in two periods. Starting with \((\bar{h}_{-T}, \bar{k})\), the solution decumulates the residential capital in \( T + 1 \). Thus, this claim also proves the statement in the proposition if the economy initially starts with \((\bar{h}_{-T}, \bar{k})\) for some integer \( T \geq 1 \).

To prove this claim, note that the sequence satisfies the equations in \((B.8)\) by construction. It remains to show that it also satisfies the constraints in \((B.9)\), so that it represents the unique solution to the difference system. Note that the equality constraint in \((B.9)\), as well as \( k_+ \leq \bar{k} \), already hold by construction. Thus we need to establish the remaining inequality constraint, \( h_+ \geq h^* \). To prove this, note that \( \bar{h}_0 > h^* \) follows from the analysis in the main text. Using this observation, along with the inequality \( \tau_{-T} > \ldots > \tau_0 \) and the feasibility constraint \((B.10)\), we further obtain \( \bar{h}_{-T} > \ldots > \bar{h}_0 > h^* \).

We have thus characterized the planning solution (and proved the proposition) for a series of initial conditions \((\bar{h}_{-T}, \bar{k})\). It can also be seen that \( \lim_{T \to \infty} \bar{h}_{-T} = \infty \) (since \( \lim_{T \to \infty} \tau_{-T} = \infty \)). It remains to characterize the planning solution for the remaining initial conditions. First consider \( h_0 \) that satisfies \( \bar{h}_{-T} \leq h_0 < \bar{h}_{-(T-1)} \) for some \( T \), along with \( k_0 = \bar{k} \). It can be seen that the solution in this case is similar to the solution with \( h_0 = \bar{h}_{-T} \), in the sense that it also decumulates the residential capital in \( T + 1 \) periods. Next consider some \( k_0 \leq \bar{k} \). In this case, if \( h_0 \) is such that \( b_0 \leq \bar{b}_0 \),
then the planning solution (just like the competitive equilibrium) decumulates the residential capital immediately. Otherwise, it can be seen that there exists a unique integer $T \geq 0$ such that $k_1 = \overline{k}$ and $h_{-T} \leq h_1 < \overline{h}_{-(T-1)}$. Our results show that, starting with $h_1$ and $k_1 = \overline{k}$, the planning allocation decumulates the residential capital in $T + 1 \geq 1$ periods. Thus, starting with $h_0$ and $k_0$, the planning solution decumulates the residential capital in $T + 2 \geq 2$ periods, completing the proof. \qed