Information Acquisition vs. Liquidity in Financial Markets

Victoria M. Vanasco

November 2013

Abstract

This paper proposes a parsimonious framework to study markets for asset-backed securities (ABS). Loan issuers acquire private information about potential borrowers, use this information to screen loans, and later design and sell securities backed by these loans when in need of funds. While information is beneficial ex-ante when used to screen loans, it becomes detrimental ex-post because it introduces a problem of adverse selection that hinders trade in ABS markets. The model matches key features of these markets, such as the issuance of senior and junior tranches, and it predicts that when gains from trade in ABS markets are ‘sufficiently’ large, information acquisition and loan screening are inefficiently low. There are two channels that drive this inefficiency. First, when gains from trade are large, a loan issuer is tempted ex-post to sell a large portion of its cashflows and thus does not internalize that lower retention implements less information acquisition. Second, the presence of adverse selection in secondary markets creates informational rents for issuers holding low quality loans, reducing the value of loan screening. This suggests that incentives for loan screening not only depend on the portion of loans retained by issuers, but also on how the market prices the issued tranches. Turning to financial regulation, I characterize the optimal mechanism and show that it can be implemented with a simple tax scheme. This paper, therefore, contributes to the recent debate on how to regulate markets for ABS.

Keywords. Security design, asset-backed securities, moral hazard, adverse selection, information acquisition, liquidity.
1 Introduction

Markets for asset-backed securities (ABS) play an important role in providing lending capacity to the banking industry. They allow banks to sell the cashflows of their loans to the market and thus reduce the riskiness of their portfolios. In 2007, more than 25 percent of consumer credit in the U.S. had been funded by ABS, through a process referred to as securitization. In the financial crash of 2008, however, in which certain ABS played a substantial role, we witnessed a collapse in the issuance of all ABS classes. Given the importance of these markets for the real economy, policy makers in the US and Europe have geared their efforts towards reviving them. In a report to the G20, the Financial Stability Board stated that “re-establishing securitization on a sound basis remains a priority in order to support provision of credit to the real economy and improve banks’ access to funding.”

Two problems have been shown to be present in the practice of securitization in the past decade. First, the increase in securitization has led to a decline in lending standards, suggesting that liquid markets for ABS reduce incentives to issue good quality loans. Second, securitizers have used private information about loan quality when choosing which loans to securitize, indicating that a problem of asymmetric information is present in ABS markets. A natural question then arises: how should ABS be designed to provide incentives to issue good quality loans and, at the same time, to preserve liquidity and trade in these markets? The literature on optimal design of ABS has studied these problems – provision of incentives and of liquidity – in isolation. However, by doing so, a fundamental trade-off between incentives and liquidity has been overlooked: while securities that provide incentives to issue good quality loans may expose the issuer to less liquid secondary markets, securities that maximize trade in these markets tend to worsen incentives to issue good loans in the first place.

This paper proposes a parsimonious framework to study ABS where both incentives and liquidity issues are considered and linked through the issuer’s information acquisition decision. I study the problem of a bank that i) privately invests in information about potential borrowers in a loan screening

---

1 And by April 2011, the market value of outstanding securitized assets in the US was larger than that of US Treasuries. See Gorton and Metrick (2013).
3 See Bernt and Gupta (2008), Dell’Ariccia et al. (2008), Elul (2009), Jaffee et al. (2009), Mian and Sufi (2009).
4 See Agarwal (2012), Calem et al. (2010), Downing et al. (2008), Jian et al. (2010), Keys et al. (2008).
stage, ii) receives private information about its borrowers once it chooses to lend, and iii) later designs and sells securities backed by its loans to realize gains from trade in secondary markets. This setup captures an important tension present in these markets, where gains from information acquisition and loan screening need to be traded-off with gains from trade in secondary markets.

This paper delivers two sets of results. First, I address some of the main forces at play in ABS markets. The model matches key features of ABS markets, such as the issuance of senior and junior tranches, and it generates new testable predictions, such as a pecking order for tranche issuance. Moreover, I find that when gains from trade are large, the bank has a problem of commitment: even though ex-ante it would like to retain some of its cashflows, ex-post, once information acquisition is sunk, it has an incentive to sell a larger portion of its loans to exploit gains from trade. In this scenario, the presence of adverse selection supports the equilibrium with information acquisition by naturally inducing retention of the bank with good loans. Consistent with this, when adverse selection is not severe, information acquisition and loan screening are inefficiently low. The second set of results characterize the inefficiencies in place and suggest interventions that improve ex-ante efficiency. In particular, I show that regulators should not only focus on retention levels for securitizers, but also on how secondary markets differentially compensate good relative to bad issuers.

The model is stylized and is yet able to capture the complexities inherent to the process of securitization. It has three periods and features a bank and a market of potential investors. The bank has an endowment that it can store or use to finance one risky project (make a loan) that pays in the final period. In the first period, the bank privately invests in information and observes two signals about project quality: while the first signal is used to screen good quality projects; the second signal is observed while holding the issued loan. By investing more in information the bank increases the precision of its private information. In the second period, given this information, the bank sells limited liability securities backed by its loan cashflows to “uninformed” investors to exploit gains from trade. In the final period, loan cashflows are realized and the bank pays investors.

When securities are designed after loan issuance, the bank faces a trade-off between the gains from selling cashflows in secondary markets and the lemon’s discount faced in the market given its private

---

6The second signal can be interpreted as the information acquired by the bank that cannot be inferred by the market through the initial screening decision: soft information, or information acquired while establishing a lending relationship (i.e. while holding the loan, as in Plantin (2009) where he introduces the concept of learning by holding.)
information. The paper provides a new rationale for the issuance of senior and junior tranches in secondary markets. In particular, I find that standard debt (the senior tranche) is the security chosen by the bank with good loans, since it minimizes the region where disagreements about the likelihood of cashflows might arise, minimizing the lemon’s discount. Consequently, banks with bad loans issue debt to receive an implicit subsidy from the bank with good loans, and issue their remaining cashflows (junior tranches) in a separate market to further exploit gains from trade. I obtain this result by departing from the literature on security design with adverse selection by imposing a No Transparency assumption. This assumption implies that in equilibrium the market is unable to fully screen the quality of the bank’s loans. That is, there is a semi-pooling equilibrium in ABS markets where all banks issue the senior tranche of their cashflows, and only banks with bad loans issue in addition a claim to their junior tranche.

The model generates predictions that match some key characteristics of markets for ABS. First, issuers of ABS should slice underlying cashflows into senior and junior tranches that are sold separately in secondary markets. Second, issuers with better quality loans should retain the junior tranches, while those with bad quality loans should sell them. Third, there is a pecking order for tranche issuance: for a given tranche sold in secondary markets, all safer tranches must be sold as well by the same issuer. Fourth, the quality of issued loans is decreasing in the fraction of cashflows being sold in secondary markets (i.e. fraction being securitized). Finally, loans for which very little information (e.g. credit cards) or a lot of information (e.g. corporate loans) is acquired in equilibrium should have more liquid secondary markets than those for which information acquisition is intermediate.

I find that when the bank and the market cannot commit to the design and price of securities ex-ante, the equilibrium is inefficient. In particular, when gains from securitization are large, the bank is tempted to sell a large portion of its cashflows ex-post, and thus information acquisition and loan screening are inefficiently low. Two separate forces drive this inefficiency. First, when the bank is tempted ex-post to sell, it does not internalize that lower retention implements less information acquisition, and thus it “under-retains” in equilibrium. Second, adverse selection in secondary markets further distorts incentives by creating informational rents for the bank holding bad loans, reducing the value of screening. However, when the adverse selection problem in secondary markets is sufficiently severe,

\[7\text{The No Transparency assumption prevents the market from enforcing retention levels on securitizers. Since retention of cashflows is essential to screen loan quality, when it cannot be enforced, loan quality cannot be screened.}\]
trade in secondary markets is inefficiently low and information acquisition too high. This suggests that the problem of provision of incentives for information acquisition and loan screening is only relevant for asset classes with liquid secondary markets and high securitization levels.

Given these inefficiencies, I characterize the optimal mechanism that is obtained when the bank and the market can commit to the design and the prices of securities chosen before loans are issued. In this case, the design of securities internalizes the effect on information acquisition and loan screening. I show that standard debt continues to be the optimal design because it minimizes the expected adverse selection and it provides the best incentives for information acquisition by exposing the bank to the most informationally-sensitive cashflows. Debt levels and market transfers are chosen to optimally trade-off gains from trade with incentives for information acquisition. I find that to improve information acquisition, the bank has to commit to retain cashflows ex-post. However, retention levels are dependent on the quality of the underlying loans. In particular, the bank with good loans underlying its ABS issuance should retain more than the one with bad loans, suggesting that retention levels imposed on securitizers should be decreasing in the quality of underlying cashflows. In addition, incentives for information acquisition are further improved by transferring ex-post all the surplus to the bank with good loans to compensate them for being exposed to a lemon’s problem.  

I show that a simple tax scheme conditional on market participation and tranche issuance decentralizes the optimal mechanism when commitment tools are not available to the bank or to the market. In particular, subsidies to participation in the market for senior tranches, together with taxes for participation in the market for the junior tranches are beneficial since they improve incentives for information acquisition at no retention cost. This policy compensates banks with good loans for the costs generated by being mimicked by those with bad loans. This result is in contrast with models that only focus on adverse selection, where transfers across banks in secondary markets would not affect ex-ante efficiency. Thus, the model suggests that regulators should not only focus on retention levels for securitizers but also on the way the market compensates good vs. bad issuers since transfer across different quality issuers in secondary market affect ex-ante efficiency by distorting incentives. Furthermore, policies that tax/subsidize debt levels (similar to imposing retention levels) can implement second-best levels of information acquisition. In particular, the issuance of senior tranches should be taxed—or retention levels

\[8\] Subject to the incentive compatibility constraints.
imposed—when markets for ABS are sufficiently liquid.

Finally, I use the model to evaluate some of the recently discussed interventions in markets for ABS. Policymakers in the US and Europe have proposed the “Skin in the Game” rule that requires issuers of asset-backed securities to retain a fraction of the underlying assets. My model rationalizes this type of intervention as a means to incentivize loan-screening only for ABS that feature high trade levels in secondary markets. The model further suggests that banks that claim to have good quality loans underlying their ABS should retain more than those that claim to have bad quality loans. As a result, policies that demand the same retention levels of all issuers impose excessive costs by hindering trade in secondary markets. This result is in contrast with the literature on security design in the presence of moral hazard, where imposing the same retention levels to all securitizers is optimal ex-ante. In addition, I find that incentives are stronger when securitizers retain the junior tranche of underlying cashflows, while proposed regulation is not specific to the type of retention.

The key trade-offs analyzed in this paper are motivated by substantial evidence that the provision of incentives in the loan screening stage and adverse selection in secondary markets are important features of the ABS market. In particular, it has been shown that credit standards in the mortgage market have fallen more in areas where lenders sold a larger fraction of the originated loans, and that performance has been worse for securitized loans (Dell’Ariccia et al. (2008), Elul (2009), Keys, Mukherjee, Seru, and Vig (2008)). Consistent with this, Bernt and Gupta (2008) find that borrowers of the syndicated loan market with more liquid secondary markets seem to under-perform in the long run. Finally, it has been found that differences in unobservable loan characteristics known by the issuer are not fully compensated by loan pricing in secondary markets (Jiang et al. (2010), Downing et al. (2008), Calem et al. (2010), and Agarwal et al. (2012)). The first set of facts suggests that provision of incentives to acquire information to issue good quality loans might be necessary. The second set of facts documents the presence of asymmetric information in ABS markets, suggesting that trade and liquidity in these markets may be affected by the issuer’s private information.

Several papers have highlighted this trade-off between incentives to issue good quality assets and secondary market liquidity. Parlour and Plantin (2008) study loan sales and show that even though liquid secondary markets are ex-post efficient, they might not be be socially desirable ex-ante, since they reduce incentives to monitor loan quality. Malherbe (2012) studies the costs and benefits of securitization
and finds that for securitization to be an efficient risk-sharing mechanism, market discipline has to be strong.\textsuperscript{9} In contrast to their work, I design the optimal securities to be sold in secondary markets given the above mentioned trade-off, and, in addition, I assume that the bank can affect the quality of its private information. Thus, in my setting, adverse selection is endogenous for two reasons: first, the bank chooses the quality of its private information; and second, by designing the issued security the bank can affect the level of adverse selection that it faces in the market. These trade-offs have also been studied in non-banking contexts by Bhide (1993), Maug (1998), Dewatripont and Tirole (1994), Winton (2001), Aghion, Bolton, and Tirole (2004), Faure-Grimaud and Gromb (2004), who focus on the relation between shareholder control on stock market liquidity.

My work builds on Myers and Majluf (1984) seminal paper, that addresses the problem of security design in the presence of adverse selection. They find that debt is superior to equity since its value is less sensitive to private information. Their results are extended by Noe and Nachman (1994), who enlarge the set of securities available to the issuer and consider signaling equilibria. They identify the conditions under which debt is the unique optimal design.\textsuperscript{10} These papers take the size of the investment, and therefore amount of funds raised in the market, as given. Instead, I follow Duffie and DeMarzo (1999), in assuming that funds raised in secondary markets are an equilibrium outcome that results from the trade-off between the lemon’s discount the market assigns to a given security and the gains from trade. Duffie and DeMarzo focus on ex-ante security design and obtain a separating equilibrium, where the issuer signals its private information by retaining a fraction of the designed security. In contrast to their paper, I study security design ex-ante and ex-post, and I take a game theoretic approach instead of focusing on competitive equilibria. By solving a screening game, I eliminate the multiplicity of equilibria that generally arises in these settings. In this sense, my paper is closely related to Biais and Mariotti (2004), where they study optimal security design by solving a screening game and find the optimal mechanism, and to DeMarzo (2005) where an ex-post security design problem is considered. I depart from the literature on security design in the presence of adverse selection by endogeneizing the decision of the issuer to acquire private information in an environment where information is desired to improve

\textsuperscript{9}In Malherbe (2012), strong market discipline implies that the securitization market outcome is able to reward diligent loan origination.

\textsuperscript{10}Brennan and Kraus (1987) and Constantinides and Grundy (1990) study the ability of an issuer to costlessly signal its private information by designing an optimal financing structure. Their results are applicable to the corporate finance literature, but not in this framework, where the issued securities and their prices can only be contingent on the cashflows of underlying assets.
the quality of underlying assets, and by imposing the No Transparency assumption that eliminates separating equilibria in secondary markets.

My paper also relates to the literature on security design in the presence of moral hazard. Innes (1990) studies a principal-agent model in which the agent needs to be offered a contract that induces him to put effort to improve the quality of an investment project. He finds that when contracts are constrained to be monotonic on underlying cashflows, as in this paper, debt is the optimal design.11 In this sense, my results are consistent with these findings. In a framework very closely to mine, Fender and Mitchell (2009) study how different contractual mechanism offered in secondary markets affect the incentives of loan originators to screen loans. They focus on different retention mechanism, and find that retention of the first-loss tranche is not always optimal in the presence of systematic risk factors affecting underlying cashflows. In contrast to this paper, I investigate the issue of incentives in a model with security design in secondary markets with adverse selection. In addition, I assume no common risk-factors affect the underlying cashflows. There has also been a growing literature that focuses on the optimal design of securities to provide incentives to investors to acquire information. Their main finding is that standard debt is the design that minimizes incentives to acquire information, and thus should be issued when information acquisition is not desired (Dang et al. (2009), Yang (2012)), while a combination of debt and equity should be issued when information acquisition is valuable (Yang and Zeng (2013)). In contrast with this literature, investors in my model do not acquire information.

Organization. In Section 2, I describe the setup of the model, and characterize the first-best of this economy. In Section 3, I study the case when securities are designed after loan issuance, as in markets for ABS. Section 4 allows for commitment and characterizes the optimal mechanism that is attained when securities are designed and priced ex-ante, before loan issuance. Section 5 uses results from the previous two sections and presents the policy implications of the model. In Section 6, some extensions to the baseline model are presented. Section 7 concludes.
2 The Model

2.1 Setup

The model has three periods, indexed by $t \in \{0, 1, 2\}$. There is a single bank and a market of potential investors. The bank is risk-neutral with a payoff function $V_0 = \theta c_1 + c_2$ where $c_t$ denotes the cashflows of the bank at time $t$, and $\theta > 1$ denotes the bank’s marginal value of funds in $t = 1$. When $\theta > 1$, the bank values funds more than investors and there are thus gains from trade in the intermediate period.\(^{12}\)

At $t = 0$, the bank has an endowment of $w_b = 1$ and it cannot borrow additional funds from the market. This assumption can be motivated by assuming that the bank is against its capital constraint and therefore can only raise funds by selling assets.

**Investment Technology.** In the initial period, the bank can store its endowment at the risk free rate, normalized to one, or invest it in risky projects (i.e. loans). There is a unit mass of risky projects that produce cashflows $X$ at $t = 2$ if they receive one unit of investment at $t = 0$. Projects can be of high or low quality, not observed by the bank nor the market. There is a fraction $\pi_H$ of high quality projects with payoff $X \sim G_H$ and a fraction $1 - \pi_H$ of low quality projects with payoff $X \sim G_L$. These distributions are related by the monotone likelihood ratio property (MLRP); that is, $\frac{g_H(x)}{g_L(x)}$ increasing in $x$. In addition, I assume that it is not profitable to invest in a project chosen at random: $\pi_H E_H [X] + (1 - \pi_H) E_L [X] < 1$; and that there are gains from learning about project quality since it is efficient to invest in high quality projects but not in low quality ones: $E_L [X] < 1 < E_H [X]$.

**Project Screening and Information Acquisition.** The bank has access to a technology to privately screen project quality.\(^{13}\) By investing $C(a)$ in information, the bank has access to signals with precision “\(a\)” about the underlying quality of projects, where $C : [\frac{1}{2}, 1] \to \mathbb{R}^+$, $C' \geq 0$, $C'' \geq 0$ and $\lim_{a \to 1} C(a) = \infty$. I assume that information acquisition is a bank’s hidden action. Privately investing $C(a)$ in information gives the bank access to two independent binary signals, $s_0, s_1 \in \{H, L\}$, where $s_0$ is observed in $t = 0$ for all available projects, and $s_1$ is observed between $t = 0$ and $t = 1$ for the project.

---

\(^{12}\)Gains from trade captured by $\theta > 1$ should be interpreted as gains from securitization not addressed in this paper. There are many reasons why a bank might want to raise funds by selling assets. If the bank is against its capital constraints, and new exclusive investment opportunities arise, it will benefit from selling a fraction of its loans to finance these new investments. Alternatively, securitization may allow the bank to share-risks with the market or to reduce bankruptcy costs by creating bankruptcy remote instruments.

\(^{13}\)Evidence of banks being special lenders can be found in Fama (1985), James (1987), and of banks having the ability to acquire private information about borrowers in Mickelson and Partch (1986), Lummer and McConnell (1989), Slovin, Sushka, Polonchek (1993), Plantin (2009), Botsch and Vanasco (2013), among others.
that received financing in $t = 0$. These signals are distributed identically and independently across projects, with conditional distributions given by $P(s = H|q = H) = a$ and $P(s = L|q = L) = a$, where $q \in \{H, L\}$ denotes project quality. The first signal, $s_0$, captures the information acquired by the bank to screen loans, while the second signal, $s_1$, captures the private information received by the bank when establishing a lending relationship.\footnote{Alternatively, the second signal can be interpreted as soft information acquired during the screening process that cannot be inferred by the market from the bank screening decisions. The binary signal structure therefore generates a useful partition between information used to screen loans, and thus inferred by the market in equilibrium, and private information that the bank cannot truthfully transmit to the market about the quality of the issued loan.} Finally, assuming that the precision of both signals is increasing in information acquisition, “$a$”, captures the fact that once a bank invests time and effort in understanding the quality of a given borrower at the screening stage, it is also better able to interpret information that is later received about that borrower.

After observing a given signal, the bank updates its beliefs about firm quality using Bayes rule. Since the bank evaluates a continuum of projects in $t = 0$, it observes a project with $s_0 = H$ with probability one, for any level of information acquisition $a$. Thus, the bank always chooses to finance a project with $s_0 = H$.\footnote{This restriction is at no loss, since I will show that in equilibrium the bank strictly prefers to lend to a firm with $s_0 = H$ if it chooses to acquire information, and is indifferent otherwise. Assuming that a high signal is always observed is a modeling device that ensures that after information is acquired, there is screening of loans in equilibrium; that is, by acquiring information the bank can always improve the expected quality of the issued loans.} The following two conditional probabilities will be used extensively throughout the paper: (i) the probability of a loan being high quality given the initial screening ($s_0 = H$), and defined as $\rho(a)$; and (ii) the probability of receiving the second high signal $s_1 = H$ for the issued loan, given the initial screening, defined as $\rho_h(a)$:

\begin{align}
\rho(a) &\equiv \mathbb{P}_a(q = H|s_0 = H) = \frac{a \pi_H}{a \pi_H + (1 - a)(1 - \pi_H)} \\
\rho_h(a) &\equiv \mathbb{P}_a(s_1 = H|s_0 = H) = a \rho(a) + (1 - a)(1 - \rho(a))
\end{align}

Finally, to ensure that there are gains to acquiring information, I assume that there exists an $a \in \left(\frac{1}{2}, 1\right]$ such that $\rho(a) \mathbb{E}_H[X] + (1 - \rho(a)) \mathbb{E}_L[X] - C(a) > \theta$. In Section 6, I extend the model by allowing the precision of the second signal to differ from that of the first one and show that the main qualitative results of the paper remain unchanged. To see why this is the case, note that the conditional distribution of the second signal, given by $\rho_h(a)$, is always a function of “$a$” through $\rho(a)$; that is, better quality screening improves the probability of observing a second high signal for an issued loan. In this sense, information
acquisition in the screening stage always has an impact on the level of informational asymmetries between the bank and the market.

Secondary Markets. At $t = 1$, the bank can raise funds by selling a portion of its loans to investors to exploit gains from trade ($\theta > 1$). In order to raise funds, the bank can issue limited-liability securities backed by its loans. The payoff of these securities can only be made contingent on the realization of loan cashflows. Thus, a security $F$ is given by some function $F : \mathbb{X} \to \mathbb{R}$ and its payoffs are given by $F(X)$. In addition, as is standard in the security design literature, I assume that the bank and the investors have limited-liability: (LL) $0 \leq F(x) \leq x$, and I restrict attention to securities whose payoffs to the bank and to investors are weakly monotone in underlying cashflows: (WM) $F(x)$ and $x - F(x)$ are weakly increasing for all $x \in \mathbb{X}$.$^{16}$ Finally, let $\Delta \equiv \{F : \mathbb{X} \to \mathbb{R} \text{ s.t. (LL) and (WM) hold}\}$ denote the set of feasible securities a bank can issue in secondary markets, and if the bank issues more than one security, where $\tilde{F}(X) \equiv \sum_i F_i(X)$, then it must be that $\tilde{F} \in \Delta$ as well.

The bank arrives to secondary markets with private information about its loan cashflows, given by the signals $s_0$ and $s_1$ and the hidden-action $a$. Let $z \in \{z_l, z_h\}$ denote the bank’s type in secondary markets, where $z_l \equiv \{s_0 = H, s_1 = L\}$ and $z_h \equiv \{s_0 = H, s_1 = H\}$ denote the bank with the bad loan and the bank with the good loan respectively.$^{17}$ Given this, the bank’s private valuation of a given security is given by $E_a[F(X)|z]$ for $z \in \{z_l, z_h\}$, where $E_a[\cdot|z]$ denotes the expectation operator over cashflows $X$, conditional on private signals $z$ and the precision of these signals $a$. I solve a screening problem in secondary markets, where uninformed investors post prices for all feasible securities $F \in \Delta$ given their beliefs about $a$ and $z$, and the $z$-type bank chooses which security to issue from the market offered menu. Therefore, the bank faces an inverse demand function $p : \Delta \to \mathbb{R}$ where $p(F)$ is the market price for security $F$ that is determined in equilibrium by the investors’ zero-profit condition.

Timing of the Game. At $t = 0$ the bank invests in information, observes signal $s_0$ and makes its lending decisions. At $t = 1$, when in need of funds and having received signal $s_1$, the bank issues feasible securities backed by its loan cashflows to investors. At $t = 2$, loan cashflows are realized and contracts are executed. The timing of the game is presented in Figure 1.

---

$^{16}$This restrictions are assumed in Nachman and Noe (1994), Duffie and DeMarzo (1999), Biais and Mariotti (2004), among others. Innes (1990) discusses the implications of restricting attention to contracts that are monotonic on realized returns in environments with moral hazard.

$^{17}$Even though $a$ could also be part of the bank’s type, since in equilibrium it is unique and inferred by the market, it simplifies the problem to keep track of $a$ and $z$ separately, even though they are both the bank’s private information.
2.2 First-Best

Before solving the model with asymmetric information, I characterize the first-best of this economy as a useful benchmark for the remainder of the paper. I solve the model by assuming that information acquisition “a” is observable, and received signals are public information. When funds are needed in $t = 1$, the bank can sell a claim to its future portfolio cashflows to the market that has the same valuation. Let $F \in \Delta$ be the security issued by the bank, and let $p(F) \in \mathbb{R}^+$ be the price the market offers for this security. The value of the $z$-type bank in $t = 1$ is given by:

$$ \theta p(F) + E_a [X - F(X) | z] = (\theta - 1) E_a [F(X) | z] + E [X | z] $$

where the last equality holds because the market values any security $F$ as the bank, and the competitive investors price securities at its expected value; that is, $p(F) = E_a[F(X) | z]$ with:

$$ E_a[F(X) | z] \equiv \rho(a) E_H [F(X)] + (1 - \rho(a)) E_L [F(X)] $$

It is straightforward that the bank chooses to issue equity, $F^*_{FI}(X) = X$, since it is the issuance that maximizes the gains from trade. Given that all claims are sold at $t = 1$, the bank chooses how much information to acquire to maximize the value of banking in $t = 0$:

$$ a^*_{FI} = \arg \max_{a \in [\frac{1}{2}, 1]} \theta [\rho(a) E_H [X] + (1 - \rho(a)) E_L [X]] - C(a) $$
When choosing how much information to acquire, the bank is fully exposed to the cashflows of its loans and the market fully compensates it for investing in information. It will be useful to keep this benchmark in mind: in the first-best, gains from trade and from information acquisition are maximized when the bank issues a claim to all of its cashflows and when the market fully compensates the bank for its investment in information.

3 Markets for ABS: The No Commitment Case

In this section, I study an economy where securities are designed after loans have been issued – at \( t = 1 \). This implicitly assumes that the bank has no commitment to securities designed in \( t = 0 \) before loan issuance. In practice, issuers of ABS design their securities after loan issuance. The reason why this might be the case is that ex-post, once an issuer has private information about the quality of its loans, nothing prevents him from re-designing a security and finding an investor to buy it. Therefore, this case is important for understanding how unregulated markets for ABS operate and what inefficiencies may arise in environments where commitment to pre-designed securities cannot be enforced. I use the results from this section to answer two main questions that are at the heart of the discussion on optimal regulation in markets for ABS. First, how does information acquisition affect the design of securities sold in secondary markets and the levels of ABS issuance in these markets? And second, how does the design of securities and trade levels in ABS markets affect incentives of the bank to acquire information and issue high quality loans in the first place? In Section 4, I study the optimal mechanism, that is attained when both the bank and investors can write contracts ex-ante and commit to securities and prices stated in the contract.

At \( t = 0 \), the bank can store its endowment or invest in information to screen and issue one loan. If the bank chooses to invest \( C(a) \) in information, it is able to identify and lend to a project with \( s_0 = H \). At \( t = 1 \), with probability \( \rho_h(a) \) the bank observes signal \( s_1 = H \) and thus is a \( z_h \)-type bank; otherwise, it observes \( s_1 = L \) and becomes a \( z_l \)-type. Let \( p_z \) and \( F_z \) denote the funds raised and cashflows sold in secondary markets by type \( z \in \{ z_l, z_h \} \), and thus \( X - F_z(X) \) are the cashflows retained until maturity.\(^{18}\)

\(^{18}\)Note that in \( F_z \) are the cashflows sold by the \( z \)-type bank, and these cashflows can potentially be sold through the issuance of more than one security in secondary markets. Consistent with this, \( p_z \) are the total funds raised in secondary markets. This clarification is important, since I will show that the bank with the bad loan issues more than one security in equilibrium.
Given this, the value of the bank with information acquisition $a$ and type $z$ at $t = 1$ is given by:

$$V_1(a, z) \equiv \theta p_z + \mathbb{E}_a [X - F_z(X)|z]$$

Consistent with this, the value of acquiring information in $t = 0$ is given by:

$$V_0(a, p_{z_l}, p_{z_h}, F_{z_l}, F_{z_h}) \equiv \rho_{h}(a) \{\theta p_{z_h} + \mathbb{E}_a [X - F_{z_h}(X) | z_h]\} + (1 - \rho_{h}(a)) \{\theta p_{z_l} + \mathbb{E}_a [X - F_{z_l}(X) | z_l]\} - C(a)$$

(3)

where the unit cost of investing in a project is incorporated into $C(a)$. The value of storing the endowment in $t = 0$ is given by $V_{store} = \theta$. Finally, let $a^e$ denote the market (investors') belief about the hidden action taken by the bank. Since in any equilibrium only one level of information acquisition is implemented, I focus on pure strategy equilibria in which market beliefs are degenerate at some level $a^e \in [0.5, 1]$. The problem is solved by backwards induction. At $t = 1$, for a given level of information acquisition $a$ and market beliefs $a^e$ about this hidden-action, a $z$-type bank designs and issues feasible securities in secondary markets to raise funds. At $t = 0$, given the secondary markets optimal strategies, the bank chooses how much information to acquire. The following definition characterizes the equilibrium with information acquisition in an economy without commitment.

**Definition 1.** An equilibrium with information acquisition is given by $\{a^e, a^*, p_{z_l}, p_{z_h}, F_{z_l}, F_{z_h}\} \in [\frac{1}{2}, 1] ^ 2 \times \mathbb{R}_+ ^ 2 \times \Delta ^ 2$ satisfying the following conditions:

1. Given $a, a^e$, $\{p_{z_l}, p_{z_h}, F_{z_l}, F_{z_h}\}$ are equilibrium outcomes in secondary markets.

2. Given $a^e$, $a^* = \arg \max_{a \in [\frac{1}{2}, 1]} V_0(a, p_{z_l}, p_{z_h}, F_{z_l}, F_{z_h})$, as defined in (3).

3. $a^* = a^e$.

For an equilibrium with information acquisition to exist it must be that:

$$V_0(a^*, p_{z_l}^*, p_{z_h}^*, F_{z_l}^*, F_{z_h}^*) \geq V_{store} = \theta$$

(4)

If condition (4) does not hold, the bank chooses to store its endowment and does not invest in information nor it extends credit to risky projects. The remainder of this section focuses on characterizing the

---

标准化的条件限制在成本函数$C(a)$上，以获得的唯一实施$a$在均衡中的水平。
equilibrium with information acquisition, and is organized as follows. First, I solve for the equilibrium outcome in secondary markets. Second, I solve for the optimal level of investment in information chosen by the bank in $t = 0$, given the previously obtained secondary market equilibrium outcomes. Finally, I discuss how results from the model are able to rationalize key features of markets for asset-backed securities, such as the tranching of underlying cashflows and the observed fall in lending standards in the years leading to the crisis.

3.1 Equilibrium in Secondary Markets

The bank arrives to secondary markets with a chosen level of information precision, $a \in [\frac{1}{2}, 1]$, which is a bank’s hidden action, and private signals $z \in \{z_l, z_h\}$. Both the hidden action and the signals determine the bank’s valuation of its loan cashflows. Conditional cashflow distributions are given by:

$$g(X|a, z_i) \equiv \pi_i(a) g_H(X) + (1 - \pi_i(a)) g_L(X), \ i = \{l, h\}$$

(5)

where $\pi_h(a) \equiv P_a(q = H|z = z_h) = \frac{a^2 \pi_H}{a^2 \pi_H + (1 - a^2)(1 - \pi_H)}$

(6)

and

$$\pi_l(a) \equiv P_a(q = H|z = z_l) = \pi_H$$

(7)

where both are computed using Bayes Rule. Note that $\frac{a^2 \pi_H}{a^2 \pi_H + (1 - a^2)(1 - \pi_H)} \geq 1$ for all $a \in [\frac{1}{2}, 1]$ and $\pi_H \in [0, 1]$, but that $\pi_l(a)$ does not depend on $a$. That is, information acquisition increases the likelihood of having good cashflows for banks with good loans only. This result will simplify the analysis, but I show in Section 6 that the qualitative results remain unchanged when $\pi_l$ also depends on $a$.

A. Strategies

Rather than defining investors’ strategies, I model the buyer side of the market as a menu of prices and securities $\{p(F), F\}_{F \in \Delta}$ offered to the bank. This menu needs to satisfy two conditions: (i) Zero Profits: investors make zero profits in expectation, and (ii) No Deals: there are no profitable deviations for an investor; that is, by offering a price different than the one on the menu for a given security, an investor cannot expect to make profits.\(^{20}\) In the remainder of the paper, I use the terms investors and

---

\(^{20}\)This approach is a useful modeling device to summarize an environment with two or more uninformed, risk-neutral, deep-pockets investors compete by posting prices for all securities. The “No Deals” condition is taken from Daley and Green (2012), and can be also be interpreted as a No Entry condition. This ”No Deals” condition needs to be imposed in environments with asymmetric information to ensure there are no profitable deviations for the buyers.
the market interchangeably. The strategy of a $z$-type bank that acquired information $a$ is to choose which securities to issue given the market posted prices.

**B. Market Beliefs**

Investors enter secondary markets with a degenerate belief $a^e$ about the bank’s hidden-action. In addition, they need to form beliefs about the bank’s type $z$. By offering a menu of securities and respective prices, the market can potentially screen the bank’s type.\(^{21}\) The idea is that the cost of retaining cashflows (i.e. of not selling them) is lower for banks with good assets than for those with bad assets, and this can be used to separate them: those with good assets retain a fraction of their cashflows while those with bad assets reveal their type to be able to sell all of their cashflows. Instead, I impose a “No Transparency” assumption that prevents the market to enforce retention levels, and thus screening bank quality is not possible in equilibrium. Gorton and Pennachi (1995) discuss the commitment to retain a given fraction when selling a loan. They argue that “... no participation contract requires that the bank selling the loan maintain a fraction, so this contract feature would also appear to be implicit and would need to be enforced by market, rather than legal, means.” This assumption is therefore motivated by behavior in ABS markets, and it generates novel predictions about potential strategies in ABS markets.\(^{22}\)

**Assumption 1. [No Transparency]** The bank cannot commit to retain cashflows. Or equivalently, balance sheet information is not verifiable and markets are anonymous.

Given the No Transparency assumption, an investor forms its beliefs about bank type only by observing the security the bank is selling to her, and cannot condition on all the securities the bank is selling in secondary markets since this is not observable. More formally, the No Transparency assumption implies that market beliefs about the bank’s type are given by some measurable function $\mu : \Delta \rightarrow [0, 1]$, where $\mu(F)$ denotes the probability of a bank being $z_h$-type if it chooses to sell security $F$. It is crucial that market beliefs are formed per security sold, and not as a function of the set of securities sold by a bank.

\(^{21}\)Separating equilibria in this type of market has been found in Duffie and DeMarzo (1999), Biais and Mariotti (2004), DeMarzo (2005), among others.

\(^{22}\)Without imposing this assumption, the ex-post security design problem is like the one presented in DeMarzo (2005), where each type issues one debt contract and retention is used to screen underlying quality. Important qualitative results remain unchanged, but transfers across types in ABS markets differs, and the issuance of multiple securities per bank type cannot be rationalized.
Consistent with this, the market valuation for a given security $F \in \Delta$ is denoted by $E_{a^e,\mu}[F(X)]$, and it is given by:

$$E_{a^e,\mu}[F(X)] = \mu(F)E_{a^e}[F(X)|z_h] + (1 - \mu(F))E_{a^e}[F(X)|z_l] \quad (8)$$

### C. Equilibrium

I assume that the bank wants to minimize the number of markets it issues in; that is, the bank prefers to issue one security than to issue several securities when both strategies have the same payoff. I rationalize this by imposing an infinitesimal cost of issuing a positive claim ($F > 0$), $c > 0$. Given this, I can assume without loss that the bank chooses to issue at most $N$ securities, where $N$ can be arbitrarily high. The equilibrium notion in secondary markets is as follows:

**Definition 2.** Given information acquisition, $a$, and market beliefs $a^e$, an equilibrium in secondary markets is given by a market menu $\{F, p(F)\}_{F \in \Delta}$, bank $z$-type strategy $\sigma(z) = \{F^1_z, ..., F^N_z\}$, and belief function $\mu: \Delta \to [0, 1]$, satisfying the following conditions:

1. **Bank’s Optimality.** Given the market posted menu $\{p(F), F\}_{F \in \Delta}$, $z$-type bank chooses $F^1, ..., F^N$ to maximize its value at $t = 1$:

$$\sum_{n=1}^{N} \{\theta p(F^n) - E[F^n(X)|z(a)]\} - c\tilde{N} \quad (9)$$

subject to $\sum_{n=1}^{N} F^n(X) \leq X$, and where $\tilde{N}$ is the number of chosen securities with $F > 0$.

2. **Belief Consistency.** $\mu(F) = P_{a^e}(z = z_h|Issue F)$ are derived from $\sigma(z)$ using Bayes rule whenever possible.

3. **Zero Profit Condition.** $p(F) = E_{a^e,\mu}[F(X)]$ for all $F \in \Delta$.

4. **No Deals.** For all $F \in \Delta$, it does not exist alternative pricing $\tilde{p}$ such that by offering to buy $F$ at price $\tilde{p}$, an investor expects to make profits.

---

23 This assumption prevents multiplicity of equilibria arising from the fact that the bank in equilibrium might be indifferent between issuing a given security or any partition of the cashflows underlying that security; and thus simply eliminates a multiplicity of payoff-equivalent equilibria.
The following Lemma presents the first important result of this section, which states that under the No Transparency assumption the bank with the good loan cannot be separated from the one with the bad loan, eliminating the possibility of screening bank quality. As a result, the issuance chosen by the bank with the good loan is always mimicked by the bank with the bad loan, and thus the bank with the good loan faces a lemon’s problem in secondary markets. Full proofs are presented in the Appendix.

**Lemma 1. [No Separation]** Under the No Transparency Assumption, fully separating equilibria in secondary markets do not exist. In particular, in any equilibrium in secondary markets the \( z_l \)-type bank mimics the issuance of the \( z_h \)-type bank.

The main idea behind the proof is that in any separating equilibrium \( \{p_{z_l}, p_{z_h}, F_{z_l}, F_{z_h}\} \), there is a profitable deviation for an investor. Note that in any separating equilibrium, \( z_l \)-type bank is identified and thus \( p(F_{z_l}) = \mathbb{E}_{a^e}[F_{z_l}(X)|z_l] \) by the zero-profit condition. Given this, consider the following deviation. An investor offers to buy security \( F' \) with cashflows \( F'(X) = X - F_{z_h}(X) \) at price \( p(F') = \mathbb{E}_{a^e}[F'(X)|z_l] - \epsilon, \epsilon > 0 \), where \( F_{z_h} \) is the security issued by \( z_h \)-type bank in the separating equilibrium. For \( \epsilon \) small enough, this offer attracts the bank with the bad loan, that now benefits from issuing a claim to all of its cashflows by issuing: \( F_{z_h} \) at price \( p(F_{z_h}) > \mathbb{E}_{a^e}[F_{z_h}(X)|z_l] \) to extract rents from the bank with the good loan, and further exploits remaining gains from trade by issuing \( F' \) at \( p(F') \). Since \( \epsilon > 0 \), the investor makes profits. Lemma 1 implies that there is pooling in the market for the securities issued by the \( z_h \)-type bank. The following proposition characterizes the security design in secondary markets.

**Proposition 1. [Security Design]** Under the No Transparency Assumption, in any equilibrium in secondary markets,

1. \( z_h \)-type bank issues one security, given by standard debt \( F_D(X) \equiv \min\{d, X\} \), where debt level \( d \) is chosen to maximize the value of the \( z_h \)-type bank in \( t = 1 \):

\[
d(a^e, a) = \arg \max_d \theta \cdot \mathbb{E}_{a^e,u}[\min\{d, X\}] - \mathbb{E}_a[\min\{d, X\}|z_h]
\]

2. \( z_l \)-type bank issues two securities: 1) standard debt \( F_D \), and 2) junior tranche \( F_J \) where \( F_J(X) \equiv \max\{X - d, 0\} \) are the remaining cashflows.
3. The market price for these securities:

\[
p(F_D) = \rho_h(a^e) \mathbb{E}_{a^e} [\min\{d, X\}|z_h] + (1 - \rho_h(a^e)) \mathbb{E}_{a^e} [\min\{d, X\}|z_l]
\]

\[
p(F_J) = \mathbb{E}_{a^e} [\min\{0, X - d\}|z_l]
\]

Four important results are presented in Proposition 1. First, standard debt is always sold in secondary markets. Second, debt levels are chosen to maximize the value of the bank with the good loan. Third, the bank with the bad loan tranches its cashflows into senior (standard debt) and junior (remaining cashflows) tranches that are sold separately in secondary markets, while the bank with the good loan only issues the senior tranche and retains its junior tranche. Finally, prices in secondary markets are such that the bank with the bad loan is subsidized by the bank with the good loan in the market for the senior tranches and it receives a fair value for its junior tranche.

**Optimality of Standard Debt.** Under the No Transparency assumption, the bank with the good loan faces a lemons problem as the one described in Akerlof (1970) when it participates in secondary markets, since the bank with the bad loan mimics its issuance. For any given security, the lemon’s discount faced by the bank with the good loan is given by the difference between its private valuation and the market valuation. Standard debt is the optimal security design because it allows the bank with the good loan to raise funds at the minimum retention cost by minimizing the region where disagreement about the likelihood of cashflows might arise. Thus, standard debt maximizes the gains from trade by minimizing the lemon’s discount since it is the design that is least informationally sensitive in the set of feasible securities. In contrast to papers on security design that obtain a separating equilibrium, the reason why high types choose to retain in this framework is not to signal underlying quality, but because the lemon’s discount is prohibitively high in the market for the junior tranche. The No Transparency assumption makes signaling through retention not credible to the market, and thus there is pooling in the market where the bank with the good loan issues. As a result, the \(z_h\)-type bank implicitly subsidizes the \(z_l\)-type in the market for standard debt.

**Tranching.** The bank with the bad loan tranches underlying cashflows into a senior tranche –i.e. standard debt– and a junior tranche –i.e. remaining cashflows,– and sells both securities in the market. It does so to receive an implicit subsidy in the market for the senior tranche and rip remaining gains from
$\theta = 1.03$, $\pi_H = 0.5$, the distribution of $X$ is given by a truncated normal in $[0, 2]$ with $E_H[X] = 1.2$, $E_L[X] = 0.7$, $V_G[X] = V_B[X] = 0.2$ respectively for good and bad projects.

trade by issuing its junior tranche simultaneously. This result strongly relies on the No Transparency assumption, since the bank with the bad loan can issue its junior tranche without being punished in the market for standard debt for doing so.

**Optimal Debt Levels.** Debt levels are chosen to maximize the value of the bank with the good loan in $t = 1$. Figure 2 plots (a) the payoff of the good bank in $t = 1$ as a function of different debt levels issued in secondary markets, and (b) optimal debt levels, both as a function of different equilibrium levels of information acquisition. Simulations are done to ease the exposition of results since qualitative results do not depend on specific functional forms not parameters (specified in bottom of each Figure).

In the Appendix, I show that highlighted properties hold for general distributions and parameters. As we can see from Figure 2, optimal debt levels are non-monotonic in information precision. For a given funding need $\theta$, debt levels are maximized when adverse selection is low. This occurs when information precision is low, and thus private information is not too valuable (see $a = 0.5$ case), and when information precision is high, and thus the quality of initial loan screening is sufficiently high to make private information not valuable (see $a = 1$ case). The following Lemma characterizes optimal debt levels for given equilibrium levels of information acquisition.
Lemma 2. Let \( a^* \) be the equilibrium level of information acquisition. Then, in any equilibrium in secondary markets, for \( \theta \rho (a^*) - \pi_h (a^*) < 0 \) optimal debt levels \( d (a^*) \) are given by the solution to:

\[
\theta \rho (a^*) - \pi_h (a^*) \left[ G_L (d) - G_H (d) \right] + (\theta - 1) \left[ 1 - G_L (d) \right] = 0
\]

(13)

Otherwise, the both \( z \)-type banks issue equity; that is, \( F_D = X \).

Debt levels are continuous, differentiable, and convex in the equilibrium level of information acquisition, \( a^* \), and increasing in funding needs, \( \theta \). The bank with the good loan chooses to retain some of its cashflows when \( \theta \rho (a^*) - \pi_h (a^*) < 0 \). Note that \( \rho (a^*) \) is the probability the market assigns to loan cashflows being high quality, while \( \pi_h (a^*) \) is the probability the \( z_h \)-type assigns to this event. We know that \( \rho (a^*) \leq \pi (a^*) \), with strict inequality when \( a^* \in (\frac{1}{2}, 1) \).\(^{24}\) When funding needs are high enough to compensate for the low probability the market assigns to high cashflows, \( z_h \)-type bank issues equity. Otherwise, it optimally chooses to retain cashflows (i.e. its junior tranche).

**Existence of Equilibrium.** I have shown that in any equilibrium with information acquisition, the bank with the good loan issues standard debt in secondary markets at average valuations, and the bank with the bad loan issues both standard debt at average valuations and its remaining cashflows at low valuations, where optimal debt levels are given by Lemma 2. Given this, I show that an equilibrium in secondary markets always exists. For example, for \( \mu (F) = 0 \) for all \( F \in \Delta \neq F_D \), there are no profitable deviations for the bank in secondary markets or in \( t = 0 \). By construction, there are no profitable deviations to investors. An equilibrium can also be supported with less stringent off-equilibrium beliefs.

### 3.2 Information Acquisition

The previous subsection characterized secondary market equilibrium outcomes for a given level of information acquisition \( a \) and market beliefs \( a^e \). Now, I proceed to find the optimal level of information acquisition and the determination of market beliefs, given secondary market equilibrium outcomes. At \( t = 0 \), the bank chooses how much information to acquire to maximize \( V_0 \) given by (3). The following proposition characterizes optimal levels of investment in information, and completes the characterization of equilibrium allocations.

\(^{24}\)Since \( \rho (a) = \mathbb{P} (q = G | s_0 = G) \) while \( \pi (a) = \mathbb{P} (q = G | s_0 = G, s_1 = G) \).
Proposition 2. In any equilibrium without commitment and with information acquisition:

1. Optimal investment in information, $a^*$, is given by the solution to:

\[ \rho_h(a) \pi'_h(a) \{ \mathbb{E}_H[\max\{X - d(a), 0\}] - \mathbb{E}_L[\max\{X - d(a), 0\}] \} + \rho'_h(a) \{ \mathbb{E}[\max\{X - d(a), 0\}|z_h] - \theta \mathbb{E}[\max\{X - d(a), 0\}|z_l] \} = C'(a) \] (14)

where $d(a)$ is the implicit function defined by (13).

2. Optimal debt level is given by $d^* = d(a^*)$.

Since the bank’s information acquisition choice is a hidden-action, by choosing more or less information, the bank cannot directly affect investor’s beliefs. The bank has two motives to acquire information: (i) to improve the quality of the tranches that it expects to retain, and (ii) to affect the probability of being a bank with a good loan, $z_h$-type, in secondary markets.

Retention of Cashflows. Retention of cashflows improves incentives for information acquisition, since by investing in information the bank can increase the quality of the tranches that it expects to retain. This motive for information acquisition is well understood, and is the rationale behind proposed regulation for securitizers in the U.S. and Europe. Retention levels, however, are determined ex-post in this environment, and depend only on the gains from trade, measured by $\theta$, and the level of adverse selection in secondary markets, given by the level of asymmetric information between the bank and the market.

Secondary Market Payoffs. For a given retention level, the differential payoff between $z_h$ and $z_l$ types in secondary markets also affects incentives for information acquisition. The higher the benefits associated with being a bank with a good loan ex-post –i.e. higher relative payoff to $z_h$-type bank,– the higher the incentives to acquire information to screen loans ex-ante. Note that the $z_h$-type bank is not fully compensated in secondary markets: it implicitly subsidizes the $z_l$-type bank in the market for debt, and it loses access to the market for its junior tranches where the lemon’s discount is prohibitively high. Even though this motive is always positive, it is non-monotonic in retention levels. An important result is therefore that transfers across different bank types in secondary markets do affect ex-ante efficiency by distorting incentives for information acquisition.
The distribution of $X$ is given by a truncated normal in $[0, 2]$ with $E_H[X] = 1.2, E_L[X] = 0.7, V_G[X] = V_B[X] = 0.2$ respectively for good and bad projects, $\pi_H = 0.5$, and information costs are given by $C(a) = -\chi (0.5 - a)^2 / (1 - a)$. Panels (a) and (b) are computed for $\chi = 0.1$, and (c) and (d) for $\theta = 1.03$.

**The Value of Adverse Selection.** Both of these motives are positive only when the bank expects to retain cashflows in secondary markets, which only occurs when adverse selection is sufficiently high. To see this, note that when adverse selection is secondary markets is not severe, the bank with the good loan chooses ex-post to issue a full claim to its cashflows. In this scenario, there is no retention, and therefore the bank has no incentives to acquire information. When $a^* = 0.5$, there is no screening in equilibrium, and thus the bank prefers to store its endowment. Therefore, with lack of commitment, the presence of adverse selection is essential to sustain an equilibrium with information acquisition, since it implicitly makes the bank with the good loan *commit* to retain its junior tranche.

### 3.3 Discussion

I have fully characterized equilibrium outcomes in an economy where loan-backed securities are designed and priced in secondary markets, after loan issuance. The environment is stylized, but rich enough to generate several predictions and new insights. Figure 3 shows optimal information acquisition...
and debt levels as a function of gains from trade, $\theta$, and of costs of information acquisition, $\chi$, where $C(a) = -\chi (0.5 - a)^2 / (1 - a)$. As gains from trade in secondary markets increase, the bank optimally chooses to increase its issuance of ABS in secondary markets. As a result, information acquisition falls and the quality of the issued loan is worsened. This prediction is consistent with what was observed in the decade leading to the crisis: where a rapid increase in securitization was accompanied by a decrease in the quality of issued loans.\textsuperscript{25}

I now address the two main questions asked at the beginning of this section. First, how does information acquisition affect the design of securities sold in secondary markets and the levels of ABS issuance in these markets? Standard debt is the optimal design for all levels of information acquisition. Debt levels, however, are shown to be non-monotonic on the precision of acquired information. That is, improving initial loan screening does not always increase liquidity and trade secondary markets (See Figure 2). This result relies on the dual effect of information acquisition, and predicts that trade is maximized for low and high levels of information precision. This result suggests that secondary markets for loans for which the bank acquires too little or too much information in the issuance stage should be more liquid.

Second, how does the design of securities sold in secondary markets affect incentives of the bank to acquire information and issue high quality loans in the first place? There are two aspects of secondary markets that affect the bank’s decision to acquire information. First, to have a relevant level of information acquisition the bank has to retain some of its cashflows –or expect to retain,– in secondary markets. In the absence of commitment, this only occurs when adverse selection in secondary markets is severe enough to have the bank with the good loan not off-loading its entire loan. Consistent with this, larger expected retention levels generate higher levels of information acquisition. The second aspect is related to the payoff received in the market for the securities sold: standard debt and junior tranche. Ex-ante, by acquiring information, the bank can affect the likelihood of showing up in secondary markets with a good loan. Thus, the differential payoff between the bank with the good loan relative to the bank with the bad loan in secondary market matters. As this relative payoff increases, incentives for information acquisition improve; this relative payoff, however, is non-monotonic in retention levels. This force, however, tends to be dominated by the incentives to acquire information as retention levels increase.

\textsuperscript{25}Jaffee et al. (2009), Dell’Ariccia, Igan and Laeven (2008), Mian and Sufi (2009), Bernd and Gupta (2008), provide empirical evidence of this fact.
In the previous section, I have fully characterized equilibrium allocations in an economy where ABS are designed after loan issuance. To highlight the inefficiencies that arise with lack of commitment, I now characterize the optimal mechanism that is obtained when the bank and the market can commit at $t = 0$ to the design and price of securities to be issued in secondary markets. This case is therefore useful to understand which securities and which levels of information acquisition a regulator would want to implement to increase ex-ante efficiency in markets for ABS. The results from this section motivate the policy interventions proposed in Section 5.

As in the case without commitment, I model the market as a menu $\{F, p(F)\}_{F \in \Delta}$ that satisfies the Zero Profit and No Deals conditions, now imposed at $t = 0$. By the Revelation Principle, we know that for any Bayesian-Nash equilibrium there exists a direct mechanism that is payoff-equivalent and where truthful revelation is an equilibrium. Therefore, I focus on direct revelation mechanisms that stipulate a transfer and a security to be issued as a function of the reported type of the bank, $\hat{z}$; that is, the market offers the bank a menu $(p(\hat{z}), F(\hat{z})) : Z \rightarrow \mathbb{R}^+ \times \Delta$. Let $\{p_l, F_l\}$ and $\{p_h, F_h\}$ denote the payments made to and the security assigned to the bank that reports type $z_l$ and $z_h$ respectively.

**Definition 3.** An equilibrium with commitment is given by $\{a^*, p_l, p_h, F_l, F_h\} \in [\frac{1}{2}, 1] \times \mathbb{R}_+^2 \times \Delta^2$ chosen to maximize the value of the bank in $t = 0$:

$$
\rho_h(a^*) \left[ \theta p_h + \mathbb{E}_{a^*}[X - F_h(X)|z_h] \right] + (1 - \rho_h(a^*)) \left[ \theta p_l + \mathbb{E}_{a^*}[X - F_l|z_l] \right] - C(a^*)
$$

subject to:

1. **The incentive compatibility constraints:**

   $$
   \theta p_l - \mathbb{E}_{a^*}[F_l(X)|z_l] \geq \theta p_h - \mathbb{E}_{a^*}[F_h(X)|z_l]
   \quad \theta p_h - \mathbb{E}_{a^*}[F_h(X)|z_h] \geq \theta p_l - \mathbb{E}_{a^*}[F_l(X)|z_h]
   \quad \text{(16)}
   $$

2. **The ex-post participation constraints:**

   $$
   \theta p_h - \mathbb{E}_{a^*}[F_h(X)|z_h] \geq 0
   \quad \theta p_l - \mathbb{E}_{a^*}[F_l(X)|z_l] \geq 0
   \quad \text{(17)}
   $$

3. **Zero-Profit Condition:**

   $$
   \rho_h(a^*) \left[ \mathbb{E}_{a^*}[F_h(X)|z_h] - p_h \right] + (1 - \rho_h(a^*)) \left[ \mathbb{E}_{a^*}[F_l(X)|z_l] - p_l \right] = 0
   \quad \text{(18)}
   $$
4. The incentive compatibility constraint for information acquisition:

\[ a^* = \arg \max_{a \in [\frac{1}{2}, 1]} \rho_h(a) \left[ \theta p_h + \mathbb{E}_a [X - F_h(X) | z_h] \right] + (1 - \rho_h(a)) \left[ \theta p_l + \mathbb{E}_a [X - F_l(X) | z_l] \right] - C(a) \]  

This problem is similar to the one presented in Biais and Mariotti (2004). They study optimal mechanism design in the presence of adverse selection, where an issuer with private information about asset quality has to issue a security to uninformed competitive liquidity providers. The main difference between their framework and mine is that in their setup, the quality of underlying assets and of the private information held by the issuer are exogenously determined, while in this problem both elements are dependent on information acquisition, which is a bank’s hidden action. Therefore, the problem internalizes the effect that different securities have on incentives to acquire information, and the impact that information has on loan screening and on issuance levels in secondary markets.

The No Deals condition is no longer imposed. Since the menu is accepted by the bank at \( t = 0 \), when there is no asymmetric information, there is no need to impose an extra constraint, as in the ex-post menu design problem. Finally, I impose ex-post participation constraints for the bank. By doing this, I am implicitly assuming that even though the bank can commit to the design of securities, it cannot commit to issue a security if doing so generates a negative payoff. In other words, the bank always has the option to not participate in secondary markets. The rest of the constraints are standard.

Lemma 3 incorporates binding and slack constraints to the optimal mechanism design problem. I show in the Appendix that without loss of generality we can focus on mechanisms where the incentive compatibility for the bank with the bad loan binds in equilibrium. Given this, the participation constraint of the bad types is slack, and the incentive compatibility for the good types can be replaced by (20). Finally, using the first-order approach, the incentive compatibility for implementable investment in information levels (19) can be replaced by its first-order condition. By plugging the binding incentive compatibility constraint for the bad type into the obtained first-order condition, constraint (22) is obtained.

**Lemma 3.** Equilibrium allocations with commitment, \( \{a^*, p_l, p_h, F_l, F_h\} \) solve the following problem:

\[ \max_{p_l, p_h, F_l, F_h} \rho_h(a^*) \left[ \theta p_h + \mathbb{E}_{a^*} [X - F_h(X) | z_h] \right] + (1 - \rho_h(a^*)) \left[ \theta p_l + \mathbb{E}_{a^*} [X - F_l(X) | z_l] \right] - C(a^*) \]
subject to:

\[ \theta p_h \geq \mathbb{E}_{a^*} [F_h(X)|z_h] \]  

(20)

\[ \mathbb{E}_{a^*} [F_l(X) - F_h(X)|z_h] \geq \mathbb{E}_{a^*} [F_l(X) - F_h(X)|z_l] \]  

(21)

\[ \rho'(a^*) \left( \mathbb{E}_H [X - F_h(X)] - \mathbb{E}_L [X - F_h(X)] \right) - C'(a^*) = 0 \]  

(22)

where transfers \( p_l, p_h \) are given by the binding incentive compatibility constraint of the \( z_l \)-type (17a) and the Zero Profit condition (18).

The following results follow from Lemma 3. First, the incentive compatibility of the \( z_l \)-type bank binds in equilibrium because transfers from the bank with a bad loan to the one with the good loan are always desired. These transfers relax the \( z_h \)-type bank participation constraint (20) and reduce the retention costs associated with an implementable level of information acquisition. That is, they compensate the bank with the good loan—as much as possible. Second, to satisfy the \( z_h \)-type incentive compatibility constraint, the \( z_l \)-type bank has to issue a claim to at least as many cashflows as the \( z_h \)-type bank; that is, the bank with the good loan retains at least as many cashflows as the bank with the bad loan, given by constraint (21). Finally, to provide incentives for information acquisition, it is only necessary to have the bank with the good loan retaining a fraction of its underlying cashflows; that is, retention of the bank with the bad loan gives no incentives for information acquisition. This last result strongly depends on the symmetry of signals, that implies that the quality of the bad loan is independent of information acquisition. –signals \( s_0 = H \) and \( s_1 = L \) cancel each other. I address this point after the presentation of the main results in Proposition 3.

Using the results from Lemma 3, we know that transfers are given by the binding incentive compatibility constraint of the bad type, and by the zero profit condition. Combining these two constraints, we get that transfers are given by:

\[ p_h = \{ \rho_h(a) \mathbb{E} [F_h|z_h] + (1 - \rho_h(a)) \mathbb{E} [F_l|z_l] \} - (1 - \rho_h(a)) \frac{1}{\theta} \left[ \mathbb{E} [F_l|z_l] - \mathbb{E} [F_h|z_l] \right] \]  

(23)

\[ p_l = \{ \rho_h(a) \mathbb{E} [F_h|z_h] + (1 - \rho_h(a)) \mathbb{E} [F_l|z_l] \} + \rho_h(a) \frac{1}{\theta} \left[ \mathbb{E} [F_l|z_l] - \mathbb{E} [F_h|z_l] \right] \]  

(24)

and therefore securities are chosen to maximize \( V_0 \) subject to (20), (21), (22), (23) and (24). The
following proposition characterizes the optimal security design in the presence of commitment.

**Proposition 3.** In the equilibrium with commitment,

1. $z_h$-type bank issues standard debt with debt level $d$; that is, $F_h(X) = \min\{d, X\}$, and

2. $z_l$-type bank issues equity; that is, $F_l(X) = X$.

The proposition states that the bank with the good loan issues standard debt, and thus retains some of its cashflows, while the bank with the bad loan issues a claim to all of its cashflows. This is because there are only gains, and no costs, from increasing the cashflows of security $F_l$. Doing this increases the value of the bank, and relaxes the remaining constraints. This, however, is not the case for the security issued by the bank with the good loan, $F_h$. There are costs associated with increasing cashflows issued by the good type: implementable information acquisition levels decrease, and its participation and incentive compatibility constraints get tighter. Therefore, the bank with the good loan might retain some of its cashflows.

Standard debt is optimal because i) given a level of information acquisition $a$, standard debt minimizes the required retention necessary to implement it, and this is good because retention of cashflows is costly – forgo gains from trade; – and ii) it relaxes the participation and incentive compatibility constraints of the bank with the good loan. As in the no commitment case, standard debt allows the bank to raise funds by loading on payments for which there is less disagreement, and thus less adverse selection in secondary markets. In addition, when securities are designed ex-ante they incorporate the impact on information acquisition, and thus standard debt is also preferable because it exposes the bank to the most informationally sensitive cashflows, improving incentives for information acquisition.

In this economy, demanding the same retention levels for all type of issuers is inefficient, since it reduces gains from trade without improving incentives. In particular, in the optimal mechanism, no retention is required for the bank with the bad loan, the $z_l$-type bank. Only the retention of the $z_h$-type bank is necessary since information acquisition only affects the expected quality of the loan held by the bank with the good loan. In Section 6, I extend the model to admit for the precision of the second signal to differ from that of the first one, and find that in the optimal mechanism retention of the bank with the bad loan may be desired, but that it is always lower than the one required from the bank with the good loan. It is never optimal to impose the same retention levels to all ABS issuers.
It remains to show how debt levels are determined. Let \( a(d) \) be the implicit function generated by the incentive compatibility of investment in information (22), once we take into account that \( z_h \)-type bank issues standard debt. Function \( a(d) \) is continuous, differentiable, and decreasing in \( d \) due to the MLRP. The following Proposition concludes the characterization of the equilibrium with commitment.

**Proposition 4.** In any equilibrium with commitment,

1. When the participation constraint of the \( z_h \)-type bank (20) does not bind in equilibrium, optimal debt levels \( d^* \) are given by:

\[
\theta \frac{\partial}{\partial a} [\rho_h(a(d)) p_h + (1 - \rho_h(a(d))) p_l] a'(d) + (\theta - 1) \rho_h(a) \int_d^\infty f(X|z_h) dX = 0
\]

Marginal Cost of ↑ \( d \)

Marginal Gain from ↑ \( d \)

When the participation constraint binds in equilibrium, optimal debt levels are given by the binding participation constraint:

\[
\theta p_h - \mathbb{E}_{a(d)} \left[ \min \{d, X\} | z_h \right] = 0
\]

2. Optimal investment in information is given by \( a^* = a(d^*) \).

By committing to lower debt levels ex-ante, the bank can commit to a certain level of information acquisition, affecting market beliefs. In particular, lower debt levels imply higher market beliefs, which are translated into higher ex-post transfers. This is the first term of equation (25), and it reflects the costs associated with increasing the debt level \( d \) marginally. The interpretation of the second term is straightforward: gains from trade are increased by increasing debt level \( d \). If the participation constraint of \( z_h \)-type is not binding in equilibrium, debt levels are chosen to optimally trade-off the gains from trade with the gains from information acquisition. If the participation constraint is violated for the solution given by (25), however, optimal debt levels are given by the binding participation constraint and retention occurs due to the presence of adverse selection. In this scenario, debt levels required to make the bank with the good loan participate are lower –and thus retention levels higher– than the one that implements the desired level of investment in information and thus first-order condition (25) is positive at \( \{a^*, d^*\} \).
Figure 4: Comparative Statics

The distribution of $X$ is given by a truncated normal in $[0, 2]$ with $E_H[X] = 1.2$, $E_L[X] = 0.7$, $V_G[X] = V_B[X] = 0.2$ respectively for good and bad projects, $\pi_H = 0.5$, and information costs are given by $C(a) = -\chi (0.5 - a)^2 / (1 - a)$ for $\chi = 0.1$

The presence of severe adverse selection in secondary markets alleviates the moral hazard problem. When the lemon’s discount faced by the bank with the good loan in secondary markets is large, debt levels are lower than the ones that implement the desired level of information acquisition. This suggests that imposing retention levels for the purpose of incentives is only necessary for ABS classes with liquid secondary markets—and thus high issuance levels. Otherwise, the bank naturally chooses to retain a large fraction of its cashflows. Which force dominates, and therefore determines retention levels, will depend on fundamentals that determine how important the provision of incentives vs. the adverse selection problem in secondary markets is for a given asset class.

4.1 Discussion

There are two key differences between the allocations obtained in the optimal mechanism and those found in Section 3, where securities were designed and priced after loan issuance as in markets for ABS. First, in the optimal mechanism, the design of securities internalizes its effect on the equilibrium level of
information acquisition. Although standard debt continues to be the optimal design, gains from trade may now be sacrificed to implement more information acquisition and better loan screening. Second, because in the optimal mechanism the market Zero Profit condition holds in expectation, there is room to exploit type-contingent transfers. In particular, I have shown that it is optimal to transfer all surplus to the bank with the good loan subject incentive compatibility constraints. These transfers improve the bank’s incentives for information acquisition for any given retention level, since they compensate the bank with the good loan for its sold tranches.

Figure 4 plots equilibrium debt levels and information acquisition for the commitment (optimal mechanism) and the no commitment (ABS markets) cases, as a function of gains from trade $\theta$. The bottom panel plots the percentage gain in ex-ante welfare arising from commitment. When gains from trade are low, ABS markets have inefficiently low levels of trade, and as a consequence inefficiently high levels of information acquisition. In these cases, the optimal mechanism implements higher issuance in secondary markets (higher debt levels). As $\theta$ increases, the bank’s incentives to issue ABS ex-post becomes larger. For intermediate levels of gains from trade, the commitment and the no commitment allocations match, although welfare is still higher for the commitment case because transfers are optimally set.

Finally, and most interestingly, when gains from trade are large, the no commitment case implements too much issuance in ABS markets and, as a result, inefficiently low levels of information acquisition. In the extreme case where gains from trade are very large, the bank chooses ex-post to issue a full claim to its loans. In this scenario, there is no information acquisition, and thus lack of commitment generates a collapse in secondary market trading and loan issuance – the bank optimally chooses to store ex-ante. This is the region where gains from commitment are large. Therefore, implementing the optimal mechanism by forcing banks to commit to retain cashflows to provide incentives for information acquisition is desired in markets that exhibit high issuance of ABSs – that is, for asset classes with liquid secondary markets. Policy implications are discussed in the following section.

5 Policy Implications: Regulating Markets for ABS

In this section I show that a simple tax scheme can implement the optimal mechanism and therefore improve ex-ante efficiency in markets for ABS. The policy prescriptions presented in this section are

---

26 When $a^* = 0.5$, the value of banking is maximized with storage.
only necessary when there are no commitment tools available to the bank and to the market. The following Lemma characterizes the policy intervention.

**Lemma 4.** Transfers \( \{T_l, T_h\} \) conditional on market participation and debt levels are sufficient to implement the optimal mechanism. In particular,

1. The bank that issues standard debt with debt level \( d \) receives transfer:

\[
T_h = T + \Gamma_h(d) + \gamma \times d
\] (27)

2. The bank that issues the junior tranche receives transfers:

\[
T_l = \Gamma_l(d)
\] (28)

Remember that in the optimal mechanism all available surplus is transfered ex-post to the bank with the good loan subject to incentive compatibility constraints. A policy that taxes the participation in the market for junior tranches, \( \Gamma_l \), and subsidizes the issuance of senior tranches, \( \Gamma_h \), is able to attain this. Optimal debt levels can be implemented by imposing a marginal tax for units of debt issued, \( \gamma \in [0, 1] \), returned as a lump sum transfer \( T \). The following proposition characterizes optimal regulation.

**Proposition 5.** An optimal policy is given by quadruple \( \{T, \Gamma_l, \Gamma_h, \gamma\} \in \mathbb{R}^3 \times [0, 1] \) given by:

1. Optimal Transfers:

\[
\Gamma^*_h = (1 - \rho_h(a)) \left( \frac{\theta - 1}{\theta} \right) \mathbb{E} \left[ \max \{0, X - d\} \mid z_l \right]
\] (29)

\[
\Gamma^*_l = -\rho_h(a) \left( \frac{\theta - 1}{\theta} \right) \mathbb{E} \left[ \max \{0, X - d\} \mid z_l \right]
\] (30)

2. Optimal Marginal Tax:

\[
\gamma^* = -\frac{1}{\theta} \left[ (\theta - 1) G_L (d^*_c) - [\theta \rho (a^*_c) - \pi_h (a^*_c)] [G_H (d^*_c) - G_L (d^*_c)] \right]
\] (31)

3. Budget Constraint:

\[
\rho_h(a^*_c)(T^* + \Gamma^*_h + \gamma^* d^*_c) + (1 - \rho_h(a^*_c))\Gamma^*_l = 0 \quad \Rightarrow \quad T^* = -\gamma^* d^*_c
\] (32)
where \( \{a^*_c, d^*_c\} \) are the outcomes of the optimal mechanism that are implemented with this policy.

Note that \( \Gamma_h \geq 0 \) and \( \Gamma_l \leq 0 \); that is, the optimal policy subsidizes retention and taxes the issuance of junior tranches. These transfers are found to make \( p^*_j = p^{nc}_j + \Gamma_j \) for \( j = 0, 1 \) where \( c \) and \( nc \) are used to denote the type contingent transfers received in secondary markets in the commitment and no commitment case respectively. By the Zero Profit condition of the optimal mechanism, these transfers are self-financed. As explained in the previous section, by imposing these transfers incentives for information acquisition are improved for all retention levels. Equation (31) is derived using the first-order conditions for debt levels in ABS markets, and \( \gamma \) is chosen so that the bank with the good loan naturally chooses to issue debt level \( d^*_c \). In particular, debt levels (issuance) should be taxed ex-post, \( \gamma^* < 0 \), when there is too much issuance in ABS markets relative to the optimal mechanism –i.e. \( d^*_c < d^{nc}_c \).

Regulators in the US and in Europe are in the process of implementing risk retention rules for all issuers of asset-backed securities. The rules demand all securitizers to retain at least 5 percent of a risk exposure to the cashflows underlying the issued securities, with some exceptions in place. This intervention is usually referred to as the “Skin in the Game” rule and is suggested in the Dodd-Frank Act in the US, and by the EU Capital Requirements Directive (CDR) in Europe. These rules intent to deal with the misalignment of interest between loan originators and investors, believed to have contributed to the financial crash of 2008. My model, by incorporating the frictions that lead to a conflict of interest as the one concerning regulators, is able to rationalize the demand of retention levels as a way to give incentives to improve loan screening standards. However, the model suggests that demanding the same retention levels to all issuers is, in general, inefficient. In particular, retention levels should be larger for issuers that claim to have good assets underlying their securities. Requesting the same retention for issuers that claim to have bad assets underlying their ABS reduces gains from trade without improving incentives. In addition, the model suggests that incentives are better provided when securitizers retain the first-lost piece (junior tranche) of the underlying assets, while the proposed regulation allows issuers to freely choose to which cashflows to be exposed to.\(^{27}\)

In the US, the Dodd-Frank Act establishes that all issuers of asset-backed securities should retain

\(^{27}\)Vertical slice, horizontal slice, originator’s share, random selection of assets, or even exposure to assets that have the same underlying characteristics as the one backing the issued ABS.
a fraction of underlying cashflows. In Europe, however, the rule imposed by the EU CRD specifies that banks can only have an exposure to securitized assets for which the originator or sponsor has a 5 percent exposure. In other words, securitizers are free to issue securities without retaining any of the risk, but banks can only invest in asset-backed securities for which the originator retains some of the risk. It is a hard task for banks to monitor the risk exposure of the originator or sponsor. One concern is that while the bank can ensure that the sponsor retains 5 percent of the risk at the time of the transaction, it might be cumbersome to monitor that they do not sell or hedge this exposure in the future. This concern is related to the No Transparency assumption made in this paper, that suggest that implementing the “Skin in the Game” rule in Europe will only be possible if the banks can enforce retention levels from originators or sponsors.

In addition, the model suggests that there are gains from subsidizing the issuance of safer tranches by taxing the issuance of risker ones. This type of policy is relatively easy to implement, but it has not been discussed in policy circles. Once the notion of adverse selection in ABS markets is introduced, transfers across issuers of ABS with different quality assets also affects incentives for information acquisition for any given retention level. Thus, the model suggests that regulators should not only focus on retention levels for securitizers but also on the way the market compensates good vs. bad issuers.

Finally, regulation on disclosure requirements and originators due diligence is also being implemented. First, it is required that all information regarding the retention and risk exposure levels of originators/sponsors is made available to investors. Second, investors and potential investors need to have access to all material that is relevant to be able to assess the credit quality and performance of the assets underlying the issued securities, and all information that is necessary to perform stress-tests on the values of cashflows and collateral. It stands to reason that this type of regulation is beneficial if possible to fully implement. Giving easy access to all the information required to evaluate underlying cashflows would solve both the moral hazard and the adverse selection problem; retention of underlying cashflows would not be necessary. All policies that address the problem of asymmetric information between originators and investors are, in the environment described in this paper, welfare improving.
6 Extensions

I generalize the model in two main directions, and show that qualitative results presented in this paper remain valid. First, I allow the bank to make multiple loans and to issue securities backed by the pool of these loans in secondary markets. This extension is motivated by the fact that most ABS are backed by pools of loans and not individual loans. Second, I generalize the signal structure by removing the symmetry assumption. By doing this, I can explicitly show how my model incorporates single friction models known in the literature, and I can also characterize policy implications as a function of the severity of the hidden-action problem in the issuance stage vs. adverse selection in secondary markets.

6.1 Pooling and Tranching: Multiple Loans

In this section, I extend the previous model to admit more than one loan issuance in primary markets. Let \( n \) be the number of loans made by the bank in \( t = 0 \); that is, \( w_b = n > 1 \). I continue to assume that at least \( n \) good projects can be identified after investing in information; that is, incentives to originate are not in place.\(^{28}\) The bank therefore issues \( n \) loans with \( S_0 = H \). Let \( Y = \frac{1}{n} \sum_{i=1}^{n} X_i \) denote the cashflows of the bank portfolio at \( t = 2 \) per loan issued at \( t = 0 \), where \( Y \sim f_Y(Y) \).\(^{29}\)

The bank issues a security backed by the entire pool of loans, \( Y \). That is, the bank is not allowed to choose which loans back the securities it issues in secondary markets and which ones it keeps on its portfolio, a behavior commonly referred to as “cherry picking”. The problem of security design with asymmetric information when the issuer is allowed to pick which assets back the issued securities is a complicated one. I abstract from this at the moment, and instead focus on understanding the effects of pooling and of having more than two types. At the end of this section, I discuss the complications that arise when “cherry picking” is allowed.

A bank arrives to secondary markets with private information about each loan in its portfolio \( \{z^1, z^2, ..., z^n\} \) with each \( z \in \{z_l, z_h\} \). To deal with this, I redefine a bank’s type in secondary markets by \( \zeta \in \{0, 1, ..., n\} \), where a bank’s type denotes the number of loans in its portfolio that received

\(^{28}\)Note that if the bank had funds \( w_b \) greater than the number of good projects identified, then the market would understand the probability of a bank issuing a bad loan, and would demand a discount in secondary markets. Incentives to originate worsen the adverse selection problem, but the mechanism discussed in this paper and in this section would still be in place. I abstract from analyzing the impact of incentives to originate in this paper.

\(^{29}\)To make this section comparable to the main section, I analyze the payoff to the bank per issued loan.
\( s_1 = G \), and therefore \( n - \zeta \) is the number of loans that received \( s_1 = H \). The distribution of types is now given by a binomial distribution with probability of success given by \( \rho_{h(a)} \); that is \( \zeta \sim B(n, \rho_{h(a)}) \), and thus:

\[
\rho_k(a) \equiv \mathbb{P}_a(\zeta = k|s_0 = H) = \binom{n}{k} \rho_{h(a)}^k (1 - \rho_{h(a)})^{n-k} \tag{33}
\]

Given the distribution of types, and the fact that the bank issues securities backed by the entire pool of loans, the value of the bank in \( t = 0 \) is given by:

\[
V_0(a, \{p_\zeta, F_\zeta\}) = \sum_{\zeta=0}^{n} \rho_\zeta(a) \left[ \theta p_\zeta + \mathbb{E}_a[Y - F_\zeta(Y)|\zeta] \right] - C(a) \tag{34}
\]

where \( F_\zeta(Y) \) is the sum of the cashflows of all securities issued by the \( \zeta \)-type bank, and \( p_\zeta \) is the sum of the prices received in each sale. The expectations operator \( \mathbb{E}_a[\cdot] \) is now used to refer to expectations over cashflows \( Y \). There are two main differences with the baseline model with one loan: first, the distribution of \( Y \) has less variance than that of \( X \), and thus there’s potentially less adverse selection in secondary markets; and second, there are more than two types.

The definition and construction of the equilibrium are as the ones described in the previous section for the commitment and the no commitment case respectively. In the remainder of this section, I use results obtained for the one loan case and extend them to admit multiple types. Proofs are presented in the Appendix.

**Markets for ABS: The No Commitment Case.** Given the No Transparency Assumption 1, type \( \zeta = k < n \) matches the issuance of higher types \( \zeta = k+1, \ldots, n \) in secondary markets. Let \( Y_k = Y - (F_{k+1} + F_{k+2} + \ldots + F_n) \) denote the remaining cashflows type \( \zeta = k \) has after mimicking the issuance of higher types, where \( Y_n = Y \). Given the Zero Profit and the No Deals condition, security \( F_\zeta \) is given by the solution to:

\[
\max_{0 \leq F \leq Y_\zeta} \theta \mathbb{E}_{a^e, \mu}[F(Y)] + \mathbb{E}_a[Y_\zeta - F(Y)|\zeta] \tag{35}
\]

Market beliefs \( \mu(F) \) are given by a probability distribution over types \( \zeta \in \{0, 1, \ldots, n\} \) conditional on issuance \( F \). Using the just described strategies, market valuation for security \( F_\zeta \) for \( \zeta = k \) are given by:

\[
\mathbb{E}[F_k|\mu(F_k)] \equiv \sum_{\zeta=0}^{k} \left[ \frac{\rho_\zeta(a)}{G(k; a)} \mathbb{E}_{a^e}[F_\zeta(Y)|\zeta] \right] \tag{36}
\]
where \( G(k; a) = \mathbb{P}_a(\zeta \leq k) \) is the unconditional cdf for types, given information acquisition \( a \). The following proposition characterizes equilibrium in secondary markets for the case without commitment.

**Proposition 6.** In any equilibrium without commitment,

1. Type \( \zeta \in \{0, 1, ..., n\} \) mimics the issuance of types \( k > \zeta \), and issues standard debt backed by remaining cashflows \( Y_\zeta \). Debt levels \( d_\zeta \) are chosen to maximize the value of the \( \zeta \)-type bank in \( t = 1 \):

\[
\max_{d_\zeta} \theta \mathbb{E}_{a^*, \mu} \left[ \min\{d_\zeta, Y_\zeta\} \right] - \mathbb{E}_{a} \left[ \min\{d_\zeta, Y_\zeta\} | \zeta \right]
\]

(37)

2. Given optimal debt levels as a function of \( a \) and \( a^* \), equilibrium level of information acquisition \( a^* \) solves:

\[
\sum_{\zeta=0}^{n} \rho_\zeta (a) \left[ \theta p_\zeta + \mathbb{E}_a \left[ \max \{Y - d_\zeta, 0\} | \zeta \right] \right] + \sum_{\zeta=0}^{n} \rho_\zeta (a) \frac{\partial}{\partial a} \mathbb{E}_a \left[ \max \{Y - d_\zeta, 0\} | \zeta \right] - C' (a) = 0
\]

(38)

3. Zero Profits:

\[
p(F) = \mathbb{E}_{a^*, \mu} [F(Y)]
\]

(39)

The presence of multiple asset qualities in secondary markets rationalizes the high number of tranches issued for a given pool of loans. As in the one loan case, cashflows sold are decreasing in underlying quality; that is, \( d_n \leq d_{n-1} \leq ... \leq d_0 \). Note that the model does not predict that there are as many tranches as types, since types with an average portfolio quality are very likely to issue the junior tranche if adverse selection is not severe. The intuition behind tranching, however, is the same as the one described in the one loan baseline case. Comparative statics remain unchanged. I find that information acquisition is increasing in expected retention, and on the differential payoff higher types receive in secondary markets relative to lower types.

As the number of loans \( n \) in the pool increases, the volatility of cashflows \( Y \) decreases.\(^{30}\) If this reduction in volatility reduces the expected adverse selection in secondary markets, expected retention levels should be therefore lower for larger pools of loans. This suggests that issuing securities backed by large pools of loans decreases incentives for information acquisition by reducing the adverse selection.

\(^{30}\)Note that ex-ante, \( \mathbb{E}[Y] = \mathbb{E}[X] \) and that \( \mathbb{V}[Y] = \frac{1}{n} \mathbb{V}[X] \).
problem the bank expects to face when issuing an ABS. I continue this discussion when addressing the decision to pool loans.

Figure 5 plots the resulting issuance in an environments with multiple loans. In this scenario, the best type $\zeta = n$ issues the senior tranche $F_n(Y) = \min\{d_n, Y\}$. The second highest type, $\zeta = n - 1$, issues the senior tranche $F_n$ and the mezzanine tranche, $F_{n-1}(Y) = \min\{d_{n-1}, Y - F_n(Y)\}$. Type $\zeta = n - 2$ mimics the issuance of types $n$ and $n - 1$, and issues the second mezzanine tranche. All types $\zeta < n - 2$ issue a claim to all of their cashflows by selling senior and mezzanine tranches, and the reaming junior tranche. In what follows, allocations in environments with commitment are characterized.

The Optimal Mechanism: The Commitment Case. As in the one loan case, it can be shown that each type chooses to sell standard debt, with debt levels decreasing in the quality of underlying cashflows. The following proposition characterizes the solution to the commitment case.

**Proposition 7.** In any equilibrium with commitment,

1. For given debt levels, information acquisition solves:

$$\sum_{\zeta=0}^{n} \rho_{\zeta}(a) \frac{\partial}{\partial a} E_{\zeta} \left[ \max \{ Y - d_{\zeta}, 0 \} \right] + \sum_{\zeta=0}^{n} \rho_{\zeta}(a) \left\{ \theta p_{\zeta} + E_{\zeta} \left[ \max \{ Y - d_{\zeta}, 0 \} \right] \right\} - C'(a) = 0$$
2. If the participation constraint for type \( k \in \{0, 1, ... n\} \) does not bind in equilibrium, debt level \( d_k \) is given by the solution to:

\[
\theta \left( \sum_{\zeta=0}^{n} \rho_{\zeta}^2 \left( a \right) \frac{\partial p_{\zeta}}{\partial a} \frac{\partial a}{\partial d_k} \right) + \rho_k \left( a \right) \int_{d_k}^{\infty} f_Y \left( y | \zeta = k \right) dy = 0
\]  

if it \( \exists \), if not \( d_k = \infty \), where \( \frac{\partial a}{\partial d_k} \) is the implicit derivative given by (40). Otherwise, \( d_k \) is given by the binding participation constraint:

\[
\theta p_k + \mathbb{E}_a \left[ Y - \min \left\{ d_k, Y \right\} | k \right] = 0
\]  

3. Zero Profits:

\[
\sum_{\zeta=0}^{n} \rho_{\zeta} \left( a \right) \left[ \mathbb{E}_a \left[ F_{\zeta}(Y) | \zeta \right] - p_{\zeta} \right] = 0
\]  

4. Incentive Compatibility:

\[
\theta p_{\zeta} + \mathbb{E}_a \left[ Y - F_{\zeta}(Y) | \zeta - 1 \right] = \theta p_{\zeta-1} + \mathbb{E}_a \left[ Y - F_{\zeta-1}(Y) | \zeta - 1 \right] \quad \zeta = 1, ..., n
\]  

Transfers \( p_{\zeta} \) are given by the Zero Profits and the Incentive Compatibility conditions. Information acquisition is chosen to improve the quality of expected retention and to affect the likelihood of holding a given pool quality in secondary markets. Debt levels are chosen to trade-off the gains from information acquisition with the gains from trade in secondary markets, given by the first and second term of equation (41) respectively. The intuition behind these results is the same as the one obtained for the two types baseline case. The results obtained in the baseline case with one loan are robust when the bank issues securities backed by pools of loans. The problem was solved under the assumption that all securities issued are backed by the sum of individual loan’s cashflows, and that the bank cannot choose when to pool or not. In what follows, I discuss the implications of giving the bank the ability to choose whether to pool or not.

**The Decision to Pool Loans.** In this environment, it is not straightforward that pooling cashflows is an optimal decision. In the case with commitment, the benefits associated with pooling are understood and are given by the reduction in the volatility of underlying cashflows. However, while issued securities
are concave, retained tranches are convex in underlying cashflows and, therefore, reducing the volatility of cashflows increases the value of sold tranches, but reduces the value of retained ones. While the first effect increases gains from trade, the second one might decrease incentives for information acquisition. Pooling is desired ex-ante when the first force dominates.

In the case without commitment, the decision to pool is done after the arrival of private information, and thus its impact on information acquisition is not considered. In this scenario, however, cherry picking arises, since the bank is able to select the most desirable loans to pool. While a bank with an average pool of loans would definitely choose to pool to reduce the adverse selection in secondary markets, the same is not true for banks that had a good draw and hold good loans. While pooling, by reducing the variance of underlying cashflows, increases the value of the sold security, it also increases the private valuation of this security, and thus conditions need to be imposed to ensure that the value of the bank in \( t = 1 \) is concave in underlying cashflows for all \( \zeta \)-type banks. Without imposing these conditions, the problem is complicated since the decision to pool or not can also be used to screen types. DeMarzo (2005) finds that pooling is always optimal ex-post. The main difference is that in his scenario, there is a separating equilibrium, and thus private and market valuations are the same, therefore pooling is always beneficial. In my scenario, given the No Transparency assumption, the bank faces adverse selection ex-post and private and market valuations are not the same, and therefore the result does not follow. Studying equilibrium allocations in the presence of cherry picking is out of the scope of this paper, but it is an interesting problem that, to my knowledge, has not been studied in environments with pooling equilibria in secondary markets.

### 6.2 General Signal Structure

In this section, I remove the assumption that received signals are symmetric by allowing the bank to receive two signals with the following conditional distributions:

\[
P(s_0 = H | q = H) = P(s_0 = L | q = L) = a
\]

\[
P(s_1 = H | q = H) = P(s_1 = L | q = L) = \tau(a)
\]

where the only constraint is given by \( \tau'(a) \geq 0 \). Thus, the precision of the second signal can be independent of the initial level of investment in information (i.e. \( \tau'(a) = 0 \)), or increasing in it (i.e. \( \tau'(a) > 0 \)).
\( \tau'(a) > 0 \). This provides flexibility to the model where now the importance of the incentives problem in the loan issuance stage vs. the adverse selection in secondary markets can be calibrated.

In this scenario, since only the precision of the second signal has been changed, \( \rho(a) = P(q = H|s_0 = H) \) remains unchanged, and the following conditional probabilities need to be re-computed as follows:

\[
\rho_h(a) = P(z = z_h|s_0 = H) = \tau(a)a + (1 - \tau(a))(1 - a) \tag{47}
\]
\[
\pi_h(a) = P(q = H|z = z_h) = \frac{\tau(a)a}{\tau(a)a + (1 - \tau(a))(1 - a)} \tag{48}
\]
\[
\pi_l(a) = P(q = H|z = z_l) = \frac{(1 - \tau(a))a}{(1 - \tau(a))a + \tau(a)(1 - a)} \tag{49}
\]

Note, however, that most of the results presented in this paper were given as a function of this conditional probabilities, and do not in general depend on their actual form. In particular, it continues to be true that \( \rho'_h(a) > 0 \) and that \( \pi'_l(a) > 0 \), the main difference being that now it is possible to have \( \pi'_l(0) \neq 0 \); that is, by investing in information the bank can affect the return of the bad loan as well.

**Markets for ABS: The No Commitment Case.** All qualitative results presented in Section 3 remain unchanged. Determination of debt levels in secondary markets is given by equation (10) and the choice of information acquisition is given by equation (14). Thus, the effect of generalizing the signal structure only affects quantitative results, where now equilibrium debt and information acquisition levels will vary depending on \( \tau(a) \). Figure 6 shows how information acquisition and debt levels in equilibrium change as \( \tau(a) \) changes. In particular, I model \( \tau(a) = c + \tau \cdot (a - c) \) and study changes in both \( c \) and \( \tau \).

**The Optimal Mechanism: The Commitment Case.** The security design problem and thus determination of retention levels in the optimal mechanism changes slightly once the symmetry assumption is relaxed. To see this, note first that the choice of information acquisition continues to be given by the solution to (19), which is now given by:

\[
\phi_h(a^*) \left( \mathbb{E}_H[X - F_h] - \mathbb{E}_L[X - F_h] \right) + \phi_0(a^*) \left( \mathbb{E}_H[X - F_l] - \mathbb{E}_L[X - F_l] \right) = C'(a^*) \tag{50}
\]

where \( \phi_h(a) \equiv \rho_h'(a)(\pi_h(a) - \pi_l(a)) + \rho_h(a)\pi'_h(a) \) and \( \phi_0(a) \equiv (1 - \rho_h(a))\pi'_l(a) \). Note that \( \phi_h(a) \neq \phi_0(a) \) a.s. when \( \tau'(a) > 0 \). The following cases arise: (i) If \( \pi'_l(a) \leq 0 \), it not optimal for the bad type to
retain. If \( \pi_l'(a) > 0 \), and \( \phi_0(a) < \phi_h(a) \) it is optimal for the bad type to retain less than the good type. If \( \pi_l'(a) > 0 \), and \( \phi_0(a) > \phi_h(a) \) it is optimal for the bad type to retain as much as the good type, since (21) has to hold.

In Case (i), the precision of the second signal is highly dependent on information acquisition. Thus, a very precise second low signal reduces the expected quality of the cashflows of the bad loan, in which case retention worsens incentives. As in the baseline case, the bank with the bad loan does not retain in this scenario and qualitative results are unaffected. For Cases (ii) and (iii), some retention from the bank with the bad loan is desired since expected quality of retained tranches is increasing in “\( a \)” for all bank types. However, by the incentive compatibility constraints of the optimal mechanism, we know that retention can never be higher for the bank with the bad loan. Therefore, when a more general signal structure is allowed, retention levels are decreasing in the quality of underlying cashflows. Since debt continues to be the design that implements a given level of information acquisition at the lowest retention cost, securities issued by the banks with bad loans in cases (ii) and (iii) continue to be standard debt –even though case (iii) does not arise for reasonable parameter values. Optimal debt levels are chosen with the same rationale as in the baseline model (see equation (25)) where now there is one first-order condition for each debt level.

Figure 6 compares equilibrium allocations for markets for ABS and for the optimal mechanism as \( \tau(a) = c \) changes. That is, the precision of the second signal is a constant that does not depend on initial levels of information acquisition. As we can see, welfare gains from implementing the optimal mechanism are larger when the precision of the second signal is small or large, and thus intermediate levels of adverse selection in ABS markets naturally implement welfare levels close the ones obtained with the optimal mechanism. In addition, note that issuance in ABS markets is inefficiently high and thus information acquisition and loan screening inefficiently low when adverse selection in secondary markets is low –low \( c \). When adverse selection is not severe, and there are gains from trade, the bank with no commitment chooses ex-post to issue a large claim to its underlying cashflows, and thus equilibrium level of information acquisition is low. Conversely, when adverse selection is severe, ABS markets feature inefficiently low levels of trade and the problem is not one of incentives, but one where regulators should incentive issuance in ABS markets.

---

\[31\] This is the case when \( \tau'(a) > \frac{\tau(a)(1-\tau(a))}{a(1-a)} \).
The distribution of $X$ is given by a truncated normal in $[0, 2]$ with $E_H[X] = 1.2$, $E_L[X] = 0.7$, $V_G[X] = V_B[X] = 0.2$ respectively for good and bad projects, $\pi_H = 0.5$, and information costs are given by $C(a) = -\chi (0.5 - a)^2 / (1 - a)$ for $\chi = 0.1$

### 6.3 Other Extensions

In this section, I discuss how results presented in this model might change once other dimensions of markets for ABS are considered. Even though these extensions are not addressed formally in this paper, I believe they are promising questions to address in future research.

**Rating Agencies.** The role of rating agencies in this environment is straightforward, since it would overcome both the hidden-action and the adverse selection problem. Having the ability to send uninformed investor unbiased signals about loan quality would allow the bank to increase trade in secondary markets, and to be compensated from its investment in information (as long as signals are precise enough). Allocations in the presence of rating agencies might approach (or even attain), first-best allocations. Given the beneficial role that rating agencies have in this environment, it would be interesting to incorporate them by including the agency problems that arise in markets with rating shopping or rating inflation, as modeled by Bolton, Freixas, and Shapiro (2012).
Securitization with Recourse. Securitization with recourse gives investors the ability to seek payment against a loan to the originator of the loan. Securitization with recourse could then help overcome some of the information frictions present in markets for ABS, since the bank is exposed to the cashflows of the sold loans by the guarantees given to the investor of the ABS. There is, however, a cost of securitizing with recourse not captured in my model since the bank is not able to fully share risks with the market –the bank continues to be exposed to the cashflows of the issued ABS. The analysis of how different forms of credit enhancements could help overcome the frictions present in this model is necessary. More formally, it requires removing the assumption that cashflows of the issued ABS can only be backed by the cashflows of the underlying loans; that is, there is no limited liability on the bank. For a discussion on effects of securitization with recourse, see Benveniste and Berger (1987).

Investors Heterogeneity. Heterogeneity in investors preferences is used to rationalize the sophisticated types of tranching observed in practice. There is substantial evidence to suggest that this is the case, and that tranches are designed to tailor different type of investors. This is in addition to the results presented in this paper. By incorporating investors’ heterogeneity into this model a richer set of securities might be obtained, but the presented frictions should not be affected by this extension. For the role of investor’s heterogeneity see Boot and Thakor (1993), Gorton and Pennacci (1990), Pagano and Volpin (2009), Chemla and Hennessy (2011), and for a richer discussion on tranching Farhi and Tirole (2012).

Investor’s Ability to Acquire Information. I have assumed in this paper that investors are not able to invest in information about bank quality. An interesting extension would then be to allow investors to acquire information as well, and study the role the market has on disciplining the bank’s behavior. In this scenario, securities will be designed to provide incentives to investors to acquire information about bank quality, and by doing so, the informational frictions might be overcome. Using the predictions of Yang and Zeng (2013), where securities are designed to provide incentives to investors to acquire information in a production economy, we should expect securities in this scenario to differ from standard debt. In their paper, they find that a combination between debt and equity is desired. This suggests that issued ABS should then be more informationally sensitive than debt to enhance investor’s incentives for information acquisition and bank monitoring.
7 Conclusions

In this paper, I have proposed a parsimonious framework to study markets for asset-backed securities (ABS). The model incorporates some of the key features of these markets, and it exploits the tension between incentives to acquire information to screen loans and liquidity in markets where ABS are issued. Loan issuers acquire private information about borrower quality, and while this information is beneficial ex-ante when used to screen loans, it becomes detrimental ex-post as it hinders gains from trade in markets where ABS are designed and traded. I have highlighted two inefficiencies that arise in these markets. First, the design of securities does not internalize its impact on the issuer’s incentives to screen good quality loans. Second, markets for ABS distort the issuer’s incentives by implicitly subsidizing issuers with bad loans at the expense of those with good loans (lemon’s problem). In the optimal mechanism, these problems are addressed by committing to the design of securities ex-ante and by the appropriate design of transfers in secondary markets across banks with different loan quality.

I show that the optimal mechanism can be decentralized with simple tax scheme. In particular, subsidies to participation in the market for senior tranches, together with taxes for participation in the market for the junior tranches are beneficial since they improve incentives for information acquisition at no retention cost. This policy compensates banks with good loans for being mimicked by those with bad loans in secondary markets. Furthermore, policies that tax/subsidize debt levels can implement second-best levels of information acquisition. In particular, retention levels should be imposed when markets for ABS are sufficiently liquid.

The result of this paper shed light on the costs and benefits of policy proposals for securitization: the “Skin in the Game” rule that requires issuers of asset-backed securities to retain a fraction of the underlying assets. My model rationalizes this type of intervention as a means to give incentives to improve loan screening only in markets with liquid secondary markets. The model further suggests that banks that claim to have good quality loans underlying their ABS should retain more than those that claim to have bad quality loans. As a result, policies that demand the same retention levels of all issuers impose excessive costs by hindering trade in secondary markets.
8 Appendix

Markets for ABS: The No Commitment Case

Equilibrium in Secondary Markets and Security Design

**Lemma (Zero Profit Condition).** In any equilibrium, investors must earn zero expected profits in each market.

*Proof.* Assume not. Investor $j$ is making positive profits in market $\{F, p_j(F)\}$, in equilibrium. If this is the case, it has to be buying at a price lower than its valuation; that is, $p_j(F) < \mathbb{E}[F|\mu(F)]$. Since for profits to be made, the bank has to be issuing in this market, it must be true that $p_i(F) \leq p_j(F), \forall i \in I$. Let $\Pi > 0$ denote the investors aggregate profits in this market. Then, one investor must be making no more than $\Pi/I$. Consider the deviation of this investor to open market $\{p_j(F) - \epsilon, F\}, \epsilon > 0$. This market will attract the bank that was issuing in market $\{F, p_j(F)\}$, without affecting their decisions to participate in other markets. Since $\epsilon$ can be chosen to be arbitrarily low, this deviation yields the investor almost $\Pi$ profits, and so the investor has a profitable deviation. Then, we must have $\Pi \leq 0$ in each market. Because investors cannot incur a loss in any equilibrium (it can always earn zero by posting price zero), all investors in fact earn zero profits.

**Lemma (No Separation).** Under the No Transparency Assumption, separating equilibria do not exist.

*Proof.* Assume there is a separating equilibrium. Let $F_z$ be a security issued by the $z$-type bank in this equilibrium. Separation implies that $\mu(F_{zh}) = 1$ and $\mu(F_{zl}) = 0$. By the Zero-Profit Condition, the payoff to type $z_l$ is given by $(\theta - 1)\mathbb{E}[F_{zl}(X)|z_l]$, and investors make zero profits. Investor $j$ has a profitable deviation to offer to buy security $H$ with cashflows $H(X) = [F_{zl}(X) - F_{zh}(X)]^+$, at price $p(H) = \mathbb{E}[H|z_l] - \epsilon$ for $\epsilon > 0$. Note that by the incentive compatibility that is required for any separating equilibria: $\mathbb{P}(H(X) > 0) > 0$. This market will attract the $z_l$-type bank. To see this, note that the $z_l$-type’s optimal response is now to issue in market $\{F_{zh}, p(F_{zh})\}$, and sell the remaining cashflows $H = [F_{zl} - F_{zh}]^+$ at $p(H)$, since for $\epsilon$ small enough, this strategy generates a higher payoff: $\theta(\mathbb{E}[F_{zh}(X)|z_h] + \mathbb{E}[H(X)|z_l] - \epsilon) - \mathbb{E}[F_{zh}(X) + H(X)|z_l] > (\theta - 1)\mathbb{E}[F_{zl}(X)|z_l]$. Then, investor $j$ attracts the $z_l$-type bank and makes profits.

**Lemma.** In any equilibrium in secondary markets, the $z_h$-type bank issues one security, $F_{zh} \in \Delta$, at price $p(F_{zh}) = \mathbb{E}_{a^\epsilon, \mu}[F_{zh}(X)]$ where $\mu(F_{zh}) = \rho_h(a^\epsilon) < 1$.

*Proof.* Assume the $z_h$-type type bank is issuing $F_h^1 > 0$ and $F_h^2 > 0$ in equilibrium, where it must be that $F_h^1(X) + F_h^2(X) \leq X \forall X$, and investors make zero profits. Note that by the No Separation Lemma, the $z_h$-type bank is always mimicked by the $z_l$-type. Since I focus on pure strategy equilibria, $\mu(F_h^1) = \mu(F_h^2) = \rho_h(a^\epsilon)$. Consider the following deviation for an investor $j$. Post security $F_h = F_h^1 + F_h^2$ at $p(F_h) = \mathbb{E}_{a^\epsilon, \mu}[F_h(X)] - \epsilon$ where $c > \epsilon > 0$, with $\mu(F_h) = \rho_h(a^\epsilon)$. The $z_h$-type bank strictly prefers to issue $F_h$ since $c > \epsilon$. Note that this is a profitable deviation for investor $j$ for $\epsilon > 0$. By Lemma 8, $z_l$-type also deviates to issue $F_h$. Thus, when security $F_h$ is issued, the market infers that with probability $\rho_h(a^\epsilon)$ the bank is a $z_h$-type. By the Zero Profit Condition, $p(F_{zh}) = \mathbb{E}_{a^\epsilon, \mu}[F_{zh}(X)] = \rho_h(a^\epsilon)$ in equilibrium.
Lemma. Let $F_{zh} \in \Delta$ be the security issued by $z_h$-type bank in equilibrium. In any equilibrium in secondary markets, junior tranches $F_j(X) = X - F_{zh}(X)$ are sold by the $z_1$-type bank at lower valuations; that is, $p(F_j) = \mathbb{E}_{a^e} [X - F_{zh}(X)|z_h]$. This implies that the $z_1$-type bank is fully identified in the market for the junior tranches: $\mu(F_j) = 0$.

Proof. Let security $H$ have cashflows $H(X) = X - F_{zh}(X) > 0$ for some positive measure set of $X$. By No Separation, we know the $z_1$-type bank is also issuing $F_{zh}$. All types are free to sell their remaining cashflows given the No Transparency assumption. However, by revealed preference, the $z_h$-type bank prefers not to issue $X - F_{zh}(X)$ at average valuation $\mu(H) = \rho_h(a^e)$, and thus is not willing to issue if $\mu(H) < \rho_h(a^e)$. $z_1$-type bank, however, can realize gains from trade by issuing cashflows $H$ at any valuation, since in equilibrium this strategy generates at least payoff: $(\theta - 1)\mathbb{E}_a [H|z_h] > \mathbb{E}_a [H|z_h]$. By the zero-profit condition, junior tranches are priced at lower valuations: $p(F_j) = \mathbb{E}_{a^e} [F_j(X)|z_h]$. \hfill \square

For the following proofs, let

$$\Pi_{zh} (a, a^e, F) \equiv \theta \mathbb{E}_{a^e, \mu} [F(X)] - \mathbb{E}_a [X - F(X)|z_h]$$

denote the value of banking for the $z_h$-type bank that invested $a$ in information acquisition, given market beliefs $a^e$.

Lemma. Assume there exists $F^* \in \Delta$, s.t. $\Pi_{zh} (a, a^e, F^*) = \sup_{F \in \Delta} \Pi_{zh} (a, a^e, F)$. Then, in any equilibrium in secondary markets, for given information acquisition $a$ and market beliefs $a^e$, the $z_h$-type bank issues security $F_{zh} \in \arg \sup_{F \in \Delta} \Pi_{zh} (a, a^e, F)$ at price $p(F_{zh}) = \mathbb{E}_{a^e, \mu} [F_{zh}(X)]$.

Proof. $F^*$ is the optimal security for the $z_h$-type bank among all securities in compact set $\Delta$ priced with valuations $\mu(F) = \rho_h(a^e)$. Assume the $z_h$-type is issuing $F_{zh} \in \Delta$, and that $\Pi_{zh} (a, a^e, F_{zh}) < \Pi_{zh} (a, a^e, F^*)$. Consider the following deviation for an investor $j$: offer to buy security $F^*$ at price $p(F^*) = \mathbb{E}_{a^e, \mu} [F^*(X)] - \frac{\epsilon}{\theta}$, $\epsilon > 0$, where $\mu(F^*) = \rho_h(a^e)$. This attracts the $z_h$-type bank, since $\theta \mathbb{E}_{a^e, \mu} [F^*(X)] - \mathbb{E}_a [F^*(X)|z_h] - \epsilon > \theta \mathbb{E}_{a^e, \mu} [F_{zh}(X)] - \mathbb{E}_a [F_{zh}(X)|z_h]$ for $\epsilon$ small enough. By Lemma No Separation, $z_1$-type bank is also attracted to buy $F^*$, and therefore the market prices at average valuations: $\mu(F^*) = \rho_h(a^e)$. For $\epsilon > 0$, investor $j$ makes profits. Therefore, it must be that $F_{zh} \in \arg \sup_{F \in \Delta} \theta \mathbb{E}_{a^e, \mu} [F(X)] - \mathbb{E}_a [F(X)|z_h]$ and by the zero profit condition, $p(F_{zh}) = \mathbb{E}_{a^e, \mu} [F_{zh}(X)]$. \hfill \square

Lemma. In any equilibrium in secondary markets, standard debt is the optimal security issued by the $z_h$-type bank. In particular, $F^* \in \arg \sup_{F \in \Delta} \Pi_{zh} (a, a^e, F)$ exists, it is unique, and it is given by $F^*(X) = \min \{ d, X \}$ where

$$d(a, a^e) \in \arg \max_d \theta \mathbb{E}_{a^e, \mu} [\min \{ d, X \}] - \mathbb{E}_a [\min \{ d, X \}|z_h]$$

(51)

Proof. We are interested in finding the solution to:

$$\theta \mathbb{E}_{a^e, \mu} [\min \{ d, X \}] - \mathbb{E}_a [\min \{ d, X \}|z_h]$$
By the law of iterated expectations, market valuation of security $F$ can be written as

$$
E_{a^r,\mu}[F(X)] = \rho_h(a^r) E_{a^r}[F(X)|z_h] + (1 - \rho_h(a^r)) E_{a^r}[F(X)|z_l] 
= \rho(a^r)[E_H[F(X)] - E_L[F(X)]] + E_L[F(X)]
$$

and also remember that:

$$
E_a[F(X)|z_h] = \pi_h(a)[E_H[F(X)] - E_L[F(X)]] + E_L[F(X)]
$$

and thus the problem can be re-written as follows:

$$
\max_{F \in \Delta} (\theta \rho(a^r) - \pi_h(a)) [E_H[F(X)] - E_L[F(X)]] + (\theta - 1) E_L[F(X)]
$$

For $\theta \rho(a^r) \geq \pi_h(a)$, the value of the $z_h$-type bank is increasing in the cashflows of $F$, and thus $F^*_{z_h}(X) = X$. In this case, we say the bank issues standard debt with $d = \infty$. For $\theta \rho(a^r) < \pi_h(a)$, the $z_h$-type faces adverse selection since it values cashflows more than the market. Let $G$ be any feasible security, and let $g = E_a[G(X)|z_h]$ and $g_m = E_{a^r,\mu}[G(X)]$, denote the private and the market valuations respectively. Now consider a standard debt security $F_D(X) = \min \{d, X\}$. Let $f = E_a[\min \{d, X\}|z_h]$ and $f_m = E_{a^r,\mu}[\min \{d, X\}]$. Given the continuity of $f_m$ on $d$, pick $d$ so that $g_m = E_{a^r,\mu}[\min \{d, X\}]$. Let $H = G - F$ and let $h(\mu) = E_{a^r,\mu}[G(X) - F_D(X)]$, where $h(\rho_h(a^r)) = 0$ by construction –note that $\mu$ here refers to the probability assigned to the $z_h$-type cashflows. Given the monotonicity of $G$, and the fact that $G(X) \leq X$, $\exists x^* \ s.t. \ H(X) = G(X) - \min \{d, X\} > 0$ iff $X > x^*$. Then, note that $E_a[H(X)|z_h] > h(\rho_h(a^r)) = 0$, where the first inequality is due to the fact that $\rho(a^r) < \pi_h(a)$ and to stochastic-dominance. This implies that $g = f + h \geq f$, and thus $\Pi_{z_h}(a, a^r, G) \leq \Pi_{z_h}(a, a^r, F)$. Because $G$ was arbitrary, the optimal security preferred by the $z_h$-type is standard debt. Given this, it is straightforward that debt level $d$ need to $\max_d \theta E_{a^r,\mu}[\min \{d, X\}] - E_a[\min \{d, X\}|z_h]$, where the solution to this exists and is unique (where $d = \infty$ is a possible solution).

In what follows, to characterize equilibrium debt levels, I impose the equilibrium condition $a = a^r$.

**Lemma.** For any $\theta > 1$, there $\exists \underline{a}(\theta), \overline{a}(\theta) \in \left[\frac{1}{2}, 1\right]$ s.t. $\forall a \in \left[\frac{1}{2}, \underline{a}(\theta)\right] \cup [\overline{a}(\theta), 1]$, equity is the only security issued in secondary markets. Threshold $\underline{a}(\theta)$ ($\overline{a}(\theta)$) is increasing (decreasing) in funding needs $\theta$.

**Proof.** By the previous Lemma, equity is chosen in equilibrium by both bank types when $\theta \rho(a) \geq \pi_h(a)$. i) Existence of $\underline{a}(\theta)$. Note that for $a = \frac{1}{2}$, the signal is uninformative, and thus $\rho(a) = \pi_h(a) = \pi_H$, the constraint is satisfied since $\theta > 1$. Using continuity and monotonicity of the RHS on $a$, the constraint must hold in an interval close to $a = \frac{1}{2}$, given by $\left[\frac{1}{2}, \underline{a}(\theta)\right]$. To see that the threshold is increasing, note that higher $\theta$ makes the constraint less binding. ii) Existence of $\overline{a}(\theta)$. Note that for $a = 1$, both signals are fully informative, and thus the initial screening excludes all bad firms, i.e. $\rho(1) = 1$, and thus the constraint is again satisfied for any $\theta > 1$. Again by continuity and monotonicity of the RHS on $a$, the constraint must hold for an interval close to $a = 1$, denoted by $[\overline{a}(\theta), 1]$. To see that $\overline{a}(\theta)$ is decreasing in $\theta$, note that the constraint is again less binding for higher $\theta$. Finally, note that if the $z_h$-type issues equity, so does the $z_l$-type. 

48
Lemma. For given market beliefs, \( a^e \), debt levels are decreasing in information acquisition.

Proof. Note that debt levels are given by the FOC, and thus implicit function \( d(a, a^e) \) is given by:

\[
\frac{\pi_h(a) - \theta \rho(a^e)}{\theta - 1} = \frac{1}{\frac{1-G_H(d)}{1-G_L(d)} - 1}
\]

i) The RHS is continuous, differentiable, and decreasing in \( d \). The MLRP implies a hazard rate ordering and thus \( \frac{1-G_H(X)}{1-G_L(X)} \) is increasing in \( X \), the continuity and differentiability are given by the continuity and differentiability of the cumulative distributions. ii) The LHS is continuous, differentiable, and increasing in \( a \). This follows from the \( \pi_h(a) \) being continuous, differentiable, and increasing in \( a \). Therefore, there exists an implicit function \( d(a,a^e) \) that is continuous, differentiable, and decreasing in \( a \).

Corollary 1. Debt levels are continuous, differentiable on the equilibrium level of information acquisition \( a^* \). For \( \pi_H \geq 0.5 \), debt levels are convex on \( a^* \).

This follows from the fact that in equilibrium, \( a = a^e \). In this scenario, it can be shown that \( \Phi(a) = \pi_h(a) - \theta \rho(a) \) is concave on \( a \). For \( \theta \in [0,2] \), and \( \pi_H \geq 0.5 \), \( \Phi''(a) \leq 0. \)

Proposition. Let \( a^* \) denote the equilibrium level of information acquisition. Then, under the No Transparency Assumption, in any equilibrium in secondary markets:

- \( z_h \)-type bank issues standard debt \( F^*_D = \min \{d(a^*),X\} \) where \( d(a^*) \) is given by (51), and receives transfer \( p_D = \mathbb{E}_{a^*,\mu}[\min \{d(a^*),X\}] \).
- \( z_l \)-type bank issues standard debt \( F_D \) and junior tranche \( F^*_J(X) = X - F^*_D(X)(a) \) and receives transfers \( p_D \) and \( p_J = \mathbb{E}_{a^*}[X - \min \{d(a^*),X\} | z_l] \).

Existence. The following off equilibrium beliefs support an equilibrium in secondary markets. For all \( G \in \Delta \) s.t. \( G(X) \leq \min \{d^*(a^e),X\} \), \( \mu(G) = \rho_h(a^e) \), otherwise, \( \mu(G) = 0 \). That is, securities with “less” cashflows than the one issued by the \( z_h \)-type in equilibrium are evaluated at average valuations, while securities with claims to more cashflows (for a positive measure of outcomes \( X \)), are priced at the lowest valuation. Note that for given \( a^e \), the market posts the described menu, and by construction there are no profitable deviations for the market. The bank chooses which security to issue, given the posted menu, and thus there is no room for signaling, the bank has access to the whole set of securities in \( \Delta \) and issues the one that maximizes the value of banking in \( t = 0 \).

Choice of Information Acquisition

Given the previously constructed equilibrium outcome in secondary markets, I now focus on the choice of information acquisition done by the bank in \( t = 0 \). The bank cannot affect market beliefs \( a^e \), and we know standard debt \( F_D(a,a^e) \) and junior tranche \( F_J(a,a^e) \) are the securities issued in secondary markets for a given level of information acquisition and corresponding market beliefs. The bank’s expected utility in \( t = 0 \) is given by:
\[ V_0(a, a^e) \equiv \rho_h(a) \{ \theta p(F_D) + \mathbb{E}_a[X - F_D(a, a^e)|z_h] \} + \\
\{ 1 - \rho_h(a) \} \{ \theta (p(F_D) + p(F_J)) \} - C(a) \]  
\quad \text{(52)}

Note that we can use the Envelope Condition to abstract from the impact \( a \) has on the choice of security \( F(a, a) \), since securities are chosen ex-post to maximize the value of the bank in \( t = 1 \). Therefore, optimal investment in information is given by the solution to:

\[ \rho'(a) [\mathbb{E}_H [X - F_D(X)] - \mathbb{E}_L [X - F_D(X)]] = \rho_h(a) (\theta - 1) \mathbb{E}_a [X - F_{z_h}|z] + C'(a) \]  
\quad \text{(54)}

**Lemma 5.** An equilibrium without commitment always exists.

Let off-equilibrium beliefs be given by:

\[ \mu(F) = \begin{cases} 
\rho_h(a^e) & F \leq F_{z_h}^* \\
0 & \text{o.w.}
\end{cases} \]  
\quad \text{(55)}

where \( F \leq F_{z_h}^* \) mean that \( F(X) \leq \min \{ d^*, X \} \equiv F_{z_h}^*, \forall X \). Let \( \{ \tilde{a}, \tilde{d} \} \) be a profitable deviation to \( \tilde{a} > a^* \), where \( a^e = a^* \). Note that this deviation implies that \( \tilde{d} \leq d^* \). In particular, for \( \tilde{d} = d^* \), we know the deviation is not profitable by construction, therefore, it must be that \( \tilde{d} < d^* \) and thus we know \( \mu(\min \{ \tilde{d}, X \}) = \rho_h(a^*) \). The payoff to the bank from this deviation is given by:

\[ \rho_h(\tilde{a}) [\theta \mathbb{E}[\min \{ \tilde{d}, X \} | \rho_h(a^*)] + \mathbb{E}[X - \min \{ \tilde{d}, X \} | z_h(\tilde{a})] ] + (1 - \rho_h(\tilde{a})) V_0^* \leq ... \]

\[ \rho_h(\tilde{a}) [\max_d \theta \mathbb{E}[\min \{ d, X \} | \rho_h(a^*)] + \mathbb{E}[X - \min \{ d, X \} | z_h(\tilde{a})] ] + (1 - \rho_h(\tilde{a})) V_0^* \leq ... \]

\[ \max \rho_h(\tilde{a}) [\max_d \theta \mathbb{E}[\min \{ d, X \} | \rho_h(a^*)] + \mathbb{E}[X - \min \{ d, X \} | z_h(\tilde{a})] ] + (1 - \rho_h(\tilde{a})) V_0^* = ... \]

\[ \rho_h(a^*) [\theta \mathbb{E}[\min \{ d^*, X \} | \rho_h(a^*)] + \mathbb{E}[X - \min \{ d^*, X \} | z_h(a^*)] ] + (1 - \rho_h(\tilde{a})) V_0^* \]

Therefore, this deviation is not profitable. Now, consider a deviation to \( \{ \tilde{a}, \tilde{d} \} \) where \( \tilde{a} < a^* \), this deviation can only be profitable if extra cashflows \( G \) are issued at lower valuations. The payoff from this deviation is given by:

\[ \rho_h(\tilde{a}) [\theta \{ \mathbb{E}[\min \{ d^*, X \} | \rho_h(a^*)] + \mathbb{E}[G | z_i] \} + \mathbb{E}[X - \min \{ d^*, X \} - G | z_h(\tilde{a})] ] + (1 - \rho_h(\tilde{a})) V_0^* \leq ... \]

\[ \rho_h(\tilde{a}) [\theta \{ \mathbb{E}[\min \{ d^*, X \} + G | \rho_h(a^*)] + \mathbb{E}[X - \min \{ d^*, X \} - G | z_h(\tilde{a})] ] + (1 - \rho_h(\tilde{a})) V_0^* \leq ... \]

\[ \rho_h(\tilde{a}) [\max_{F \in \Delta} \{ \theta \mathbb{E}[F | \rho_h(a^*)] + \mathbb{E}[X - F | z_h(\tilde{a})] ] + (1 - \rho_h(\tilde{a})) V_0^* \leq ... \]

\[ \max_{\tilde{a}} \rho_h(\tilde{a}) [\max_{F \in \Delta} \{ \theta \mathbb{E}[F | \rho_h(a^*)] + \mathbb{E}[X - F | z_h(\tilde{a})] ] + (1 - \rho_h(\tilde{a})) V_0^* = ... \]

\[ \rho_h(a^*) [\theta \{ \mathbb{E}[\min \{ d^*, X \} | \rho_h(a^*)] + \mathbb{E}[X - \min \{ d^*, X \} | z_h(a^*)] ] + (1 - \rho_h(\tilde{a})) V_0^* \]
Proof. That is, \( \theta \rho_h(\hat{a}) \in \Delta \), that is, expected transfers remain unchanged. We can re-write \( \rho_h(\hat{a})p_h + (1 - \rho_h(\hat{a}))p_l = \rho_h(\hat{a})E[F_h|z_h] + (1 - \rho_h(\hat{a}))E[F_l|z_l] \)

Let \( p'_l = p_l - \epsilon \), with \( \epsilon \) small enough that the \((IC_l)\) is still satisfied. Let \( p'_h = p_h + \epsilon \), with \( \epsilon = \frac{\rho_h(\hat{a}) - \rho_l(\hat{a})}{\rho_h(\hat{a})} \), so that \( \rho_h(\hat{a})p'_h + (1 - \rho_h(\hat{a}))p'_l = \rho_h(\hat{a})p_h + (1 - \rho_h(\hat{a}))p_l \); that is, expected transfers remain unchanged. We can re-write the \((IC_a)\) constraint as follows,

\[
a = \arg \max_a \theta \rho_h(\hat{a})p_h + (1 - \rho_h(\hat{a}))p_l = \rho_h(\hat{a})E[F_h|z_h] + (1 - \rho_h(\hat{a}))E[F_l|z_l] - C(\hat{a})
\]

and therefore mechanism \( \{p'_l, p'_h, F_l, F_h\} \) implements \( \hat{a} \) as well. Note also that by construction, the objective function remains unchanged, and \( V_0 \) is the same given the above done transfers. However, the participation and the incentive compatibility constraints of the good type are relaxed, since \( p'_h > p_h \). Therefore, if for mechanism \( \{p_l, p_h, F_l, F_h\} \) the \((PC_h)\) or the \((IC_h)\) were binding, this transfer is welfare improving since it relaxes the problem, and thus \( \{p_l, p_h, F_l, F_h\} \) cannot be an optimal mechanism. If, on the other hand, none of the constraints were binding in the optimal mechanism, these transfers are done without loss \( \square \)

The binding \((IC_l)\) makes the incentive compatibility for high types, \((IC_h)\) slack, meaning that there is no need to impose an extra constraint. Note that the Binding \((IC_l)\) implies:

\[
\theta p_l - E[F_l|z_l] = \theta p_h - E[F_h|z_l] \Rightarrow E[F_h|z_l] - E[F_l|z_l] = \theta (p_h - p_l)
\]

First, note that the \((PC_l)\) is slack:

\[
\theta p_l - E[F_l|z_l] = \theta p_h - E[F_h|z_l] \geq \theta p_h - E[F_h|z_h] \geq 0
\]

**The Optimal Mechanism: The Case of Commitment.**

To simplify on notation, from now on \( E[F] \equiv E[F(X)] \), and when not indicated, these expectations are computed for implementable levels of information \( a^* \). The following Lemmas are needed for the results of the main proposition of this section.

**Lemma.** It can be assumed without loss that the incentive compatibility for the \( z_l \)-type bank binds in equilibrium; that is, \( \theta p_l - E[F_l|z_l] = \theta p_h - E[F_h|z_l] \).

Proof. Suppose that we have an optimal mechanism \( \{p_h, p_l, F_l, F_h\} \) that implements \( \hat{a} \) where the \((IC_l)\) does not bind; that is, \( \theta p_l - E[F_l|z_l] > \theta p_h - E[F_h|z_l] \). Note that in any optimal mechanism, the participation constraint of the market binds. By the binding \((PC_m)\), we know that:

\[
\rho_h(\hat{a})p_h + (1 - \rho_h(\hat{a}))p_l = \rho_h(\hat{a})E[F_h|z_h] + (1 - \rho_h(\hat{a}))E[F_l|z_l]
\]

Contradiction. Since \( a^* \) is the solution to:

\[
a^* = \arg \max_a \rho_h(\hat{a})[\max_{F \in \Delta} \theta E[F|\rho_h(a^*)] + E[X - F|z_h(\hat{a})]] + (1 - \rho_h(\hat{a}))V_0^*
\]

And thus, there are no profitable deviations from equilibrium \( \{a^*, d^* = d(a^*, a^*)\} \).
Now, all remaining funds are transferred to the good type, and therefore the \((IC_h)\) is slack. To see this, note that from \((IC_h)\) we get:

\[
\theta p_h - \mathbb{E}[F_h | z_h] \geq \theta p_l - \mathbb{E}[F_l | z_h] \iff \theta(p_h - p_l) \geq \mathbb{E}[F_h | z_h] - \mathbb{E}[F_l | z_h]
\]

using the binding \((IC_l)\)

\[
\iff \mathbb{E}[F_h | z_l] - \mathbb{E}[F_l | z_l] \geq \mathbb{E}[F_h | z_h] - \mathbb{E}[F_l | z_h]
\]

\[
\iff \mathbb{E}[F_l - F_h | z_h] \geq \mathbb{E}[F_l - F_h | z_l]
\]

High types need to retain as much as low types in any equilibrium; that is, \(\mathbb{E}[F_l - F_h | z] > 0, \forall z\).

**Lemma.** In the optimal mechanism, the level of information acquisition that can be implemented, \(a\), only depends on the security issued by the \(z_h\)-type bank, \(F_h\).

**Proof.** By previous Lemmas, we know that the \((IC_l)\) binds in equilibrium. The \((IC_a)\) determines the implementable level of information acquisition, \(a\), which is given by the following FOC:

\[
\rho_h'(a)\{\theta(p_h - p_l) + \mathbb{E}[X - F_h | z_h] - \mathbb{E}[X - F_l | z_l]\} - C'(\hat{a}) + ...
\]

\[
\rho_h(\hat{a})\pi_h'(a)\{\mathbb{E}_H[X - F_h] - \mathbb{E}_L[X - F_h]\} = 0
\]

Using the binding \((IC_l)\), \(\theta p_l - \mathbb{E}[F_l | z_l] = \theta p_h - \mathbb{E}[F_h | z_l]\), and the fact that \(\pi_l(a) = \pi_H\) we get:

\[
\rho_h'(a)\{\theta p_h - \mathbb{E}[F_h | z_l] + \mathbb{E}[F_h | z_l] + \mathbb{E}[X - F_h | z_h] - [\theta p_l + \mathbb{E}[X - F_l | z_l]]\} - C'(\hat{a}) + ...
\]

\[
\rho_h(\hat{a})\pi_h(\hat{a})\{\mathbb{E}_H[X - F_h] - \mathbb{E}_L[X - F_h]\} - C'(\hat{a}) + \rho_h(\hat{a})\pi_h'(a)\{\mathbb{E}_H[X - F_h] - \mathbb{E}_L[X - F_h]\} = 0
\]

\[
\rho'(a)\{\mathbb{E}_H[X - F_h] - \mathbb{E}_L[X - F_h]\} = C'(a)
\]

since \(\rho'(a) = \rho_h'(a)\pi_h(a) - \pi_H + \rho_h(\hat{a})\pi_h'(a)\). Thus, \(a\) only depends on \(F_h\). \(\Box\)

Using the results from the previous Lemmas, the optimal mechanism is given by the solution to the following simplified problem:

\[
\max_{\{p_l, p_h, F_l, F_h\} \in \mathbb{R}_+^2 \times \mathbb{R}^2} \rho_h(a^*)\{\theta p_h + \mathbb{E}[X - F_h | z_h(a^*)]\} + (1 - \rho_h(a^*))\{\theta p_l + \mathbb{E}[X - F_l | z_l(a^*)]\} - C(a^*)
\]
Lemma. In the equilibrium with commitment, the $z_l$-type bank issues equity, $F_l = X$.

Proof. The objective function can be re-written by plugging in the binding ($PC_m$) as follows:

\[ V_0 = (\theta - 1)[\rho_h(a)\mathbb{E}[F_h|z_h] + (1 - \rho_h(a))\mathbb{E}[F_l|z_l]] + \rho_h(a)\mathbb{E}[X|z_h] + (1 - \rho_h(a))\mathbb{E}[X|z_l] - C(a) \]

The value of the bank increases with the cash-flows in $F_l$. From the binding ($IC_i$) and ($PC_m$), we can solve for the transfers made to each type as a function of chosen securities and implementable investment levels:

\[ p_l = \rho_h(a)\mathbb{E}[F_h|z_h] + (1 - \rho_h(a))\mathbb{E}[F_l|z_l] + \frac{1}{\theta}[\mathbb{E}[F_l|z_l] - \mathbb{E}[F_h|z_l]] \]

\[ p_h = \rho_h(a)\mathbb{E}[F_h|z_h] + (1 - \rho_h(a))\mathbb{E}[F_l|z_l] - (1 - \rho_h(a))\frac{1}{\theta}[\mathbb{E}[F_l|z_l] - \mathbb{E}[F_h|z_l]] \]

Therefore, increasing the cashflows in $F_l$ also relaxes the ($PC_h$) by increasing the transfers made to the good type bank. Finally, the ($IC_a$) constraint is unaffected. Therefore, since there are only gains from increasing the cash-flows in $F_l$, it must be that in the optimal mechanism, $F_l = X$. \qed

Lemma. In the equilibrium with commitment, the $z_h$-type bank issues standard debt, $F_h = \min\{d_h, X\}$.

Proof. Let \{\(p_l, p_h, F_l, F_h\)\} be an optimal mechanism where $F_h$ is not standard debt. As shown in the previous Lemmas, ($IC_i$) and ($IC_0$) bind, and $F_l = X$ in equilibrium. Therefore, the bank’s objective function is maximized:

\[ V_0 = (\theta - 1) [\rho_h(a)\mathbb{E}[F_h|z_h] + (1 - \rho_h(a))\mathbb{E}[F_l|z_l]] + \rho_h(a)\mathbb{E}[X|z_h] + (1 - \rho_h(a))\mathbb{E}[X|z_l] - C(a) \]

subject to:

\[ \theta p_h - \mathbb{E}[F_h|z_h] \geq 0 \]

\[ \mathbb{E}[F_l - F_h|z_h] \geq \mathbb{E}[F_l - F_h|z_l] \]

\[ \rho'(a)\{\mathbb{E}_H[X - F_h] - \mathbb{E}_L[X - F_h]\} = C'(a) \]

\[ p_h = \rho_h(a)\mathbb{E}[F_h|z_h] + \frac{1}{\theta} \{(\theta - 1)(1 - \rho_h(a))\mathbb{E}[X|z_l] + (1 - \rho_h(a))\mathbb{E}[F_h|z_l]\} \]

\[ p_l = \rho_h(a)\mathbb{E}[F_l|z_l] + \frac{1}{\theta} \{(\theta(1 - \rho_h(a)) + \rho_h(a))\mathbb{E}[X|z_l] - \rho_h(a)\mathbb{E}[F_h|z_l]\} \]
Let $F_h = G$ be an arbitrary $G \in \Delta$, with cashflows $v(X)$, different than standard debt. Let $F = \min \{d, X\}$ and choose $d$ so that $\mathbb{E}[G|z_h] = \mathbb{E}[F|z_h]$. Let $H = G - F$, and let $h(z) = \mathbb{E}[G - F|z]$, where by construction $h(z_h) = 0$. Note that since $v(X) \leq X$, $H(x) > 0$ if $x \geq x^*$ for some $x^* \in \Omega$. Therefore, given the MLRP $(G_H(X)/G_L(X)$ is increasing in $X)$, $\mathbb{E}_H[H] - \mathbb{E}_L[H] > 0$. Therefore,

$$\mathbb{E}_H[G] - \mathbb{E}_L[G] > \mathbb{E}_H[F(X)] - \mathbb{E}_L[F(X)]$$

$$\mathbb{E}_H[X - G] - \mathbb{E}_L[X - G] < \mathbb{E}_H[X - F] - \mathbb{E}_L[X - F]$$

And thus, security $F$ implements the same level of information acquisition at lower retention costs. Also note that since $h(z_l) < h(z_h) = 0$,

$$\mathbb{E}[H|z_l] < 0 \Rightarrow \mathbb{E}[G|z_l] < \mathbb{E}[F|z_l] \rightarrow p_h(F) > p_h(G)$$

and the $(PC_h)$ is relaxed. Since by construction $h(z_h) = 0$, the objective function and the remaining constraints are unaffected by this change. Therefore, mechanism $(p_t, p_h, F, F)$ reduces the cost associated with implementing a given level of information acquisition, and relaxes the $(PC_h)$ and the $(IC_h)$ without affecting the remaining constraints. Since $G \in \Delta$ was an arbitrary security different than debt, it must be that the good types issues standard debt in equilibrium; that is $F = \min \{d, X\}$. 

Let $a(d)$ be the implicit function given by the $(IC_a)$ constraint:

$$\rho'(a)\{\mathbb{E}_H[X - \min\{d, X\}] - \mathbb{E}_L[X - \min\{d, X\}]\} = C'(a)$$

Note that $a(d)$ is continuous, differentiable, and decreasing in $d$ given that the MLRP. Incorporating this implicit function, the problem becomes:

$$\max_{\{p_t, p_h, F, F_h\} \in \mathbb{R}_+^2 \times \theta^2} \rho_h(a^*)[\theta p_h + \mathbb{E}[X - F_h|z_h(a^*)]] + (1 - \rho_h(a^*))[\theta p_t + \mathbb{E}[X - F_t|z_l(a^*)]] - C(a^*)$$

subject to:

$$\rho'(a^*)\{\mathbb{E}_H[X - \min\{d, X\}] - \mathbb{E}_L[X - \min\{d, X\}]\} = C'(a^*)\text{arrow}a^*(d)$$

$$\theta p_h - \mathbb{E}[F_h|z_h] \geq 0 \quad (\lambda)$$

$$p_h = \rho_h(a)\mathbb{E}[F_h|z_h] + \frac{1}{\theta}((\theta - 1)(1 - \rho_h(a))\mathbb{E}[X|z_l] + (1 - \rho_h(a))\mathbb{E}[F_h|z_l])$$

$$p_t = \rho_h(a)\mathbb{E}[F_t|z_h] + \frac{1}{\theta}((\theta(1 - \rho_h(a)) + \rho_h(a))\mathbb{E}[X|z_l] - \rho_h(a)\mathbb{E}[F_h|z_l])$$

If $\lambda^* = 0$:

$$\theta \frac{\partial}{\partial a^*} \{\rho_h(a^*)p_h(a^*) + (1 - \rho_h(a^*))p_t(a^*)\} \frac{\partial a^*}{\partial d} + \theta[\rho_h(a^*)\frac{\partial p_h(a^*)}{\partial d} + (1 - \rho_h(a^*))\frac{\partial p_t(a^*)}{\partial d}] - \rho_h(a^*) \int_{d}^{\infty} f(X|z_h)dX = 0$$
where

\[
\frac{\partial p_h}{\partial d} = \rho_h(a) \int_d^\infty f(X|z_h)dX + \frac{1}{\theta}(1 - \rho_h(a)) \int_d^\infty f(X|z_l)dX
\]

\[
\frac{\partial p_l}{\partial d} = \rho_h(a) \int_d^\infty f(X|z_h)dX - \frac{1}{\theta} \rho_h(a) \int_d^\infty f(X|z_l)dX
\]

and thus,

\[
\theta[\rho_h(a)] \frac{\partial p_h}{\partial a^*} + (1 - \rho_h(a)) \frac{\partial p_l(a)}{\partial a^*} a'(d) + (\theta - 1) \rho_h(a) \int_d^\infty f(X|z_h)dX = 0
\]

Finally,

\[
\frac{\partial p_h}{\partial a} = \rho'(a)[\mathbb{E}_H[F_h] - \mathbb{E}_L[F_h]] - \rho'_h(a)(\theta - 1)\mathbb{E}[X - F_h|z_l]
\]

\[
\frac{\partial p_l}{\partial a} = \rho'(a)[\mathbb{E}_H[F_h] - \mathbb{E}_L[F_h]] - \rho'_h(a)(\theta - 1)\mathbb{E}[X - F_h|z_l]
\]

Therefore,

\[
\{\theta \rho'(a)[\mathbb{E}_H[F_h] - \mathbb{E}_L[F_h]] - (\theta - 1)\rho'_h(a)\mathbb{E}[X - F_h|z_l]\} a'(d) + (\theta - 1) \rho_h(a) \int_d^\infty f(X|z_h)dX = 0
\]

If the \{d, a(d)\} given by the previous FOC satisfy the (PC_h), then \(d\) is given by the first-order condition, and \{\(p_l, p_h\)\} are given by the binding (PC_m) and the (IC_0), and investment in information \(a(d)\) is implemented. If the \{d, a(d)\} given by the previous FOC violate the (PC_h); then \(\lambda^* > 0\) and \(d\) is given by the binding (PC_h), transfers are given by the binding (PC_l) and (IC_0) and \(a(d)\) is implemented. Clearly, when the (PC_h) binds, the previous FOC are positive evaluated at the optimum \{d^*, a(d^*)\}.

**Policy Implications**

Let \{\(\Gamma_0, \Gamma_h, \gamma, T\)\} be the transfers, marginal and lump-sum transfers that the regulator uses to implement the commitment (optimal mechanism) allocations: \{\(d^*_c, a^*_c\)\}. Let

* Transfers Across Types. Let \(\Gamma_0, \Gamma_h\) be the transfers received when issuing senior and junior tranches respectively. Transfers need to be set so that:

\[
p_{h,nc}^* + \Gamma_h^* = p_{h,c}^*
\]

\[
p_{l,nc}^* - \Gamma_0^* = p_{l,c}^*
\]

Note that given the previous transfers, for a given debt level, information acquisition is given by:

\[
\rho'(a)[\mathbb{E}_H[\min\{d, X\}] - \mathbb{E}_L[\min\{d, X\}]] = C'(a) \rightarrow a(d)
\]
Marginal Tax $\gamma$ on debt levels. Choose $\gamma$ so that the FOC of the security design problem in $t = 1$ for $a = a^*_c$ is zero at $d^*_c$. The problem at $t = 1$:

$$\max_d \theta \{ \mathbb{E} \left[ \min \{d, X\} \right| \rho_h(a) \} + \gamma \times d \} - \mathbb{E} \left[ \min \{d, X\} \right| z_h(a) \}$$

with FOC:

$$(\theta \rho (a^*_c) - \pi_h (a^*_c)) \frac{1}{\theta} \left[ F_H (d^*_c) - F_L (d^*_c) \right] + \frac{\theta - 1}{\theta} F_L ((d^*_c)) - \gamma^* = 0$$

$$\gamma^* = \frac{1}{\theta} \left\{ (\theta \rho (d^*_c) - \pi_h (d^*_c)) \left( G_H (d^*_c) - G_L (d^*_c) \right) - (\theta - 1) G_L (d^*_c) \right\}$$

where $a^*_c = a (d^*_c)$ once transfers are made. Note that when $d^*_nc > d^*_c$, $\gamma^* < 0$; that is, debt levels are taxed, or equivalently, retention levels are subsidized. Participation transfer. Transfer $\Gamma = -\rho_h (a^*_c) \gamma^* d^*_c$ is given to the bank if it participates in secondary markets. Note the bank agrees with this policy since it increases its ex-ante efficiency. Finally, note that by construction, the budget constraint of the regulator is satisfied, that is:

$$\rho_h (a^*_c) [\Gamma^*_h + \gamma d^*_c] + (1 - \rho_h (a^*_c)) \Gamma^*_h + T^* = 0$$

I proceed to compute the transfers.

$$p^nc = \mathbb{E} [X | z_i] + \rho_h(a) \{ \mathbb{E} [F_h | z_h] - \mathbb{E} [F_h | z_i] \}$$

$$p^nc = \mathbb{E} [F_h | z_h] - (1 - \rho_h(a)) \{ \mathbb{E} [F_h | z_h] - \mathbb{E} [F_h | z_i] \}$$

$$p^c = \rho_h(a) \mathbb{E} [F_h | z_h] + (1 - \rho_h(a)) \mathbb{E} [F_i | z_i] + \rho_h(a) \frac{1}{\theta} \mathbb{E} [F_i | z_i] - \mathbb{E} [F_h | z_i]$$

$$p^h = \rho_h(a) \mathbb{E} [F_h | z_h] + (1 - \rho_h(a)) \mathbb{E} [F_i | z_i] - (1 - \rho_h(a)) \frac{1}{\theta} \mathbb{E} [F_i | z_i] - \mathbb{E} [F_h | z_i]$$

Therefore, $\Gamma^*_h = p^c - p^nc$ is given by:

$$\Gamma^*_h = (1 - \rho_h(a)) (\frac{\theta - 1}{\theta}) \mathbb{E} [\max \{0, X - d\} | z_i]$$

And $\Gamma^*_0 = p^c - p^nc = \Gamma^*_0 = -\rho_h(a) (\frac{\theta - 1}{\theta}) \mathbb{E} [\max \{0, X - d\} | z_i]$

**Extensions**

**Pooling and Tranching**

The bank issues $n$ loans in $t = 0$; that is, $\omega_b = n < \infty$. I continue to assume that incentives to originate are not in place, and thus the bank lends to $n$ firms with $s_0 = G$. In $t = 1$, the bank issues a security backed by the pool of loans $Y \sim f_y(y)$, where $Y = \frac{1}{n} \sum_{i=1}^{n} X_i$ denotes the cash-flows of the bank in $t = 2$ per loan issued. It is out of the scope of this paper to investigate whether issuing a security backed by pool $Y$ is chosen over issuing $n$ securities backed by $X$, or pooling $m < n$ securities, and
issuing $n - m$ individual securities backed by a loan. When the bank has private information, giving the freedom to choose which assets to pool severely complicates the problem, since the decision to pool vs. not pool signals something to the market. Instead, I assume that the bank issues $n$ loans and pools them to issue securities in secondary markets.

Let $z^i$ be the information held by the bank in $t = 1$ about firm $i$, then $\{z^i\}_{i=1,...,n} \in \mathbb{Z}^n$ is the bank’s private information. Then $\zeta \in \mathbb{N}$ denotes the bank’s type and it is given by the number of loans $z_h$ received for the loans in the pool. Finally, let $\rho_\zeta (a)$ denote the probability of receiving $\zeta$ signals $s_1 = G$, given that $s_0 = G$, where $\sum_{\zeta=0}^{\zeta} \rho_\zeta (a) = 1$. The following properties hold:

- $f_y (y)$ is given by the convolution of $n$ pdfs $f_x \left( \frac{x}{n} \right)$ and $f_y (y|z)$ is given by the convolution of $f_x \left( \frac{x}{n} | z^1 \right), f_x \left( \frac{x}{n} | z^2 \right), ..., f_x \left( \frac{x}{n} | z^n \right)$ for $z = \{z^1, z^2, ..., z^n\}$, with $\mathbb{E} [Y] = \mathbb{E} [X]$ and $\mathbb{E} [Y|z] = \sum_{i=1}^{n} \mathbb{E} [X|z_i]$ and $V (Y) = \frac{1}{n} V (X)$ and $V (Y|z) = \frac{1}{n} \sum_{i=1}^{n} V (X|z_i)$.

- Bank type $\zeta$ is distributed with $\zeta \sim Binomial (\rho_h (a), n)$ and thus the probability of the bank being type $\zeta$ conditional on initial investment in information $a$ is given by:

$$g (\zeta; n, \rho_h (a)) = \rho_\zeta (a) = \binom{n}{\zeta} \rho_h (a)^\zeta (1 - \rho_h (a))^{n-\zeta}$$

with cumulative distribution $G (k; n, p)$ (denoted by $G (k)$ from now on).

- The value of the bank in $t = 0$ is given by:

$$\sum_{\zeta=0}^{n} \rho_{h,\zeta} (a) [p (\zeta) + \mathbb{E} [X - F (\zeta) | \zeta]]$$

where $F (\zeta)$ are the cash-flows sold by type $\zeta$, and $p (\zeta)$ the funds raised by this type in secondary markets.

**Markets for ABS: No Commitment.** The definition of equilibrium of the full game, and of equilibrium in secondary markets remains unchanged. I proceed with the security design problem solved by the best type $\zeta = n$. Using the results from the baseline section, the security design problem with multiple types is as follows.

1. By our construction of the two-types equilibrium, we know the high type would choose to issue one security. The problem faced by the high type is given by:

$$\max_{F \in \Delta} \theta p_n - \mathbb{E} [F_n (Y) | n]$$

2. As in the baseline case, this type is mimicked by lower types in secondary markets. Therefore, since it faces adverse selection, the optimal security continues to be standard debt: $F_n (Y) =$
\[
\min \{d_n, Y\}, \text{ and that } p_n = \sum_{k=0}^{n} \rho_k (a) \mathbb{E} [\min \{d_n, Y\} | k] \text{ since all types } k < n \text{ mimic this issuance } n.
\]

3. Since the high type is mimicked by lower types in equilibrium, the problem can be written as:

\[
\max_d \theta \left[ \sum_{k=0}^{n} \rho_k (a) \mathbb{E} [\min \{d_n, Y\} | k] \right] - \mathbb{E} [\min \{d_n, Y\} | n]
\]

with FOC

\[
\int_{d}^{\infty} \left[ \theta \left( \sum_{k=0}^{n-1} \rho_k (a) f_{y} (y|k) \right) + (\theta \rho_n (a) - 1) f_{y} (y|n) \right] dy = 0
\]

If for \( d = \infty \), the LHS is still positive, then all types issue equity. If not, \( d_n < 0 \) is chosen to satisfy the FOC.

4. I continue to solve the problem of the next highest type: \( \zeta = n - 1 \) has remaining cashflows \( Y_{n-1} = Y - \min \{d_n, Y\} \), also monotonic in \( Y \). Bank type \( n - 1 \) solves the same problem, and issues issues debt contract \( d_{n-1} \) backed by \( Y_{n-1} \). It issues the safe tranche \( F_n = \min \{d_n, Y\} \) and the mezzanine tranche \( F_{n-1} = \min \{d_{n-1}, Y_{n-1}\} \). The latter issuance is mimicked by types \( \zeta \leq \zeta_{n-1} \) and therefore the market price is given by:

\[
p_{n-1} = \frac{1}{G(n-1)} \sum_{k=0}^{n-1} \rho_k (a) \mathbb{E} [\min \{d_{n-1}, Y_{n-1}\} | \zeta_k]
\]

Optimal threshold level \( d_{n-1} \) is chosen to maximize:

\[
\theta \sum_{k=0}^{n-2} \frac{\rho_k (a)}{G(n-1)} \mathbb{E} [\min \{d_{n-1}, Y_{n-1}\} | \zeta_k] + \left( \frac{\theta \rho_{n-1} (a)}{G(n-1)} - 1 \right) \mathbb{E} [\min \{d_{n-1}, Y_{n-1}\} | \zeta_{n-1}]
\]

This problem continues until type \( k \geq 0 \) issues equity.

5. There exists a type \( k \geq 0 \) that issues a claim to all of its cash-flows. It mimicks issuance of types \( \{\zeta_{k+1}, \ldots, \zeta_n\} \) and issues equity tranche \( F_k = Y - \min \{d_{k+1}, Y\} \) at valuation \( p_k = \sum_{i=1}^{k} \frac{\rho_i (a)}{1 - C(a)} \mathbb{E} [\min \{d, \max \{Y - d_{k+1}, 0\}\} | \zeta_i] \). Note that type \( k = 0 \) does not face a lemons discount and thus issues an equity tranche.

The choice of information acquisition:

\[
a = \arg \max_{a \in [\frac{1}{2}, 1]} \left\{ \sum_{k=0}^{n} \rho_k (a) [\theta p (F (k)) + \mathbb{E} [Y - F (k) | \zeta_k (a)] - C (a)] \right\}
\]

where \( p (F (k)) = \sum_{j=k}^{n} p_j \) and \( F (k) = \min \{d_k, Y\} \). As in the two-types case, the choice of information acquisition is done to increase the expected value of the retained tranches, and to affect the
distribution of types. The FOC:

\[
\sum_{k=0}^{n} \rho_k' (a) [\theta p (F (k)) + \mathbb{E} [X - F (k) | \zeta_k (a)]] + \sum_{k=0}^{n} \rho_k (a) \pi_k' (a) (\mathbb{E}_H [Y - F (k)] - \mathbb{E}_L [Y - F (k)]) - C' (a) = 0
\]

\[
\sum_{k=0}^{n} \rho_k (\hat{a}) [\theta p (F (k)) + \mathbb{E} [Y - F (k) | \zeta_k (\hat{a})]] + \sum_{k=0}^{n} \rho_k (\hat{a}) \frac{\partial}{\partial a} \mathbb{E} [Y - F (k) | \zeta_k (\hat{a})] - C (\hat{a}) = 0
\]

For \( n \to \infty \), the ex-ante probability of issuing junior tranches increases, while the probability of being of a higher type and retain decreases. Therefore, incentives for info acquisition are very likely to be decreasing in \( n \). While pooling increases the expected gains from trade, it is detrimental since it worsens incentives for information acquisition.

The Optimal Mechanism: Commitment. As in the baseline case, the equilibrium with commitment is given by \( \{a, \{p_k, d_k\}_{k=0}^{n}\} \) chosen to:

\[
\max_{\{F_\zeta, p_\zeta\}} \left\{ \sum_{\zeta=0}^{n} \rho_\zeta (a) [\theta p_\zeta + \mathbb{E} [Y - F_\zeta | \zeta (a)]] - C (a) \right\}
\]

subject to:

1. Incentive compatibility:

\[
\zeta = \arg \max_{\zeta \in \{0, \ldots, n\}} \theta p_\zeta + \mathbb{E} [Y - F_\zeta | \zeta]
\]

2. Ex-Post Rationality Constraints are satisfied.

\[
\theta p_\zeta + \mathbb{E} [Y - \min \{d_k, Y\} | \zeta (a)] = 0 \quad \forall \zeta \in \{0, \ldots n\}
\]

3. Zero-Profit Condition:

\[
\sum_{\zeta=0}^{n} \rho_\zeta (a) [\mathbb{E} [F_\zeta | \zeta] - p_\zeta] = 0
\]

Debt continues to be the optimal design for all types, the arguments used in the baseline case follow through. It can also be shown that transfer of funds to higher types improves can be done at no loss subject to incentive compatibility constrains. Therefore, \( p_\zeta \) are given by the binding (IC) and the Zero Profit condition. The choice of information acquisition for schedule \( \{F_\zeta, p_\zeta\} \) is given by:

\[
\max_{a} \left\{ \sum_{\zeta=0}^{n} \rho_\zeta (a) [\theta p_\zeta + \mathbb{E} [Y - F_\zeta | \zeta (a)]] - C (a) \right\}
\]
\[
\sum_{\zeta=0}^{n} \rho_{\zeta} (a) [\theta p_{\zeta} + \mathbb{E} \{\max \{Y - d_{\zeta}, 0\} \mid \zeta (a)\}] + \sum_{\zeta=0}^{n} \rho_{\zeta} (a) \frac{\partial}{\partial a} \mathbb{E} \{\max \{Y - d_{\zeta}, 0\} \mid \zeta (a)\} - C' (a) = 0
\]

where \( \frac{\partial}{\partial a} \mathbb{E} [Y - F_{\zeta} | \zeta (a)] = \int_{d}^{\infty} (y - d) \left( \frac{\partial}{\partial a} f_{Y} (y | \zeta (a)) \right) dy \).

As before, the choice of information acquisition ex-ante is done to affect the quality of retained tranches, and to affect the distribution of types. The previous function generates an implicit function of \( a^e = a (d_0, d_1, ..., d_n) \). Given this, when the participation constraint of type \( k \) does not bind in equilibrium, debt levels are chosen:

\[
\max_{d} \left\{ \sum_{\zeta=0}^{n} \rho_{\zeta} (a) [\theta p_{\zeta} (a^e) + \mathbb{E} [Y - F_{\zeta} | \zeta (a)]] - C (a) \right\}
\]

\[
(d_k) \quad \theta \left( \sum_{\zeta=0}^{n} \rho_{\zeta} (a) \frac{\partial p_{\zeta}}{\partial a^e} \frac{\partial a^e}{\partial d_k} \right) + \rho_{k} (a) \int_{d_k}^{\infty} f_{Y} (y | \zeta = k) dy \geq 0 \quad \forall d_k, k = 1, 2, ...n
\]

otherwise, debt levels are given by the binding participation constraint, as in the baseline case. The value of retention is higher for those types that have a large impact on incentives. For this, these types need to be also likely ex-ante, that is, relatively large \( \rho_{\zeta} (a) \). The two previous FOC, together with binding IC and PC constraints and zero profit solve the problem.