Market Exposure and Endogenous Firm Volatility over the Business Cycle∗

Ryan Decker  Pablo N D’Erasmo  Hernan Moscoso Boedo
University of Maryland  Federal Reserve Bank  Carl H. Lindner College of Business
of Philadelphia  University of Cincinnati

This Draft: January 30, 2015,
First Draft: November 1, 2011

Abstract

We propose a theory of endogenous firm-level risk over the business cycle based on endogenous market exposure. Firms that reach a larger number of markets diversify market-specific demand shocks at a cost. The model is driven only by total factor productivity shocks and captures the observed countercyclicality of firm-level risk. Using a panel of U.S. firms (Compustat) matched to Compustat’s Segment data and the U.S. Census Bureau’s Longitudinal Business Database (LBD) we show that, consistent with our model, measures of market reach are procyclical, and the countercyclicality of firm-level risk is driven mostly by those firms that adjust the number of markets to which they are exposed which on average are larger than those that do not. This finding is explained by the negative elasticity between various measures of market exposure and firm-level idiosyncratic risk we uncover.

Keyword: Endogenous idiosyncratic risk
JEL Classifications: D21, D22, E32, L11, L25

∗Previously circulated as “Intangibles and Endogenous Firm Volatility over the Business Cycle.” We thank Nick Bloom, Rui Castro, Larry Christiano, Gian Luca Clementi, Max Croce, John Haltiwanger, and Jonathan Willis; the seminar participants at the 2011 LACEA meeting in Chile, the 2013 SED meeting in Seoul, the NBER SI 2012, the Econometric Society Meetings 2012, and the Fall Midwest Macroeconomics Meetings 2013, the Federal Reserve Banks of Atlanta, St Louis, and Philadelphia; and the University of Virginia and the University of Maryland for their very helpful comments. We thank Javier Miranda for technical guidance on U.S. Census Bureau data. All errors are ours. Certain data included herein are derived from the Kauffman Firm Survey release 4.0. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the Ewing Marion Kauffman Foundation. Any opinions and conclusions expressed herein are those of the authors and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential information is disclosed. The views expressed in this paper are those of the author and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. Hernan Moscoso Boedo gratefully acknowledges financial support from the Bankard Fund for Political Economy.
1 Introduction

Firm-level risk has been documented to be countercyclical. In a seminal paper, Bloom (2009) and extensive literature that followed have analyzed how exogenous changes in uncertainty (or second-moment shocks) can be key drivers of the business cycle. In this paper, we evaluate the possibility that causality operates in the opposite direction. Our hypothesis is that part of the observed change in measured firm-level risk over the business cycle is an endogenous response to first-moment shocks.

We propose a theoretical model in which firms endogenously determine the number of markets they participate in and thereby expose themselves to market-specific demand shocks. As long as the market-specific shocks are not perfectly correlated, the endogenous dynamics of market exposure implies changes in firm-level volatility. A continuum of competitive firms that face stochastic market demands differ in their level of idiosyncratic productivity and face an aggregate productivity shock. They can participate in many markets by incurring selling expenses before demand shocks are realized. Thus, these endogenous per-period sunk costs determine the pool of suppliers in each market. Incentives to expand are higher in good times (i.e., when aggregate productivity is high) than in bad times. Since a larger market exposure results in lower firm-level volatility, the model captures the observed countercyclicality of firm-level risk (between -0.20 and -0.42 in the model versus -0.46 in the data).

We test the model in several dimensions and find that it is broadly consistent with the evidence. In order to do so, we work with novel data on market presence that link Compustat with the U.S. Census Bureau’s Longitudinal Business Database (LBD). Though the firms in our model are risk neutral and lack any risk diversification objective, we find that they increase their revenues and diversify market-specific shocks by reaching more markets. The model is consistent with the procyclicality of measures of market exposure as well as the observed negative elasticity of firm-level risk to measures of market expansion (that ranges from -7.5% to -30.1% in the data and between -5.4% and -57.9% in the model).

1 We adopt a broad definition of “market” that applies to the market-product space, the market-location space, and a combination of the two. The empirical measures we use are intended to capture this broad definition of “market.”

2 In the data, we follow Castro, Clementi, and MacDonald (2009) and measure firm-level idiosyncratic risk as the portion of growth in sales that cannot be explained by firm-level characteristics (such as age or size), industry, or year effects. The core data set for our empirical analysis is Compustat Fundamental (a sizable panel of large, public U.S. firms). We construct two samples based on the core Compustat file: in one sample, we match Compustat Fundamental with Compustat Segment or “line of business” data; in another sample, we add matched firm-year data from the LBD. See the following section as well as the Appendix for a detailed description of each data source and the matching procedure.
A key prediction of the model is that the negative correlation between firm-level risk and the business cycle is mostly driven by those firms that adjust the number of markets they operate over time which happen to be, on average, larger than those firms that do not expand. We test this prediction in the data and find that, consistent with the model, among firms that adjust market exposure, the correlation between firm-level risk and detrended GDP is between -0.311 and -0.422, depending on our market definition, while risk is acyclical for firms that do not adjust. Also consistent with our theory, when we split the sample of firms by size, we find that firm level risk for large firms is countercyclical while it is not for small firms. In addition, the model presented here is consistent with the evidence at levels of aggregation other than that of the firm. First, it captures the fact that the distribution of prices is countercyclical, as described in Berger and Vavra (2011). Second, it generates a countercyclical cross-sectional variance of plant-level productivity, as reported by Kehrig (2011). Finally, the model captures the fact that firm-level market exposure is procyclical. This has been documented by Broda and Weinstein (2010) based on the number of products per firm (derived from bar code data) over the cycle. We also find that the number of establishments per firm, another correlate of market exposure (especially for large firms), is procyclical.

This work brings together two relatively recent streams in the literature. The first is the literature regarding business cycles and uncertainty that began with the work by Bloom (2009) but also including Arellano, Bai, and Kehoe (2012); Bloom et al. (2012); Bachmann and Bayer (2013 and 2014); Christiano, Motto, and Rostagno (2014); Chugh (2014); and Schaal (2012). In this literature, in contrast to our paper, exogenous changes in volatility are key to generating business cycles. The second stream of the literature is composed of studies that empirically analyze firm-level risk.³ Leahy and Whited (1996) analyzed the relationship between uncertainty (measured by the volatility of stock returns) and firm investment. Castro, Clementi, and Lee (2014) attribute differences in firm-level volatility to differences in the sectors in which firms operate. In contrast, we uncover the relationship between firm-level risk and total market exposure and associated expenditures (after controlling for industry effects). Comin and Philippon (2006) document the increasing trend in firm-level volatility using Compustat, whereas Davis et al. (2007) show that the increase in firm-level risk is related to a selection issue, since public firms are more volatile.

Our paper is related to previous work analyzing the possibility of reverse causation be-

³Bachmann and Bayer (2013 and 2014) show that if countercyclical firm level risk is imposed as a second driving force and propagated through a wait-and-see mechanism in capital adjustment costs, it does not generate large aggregate fluctuations.
tween measured uncertainty and business cycles, which has already been documented by Bachmann, Elstner, and Sims (2013), and Bachmann and Bayer (2013 and 2014).\textsuperscript{4} We offer an alternative explanation to Van Nieuwerburgh and Veldkamp (2006), Bachmann and Moscarini (2012), and Tian (2012). In Van Nieuwerburgh and Veldkamp (2006), procyclical learning about productivity generates the observed countercyclicality in firm-level volatility. In Bachmann and Moscarini (2012), downturns offer the opportunity to experiment and learn the firm-specific demand function; that experimentation is the driver of additional volatility. In Tian (2012), periods of recession are accompanied by more risk-taking behavior at the firm level. In our model, positive first-moment shocks (TFP) enable firms to expand into more markets and expose firms to an increased number of market-specific shocks, reducing volatility through a standard diversification mechanism. More recently, Alessandria, Choi, Kaboski and Midrigan (2014) study the role of exogenous first and second moment shocks to productivity as drivers of export dynamics and business cycles.

The notion that agents are exposed to a limited number of shocks, and therefore, the law of large numbers does not apply, is not unique to our work. Among the papers that use this assumption are Gabaix (2011), Acemoglu et al. (2012), and Koren and Tenreyro (2013). These papers argue that a small group of firms (as in Gabaix (2011)), a small number of sectors (as in Acemoglu et al. (2012)), or a small number of inputs (as in Koren and Tenreyro (2013)) are the drivers of aggregate volatility. We also build on the literature of multiproduct firms. For example, Bernard, Redding, and Schott (2010) allow for the endogenous expansion of the firm but do not consider the risk dimension of this activity. Other related papers include Arkolakis (2010), Bloom et al. (2013), and Gourio and Rudanko (2014). Arkolakis (2010) develops a model of customer capital through advertisement, which is one of the elements of our intangible expenditures measure. Bloom et al. (2013) measure the effects of management expenditures (also within our definition of market exposure costs) on Indian firms, and Gourio and Rudanko (2014) develop a search model to analyze how intangible expenses affect firm dynamics.

The paper is organized as follows: Section 2 presents the empirical facts regarding the risk distribution across firms and over the business cycle, using Compustat and Kauffman Firm Survey (KFS) data. Sections 3 and 4 present a firm dynamics model with endogenous expansion and contraction of firms to capture the evidence presented in Section 2. Section 5 calibrates the model to the distribution of firms in the United States and discusses the workings of the model. Section 6 presents the main results and compares the empirical\textsuperscript{4}Baker and Bloom (2013) tackle the issue of causality between first- and second-moment shocks using disasters as natural experiments, finding that both are significant in explaining GDP growth.

\textsuperscript{4}Baker and Bloom (2013) tackle the issue of causality between first- and second-moment shocks using disasters as natural experiments, finding that both are significant in explaining GDP growth.
evidence with model outcomes. Section 7 focuses on the relationship between volatility and measures of market exposure. Section 8 concludes the paper.

2 Idiosyncratic Risk and Business Cycles

In this section, we present evidence on the level of idiosyncratic risk and its cyclical components using our sample. These are well-known facts in the literature; specifically, firm-level risk is countercyclical and is related to firm size (larger firms tend to be less volatile).

Our main empirical facts come from Compustat and consist of annual accounting data for publicly listed U.S. firms.⁵ We use data from 1960 to 2012, consisting of an unbalanced panel of more than 8,400 firms for a total of 241,308 firm-year observations. Compustat data are subject to selection bias as described by Davis et al. (2007). Because these firms are relatively larger and older than those that are not in Compustat, they are likely to be less volatile (see Castro, Clementi, and Lee (2014)). We try to address these differences by controlling for age and size and by using data from the KFS, which is based on a sample of small firms, to derive some of our results.⁶ The KFS provides a large panel of data on “young” businesses. Firms in the sample were founded in 2004 and have been tracked annually.⁷ This panel was created using a random sample from Dun and Bradstreet’s database of new businesses. The target population consisted of new businesses that were started in 2004 in the United States and excluded any branch or subsidiary that was owned by an existing business or was inherited from someone else. The sample for the first survey consisted of 4,928 businesses.

As in Castro, Clementi, and MacDonald (2009), our proxy for firm-level idiosyncratic risk is the portion of sales’ growth that is not explained by industry effects, time effects, or

---

⁵Appendix A1 provides a detailed description of our sample and the construction of the key variables and the matching procedure.

⁶As we discuss in Section 6.4, we combine Compustat Fundamental with Compustat Segment data and the U.S. Census Bureau’s LBD to provide direct evidence on the mechanism of the model. While the LBD contains nearly every firm in the U.S. economy, it does not provide information on total revenues at the firm level (and, regardless, we only report results from LBD firms that have been matched to Compustat data). The Census Bureau data sets, such as the Longitudinal Research Database (LRD), include revenue data but consist of limited samples for specific sectors only. These limitations prevented us from conducting the full experiment (all sectors, all firms) using only the LBD and/or the other data sources.

⁷Data are currently available for the years through 2008. See http://www.kauffman.org/kfs/ for a detailed description of the data and for the public-use microdata itself.
firm characteristics associated with growth such as age or size (measured by employment). The first step toward obtaining our measure of idiosyncratic risk is to estimate the following equation:

\[
\Delta \ln(sales_{ijt}) = \mu_i + \delta_{jt} + \beta_1 \ln(size)_{ijt} + \beta_2 \ln(age)_{ijt} + \epsilon_{ijt},
\]

where \(\Delta \ln(sales_{ijt})\) is the growth of real sales for firm \(i\), in industry \(j\), between period \(t\) and period \(t + 1\). The variable \(\mu_i\) is a firm fixed effect that accounts for unobserved persistent heterogeneity at the firm level (such as higher productivity or higher human capital of the entrepreneur). The variable \(\delta_{jt}\) denotes a full set of time- and industry-specific fixed effects. We allow for industry-specific size effects. The estimation of equation (1) is done using the fixed effects panel estimator with robust standard errors. In the KFS sample, we use revenues from sales of goods, services, or intellectual property as our measure of sales. In the Compustat Fundamental sample, our measure of sales is item #12, net sales. As is standard in the literature, size is defined in both samples as the number of employees. Age corresponds to the amount of time since a firm first appeared in the sample.

Once equation (1) is estimated, we can compute the error, or the pure idiosyncratic and unpredictable component of firms’ sales growth, \(\epsilon_{ijt}\). Following Castro, Clementi, and MacDonald (2009), we proxy firm-level risk by \(\epsilon^2_{ijt}\).

Figure 1 shows the relationship between detrended log-real GDP and two different aggregate measures of idiosyncratic risk: the detrended log-median \(\epsilon^2_{ijt}\) and the detrended cross-sectional standard deviation of \(\epsilon_{ijt}\).

---

8 Results are robust to a measure of idiosyncratic risk derived from total factor productivity (TFP) at the firm level. However, due to measurement issues associated with physical capital and factor shares in Compustat Fundamental and KFS data, our preferred firm-level volatility measure is based on sales growth. TFP results are available upon request.

9 We are able to explicitly control for age in our Compustat sample; however, because all firms in the KFS are of the same age (all firms began operating in 2004), this effect has been already factored in.

10 We use two-digit NAICS codes for firms in our KFS and Compustat samples.

11 The sample selection and the definition of all variables used in the analysis are described in detail in Appendix A1. Nominal variables are deflated using the two-digit sector-specific price deflator for value added from the U.S. Bureau of Economic Analysis (BEA).

12 The estimated dispersion for the Compustat sample is consistent with the estimates in Castro, Clementi, and MacDonald (2009) and Castro, Clementi, and Lee (2014). Consistent with the estimates in Comin and Philippin (2006), and Davis et al. (2007), we find that idiosyncratic risk for publicly traded firms increased for several decades until the early 2000s.
Note: This figure shows the correlation between the detrended log-real GDP and the detrended cross-sectional standard deviation of $\epsilon_{ijt}$ and between the detrended log-real GDP and the median $\ln(\epsilon_{ijt}^2)$, where $\epsilon_{ijt}$ is the unexplained portion of sales growth from equation (1). All variables are detrended using the Hodrick-Prescott (H-P) filter with a parameter of 6.25. Sources: Firm-level data are from Compustat Fundamental. Log-real GDP data are from the Federal Reserve Economic Data (FRED) of the Federal Reserve Bank of St. Louis.

The correlation between log-real GDP and the median $\ln(\epsilon_{ijt}^2)$ and between the log-real GDP and the cross sectional standard deviation of $\epsilon_{ijt}$ (our estimated measures of idiosyncratic risk) equals -0.46 ($p$-value=0.00) and -0.23 ($p$-value=0.09), respectively. The 10% confidence interval for these correlations is $[-0.62, -0.26]$ and $[-0.44, -0.01]$. The finding of countercyclical risk at the firm level is common in the literature. A survey of this literature can be found in Bloom (2014).

We present our results based on the median log($\epsilon_{ijt}^2$) and cross sectional standard deviation of $\epsilon_{ijt}$. The results are robust to different definitions of volatility. In particular, the correlation between the average log($\epsilon_{ijt}^2$) and the log-real detrended GDP is -0.22 (significant at the 10% level) and the correlation between the sales-weighted standard deviation of $\epsilon_{ijt}$ and the log-real detrended GDP is -0.09 (significant only at the 25% level).
In what follows, we explore the relationship between firm-level risk and the business cycle through the lens of our model.

3 Environment

We study an economy with $N$ markets (where $N$ is large but finite), a representative consumer, and a continuum of competitive firms. Time is discrete, and a period is set to one year. Firms can service each of the different markets by incurring sales and marketing expenses. We adopt a broad definition of “market” that applies to the market-product space, the market-location space, or a combination of the two.

3.1 Household Preferences and Endowments

The representative household derives utility from the consumption of the composite good $C_t$. More specifically, its preferences are given by $U(C_t)$, where $C_t$ is a composite of the consumption goods associated with each market $n$:

$$ C_t = \left[ \sum_{n=1}^{N} \left( \xi_{n,t} c_{n,t} \right)^{\rho} \right]^{1/\rho}, \quad 1 > \rho > 0, $$

(2)

where $c_{n,t}$ refers to consumption in market $n$, $\xi_{n,t}$ is a taste shock associated with market $n$ in period $t$, and $1/(1 - \rho) > 1$ is the elasticity of substitution across different markets. It is assumed that $\log \left( \xi_{n,t}^{\frac{1}{\rho}} \right) \sim N(0, \sigma^2_\xi)$, where $\alpha$ is the degree of decreasing returns to scale.

The household is endowed with one unit of labor that it supplies inelastically every period at wage $w_t$ and receives dividends $D_t$ through ownership of firms in the economy.

The ideal Dixit-Stiglitz price index is then,

$$ P_t = \left[ \sum_{n=1}^{N} \left( \frac{p_{n,t}}{\xi_{n,t}} \frac{\xi_{n,t}^{\frac{1}{\rho}}}{\left( \xi_{n,t}^{\frac{1}{\rho}} \right)} \right)^{\frac{\rho - 1}{\rho}} \right]. $$

(3)

Thus, the budget constraint that consumers face is

$$ P_tC_t \leq w_t + D_t. $$

(4)

\[\text{This normalization of the exponent of } \xi_{n,t} \text{ only makes the analysis cleaner further along.}\]

\[\text{Note that firms will make profits given the assumption of decreasing returns to scale.}\]
3.2 Firms and Technology

Firms are described by their productivity parameter $s$, which is constant over time. Production requires only one factor, labor. Given aggregate productivity $z_t$, a firm that has productivity $s$ and supplies to market $n$ produces with technology given by

$$q_{n,t}(s) = z_t s \ell_{n,t}^\alpha,$$

where $\ell_{n,t}$ is labor employed in the production of goods in period $t$. We assume that firm-level productivity takes values on a finite set $S$, is drawn from a distribution with probability distribution function (pdf) equal to $\mu(s)$, and is constant over the lifespan of the firm.

Firms can reach and sell to consumers in market $n$ by incurring sales, marketing, and other intangible expenses. We assume that these expenses are measured in units of labor and are increasing in the number of markets that the firms serve.\footnote{One interpretation of the demand differences corresponds to geographical distance or differences in products. Another interpretation is an increasing cost that arises from the complexity of serving many markets.} The total cost paid, measured in labor units, by a firm that serves $m$ markets is

$$w_t \Phi_t(m) = w_t \frac{\psi}{z_t} (m - 1)^{1+\nu}.$$

Firms have incentives to participate in more markets to access more customers; this results in diversification of market-specific risk even though diversification is not the firm’s objective. We are assuming that the firm runs an establishment (or has a physical presence) in each location/market it serves (a reasonable assumption for most industries with the possible exceptions of manufacturing, online trade, and certain sectors of the finance, and insurance, and information industries). The assumption that marketing and sales expenses are increasing in the number of markets that a firm serves reflects the notion that complexity in management is tied to some resource that is in fixed supply. This is consistent with the evidence that shows that a considerable set of firms operates in a small number of markets, but the distribution does not place all mass in only one point.\footnote{In the quantitative section of the paper, we use SIC codes, MSAs, and the number of establishments as different measures of market exposure. All three measures present a distribution of firms in a wide range with median to mean ratios significantly below one.} A different assumption (such as a linear or strictly concave functional form) would result in all firms expanding to all markets. Moreover, while we do not have access to a direct measure of the total cost of expansion to a new market, we observe selling, general and administrative expenses (SGA)
that are a proxy of market expansion costs since they refer to expenses on, for example, advertising, marketing, brand development, and research and development. Using this information and our measures of market presence, we estimate a cost function that links changes in SGA with firm size and changes in market presence. The empirical evidence is broadly consistent with a convex functional form.\footnote{More specifically, we classify firms according to their market presence into three categories: small, medium, and large, and we estimate how the change in expenses is correlated to the change in market exposure for firms in the different size categories. We find a convex form (either across all size categories or at least between two of them) for the three measures of market exposure (i.e., SICs, MSAs, and establishments). See Appendix A1.5 for details.}

In the model, sales and marketing expenses are treated as expenditures, which is a reflection of their large depreciation rate. Landes and Rosenfield (1994) observe that, for advertising, the annual depreciation rate was between 55\% and 100\%. Note that the aggregate shock $z_t$ appears in the expansion cost function, as we assume that workers employed in market expansion activities are affected by changes in $z_t$ just as their production counterparts. That is, we analyze the impact of productivity changes affecting all the inputs in the economy and not a relative change of productivity in production versus management technology. Note that it is critical for the results to have $z_t$ in the expansion cost function. If the expansion cost is independent of TFP, the general equilibrium effects on wages cancel the changes in productivity, and the model predicts no change in the firm-level market exposure and risk.

Firms maximize dividends, acting as price takers in each location in which they participate. Our model can also be interpreted as one in which the shocks are location-specific productivity shocks, with an immobile labor force and perfectly flexible demand. However, Foster, Haltiwanger, and Syverson (2012) find that much of the differences across firms is better explained by demand factors than productivity; we base our model’s demand shock setup on this observation.

### 3.3 Timing

The timing within a period is as follows:

1. $z_t$ is realized.

2. Firms choose the number of markets in which to operate.

3. Taste shocks $\xi_{n,t}$ are realized.
4. Taking prices as given, firms choose labor and produce.

5. Households consume.

This assumed timing simplifies the model solution because it abstracts from the specific market in which the firm chooses to participate and reduces the problem to choosing the number of markets the firm wants to reach as a function of the aggregate productivity $z_t$ and its own idiosyncratic productivity $s$. These assumptions imply that the solution to the dynamic problem of the firm boils down to solving a sequence of one-period problems.

4 Equilibrium

In this section, the paper presents the solution, the definition, and a characterization of the Competitive Equilibrium of the model.

4.1 Consumer’s Problem

The household’s optimal conditions imply that its demand for consumption good in location $n$ in period $t$ is:

$$c_{n,t} = \xi_{n,t}^{\frac{\varphi}{1-\varphi}} \left( \frac{p_{n,t}}{P^\rho} \right)^{\frac{1}{\varphi-1}} [w_t + D_t].$$  \hfill (7)

4.2 Firm’s Problem

Firms are perfect competitors in each market in which they participate. It is most intuitive to start by solving the firm’s problem at the production stage and then to derive the optimal condition for the number of markets. After the shocks $z_t$ and $\xi_{n,t}$ are revealed, the firm optimizes over the amount of labor to demand in each market they have previously chosen to serve.

The profit function for a firm in market $n$ is given by:

$$\pi_{n,t}(s) = \max_{l_{n,t}} \{ p_{n,t}q_{n,t}(s) - w_t l_{n,t} \}$$ \hfill (8)

subject to

$$q_{n,t}(s) = z_t s^{\ell_{n,t}}.$$ \hfill (9)
This delivers a standard labor demand in market $n$ for a firm with productivity $s$,

$$l_{n,t}(s) = \left( \frac{w_t}{p_{n,t} sz_t \alpha} \right)^{\frac{1}{\alpha-1}}. \quad (10)$$

This implies that profits for a firm with productivity $s$ in market $n$ are the following:

$$\pi_{n,t}(s) = (p_{n,t} sz_t)^{1-\alpha} w_t^{\alpha} \left( \alpha^{\frac{1}{\alpha-1}} - \alpha^{\frac{1}{1-\alpha}} \right). \quad (11)$$

At the beginning of the period (i.e., before $\xi_{n,t}$ are revealed but after firms observe $z_t$), using the previous expression in expected value, firms can derive the optimal number of markets they would like to serve. More specifically, firms enter the $m^{th}$ market as long as

$$E(\pi_{m,t}(s)) \geq w_t \left( \Phi(m) - \Phi(m-1) \right). \quad (12)$$

In other words, the firm will expand into $m$ markets as long as the expected profit in the last market is larger than the additional cost required to manage the last market. We denote by $m_t(s)$ the number of markets in which each firm, of productivity $s$, chooses to participate in period $t$.

### 4.3 Definition of Equilibrium

In any given period $t$, the Competitive Equilibrium is a set of labor $l_{n,t}(s)$ and number of markets $m_t(s)$ decision rules, a wage rate $w_t$, a vector of goods prices $\{p_{n,t}\}_{n=1}^N$, and a vector of distribution of firms with productivity $s$, participating in each market $n$, $\{\lambda_{n,t}(s)\}_{n=1}^N$, such that:

1. At the given wage rate, prices, each firm’s labor decision rule, and each firm’s optimal number of markets are the solution to problems (8) and (12).

2. The distribution of firms in market $n$ equals

$$\lambda_{n,t}(s) = \frac{\mu(s)m_t(s)}{N}. \quad (13)$$

\[\text{Our convexity assumption on the cost function } \Phi(m) \text{ ensures that the solution to the firm’s problem is unique.}\]
3. The labor market clears, that is,
\[ \sum_{s=\mathcal{S}} \sum_{n=1}^{N} \lambda_{n,t}(s) l_{n,t}(s) + \sum_{s=\mathcal{S}} \mu(s) \Phi(m_t(s)) = 1. \] (14)

4. The price \( p_{n,t} \) is such that it clears the \( n^{th} \) market, that is,
\[ \sum_{s=\mathcal{S}} \lambda_{n,t}(s) q_{n,t}(s) = c_{n,t}, \] (15)
where \( c_{n,t} \) is given by equation (7).

5. Aggregate dividends are
\[ D_t = \Pi_t - w_t \sum_{s=\mathcal{S}} \mu(s) \Phi(m_t(s)), \] (16)
where \( \Pi_t \) denotes the sum of profits across markets and is given by
\[ \Pi_t = \sum_{s=\mathcal{S}} \sum_{n=1}^{N} \lambda_{n,t}(s) \pi_{n,t}(s). \] (17)

With this definition established, we can characterize firms’ behavior and the aggregate equilibrium objects.

### 4.4 Characterization of the Equilibrium

From the price market clearing condition (equation (15)) and the optimal demand of goods (equation (7)), the equilibrium price in market \( n \) is
\[ p_{n,t} = \bar{\xi}_{n,t}^{\alpha(1-\alpha)} A_t, \] (18)
where \( A_t = \left[ P_t \left( \frac{(w_t + D_t) \tilde{s}_{t}^{-\alpha}}{w_t \tilde{s}_{t}^{1-\alpha} \sigma^{1-\alpha}} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \) and \( \tilde{s}_{t} = \frac{1}{N} \sum_{s=\mathcal{S}} \mu(s) m_t(s) s^{\frac{1}{1-\alpha}} \). Note that \( A_t \) is a function of aggregate productivity as well as the endogenous wage. Under the calibrated parameters that follow, \( A_t \) is countercyclical and is one of the driving forces of the countercyclicality of the dispersion of prices across markets.
Combining equation (12) (i.e., the equation that determines the number of markets for a given firm of productivity $s$) with the equation of the market clearing price in market $n$ that we just derived (equation 18), we find that firms will enter market $m$ only if

$$\frac{s^\frac{1}{\alpha} B_t}{\text{Expected Marginal Profit}} \geq w_t \left( \Phi(m) - \Phi(m - 1) \right),$$

where $B_t = e^{\frac{\sigma^2}{2}} \frac{1}{\alpha} \frac{\alpha}{1-\alpha} \left( \frac{1}{\alpha^{1-\alpha}} - \frac{1}{1-\alpha} \right) A_t^{\frac{1}{\alpha}}$. The expected marginal profit has two components: One is firm-specific and is a function of its productivity $s$, and the other has to do with the economy as a whole and depends on parameters in time $t$ (such as the wage rate $w_t$ and aggregate productivity $z_t$). The larger the firm-specific productivity, the larger the expected profit, given the assumption of decreasing returns to scale. The effects from the economy wide parameters go in the following direction: Higher levels of aggregate productivity generate higher expected profits, while higher wages reduce expected profits. Both aggregate effects are multiplied by the firm-specific productivity, generating an asymmetric response of productivity to the aggregate environmental parameters.

The labor market clearing condition (equation (14)) implies that

$$w_t + D_t = \frac{w_t}{\alpha} \left[ 1 - \sum_{s=1}^{\bar{s}} \mu(s) \Phi(m_t(s)) \right],$$

$$\Rightarrow \frac{\Pi_t}{w_t} = \left[ 1 - \sum_{s=1}^{\bar{s}} \mu(s) \Phi(m_t(s)) \right] \left( \frac{1}{\alpha} - 1 \right),$$

so, in equilibrium, the price index $P_t$ becomes

$$P_t = \left[ 1 - \sum_{s=1}^{\bar{s}} \mu(s) \Phi(m_t(s)) \right]^{1-\alpha} \frac{w_t}{\alpha z_t} \left( N e^{\frac{\sigma^2}{2}} \right)^{\frac{\alpha\rho-1}{\rho}}. \quad (20)$$

To solve for an equilibrium, we need to solve a system of three aggregate equations ((14),(16),(17)) and three unknowns $\{\sum_{s=1}^{\bar{s}} \mu(s) \Phi(m_t(s)), w_t, s_t\}$ such that they are consistent with firm-level decisions.

### 4.4.1 Elasticity of Firm/Plant-Level Volatility and Business Cycle Properties

In this section, we analyze the coefficient of variation of firm-level total factor productivity (TFPR) because it gives a compact expression that is useful for building intuition, and
generates closed form solutions to moments related to firm-level risk.\textsuperscript{20} Conditional on the aggregate shock $z_t$, the model predicts a relationship between the firm’s idiosyncratic productivity $s$ and its volatility. The coefficient of variation of the weighted sum of TFPR to which the firm is exposed, conditional on serving $m$ markets, is

$$CV_t(s) = \frac{\sqrt{\text{Var}(\sum_{n=1}^m (sz_n p_{n,t})^{1/(1-\alpha)})}}{E(\sum_{n=1}^m (sz_n p_{n,t})^{1/(1-\alpha)})}.$$  \hfill (21)

Note that equation (21) is presented as the coefficient of variation for a given firm with productivity $s$. However, an identical expression can be derived if we focus on the coefficient of variation across establishments, conditional on firm-level productivity. The analysis that follows is consistent with either interpretation.\textsuperscript{21} From equation (19), it is evident that the firm will participate in an increasing number of markets as a function of its productivity $s$. Therefore, the coefficient of variation of the firm’s TFPR is a function of the firm’s productivity through its effect on the optimal number of markets that the firm will serve. Then, using the optimal market exposure decision, the coefficient of variation for a firm with productivity $s$ can be written as

$$CV_t(s) = \sqrt{\frac{e^{\sigma_x^2} - 1}{m_t(s)}}.$$  \hfill (22)

This result is based on the assumption that the shocks $\xi_{n,t}$ are i.i.d. However, as long as the shocks are not perfectly correlated (which would make them, in fact, one unique shock), the coefficient of variation decreases as the firm is exposed to an increasing number of shocks. This can be seen by analyzing the variance-covariance matrix of the shocks $\xi_{n,t}$. Given that they have the same variance, the variance-covariance matrix can be rewritten in terms of the correlation coefficient between two shocks multiplied by the common variance term. The coefficient of variation is then given by the following expression for the case of a firm that serves $m_t^*(s)$ markets:

$$CV_t(s) = \sqrt{(e^{\sigma_x^2} - 1) \sum_{u=1}^n \sum_{v=1}^n \rho_{uv} m_t(s)},$$  \hfill (23)

\textsuperscript{20}The next section presents a set of testable implications to connect the model with the empirical evidence using a set of moments that are more reliable empirically but for which it is not possible derive closed form expressions.

\textsuperscript{21}As we discuss in the next section, consistent with the model, the data show that volatility is countercyclical at both levels of aggregation.
where $\rho_{uv}$ is the correlation coefficient between the shocks $u$ and $v$. In the case of i.i.d shocks, the double sum equals the number of shocks, and in the case of perfectly correlated shocks, it equals the square of the number of shocks. Anything in between means that the coefficient of variation drops as the number of varieties increases.

A key prediction of the model that can be derived from equation (22) is that for firms that expand in booms and contract in recessions, the coefficient of variation at the firm level is countercyclical. Thus, the model asks us to split the data sample between those firms that adjust the number of markets to which they are exposed and those that do not (as opposed to, for example, splitting the sample by firm size). We perform this critical empirical test in Section 5.1 and show that the model is consistent with the empirical evidence. Moreover, we also observe that the variance of the weighted sum of TFPR to which the firm is exposed is countercyclical, given that the variance of prices follows the term $A_t$ in equation (18), which, at the calibrated parameters, is countercyclical. The fact that $A_t$ is countercyclical implies a countercyclical variance in prices across markets, which is consistent with the evidence provided by Berger and Vavra (2011). Further, under our assumption of one establishment per market, the cross-sectional variance of TFPR at the plant level is countercyclical, as reported by Kehrig (2011).

## 5 Calibration

This section presents the calibration of the model. Using this calibration, we then explore further the workings of the model to study the business cycle properties of firm-level risk.

We assume that firm-level productivity is distributed following a log-normal distribution with mean $\bar{s}$ and standard deviation $\sigma_s$, so $\log(s) \sim \mathcal{N}(\bar{s}, \sigma_s^2)$. The number of markets, $N$, only determines the scale of the problem. We set its value to 100, but this number is irrelevant to our results. We assume that $z_t \in \{z_B, z_G\}$ with transition probability $\Gamma(z', z)$ and denote by $\Gamma_{kj}$, the $(j,k)$th element of $\Gamma(z', z)$. We normalize $z_G = 1$. This leaves us with 10 parameters to calibrate:

$$\{\rho, \sigma_\xi, \alpha, \nu, \psi, \bar{s}, \sigma_s, z_B, \Gamma_{GG}, \Gamma_{BB}\}. \quad (24)$$

We calibrate the preference parameter $\rho$ to 0.83, a standard parameter in the trade literature.\footnote{In a model with monopolistic competition, this would imply a 20% markup. Recall, though, that there is no markup in our model since we look for a Competitive Equilibrium.} We set $\alpha = 0.64$, also a standard value in the literature, which matches the labor
share of output. Once we have \( \rho \) and \( \alpha \), we use equation (22) to determine \( \sigma_{\xi} \). More specifically, we set \( \sigma_{\xi} = 3.04 \) to match the standard deviation of \( \log(\hat{\epsilon}^{1-\alpha \rho}) \), where \( \hat{\epsilon} \) is estimated from our KFS sample using equation (1). This is a good approximation under the assumption that these very small firms are exposed to only one market, and it allows us to pin down the dispersion of market-specific risk.\(^{23}\) To calibrate \( \Gamma_{GG} \) and \( \Gamma_{BB} \), we estimate the fraction of booms and recessions with data from the NBER. More specifically, for a given year, we set a recession indicator to one if two or more quarters in that year were dated as part of a recession by the NBER. Then, we identify years in which the indicator equals one with our periods of \( z = z_B \) and construct a transition matrix. The estimate of \( \Gamma_{kj} \), the \((j,k)\)th element of the aggregate state transition matrix, is the ratio of the number of times the economy switched from state \( j \) to state \( k \) to the number of times the economy was observed to be in state \( j \). We find that \( \Gamma_{GG} \) is 0.86 and that \( \Gamma_{BB} \) is 0.43. This implies that the unconditional probabilities of \( z_G \) and \( z_B \) are 0.80 and 0.20, respectively. Finally, we set \( z_B = 0.90 \). This amplitude of the support for \( z \) is in line with the data presented by Gordon (2005), where the average peak-to-trough distance in terms of output gap is 7.9% for the period 1945-2005.\(^{24}\)

The four remaining parameters \{\( \bar{s}, \sigma_s, \nu, \psi \}\} (the mean and standard deviation of the distribution of productivity (at the firm level) as well as the two parameters that control the cost of firm expansion respectively) are jointly calibrated so that the average model firm size distribution (in terms of the number of employees) matches the year 2008 firm size distribution from the Census Bureau’s Business Dynamics Statistics (BDS) data series.\(^{25,26}\) The identification of these parameters derives from the fat tail in the firm size distribution.\(^{27}\)

---

\(^{23}\)The KFS sample does not contain information on market exposure. The KFS focuses strictly on new businesses that most likely operate in only one market. Note also that from our Compustat-LBD link, we observe firms with only one establishment (i.e., exposed to only one market). They represent approximately 1% of total sales and total workers in 2011. However, since Compustat focuses on publicly listed firms, it makes us believe that these are not representative firms exposed to a single market. Moreover, due to data limitations, the category with only one establishment could potentially include firms in Compustat that are not perfectly matched with establishments in the LBD sample introducing an additional factor of measurement error to the calibration.

\(^{24}\)This is computed by averaging the peaks and the troughs during the sample time period and taking their difference. Note that this time period excludes the 2007-2009 recession and the Great Depression, as well as all the previous recessions that had a much stronger impact in terms of GDP.

\(^{25}\)The BDS covers all employer firms (about 5 million) that, in total, employ approximately 120 million workers. For 2008, we observe 5,186 firms in our Compustat Fundamental sample, which is less than 0.1% of the total number of firms.

\(^{26}\)In particular, we choose the parameters by minimizing the sum squared error of the distance between the model moments to the data moments where each moment is the fraction of firms in different size bins (as specified by BDS). Effectively, since we have four parameters and six moments, this is an overidentified model.

\(^{27}\)To compute the average distribution in the model, we draw 100,000 firms from the idiosyncratic pro-
Table 1 describes the main parameters of the model.

Table 1: Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preference Parameter</td>
<td>$\rho$</td>
<td>0.83 Elasticity of Substitution</td>
</tr>
<tr>
<td>Dispersion Taste Shock</td>
<td>$\sigma_\xi$</td>
<td>3.04 $SD(\log(\hat{\epsilon}^{\frac{\alpha}{\rho}}))$ from KFS</td>
</tr>
<tr>
<td>Labor Share</td>
<td>$\alpha$</td>
<td>0.64 Standard Value</td>
</tr>
<tr>
<td>Aggregate Prod.</td>
<td>$z_G$</td>
<td>1 Normalization</td>
</tr>
<tr>
<td>Aggregate Prod.</td>
<td>$z_B$</td>
<td>0.90 Peak to Trough Amplitude</td>
</tr>
<tr>
<td>Transition Prob.</td>
<td>$\Gamma_{GG}$</td>
<td>0.86 NBER Boom/Recession</td>
</tr>
<tr>
<td>Transition Prob.</td>
<td>$\Gamma_{BB}$</td>
<td>0.43 NBER Boom/Recession</td>
</tr>
<tr>
<td>Cost Function</td>
<td>$\nu$</td>
<td>0.56 Firm Size Dist. (see Table 2)</td>
</tr>
<tr>
<td>Cost Function</td>
<td>$\psi$</td>
<td>0.46 Firm Size Dist. (see Table 2)</td>
</tr>
<tr>
<td>Mean Productivity</td>
<td>$\bar{s}$</td>
<td>ln(1.7) Firm Size Dist. (see Table 2)</td>
</tr>
<tr>
<td>Std. Dev. Productivity</td>
<td>$\sigma_s$</td>
<td>0.4 Firm Size Dist. (see Table 2)</td>
</tr>
</tbody>
</table>

The accuracy of the match for the distribution of employees across firms is shown in Table 2.28

Table 2: Firm Size Distribution - Number of Employees

<table>
<thead>
<tr>
<th>Employment Size</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firms with 1 to 4 employees</td>
<td>0.610</td>
<td>0.601</td>
</tr>
<tr>
<td>Firms with 5 to 9 employees</td>
<td>0.176</td>
<td>0.209</td>
</tr>
<tr>
<td>Firms with 10 to 19 employees</td>
<td>0.107</td>
<td>0.100</td>
</tr>
<tr>
<td>Firms with 20 to 99 employees</td>
<td>0.089</td>
<td>0.064</td>
</tr>
<tr>
<td>Firms with 100 to 499 employees</td>
<td>0.015</td>
<td>0.021</td>
</tr>
<tr>
<td>Firms with 500 or more employees</td>
<td>0.003</td>
<td>0.005</td>
</tr>
</tbody>
</table>

We observe that the model adequately replicates the firm size distribution for all size classes.

The model also performs well when compared with other moments related to the core story of the paper; in particular, to moments related to the joint distribution of markets and productivity distribution and simulate the model 20 times for 50 periods in each simulation in which aggregate shocks are drawn from $\Gamma(z, z')$.

28Given that in the model we have a measure of firms equal to one, the model distribution reported was adjusted for the difference in mean.
employment. In the data, 90% of firms have fewer than 20 employees and are almost all single establishment (in fact, the average number of establishments per firm, conditional on firms employing fewer than 20 employees is 1.01 establishments). Then, in the data (BDS) the average number of establishments per firm is equal to 1.267. The model generates 91% of single establishment firms and an average of 1.186 establishments per firm. These numbers reassure us that the functional forms and calibrated parameters within the model are generating reasonable quantitative results in the dimensions relevant for the main results. Other dimensions worth looking at are related to the business cycle dynamics. The relevant ones are real wages and dispersion of productivity and prices. Real wages in the model are procyclical, and, at the calibrated parameters, the term $A$ in equation 18 is countercyclical, generating countercyclical cross-sectional variances in prices, and also in TFPR at the firm level. Moreover, consumption, investment, and productivity are by construction procyclical. Finally, the price level is countercyclical. All these are in line with the business cycle evidence in the US for the post-war period.\footnote{They are close to those generated by the standard business cycle model.}

5.1 Workings of the Model

In this section, we further explore the workings of the model and present intuition for the main result and a set of testable implications that we compare with the data in the following section.

Our first step is to examine how a movement in $z$ affects market exposure and the cross-sectional distribution of firms in the model. Figure 2 shows the effect of the aggregate shock on the endogenous number of markets that each firm serves.
Figure 2: Market Exposure over the Business Cycle

Note: This figure presents the optimal number of markets conditional on firm productivity over the business cycle $m_t(s|z_t)$ for $z_t \in \{z_L, z_H\}$ as well as the productivity distribution of firms $\mu(s)$. $Boom = m_t(s|z_H)$, and $Bust = m_t(s|z_L)$.

Changes in $z$ have a nonmonotone impact on firm-level decisions. The most productive firms expand in response to an increase in the aggregate shock, whereas the less productive firms change their market exposure only slightly. This cyclical expansion and contraction is in line with the procyclical net entry rate found for the manufacturing sector by Lee and Mukoyama (2012). Moreover, the uneven response of the change in the number of markets by firm size is also consistent with the data (see Figure 4 and Table 5 that follow).

Consistent with the characterization of the equilibrium, changes in $z$ generate an endogenous change in firm-level risk. The impact of changes in $z_t$ on the coefficient of variation from equation (22), by productivity level, is depicted in Figure 3.
It is clear that the impact of TFP shocks on the coefficient of variation is not monotone by productivity level. The average firm-level volatility decreases by 0.023%, and the average firm-level risk for the top 10% and 1% of firms falls by 1.4% and 3.9%, respectively. This uneven change in volatility is a direct consequence of firms’ asymmetric response to variations in $z$. The fixed cost of expanding to an additional market creates regions of inaction. We observe that those regions of inaction become smaller as productivity increases. Moreover, the impact of the cycle on the coefficient of variation decreases with productivity in the regions in which firms adjust the number of markets in which they operate, since conditional on $z$, high-productivity firms are better diversified than those with low productivity. From this discussion, we conclude that (a) the idiosyncratic volatility of firms engaged in market expansions and contractions should be countercyclical, and (b) the idiosyncratic volatility of those firms not adjusting should be acyclical. As we show in Section 6.2, this is consistent with the empirical evidence.

In this model with firm heterogeneity, the endogenous variation in the number of markets also has an effect on measured aggregate TFP. As is standard, aggregate measured TFP is computed as aggregate production over aggregate labor (the only input of production) to
the power $\alpha$. In this model, with a unit measure of labor, measured aggregate TFP in period $t$ equals total output $\sum_{n}^{N} \sum_{s} q_{t,n}(s) \lambda_{t,n}(s)$. As $z$ increases, and more productive firms expand proportionally to a larger set of markets, there is an additional positive effect on measured TFP over the change in $z$. This endogenous amplification effect on measured TFP is nonnegligible and amounts to a further 13% increase in measured TFP beyond the effect of the aggregate shock $z$.

In summary, the model predicts that

(i) Firm level risk is countercyclical.

(ii) Market exposure is procyclical.

(iii) Those firms adjusting the number of markets are larger than those not adjusting.

(iv) Small and less productive firms display larger firm level risk than large firms.

(v) Firm level risk is countercyclical only for those adjusting the number of markets but not for those exposed to the same number of markets over the business cycle.

(vi) A similar cyclical pattern should be observed when comparing large vs small firms with large firms’ risk being countercyclical.

(vii) The elasticity of firm level risk to market exposure is negative.

We confront these model predictions with the data in the following section.

6 Main Results and Testable Implications

In this section, we present the main results of the paper —cyclical properties of firm level risk and market participation measures— together with a set of testable implications where we compare the predictions of the theoretical model with the empirical evidence.

When looking at the data, we combine several sources to cover many angles.\textsuperscript{30} As we discussed in Section 2, we derive our measures of firm-level risk from the Compustat Fundamental and KFS data sets after estimating equation (1). Since the core Compustat sample covers five decades, by focusing on that data set, we can analyze how firm-level risk moves over the business cycle. Furthermore, we match our Compustat Fundamental panel with

\textsuperscript{30}We direct the interested reader to the Appendix for a detailed explanation of how our sample is constructed and how the match across data sets is performed.
two other data sets. First, we link Compustat Fundamental with the Compustat Segment data. The Segment data provide information on sales for each firm by four-digit SIC codes, at an annual frequency for most years in our sample.\textsuperscript{31} Like Bloom, Schankerman, and Van Reenen (2013), we use line of business (i.e., SIC codes) information as one of our direct measures of product market exposure.\textsuperscript{32} Second, we match Compustat Fundamental with the U.S. Census Bureau’s LBD data set. This considerable task allowed us to obtain information on the number of establishments, as well as their location (at the Metropolitan Statistical Area (MSA) level), for firms in our Compustat sample. The number of establishments and number of MSAs in which a firm is operating provide two new measures of market exposure.\textsuperscript{33} Finally, we also use the Census Bureau’s BDS data, since it is possible to derive the average number of establishments by firm size (employment), year by year starting in 1977, and analyze its cyclical properties.

6.1 Firm-Level Risk and Business Cycles

We start by presenting our main result, the negative correlation between firm-level risk and the business cycle. Using the pseudo panel of firms from the model, we estimate firm-level volatility as we did in the data. That is, we compute the sales’ growth (in logs) from period $t$ to $t+1$ and regress it against a firm fixed effect, size (in terms of number of employees), and a time dummy capturing aggregate conditions (booms or recessions). We estimate

$$\Delta \ln(sales_{it}) = \delta_0 s_i + \delta_1 z_t + \delta_2 \ln(size)_{it} + \epsilon_{it},$$  \hspace{1cm} (25)$$

and obtain the errors $\epsilon_{it}$ from equation (25) to derive our measure of firm-level risk $\ln(\epsilon^2_{it})$, as in Castro, Clementi, and MacDonald (2009). We study the cyclical properties of this measure of firm-level risk. The findings are summarized in Table 3.

\textsuperscript{31}The sample starts in 1977 and provides 183,991 firm/year observations.

\textsuperscript{32}On average, each firm reports 2.68 industry codes every year (approximately five industry codes per year when weighted by sales).

\textsuperscript{33}We understand that these measures of market reach (SIC codes, establishments, and geographical location) have weaknesses. For example, business lines, even at the four-digit level, can be associated with R&D; the construction of a plant has an investment component that we do not consider in the model, and firms in some industries might ship their goods to various locations from only one establishment (this is more problematic in manufacturing than in services or retail). However, the fact that most of our results are robust across market exposure measures is evidence that the mechanism in the model is present in the data.
We find that when we look at all the entire sample as well as the top 5% of firms from the model (closest to the Compustat sample), measures of firm-level risk are countercyclical and correlations are significantly different from zero. More specifically, the correlation between median $\log(\epsilon_{it}^2)$ and GDP equals -0.20 (significant at the 10% level), and the correlation between the cross-sectional std($\epsilon_{it}$) and GDP is -0.345 (significant at the 1% level). These correlations are -0.421 (significant at the 1%) and -0.179 (significant at the 7%) if we compute them using all the firms from the model. In our panel of U.S. firms (i.e., Compustat Fundamental), as we discussed in Section 2, the correlation between median idiosyncratic risk and GDP is -0.46 with a 90% interval equal to $[-0.625, -0.257]$, and the correlation between the cross-sectional standard deviation of risk and GDP is -0.23 with a 90% interval equal to $[-0.439, -0.017]$.

### 6.2 Market Exposure and Business Cycles

The model predicts that high-productivity firms respond to the changes in aggregate productivity by expanding and contracting the number of markets in which they operate (with the corresponding change in selling expenses), but these observed changes in market participation do not translate into a monotone relationship in terms of the cyclicality of the volatility as shown in Figure 3. The reason is that high-productivity firms are already exposed to a large number of markets even in bad times. Therefore, their reaction to aggregate productivity changes does not affect their volatility much, but the firms that operate in relatively few markets during recessions and expand in booms are the ones that experience large fluctuations in their volatility over the cycle.

---

34 This measure of firm-level risk derived from sales’ growth has been widely used in the literature. However, firm-level risk can also be derived from an autoregressive model of $\log(sales)$ such as an AR(1). In appendix Section A1.6 in the Appendix, we present evidence that shows these alternative measures are also countercyclical.
As a first step, we test whether market exposure is procyclical for those firms engaged in market expansions and contractions (i.e., predictions derived from Figure 2). To do so, Figure 4 reports how the average number of markets that firms participate in (as measured by four-digit SIC codes, number of establishments, number of MSAs, and the product of SIC codes and establishments) moves for firms that change (firms that we refer to as “changers”).

Figure 4: Average Change in Market Exposure for “Changers”

Note: Detrended GDP corresponds to detrended log-real GDP. The market exposure measure corresponds to the number of four-digit industry codes (i.e., line of business), number of establishments a firm operates, number of MSAs in which a firm has establishments, and the product of establishments and SIC codes. “Changers” refers to firms that change market exposure measure in a given period. We report the average change in the market exposure conditional on being a “changer.” GDP is detrended using a Hodrick-Prescott (H-P) filter with a parameter 6.25. Source: Data are from the link Compustat Fundamental to Compustat Segment and the link Compustat Fundamental to LBD data.

As predicted by the theory, the average change of the changers behaves procyclically for
all of our definitions of market participation. The correlation coefficients between detrended GDP and average change in market participation for changing firms is 0.28 for SIC codes, 0.48 for establishments, 0.51 for MSAs, and 0.31 for SICs×establishments with respect to detrended GDP.

While not as direct as the measures of market exposure presented so far, guided by our model, we also look at how expenses associated with market reach move with the business cycle. In the model, the cost of market exposure is given by $w_t \Phi_t(m)$ and is predicted to be procyclical given that it is a function of $m$. In our Compustat sample, we look at Selling, General and Administrative (SGA) expenditures as the measure associated with operating the firm and as a function of the firm’s complexity. We also look at the advertising component within SGA expenditures, as it should closely follow the market reach of the firm. Figure 5 shows the correlation between our indirect measures of market exposure and GDP.

Figure 5: Market Reach Expenses and Business Cycles

Note: Log-real (SGA and Advertising) expenses correspond to the median of the observed distribution in any given year. Series are detrended using a Hodrick-Prescott (H-P) filter with parameter 6.25. GDP data are available since 1947. In Panel (i), log-real expenses are measured by Selling, General, and Administrative expenses. In Panel (ii), log real expenses are measured by Advertising Expenses. Source: Firm-level data come from the Compustat Fundamental sample and log-real GDP from the Federal Reserve Economic Data (FRED) of the Federal Reserve Bank of St. Louis.
Figure 5 shows that log-real GDP and our measures of market reach expenses are positively correlated. The correlation is 0.283 (significant at the 5% level) and 0.149 (significant at the 10% level) when expenses are measured as SGA expenses and Advertising expenses, respectively. In the model, as a result of the contractions and expansions of selling expenses over the business cycle, the labor force that is employed in market reach activities accounts for 17.8% of the total labor force during boom years and is reduced by 2.65 percent in the low-TFP years; however, the model overpredicts the correlation between GDP and median selling expenses since this correlation for the most productive 5% of firms in the model is 1.35

6.3 Link between Market Exposure and Firm Size

Another implication of the model is that firms that react to the cycle by changing the number of markets in which they participate are larger, on average, than those that do not change their market exposure. Table 4 shows descriptive statistics for the linked Compustat Fundamental data to Compustat Segment data and our combined Compustat Fundamental-LBD data set, for the “changers” and “non changers” groups. Also, consistent with the implications described in Figure 2, we find that the fraction of firms that adjust their market participation is acyclical. The fraction of firms that adjust their market participation is between 20% and 56% of the total number of firms, depending on the definition of market, and this fraction is uncorrelated with detrended GDP.36

35Sampling from the top of the model’s distribution resembles the Compustat Fundamental data sample since Compustat corresponds to the right tail of the firm-size distribution.
36Market participation is measured by the number of products (or SIC codes) for which they report sales, the number of establishments they operate, the number of MSAs in which they have a presence, or the product of establishments and SIC codes. The fraction of changing firms corresponds to 20%, 52%, 42%, and 56% of the total number of firms, respectively, and this fraction is uncorrelated with GDP.
Table 4: Descriptive Statistics - “changers” versus “non-changers”

<table>
<thead>
<tr>
<th></th>
<th>All Sample</th>
<th>“Changers”</th>
<th>“Non-changers”</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>std dev</td>
<td>mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mean</td>
</tr>
<tr>
<td>Number of SIC codes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1607.06</td>
<td>8678.85</td>
<td>2739.44</td>
</tr>
<tr>
<td>Emp.</td>
<td>8.42</td>
<td>33.90</td>
<td>11.72</td>
</tr>
<tr>
<td>SGA</td>
<td>268.78</td>
<td>1416.04</td>
<td>464.69</td>
</tr>
<tr>
<td>Adv.</td>
<td>50.12</td>
<td>272.79</td>
<td>80.45</td>
</tr>
<tr>
<td>SICs</td>
<td>2.62</td>
<td>1.86</td>
<td>3.32</td>
</tr>
<tr>
<td>Number of Establishments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1553.35</td>
<td>8307.57</td>
<td>2316.25</td>
</tr>
<tr>
<td>Emp.</td>
<td>8.33</td>
<td>33.44</td>
<td>12.30</td>
</tr>
<tr>
<td>SGA</td>
<td>260.00</td>
<td>1370.46</td>
<td>399.23</td>
</tr>
<tr>
<td>Adv.</td>
<td>48.64</td>
<td>264.46</td>
<td>77.22</td>
</tr>
<tr>
<td>Establ.</td>
<td>115.85</td>
<td>564.74</td>
<td>215.03</td>
</tr>
<tr>
<td>Number of MSAs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1553.35</td>
<td>8307.57</td>
<td>2582.83</td>
</tr>
<tr>
<td>Emp.</td>
<td>8.33</td>
<td>33.44</td>
<td>13.66</td>
</tr>
<tr>
<td>SGA</td>
<td>260.00</td>
<td>1370.46</td>
<td>438.54</td>
</tr>
<tr>
<td>Adv.</td>
<td>48.64</td>
<td>264.46</td>
<td>85.73</td>
</tr>
<tr>
<td>MSAs</td>
<td>32.24</td>
<td>92.75</td>
<td>65.52</td>
</tr>
<tr>
<td>Number of SIC * Establishments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1553.35</td>
<td>8307.57</td>
<td>2108.33</td>
</tr>
<tr>
<td>Emp.</td>
<td>8.33</td>
<td>33.44</td>
<td>11.39</td>
</tr>
<tr>
<td>SGA</td>
<td>260.00</td>
<td>1370.46</td>
<td>363.59</td>
</tr>
<tr>
<td>Adv.</td>
<td>48.64</td>
<td>264.46</td>
<td>70.89</td>
</tr>
<tr>
<td>SIC*Establ.</td>
<td>405.99</td>
<td>2552.05</td>
<td>704.83</td>
</tr>
</tbody>
</table>

Note: Sales; Selling, General and Administrative expenses (SGA); and Advertising Expenses (Adv.) are expressed in millions of dollars deflated using BEA two-digit SIC price deflators for value added. Employment (Emp.) is expressed in thousands of employees. Source: Data are from the link Compustat Fundamental to Compustat Segment and the link Compustat Fundamental to LBD data. We formally test for statistically significance difference to find that all the moments for “changers” are significantly different than the moments for “non-changers” at the 5% level.

We find that, when conditioning on the number of SIC codes, the “changers” are about twice as big as the “non-changers” in terms of sales, SGA expenses, and advertising expenses. They are 50% larger in terms of the number of employees and 33% larger in terms of the
number of products they offer (as measured by the number of SIC codes for which they report sales). But the “changers” are between five and nine times larger than “non changers” when we condition on the number of establishments each firm operates or the number of MSAs in which each firm is present and look at sales, employees, or expenditures. “Changers” are 26 times larger than “non-changers” in terms of the number of establishments, and 31 times larger in terms of the product of SIC codes and establishments.

We can split the sample by size and look at whether the cyclicality of market exposure for large firms is consistent with that of “changers”. In this case, the measure of market exposure corresponds to the number of active establishments by firm. Using the publicly available BDS data, it is possible to derive the average number of establishments by firm size (employment), year by year from 1977 to 2009, and analyze its cyclical properties. The last three columns of Table 5 present evidence on the change in the number of establishments (plants) by firm size between periods when log-real GDP is above and below trend as well as the correlation between the number of establishments per firm and GDP (conditional on firm size).

This table shows that for most size categories, there is a minimal change in the number of establishments per firm between periods when GDP is above trend and those when it is below trend (the correlation with detrended GDP presents similar results). However, for large firms (those with with at least 5,000 workers), the number of plants is larger when GDP is above trend than when GDP is below trend, and the elasticity with detrended GDP is close to 1. This difference is economically and statistically significant. This is relevant in terms of activity and observed dispersion because, as Table 5 shows, these firms account for about 30% of total employment and represent the 1,450 largest firms in the economy (just under half of the number of firms included in our Compustat sample). Furthermore, since the change in the number of plants is approximately eight between periods when GDP is above trend and periods when GDP is below trend in these 1,450 firms, and these firms employ about 75 workers per plant, the change in employment coming only from this margin amounts to 1.29% of total private nonfarm employment.

37The BDS data are compiled from the Longitudinal Business Database (LBD). The LBD is a longitudinal database of business establishments and firms covering the years from 1977 on. The BDS series provide annual statistics by firm size on the number of establishments as well as gross job flows for the entire economy.

38This is consistent with the evidence presented in Moscarini and Postel-Vinay (2012) that the net job creation of large firms or establishments comoves negatively and more strongly with aggregate unemployment than the net job creation of small employers at business cycle frequencies.

39Note that the reported numbers in Table 5 correspond to detrended averages from the period 1977-2009. The total number of firms for the year 2009 was around 3,000.

40Another margin of adjustment for firms is the number of workers per plant. As we show in Table 20 in
Table 5: Number of Plants per Firm over the Business Cycle

<table>
<thead>
<tr>
<th>Firm Size (# Workers)</th>
<th>Avg. # of Firms (in 1000’s)</th>
<th>Fraction Total Emp. (%)</th>
<th>Avg. Emp. per Plant</th>
<th>Cyclical Prop. # Plants per Firm when GDP Below Trend</th>
<th>Cyclical Prop. # Plants per Firm above Trend</th>
<th>Elast. w/ GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 4</td>
<td>1908.71</td>
<td>6.55</td>
<td>2.31</td>
<td>1.00</td>
<td>1.00</td>
<td>-0.009</td>
</tr>
<tr>
<td>5 to 9</td>
<td>736.37</td>
<td>7.32</td>
<td>6.59</td>
<td>1.03</td>
<td>1.03</td>
<td>-0.018</td>
</tr>
<tr>
<td>10 to 19</td>
<td>407.70</td>
<td>8.40</td>
<td>12.68</td>
<td>1.10</td>
<td>1.09</td>
<td>0.048</td>
</tr>
<tr>
<td>20 to 49</td>
<td>237.02</td>
<td>11.01</td>
<td>24.43</td>
<td>1.28</td>
<td>1.27</td>
<td>0.151</td>
</tr>
<tr>
<td>50 to 99</td>
<td>71.04</td>
<td>7.54</td>
<td>40.23</td>
<td>1.77</td>
<td>1.73</td>
<td>0.023</td>
</tr>
<tr>
<td>100 to 249</td>
<td>35.00</td>
<td>8.20</td>
<td>51.44</td>
<td>3.02</td>
<td>2.95</td>
<td>-0.436</td>
</tr>
<tr>
<td>250 to 499</td>
<td>9.77</td>
<td>5.16</td>
<td>59.00</td>
<td>5.85</td>
<td>5.74</td>
<td>-0.194</td>
</tr>
<tr>
<td>500 to 999</td>
<td>4.68</td>
<td>4.81</td>
<td>63.17</td>
<td>10.54</td>
<td>10.63</td>
<td>0.179</td>
</tr>
<tr>
<td>1,000 to 2,499</td>
<td>3.04</td>
<td>6.52</td>
<td>66.13</td>
<td>21.56</td>
<td>21.28</td>
<td>-0.096</td>
</tr>
<tr>
<td>2,500 to 4,999</td>
<td>1.11</td>
<td>4.77</td>
<td>60.53</td>
<td>46.69</td>
<td>46.77</td>
<td>0.378</td>
</tr>
<tr>
<td>5,000 +</td>
<td>1.45</td>
<td>29.74</td>
<td>75.81</td>
<td>167.43</td>
<td>175.19</td>
<td>0.953*</td>
</tr>
</tbody>
</table>

Note: We extract a linear trend component to all variables. The “Avg. # of Firms” corresponds to the average number of firms in each size bin over our sample (in thousands). “Fraction Total Emp.” is computed as the average of total employees in each size bin divided by total employment. “Avg. Emp. per Plant” corresponds to the average number of employees per establishment in each size category. “Avg. # of Plants when GDP Below Trend or Above Trend” is derived from a linear regression of the average number of establishments by firm size on a constant, a linear trend, and a dummy that identifies periods in which GDP is above trend. GDP corresponds to log-real GDP. The trend for GDP is computed using the Hodrick-Prescott (H-P) filter with a parameter of 6.25. “Elast. w/ GDP” corresponds to the elasticity between the average number of plants and detrended GDP. This elasticity is derived from a linear regression of log average number of establishments by firm size on a constant, a linear trend, and log-real detrended GDP. * denotes significant at the 10% level. Source: U.S. Census Bureau, Business Dynamics Statistics (BDS) data tables.

Also, within-firm expansions and contractions seem to be correlated with the business cycle. Broda and Weinstein (2010) report that the product portfolio of firms is procyclical, whereas the product count is based on the “bar codes” a firm uses. This measure is consistent with our evidence and our model. In bad times, firms contract their product mix; in good times, firms expand their product mix and expose themselves to more markets.

We can exploit the relationship between market exposure and size further in order to
test the model. In the model, low-productivity firms (on average small) are more volatile than high-productivity firms (on average large) since they are exposed to a lower number of markets. If consistent with the model, the previous two tables imply that this negative relationship between firm size and firm-level risk should also be present in the data. Figure 6 presents the estimated distribution of idiosyncratic risk for our two samples (i.e., the KFS and Compustat).

Figure 6: Distribution Firm Risk across Samples

![Distribution Firm Risk across Samples](image)

Note: Idiosyncratic dispersion is based on sales growth. Sources: Compustat Fundamental and KFS

As in the model, when looking at firm-level risk across our two samples to collect evidence on firm level risk across firms of different size, we find that small firms (i.e., those in KFS) are considerable more volatile than large firms (i.e., those in Compustat). This is consistent with the evidence presented in Haltiwanger et al. (2013). The median dispersion in the KFS is more than five times the median dispersion in the Compustat sample.

To continue exploring the link between firm-level risk and firm size, Table 6 presents several moments of the distribution of firm-level risk conditional on firm size. To construct this table, we condition on a particular size bin and then compute moments of the distribution of firm-level risk.

41Some firms in our Compustat sample correspond to one establishment firms (they represent 4% of employment and 5% of sales). However, by definition, since Compustat corresponds to publicly traded firms we decide not to use these firms for the comparison because they are far from a “representative” small firm.
Table 6: Firm-Level Risk (Conditional on Size)

<table>
<thead>
<tr>
<th>Employment Size</th>
<th>Moments Distribution log(ε²)</th>
<th>Mean</th>
<th>Median</th>
<th>Bottom 99%</th>
<th>Bottom 90%</th>
<th>Top 10%</th>
<th>Top 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 4</td>
<td></td>
<td>-1.97</td>
<td>-1.75</td>
<td>-10.00</td>
<td>-5.26</td>
<td>1.19</td>
<td>3.05</td>
</tr>
<tr>
<td>5 to 9</td>
<td></td>
<td>-2.31</td>
<td>-1.96</td>
<td>-9.76</td>
<td>-5.86</td>
<td>0.85</td>
<td>2.67</td>
</tr>
<tr>
<td>10 to 19</td>
<td></td>
<td>-2.47</td>
<td>-2.21</td>
<td>-10.37</td>
<td>-5.78</td>
<td>0.66</td>
<td>2.62</td>
</tr>
<tr>
<td>20 to 99</td>
<td></td>
<td>-3.35</td>
<td>-3.09</td>
<td>-11.39</td>
<td>-6.75</td>
<td>-0.26</td>
<td>2.08</td>
</tr>
<tr>
<td>100 to 499</td>
<td></td>
<td>-4.18</td>
<td>-3.87</td>
<td>-12.14</td>
<td>-7.41</td>
<td>-1.32</td>
<td>0.96</td>
</tr>
<tr>
<td>500 or more</td>
<td></td>
<td>-5.26</td>
<td>-4.94</td>
<td>-13.03</td>
<td>-8.45</td>
<td>-2.46</td>
<td>-0.44</td>
</tr>
</tbody>
</table>

Note: Idiosyncratic dispersion is based on sales growth. Source: Data are from the link Compustat Fundamental to Compustat Segment and the link Compustat Fundamental to LBD data.

Table 6 shows that all moments of the distribution are decreasing in firm size. That is, consistent with the predictions of the model, we find that large firms tend to be less volatile than small firms. For example, the median values of log(ε²) imply that a firm with 10 to 19 employees faces 36.87% less risk than a firm with one to four employees and that a firm with 20 to 99 employees faces 73.82% less risk also than a firm in the one to four employees category.42

6.4 Conditional Firm-Level Risk over the Business Cycle

This subsection presents evidence on the cyclicality of firm level risk when splitting the sample in two different ways. First, we look at the difference in cyclicality of firm level risk between “changers” and “non-changers”. Second, we look at differences in the cyclicality of firm level risk conditional on firm size. The model implies that “changers” and large firms’ level risk should be countercyclical while it should be acyclical for “non-changers” and small firms. The evidence is consistent with the model.

6.4.1 Cyclical Properties of Firm Level Risk: “changers” vs “non-changers”

We start by testing whether, by reacting to the business cycle, firms that change their market exposure experience a countercyclical pattern of volatility, whereas the firms that do not change their market exposure have an acyclical pattern (i.e., we find evidence of the nonmonotone relationship depicted in Figure 3). Figure 7 presents the evolution of our

42These can be computed by dividing the values for the conditional median. More specifically, exp(-2.21)/exp(-1.75) − 1 = −0.3687 and exp(-3.09)/exp(-1.75) − 1 = −0.7382.
measure of firm level risk over the business cycle conditional on being a “changer” and a “non-changer” as well as the correlation of each series with detrended GDP.

Figure 7: Volatility of “changers” versus “non-changers”

Note: Detrended GDP corresponds to detrended log-real GDP. The market exposure measure corresponds to the number of four-digit industry codes (i.e., line of business), the number of establishments a firm operates, the number of MSAs in which the firms are present, and the product of establishments times SIC codes. “Changers” refers to firms that change market exposure measure in a given period. “Non-changers” refers to firms that do not change their market exposure measure. All series are detrended using an H-P filter with parameter 6.25. Source: Data are from the link Compustat Fundamental to Compustat Segment and the link Compustat Fundamental to LBD data.
We find that the data is consistent with the predictions of the model. Once we split the sample into “changers” and “non-changers” (using our four different definitions of “market”) we see that the correlation of median \( \epsilon^2 \) and GDP for “changers” and “non-changers” is different. Exactly as predicted by the model, in each of the market definitions, the “changers”, median \( \epsilon^2 \) is countercyclical, with a correlation between -0.31 and -0.422 (significant at the 5% level) with respect to GDP. On the other hand, the correlation of the median \( \epsilon^2 \) and GDP in the case of the “non-changers” is between -0.098 and -0.247 and is not significantly different from zero in all cases. Figure 7 shows the cyclical behavior for “changers” and “non-changers” as described above.\(^{43}\)

### 6.4.2 Cyclical Properties of Firm-Level Risk by Firm Size

The model predicts a nonmonotone but decreasing relationship between firm-level risk and firm size (see Figures 2 and 3). However, on average, large firms react to the business cycle by expanding while small firms do not, so firm level risk conditional on firm size should display different cyclical properties across size categories.

Figure 8 presents the correlation of firm-level risk conditional on firm size. To construct this figure we rank firms by their size (employment) and label as “small” those firms in the bottom 25% of the size distribution, as “medium” those firms in the 25%-75% range of the distribution and as “large” those firms in the top 25% of the size distribution.\(^{44}\)

\(^{43}\)In Appendix A1.7 we show that these findings are robust to a different measure of firm level risk derived from the time series standard deviation of \( \epsilon_{ijt} \).

\(^{44}\)As a robustness check we computed similar correlations for a different size definition. In particular, we obtained similar results when we label as “small” firms in the bottom 10% of the size distribution, “large” as firms in the top 10% of the size distribution, and “medium” the remaining firms. The business cycle correlations of firm level risk are -0.22 (\( p-value=0.21 \)) and -0.405 (\( p-value=0.02 \)) for “small” and “large,” respectively. See Section A1.8 in the Appendix for further details.
Figure 8: Firm Risk over the Business Cycle (Conditional on Size)

Note: Idiosyncratic risk based on sales growth. Sources: Compustat Fundamental and link Compustat Fundamental to LBD data.

The evidence is consistent with our model and shows that the correlation is negative and stronger for large firms than for small firms.\textsuperscript{45} This is not surprising based on the model we presented since, as we discussed in the previous section, not only “changers” are large but also large firms tend to be “changers”. The correlation of the median($\epsilon^2$) and GDP equals 0.111 and −0.2187 for the bottom 25% of firms, and top 25% of firms respectively (within the top 5% of firms, which is our model analog of Compustat).\textsuperscript{46}

\textsuperscript{45}Section A1.8 in the Appendix also shows that this result is robust to a measure of firm level risk derived from an autoregressive model of log(sales). In particular, the correlation for “small” firms is -0.229 (p-value=0.20), and -0.386 (p-value=0.03) for “small” and “large,” respectively.

\textsuperscript{46}The relationship is also present when we consider all the firms. The correlation between median($\epsilon^2$) and GDP is −0.2796, and −0.2648 for the bottom 25% and top 25% of firms, respectively.
7 Determinants of Firm-Level Risk

To understand the properties of firm-level volatility and market exposure, we derive a testable implication that links the number of markets $m_t(s)$, selling expenses, and firm-level volatility $\ln(\epsilon^2_{it})$ derived from equation (25). The model predicts that market exposure and selling expenses are key to understanding the evolution of firm-level risk. As we describe in detail in the following section, in the data, we observe several relatively direct measures of market exposure as well as selling expenses (an indirect measure of market exposure), which in our model correspond to $m^*(s)_t$ and $w_t\Phi(m^*(s))$, respectively. Therefore, we estimate the regression

$$\ln(\epsilon^2_{it}) = \gamma_0 s_i + \gamma_1 \ln(x_{it}) + u_{it},$$

where $\ln(\epsilon^2_{it})$ is our measure of firm-level risk, $\gamma_0 s_i$ is a firm fixed effect, and $x_{it}$ represents either the number of markets or selling expenses. Table 7 summarizes our findings.

Table 7: Model Firm-Level Idiosyncratic Volatility and Market Exposure

<table>
<thead>
<tr>
<th></th>
<th>Dependent Variable $\ln(\epsilon^2_{ijt})$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Sample</td>
</tr>
<tr>
<td>$\ln(m_{ijt})$</td>
<td>-0.579</td>
</tr>
<tr>
<td>Std. Error</td>
<td>0.0034***</td>
</tr>
<tr>
<td>$\ln(\text{expenses}_{ijt})$</td>
<td>-0.054</td>
</tr>
<tr>
<td>Std. Error</td>
<td>-0.0003***</td>
</tr>
</tbody>
</table>

Note: *** denotes significant at the 1% level, ** at the 5% level and * at the 10% level. $\ln(\epsilon^2_{it})$ is constructed from the estimated residual of equation (25).

The elasticities between firm-level risk and the number of markets and between firm-level risk and selling expenses are -0.579 (significant at 5%) and -0.054 (significant at 5%), respectively, when looking at a sample that includes all the firms from our model. Restricting attention to the top 5% of firms (which is our model counterpart to the firms included in Compustat) delivers firm-level risk elasticities of -0.210, with respect to markets, and -0.145, with respect to expenses. These values are very close to the ones reported in Table 8 (constructed from real-world data).

We now turn our attention to the determinants of firm-level risk in the data. With our estimate of idiosyncratic risk $\epsilon_{ijt}$ from equation (1), as in Castro, Clementi, and MacDonald (2009), we proxy its variance at the firm level by $\ln(\epsilon^2_{ijt})$ and study how it is related to our
different market exposure measures once industry-specific factors are accounted for.\textsuperscript{47} In particular, we estimate the following log-linear equation (as we do for the model):

\[
\ln(\epsilon_{ijt}^2) = \gamma_i + \theta_{tj} + \alpha_1 \ln(X_{ijt}) + \alpha_2 t + u_{ijt}, \tag{27}
\]

where \(\gamma_i\) is a firm fixed effect, \(\theta_{tj}\) is an industry- and year-specific component, \(\ln(X_{ijt})\) is the measure of market exposure for firm \(i\) in sector \(j\) at time \(t\), and \(t\) is a time trend.\textsuperscript{48} We use many different market exposure measures as our \(X_{ijt}\) from different sources. We use Compustat Fundamental linked to the LBD to obtain the number of establishments each firm operates at each point in time. Also, given that we have the location of the establishment, we can identify the number of MSAs in which a firm is operating at each point in time. Using the link Compustat Fundamental with Compustat Segment, we also look at the number of SIC codes for which a firm reports sales in a given year as a measure of product market exposure. We also look at the interaction between geographic locations (establishments) and product markets. Finally, we consider indirect measures of market exposure, such as SGA expenses and advertising expenses in the Compustat Fundamental sample and SGA in the KFS sample in the case of the small firms.\textsuperscript{49}

Table 8 reports the results of a selected number of regressions.

\textsuperscript{47}This specification for the variance allows us to identify a value for the variance for every firm in industry \(j\) and year \(t\), and it is consistent with the multiplicative heteroscedasticity model analyzed by Harvey (1976). More specifically, this formulation results from assuming that \(\sigma_{ijt}^2\), the variance of the disturbance in equation (1) (i.e., the variance of \(\epsilon_{ijt}\)), takes the following form: \(\sigma_{ijt}^2 = \exp(\gamma_i + \theta_{tj} + \alpha_1 \ln(X_{ijt}) + \alpha_2 t)\).

\textsuperscript{48}We will show below that, consistent with the evidence presented in Comin and Philippon (2006), a time trend is necessary because the variance of idiosyncratic risk for public firms has been rising during the last 30 years.

\textsuperscript{49}We perform a large set of robustness checks, which are reported in the appendix.
Table 8: Dependent Variable $\ln(\epsilon_{ijt}^2)$

<table>
<thead>
<tr>
<th>$\ln(X)$</th>
<th>Estabs.</th>
<th>MSAs</th>
<th>SICs</th>
<th>Est*SIC</th>
<th>Adv</th>
<th>SGA</th>
<th>SGA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln(X)$</td>
<td>-0.081</td>
<td>-0.094</td>
<td>-0.137</td>
<td>-0.075</td>
<td>-0.134</td>
<td>-0.301</td>
<td>-0.085</td>
</tr>
<tr>
<td>S.E.</td>
<td>0.0088</td>
<td>0.0102</td>
<td>0.0212</td>
<td>0.0080</td>
<td>0.012</td>
<td>0.009</td>
<td>0.036</td>
</tr>
<tr>
<td>N</td>
<td>129724</td>
<td>129724</td>
<td>155175</td>
<td>124433</td>
<td>66962</td>
<td>177178</td>
<td>2547</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.0180</td>
<td>0.0180</td>
<td>0.0178</td>
<td>0.0183</td>
<td>0.0423</td>
<td>0.0435</td>
<td>0.16</td>
</tr>
<tr>
<td>Source</td>
<td>LBD</td>
<td>LBD</td>
<td>Comp Seg</td>
<td>LBD</td>
<td>Comp Seg</td>
<td>Comp</td>
<td>KFS</td>
</tr>
</tbody>
</table>

All results are significant at 1% level except for those from the KFS, which are significant at 5% level. All regressions include firm fixed effects, industry-year controls, and time trends. Sources: Number of establishments and MSAs taken from the LBD, Number of SICs from Compustat segment database. $\ln(\epsilon_{ijt}^2)$ is derived from estimating equation (1) on our Compustat Fundamental Sample. Regressions are run after performing the corresponding links with LBD and Compustat Segment data sets.

The results are very close in terms of magnitude of the regressions that use establishments, MSAs, SIC codes, and their interaction. The elasticity of firm-level volatility and these measures is between 7.5% and 13.7%. Even using advertising expenses as the measure for market reach delivers an estimate that is very close to those obtained using direct measures of market reach. All of the measures reported in the first five regressions use either Compustat Fundamental linked to Compustat segment data or our Compustat Fundamental LBD link data set, so the model counterpart for these firms must be the top end of the firm-size distribution. In the model, when we look at the top 5% of firms by productivity, the elasticity of risk to market exposure is -0.21, which is close to the numbers reported above.\(^{50}\)

Finally, when looking at the indirect measures of market exposure, namely SGA expenses, we find that the elasticity is -0.3 for the Compustat Fundamental sample, and the same number changes to -0.085 on the other end of the firm size distribution (i.e., for the KFS sample). In this dimension, the model also performs well. It delivers an elasticity of -0.14 when we restrict the attention to the top 5% of firms, and the elasticity changes to -0.05 if we consider the full universe of firms.\(^{51}\)

---

\(^{50}\)In Appendix A1.9.1, we show that the estimated negative elasticity is robust to various definitions of firm-level volatility. In particular, we present results where we regress the time series deviation of $\epsilon_{ijt}$ for each firm against the different measures of market exposure, as well as a regression with the (five-period) rolling window standard deviation of $\epsilon_{ijt}$ for each firm as a dependent variable against the same controls to show that the elasticity is negative and significant in every specification ranging from -2.1 percent to -27.9 percent.

\(^{51}\)In Appendix A1.9.2, we show that the estimated firm-level volatility $\hat{\sigma}_{ijt}^2 = \exp(\hat{\gamma}_t + \hat{\theta}_{ij} + \hat{\alpha}_1 \ln(X_{ijt}) + \ldots)$.
8 Conclusion

Consistent with previous literature, using a panel of U.S. firms (data from Compustat Fundamental), we document the countercyclical nature of idiosyncratic firm-level risk. We propose a theory of endogenous volatility over the business cycle based on firm-level market exposure to explain this fact. In our model, firms pay a cost to be able to expand to a larger number of markets. The result is that high-productivity firms expand to a larger set of markets, making them less volatile than their low-productivity counterparts. Notably, low-productivity firms do not react to the cycle, whereas medium scale and large firms do, explaining the cyclical properties of firm-level volatility.

From the model, we derive a set of testable implications for measures of market exposure and firm-level volatility to then show that the empirical evidence is broadly consistent with the theory. Specifically, using firm-level data from Compustat Fundamental, Compustat Segment data, and BDS data, we show that measures of market exposure (both direct measures, such as line of business, number of establishments or their geographical location, and indirect measures such as selling and advertisement expenses) are procyclical and that the volatility of firms that expand and contract is countercyclical (as opposed to acyclical for those not engaged in market expansions and contractions). Moreover, using data from Compustat Fundamental, Compustat Segment, the LBD, and the KFS, we show that firm-level idiosyncratic risk is negatively correlated with all measures of market exposure, even after controlling for firm, year, and industry fixed effects.

\( \hat{\alpha}_{i2t}E(\hat{u}_{ijt}) \) is countercyclical as all our other measures of firm level risk.
9 References


Bloom, Nicholas (2014). “Fluctuations in Uncertainty,” *Journal of Economic Perspec-


A1 Appendix

A1.1 Kauffman Firm Survey Sample

The Kauffman Firm Survey (KFS) provides a large panel of data on “young” businesses. Firms in the sample were founded in 2004 and have been tracked annually.\textsuperscript{52} This panel was created using a random sample from Dun & Bradstreet’s database of new businesses. The target population consists of all new businesses that were started in 2004 in the United States, and excludes any branch or subsidiary owned by an existing business or what was inherited from someone else. The sample for the first survey consisted of 4,928 businesses.

The KFS provides us with the unique opportunity to study a panel of new businesses from startup, using available data on their revenues and expenses, number of workers, products, services, innovations that they possessed and developed in their early years of existence, and the extent to which these business are involved in innovative activities. One drawback of the publicly available KFS data is that some variables, such as assets (and its components) and sales, are only reported within certain ranges.\textsuperscript{53} We set the value of the corresponding variables to the middle value of the reported range.\textsuperscript{54}

Our unit of observation is the firm, as defined by the KFS. The change in sales is constructed from total revenues from sales of goods, services, or intellectual property. As is standard in the literature, size is defined as the number of employees. We use two-digit NAICS codes to control for industry effects. All variables are deflated using two-digit industry deflators. Our variable expenses is defined as expenses that do not correspond to production inputs. It is constructed as total expenses in selling, administrative and general expenses (SGA) which include expenses associated with, for example, design of new products, brand development, advertising, marketing, organizational development, or management consulting. For firm/year observations with missing values of SGA expenses, we compute the average ratio of SGA expenses to total expenses and input SGA expenses from this ratio and total expenses.

Table 9 presents the distribution of real sales and real SGA expenses for newborn firms (i.e., the distribution of firms in 2004) and for firms that survive until the end of our sample

\textsuperscript{52}\hspace{1em} Data available at the time this paper is being written extend through 2008. Firms will continue to be tracked through 2011. See http://www.kauffman.org/kfs/ for a detailed description of the data and the public-use microdata itself.

\textsuperscript{53}\hspace{1em} For example, ranges for revenues are 0, $1-$1000, $1,001-$5,000, $5,001-$10,000, $10,001-$25000, $25001-$100,000, and $100,001 or more.

\textsuperscript{54}\hspace{1em} The set of variables we use that present this problem are: revenue from sales of goods, services, or intellectual property, expenses, wages, and assets (and their components).
Table 9: Distribution of Sales and Expenses (%)

<table>
<thead>
<tr>
<th>Thousands of $</th>
<th>Year 2004</th>
<th>Year 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sales</td>
<td>SGA</td>
</tr>
<tr>
<td>$0−$3</td>
<td>14.52</td>
<td>55.09</td>
</tr>
<tr>
<td>$3−$10</td>
<td>14.39</td>
<td>26.40</td>
</tr>
<tr>
<td>$10−$50</td>
<td>14.59</td>
<td>0.00</td>
</tr>
<tr>
<td>$50−$100</td>
<td>28.58</td>
<td>16.64</td>
</tr>
<tr>
<td>$&gt;100</td>
<td>27.92</td>
<td>1.87</td>
</tr>
<tr>
<td># Firms</td>
<td>3,037</td>
<td>4,382</td>
</tr>
</tbody>
</table>

Note: Sales and SGA are deflated using the GDP deflator. Source: The KFS.

Observe that many firms are relatively small, with sales and selling expenses below $10,000. This is still the case even after four years of existence. However, a non-trivial number of new firms have sales and SGA above $100,000. The distributions clearly shift upward as the cohort of firms becomes older and grows and as selection takes place.

Table 10 reports the distribution of newly created firms as seen in the KFS, a comparison with the size distribution of new firms from the Census Bureau’s data, and the distribution of firms over employment for our cohort of firms in 2008.\(^{55}\)

Table 10: Distribution of Workers (%)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1−4</td>
<td>74.4</td>
<td>76.7</td>
<td>64.8</td>
</tr>
<tr>
<td>5−9</td>
<td>15.3</td>
<td>13.0</td>
<td>17.8</td>
</tr>
<tr>
<td>10−19</td>
<td>6.6</td>
<td>6.0</td>
<td>9.5</td>
</tr>
<tr>
<td>20−99</td>
<td>3.4</td>
<td>3.8</td>
<td>2.9</td>
</tr>
<tr>
<td>100−499</td>
<td>0.3</td>
<td>0.4</td>
<td>5.0</td>
</tr>
<tr>
<td>500+</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>


Table 10 shows that a large fraction of firms start with only a few workers. More than 70% of new firms hire between one and four workers. As a comparison, we report the distribution

\(^{55}\)For comparison, we report the distribution conditional on firms having more than one worker. In the KFS data, we find that in 2004, 58% of active firms hired zero workers; this value equaled 44% in 2008.
of new firms from the Census Bureau’s data; note that the distributions are very similar. This reassures us that we have a representative sample of new firms, despite some differences in the distribution of new firms across industries and the different methodologies used across sources. Finally, and consistent with the evidence presented in Table 9, among active firms in the KFS in 2008, there is a sizable reduction in the fraction of firms with less than four workers and an increase in the fraction of firms with more than 10 workers.

Table 11 displays the distribution of firms across some representative industries and their one-year survival rates.

<table>
<thead>
<tr>
<th>Industry</th>
<th>Fraction of Firms (%)</th>
<th>One Year Survival Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction</td>
<td>10.0</td>
<td>91.9</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>7.1</td>
<td>92.0</td>
</tr>
<tr>
<td>Wholesale</td>
<td>5.4</td>
<td>88.7</td>
</tr>
<tr>
<td>Retail</td>
<td>15.9</td>
<td>86.1</td>
</tr>
<tr>
<td>Transportation and Warehousing</td>
<td>3.4</td>
<td>84.7</td>
</tr>
<tr>
<td>Information</td>
<td>2.7</td>
<td>84.6</td>
</tr>
<tr>
<td>Finance and Insurance</td>
<td>4.7</td>
<td>95.8</td>
</tr>
<tr>
<td>Administration and Support</td>
<td>9.6</td>
<td>91.7</td>
</tr>
<tr>
<td>Accommodation and Food Services</td>
<td>4.3</td>
<td>77.7</td>
</tr>
</tbody>
</table>

Source: Kauffman Firm Survey

**A1.2 Compustat and Compustat-Segment Sample**

We use Compustat’s fundamental and segment annual data.\(^{56}\) Our choice of firm identifier is GVKEY, and this is the variable we use for matching the Compustat segment file to the fundamentals file. The sample period for the fundamentals data ranges from 1960 to 2012, but segment data exist only from 1977 to 2012. Not all firms have segment data. Our year variable is extracted from the variable DATADATE (for both the fundamentals and the segments files). We exclude financial firms with SIC codes between 6000 and 6999, utility firms with SIC codes between 4900 and 4999, and firms with SIC codes greater than 9000 (residual categories). Observations are deleted if they do not have a positive book value of assets or if gross capital stock or sales are either zero, negative, or missing. The final sample

\(^{56}\)All variable names correspond to the Wharton Research Data Services (WRDS) version of Compustat.
is an unbalanced panel with more than 21,600 firms and 241,000 firm/year observations; of these, there are 18,700 firms and 184,000 firm/year observations with segment data.

Our data variables are defined as follows. The change in sales is constructed from the variable SALE. As is standard in the literature, firm size is defined as the number of employees, using the variable EMP. We use two-digit NAICS codes to control for industry effects. Firm age is proxied by the number of years since the firm’s first-year observation in Compustat. All nominal variables are deflated using the BEA’s two-digit, sector-specific price deflator for value added.

Segment counts reflect the sum of primary and secondary four-digit SIC codes reported in the Compustat variables SICS1 and SICS2. Compustat reports four-digit SIC codes for segments throughout the time sample. NAICS codes are also reported in later years; there are no observations in which a NAICS code is reported but a SIC code is not. BEA deflators for value added are only given for SIC codes until 1998; at that time, the BEA began reporting deflators for NAICS codes. Therefore, when possible, we deflate segment-level sales using 10 sector-level SIC code deflators; elsewhere, we deflate with 24 two-digit NAICS sector codes. Thus, our SIC deflators reflect lower industry detail than our NAICS deflators due to the lack of one-to-one mapping between NAICS and SIC; for this reason, we verified that our results are robust to using SIC deflators at the next possible level of detail, for which there are more than 80 SIC codes. For our reported results, we used the sector-level SIC deflators.

Table 12 reports the distribution of real sales and real SGA expenses for firms in 1980 and 2008.\textsuperscript{57}

\textsuperscript{57}Our data extend to 2012, but we present 2008 to allow a comparison with the last year of our KFS sample.
Table 12: Distribution of Sales and Expenses (%)

<table>
<thead>
<tr>
<th>In millions of $</th>
<th>Year 1980</th>
<th>Year 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sales</td>
<td>SGA</td>
</tr>
<tr>
<td>$&lt; 10</td>
<td>19.71</td>
<td>27.73</td>
</tr>
<tr>
<td>$10−$20</td>
<td>9.39</td>
<td>12.41</td>
</tr>
<tr>
<td>$20−$50</td>
<td>13.99</td>
<td>16.80</td>
</tr>
<tr>
<td>$50−$100</td>
<td>11.79</td>
<td>11.98</td>
</tr>
<tr>
<td>$100−$250</td>
<td>14.32</td>
<td>12.94</td>
</tr>
<tr>
<td>$&gt;$250</td>
<td>30.8</td>
<td>18.14</td>
</tr>
<tr>
<td># Firms</td>
<td>4,581</td>
<td>4,150</td>
</tr>
</tbody>
</table>

Note: Sales and Expenses are deflated using the BEA’s two-digit SIC price deflators for value added. Source: Compustat Fundamental

Note that firms’ sales and SGA expenses are considerably larger than those in the KFS sample.

Table 13 reports the distribution of employment size for 1980 and 2008. To simplify the comparison, the size bins are the same as the ones we used for the KFS sample.

Table 13: Distribution of Workers (%)

<table>
<thead>
<tr>
<th>Number of Employees</th>
<th>All Firms</th>
<th>Segment Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 − 4</td>
<td>1.64</td>
<td>1.38</td>
</tr>
<tr>
<td>5 − 9</td>
<td>1.75</td>
<td>1.80</td>
</tr>
<tr>
<td>10 − 19</td>
<td>2.49</td>
<td>3.14</td>
</tr>
<tr>
<td>20 − 99</td>
<td>11.07</td>
<td>13.26</td>
</tr>
<tr>
<td>100 − 499</td>
<td>23.31</td>
<td>21.63</td>
</tr>
<tr>
<td>500 +</td>
<td>59.75</td>
<td>58.79</td>
</tr>
<tr>
<td># Firms</td>
<td>4,581</td>
<td>5,219</td>
</tr>
</tbody>
</table>

Source: Compustat Fundamental and Compustat Segment

Most firms in the Compustat sample, have more than 500 workers, whereas in the KFS sample this value is less than 1%. Table 14 reports the distribution of firm age (computed as the number of years in the sample).
<table>
<thead>
<tr>
<th>Firm’s Age</th>
<th>All Firms</th>
<th>Segment Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 5</td>
<td>18.05</td>
<td>26.38</td>
</tr>
<tr>
<td>6 – 10</td>
<td>41.06</td>
<td>18.85</td>
</tr>
<tr>
<td>11 – 15</td>
<td>15.06</td>
<td>17.34</td>
</tr>
<tr>
<td>16 – 20</td>
<td>10.43</td>
<td>10.92</td>
</tr>
<tr>
<td>21 – 25</td>
<td>0.00</td>
<td>7.86</td>
</tr>
<tr>
<td>26 +</td>
<td>0.00</td>
<td>14.60</td>
</tr>
<tr>
<td>Top Censored</td>
<td>15.39</td>
<td>4.04</td>
</tr>
<tr>
<td># Firms</td>
<td>4,581</td>
<td>5,219</td>
</tr>
</tbody>
</table>

Note: “Top Censored” corresponds to firms that are in our sample starting in 1960. Source: Compustat Fundamental and Compustat Segment

We employed the following rules when constructing the Compustat Fundamental - Compustat Segment data set. When multiple data source dates (SRCDATE) existed for one firm/data date/segment combination, we kept only the most recent source date. When multiple data dates existed for one firm-year-segment combination, we kept only the later data date unless its sales figure was missing (in which case we kept the earlier data date). When multiple segment identifiers existed for one four-digit SIC code, we combined the segments: Segment counts reflect the number of unique four-digit SIC codes, and segment-level employment reflects the sum of all reported segments within a four-digit SIC code.

Finally, Table 15 shows the correlations between the variables used from the Compustat Fundamental - Compustat Segment data set.
### Table 15: Correlations Table - Compustat Segment

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Emp.</th>
<th>SGA</th>
<th>Adv</th>
<th>SICs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>All Sample - Compustat Segment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp.</td>
<td>0.7974</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA</td>
<td>0.8927</td>
<td>0.7486</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv</td>
<td>0.641</td>
<td>0.4838</td>
<td>0.7162</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>SICs</td>
<td>0.227</td>
<td>0.2331</td>
<td>0.2284</td>
<td>0.2258</td>
<td>1</td>
</tr>
</tbody>
</table>

| **“Changers”** |       |      |     |     |      |
| Sales    | 1     |      |     |     |      |
| Emp.     | 0.7888 | 1    |     |     |      |
| SGA      | 0.8736 | 0.728 | 1   |     |      |
| Adv      | 0.6993 | 0.533 | 0.764 | 1 |
| SICs     | 0.205  | 0.2402 | 0.1993 | 0.206 | 1 |

| **“Non-Changers”** |       |      |     |     |      |
| Sales    | 1     |      |     |     |      |
| Emp.     | 0.7994 | 1    |     |     |      |
| SGA      | 0.8976 | 0.7531 | 1   |     |      |
| Adv      | 0.6243 | 0.4702 | 0.7028 | 1 |
| SICs     | 0.229  | 0.2267 | 0.2313 | 0.2275 | 1 |

Note: All nominal variables are deflated using the BEA’s two-digit, sector-specific price deflator for value added. Source: Compustat Fundamental and Compustat Segment.

### A1.3 LBD-Compustat Fundamental Link

The LBD is constructed from the business register of the U.S. Bureau of the Census (see Jarmin and Miranda (2002)). It includes all nonfarm private sector employer establishments and firms in the United States from 1976 to 2011 and provides information on location, industry, and employment. Employment information reflects the status of establishments as of March 12 of a given year. The LBD links establishments as firms; firm identifiers reflect operational control and can span across state lines.

Both Compustat and the LBD include various firm identifiers that can be used for matching: employer identification numbers (EINs), two alternative business names (in Compustat, these are given by CONM and CONML), and addresses. We obtained further match flexibility by employing the SAS DQMATCH system. We linked Compustat to the LBD by using...
successive “passes” that matched firms using these identifiers with varying degrees of specificity. Early match passes relied on EINs and full business names and addresses (which have been standardized). Subsequent passes utilized algorithms that evaluate name similarity conditional on geographic matches. Final passes employed DQMATCH descriptors. We utilize both alternative name variables from each data set, thus allowing for potential matches along any combination of name variables. Only residual nonmatched CUSIPs are retained after each pass; by ordering passes such that more specific match criteria are tested earlier, we ensure that the final linked data set is based on the highest possible match quality for each firm.

We eliminate firm-year matches that are out of scope for Compustat activity (as determined by IPODATE and DLDTE when available or by time periods of positive employment, sales, or share price when the former variables are missing). Instances in which a CUSIP was paired with multiple LBD firms were resolved by first dropping LBD firms with only one operating unit and then choosing the LBD firm with reported employment closest to Compustat reported employment. Since many firms have time series gaps in EIN coverage, and since business names in the LBD refer to establishments rather than firms (and occasionally change over time), we make additional matches by rolling firm-year matches across years when appropriate.

We find matches for about 80% of relevant Compustat CUSIPs in the LBD source data; on average, we have about 4,200 observations with relevant nonmissing data per year in our LBD-Compustat Fundamental linked data spanning from 1977 to 2011. The resulting comingled data set includes sales and industry data from Compustat with employment and geographic data from the LBD. In the LBD, an establishment is a single business location with one or more employees. We classify establishment locations using the Census Bureau’s 2009 definitions of MSA, which comprised entire counties (i.e., an MSA is a group of counties). For our estimation purposes, any county that is not included in an MSA is classified as its own MSA. Table 16 presents summary statistics.

58 For matching purposes, we first discard from Compustat all exchange traded funds (ETFs) that can be easily identified, American depositary receipts (ADRs) and American depositary shares (ADSs), CUSIPs with non-U.S. geographical identifiers, and firms that operate only outside of North America (as identified by IDBFLAG).

59 An alternative to this classification is to collect all counties within a state that are not included in an MSA and define that collection as an MSA so that each state has a single residual “MSA” in addition to proper MSAs. We also performed our analyses on data constructed with this definition, but it did not substantially alter the results.
Table 16: Summary Statistics for LBD variables

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Corr. with SGA exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of estabs</td>
<td>124.67</td>
<td>617.30</td>
<td>0.35</td>
</tr>
<tr>
<td>Number of MSAs</td>
<td>34.21</td>
<td>99.14</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Finally, Tables 17, 18, and 19 show the correlations between the variables used from the Compustat-LBD data for the different market definitions.

Table 17: Correlations Table - Compustat-LBD: Number of Establishments

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Emp.</th>
<th>SGA</th>
<th>Adv.</th>
<th>Establ</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>All Sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp.</td>
<td>0.8</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA</td>
<td>0.8828</td>
<td>0.7494</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv.</td>
<td>0.656</td>
<td>0.4616</td>
<td>0.7266</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Establ</td>
<td>0.3475</td>
<td>0.4483</td>
<td>0.37</td>
<td>0.2301</td>
<td>1</td>
</tr>
<tr>
<td><strong>“Non-Changers”</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp.</td>
<td>0.8365</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA</td>
<td>0.8638</td>
<td>0.6635</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv.</td>
<td>0.6962</td>
<td>0.5647</td>
<td>0.7872</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Establ</td>
<td>0.4473</td>
<td>0.2846</td>
<td>0.5497</td>
<td>0.3556</td>
<td>1</td>
</tr>
<tr>
<td><strong>“Changers”</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp.</td>
<td>0.7927</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA</td>
<td>0.8807</td>
<td>0.7431</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv.</td>
<td>0.6551</td>
<td>0.4498</td>
<td>0.7265</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Establ</td>
<td>0.326</td>
<td>0.4393</td>
<td>0.345</td>
<td>0.2079</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: All nominal variables are deflated using the BEAs two-digit, sector-specific price deflator for value added. Source: Compustat Fundamental and LBD
Table 18: Correlations Table - Compustat-LBD: Number of MSAs

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Emp.</th>
<th>SGA</th>
<th>Adv.</th>
<th>MSA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>All Sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp.</td>
<td>0.8</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA</td>
<td>0.8828</td>
<td>0.7494</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv.</td>
<td>0.656</td>
<td>0.4616</td>
<td>0.7266</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>MSA</td>
<td>0.3674</td>
<td>0.4788</td>
<td>0.3847</td>
<td>0.2368</td>
<td>1</td>
</tr>
<tr>
<td><strong>“Non-Changers”</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp.</td>
<td>0.8009</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA</td>
<td>0.8745</td>
<td>0.6527</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv.</td>
<td>0.6802</td>
<td>0.4865</td>
<td>0.7661</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>MSA</td>
<td>0.3673</td>
<td>0.4098</td>
<td>0.3601</td>
<td>0.2304</td>
<td>1</td>
</tr>
<tr>
<td><strong>“Changers”</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp.</td>
<td>0.7931</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA</td>
<td>0.8819</td>
<td>0.7491</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv.</td>
<td>0.6562</td>
<td>0.4508</td>
<td>0.7239</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>MSA</td>
<td>0.3411</td>
<td>0.4619</td>
<td>0.3577</td>
<td>0.2032</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: All nominal variables are deflated using the BEAs two-digit, sector-specific price deflator for value added. Source: Compustat Fundamental and LBD.
Table 19: Correlations Table - Compustat-LBD: Establishments * SICs

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Emp</th>
<th>SGA</th>
<th>Adv</th>
<th>Est*SIC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>All Sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp.</td>
<td>0.8024</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA</td>
<td>0.8837</td>
<td>0.7476</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv.</td>
<td>0.6578</td>
<td>0.4652</td>
<td>0.7274</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Est*SIC</td>
<td>0.381</td>
<td>0.4444</td>
<td>0.391</td>
<td>0.2577</td>
<td>1</td>
</tr>
<tr>
<td><strong>“Non-Changers”</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp.</td>
<td>0.8386</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA</td>
<td>0.8553</td>
<td>0.6367</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv.</td>
<td>0.7008</td>
<td>0.5772</td>
<td>0.8003</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Est*SIC</td>
<td>0.4896</td>
<td>0.2524</td>
<td>0.6153</td>
<td>0.3827</td>
<td>1</td>
</tr>
<tr>
<td><strong>“Changers”</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp.</td>
<td>0.7975</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA</td>
<td>0.8821</td>
<td>0.7401</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv.</td>
<td>0.6642</td>
<td>0.4614</td>
<td>0.7315</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Est*SIC</td>
<td>0.3615</td>
<td>0.4448</td>
<td>0.3625</td>
<td>0.2409</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: All nominal variables are deflated using the BEAs two-digit, sector-specific price deflator for value added. Source: Compustat Fundamental and LBD.

All reported statistics based on the LBD-Compustat link were reviewed and do not disclose confidential information.

A1.4 BDS Sample

Table 20 presents the cyclical properties of workers per establishment computed from BDS data. Since most variables in this sample have a trend component, we detrended them using a linear trend when reporting the averages in Tables 5 and 20. Table 21 reports the detrended and the nondetrended average (i.e., a simple average) for the variables of interest.
Table 20: Number of Workers per Establishment over the Business Cycle

<table>
<thead>
<tr>
<th>Firm Size (# workers)</th>
<th>Avg. # of Firms (in 1000’s)</th>
<th>Fraction Total Emp. (%)</th>
<th>Avg. Emp. per Plant</th>
<th>Cyclical Prop. # Workers per Est.</th>
<th>Avg. # worker per Est. w/ gdp Below Trend</th>
<th>Above Trend</th>
<th>Elas. w/ GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 4</td>
<td>1908.71</td>
<td>6.55</td>
<td>2.31</td>
<td>2.3065</td>
<td>2.315</td>
<td>0.137</td>
<td></td>
</tr>
<tr>
<td>5 to 9</td>
<td>736.37</td>
<td>7.32</td>
<td>6.59</td>
<td>6.561</td>
<td>6.6208</td>
<td>0.385*</td>
<td></td>
</tr>
<tr>
<td>10 to 19</td>
<td>407.70</td>
<td>8.40</td>
<td>12.68</td>
<td>12.6129</td>
<td>12.759</td>
<td>0.346*</td>
<td></td>
</tr>
<tr>
<td>20 to 49</td>
<td>237.02</td>
<td>11.01</td>
<td>24.43</td>
<td>24.2209</td>
<td>24.651</td>
<td>0.302</td>
<td></td>
</tr>
<tr>
<td>50 to 99</td>
<td>71.04</td>
<td>7.54</td>
<td>40.23</td>
<td>39.6674</td>
<td>40.8219</td>
<td>0.452</td>
<td></td>
</tr>
<tr>
<td>100 to 249</td>
<td>35.00</td>
<td>8.20</td>
<td>51.44</td>
<td>50.5698</td>
<td>52.3573</td>
<td>0.897*</td>
<td></td>
</tr>
<tr>
<td>250 to 499</td>
<td>9.77</td>
<td>5.16</td>
<td>59.00</td>
<td>58.205</td>
<td>59.8447</td>
<td>0.706*</td>
<td></td>
</tr>
<tr>
<td>500 to 999</td>
<td>4.68</td>
<td>4.81</td>
<td>63.17</td>
<td>63.037</td>
<td>63.3069</td>
<td>0.312</td>
<td></td>
</tr>
<tr>
<td>1000 to 2499</td>
<td>3.04</td>
<td>6.52</td>
<td>66.13</td>
<td>65.4618</td>
<td>66.8366</td>
<td>0.479*</td>
<td></td>
</tr>
<tr>
<td>2500 to 4999</td>
<td>1.11</td>
<td>4.77</td>
<td>60.53</td>
<td>60.1928</td>
<td>60.8968</td>
<td>0.080</td>
<td></td>
</tr>
<tr>
<td>5000 +</td>
<td>1.45</td>
<td>29.74</td>
<td>75.81</td>
<td>75.7357</td>
<td>75.899</td>
<td>0.127</td>
<td></td>
</tr>
</tbody>
</table>

Note: We extract a linear trend component to all variables. The “Avg. # of Firms” corresponds to the average number of firms in each size bin over our sample (in thousands). “Fraction Total Emp.” is computed as the average of total employees in each size bin divided by total employment. “Avg. Emp. per Plant” corresponds to the average of the number of employees per establishment in each size category. “Avg. # Worker per Est. when GDP is below and above trend” is derived from a linear regression of average number of workers per establishment by firm size on a constant, a linear trend and a dummy that identifies periods where GDP is above trend. The value reported is the parameter on this dummy. GDP corresponds to log-real GDP. The trend for GDP is computed using the H-P filter with parameter 6.25. “Elas. w/ GDP” corresponds to the elasticity between the average number of workers per establishment by firm size on a constant, a linear trend and log real detrended GDP. * denotes significant at the 10%. Source: U.S. Census Bureau, Business Dynamics Statistics (BDS) Data Tables.
Table 21: Averages with and without a time trend

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 4</td>
<td>1908.71</td>
<td>2437.26</td>
<td>6.55</td>
<td>5.78</td>
<td>2.31</td>
<td>2.23</td>
</tr>
<tr>
<td>5 to 9</td>
<td>736.37</td>
<td>941.08</td>
<td>7.32</td>
<td>6.63</td>
<td>6.59</td>
<td>6.54</td>
</tr>
<tr>
<td>10 to 19</td>
<td>407.70</td>
<td>541.73</td>
<td>8.40</td>
<td>7.82</td>
<td>12.68</td>
<td>12.75</td>
</tr>
<tr>
<td>20 to 49</td>
<td>237.02</td>
<td>328.85</td>
<td>11.01</td>
<td>10.62</td>
<td>24.43</td>
<td>24.62</td>
</tr>
<tr>
<td>50 to 99</td>
<td>71.04</td>
<td>102.05</td>
<td>7.54</td>
<td>7.47</td>
<td>40.23</td>
<td>40.07</td>
</tr>
<tr>
<td>100 to 249</td>
<td>35.00</td>
<td>53.62</td>
<td>8.20</td>
<td>8.54</td>
<td>51.44</td>
<td>49.89</td>
</tr>
<tr>
<td>250 to 499</td>
<td>9.77</td>
<td>15.39</td>
<td>5.16</td>
<td>5.46</td>
<td>59.00</td>
<td>54.07</td>
</tr>
<tr>
<td>500 to 999</td>
<td>4.68</td>
<td>7.25</td>
<td>4.81</td>
<td>5.01</td>
<td>63.17</td>
<td>58.40</td>
</tr>
<tr>
<td>1000 to 2499</td>
<td>3.04</td>
<td>4.64</td>
<td>6.52</td>
<td>6.81</td>
<td>66.13</td>
<td>62.86</td>
</tr>
<tr>
<td>2500 to 4999</td>
<td>1.11</td>
<td>1.76</td>
<td>4.77</td>
<td>5.15</td>
<td>60.53</td>
<td>58.93</td>
</tr>
<tr>
<td>5000 +</td>
<td>1.45</td>
<td>2.18</td>
<td>29.74</td>
<td>30.72</td>
<td>75.81</td>
<td>61.80</td>
</tr>
</tbody>
</table>

Note: The “Avg. # of Firms” corresponds to the average number of firms in each size bin over our sample (in thousands). “Fraction Total Emp.” is computed as the average of total employees in each size bin divided by total employment. “Avg. Emp. per Plant” corresponds to the average of the number of employees per establishment in each size category. Source: U.S. Census Bureau, Business Dynamics Statistics (BDS) Data Tables.
A1.5 Evidence on Market Exposure Expenses

In this section, we present evidence about the relationship between market exposure and expenses. The model assumes that marketing and sales expenses are increasing in the number of markets that a firm serves. The evidence is broadly consistent with this assumption and reflects the notion that complexity in management is tied to some resource that is in fixed supply.

More specifically, we have access to selling, administrative and general expenses (SGA) that are a proxy of market expansion costs since they refer to expenses on, for example, advertising, marketing, brand development, and research and development. Using this information and our measures of market exposure (SICs, Establishments, and MSAs), we estimate a cost function that links changes in SGA with firm size and changes in market presence. For each measure of market presence, we classify firms according to how broad their market presence is: “small”, “medium” and “large.” We then estimate how the change in expenses is correlated to the change in market exposure plus the change in market exposure interacted with a dummy for the size category after conditioning for firm size (labor). We leave out the medium size category, so a full convex functional form would result in a negative coefficient for the interaction term between changes in market exposure and the small category and a positive coefficient term for the interaction term between changes in market exposure and the large category. Table 22 presents the results.

Table 22: Cost of Expansion and Market Exposure

<table>
<thead>
<tr>
<th>Market Exposure Measure</th>
<th>Dep. Var. ΔExpenses</th>
<th>SIC</th>
<th>Estab.</th>
<th>MSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>0.0017</td>
<td>0.0023</td>
<td>0.0023</td>
<td></td>
</tr>
<tr>
<td>s.e.</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Δ Markets</td>
<td>9.4763</td>
<td>0.2976</td>
<td>0.0035</td>
<td></td>
</tr>
<tr>
<td>s.e.</td>
<td>1.16</td>
<td>0.01</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Δ Markets × Small</td>
<td>-1.1235</td>
<td>-0.3117</td>
<td>2.1085</td>
<td></td>
</tr>
<tr>
<td>s.e.</td>
<td>0.241</td>
<td>0.01</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Δ Markets × Large</td>
<td>8.7281</td>
<td>-0.2656</td>
<td>0.3675</td>
<td></td>
</tr>
<tr>
<td>s.e.</td>
<td>2.40</td>
<td>0.01</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.0522</td>
<td>0.0941</td>
<td>0.0878</td>
<td></td>
</tr>
<tr>
<td># Obs.</td>
<td>135300</td>
<td>111300</td>
<td>111300</td>
<td></td>
</tr>
</tbody>
</table>

Source: Data are from the link Compustat Fundamental to Compustat Segment and the link Compustat Fundamental to LBD data.
This table shows that we find a convex form, either across all size categories or at least between two of them, for the three measures of market exposure. In particular, when using SICs as the measure of market expansion the coefficients \( \Delta \text{Markets} \times \text{Small} \) and \( \Delta \text{Markets} \times \text{Large} \) are both negative. When using Establishments or MSA as a measure of market exposure, the cost function shows signs of being convex only across two size categories (in the small-medium portion for Establishments and in the medium-large portion for MSAs).

A1.6 Firm-Level Risk over the Business Cycle: Robustness Checks

Our preferred proxy for firm-level idiosyncratic risk is the portion of growth in sales that is not explained by industry effects, time effects, or firm characteristics associated with growth such as age or size (measured by employment). However, an alternative measure can be derived from an autoregressive model of \( \log(sales) \). In particular, we estimate the following AR(1) process for \( \log(sales) \) for firm \( i \), in industry \( j \), between period \( t \)

\[
\ln(sales)_{ijt} = \rho \ln(sales)_{ijt-1} + \mu_i + \delta_{jt} + \beta_{1j} \ln(size)_{ijt} + \beta_{2j} \ln(age)_{ijt} + \epsilon_{ijt}, \quad (A.1.1)
\]

where \( \mu_i \) is a firm fixed effect that accounts for unobserved persistent heterogeneity at the firm level (such as higher productivity or higher human capital of the entrepreneur). The variable \( \delta_{jt} \) denotes a full set of time- and industry-specific fixed effects.\(^{60}\) We allow for industry-specific size and age effects. The estimation of equation (1) is done using the fixed effects panel estimator with robust standard errors.

Once equation (A.1.1) is estimated, we can compute the error, or the pure idiosyncratic and unpredictable component of firms’ sales \( \epsilon_{ijt} \) and proxy firm-level risk by \( \epsilon_{ijt}^2 \). Figure 9 presents the evolution of detrended log-median \( \epsilon_{ijt}^2 \).

---

\(^{60}\)We use two-digit NAICS codes for firms in our KFS and Compustat samples.
Consistent with the measure of firm level risk derived from sales' growth, detrended log-median $\epsilon^2_{ijt}$ derived from equation (A.1.1) is countercyclical (correlation equal to -0.319) and significant at the 10% level.

A1.7 Conditional Firm-Level Risk over the Business Cycle: Robustness Checks

An alternative measure of firm-level risk can be derived using the time series standard deviation of $\epsilon_{it}$ for each firm. In order to obtain a value at the firm level that is still suitable for computing business cycle correlations (i.e., that does not collapses to a single number) and that allows us to condition on whether a firm is a “changer” or a “non-changer” we use a five-period rolling window for each firm. More specifically, the estimated $\sigma_e$ in period $t$ for
firm \(i\) is the five period standard deviation of \(\epsilon_{it}\) computed centered at period \(t\). Figure 10 presents the cyclical properties of the median \(\epsilon_{it}\) conditional on ‘changer” and “non-changer”.

59
Figure 10: Firm-Level Risk “changers” versus “non-changers”

Panel (i): Market Exposure = Number of SIC Codes

Panel (ii): Market Exposure = Number of Establishments

Panel (iii): Market Exposure = Number of MSAs

Panel (iv): Market Exposure = Number of Establishments × SIC Codes

Note: Detrended GDP corresponds to detrended log-real GDP. The market exposure measure corresponds to the number of four-digit industry codes (i.e., line of business), number of establishments a firm operates, number of MSAs in which the firms are present, and the product of establishments times SIC codes. “Changers” refers to firms that change market exposure measure in a given period. “Non-changers” refers to firms that do not change their market exposure measure. All series are detrended using an H-P filter with parameter 6.25. Source: Data are from the link Compustat Fundamental to Compustat Segment and the link Compustat Fundamental to LBD data.
This figure shows that the cyclical properties of the standard deviation of $\epsilon_{it}$ are qualitatively (and for most part quantitatively consistent) with our preferred measure of firm level risk. In particular, firm-level risk for “changers” is countercyclical and for “non-changers” less countercyclical or acyclical.

**A1.8 Firm Size and Conditional Firm Level Risk: Robustness Checks**

Figure 11 presents the correlation of firm-level risk conditional on firm size for a different size category than that presented in the main body of the text. In particular, to construct this figure, we rank firms by their size (employment) and label as “small” those firms in the bottom 10% of the size distribution, as “medium” those firms in the 10%-90% range of the distribution and as “large” those firms in the top 10% of the size distribution.

Figure 11: Firm-Level Risk over the Business Cycle (Conditional on Size)

![Figure 11: Firm-Level Risk over the Business Cycle (Conditional on Size)](image)

Note: Idiosyncratic risk based on sales growth. Sources: Compustat Fundamental and link CompuStat Fundamental to LBD data
The evidence is consistent with our model and shows that the correlation is negative and stronger for large firms than for small firms. The business cycle correlations of firm level risk are -0.22 (p-value=0.21), and -0.321 (p-value=0.06) for “small” and “large” respectively. This is not surprising based on the model we presented since, as we discussed in the previous section, not only “changers” are large but also large firms tend to be “changers”.

In this section, we also provide evidence on the conditional (on size) business cycle correlations of firm-level risk derived from equation A.1.1. We label firms by their size following the ranking in the body of the paper. That is, we defined as “small” those firms in the bottom 25% of the size distribution, as “medium” those firms in the 25%-75% range of the distribution, and as “large” those firms in the top 25% of the size distribution.\(^61\) Figure 12 presents the evolution of median log(\(\epsilon^2\)) conditional on size.

Figure 12: Firm Risk over the Business Cycle (conditional on size)

Note: Idiosyncratic risk derived from equation A.1.1. Sources: Compustat Fundamental and link Compustat Fundamental to LBD data

As with the benchmark definition of firm level risk, we find that the correlation is negative

\(^{61}\)Results are robust to different definitions of “small,” “medium,” and “large.”
and stronger for large firms than for small firms. To further explore the link between firm level risk and firm size, Table 23 presents several moments of the distribution of firm-level risk conditional on firm size when firm-level risk is derived from equation A.1.1. To construct this table, we condition on a particular size bin and then compute the moment of the distribution of firm-level risk.

Table 23: Firm-Level Risk (Conditional on Size)

<table>
<thead>
<tr>
<th>Employment Size</th>
<th>Mean</th>
<th>Median</th>
<th>Bottom 99%</th>
<th>Bottom 90%</th>
<th>Top 10%</th>
<th>Top 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 4</td>
<td>-2.03</td>
<td>-1.68</td>
<td>-9.77</td>
<td>-5.30</td>
<td>0.99</td>
<td>2.81</td>
</tr>
<tr>
<td>5 to 9</td>
<td>-2.35</td>
<td>-2.08</td>
<td>-9.41</td>
<td>-5.70</td>
<td>0.61</td>
<td>2.44</td>
</tr>
<tr>
<td>10 to 19</td>
<td>-2.63</td>
<td>-2.35</td>
<td>-10.58</td>
<td>-6.03</td>
<td>0.40</td>
<td>2.33</td>
</tr>
<tr>
<td>20 to 99</td>
<td>-3.39</td>
<td>-3.12</td>
<td>-11.08</td>
<td>-6.68</td>
<td>-0.44</td>
<td>1.76</td>
</tr>
<tr>
<td>100 to 499</td>
<td>-4.16</td>
<td>-3.84</td>
<td>-11.91</td>
<td>-7.33</td>
<td>-1.38</td>
<td>0.69</td>
</tr>
<tr>
<td>500 or more</td>
<td>-5.08</td>
<td>-4.72</td>
<td>-12.88</td>
<td>-8.23</td>
<td>-2.35</td>
<td>-0.55</td>
</tr>
</tbody>
</table>

Note: Idiosyncratic risk derived from equation A.1.1. Source: Data are from the link Compustat Fundamental to Compustat Segment and the link Compustat Fundamental to LBD data.

A1.9 Market Exposure and Volatility: Robustness Checks

Table 24 presents the estimates from equation (27) using the number of SIC codes, as well as SGA expenses and Advertising expenses using the Compustat Segment database. We incorporated size and age as additional controls (in addition to the firm fixed effects, the year-industry fixed effects, and the time trend).

Table 25 presents the estimates from equation (27) using the number of establishments, MSAs, and the product of SIC codes and establishments using the computstat-LBD database. In this table, too, we incorporated size and age as additional controls (in addition to the firm fixed effects, the year-industry fixed effects and the time trend).

Observe that the relationship between our measures of market exposure (SIC codes, establishments, MSA, SIC×establishments, SGA expenses, and advertising expenses) and firm-level volatility is robust to the incorporation these additional controls. The coefficient on the appropriate market exposure measure is negative in all our specifications other than in the case of the number of SIC codes, and the product of SIC codes and establishments when size is used as a control. Moreover, the introduction of size as a control makes the estimates on market exposure nonsignificant. Note that this is an expected result from our theoretical model given the high correlation between the number of markets a firm decides
to participate in and the total number of employees the firm has. In the data, the measures of market exposure and employment are also highly correlated with correlation coefficients of up to 0.47.

Finally, Table 26 reports the elasticity of the firm-level volatility to additional number of market exposure measures. We look at the number of SIC codes, number of establishments, number of MSAs, as well as their interactions. All these variations of the market exposure measure deliver very close elasticities between 9% and 13%.
Table 24: Market Exposure and Firm-Level Idiosyncratic Volatility I

<table>
<thead>
<tr>
<th></th>
<th>Dependent Variable $\ln(\varepsilon_{ijt}^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln(numberSICs_{ijt})$</td>
<td>- - -</td>
</tr>
<tr>
<td>Std. Error</td>
<td>- - -</td>
</tr>
<tr>
<td>$\ln(expenses_{ijt})$</td>
<td>-0.299 -0.228 -0.096</td>
</tr>
<tr>
<td>Std. Error</td>
<td>0.010*** 0.01*** 0.015***</td>
</tr>
<tr>
<td>$\ln(advertising_{ijt})$</td>
<td>- - - -0.126 -0.095 -0.011</td>
</tr>
<tr>
<td>Std. Error</td>
<td>0.012*** 0.012*** 0.014</td>
</tr>
<tr>
<td>$\ln(size_{ijt})$</td>
<td>- - -0.252 -0.304</td>
</tr>
<tr>
<td>Std. Error</td>
<td>0.014*** -0.021*** 0.010***</td>
</tr>
<tr>
<td>$\ln(age_{ijt})$</td>
<td>- - -0.451 -0.541 -0.559</td>
</tr>
<tr>
<td>Std. Error</td>
<td>-0.014*** -0.026*** 0.016***</td>
</tr>
</tbody>
</table>

| Firm Fixed Effect      | Yes Yes Yes                                   |
| Industry-Year          | Yes Yes Yes                                   |
| Time Trend             | Yes Yes Yes                                   |

| N. Obs. | 184548 | 184548 | 184548 | 70003 | 70003 | 70003 | 155175 | 155175 | 155175 |
| R-squared | 0.0455 | 0.0511 | 0.0473 | 0.0348 | 0.0417 | 0.0382 | 0.0178 | 0.0264 | 0.0243 |

Sample: Compustat
Years: 1960-2012

Note: *** denotes significant at the 1% level, ** at the 5% level, and * at the 10% level. $\ln(\varepsilon_{ijt}^2)$ is constructed from the estimated residual of equation (1). $\ln(expenses_{ijt})$ is constructed as log real selling, general and administrative expenses (SGA) and $\ln(advertising_{ijt})$ corresponds to Advertising Expenses (XAD). Industry deflators are used in every case. $\ln(size_{ijt})$ corresponds to log-employment as in equation (1). The age of the firm corresponds to the number of years in the Compustat Fundamental sample. $\ln(numberSICs_{ijt})$ corresponds to four-digit SIC codes as reported in the Compustat Segment Data.
Table 25: Market Exposure and Firm-Level Idiosyncratic Volatility II

<table>
<thead>
<tr>
<th></th>
<th>Dependent Variable $\ln(\epsilon^2_{ijt})$</th>
<th>$-0.0808$</th>
<th>$-0.0461$</th>
<th>$-0.0048$</th>
<th>$0.00882^{***}$</th>
<th>$0.00887^{***}$</th>
<th>$0.00944$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln(numberEstabl_{ijt})$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$0.00805^{***}$</td>
<td>$0.00813^{***}$</td>
<td>$0.00875$</td>
</tr>
<tr>
<td>Std. Error</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln(numberMSA_{ijt})$</td>
<td>$-0.0941$</td>
<td>$-0.0525$</td>
<td>$-0.00179$</td>
<td>$-0.075$</td>
<td>$-0.0368$</td>
<td>$0.00437$</td>
<td></td>
</tr>
<tr>
<td>Std. Error</td>
<td>$0.010^{***}$</td>
<td>$0.0103^{***}$</td>
<td>$0.0109$</td>
<td>$0.012^{***}$</td>
<td>$0.012^{***}$</td>
<td>$0.012^{***}$</td>
<td></td>
</tr>
<tr>
<td>$\ln(SIC \times Est_{ijt})$</td>
<td>-</td>
<td>-</td>
<td>$-0.282$</td>
<td>$-0.558$</td>
<td>$-0.558$</td>
<td>$-0.558$</td>
<td></td>
</tr>
<tr>
<td>Std. Error</td>
<td>-</td>
<td>-</td>
<td>$0.012^{***}$</td>
<td>$0.019^{***}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln(size_{ijt})$</td>
<td>-</td>
<td>-</td>
<td>$-0.282$</td>
<td>$-0.558$</td>
<td>$-0.558$</td>
<td>$-0.558$</td>
<td></td>
</tr>
<tr>
<td>Std. Error</td>
<td>-</td>
<td>-</td>
<td>$0.012^{***}$</td>
<td>$0.019^{***}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln(age_{ijt})$</td>
<td>$-0.558$</td>
<td>-</td>
<td>-</td>
<td>$-0.558$</td>
<td>$-0.558$</td>
<td>$-0.558$</td>
<td></td>
</tr>
<tr>
<td>Std. Error</td>
<td>$-0.558$</td>
<td>-</td>
<td>$0.019^{***}$</td>
<td>$0.019^{***}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Firm Fixed Effect | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
Industry-Year     | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
Time Trend         | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
N. Obs.            | 129724 | 129724 | 129724 | 124433 | 124433 | 124433 | 129724 | 129724 | 129724 |
R-squared          | 0.018 | 0.0251 | 0.0227 | 0.0183 | 0.0253 | 0.0228 | 0.018 | 0.02561 | 0.0227 |
Sample Years       | Compustat - LBD | 1977-2011 |

Note: *** denotes significant at the 1% level, ** at the 5% level, and * at the 10% level. $\ln(\epsilon^2_{ijt})$ is constructed from the estimated residual of equation (1). Industry deflators are used in every case. $\ln(size_{ijt})$ corresponds to log-employment as in equation (1). The age of the firm corresponds to the number of years in the Compustat LBD sample. $\ln(numberSICs)$ corresponds to four-digit SIC codes as reported in the Compustat LBD Data. The source for the number of establishments and MSAs is the Compustat-LBD database.
Table 26: Market Exposure and Firm-Level Idiosyncratic Volatility III

<table>
<thead>
<tr>
<th></th>
<th>Dependent Variable $\ln(\epsilon_{ijt}^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(numberSICs)</td>
<td>-0.137</td>
</tr>
<tr>
<td>Std. Error</td>
<td>0.0212***</td>
</tr>
<tr>
<td>ln(numberestabs.)</td>
<td>-0.081</td>
</tr>
<tr>
<td>Std. Error</td>
<td>0.009***</td>
</tr>
<tr>
<td>ln(numberMSAs)</td>
<td>-0.094</td>
</tr>
<tr>
<td>Std. Error</td>
<td>0.01***</td>
</tr>
<tr>
<td>ln(SICs * estabs.)</td>
<td>-0.075</td>
</tr>
<tr>
<td>Std. Error</td>
<td>0.008***</td>
</tr>
<tr>
<td>ln(SICs * MSAs)</td>
<td>-0.085</td>
</tr>
<tr>
<td>Std. Error</td>
<td>0.009***</td>
</tr>
<tr>
<td>ln(estabs. * MSAs)</td>
<td>-0.045</td>
</tr>
<tr>
<td>Std. Error</td>
<td>0.005***</td>
</tr>
<tr>
<td>Firm Fixed Effect</td>
<td>Yes</td>
</tr>
<tr>
<td>Industry-Year Control</td>
<td>Yes</td>
</tr>
<tr>
<td>Time Trend</td>
<td>Yes</td>
</tr>
<tr>
<td>N. Obs.</td>
<td>155175</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.0178</td>
</tr>
<tr>
<td>Sample</td>
<td>Compustat</td>
</tr>
<tr>
<td>Years</td>
<td>1960-2012</td>
</tr>
</tbody>
</table>

Note: *** denotes significant at the 1% level, ** at the 5% level, and * at the 10% level. $\ln(\epsilon_{ijt}^2)$ is constructed from the estimated residual of equation (1). $\ln(numberSICs)$ corresponds to four-digit SIC codes as reported in the Compustat Segment Data. $\ln(numberestabs.)$ corresponds to the number of establishments each firm owns as derived from our sample that links Compustat with the LBD. $\ln(numberMSAs)$ corresponds to the number of MSA’s where the establishments a firm owns are located also from the Compustat-LBD link. Some variables were automatically omitted due to collinearity.
A1.9.1 Alternative Measures of Volatility

In this section, we present evidence of the elasticity of firm-level volatility and market exposure. We introduce two alternative measures of firm level volatility. The first one corresponds to the time series standard deviation of $\ln(\epsilon_{ijt}^2)$, denoted $\sigma_\epsilon$ for each firm $i$. We regress this measure of volatility to the different measures of Market Exposure. Note that since we are aggregating time series information, this regression is a cross-sectional regression and we choose the average value of each market exposure variable as controls. The second measure of volatility corresponds to a five-period rolling window standard deviation of $\ln(\epsilon_{ijt}^2)$ for each firm $i$. This measure allows us to keep the panel dimension in place. Table 27 presents the results.

Table 27: Firm-Level Volatility and Market Exposure

<table>
<thead>
<tr>
<th></th>
<th>Dep. Var. time series $\sigma_\epsilon$</th>
<th></th>
<th>Dep. Var. Rolling Window $\sigma_\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>Estab.</td>
<td>MSAs</td>
<td>SICs</td>
</tr>
<tr>
<td>log($X$)</td>
<td>-0.154***</td>
<td>-0.183***</td>
<td>-0.279***</td>
</tr>
<tr>
<td>S.E.</td>
<td>0.0008</td>
<td>0.0009</td>
<td>0.0031</td>
</tr>
<tr>
<td>N</td>
<td>194800</td>
<td>194800</td>
<td>210600</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.249</td>
<td>0.250</td>
<td>0.125</td>
</tr>
<tr>
<td>Source</td>
<td>LBD-Compustat Link</td>
<td></td>
<td>LBD-Compustat Link</td>
</tr>
</tbody>
</table>

Note: All results are significant at the 1% level. Sources: The number of establishments and MSAs were taken from the LBD, the number of SICs are from the Compustat Segment database. $\ln(\epsilon_{ijt}^2)$ is derived from estimating equation (1) on our Compustat Fundamental Sample. Regressions are run after performing the corresponding links with LBD and Compustat Segment data sets.

Consistent with the estimates presented in the main body of the text, we find that the elasticity of volatility to market exposure is negative and significant in every case ranging from -2.1% to -27.9%.
A1.9.2 Conditional Firm-Level Volatility over Business Cycle

In this section, we present how the estimated value for the conditional variance of $\ln(\epsilon_{ijt}^2)$, $\hat{\sigma}_{ijt}^2 = \exp(\hat{\gamma}_i + \hat{\theta}_{ij} + \hat{\alpha}_1 \ln(X_{ijt}) + \hat{\alpha}_2 t) \hat{E}(\hat{u}_{ijt})$, moves with the business cycle. Note that we focus on business cycle fluctuations, so we do not need to estimate the scale factor $\hat{E}(\hat{u}_{ijt})$ since it is constant across firms and time. Figure 13 presents the evolution of firm-level volatility when estimated using Establishments as our measure of market exposure.\footnote{Results are robust to every definition of market exposure (i.e., MSAs, SICs, SICs×Establishments).}

Figure 13: Firm Risk over the Business Cycle

Note: Idiosyncratic volatility is based on the fitted value of equation (27). Sources: Compustat Fundamental and link Compustat Fundamental to LBD data

Figure 13 shows that this measure of volatility is countercyclical as our previous measures of firm-level risk.