These notes cover selected topics.

I. Introduction

1. What do we study

- How the political nature of policymaking affect economic outcomes.
  - For example, what explains the divergence between textbook models of economic policymaking (where the question is: what are optimal policies?) and real world experience (in which these policies are often not adopted)?

- How economic models may help explain political phenomena

1.1 Some definitions

**Economics** = optimal use of scarce resources

**Politics** = study of power and authority

**Power** = ability of individual or group to achieve its aims

1.2 Heterogeneity of interests as key

The key point is that there is only an issue when there is heterogeneity of interests or a conflict of interests in a society, so that it must find ways to make collective choices.

Consider two general types of heterogeneity of interests:

- Differences in tastes, endowments, etc. implying different preferred policies. (“ex ante” heterogeneity)

- Distributional conflict among agents with similar “ex ante” objectives.

The conflict of interests may be between citizens, as in the following example, or between citizens and policymakers.
1.2.1 An Illustration

Take the following example from an optimal capital accumulation problem. For a representative agent, we write:

$$Max \sum_{t=0}^{\infty} \beta^t u(c_t)$$ (1)

subject to

$$k_{t+1} + c_t \leq f(k_t)$$ (2)

where $c_t, k_{t+1} \geq 0$ and $k_0$ is given. This is a dynamic programming problem.

To model ex ante heterogeneity, suppose two different agents $R$ and $L$ with different discount factors, say $\beta_R \neq \beta_L$, so that they would like different paths for $\{c_t, k_{t+1}\}_{t=0}^T$. If we assign weights $\alpha$ and $1 - \alpha$ to the two types of agents in the social welfare function, we could derive the optimal path. That is, replace (1) with

$$Max \sum_{t=0}^{\infty} \alpha \beta_R^t u\left(\frac{1}{2}c_t\right) + (1 - \alpha) \beta_L^t u\left(\frac{1}{2}c_t\right)$$ (3)

(where it is assumed both agents receive half of the consumption pie) and maximize (3) subject to (2).

To model distributional conflict, suppose the two agents have exactly the same policy preferences, discount rate, ..., but each would like more of the pie. That is, let total consumption be divided according to

$$c^R + c^L = c_t$$ (4)

where $c^R_t$ need not equal $c^L_t$. The case of distributional conflict could be represented by maximizing

$$Max \sum_{t=0}^{T} \beta^t \left(\alpha u(c^R_t) + (1 - \alpha) u(c^L_t)\right)$$ (5)

subject to

$$k_{t+1} + c^R_t + c^L_t \leq f(k_t)$$ (6)

Note that in both problems the actual path depends on $\alpha$, and agents have obvious (polar) preferences over $\alpha$.

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$^1$Given the concavity of $u(\cdot)$, if $c^R_t \neq c^L_t$, the agent’s may want different time paths for $k_t$, but will want the same steady state.
1.3 Some Notes

1. Why associate heterogeneity with politics? Isn’t heterogeneity also key to much of economics? (Consider the need to trade among heterogeneous agents, leading to supply and demand and the use of markets.) There are two differences: first, conflict of interest is necessary for a political problem, but not for there to be an economic problem (as in the case of Robinson Crusoe); second, the mechanisms by which conflicts are resolved are different (compare a price mechanism to allocate goods versus lobbying or voting over allocation or distribution).²

2. Aren’t the multi-agent problems in section 1.2 simply problems in multi-person welfare economics? It depends who chooses $\alpha$. In welfare economics we think of a social planner doing the maximization subject to given $\alpha$ (or to $\alpha$ determined outside the model). In contrast, politics would focus on how $\alpha$ would be determined – who has the power to determine $\alpha$ and how different political mechanisms translate into different values of $\alpha$. Hence, the welfare economics approach might be thought of as leaving out exactly the problem that is the focus of politics.

3. We could also illustrate the two topics of political economy mentioned above in terms of this example. Combining the political determination of the $\alpha$ with the economic analysis of the relation of $\alpha$ (and of how they are chosen) to economic outcomes illustrates the first topic, economic modeling of determination of $\alpha$ the second.

4. These examples stress how the politically determined outcome will differ from the social planner’s outcome, but in both cases the solution is Pareto efficient. Political choice mechanism may lead to inefficient outcomes, as the example of hyperinflation illustrates.

2 Political-Economic Equilibrium

2.1 Direct Democracy

To further illustrate the first topic, consider a three-stage political-economic equilibrium in a direct democracy, that is where decisions are not delegated to policymakers:

(1) Agents optimize over their economic choice variables given policy (for example, optimal labor supply and consumption as a function of the tax system), given their beliefs and information;

²An interesting perspective is in Hirschman, Exit, Voice, and Loyalty, comparing Exit (some customers stop buying the firm’s products; some members leave the organization) and Voice (the firm’s customers or the organization’s members express their dissatisfaction directly to management).
(2) On the basis of how it affects him (that is, the individual optimization in (1)), each individual
determines his preferred policy choice (for example, tax system), given his beliefs and information;
(3) Aggregation of the policy preferences in (2) into an economy-wide policy via some collective choice
mechanism.

2.1.1 Some notes

The first and second steps can be summarized by each individual calculating the policy that maximizes
his expected utility. Sometimes, this calculation is simple, as in our example above. Aggregation in
the third stage need not be via voting – an alternative would be agreement by all parties (consensus)
– but this is the leading aggregation mechanism. Riker calls voting the “central act of democracy.”
(Democracy implies voting, but voting does not imply democracy, as voting can be a sham. There must
be a genuine choice.)

2.2 Representative Democracy

In a representative democracy, citizens don’t choose policies directly, but choose policymakers to whom
the delegate choice of policies. We will concentrate on voting over candidates as the obvious aggregation
mechanism here, but it could be combined with, for example, contributing to candidates which would
also influence the equilibrium collective policy. The policy choice mechanism in a representative democ-

The riker’s central question is whether it is true, as much democratic theory argues, that democratic ends can be achieved
by democratic means. We will return to this.

Equilibrium often also specifies how beliefs are formed and require that they satisfy some consistency requirements,
but we omit this for now.
A voting rule determines which candidate becomes the policymaker (or policymakers), who then choose(s) policy.\(^5\)

### 2.2.1 Some notes

In the first part of the definition, voters need not vote for the candidate who maximizes their utility if there are more than two candidates, which is where the behavior of other voters comes in. Nor need they vote at all – the optimal behavior to maximize utility may be to abstain. In part (2) the objective function of candidates – social welfare, being in office, preferred policy, maximizing campaign contributions – will obviously affect the nature of the equilibrium. In part (3), voters may choose representatives in a legislature, who would subsequently choose policy via some aggregation mechanism.

Both of these definitions of equilibrium may be quite useful in helping to organize the various topic to be covered.

\(^5\)In a model where policy is determined by the interaction of policymakers and lobbies, we would replace “citizens” and “voters” by “lobbies”, actions in part (1) would be contributions (or contribution schedules), and the aggregation mechanism in part (3) would be some sort of bargaining.
II. Elections and Voting

We begin with voting as the method for aggregation of preferences.

1 Voting Over Two Options

If there are only two options, then things are simple. Take a “fair” vote over the 2 options, with the option or candidate with the majority being chosen. Riker, chapter 2, discusses a number of attractive properties of majority rule.

The basic problem, as pointed out by Riker, is in limiting options or candidates to two. It is extremely rare that two options naturally exist; methods to limit are often themselves unfair.

2 More than Two Options – Condorcet’s Paradox

Consider three voters $A$, $B$, and $C$ and three policy options, $\pi_1$, $\pi_2$, and $\pi_3$. Suppose each voter’s preference are as follow (where “$\succ$” means “is preferred to”)

\[
A : \quad \pi_1 \succ \pi_2 \succ \pi_3 \\
B : \quad \pi_2 \succ \pi_3 \succ \pi_1 \\
C : \quad \pi_3 \succ \pi_1 \succ \pi_2 
\]

Using majority rule to aggregate preferences (where the winner of the first contest faces the policy not in the first vote) and denoting by $\succ^M$ the winner in a majority vote, one has

\[
\pi_1 \succ^M \pi_2 \succ^M \pi_3 \succ^M \pi_1 \tag{1}
\]

Hence, there is “cycling” with the order of voting determining the winner. Agenda setters can determine the outcome. Or, equivalently, with more than two options, there may be no determinate winner in a simple majority system.\footnote{We return to Riker’s question of whether democratic means can produce democratic results. First, can voting restrain officials if the outcome of voting is inconsistent? Second, if the outcome of voting depends on the order of voting, “does not an accident of institutions, rather than popular taste, select the winner?”}

How can majority voting produce a consistent outcome?
3 Restricting Preferences

One possibility is to make assumptions about individual preferences to eliminate the Condorcet paradox.

3.1 One Dimension

3.1.1 Single-Peaked preferences.

Consider taste for guns versus butter, as in Drazen (2000).

If choices are over primitives in \( u(\cdot) \), then simple concavity of \( u(\cdot) \) is sufficient. But when choices depend on policy (for example, supply of labor as a function of the tax rate) concavity of \( u(c, 1-l) \) does not ensure concavity of indirect utility function \( V(\tau) \) and hence single-peakedness.

3.1.2 Median voter theorem – MVT

See Drazen (2000) for a discussion and informal proof.

Drawbacks –

- participation
- strategic voting – Grossman-Helpman, 47-8

3.1.3 Single-crossing property

Preferences will often satisfy this even if they aren’t single peaked. This will allow MVT to go through.

3.2 Two or More Dimensions

Condorcet winner must be median in "all directions" for median voter to hold.

An example (in class) will make clear how unlikely this is.

4 Alternative Voting Rules


5 Restricting Institutions

*Probabilistic voting* model – discussed below.

These latter two flow from the observation that policy choices are not made via direct democracy, but are delegated to elected representatives.

5.1 Structure-Induced Equilibrium

“Decisions by a group of representatives, with given policy preferences, which is in charge of making policy decisions in a committee or a legislature. The political institution prescribes some procedure for reaching a consensus. Specifically, consider a situation in which the decision can be split in different stages, each stage being under the jurisdiction of a specific committee or being the outcome of a separate vote.” Persson and Tabellini (2000, p. 35). See their discussion.

5.2 Agenda-Setting

“In many political decisions, however, specific politicians or bureaucrats do have a great deal of influence on the alternatives the decision makers face: they may not only have the power to propose, but also to prevent amendments from being made (gatekeeping power), such that a closed-agenda process is a better description of reality. Successful positive modeling of political decisions will then have to take these powers into account. The same argument applies, of course, irrespective of whether the policy issue at hand is one-dimensional and a Condorcet winner exists. Suppose it does. Then if the agenda setter’s preferred policy does not coincide with the pivotal voter’s, neither will the equilibrium policy.” Persson and Tabellini (2000, p. 37).
III. Voting Equilibrium in A Representative Democracy

We now consider the existence of equilibrium in a representative democracy.

1 Opportunistic Candidates – Downs

Candidates care only about being elected to office. In this case, the general result is that policies converge to that favored by the median voter. This is a key uneasiness with the approach. See Roemer (2001) *Political Competition* (Harvard University Press) for a critique of the Downs approach in this respect and an extensive exposition of policy-motivated candidates, as discussed below.

1.1 A basic result

Let \( p(\tau_A, \tau_B) \) denote the probability that candidate A wins.

Call \( S(\tau, \tau') \) set of voters who prefer \( \tau \) to \( \tau' \).

**Definition:** A pair of platforms \( (\tau^*_A, \tau^*_B) \) is an equilibrium if

(i) \( \tau^*_A = \arg\max p(\tau^*_A, \tau^*_B) \)

(ii) \( \tau^*_B = \arg\max (1 - p(\tau^*_A, \tau^*_B)) \)

**Proposition:** A pair of platforms \( (\tau^*_A, \tau^*_B) \) is an equilibrium if and only if, for each candidate \( J \), \( \tau^*_J \) is a Condorcet winner.

**Proof:** 1) Suppose that \( (\tau^*_A, \tau^*_B) \) is an equilibrium. Then clearly \( p(\tau^*_A, \tau^*_B) = \frac{1}{2} \). Suppose that (say) \( \tau^*_A \) were not a Condorcet winner. Then there would exist some \( \tau' \) that would allow \( B \) to win with probability 1 (that is, \( \#S(\tau', \tau^*_A) > \#S(\tau^*_A, \tau') \)) - a contradiction.

2) Suppose that for each candidate \( J \), \( \tau^*_J \) is a Condorcet winner. Then it must be the case \( \#S(\tau^*_B, \tau^*_A) > \#S(\tau^*_A, \tau^*_B) \), so that if each candidate \( J \) selects \( \tau^*_J \) the candidates would tie; \( p(\tau^*_A, \tau^*_B) = \frac{1}{2} \). and no candidate could do better by deviating. Thus, \( (\tau^*_A, \tau^*_B) \) is an equilibrium. \( \square \)

**Corollary:** There exists an equilibrium if and only if there exists a Condorcet winner. Condorcet winners do not typically exist unless the policy space is one dimensional and voters’ policy preferences are single-peaked.

**Corollary:** The same result will hold true if candidates maximize their vote share.


2 Probabilistic Voting

Thus, the Downsian model typically fails to deliver a prediction when policy spaces are multi-dimensional. The most popular “fix” is to assume that voters vote probabilistically. Thus, the probability that voter $i$ votes for candidate $A$ over candidate $B$ is an increasing, but smooth function of the utility difference $u(\tau_A^*) - u(\tau_B^*)$. This can give equilibrium. Note, however, that in a single dimension, there is still convergence to the median voter’s policy with probabilistic voting. You should demonstrate this to yourself.

Voter $i$ votes for $A$ over $B$ if $u(\tau_A) - u(\tau_B) \geq \eta$, where $\eta$ is the realization of a Random Variable from CDF $F_i(\eta)$. Hence the candidates’ perspective, the probability that voter $i$ votes for candidate $A$ is (NOT SAME $p$ as above!)

$$p_i(\tau_A, \tau_B) = F_i\left(u(\tau_A) - u(\tau_B)\right)$$

The expected number of votes for candidate $A$ is $\sum_i p_i(\tau_A, \tau_B)$. Each candidate seeks to maximize his expected votes.

Definition: A pair of policy platforms $(\tau_A, \tau_B)$ is an equilibrium if

(i) $\tau_A = \arg \max_{\tau \in \Upsilon} \sum_i p_i(\tau, \tau_B)$ and

(ii) $\tau_B = \arg \max_{\tau \in \Upsilon} N - \sum_i p_i(\tau_A, \tau)$

EXAMPLE: Dixit and Londregan, Lindbeck and Weibull

3 Policy-Motivated Candidates

Purely opportunistic candidates is empirically unsatisfactory both on \textit{a priori} grounds but also because the implications in terms of complete platform convergence are wrong.

It is logically unsatisfactory because candidates must be citizens and citizens are presumed to have policy preferences.

The main result was that the median voter theorem was robust to the extension of allowing opportunistic candidates to have preferences over policy. (See Grossman and Helpman.)

Later work introduced candidate uncertainty in the location of the median voter and showed that this robustness conclusion was not always correct. To see this consider the following example, following Coate. Roemer \textit{Political Competition} section 3.3 also presents a good example.
3.1 An Example

Assume a one-dimensional policy space of $\tau \in (0, 1)$, where the payoff to voter $i$ from policy $\tau^J$ is $u^i(\tau^J, \tau^i) = - (\tau^J - \tau^i)^2$ and the payoff to the candidates is $u^P(\tau^J, \tau^P) = \chi - (\tau^J - \tau^P)^2$ for $P = L, R$. $\tau^L = 1 - \tau^R = t$, where $t < \frac{1}{2}$. Candidates believe that the ideal point of the median voter $\mu$ is uniformly distributed on $[\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon]$, where $\varepsilon \in (0, \frac{1}{2} - t)$. If the candidates choose platforms $\tau^L$ and $\tau^R$, the indifferent voter is one whose most preferred policy is at the midpoint $\tau^{mid}(\tau^L, \tau^R) = \frac{\tau^L + \tau^R}{2}$, so that $L$ wins if $\mu < \tau^{mid}(\tau^L, \tau^R)$. Therefore, assuming that $\frac{\tau^L + \tau^R}{2} \in (\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon)$, the probability that candidate $L$ wins is

$$\pi(\tau^L, \tau^R) = \Pr(\mu < \tau^{mid}(\cdot)) = \frac{\tau^{mid}(\cdot) - (\frac{1}{2} - \varepsilon)}{(\frac{1}{2} + \varepsilon) - (\frac{1}{2} - \varepsilon)} = \frac{\tau^L + \tau^R - 1}{4\varepsilon}$$

A pair of ideologies $(\tau^{L*}, \tau^{R*})$ is an equilibrium if

$$\tau^{L*} = \arg \max_{\tau^L} \pi(\tau^L, \tau^{R*}) \left[ \chi - (\tau^L - t)^2 \right]$$

$$- (1 - \pi(\tau^L, \tau^{R*})) \left[ (\tau^{R*} - t)^2 \right]$$

and

$$\tau^{R*} = \arg \max_{\tau^R} -\pi(\tau^{L*}, \tau^R) \left[ (\tau^{L*} - (1 - t))^2 \right]$$

$$+ (1 - \pi(\tau^{L*}, \tau^R)) \left[ \chi - (\tau^R - (1 - t))^2 \right]$$

A symmetric equilibrium is one where $\tau^{L*} = 1 - \tau^{R*}$. If $\chi < 2\varepsilon (1 - 2t)$ (that is, if holding office is not worth too much), then the equilibrium is symmetric with

$$\tau^{L*} = 1 - \tau^{R*} = \frac{\chi + \frac{1}{2} - t (1 - 2\varepsilon)}{2\varepsilon + 1 - 2t}$$

Hence, candidates move toward the expected most preferred policy of the median voter (since $\frac{\chi + \frac{1}{2} - t (1 - 2\varepsilon)}{2\varepsilon + 1 - 2t} > t$), but not all the way as long as $\chi < 2\varepsilon (1 - 2t)$. The degree to which candidates converge is positively related to the value of holding office $\chi$ and negatively related to the degree of uncertainty in the location of the median voter, represented by $\varepsilon$. It is also the case that the equilibrium is increasing in $t$. 

3
IV. Disciplining Policymakers

Elections may also serve as a device for disciplining policymakers. Here, the rational voter would be retrospective (deciding whether or not to vote for the incumbent based on his past performance) rather than prospective (deciding whether or not to vote for a candidate depending on expected utility if that candidate is elected).

1 Ferejohn (1986)

The basic model is due to Ferejohn (1986) which is covered in Drazen (2000).

2 Discipline with Candidate Types

Banks and Sundaram (1983), following Stephen Coate’s lecture notes.

2.1 Set-up

Unlike Ferejohn, consider two types of politicians ("competence"), but where candidates take actions, so that there is a discipline (or moral hazard problem). Politician type is unobserved. Assume just two politician types $\omega^1$ and $\omega^2$, where $\omega^2 > \omega^1$.

Higher types like more effort and get more utility from holding office for any given effort $a$. Say $u(a, \omega) > 0$ for all $a$. The prior on the fraction of the high type in the population is $\pi > 0$. A politician not in office gets a per-period payoff of 0.

Suppose also that there are just two outcome levels $y_t \in \{y^L, y^H\}$ where $y^L < y^H$. Assume continuous effort choice and let the probability that $y_t = y^H$ be $F(a)$ where $F'(a) > 0$ and $F''(a) < 0$.

The voter’s strategy can be summarized is a probability of voting for incumbent when $y_t = y^L, y^H$ namely

$$(\sigma(y^L), \sigma(y^H)) \in \{0,1\}^2.$$ For example, consider $(\sigma(y^L) = 0, \sigma(y^H) = 1)$. The voter’s beliefs can be summarized by a pair $(B(y^L), B(y^H))$ where $B(y)$ is the probability that the voter assigns a $\omega = \omega^2$ given a record $y$. Assume politician’s employ a stationary strategy $a^2 = \gamma^2$ and $a^1 = \gamma^1$. Earlier assumptions imply that $\gamma^2 > \gamma^1$, so that $F(\gamma^2) > F(\gamma^1)$.

2.2 Voter Behavior

Suppose a voter observes $y_t$. He knows that the incumbent of type $j = 1, 2$ is using strategy $\gamma^j$. Thus, by Bayes Rule

$$B(y^H) = \Pr(\omega = \omega^2 | y = y^H) = \frac{\pi F(\gamma^2)}{\pi F(\gamma^2) + (1-\pi)F(\gamma^1)}$$

and

$$B(y^L) = \frac{\pi(1 - F(\gamma^2))}{\pi(1 - F(\gamma^2)) + (1-\pi)(1 - F(\gamma^2))}$$

On this basis he will choose his cut-off rules.

How does one solve for an equilibrium? Consider $(\sigma(y^L) = 0, \sigma(y^H) = 1)$. For the politician we find the optimal $a_t$ given the voter’s behavior, where he chooses over multi periods where re-election has a value. That is, in a two period case, he chooses $a_1$ to maximize objectives

$$u(a_1; \omega^j) + \beta [F(a_1)\sigma(y^H) + (1 - F(a_1))\sigma(y^L)] u(\gamma_2, \omega^j)$$

which in the case of $(\sigma(y^L) = 0, \sigma(y^H) = 1)$, this becomes ...This ties down $\gamma_{t=1}$.

Then show that given this incumbent behavior, $(\sigma(y^L) = 0, \sigma(y^H) = 1)$ is optimal, meaning voter has no incentive to deviate.