

# ADVANCES IN NOWCASTING ECONOMIC ACTIVITY: SECULAR TRENDS, LARGE SHOCKS AND NEW DATA

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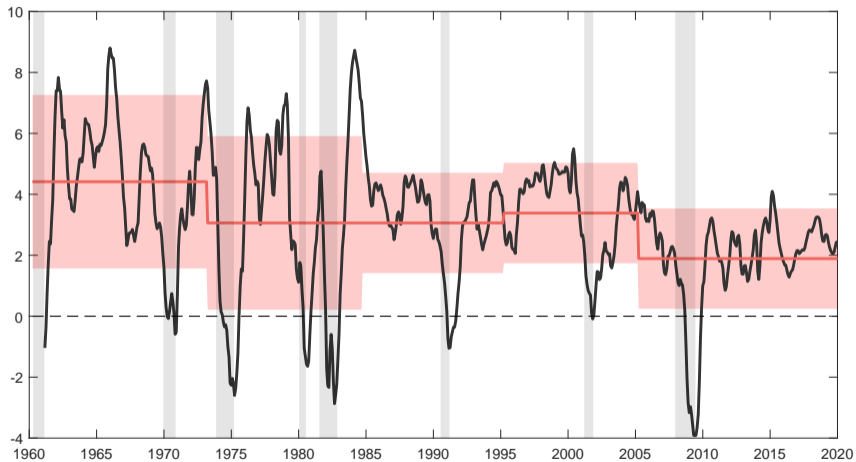
<sup>3</sup>Warwick Business School and CEPR

## CONTRIBUTION OF THIS PAPER

- ▶ This paper is about nowcasting economic activity
- ▶ Propose Bayesian dynamic factor model (DFM), which takes seriously key features of macroeconomic data:
  1. Low-frequency variation in the mean and variance
  2. Heterogeneous responses to common shocks (leads/lags)
  3. Fat tails (outliers and “large” shocks)
- ▶ Evaluate model and its components in comprehensive out-of-sample exercise
  - ▶ On fully real-time, unrevised data US data 2000-2019
  - ▶ Point and density forecasting
  - ▶ Taking advantage of cloud computing
- ▶ Apply model out of sample to track the Great Lockdown of 2020 (in progress)

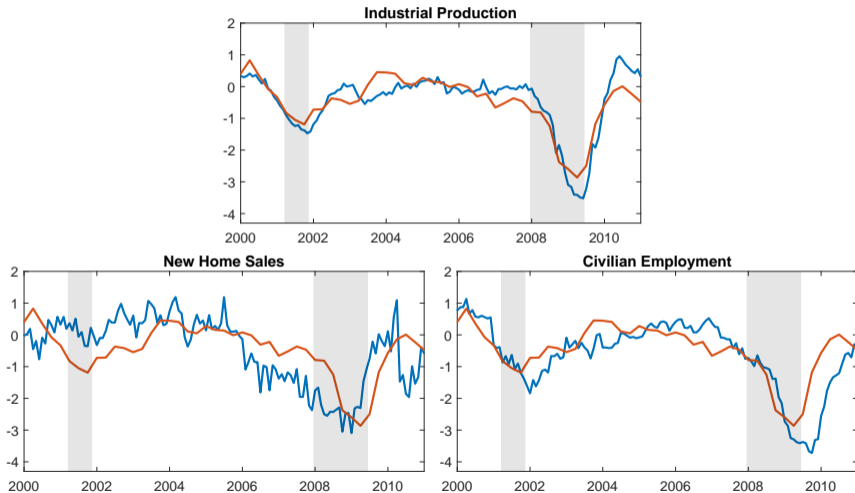
# MOTIVATION #1

## TREND AND VOLATILITY CHANGES IN US REAL GDP GROWTH



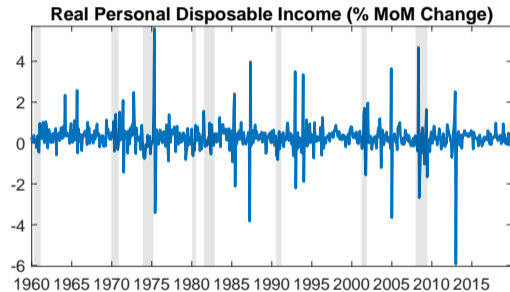
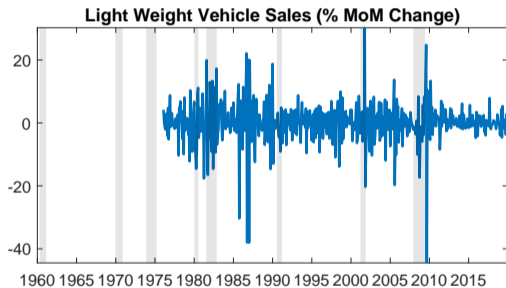
# MOTIVATION #2

## LEAD AND LAG DYNAMICS IN MACRO DATA



# MOTIVATION #3

## OUTLIERS IN MACRO DATA



# PLAN OF THE TALK

1. The Model
  - ▶ Lay out how data features are formally captured in DFM
  - ▶ Highlight what the related novel model components achieve
2. Data and estimation algorithm
3. Setup of comprehensive real-time evaluation exercise
4. Evaluation results
5. Application to the Great Lockdown (in progress)
6. Conclusion

## THE MODEL

## THE MODEL: SPECIFICATION OF BASELINE

- ▶ Start from familiar specification of a DFM (e.g. Giannone, Reichlin, and Small, 2008 and Banbura, Giannone, and Reichlin, 2010)
- ▶ An  $n$ -dimensional vector of quarterly and monthly observables  $\mathbf{y}_t$  follows

$$\begin{aligned}\Delta(\mathbf{y}_t) &= \mathbf{c} + \boldsymbol{\lambda}\mathbf{f}_t + \mathbf{u}_t \\ (I - \Phi(L))\mathbf{f}_t &= \boldsymbol{\varepsilon}_t \\ (1 - \rho_i(L))u_{i,t} &= \eta_{i,t}, \quad i = 1, \dots, n\end{aligned}$$

$$\begin{aligned}\boldsymbol{\varepsilon}_t &\stackrel{iid}{\sim} N(0, \boldsymbol{\Sigma}_\varepsilon) \\ \eta_{i,t} &\stackrel{iid}{\sim} N(0, \sigma_{\eta_i}^2), \quad i = 1, \dots, n\end{aligned}$$



## THE MODEL: SPECIFICATION OF TREND

- ▶ Consider  $n$ -dimensional vector of observables  $\mathbf{y}_t$ , which follows

$$\Delta(\mathbf{y}_t) = \mathbf{c}_t + \lambda \mathbf{f}_t + \mathbf{u}_t,$$

with

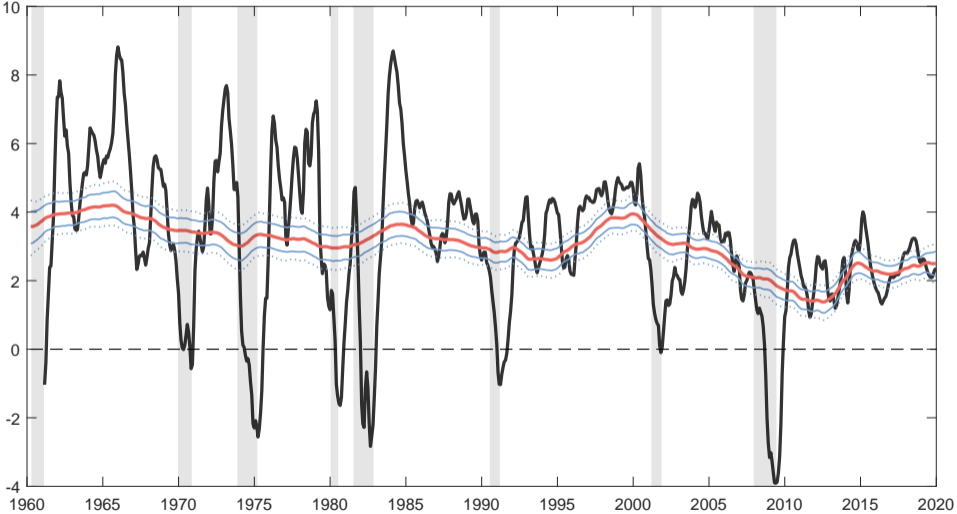
$$\mathbf{c}_t = \begin{bmatrix} \mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{c} \end{bmatrix} \begin{bmatrix} \mathbf{a}_t \\ 1 \end{bmatrix},$$

- ▶ Common factor and idiosyncratic components follow

$$\begin{aligned} (I - \Phi(L))\mathbf{f}_t &= \boldsymbol{\varepsilon}_t, \\ (1 - \rho_i(L))u_{i,t} &= \eta_{i,t}, \quad i = 1, \dots, n \end{aligned}$$

- ▶ Builds on [Antolin-Diaz, Drechsel, and Petrella \(2017\)](#)

# ESTIMATED TREND



## THE MODEL: SPECIFICATION OF SV

- ▶ Consider  $n$ -dimensional vector of observables  $\mathbf{y}_t$ , which follows

$$\Delta(\mathbf{y}_t) = \mathbf{c}_t + \lambda \mathbf{f}_t + \mathbf{u}_t,$$

with

$$\mathbf{c}_t = \begin{bmatrix} \mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{c} \end{bmatrix} \begin{bmatrix} \mathbf{a}_t \\ 1 \end{bmatrix},$$

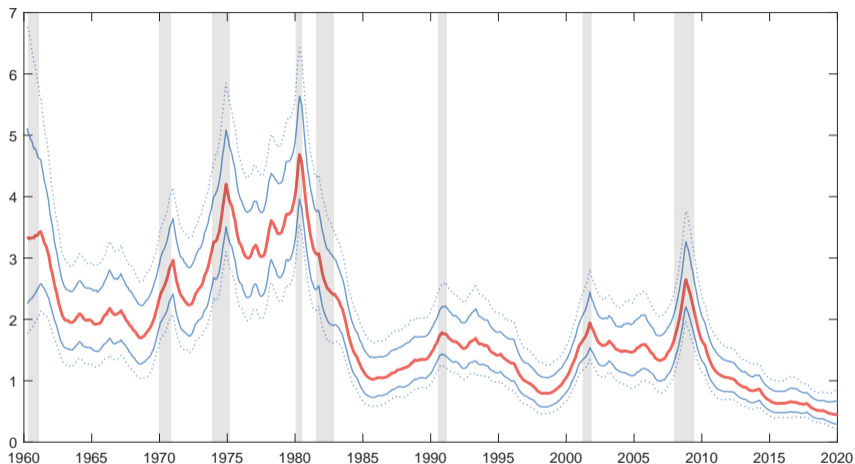
and

$$\begin{aligned} (I - \Phi(L))\mathbf{f}_t &= \sigma_{\varepsilon_t} \varepsilon_t, \\ (1 - \rho_i(L))u_{i,t} &= \sigma_{\eta_{i,t}} \eta_{i,t}, \quad i = 1, \dots, n \end{aligned}$$

- ▶ The time-varying parameters are specified as random walk processes
- ▶ Builds on [Antolin-Diaz, Drechsel, and Petrella \(2017\)](#)

TVP processes

## ESTIMATED VOLATILITY OF THE FACTOR



- ▶ SV captures both secular (McConnell and Perez-Quiros, 2000) and cyclical (Jurado et al., 2014) movements in volatility

## THE MODEL: ADDING HETEROGENEOUS DYNAMICS

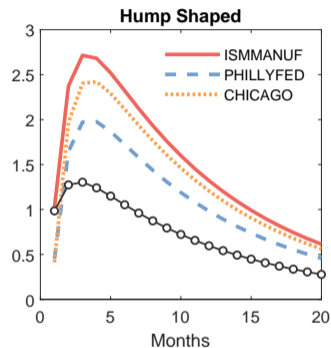
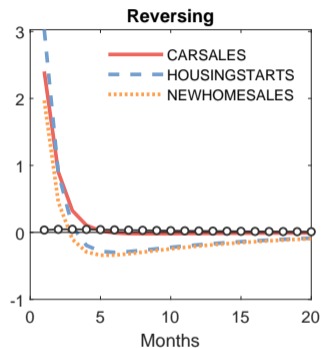
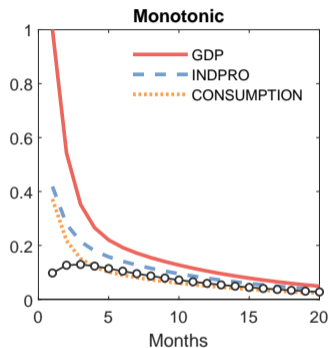
- ▶ Modify the observation equation to be

$$\Delta(\mathbf{y}_t) = \mathbf{c}_t + \mathbf{\Lambda}(\mathbf{L})\mathbf{f}_t + \mathbf{u}_t,$$

where  $\mathbf{\Lambda}(\mathbf{L})$  contains the loadings on contemporaneous and lagged factors

- ▶ Camacho and Perez-Quiros (2010) first noticed that survey data was better aligned with a distributed lag of GDP
- ▶ D'Agostino et al. (2015) show that adding lags improves performance in the context of a small model

# ESTIMATED HETEROGENEOUS DYNAMICS



- ▶ Substantial heterogeneity in IRFs of to innovations in the cyclical factor

## THE MODEL: ALLOWING FOR OUTLIERS

- ▶ Modify the observation equation to be

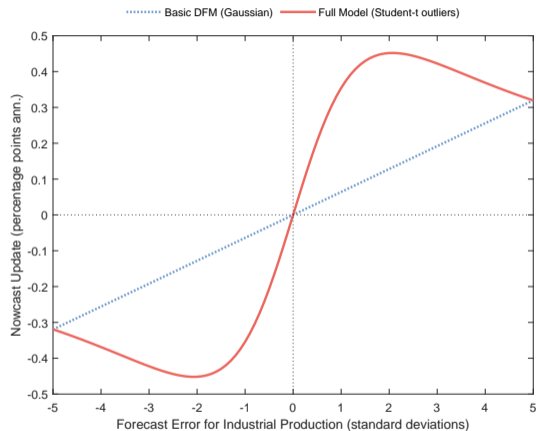
$$\Delta(\mathbf{y}_t - \mathbf{o}_t) = \mathbf{c}_t + \mathbf{\Lambda}(\mathbf{L})\mathbf{f}_t + \mathbf{u}_t,$$

where the elements of  $\mathbf{o}_t$  follow  $t$ -distributions:

$$o_{i,t} \stackrel{iid}{\sim} t_{\nu_i}(0, \omega_{o,i}^2), \quad i = 1, \dots, n$$

- ▶ The degrees of freedom of the  $t$ -distributions,  $\nu_i$ , are estimated jointly with the other parameters of the model

# NEWS DECOMPOSITIONS: WHAT FAT TAILS ACHIEVE



News decomposition details

All variables



## THE MODEL: SUMMARY OF NOVEL FEATURES

1. Macro data: low-frequency variation in mean and variance
  - ▶ Model: **time-varying parameters**
2. Macro data: different leads and lags across indicators
  - ▶ Model: **variables load on factor lags**
3. Macro data: recurring outliers in level and difference
  - ▶ Model: **t-distributed component**

## ESTIMATION

## ESTIMATION: DATA SETS

- ▶ We construct a data set comprising US quarterly and monthly time series
- ▶ Indicators of real economic activity, excluding prices and financial variables
- ▶ We use both hard and soft indicators and
- ▶ Choice is guided by timeliness and coincidence with GDP
- ▶ In each economic category we include series at the highest level of aggregation
- ▶ This results in a panel of 28 series

# US DATA SET

	Frequency
<i>Hard Indicators</i>	
GDP (Chained \$)	Q
GDI (Chained \$)	Q
Consumption (Chained \$, Non Dur. + Serv.)	Q
Investment (Chained \$, Fixed + Cons. Dur.)	Q
Aggregate Hours Worked (Total Economy)	Q
Industrial Production	M
Payroll Employment (Establishment Survey)	M
Real Retail Sales Food Services	M
Real Personal Income less Transfer Payments	M
New Orders of Capital Goods	M
Light Weight Vehicle Sales	M
Real Exports of Goods	M
Real Imports of Goods	M
Building Permits	M
Housing Starts	M
New Home Sales	M
Civilian Employment (Household Survey)	M
Unemployed	M
Initial Claims for Unemployment Insurance	M
<i>Soft Indicators</i>	
Markit Manufacturing PMI	M
ISM Manufacturing PMI	M
ISM Non-manufacturing PMI	M
Conference Board: Consumer Confidence	M
University of Michigan: Consumer Sentiment	M
Richmond Fed Mfg Survey	M
Philadelphia Fed Business Outlook	M
Chicago PMI	M
NFIB: Small Business Optimism Index	M
Empire State Manufacturing Survey	M

## ESTIMATION: SETTINGS AND PRIORS

- ▶ Our methods are Bayesian
- ▶ We use conservative priors that shrink the model towards a more parsimonious specification without the new features
- ▶ This has the appeal that we can let the data speak about to what extent the additional components are required

[More details](#)

## ESTIMATION: ALGORITHM

- ▶ Model specified at monthly frequency. Observed quarterly growth rates related to unobserved monthly ones using weighted mean ([Mariano and Murasawa, 2003](#))
- ▶ Hierarchical implementation of a Gibbs Sampler ([Moench, Ng, and Potter, 2013](#)) which iterates between
  - ▶ Small DFM on the outlier adjusted data
  - ▶ Univariate measurement equations⇒ large computational gains from parallelization
- ▶ SVs are sampled following [Kim et al. \(1998\)](#), the Student-t component is sampled following [Jacquier et al. \(2004\)](#)
- ▶ Vectorized implementation of the Kalman filter

## REAL-TIME OUT-OF-SAMPLE EVALUATION

## DETAILS OF DATA BASE CONSTRUCTION

- ▶ We construct a real-time data base for the US from ALFRED
- ▶ For each vintage, sample start is Jan 1960, appending missing observations to any series which starts after that date
- ▶ Use appropriate deflators for nominal-only vintages
- ▶ Splice data for series with methodological changes
- ▶ Apply seasonal adjustment in real time for survey data



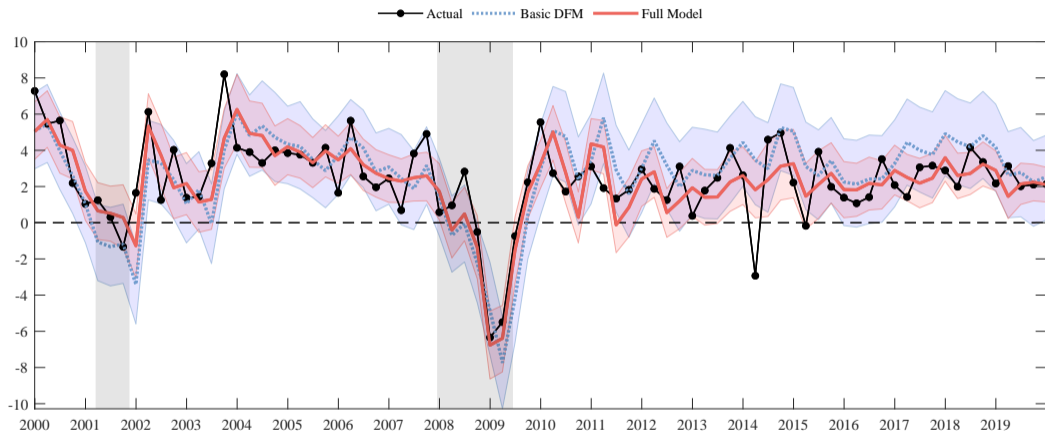
## IMPLEMENTATION OF THE EXERCISE

- ▶ **The model is fully re-estimated every time new data is released/revised**
- ▶ The exercise starts in Jan 2000 and ends in Dec 2019: on average there is a data release on 15 different dates every month  $\Rightarrow$  3600 vintages of data
- ▶ Thanks to efficient implementation, it takes just 20 min Gibbs sampler on a single computer (we use 8,000 iterations/draws)
- ▶ However this would mean almost 2 months of time to run the evaluation
- ▶ Made feasible by using Amazon Web Services cloud computing platform

## SELECTED EVALUATION RESULTS

# EVALUATION RESULTS

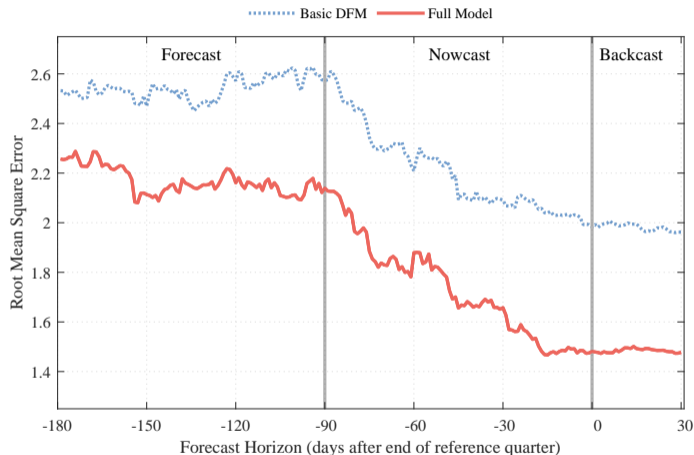
## FORECASTS VS. ACTUAL OVER TIME (US)



- ▶ Long run trend eliminates the upward bias in GDP forecasts after the crisis
- ▶ Lead-lag dynamics improve the model's performance around turning points

# GDP POINT FORECASTS

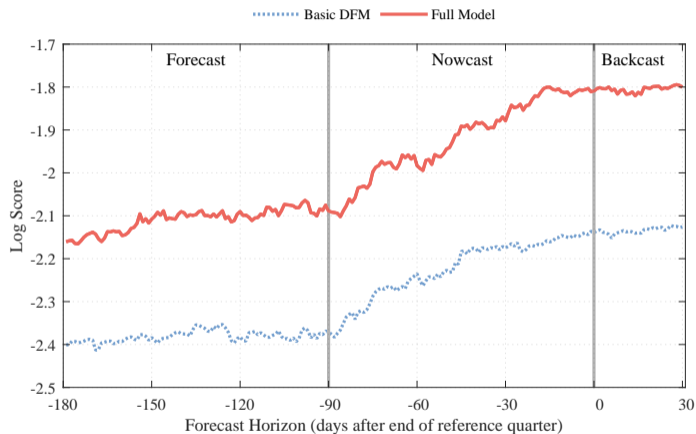
## RMSE ACROSS HORIZONS



- Point forecasting performance significantly improved across horizons Results for MAE

# GDP DENSITY FORECASTS

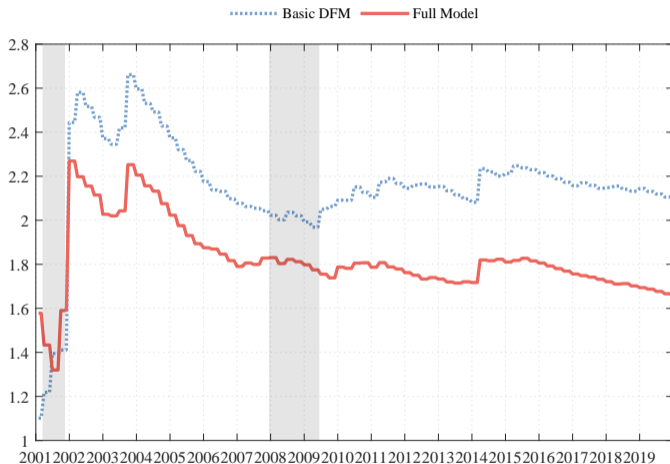
## LOG SCORE ACROSS HORIZONS



- Density forecasting performance significantly improved across horizons Results for CRPS

# GDP POINT FORECASTS

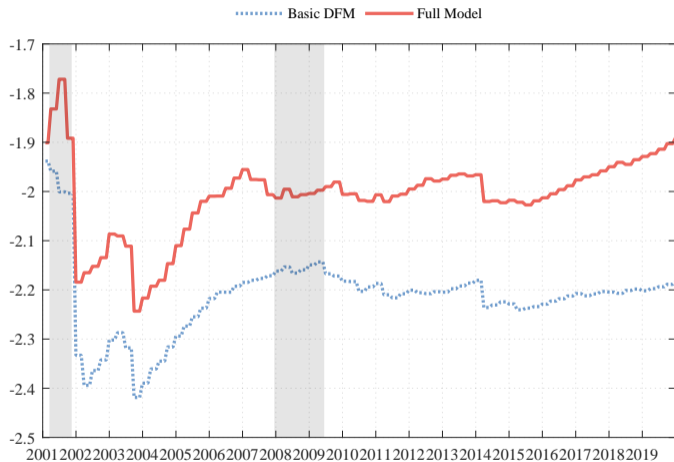
RMSE THROUGH TIME



► Proposed model is favored early on in the evaluation period Results for MAE

# GDP DENSITY FORECASTS

LOG SCORE THROUGH TIME



► Proposed model is favored early on in the evaluation period Results for CRPS

## CONTRIBUTION OF DIFFERENT MODEL COMPONENTS

<b>RMSE across horizons</b> <b>[p-value against AR(1)]</b>	<b>AR(1)</b>	<b>Basic DFM</b>	<b>Trend &amp; SV</b>	<b>Lead/lag</b>	<b>Fat tails</b>
-180 days	2.4	2.5 [0.80]	2.4 [0.63]	2.3 [0.37]	2.3 [0.36]
-90 days (start reference quarter)	2.3	2.6 [0.75]	2.4 [0.59]	2.2 [0.30]	2.1 [0.28]
-60 days	2.1	2.2 [0.58]	2.0 [0.39]	1.9 [0.25]	1.9 [0.23]
-30 days	2.1	2.1 [0.48]	1.9 [0.29]	1.7 [0.12]	1.7 [0.10]
0 days (end reference quarter)	2.1	2.0 [0.33]	1.8 [0.18]	1.5 [0.05]	1.5 [0.04]
+30 days (first release)	2.1	1.9 [0.28]	1.8 [0.13]	1.5 [0.04]	1.5 [0.03]

Table for Log Score

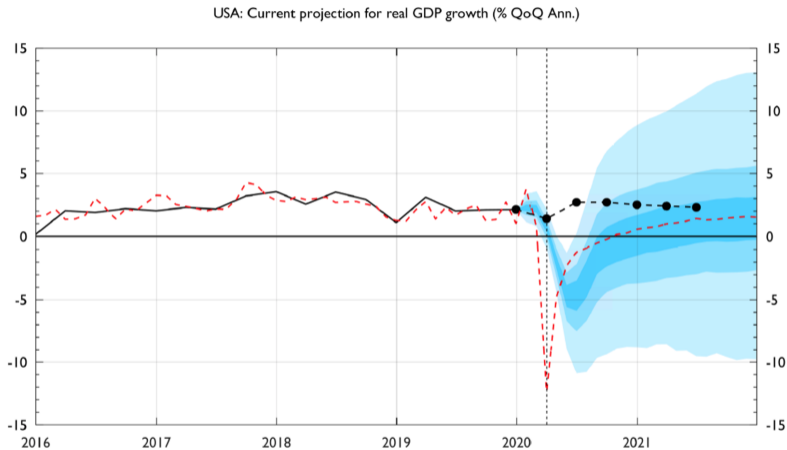


## THE GREAT LOCKDOWN

## NOWCASTING DURING THE GREAT LOCKDOWN

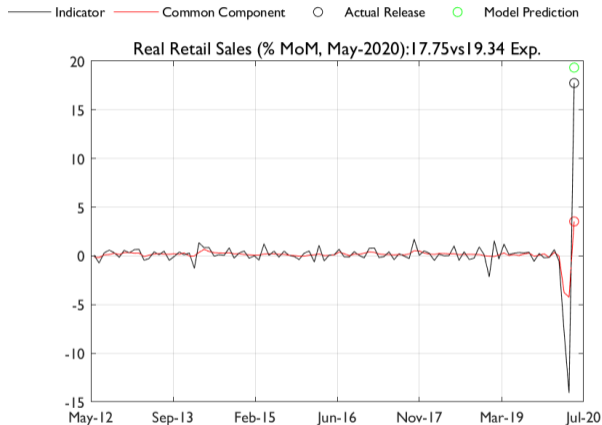
- ▶ Covid-19 pandemic caused recession of unprecedented magnitude
- ▶ Many formal models simply produce nonsensical results
- ▶ We have been exploring two avenues
  1. How novel model components help tracking activity in 2020
    - ▶ In particular heterogeneous dynamics and fat tails
  2. How to incorporate 'alternative data' in the DFM machinery
    - ▶ Novel data sources with very small history have become available
- ▶ Both avenues are work in progress
- ▶ A few pictures to on no 1. to follow ...

# NOWCASTS AS OF 30 MARCH 2020



- ▶ SV captures massive increase in uncertainty

# FAT TAILED OBSERVATIONS



- ▶ Model correctly captured rebound in retail sales based on history of similar outliers

# NOWCASTS AS OF JUNE 2020

BASIC DFM (LEFT) VS. FULL MODEL (RIGHT)



- ▶ Persistent decline or more V-shaped recovery?
- ▶ Heterogeneous dynamics capture rebound in GDP despite persistent decline in other series (in particular surveys)

## CONCLUSION

## CONCLUSION

- ▶ We propose a bayesian DFM, which explicitly incorporates:
  1. Low-frequency variation in the mean and variance
  2. Heterogeneous responses to common shocks
  3. Outlier observations and fat tails
- ▶ We provide a thorough evaluation of the novel model features for the nowcasting process and demonstrate how they improve point and density nowcasts in real time
- ▶ Assessment of US activity in 2020 is in progress

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## APPENDIX SLIDES

## THE MODEL: TIME-VARYING PARAMETER PROCESSES

- ▶ Model's time-varying parameters are specified to follow driftless random walks:

$$\begin{aligned}a_{j,t} &= a_{j,t-1} + v_{a_{j,t}}, & v_{a_{j,t}} &\stackrel{iid}{\sim} N(0, \omega_{a,j}^2) \quad j = 1, \dots, r \\ \log \sigma_{\varepsilon_t} &= \log \sigma_{\varepsilon_{t-1}} + v_{\varepsilon,t}, & v_{\varepsilon,t} &\stackrel{iid}{\sim} N(0, \omega_{\varepsilon}^2) \\ \log \sigma_{\eta_{i,t}} &= \log \sigma_{\eta_{i,t-1}} + v_{\eta_{i,t}}, & v_{\eta_{i,t}} &\stackrel{iid}{\sim} N(0, \omega_{\eta,i}^2) \quad i = 1, \dots, n\end{aligned}$$

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## NEWS DECOMPOSITIONS

Banbura and Modugno (2014) show in a Gaussian model that the impact of a new release on the nowcasts can be written as a linear function of the *news*:

$$E(y_{k,t_k}|\Omega_2) - E(y_{k,t_k}|\Omega_1) = \mathbf{w}_j (y_{j,t_j} - E(y_{j,t_j}|\Omega))$$

$$\mathbf{w}_j = \frac{\Lambda_k E((f_{t_k} - f_{t_k|\Omega})(f_{t_j} - f_{t_j\Omega})) \Lambda_j'}{\Lambda_j E((f_{t_j} - f_{t_j|\Omega})(f_{t_j} - f_{t_j\Omega})) \Lambda_j' + \sigma_{\eta_{j,t_j}}^2}$$

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## NEWS DECOMPOSITIONS

We show that with the Student- $t$  distribution the weights are no longer linear, but depend on the value of the forecast error itself:

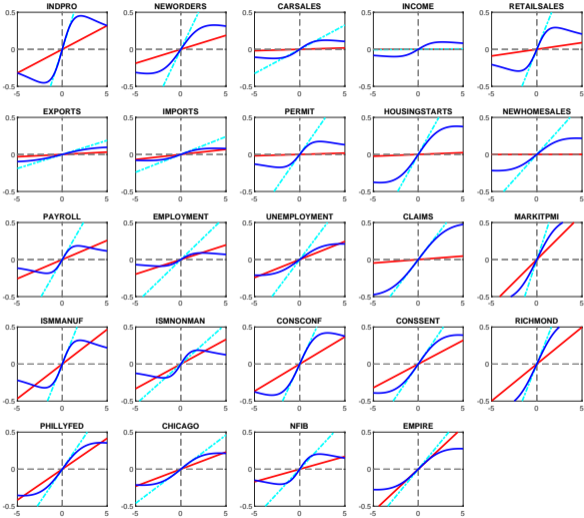
$$E(y_{k,t_k}|\Omega_2) - E(y_{k,t_k}|\Omega_1) = \mathbf{w}_j(y_{j,t_j}) (y_{j,t_j} - E(y_{j,t_j}|\Omega))$$

$$\mathbf{w}_j(y_{j,t_j}) = \frac{\Lambda_k E((f_{t_k} - f_{t_k|\Omega})(f_{t_j} - f_{t_j|\Omega})) \Lambda'_j}{\Lambda_j E((f_{t_j} - f_{t_j|\Omega})(f_{t_j} - f_{t_j|\Omega})) \Lambda'_j + \sigma_{\eta_{j,t_j}}^2 \delta_{j,t_j}}$$

$$\delta_{j,t_j} = (((y_{j,t_j} - E(y_{j,t_j}|\Omega))^2 / \sigma_{\eta_{j,t_j}}^2 + v_{o,j}) / (v_{o,j} + 1))$$

Large errors are discounted as outlier observations containing less information.

# INFLUENCE FUNCTIONS FOR ALL VARIABLES



# ESTIMATION

## DETAILS ON MODEL SETTINGS AND PRIORS (1/2)

- ▶ Number of lags in polynomials  $\Lambda(\mathbf{L})$ ,  $\phi(L)$ , and  $\rho(L)$ : Set to  $m = 1$ ,  $p = 2$ , and  $q = 2$
- ▶ For  $\Lambda(L)$ , prior mean is set to 1 for first lag, and zero in subsequent lags, which shrinks factor to being cross-sectional average, see [D'Agostino et al. \(2015\)](#)
- ▶ For  $\phi(L)$  prior mean is set to 0.9 for first lag, and zero in subsequent lags, which reflects a belief that factor captures highly persistent but stationary business cycle process
- ▶ For AR coefficients of idiosyncratic components,  $\rho_i(L)$  prior is set to zero for all lags, shrinking model towards no serial correlation in  $u_{i,t}$

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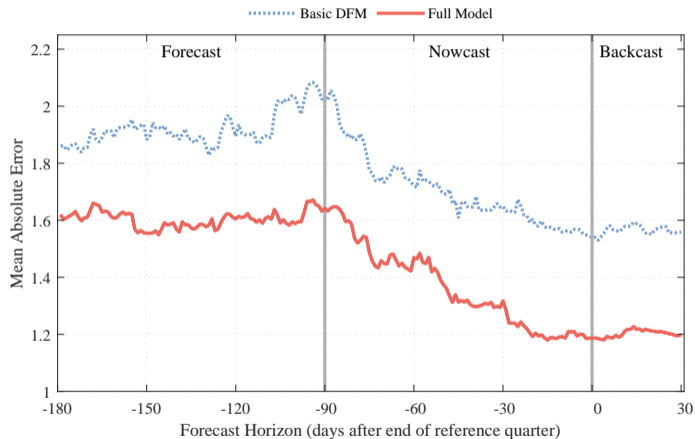
# ESTIMATION

## DETAILS ON MODEL SETTINGS AND PRIORS (2/2)

- ▶ Variance on priors set to  $\frac{\tau}{h^2}$ , where  $\tau$  governs tightness of prior, and  $h$  ranges over lag numbers  $1 : p, 1 : q, 1 : m + 1$ .
- ▶ Following D'Agostino et al. (2015), we set  $\tau = 0.2$ , a value which is standard in the Bayesian VAR literature.
- ▶ Shrink  $\omega_a^2, \omega_\epsilon^2$  and  $\omega_{\eta,i}^2$  towards zero (standard DFM). For  $\omega_a^2$  set IG prior with one d.f. and scale 1e-3. For  $\omega_\epsilon^2$  and  $\omega_{\eta,i}^2$  set IG prior with one d.f. and scale 1e-4 (see Primiceri, 2005)

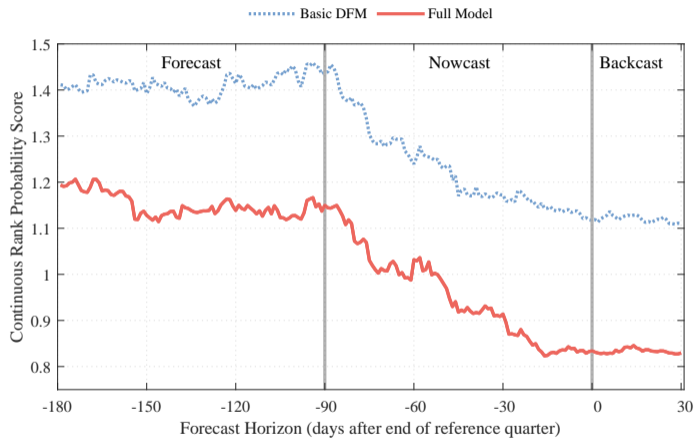
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# MAE INSTEAD OF RMSE (ACROSS HORIZON)



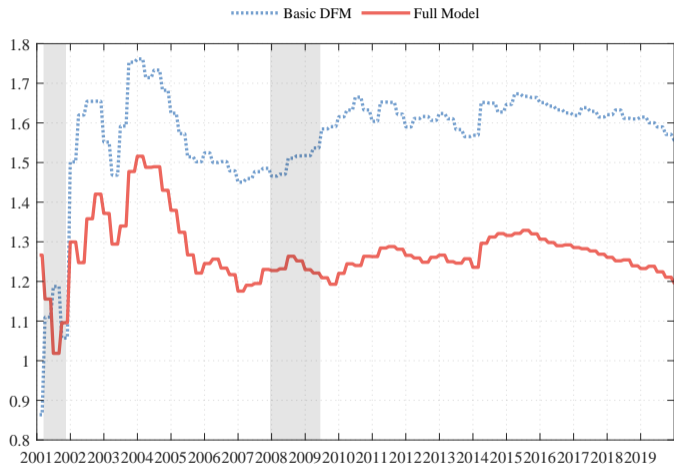


# CRPS INSTEAD OF LOG SCORE (ACROSS HORIZON)

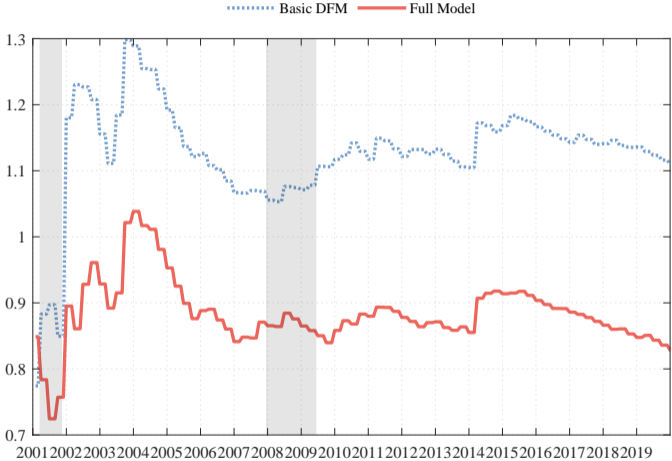


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## MAE INSTEAD OF RMSE (THROUGH TIME)



# CRPS INSTEAD OF LOG SCORE (THROUGH TIME)



## CONTRIBUTION OF DIFFERENT MODEL COMPONENTS

<b>Logscore across horizons</b> <b>[p-value against AR(1)]</b>	<b>AR(1)</b>	<b>Basic DFM</b>	<b>Trend &amp; SV</b>	<b>Lead/lag</b>	<b>Fat tails</b>
-180 days	-2.42	-2.40 [0.36]	-2.16 [0.00]	-2.16 [0.00]	-2.16 [0.00]
-90 days (start reference quarter)	-2.40	-2.37 [0.33]	-2.21 [0.03]	-2.10 [0.00]	-2.09 [0.00]
-60 days	-2.34	-2.24 [0.09]	-2.05 [0.00]	-1.99 [0.00]	-1.98 [0.00]
-30 days	-2.34	-2.18 [0.02]	-2.00 [0.00]	-1.90 [0.00]	-1.88 [0.00]
0 days (end reference quarter)	-2.34	-2.14 [0.00]	-1.98 [0.00]	-1.83 [0.00]	-1.81 [0.00]
+30 days (first release)	-2.35	-2.13 [0.00]	-1.96 [0.00]	-1.81 [0.00]	-1.80 [0.00]

# DYNAMIC VS. STATIC FACTORS

## AN ALTERNATIVE BENCHMARK?

- ▶ A dynamic factor model (with 1 lag and 1 factor) can be always rewritten as a static factor model (with 2 static factors, with a rank restriction on the variance of the transition equation).
- ▶ So the question is: How close to rank deficient is the static factor representation?
- ▶ To answer this question we look at the relative size of the eigenvalues of the variance in the transition equation of the (unrestricted) static factor representation of the model (a) using real data and (b) from data simulated from a ( $s=1$ ,  $r=1$ ) dynamic factor model (with parameters chosen so as to be in line with the estimation of our model).

# DYNAMIC VS. STATIC FACTORS

## AN ALTERNATIVE BENCHMARK?

- ▶ Two static factors:

$$\begin{bmatrix} F_t^1 \\ F_t^2 \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} F_{t-1}^1 \\ F_{t-1}^2 \end{bmatrix} + \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix},$$

- ▶ One Dynamic factor:

$$\begin{bmatrix} f_t \\ f_{t-1} \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} f_{t-1} \\ f_{t-2} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \eta_t,$$

- ▶ The latter specification implies 2 static factors representation, with a reduced rank covariance matrix restriction on the shocks  $\epsilon_t$
- ▶ A static factor model can always be rotated into a dynamic factor provided that the rank restriction is satisfied.

# DYNAMIC VS. STATIC FACTORS

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# WHICH SPECIFICATION IS PREFERRED BY THE DATA?

EVIDENCE THAT SINGLE DYNAMIC FACTOR IS PREFERRED

