Macroprudential policy with earnings-based borrowing constraints

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Abstract

A large literature has studied optimal regulatory policy in macroeconomic models with asset-based collateral constraints. A common conclusion is that agents 'over-borrow' and optimal policy reduces debt positions through taxes. The reason is that agents do not internalize the effects of their choices on *asset prices*. However, recent empirical evidence shows that firms primarily borrow against their earnings rather than their assets. This paper studies optimal macroprudential policy with earnings-based borrowing constraints, both in closed and open economies. We reach the opposite conclusion to the previous literature. Agents 'over-save' (and 'under-borrow') relative to the social optimum, as they do not internalize changes in *wages*, which in turn affect firms' earnings. A numerical model exercise demonstrates that incorrectly rolling out a tax policy derived under the assumption of asset-based constraints in an economy where firms actually borrow based on earnings leads to a consumption equivalent welfare loss of up to 2.55%. Optimal macroprudential policy thus critically depends on the specific form of financial constraints.

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1. Introduction

Should financial markets be regulated? If so, why and how? A large literature studies how the presence of borrowing constraints affects optimal regulatory policy (e.g. Dávila and Korinek, 2018, Bianchi and Mendoza, 2018). Most of this literature focuses on asset-based collateral constraints, which tie credit access to the resale value of an asset, such as a building or machine. The price of the asset can be a source of a pecuniary externality. Households or firms do not realize that their choices move asset prices in equilibrium,

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which in turn affects borrowing limits in the economy. A common conclusion is that agents borrow more than a social planner would prescribe. Optimal macroprudential policy therefore aims to limit debt, for example by imposing taxes on borrowing.

Meanwhile, a growing branch of research studies macroeconomic models with earnings-based borrowing constraints (e.g. Drechsel, 2023). These constraints link firms' ability to obtain funds to their earnings, usually measured before interest, taxes, depreciation and amortization (EBITDA). Although earnings-based constraints are more prevalent for US corporations than asset-based constraints (Lian and Ma, 2020), there is still a limited understanding of how macroprudential policy should be conducted in their presence.[1,2]

The contribution of this paper is to advance our understanding of the normative consequences of earnings-based borrowing constraints in a theoretical framework. We provide analytical proofs under minimal assumptions, as well as a numerical analysis in a more general model. We contrast our insights with borrowing constraints that are commonly studied in the existing literature, for both closed and open economies.

Our findings are the following. First, in a simple closed economy setting we show how an earnings-based borrowing constraint leads to 'over-saving' and 'under-borrowing' from a welfare point of view. The intuition is that when saving increases (borrowing decreases) in the current period, saver (borrower) net worth will be higher next period. Under relevant economic conditions, which our analysis examines closely, such an increase in net worth leads real wages to rise next period. A higher real wage means higher costs and lower earnings for firms, which through the earnings-based borrowing constraint allows for less credit. However, when agents save or borrow today, they do not take into account this negative impact of their decisions today on the future borrowing limit through wages. Therefore agents save a larger (borrow a smaller) amount in the current period than what

[1] There are a few exceptions, that is, normative analyses in which earnings do play some role in credit constraints, e.g. Bianchi (2016). We explain the differences to these formulations of financial constraints.

[2] We define macroprudential policy as regulatory policy that eliminates pecuniary externalities through *ex-ante* taxes. This includes policies that, if optimal, support borrowing through negative taxes (subsidies).

a social planner would implement as a constrained efficient allocation.

Second, this result is the opposite to what holds under asset-based borrowing constraints, which we analyze in our setting for comparison. In essence, in an earnings-based credit constraint an *input price* (through the wage bill) enters with a negative sign, while in an asset-based constraint an *asset price* (through the value of capital) enters with a positive sign. When future wages and capital prices respond with the same sign to current saving and borrowing decisions, then the directions of the pecuniary externalities are the opposite for the two constraints.

Third, we compare earnings-based borrowing constraints to income-based borrowing constraints in a small open economy (SOE) setting with tradable and nontradable goods. With an income-based constraint, the external debt position of an economy is limited by its total income. As the wage bill is a payment from domestic producers to domestic employees, the wage does not affect total income and the relative price of nontradable goods is the only price that gives rise to a pecuniary externality. In contrast, an earnings-based constraint in the same economy determines borrowing capacity based on operating profits of producers, so both the price of nontradable goods and the wage give rise to pecuniary externalities. We show that prices of nontradable goods and wages respond with the same sign to current saving and borrowing decisions but enter with opposite signs in the earnings-based constraint. In consequence, there is an under-borrowing force through wages on top of a the over-borrowing force through nontradable goods prices that the literature has pointed out in this class of models.[3]

Finally, we study a numerical application in a general model with a wider array of economic channels. This includes additional externalities that work through redistribution, which are generally difficult to sign (Dávila and Korinek, 2018), but can be important in the

[3] We also examine working capital constraints (Bianchi and Mendoza, 2010; Jermann and Quadrini, 2012; Bianchi, 2016; Bocola and Lorenzoni, 2023). We find that when firms need to pre-finance wages and also face earnings-based limits on credit, the pecuniary externality through wages is magnified.

context of collateral constraints (Lanteri and Rampini, 2021). In our main experiment, a planner calculates optimal taxes assuming that the economy features asset-based borrowing constraints. In an equally calibrated economy where firms actually borrow based on earnings, we impose these 'incorrect' taxes. We find that they lead to large welfare losses. For example, relative to imposing the optimal policy, the wrongly designed tax policy leads to a loss of up to 2.55% in aggregate consumption. In light of comparable magnitudes in the literature, this is very sizable effect. Our findings make clear that optimal macroprudential policy critically depends on the specific form of financial constraints.

Our work contributes to two strands of research. The first strand studies pecuniary externalities with financial constraints.[4] Our approach is similar to Dávila and Korinek (2018) but considers a labor market and examines additional types of constraints. The introduction of a labor market provides new challenges in signing externalities, and a contribution of this paper is to determine relevant model restrictions. Our insight that higher wages tighten financial constraints is complementary to the mechanism in Bianchi (2016), where firms face working capital and equity constraints, and do not internalize that when they hire workers, wages increase, which in turn tightens equity constraints.[5] A few other studies consider income-based rather than asset-based credit constraints in normative analysis, for example Bianchi (2011) where tradable and nontradable income restrict the economy's external debt position. We contrast our results with the ones arising under those constraints. Benigno et al. (2013) and Schmitt-Grohé and Uribe (2020) also note the possibility of under-borrowing, but through channels different from ours. In Benigno et al.

[4]Important contributions include Mendoza (2006, 2010), Lorenzoni (2008), Jeanne and Korinek (2010), Korinek (2011), Bianchi (2011), Benigno et al. (2013), Bianchi (2016), Bianchi and Mendoza (2018). A related line of research studies *aggregate demand externalities* (Schmitt-Grohé and Uribe, 2016; Farhi and Werning, 2016). These do not work through financial constraints, but through the combination of nominal rigidities and other constraints, such as a fixed exchange rate. Wolf (2020) studies pecuniary externalities that arise from wage rigidities independently of financial constraints and aggregate demand channels.

[5]The pecuniary externality in Bianchi (2016) works through higher *labor demand* having a negative effect on other firms' dividend constraints. In our framework, the pecuniary externality arises from firms' current borrowing exerting a positive effect on future credit limits through *labor supply*.

(2013), when the planner can use an ex-post stabilization tool, the constrained efficient allocation features more borrowing than the decentralized equilibrium. In Schmitt-Grohé and Uribe (2020) under-borrowing is a result of precautionary savings in the face of self-fulling crises. Fazio (2021) proposes a framework with earnings-based constraints to study a credit crunch at the zero lower bound (ZLB) on interest rates. What distinguishes our paper from all of the above is that we compare a variety of credit constraints and systematically study the different policy implications. Another aspect that differentiates our paper is that we examine pecuniary externalities in a general labor market structure, with an explicit analysis of both labor demand and labor supply effects. Bianchi and Mendoza (2010), Bianchi (2016), Fazio (2021) and Bocola and Lorenzoni (2023) all focus on preferences without wealth effects on labor supply, while our setting features a more general labor supply specification. Finally, a related paper is Ottonello, Perez and Varraso (2022) which focuses on the timing of collateral constraints and shows that policy conclusions can change depending on whether current or future prices of collateral affect credit access. Instead of timing, we focus on different variables entering borrowing constraints.

The second strand of research highlights the distinction between asset-based constraints and earnings-based constraints. Drechsel (2023) studies how earnings-based borrowing constraints affect the transmission of macroeconomic shocks. Lian and Ma (2020) show that 80% of U.S. corporate debt is earnings-based. Caglio, Darst and Kalemli-Özcan (2021) show that earnings-based constraints are also prevalent for private small and medium-sized companies.[6] None of these papers consider normative implications.

The paper is organized as follows. Section 2 provides the intuition behind pecuniary externalities with earnings-based borrowing constraints in a simple setting. Section 3 compares these insights with asset-based constraints and income-based constraints in SOEs. Section 4 presents the more general model. We provide more formal proofs for

[6]di Giovanni et al. (2022) provide evidence for Spain and Camara and Sangiacomo (2022) for Argentina.

our earlier results, and carry out the numerical policy experiments. Section 5 concludes.

2. Intuition for pecuniary externalities with earnings-based constraints

This section presents a simple two-period model in which borrowers face an earnings-based borrowing constraint as formulated in Drechsel (2023). In this model, we derive our main theoretical intuition. We explain how pecuniary externalities will arise through the borrowing constraint from the way wages respond to agents' past financial decisions. We do so under different assumptions about preferences and the labor market structure.

2.1. Model setup

There are two time periods $t = 1, 2$. The economy is closed and populated by unit measures of borrowers and lenders, denoted by superscript $i \in \{b, l\}$. Agents have perfect foresight. Agent type i derives utility from consumption c_t^i in both periods and disutility from supplying labor ℓ_s^i at wage w in $t = 1$. Both agents are risk-neutral in $t = 2$. We examine different cases for risk aversion in $t = 1$. The borrower has access to a Cobb-Douglas production technology that uses labor ℓ_d and capital K as inputs in $t = 1$, and capital only in $t = 2$. The capital stock is fixed and owned by the borrower. The lender does not produce, but is endowed with resources e_t^l. Agent i can trade a risk-free bond x_2^i between the two periods at price m, where positive values of x indicate saving, negative values borrowing. The borrower faces the following earnings-based borrowing constraint:

$$-x_2^b \leq \phi_\pi (K^\alpha \ell_d^{1-\alpha} - w\ell_d) \tag{1}$$

where α is the capital share in production and $\phi_\pi > 0$ is a parameter that governs the tightness of the constraint. The difference between sales $K^\alpha \ell_d^{1-\alpha}$ and input costs $w\ell_d$ defines earnings (EBITDA) and restricts debt access (Drechsel, 2023). Agent i holds an initial asset position x_1^i. This position results from choices in period $t = 0$ which we do not model explicitly, but which as we will describe below will be relevant in driving pecuniary externality. Taken together, the maximization problem of the borrower is

$$\max\left(\frac{(c_1^b)^{1-\gamma}}{1-\gamma} - \frac{(\ell_s^b)^{1+\psi}}{1+\psi} + \beta c_2^b\right) \tag{2}$$

subject to (1) and

$$c_1^b + m x_2^b \leq K^\alpha \ell_d^{1-\alpha} - w\ell_d + x_1^b + w\ell_s^b \tag{3}$$

$$c_2^b \leq A_2 K + x_2^b \tag{4}$$

γ and ψ are the risk aversion and Frisch elasticity parameters. The lender's problem is

$$\max\left(\frac{(c_1^l)^{1-\gamma}}{1-\gamma} - \frac{(\ell_s^l)^{1+\psi}}{1+\psi} + \beta c_2^l\right) \tag{5}$$

subject to

$$c_1^l + m x_2^l \leq e_1^l + w\ell_s^l + x_1^l \tag{6}$$

$$c_2^l \leq e_2^l + x_2^l \tag{7}$$

The setting nest the special cases in which agents are risk neutral ($\gamma = 0$) and in which only lenders supply labor ($\ell_s^b = 0$). We analyze these cases below.

2.2. Decentralized equilibrium

We solve the maximization problems of borrowers and lenders. The aggregate states of the model in $t = 1$ are denoted $X \equiv (X_1^b, X_1^l)$, and we characterize a symmetric equilibrium in which $x_1^i = X_1^i$ and the borrowing constraint binds. Combining labor market clearing

$\ell_d = \ell_s^l + \ell_s^b$ with optimal choices gives

$$\left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} K = \left(\frac{\beta w}{m}\right)^{\frac{1}{\psi}} + h(w, m, x_1^b) \tag{8}$$

where the labor supply function of the borrower $h(w, m, x_1^b)$ depends positively on w, negatively on m and x_1^b.[7] Bond market clearing $-x_2^b = x_2^l$ implies that

$$\alpha\phi_\pi \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} K = \frac{1}{m}\left(e_1^l + w\left(\frac{\beta w}{m}\right)^{\frac{1}{\psi}} + x_1^l - \left(\frac{m}{\beta}\right)^{\frac{1}{\gamma}}\right) \tag{9}$$

Condition (8) and (9) allow us to write the equilibrium wage and bond price as a function of the aggregate states X_1^b and X_1^l:

$$w = L(m, X_1^b) \tag{10}$$

$$m = B(w, X_1^l) \tag{11}$$

where $\partial L/\partial m > 0$, $\partial L/\partial X_1^b > 0$, $\partial B/\partial w > 0$ and $\partial B/\partial X_1^l > 0$. (10) and (11) characterize the decentralized equilibrium in $t = 1$ in two price schedules for (w, m).

2.3. Sufficient statistics approach to pecuniary externalities

Agent i's initial asset position x_1^i results from past saving and borrowing decisions that are not explicitly modeled in this section. We study how wages change with the aggregate initial asset positions X, by determining the sign of $\partial w/\partial X$ in the equilibrium described by (10) and (11). These wage changes in turn affect the earnings-based borrowing constraint (1) because higher wages reduce earnings, all else equal. As wage changes in $t = 1$ and their effect on the constraint are not internalized by agents in $t = 0$, their past saving and borrowing decisions are not generally optimal when the borrowing constraint binds.

[7] The labor supply function $h(w, m, x_1^b)$ is implicitly defined by the borrower's optimality conditions $(l_s^b)^\psi = w(c_1^b)^{-\gamma}$ and $c_1^b = \alpha(1 + m\phi_\pi)(\frac{1-\alpha}{w})^{\frac{1-\alpha}{\alpha}} K + wl_s^b + x_1^b$.

Examining the sign of $\partial w/\partial X$ therefore provides the intuition for the direction in which pecuniary externalities in the earnings-based borrowing constraint result from saving and borrowing decisions in $t = 0$. In the more general model in Section 4, the decisions in $t = 0$ are explicitly modeled, a social planner problem is introduced, and the direction of the pecuniary externalities are proven formally.

In examining the direction of price responses to aggregate states we follow the "sufficient statistics" approach of Dávila and Korinek (2018) [henceforth 'DK18']. Similar to them, we sign the pecuniary externalities that result from past saving and borrowing decisions affecting the borrowing constraint in the current period. There are other externalities, in particular those that result from investment rather than saving and borrowing decisions and those that affect redistribution of resources across agents. It is challenging to sign these externalities in general, a result that DK18 refer to as "anything goes."

2.4. Equilibrium wage responses to past saving and borrowing decisions

To determine the sign of $\partial w/\partial X$, we examine the following three cases of our setting:

(i) lenders are risk neutral, borrowers are risk averse; only lenders supply labor

(ii) lenders and borrowers are risk averse; only lenders supply labor

(iii) lenders and borrowers are risk averse; all agents supply labor

Distinguishing between risk neutrality and risk aversion has two implications. First, with risk neutrality the interest rate in this economy is constant. Second, with $\gamma = 0$ in lenders' preferences, there is no wealth effect on labor supply.[8] Distinguishing which agents supply labor to begin with is relevant, because with earnings-based borrowing constraints the borrower is typically thought of as a firm. Therefore restricting the borrower to demanding labor and the lender to supplying it is a natural assumption. Making these distinctions about the setting helps us clarify the economic conditions under which the relevant pecuniary externalities will arise.

[8]To see this, note that the case $\left(c_1^l - \frac{(\ell_s^l)^{1+\psi}}{1+\psi}\right)$ represents Greenwood-Hercowitz-Huffman preferences.

Case (i). Risk-neutrality of lenders implies that (11) becomes $m = \beta$ and the bond price does not depend on aggregate states. When borrowers do not supply labor, the second term of the right hand side of (8) disappears, and (10) simplifies to $w = L(m)$, so the wage also does not depend on aggregate states. Past saving and borrowing decisions do not move prices, so that $\partial w/\partial X = 0$ and no pecuniary externality operates through the earnings-based constraint. Agents' financial decisions in $t = 0$ will be constrained efficient.

Case (ii). The bond price schedule (11) is now a function of X_1^l, while wages depend on X_1^l only through m in (8). Lenders' decisions in $t = 0$ shift the bond price schedule, thereby affect equilibrium wages, so that $\partial w/\partial X \neq 0$. As lenders do not internalize this effect on the borrowing constraint, their $t = 0$ saving decision is not constrained efficient. To examine the direction of the pecuniary externality, note the following condition:

$$\frac{\partial w}{\partial X_1^l} \geq 0 \Leftrightarrow \frac{\partial B^{-1}}{\partial m} > \frac{\partial L}{\partial m} \tag{12}$$

If the slope of B is steeper than the slope of L, higher lender net worth increases wages. In consequence, more saving by lenders in $t = 0$ tightens the earnings-based constraint by raising wages in $t = 1$. Figure 1 examines the equilibrium under condition (12).

On what grounds may the failure of condition (12), where $\frac{\partial B^{-1}}{\partial m} < \frac{\partial L}{\partial m}$, be ruled out? Figure 2 illustrates that in this case the equilibrium is unstable. The left panel presents a phase diagram corresponding to Figure 1, while the right panel shows a phase diagram when (12) does not hold. The equilibrium in the right panel is an unstable saddle point while the equilibrium in the left panel is fully stable. Thus, based on stability considerations, we argue that (12) is an appealing restriction. Further below, we show that this argument has an analogy under asset-based collateral constraints.

It is possible to provide a sufficient condition on the model's parameters that ensures that (12) is satisfied: if $1 + \frac{\psi}{\alpha} > \frac{1-\alpha}{\alpha}$, then $\frac{\partial w}{\partial X_1^l} \geq 0$ holds. Conditional on the capital share of production, there needs to be a sufficiently strong labor supply elasticity for more lender net worth to raise wages. The Online Appendix provides the formal derivation of this sufficient condition. This derivation also makes clear that condition (12) generally depends on other model primitives, in particular the risk aversion γ. The condition $1 + \frac{\psi}{\alpha} > \frac{1-\alpha}{\alpha}$ therefore is not necessary, but sufficient. In the more general model below, we explore the calibration of the key parameters α and ψ.

Case (iii). When borrowers also supply labor, the wage schedule becomes a function of X_1^b.[9] Now both lenders' and borrowers' decisions in $t = 0$ affect the earnings-based constraint in $t = 1$ through equilibrium wages, and their decisions is thus not constrained efficient. The relevant condition generalizes to two derivatives in a similar fashion:

$$\frac{\partial w}{\partial X_1^l}, \frac{\partial w}{\partial X_1^b} \geq 0 \Leftrightarrow \frac{\partial B^{-1}}{\partial m} > \frac{\partial L}{\partial m} \tag{13}$$

Figure 3 examines the equilibrium under condition (13) graphically.

Similar to Case (ii), condition (13) can be supported based on stability arguments. It is not possible to derive a simple parametric sufficiency condition as in Case (ii), but it is again evident that the strength of labor supply is important as X_1^b enters (8) through $h(\cdot)$.

Labor demand vs. labor supply. In our setting, inefficiencies in the decisions of lenders and borrowers in $t = 0$ arise from changes in labor supply in $t = 1$. To see this, note that in Case (ii) X_1^l enters in (11) because of wealth effects on lenders' labor supply and in Case (iii) X_1^b enters in (10) because of wealth effects on borrowers' labor supply. In both cases, labor demand is pinned down from optimal behavior within the period based on the predetermined capital stock, as the agents can always choose labor demand that maximizes their unconstrained objective as well as their borrowing capacity. Labor demand choices are thus not affected by changes in borrower net worth. Without labor supply reacting

[9]This would not be true in the absence of wealth effect on borrowers' labor supply, in which case the borrowers' decisions would be constrained efficient. We omit this intermediate case, because the logic is similar to Case (i) for lenders' labor supply without wealth effects.

to changes in net worth, the allocation under an earnings-based borrowing limit would not exhibit constraint externalities through saving and borrowing choices. Providing this reasoning for signing pecuniary externalities with labor demand and labor supply is a central insight of our analysis, and makes our mechanism distinct from that in Bianchi (2016) and Bianchi and Mendoza (2010). Further below, we show that in interaction with working capital constraints, labor demand does give rise to additional pecuniary externalities with earnings-based borrowing constraints, similar to these papers.

2.5. Over-saving and under-borrowing effects with earnings-based constraints

The above analysis makes clear that when we consider stable equilibria in $t = 1$ with risk aversion, agents' decisions might not be constrained efficient.[10] In Cases (ii) and (iii), lenders in $t = 0$ will not internalize that saving more raises wages in $t = 1$ which in turn tightens the earnings-based constraint. From the point of view of a social planner, they thus over-save relative to the optimal allocation. In Case (iii) borrower decisions are not constrained efficient. Borrowers in $t = 0$ will not internalize that borrowing more lowers wages in $t = 1$ which in turn relaxes the earnings-based constraint. From the point of view of a social planner, they thus under-borrow relative to the optimal allocation.

We make the $t = 0$ choices as well as the planner problem explicit in a more general formulation of the model in Section 4. In that section, we formally prove the over-saving (under-borrowing) result by deriving the planner's optimal taxes/subsidies on borrowing. We show that the results on over-saving and under-borrowing hold in a more general setting as long as $\partial w/\partial X > 0$. Before generalizing the setting, we contrast the insights above with common formulations of financial constraints in the literature.

[10] Our analysis of the SOE setting in Section 3 makes clear that also with a fixed interest rate (risk neutral lenders) pecuniary externalities can arise.

3. Comparison with constraints commonly studied in the literature

This section compares the implications of earnings-based borrowing constraints to those from common formulations of borrowing constraints in the macroprudential policy literature. We first focus on asset-based collateral constraints in the same setting as above. We then consider a small open economy environment and study income constraints on the economy's external debt position.

3.1. Over-borrowing effects with asset-based constraints

Suppose that capital is still in inelastic supply but can be traded at price q. The borrower faces the following commonly studied asset-based collateral constraint:

$$-x_2^b \leq \phi_k q k \tag{14}$$

where k is the capital choice and $0 < \phi_k < 1$ governs the tightness of the constraint. To demonstrate that the typical over-borrowing result holds in our setting, it is enough focus on the simplest setting with risk-neutral lenders and labor supply coming from lenders only (Case (i)). The borrower's problem becomes

$$\max \left(\frac{(c_1^b)^{1-\gamma}}{1-\gamma} + \beta c_2^b \right) \tag{15}$$

subject to (14) instead of (1) and subject to

$$c_1^b + m x_2^b + qk \leq (k^\alpha \ell_d^{1-\alpha} - w\ell_d) + x_1^b + qK \tag{16}$$

$$c_2^b \leq A_2 k + x_2^b \tag{17}$$

The lender problem remains the same as in Section 2. We can derive a capital demand function in period $t = 1$ that depends on X_1^b, and may be upward-sloping or downward sloping.[11] Figure 4 shows both cases. When $\frac{\partial q}{\partial X_1^b} < 0$ (right panel), the equilibrium is not stable. Therefore $\frac{\partial q}{\partial X_1^b} \geq 0$ (left panel) is a sensible restriction. Indeed it typically features in the literature on asset-based constraints. Dávila and Korinek (2018) also show that failure of $\frac{\partial q}{\partial X_1^b} \geq 0$ leads to multiplicity and unstable equilibria. Economically, an increase in resources, holding the amount of available capital in the economy fixed, will increase the capital demand and thus put upward pressure on its price. When $\frac{\partial q}{\partial X_1^b} \geq 0$, borrowers in $t = 0$ will not internalize that saving more, and thus reducing net worth next period, will reduce capital prices in $t = 1$ and therefore tighten borrowing constraints. In consequence, they over-borrow relative to what a social planner would prescribe.

The contrast between the earnings-based and the asset-based constraint makes clear that when w and q respond with the same sign to current asset positions, then the directions of the pecuniary externalities coming from past saving and borrowing decisions are the opposite, as w and q enter with opposite sign in each constraint. Agents over-save and under-borrow with earnings-based constraints, but over-borrow with asset-based constraints. We reach the opposite conclusion from much of the previous literature on macroprudential policy with financial constraints.[12]

3.2. Earnings-based vs. income-based constraints in small open economies

We now consider an SOE version of the two-period model in Section 2. A representative household consumes tradable goods, which are the numéraire, and nontradable goods according to a CES aggregator $c = [\theta(c^T)^\rho + (1-\theta)(c^N)^\rho]^{\frac{1}{\rho}}$. The household receives an endowment of nontradable goods y^N and produces tradable goods with a Cobb-Douglas

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[11]Formally, solving for the $t = 1$ capital choice k as a function of q, X_1^b and predetermined capital supply gives the following relation: $k = \frac{1}{q(1-\beta\theta_k)}\left[\alpha(\frac{1-\alpha}{w})^{\frac{1-\alpha}{\alpha}} K + X_1^b + qK - \left(\frac{1}{1-\beta\theta_k}(\frac{1}{q}\beta A_2 - \beta\theta_k)\right)^{-\frac{1}{\gamma}}\right]$.

[12]Our over-saving and under-borrowing results require wealth effects on labor supply. In the absence of those effects, the normative conclusions with an earnings-based constraint would still be different from the typical over-borrowing result with an asset-based constraint, as they would imply constrained efficiency.

production technology $y^T = zK^\alpha \ell^{1-\alpha}$.[13] The economy has access to a one-period bond on international markets. It is denominated in units of tradables and its exogenously fixed price is m. More details on the SOE setup are provided in the Online Appendix.

We now define income-based and earnings-based borrowing constraints and highlight the prices that enter in each constraint. Income-based constraints limit the amount that the economy can borrow externally by a fraction of *total* current income, the sum of profits, endowments, and wages, as for example in Bianchi (2011) and Benigno et al. (2013):

$$-x_2 \leq \phi_I((y_1^T - w\ell^d) + p_1 y_1^N + w\ell^s) = \phi_I(y_1^T + p_1 y_1^N), \tag{18}$$

where p_1 is the relative price of nontradable goods, and ℓ^d and ℓ^s are labor demand and supply. The key price in the income-based constraint is p_1, so $\partial p_1/\partial X$ determines the direction of the pecuniary externality. In Bianchi (2011), $\partial p_1/\partial X > 0$ and agents over-borrow under income-based borrowing constraints as they do not internalize that their debt positions shrink borrowing capacity through a lower p_1.

In contrast, earnings-based constraints are determined by a multiple of the EBITDA of *firms* rather than the *total* income of the economy. In the SOE economy this gives

$$-x_2 \leq \phi_\pi(y_1^T - w\ell^d + p_1 y_1^N). \tag{19}$$

because tradable firm earnings are $(y_1^T - w\ell^d)$ and nontradable firm earnings are $p_1 y_1^N$ (nontradable sector firms produce an endowment with zero costs). The key prices for the earnings-based constraints are the price of nontradable goods p_1 and wage w, so $\partial p_1/\partial X$

[13] In the Online Appendix, we also consider the case with tradable goods being an endowment and production of nontradable goods. We reach similar conclusions in that alternative setting.

and $\partial w/\partial X$ are the relevant sufficient statistics. (p_1, w) are determined by

$$p_1 = \frac{1-\theta}{\theta}\left(\frac{(1+\alpha m\phi_\pi)y_1^T + X}{y_1^N} + m\phi_\pi p_1\right)^{1-\rho} \tag{20}$$

$$\left(\frac{(1-\alpha)z_1}{w}\right)^{\frac{1}{\alpha}} K = \ell^{s^*}, \tag{21}$$

where ℓ^{s^*} is the optimal labor supply which depends on preferences. In theory, $\partial p_1/\partial X$ can be either positive or negative depending on parameter values.[14] We focus our analysis on the case $\partial p_1/\partial X > 0$ as we want to contrast it with the standard over-borrowing result.

We consider two cases regarding labor supply: (i) labor supply is exogenously fixed; (ii) labor supply is endogenously determined.

Case (i). As ℓ^{s^*} is fixed, the equilibrium wage does not change with aggregate net worth i.e. $\frac{\partial w}{\partial X} = 0$. A pecuniary externality emerges only through the price of nontradable goods. With $\partial p_1/\partial X > 0$, the standard over-borrowing results hold.

Case (ii). In the SOE setting with endogenous labor supply, $sign(\partial p_1/\partial X) = sign(\partial w/\partial X)$. We show this formally in the Online Appendix. As we focus on $\partial p_1/\partial X > 0$, it is also the case that $\partial w/\partial X > 0$. Based on the arguments in Section 2, $\partial w/\partial X > 0$ leads to an under-borrowing force with earnings-based constraints. Thus, there is both an over-borrowing mechanism, which goes through the relative price of nontradable goods, and under-borrowing mechanism, which operates through wages.[15]

We conclude that in SOEs with earnings-based borrowing constraints, there is an under-borrowing force that features alongside the over-borrowing force present in income-based

[14] Schmitt-Grohé and Uribe (2020) show that $\partial p_1/\partial X$ can have either sign depending on parameter values, and that the equilibrium is unique with $\partial p_1/\partial X > 0$ under the calibration of Bianchi (2011). In other cases, the model features multiple equilibria and $\partial p_1/\partial X < 0$.

[15] It could be interesting to study relative output price variation as a source of pecuniary externalities also in a closed economy setting with earnings-based constraints. Fazio (2021) explores this possibility in an environment with a manufacturing and a service sector, where manufacturing producers face a credit constraint that depends on their earnings.

constraints. Which force dominates the other is a quantitative and empirical question, which we leave as an avenue for future research. Its answer depends on whether debt positions of SOEs are taken by households, firms or governments, as these agents might feature differential constraints. With income-based constraints the literature has taken a natural starting point, as they link to the total income across all of these agents. If, however, external borrowing is primarily done by firms subject to earnings-based constraints, then the contribution of the under-borrowing force could be first-order.

3.2.1. Discussion: working capital constraints

Firms sometimes pre-finance production inputs before revenues are collected. If the access to such *working capital*, in addition to other debt, is limited by an earnings-based constraint, this enhances the strength of the externality that operates through wages. To see this, suppose a firm takes the intertemporal position x_2^b as above, and in addition pre-finances a fraction ψ of its wage bill with an intraperiod working capital loan $x_{wc} = -\psi w \ell_d$. Such a setup is chosen, for example, by Bianchi and Mendoza (2010) and Bocola and Lorenzoni (2023). When we add working capital to our framework, an earnings-based constraint on *total borrowing* takes the form

$$-(x_2^b - \psi w \ell_d) \leq \phi_\pi (K^\alpha \ell_d^{1-\alpha} - w \ell_d) \tag{22}$$

which can be rearranged to

$$-x_2^b \leq \phi_\pi K^\alpha \ell_d^{1-\alpha} - (\phi_\pi + \psi) w \ell_d \tag{23}$$

which corresponds to (1), with the only difference that the parameter multiplying the wage bill is $(\phi_\pi + \psi) > \phi_\pi$. The presence of working capital thus strengthens the externality in the earnings-based constraint, leading to a more pronounced under-borrowing effect.[16]

Recall from above that in our framework without working capital there are no inefficiencies that operate through labor demand. This changes with a working capital constraint, as lower labor demand eases the working capital constraint. In this case, higher borrower net worth from past saving and borrowing decisions increases the equilibrium wage through higher labor demand. Thus, the under-borrowing effects from earnings-based constraints are magnified with working capital through both a higher parameter in front of the wage bill and an additional labor demand channel. Interestingly, in models such as Bianchi and Mendoza (2010) and Bocola and Lorenzoni (2023) agents have GHH preferences, so constraint externalities operate exclusively through labor demand.

4. General setting, formal proofs and numerical application

This section generalizes the model of Section 2 to feature three periods and capital investment. All agents are risk averse, produce and supply labor. The model is close to DK18, but with a labor market and different credit constraints. In this setting, we formally prove the direction of the pecuniary externalities for which we developed the intuition above. We also carry out numerical model experiments.

4.1. Generalized model

There are three time periods $t = 0, 1, 2$. The state of nature is realized at date $t = 1$ and is denoted by $\theta \in \Theta$. Agent type $i \in \{b, l\}$ has a time separable utility function

$$U^i = \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t u^i(c_t^i, \ell_{st}^i) \right] \tag{24}$$

[16]To see this formally, in the proof of Proposition 2 in Section 4 a larger parameter multiplying the wage increases $\frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta}$ and thus drives $C_{N^i}^{b,\theta}$ more negative.

where $u^i(\cdot,\cdot)$ is strictly increasing and weakly concave in consumption, strictly decreasing and weakly convex in labor, and $u^i(c_0^i, \ell_{s0}^i) = u^i(c_0^i)$. There are consumption goods and capital goods. $e_t^{i,\theta}$ is the endowment of consumption goods agent i receives at date $t = 1, 2$ given state θ. Time-0 endowments are denoted by e_0^i. At date $t = 0$, agents can invest $h^i(k_1^i)$ units of consumption good to produce k_1^i units of date-1 capital goods.[17] The functions $h^i(\cdot)$ are increasing and convex and satisfy $h^i(0) = 0$. k_1^i can be used for the production of consumption goods in period $t = 1$ and be carried over for production in period $t = 2$. $k_2^{i,\theta}$ denotes the amount of capital that agent i carries from date 1 to 2. Capital fully depreciates after date 2. To produce consumption goods in $t \geq 1$, agent i employs both capital and labor to produce $F^i(k_t^{i,\theta}, \ell_{dt}^{i,\theta})$ units of the consumption good. $\ell_{dt}^{i,\theta}$ is labor demanded by agent i at date t. The production functions $F^i(\cdot,\cdot)$ are strictly increasing and weakly concave in each argument and satisfy $F^i(0, 0) = 0$.

At date $t = 0$, agents trade state-contingent assets that pay 1 unit of the consumption good in period $t = 1$ and state θ. $x_1^{i,\theta}$ denotes the date-0 state-θ purchases by agent i and m_1^θ is the corresponding asset price, taken as given by the agent. Agent i spends $\int_{\theta\in\Theta} m_1^\theta x_1^{i,\theta} d\theta$ in total on these securities. Without further uncertainty between $t = 1$ and $t = 2$, agents trade non-contingent one-period bonds $x_2^{i,\theta}$ at time $t = 1$ at price m_2^θ. There is a competitive labor market. Wages at date $t \geq 1$ and state θ are denoted by w_t^θ. There is also a market to trade capital at a price q^θ at date 1 after production has taken place. There is no trading of capital at date 2. The budget constraints of agent $i \in \{b, l\}$ are

$$c_0^i + h^i(k_1^i) + \int_{\theta\in\Theta} m_1^\theta x_1^{i,\theta} d\theta = e_0^i \tag{25}$$

$$c_1^{i,\theta} + q^\theta \Delta k_2^{i,\theta} + m_2^\theta x_2^{i,\theta} = e_1^{i,\theta} + x_1^{i,\theta} + F^i(k_1^i, \ell_{d1}^{i,\theta}) - w_1^\theta \ell_{d1}^{i,\theta} + w_1^\theta \ell_{s1}^{i,\theta}, \quad \forall\theta \tag{26}$$

$$c_2^{i,\theta} = e_2^{i,\theta} + x_2^{i,\theta} + F^i(k_2^{i,\theta}, \ell_{d2}^{i,\theta}) - w_2^\theta \ell_{d2}^{i,\theta} + w_2^\theta \ell_{s2}^{i,\theta}, \quad \forall\theta \tag{27}$$

[17]Note that $k_1^{i,\theta} = k_1^i$ since it is chosen in $t = 0$, thus not conditional on the state of nature θ.

where $\Delta k_2^{i,\theta} \equiv k_2^{i,\theta} - k_1^i$. Recall that the state θ materializes in $t = 1$ so choices in $t \geq 1$ are made conditional on the realized state of nature. There are constraints on the holdings of securities between periods $t = 0$ and $t = 1$, as well as between periods $t = 1$ and $t = 2$. At date $t = 0$, borrowers' holdings of $x_1^b = \{x_1^{b,\theta}\}_{\theta\in\Omega}$ are subject to a constraint

$$\Phi_1^b(x_1^b, k_1^b) \geq 0 \tag{28}$$

At date $t = 1$, borrowers' holdings of $x_2^{b,\theta}$ are subject to a state-dependent constraint

$$\Phi_2^{b,\theta}(x_2^{b,\theta}, k_2^{b,\theta}, \{\ell_{dt}^{b,\theta}, \ell_{st}^{b,\theta}\}_{t=1}^2; q^\theta, w_1^\theta, w_2^\theta, m_2^\theta) \geq 0, \ \forall\theta \tag{29}$$

We assume $\Phi_1^l(\cdot) = \Phi_2^{l,\theta}(\cdot) = 0$, that is, lenders are financially unconstrained.

4.1.1. Decentralized equilibrium

A decentralized equilibrium consists of asset allocations $\{x_1^{i,\theta}, x_2^{i,\theta}\}_{i\in\{b,l\},\theta\in\Theta}$, real allocations $\{c_0^i, c_1^{i,\theta}, c_2^{i,\theta}, k_1^i, k_2^{i,\theta}, \ell_{d1}^{i,\theta}, \ell_{d2}^{i,\theta}, \ell_{s1}^{i,\theta}, \ell_{s2}^{i,\theta}\}_{i\in\{b,l\},\theta\in\Theta}$ and prices $\{q^\theta, w_1^\theta, w_2^\theta, m_1^\theta, m_2^\theta\}_{\theta\in\Theta}$, such that agents solve their optimization problems and markets clear. The market clearing conditions are shown formally in the Online Appendix. The solution for the decentralized equilibrium can be obtained via backward induction. Optimal choices at time $t = 2$ are purely intratemporal decisions on consumption and labor supply and demand. In $t = 1$, two sets of variables fully characterize the state of the economy. The first is the holdings of capital by both agents k_1^i. The second one is agents' net worth $n_1^{i,\theta} \equiv e_1^{i,\theta} + x_1^{i,\theta}$.[18] Agents take aggregate states as given so we distinguish individual states $\{n_1^{b,\theta}, n_1^{l,\theta}, k_1^b, k_1^l\}$ from aggregate states $\{N_1^{b,\theta}, N_1^{l,\theta}, K_1^b, K_1^l\}$. We further define $N_1^\theta \equiv \{N_1^{b,\theta}, N_1^{l,\theta}\}$ and $K_1 \equiv \{K_1^b, K_1^l\}$, and note that the equilibrium prices are functions of the aggregate

[18]DK18 include production output as part of net worth. In our model, the quantity $F^i(k_1^i, \ell_{d1}^{i,\theta})$ is not predetermined because labor is chosen during $t = 1$. We therefore do not include it as part of $n_1^{i,\theta}$. In the Online Appendix, we formally verify that this does not alter the original results of DK18.

state variables: $q^\theta(N_1^\theta, K_1)$, $m_2^\theta(N_1^\theta, K_1)$, $w_1^\theta(N_1^\theta, K_1)$, and $w_2^\theta(N_2^\theta(N_1^\theta, K_1), K_2(N_1^\theta, K_1)) = w_2^\theta(N_1^\theta, K_1)$. The optimization problem of an individual agent i at time $t = 1$ is

$$V^{i,\theta}(n_1^{i,\theta}, k_1^i; N_1^\theta, K_1) = \max_{\{c_1^{i,\theta}, c_2^{i,\theta}, k_2^{i,\theta}, x_2^{i,\theta}, \ell_{dt}^{i,\theta}, \ell_{st}^{i,\theta}\}} \left\{ u^i(c_1^{i,\theta}, \ell_{s1}^{i,\theta}) + \beta u^i(c_2^{i,\theta}, \ell_{s2}^{i,\theta}) \right\} \tag{30}$$

$$\text{s.t.} \quad c_1^{i,\theta} + q^\theta \Delta k_2^{i,\theta} + m_2^\theta x_2^{i,\theta} = e_1^{i,\theta} + x_1^{i,\theta} + F^i(k_1^i, \ell_{d1}^{i,\theta}) - w_1^\theta \ell_{d1}^{i,\theta} + w_1^\theta \ell_{s1}^{i,\theta} \qquad [\lambda_1^{i,\theta}] \tag{31}$$

$$c_2^{i,\theta} = e_2^{i,\theta} + x_2^{i,\theta} + F^i(k_2^{i,\theta}, \ell_{d2}^{i,\theta}) - w_2^\theta \ell_{d2}^{i,\theta} + w_2^\theta \ell_{s2}^{i,\theta} \qquad [\lambda_2^{i,\theta}] \tag{32}$$

$$\Phi_2^{b,\theta}(x_2^{b,\theta}, k_2^{b,\theta}, \{\ell_{dt}^{b,\theta}, \ell_{st}^{b,\theta}\}_{t=1}^2; q^\theta, w_1^\theta, w_2^\theta, m_2^\theta) \geq 0 \qquad [\kappa_2^{i,\theta}] \tag{33}$$

where $\lambda_1^{i,\theta}$, $\lambda_2^{i,\theta}$, and $\kappa_2^{i,\theta}$ are the Lagrange multipliers. The $t = 0$ optimization problem is

$$\max_{\{c_0^i, k_1^i, x_1^{i,\theta}\}} u^i(c_0^i) + \beta \mathbb{E}_0[V^{i,\theta}(n_1^{i,\theta}, k_1^i; N_1^\theta, K_1)] \tag{34}$$

subject to (25) and (28). The Online Appendix presents the agents' first-order conditions.

4.1.2. Distributive effects and constraint effects

DK18 show that changes in aggregate states have *distributive effects* and *collateral effects*. We refer to the latter effects with a more general terminology as *constraint effects*.[19] Our Online Appendix formally characterizes the distributive and constraint effects in a symmetric equilibrium in which $n^{i,\theta} = N^{i,\theta}$ and $k_1^i = K_1^i$, by differentiating the indirect utility $V^{i,\theta}$ with respect to $N_1^{j,\theta}$ and $K_1^{j,\theta}$. The first of these derivatives is

$$V_{N_1^j}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dN_1^{j,\theta}} = \lambda_1^{i,\theta} \mathcal{D}_{1N^j}^{i,\theta} + \lambda_2^{i,\theta} \mathcal{D}_{2N^j}^{i,\theta} + \kappa_2^{i,\theta} \mathcal{C}_{N^j}^{i,\theta} \tag{35}$$

[19] This is because we study credit constraints that do not necessarily contain "collateral" in the sense of physical assets. Alternatively, one could re-label the earnings-based borrowing constraint as a "collateral constraint" in which earnings serve as collateral. We instead refer to collateral more narrowly as the presence of physical k in the borrowing constraint.

where $\mathcal{C}^{i,\theta}_{N^j}$ is a constraint effect. It collects any derivatives that multiply the shadow price on the financial constraint $\kappa^{i,\theta}_2$, and depends on price changes as follows

$$\mathcal{C}^{b,\theta}_{N^j} \equiv \frac{\partial \Phi^{b,\theta}_2}{\partial q^\theta}\frac{\partial q^\theta}{\partial N^{j,\theta}_1} + \frac{\partial \Phi^{b,\theta}_2}{\partial m^\theta_2}\frac{\partial m^\theta_2}{\partial N^{j,\theta}_1} + \frac{\partial \Phi^{b,\theta}_2}{\partial w^\theta_1}\frac{\partial w^\theta_1}{\partial N^{j,\theta}_1} + \frac{\partial \Phi^{b,\theta}_2}{\partial w^\theta_2}\frac{\partial w^\theta_2}{\partial N^{j,\theta}_1} \tag{36}$$

Instead, $\mathcal{D}^{i,\theta}_{1N^j}$ and $\mathcal{D}^{i,\theta}_{2N^j}$ in (35) are distributive effects which net out across the agents. Relative to DK18, both constraint and distributive effects feature additional economic forces in our model. In particular, (36) makes clear that wages give rise to constraint effects, which we will show leads to pecuniary externalities with earnings-based constraints.

4.1.3. Social planner problem and constrained efficient allocation

The social planner chooses allocations in $t = 0$ subject to the same period-0 constraints as the private agents, and subject to optimal behavior of the agents in periods $t = 1, 2$. This corresponds to a constrained Ramsey planner who can levy taxes in $t = 0$. Formally,

$$\max_{\{C^i_0 \geq 0, K^i_1, X^{i,\theta}_1\}} \sum_i \alpha^i \{u^i(C^i_0) + \beta \mathbb{E}_0[V^{i,\theta}(N^{i,\theta}_1, K^i_1; N^\theta, K_1)]\} \tag{37}$$

$$\text{s.t.} \sum_i [C^i_0 + h^i(K^i_1) - e^i_0] \leq 0 \quad (v_0) \tag{38}$$

$$\sum_i X^{i,\theta}_1 = 0, \ \ \forall\theta \quad (v^\theta_1) \tag{39}$$

$$\Phi^i_1(X^i_1, K^i_1) \geq 0, \ \forall i \quad (\alpha_i \kappa^i_1) \tag{40}$$

Note that α^b and α^l are Pareto weights that the social planner applies to borrowers and lenders, respectively. The variables in brackets denote Lagrange multipliers. The presence of $V^{i,\theta}(N^{i,\theta}_1, K^i_1; N^\theta, K_1)$ makes clear that the planner takes the private equilibrium of periods $t = 1, 2$ as given and internalizes the impact of changing N^θ and K_1 on prices.

The economy's constrained efficient allocation is described by quantities $(C^i_0, K^i_1, X^{i,\theta}_1)$, Pareto weights $\alpha^b/\alpha^l = \lambda^l_0/\lambda^b_0$ and shadow prices v_0, v^θ_1, and κ^i_1 satisfying the optimality conditions and constraints of the social planner's problem. This allocation can be implemented with a set of tax rates on financial asset and capital purchases. We relegate the derivations to the Online Appendix. The tax rate on saving is

$$\tau_x^{i,\theta} = -\Delta MRS_{01}^{ij,\theta}\mathcal{D}_{1N^i}^{i,\theta} - \Delta MRS_{02}^{ij,\theta}\mathcal{D}_{2N^i}^{i,\theta} - \tilde{\kappa}_2^{b,\theta}\mathcal{C}_{N^i}^{b,\theta},\ \forall i,\theta \tag{41}$$

$\Delta MRS_{0t}^{ij,\theta} \equiv MRS_{0t}^{i,\theta} - MRS_{0t}^{j,\theta}$ denotes the difference between agents' marginal rate of substitution (MRS) across time, $MRS_{01}^{j,\theta} \equiv \beta\lambda_1^{j,\theta}/\lambda_0^j$, $MRS_{02}^{j,\theta} \equiv \beta\lambda_2^{j,\theta}/\lambda_0^j$. We define $\tilde{\kappa}_2^{b,\theta} \equiv \beta\kappa_2^{b,\theta}/\lambda_0^b$ as the relative shadow price. The $\mathcal{D}$ and $\mathcal{C}$ terms correspond to the distributive and constraint effects discussed above.

4.1.4. Nature of externalities and sufficient statistics

The optimal tax (41) combined with the constraint effects $\mathcal{C}$ in (36) allow us to characterize externalities through a compact list of sufficient statistics. Externalities are determined by the product of the relative shadow price of the financial constraint $\tilde{\kappa}_2^{i,\theta}$, the sensitivity of the financial constraint to the price of capital, asset price and wages $\partial\Phi_2^{i,\theta}/\partial q^\theta$, $\partial\Phi_2^{i,\theta}/\partial m_2^\theta$, $\partial\Phi_2^{i,\theta}/\partial w_1^\theta$, $\partial\Phi_2^{i,\theta}/\partial w_2^\theta$, and the sensitivity of the equilibrium capital price, asset price and wages in periods 1 and 2 to changes in aggregate states $\frac{\partial q^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial m_2^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial w_2^\theta}{\partial N_1^{j,\theta}}$. By analyzing and interpreting price changes, we can study how market outcomes deviate from the constrained efficient allocation and how such distortions are corrected by the planner. A positive $\tau_x^{i,\theta}$ implies that agent i saves too much (borrows too little) in the market outcome, so the planner imposes a tax on savings (subsidy on borrowing).

DK18 show that distributive externalities as well as constraint externalities from changes in aggregate capital cannot generally be signed. In our formal proofs below, we therefore focus on over-/under-borrowing instead of over-/under-investment effects, and on constraint externalities rather than distributive externalities. In the numerical application, we allow for all possible forces, so the planner chooses a tax on capital purchases τ_k^i in addition to $\tau_x^{i,\theta}$, and internalizes both $\mathcal{D}$ and $\mathcal{C}$ effects.

4.2. Formal proofs for pecuniary externalities

The following conditions specialize the economic setting enough to determine the sign of the constraint externalities for the financial constraints of interest.

$$\frac{\partial w_1^\theta}{\partial N_1^{i,\theta}} \geq 0, \ \forall i \tag{42}$$

$$\frac{\partial q^\theta}{\partial N_1^{i,\theta}} \geq 0, \ \forall i \tag{43}$$

We interpret these conditions in Sections 2 and 3.1. In our numerical application below, we verify the conditions under specific functional forms for preferences and technology. We can now formally derive efficiency properties of different forms of the financial constraint (33). Consider first the case of an asset-based collateral constraint. (33) becomes

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \phi_k q^\theta k_2^{b,\theta} \geq 0 \tag{44}$$

Proposition 1. A collateral constraint as defined by (44), as long as it binds, gives rise to non-negative constraint externalities. This implies that there is an over-borrowing effect that operates through the constraint externalities.

Proof. From (44), $\phi_k > 0$ and $k_2^{b,\theta} \geq 0$ it follows that $\frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta} \geq 0$. According to condition (43), $\frac{\partial q^\theta}{\partial N_1^{i,\theta}} \geq 0$. Therefore $C_{N^i}^{b,\theta} = \frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta} \frac{\partial q^\theta}{\partial N_1^{i,\theta}} \geq 0$. If the constraint binds, $\tilde{\kappa}_2^{b,\theta}$ is non-negative. It follows that the constraint externality resulting from the constraint is non-negative, that is, $\tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta} \geq 0$. This implies that there is over-borrowing operating through the constraint externalities: as is visible in equation (41), the social planner imposes subsidies on savings $\tau_x^{i,\theta}$ in order to induce less borrowing. ■

Next, consider an earnings-based borrowing constraint. (33) is specified as

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \phi_\pi (F^b(k_1^b, \ell_{d1}^{b,\theta}) - w_1^\theta \ell_{d1}^{b,\theta}) \geq 0 \tag{45}$$

Proposition 2. An earnings-based borrowing constraint as defined by (45), as long as it binds, gives rise to non-positive constraint externalities. This implies that there is an over-saving (under-borrowing) effect that operates through the constraint externalities.

Proof. From (45), $\phi_\pi > 0$ and $\ell_{d1}^{b,\theta} \geq 0$ it follows that $\frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta} \leq 0$. According to (42), $\frac{\partial w_1^\theta}{\partial N_1^{i,\theta}} \geq 0$. Therefore, $C_{N^i}^{b,\theta} = \frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta} \frac{\partial w_1^\theta}{\partial N_1^{i,\theta}} \leq 0$. If the constraint binds, $\tilde{\kappa}_2^{b,\theta}$ is non-negative. It follows that the constraint externality resulting from the constraint is non-positive, $\tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta} \leq 0$. This implies that there is over-saving (under-borrowing) operating through the constraint externalities: as is visible in equation (41) the planner imposes taxes on savings (subsidies on borrowing) $\tau_x^{i,\theta}$ in order to induce less saving (more borrowing). ■

Propositions 1 and 2 underscore the insights of our simple model in Sections 2 and 3.1 more formally. The Online Appendix provides a graphical illustration of our proofs.[20]

4.3. Numerical application

This section conducts policy experiments in a parameterized version of the model. We quantify the welfare loss that arises from imposing an ‘incorrect’ macroprudential policy, where the true model is an economy with earnings-based borrowing constraints, but we impose tax rates that are computed as optimal under the assumption that agents face asset-based constraints. In this experiment, both distributive and constraint externalities, as well as both under- and over-borrowing and under- and over-investing, are at play.

4.3.1. Model specification

There is no uncertainty and no period-0 financial constraint. We consider the case where labor supply is inelastic and the case where it is optimally chosen. In the case of inelastic labor supply, the period utility function follows the log-utility specification $u^i(c_t^i, \ell_{st}^i) = \log(c_t^i)$. In the case of endogenously determined labor supply, the period

[20] In an earlier version of this paper (Drechsel and Kim, 2022), we also study interest coverage constraints, which restrict the ratio of interest payments to earnings. See also Greenwald (2019). An interest coverage constraint leads to either over-borrowing or under-borrowing, and can be interpreted as a mixture between an asset-based and earnings-based constraint from a welfare point of view.

utility function follows a standard separable utility specification with wealth effects on labor supply $u^i(c_t^i, \ell_{st}^i) = \log(c_t^i) - \frac{1}{1+\psi}(\ell_{st}^i)^{1+\psi}$ for $t \geq 1$. We assume a constant to returns to scale (CRS) and a decreasing returns to scale (DRS) production function for the borrower and the lender, respectively. Formally, $F^b(k_t^b, \ell_{dt}^b) = z_b(k_t^b)^\alpha(\ell_{dt}^b)^{1-\alpha}$ and $F^l(k_t^l, \ell_{dt}^l) = z_l((k_t^l)^\alpha(\ell_{dt}^l)^{1-\alpha})^\nu$ where we assume $z_b > z_l$ and $\nu < 1$. Following DK18, $h^i(k) = \frac{\eta}{2}k^2$.

4.3.2. Parameterization

Table 1 summarizes our parameterization. We set β to 0.9752 following Drechsel (2023) who targets average US corporate loan rates. The Frisch elasticity ψ and returns to scale ν are set to 2 and 0.75 as in Jungherr and Schott (2021). We set the tightness parameter of the asset-based constraint ϕ_k following Bianchi (2016), who uses the average leverage ratio of US non-financial corporations of 46% as a target. We then calibrate ϕ_π to ensure that the debt-to-output ratio is the same across the economies in which we calculate the optimal tax rates and the one in which we impose them. We do this separately for the case with inelastic labor supply and the case with endogenous labor supply. We set the remaining parameters to ensure that the borrower has a superior production technology ($z_b > z_l$), but lacks the endowments to make capital investment relative to the lender.

Validity of model restrictions. Based on the parameterization of the model, we verify numerically that the model restrictions required to derive our formal theoretical analysis above, indeed hold. That is, the calibration of the model implies $\frac{\partial q}{\partial N_1^i} \geq 0$, $\frac{\partial w_1}{\partial N_1^i} \geq 0$, $\forall i$.

4.3.3. Determining the tax schedule in asset-based economy

We first solve the planner problem in an economy with asset-based borrowing constraints. We set (α_b, α_l) to achieve the same ratio of period-0 consumption as in the corresponding decentralized equilibrium. This leads to $(\alpha_b, \alpha_l) = (0.05, 0.95)$ for the case with inelastic labor supply and $(\alpha_b, \alpha_l) = (0.20, 0.80)$ for the case with endogenous labor supply. We then compute the optimal corrective taxes $(\tau_x^b, \tau_x^l, \tau_k^b, \tau_k^l)$ at the constrained efficient allocation.[21] To separate distributive and constraint externalities, we also compute that component of optimal taxes on borrowing/saving that arises from the constraint externalities at the constrained efficient allocation, $\tau_x^{i,c.e.} = -\tilde{\kappa}_2^b \mathcal{C}_{N^i}^b. \ \forall i$.

4.3.4. Imposing the 'wrong' tax schedule in earnings-based economy

Next we consider the 'true' economy with earnings-based borrowing constraints. First, we compute the welfare gain from moving from the decentralized equilibrium to the constrained efficient allocation in this economy. This is done with the same welfare weights as in the asset-based economy. We call this the 'right' policy. Second, we compute the welfare change from imposing the corrective taxes that we optimally derived in the economy with asset-based constraints above. We call this the 'wrong' policy. Following Jones and Klenow (2016), we compute how much of permanent consumption should be inflated or deflated when we change from allocation B to allocation A, by finding λ such that

$$\begin{aligned} SW^{B,\lambda} &\equiv \alpha_b \sum_{t=0}^{2} \beta^t u((1+\lambda)c_{bt}^B, \ell_{bt}^B) + \alpha_l \sum_{t=0}^{2} \beta^t u((1+\lambda)c_{lt}^B, \ell_{lt}^B) \\ &= \alpha_b \sum_{t=0}^{2} \beta^t u(c_{bt}^A, \ell_{bt}^A) + \alpha_l \sum_{t=0}^{2} \beta^t u(c_{lt}^A, \ell_{lt}^A) \equiv SW^A. \end{aligned}$$

Under log-utility assumption, λ is derived as $\lambda = \exp\left((SW^A - SW^B)\frac{1-\beta}{1-\beta^3}\right) \times 100$ (%), where $SW^B \equiv \alpha_b \sum_{t=0}^{2} \beta^t u(c_{bt}^B, \ell_{bt}^B) + \alpha_l \sum_{t=0}^{2} \beta^t u(c_{lt}^B, \ell_{lt}^B)$. Finally, similar to Lanteri and Rampini (2021) we assume that agents are reimbursed a lump-sum amount that corresponds to the amount they paid or received through distortionary taxes.

4.3.5. Optimal corrective taxes in different economies

Table 2 shows the tax rates that implement constrained efficient allocation for each economy. The subscripts x and k indicate taxes on saving in the financial asset and saving

[21] Savings taxes τ_x^i are determined by (41). The optimal tax on capital investment is derived in an analogous way in the Online Appendix.

in capital, respectively. The table shows these two tax rates separately for the lender and the borrower, and additionally reports the component of the corrective taxes on saving due to constraint externalities only, $\tau_x^{b,c.e.}$ and $\tau_x^{l,c.e.}$. The negative sign of these tax rates in the asset-based economy, and the positive sign in the earnings-based economy with endogenous labor supply confirm our findings from above. There is over-borrowing with a collateral constraint, so the social planner levies a negative tax on saving, $\tau_x^{i,c.e.} < 0$. There is over-saving (under-borrowing) with the earnings-based constraint, so the social planner taxes saving (subsidizes borrowing) through $\tau_x^{i,c.e.} > 0$. If labor is inelastic, however, the allocation with the earnings-based constraint is already constrained efficient, so $\tau_x^{i,c.e.} = 0$.

Table 2 also shows that the fully optimal taxes $(\tau_x^b, \tau_x^l, \tau_k^b, \tau_k^l)$ are large compared to the components that address the constraint externalities only. This indicates that distributive externalities and over- and under-investment forces, which cannot be signed in general, are quantitatively large. This is in line with the findings of Lanteri and Rampini (2021).

4.3.6. Results of numerical policy experiment

We calculate how much macroprudential policy designed under imprecise assumptions about financial constraints deteriorates social welfare. Table 3, Panel (a) shows the welfare results when both distributive and constraint externalities are operational. With earnings-based borrowing constraints, the constrained efficient allocation leads to a 0.60% higher permanent consumption than the decentralized equilibrium. Importantly, when the wrong policy is rolled out, consumption equivalent welfare decreases by 1.95% and 0.52% relative to the decentralized equilibrium for the economy with inelastic and endogenous labor supply. The table also reports the difference in consumption equivalents between imposing the right and the wrong policy, which amounts to as much as 2.55% in the economy where labor supply is inelastic. To put these magnitudes into context, in Bianchi (2011) the welfare gains from correcting the externality are 0.135% of permanent consumption. In Bianchi and Mendoza (2018) the welfare gain from implementing the optimal policy is 0.3%

in permanent consumption. The wrong policy thus worsens social welfare significantly, relative to the market allocation and even more so relative to the optimal policy.

Panel (b) separately breaks out results for the effects of constraint externalities only. As there is no inefficiency through constraint externalities in the earning-based economy with inelastic labor supply, social welfare is not altered through the right policy. With endogenous labor supply, the right policy increases permanent consumption only marginally, by 0.06%. However, the wrong policy decreases permanent consumption by 0.01% and 0.47% for the economy with inelastic and endogenous labor supply. Compared to the optimal policy, a consumption loss of as much as 0.53% is incurred by the agents. These effects are still meaningful, and larger than some results in the literature. The Online Appendix provides robustness checks for the calibration underlying our numerical experiments.

5. Conclusion

This paper examines normative implications of earnings-based credit constraints. Our results have important implications for the design of an effective regulatory system. Macroprudential policy guided solely by an asset-based collateral mechanism might be counterproductive in credit markets where earnings-based borrowing constraints are dominant. The evidence motivating our analysis focuses on nonfinancial companies, so the regulation of corporate credit is where our insights are most applicable. Collateral constraints are a more central force in household mortgage markets, where real estate serves as collateral, or in trade between financial institutions, where financial assets are pledged in repurchase agreements. This paper makes the case for studying carefully which pecuniary externalities are critical in which types of credit markets, and shows that the distinction between asset and input prices in credit constraint is of first-order importance for optimal policy.

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Figures and tables

Figure 1: Wage changes in response to past financial decisions – Case (ii)

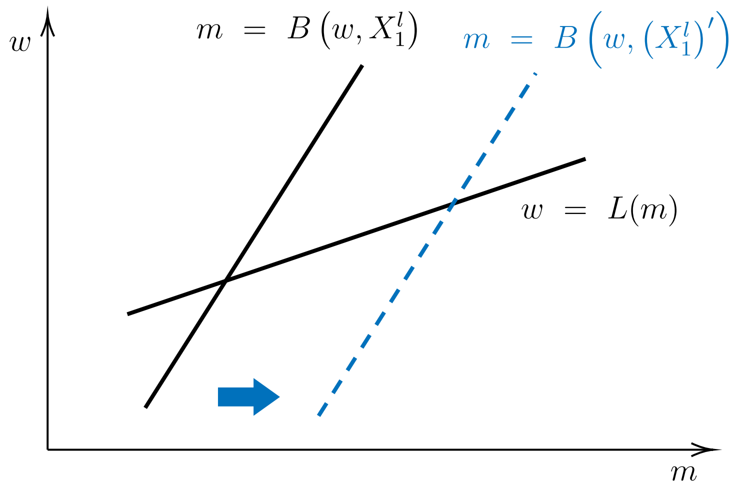


Figure 2: Equilibria with phase diagram under different conditions

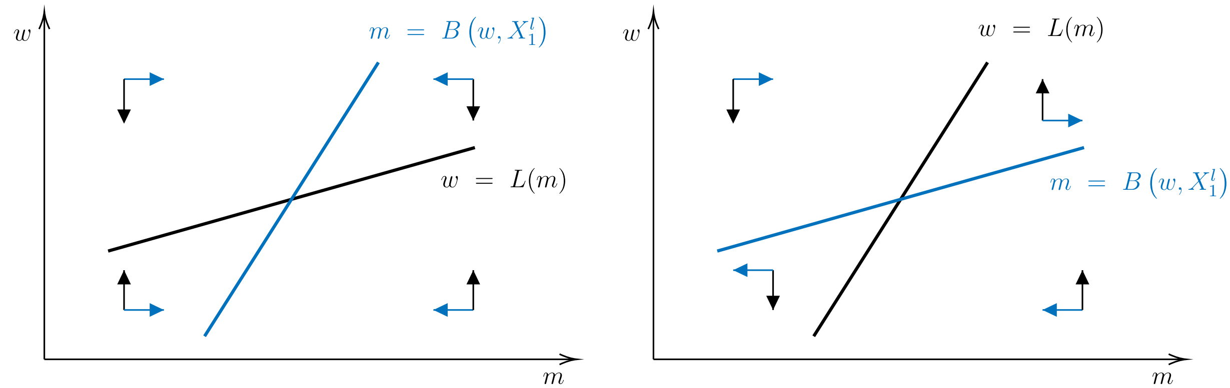


Figure 3: Wage changes in response to past financial decisions – Case (iii)

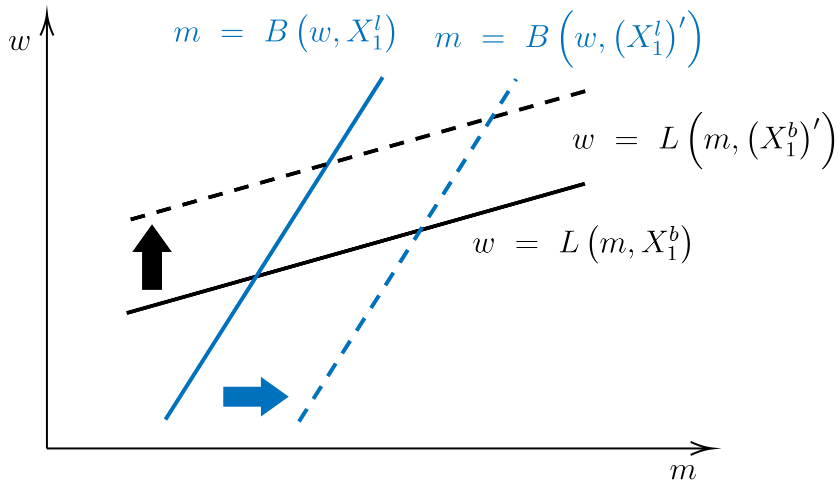


Figure 4: Capital price changes in response to past financial decisions

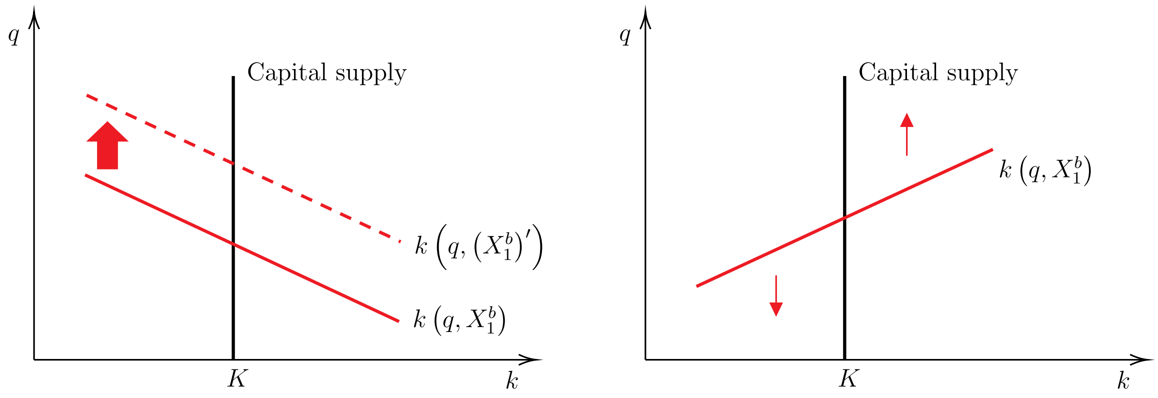


Table 1: Calibration of the model

Parameter	**Description**	**Value**	**Source / Target**
α	Capital share	0.33	Standard for US case
β	Discount factor	0.9752	Drechsel (2023)
ψ	Labor supply elasticity	2	Jungherr and Schott (2021)
ν	Returns to scale - lender	0.75	Jungherr and Schott (2021)
ϕ_k	Borrowing limit - asset	0.46	Bianchi (2016)
ϕ_π	Borrowing limit - earnings (inelastic labor)	0.534	Match debt-to-output, $\frac{-x_2^b}{y_1^b+y_1^l}$
	Borrowing limit - earnings (endogenous labor)	0.617	Match debt-to-output, $\frac{-x_2^b}{y_1^b+y_1^l}$
η	Investment technology	1	Normalization
(z_b, z_l)	Productivity	(2,1)	
(e_0^b, e_1^b, e_2^b)	Endowments - borrower	(0,0,0)	
(e_0^l, e_1^l, e_2^l)	Endowments - lender	(1,1,0)	

Table 2: Optimal corrective taxes in different economies (in %)

Economy	$\boldsymbol{\tau_x^b}$	$\boldsymbol{\tau_x^l}$	$\boldsymbol{\tau_k^b}$	$\boldsymbol{\tau_k^l}$	$\boldsymbol{\tau_x^{b,c.e.}}$	$\boldsymbol{\tau_x^{l,c.e.}}$
Collateral constraints, inelastic labor	-21.1	4.0	-29.1	-29.4	-0.3	-0.1
Earnings-based constraints, inelastic labor	-8.2	-1.3	-26.7	-12.4	0.0	0.0
Collateral constraints, endogenous labor	-1.6	-3.4	-1.0	0.6	-1.9	-3.2
Earnings-based constraints, endogenous labor	0.3	0.4	-2.6	-7.1	0.9	0.3

Table 3: Consumption equivalent welfare change in different counterfactuals

Panel (a): all types of externalities

Economy	**Right policy, λ(%)**	**Wrong policy, λ(%)**	**Δ(%)**
Earnings-based, inelastic labor	0.60	-1.95	-2.55
Earnings-based, endogenous labor	0.60	-0.52	-1.12

Panel (b): constraint externalities only

Economy	**Right policy, λ(%)**	**Wrong policy, λ(%)**	**Δ(%)**
Earnings-based, inelastic labor	0.00	-0.01	-0.01
Earnings-based, endogenous labor	0.06	-0.47	-0.53

Notes. The table shows the welfare impact of policies carried out in the 'true' economy, which features earnings-based constraints. The right policy is the solution to the social planner's problem in that economy. It moves the allocation in the decentralized equilibrium to the constrained efficient allocation. The wrong policy is calculated under the incorrect assumption that agents face asset-based borrowing constraints. It moves the allocation in the decentralized equilibrium to allocation that arises from the wrong policy.

ONLINE APPENDIX TO

Macroprudential policy with earnings-based borrowing constraints

by Thomas Drechsel and Seho Kim

Appendix A. Derivation for the sufficient condition for case (ii)

In case (ii) where the lender is the only supplier for labor, $h(w,m,x_1^b)=0$ in Equation (8). By solving Equation (8) for m and plugging in Equation (9),

$$\alpha\phi_\pi\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}K=\frac{1}{m}\left(e_1^l+w\left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}}K+x_1^l-\left(\frac{w^{(1+\frac{\psi}{\alpha})}}{(1-\alpha)^{\frac{\psi}{\alpha}}K^{\psi}}\right)^{\frac{1}{\gamma}}\right).$$

By differentiating this equation with respect to x_1^l,

$$\left[\frac{\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}-\frac{\psi}{\alpha}}\beta\phi_\pi}{K^{\psi-1}}\left((1+\frac{\psi}{\alpha})-(\frac{1-\alpha}{\alpha})\right)w^{\frac{\psi}{\alpha}-\frac{1-\alpha}{\alpha}}+(1-\alpha)^{\frac{1}{\alpha}}K(\frac{1}{\alpha}-1)w^{-\frac{1}{\alpha}}\right.$$
$$\left.+\frac{1}{\gamma}(1+\frac{\psi}{\alpha})\left(\frac{w^{(1+\frac{\psi}{\alpha})}}{(1-\alpha)^{\frac{\psi}{\alpha}}K^{\psi}}\right)^{\frac{1}{\gamma}}\frac{1}{w}\right]\frac{\partial w}{\partial X_1^l}=1.$$

As long as $(1+\frac{\psi}{\alpha})-(\frac{1-\alpha}{\alpha})\geq 0$ holds, $\frac{\partial w}{\partial X_1^l}\geq 0$.

This is a sufficient condition, not a necessary condition. To understand what necessity and sufficiency mean in this context, it is helpful to invoke Figure 1. Condition (12) holds if the function $w=L(m)$ is steeper with respect to m than the function $w=B^{-1}(m,X_1^l)$. The relative steepness of the two functions depends on many model primitives, including γ. However, $w=L(m)$ alone does not depend γ. Under the condition $1+\frac{\psi}{\alpha}>\frac{1-\alpha}{\alpha}$ this function is so "flat", that the function $w=B^{-1}(m,X_1^l)$ is steeper for any value of γ. In this case, the appropriate relative size of α and ψ alone suffices to fulfill condition (12). But this is not necessary. Even when $w=L(m)$ is less "flat" than it is under the sufficient condition, then there are γ values that can make $w=B^{-1}(m,X_1^l)$ steep enough to fulfil condition (12).

Appendix B. Additional details for the small open economy model

Appendix B.1. SOE model with tradable production and earnings-based constraints

There are two time periods $t = 1, 2$. There is a representative household who consumes tradable goods c_t^T and nontradable goods c_t^N according to a standard CES aggregator. The representative agent starts period 1 with an initial net worth X (see Section 2.3 for a discussion of aggregate net worth). The supply of nontradable goods is exogenously determined by an endowment y_t^N while tradable goods y_t^T are produced using capital and labor in period 1 and using only capital in period 2. The agent supplies labor (ℓ^s) in period 1. Capital K is is fixed. We assume risk-neutrality in period 2. International borrowing ($-x_2$) is denominated by tradable goods units with a fixed bond price m. The representative agent is subject to earnings-based borrowing constraints that are discussed in the main text. The price of nontradable goods in period t and wage are denoted by p_t and w, respectively.

The optimization problem of the representative household is

$$\max_{c_1^T, c_1^N, c_2^T, c_2^N, \ell^d, \ell^s, x_2} (u(c_1) - v(\ell^s)) + \beta c_2$$

$$s.t.$$

$$c_1^T + p_1 c_1^N + m x_2 = (y_1^T - w\ell^d) + p_1 y_1^N + w\ell^s + X$$
$$c_2^T + p_2 c_2^N = y_2^T + p_2 y_2^N + x_2$$
$$-x_2 \leq \phi_\pi((y_1^T - w\ell^d) + p_1 y_1^N)$$
$$\text{where}$$
$$c_t = \left[\theta (c_t^T)^\rho + (1-\theta)(c_t^N)^\rho\right]^{\frac{1}{\rho}}, \quad t \in \{1, 2\}, \quad \rho \in (-\infty, 1]$$
$$y_1^T = z_1 K^\alpha (\ell^d)^{1-\alpha}$$
$$y_2^T = z_2 K.$$

The market clearing conditions are:

$$c_1^T + m x_2 = y_1^T + X, \quad c_1^N = y_1^N$$
$$c_2^T = y_2^T + x_2, \quad c_2^N = y_2^N$$
$$\ell^d = \ell^s.$$

When the borrowing constraint binds, (p_1, w) are determined by the following two equations:

$$p_1 = \frac{1-\theta}{\theta}\left(\frac{(1+\alpha m\phi_\pi)y_1^T + X}{y_1^N} + m\phi_\pi p_1\right)^{1-\rho} \tag{B.1}$$

$$\left(\frac{(1-\alpha)z_1}{w}\right)^{\frac{1}{\alpha}} K = \ell^{s^*}, \tag{B.2}$$

where ℓ^{s^*} is the optimal labor supply.

We now show why $sign(\partial p_1/\partial X) = sign(\partial w/\partial X)$ holds when labor supply is endogenously determined. For a general preference $u(c_1) = \frac{1}{1-\gamma}c_1^{1-\gamma}$, $v(\ell^s) = \frac{1}{1+\psi}(\ell^s)^{1+\psi}$, the optimal labor supply ℓ^{s^*} is

$$\ell^{s^*} = \left(w\theta(\theta + (1-\theta)(\frac{1-\theta}{\theta p_1})^{\frac{\rho}{1-\rho}})^{\frac{1-\rho}{\rho}} c_1^{-\gamma}\right)^{\frac{1}{\psi}}, \tag{B.3}$$

where $c_1 = y_1^N[\theta(\frac{\theta p_1}{1-\theta})^{\frac{\rho}{1-\rho}} + (1-\theta)]^{\frac{1}{\rho}}$.

By differentiating Equation (B.2) with respect to X after plugging in equation (B.3), the following relationship holds:

$$\frac{1}{w}\left[\psi+\alpha\right]\frac{\partial w}{\partial X} = \frac{1}{p_1}\left[\alpha\epsilon + \frac{\alpha\gamma}{1-\rho}(1-\epsilon)\right]\frac{\partial p_1}{\partial X}, \tag{B.4}$$

where $\epsilon = \frac{(1-\theta)(\frac{1-\theta}{\theta p_1})^{\frac{\rho}{1-\rho}}}{\theta+(1-\theta)(\frac{1-\theta}{\theta p_1})^{\frac{\rho}{1-\rho}}} < 1$. As $\psi + \alpha > 0$ and $\alpha\epsilon + \frac{\alpha\gamma}{1-\rho}(1-\epsilon) > 0$, $sign(\partial p_1/\partial X) = sign(\partial w/\partial X)$ holds. Note that this result holds even with GHH preferences (when $\gamma = 0$).

Appendix B.2. SOE model with nontradable production and earnings-based constraints

We also consider the case where nontradable goods are produced and tradable goods are an endowment. (p_1, w) are still the key prices in this case, and we can characterize them with similar equilibrium conditions:

$$p_1 = \frac{1-\theta}{\theta}\left(\frac{(1+m\phi_\pi)y_1^T + X}{y_1^N} + \alpha m\phi_\pi p_1\right)^{1-\rho} \tag{B.5}$$

$$\left(\frac{(1-\alpha)z_1 p_1}{w}\right)^{\frac{1}{\alpha}} K = \ell^{s^*} \tag{B.6}$$

For a general specification of preferences $u(c_1) = \frac{1}{1-\gamma}c_1^{1-\gamma}$, $v(\ell^s) = \frac{1}{1+\psi}(\ell^s)^{1+\psi}$, we derive a relationship between $\partial p_1/\partial X$ and $\partial w/\partial X$

$$\frac{1}{w}\left[\psi+\gamma(1-\alpha)+\alpha\right]\frac{\partial w}{\partial X_1}=\frac{1}{p_1}\left[\psi+\alpha\epsilon+\gamma(1-\alpha)+\frac{\alpha\gamma}{1-\rho}(1-\epsilon)\right]\frac{\partial p_1}{\partial X_1}, \tag{B.7}$$

where $\epsilon=\frac{(1-\theta)(\frac{1-\theta}{\theta p_1})^{\frac{\rho}{1-\rho}}}{\theta+(1-\theta)(\frac{1-\theta}{\theta p_1})^{\frac{\rho}{1-\rho}}}<1$. As $\psi+\gamma(1-\alpha)+\alpha>0$ and $\psi+\alpha\epsilon+\gamma(1-\alpha)+\frac{\alpha\gamma}{1-\rho}(1-\epsilon)>0$, $sign(\partial p_1/\partial X)=sign(\partial w/\partial X)$ also holds under this alternative SOE model with nontradable production. Note that $\partial w/\partial X$ is not zero even with exogenously determined labor supply as labor demand changes with p_1 which changes with X. Thus, it can be shown that $sign(\partial p_1/\partial X)=sign(\partial w/\partial X)$ even in a setting with inelastic labor supply.

Appendix C. Details about the general model

Appendix C.1. Market clearing conditions

The model's market clearing conditions are the following:

$$\sum_i [c_0^i + h^i(k_1^i)] \leq \sum_i e_0^i \tag{C.1}$$

$$\sum_i c_t^{i,\theta} \leq \sum_i [e_t^i + F^i(k_t^{i,\theta}, \ell_{dt}^{i,\theta})],\ t = 1, 2,\ \forall\theta \tag{C.2}$$

$$\sum_i k_2^{i,\theta} \leq \sum_i k_1^i,\ \forall\theta \tag{C.3}$$

$$\sum_i \ell_{dt}^{i,\theta} = \sum_i \ell_{st}^{i,\theta},\ t = 1, 2, \forall\theta \tag{C.4}$$

$$\sum_i x_t^{i,\theta} = 0,\ t = 1, 2,\ \forall\theta \tag{C.5}$$

Appendix C.2. First-order conditions

The first-order conditions for the period-1 maximization problem with respect to $x_2^{i,\theta}$ and $k_2^{i,\theta}$ are

$$m_2^\theta \lambda_1^{i,\theta} = \beta\lambda_2^{i,\theta} + \kappa_2^{i,\theta}\Phi_{2x^\theta}^{i,\theta}, \tag{C.6}$$

$$q^\theta \lambda_1^{i,\theta} = \beta\lambda_2^{i,\theta} F_{2k}^{i,\theta}(k_2^{i,\theta}, \ell_{d2}^{i,\theta}) + \kappa_2^{i,\theta}\Phi_{2k}^{i,\theta},\ \forall i, \theta \tag{C.7}$$

Equations (C.6) and (C.7) are the Euler equations for the financial asset and physical investment. Remember that $\Phi_2^{b,\theta}$ is given by (29) and $\Phi_2^{l,\theta} = 0$.

Using the envelope conditions $\frac{\partial V^{i,\theta}(\cdot,\cdot)}{\partial n_1^{i,\theta}} = \lambda_1^{i,\theta}$ and $\frac{\partial V^{i,\theta}(\cdot,\cdot)}{\partial k_1^i} = \lambda_1^{i,\theta}(q^\theta + F_{1k}^{i,\theta}(k_1^i, l_{d1}^{i,\theta}))$, the first-order conditions with respect to the asset holding and capital are derived as

$$m_1^\theta \lambda_0^i = \beta\lambda_1^{i,\theta} + \kappa_1^i \Phi_{1x^\theta}^i, \tag{C.8}$$

$$h^{i\prime}(k_1^i)\lambda_0^i = \mathbb{E}_0[\beta\lambda_1^{i,\theta}(F_{1k}^{i,\theta}(k_1^i, \ell_{d1}^{i,\theta}) + q^\theta)] + \kappa_1^i \Phi_{1k}^i,\ \forall i, \theta \tag{C.9}$$

where λ_0^i is Lagrange multiplier for (25) and κ_1^i is Lagrange multiplier for (28).

Appendix C.3. Derivation of distributive and constraint effects

Lemma 1 characterizes relevant properties of the date 1 equilibrium.

Lemma 1. *The effects of changes in the aggregate state variables* $N_1^{j,\theta}$ *and* K_1^j *on agent* i*'s indirect utility at date 1 are given by*

$$V_{N_1^j}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dN_1^{j,\theta}} = \lambda_1^{i,\theta}\mathcal{D}_{1N^j}^{i,\theta} + \lambda_2^{i,\theta}\mathcal{D}_{2N^j}^{i,\theta} + \kappa_2^{i,\theta}\mathcal{C}_{N^j}^{i,\theta} \tag{C.10}$$

$$V_{K_1^j}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dK_1^{j}} = \lambda_1^{i,\theta}\mathcal{D}_{1K^j}^{i,\theta} + \lambda_2^{i,\theta}\mathcal{D}_{2K^j}^{i,\theta} + \kappa_2^{i,\theta}\mathcal{C}_{K^j}^{i,\theta} \tag{C.11}$$

where $\mathcal{D}_{1N^j}^{i,\theta}$, $\mathcal{D}_{1K^j}^{i,\theta}$, $\mathcal{D}_{2N^j}^{i,\theta}$ *and* $\mathcal{D}_{2K^j}^{i,\theta}$ *are called the distributive effects*

$$\mathcal{D}_{1N^j}^{i,\theta} \equiv -\frac{\partial q^\theta}{\partial N_1^{j,\theta}}\Delta K_2^{i,\theta} - \frac{\partial m_2^\theta}{\partial N_1^{j,\theta}}X_2^{i,\theta} - \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}\ell_{d1}^{i,\theta} + \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}\ell_{s1}^{i,\theta} \tag{C.12}$$

$$\mathcal{D}_{1K_1^j}^{i,\theta} \equiv -\frac{\partial q^\theta}{\partial K_1^{j}}\Delta K_2^{i,\theta} - \frac{\partial m_2^\theta}{\partial K_1^{j}}X_2^{i,\theta} - \frac{\partial w_1^\theta}{\partial K_1^{j}}\ell_{d1}^{i,\theta} + \frac{\partial w_1^\theta}{\partial K_1^{j}}\ell_{s1}^{i,\theta} \tag{C.13}$$

$$\mathcal{D}_{2N^j}^{i,\theta} \equiv -\frac{\partial w_2^\theta}{\partial N_1^{j,\theta}}\ell_{d2}^{i,\theta} + \frac{\partial w_2^\theta}{\partial N_1^{j,\theta}}\ell_{s2}^{i,\theta} \tag{C.14}$$

$$\mathcal{D}_{2K^j}^{i,\theta} \equiv -\frac{\partial w_2^\theta}{\partial K_1^{j}}\ell_{d2}^{i,\theta} + \frac{\partial w_2^\theta}{\partial K_1^{j}}\ell_{s2}^{i,\theta} \tag{C.15}$$

and $\mathcal{C}_{N^j}^{i,\theta}$ *and* $\mathcal{C}_{K^j}^{i,\theta}$ *are called the constraint effects*

$$\mathcal{C}_{N^j}^{b,\theta} \equiv \frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta}\frac{\partial q^\theta}{\partial N_1^{j,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial m_2^\theta}\frac{\partial m_2^\theta}{\partial N_1^{j,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta}\frac{\partial w_1^\theta}{\partial N_1^{j,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta}\frac{\partial w_2^\theta}{\partial N_1^{j,\theta}} \tag{C.16}$$

$$\mathcal{C}_{K^j}^{b,\theta} \equiv \frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta}\frac{\partial q^\theta}{\partial K_1^{j}} + \frac{\partial \Phi_2^{b,\theta}}{\partial m_2^\theta}\frac{\partial m_2^\theta}{\partial K_1^{j}} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta}\frac{\partial w_1^\theta}{\partial K_1^{j}} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta}\frac{\partial w_2^\theta}{\partial K_1^{j}} \tag{C.17}$$

$$\mathcal{C}_{N^j}^{l,\theta} = \mathcal{C}_{K^j}^{l,\theta} = 0 \tag{C.18}$$

for $i \in \{b, l\}$, $j \in \{b, l\}$ *and* $\theta \in \Theta$.

Proof. The effects of changes in the aggregate state variables (N_1^θ, K_1) on agents' indirect utility are derived by taking partial derivatives of $V^{i,\theta}$ as defined by equations (30) to (33). We make use of the envelope theorem, according to which the derivatives of $\left\{u^i(c_1^{i,\theta}, \ell_{s1}^{i,\theta}) + \beta u^i(c_2^{i,\theta}, \ell_{s2}^{i,\theta})\right\}$ with respect to the state variables are 0. We further impose a symmetric equilibrium in which $n^{i,\theta} = N^{i,\theta}$ and $k_1^i = K_1^i$. ■

Remarks on Lemma 1. $\mathcal{D}_{1N^j}^{i,\theta}$, $\mathcal{D}_{1K^j}^{i,\theta}$, $\mathcal{D}_{2N^j}^{i,\theta}$ and $\mathcal{D}_{2K^j}^{i,\theta}$ are called *distributive effects* because

$$\sum_i \mathcal{D}_{1N^j}^{i,\theta} = \sum_i \mathcal{D}_{2N^j}^{i,\theta} = \sum_i \mathcal{D}_{1K^j}^{i,\theta} = \sum_i \mathcal{D}_{2K^j}^{i,\theta} = 0 \tag{C.19}$$

from the market clearing conditions, that is, they are "zero sum" effects across agents, state by state. Such a relation does not hold for the *constraint effects* $\mathcal{C}^{i,\theta}_{N^j}$ and $\mathcal{C}^{i,\theta}_{K^j}$. These collect any derivatives that multiply the shadow price on the financial constraint $\kappa^{i,\theta}_2$. Comparing Lemma 1 to its analogue in DK18, both our inclusion of labor markets and our more general financial constraint change this characterization. In particular, wage changes generate both distributive effects and constraint effects. This observation will be important for the earnings-based constraint. Third, we also allow equation (29) to include the asset price m^θ_2 so the constraint effects include partial derivatives with respect to this variable.

Appendix C.4. Constrained efficient allocation and implementation

The economy's constrained efficient allocation is described by quantities $(C^i_0, K^i_1, X^{i,\theta}_1)$, Pareto weights $\alpha^b/\alpha^l = \lambda^l_0/\lambda^b_0$ and shadow prices v_0,v^θ_1, and κ^i_1 satisfying the optimality conditions and constraints of the social planner's problem. This allocation can be implemented with a set of tax rate on financial asset and capital purchases.

Derivation of constrained efficient allocation. These derivations correspond to Proposition 1 (a) and the associated proof in DK18. The Lagrangian of the social planner's problem can be written as

$$\mathcal{L} = \sum_i \alpha^i \{u^i(C^i_0) + \beta \mathbb{E}_0[V^{i,\theta}(N^{i,\theta}_1, K^i_1; N^\theta, K_1)] + \kappa^i_1 \Phi^i_1(X^i_1, K^i_1)\}$$
$$+ v_0 \sum_i [e^i_0 - (C^i_0 + h^i(K^i_1))] - \int_{\theta\in\Theta} v^\theta_1 \sum_i X^{i,\theta}_1 d\theta.$$

The first-order conditions of the social planner are

$$\frac{d\mathcal{L}}{dC^i_0} = \alpha^i u'^i(C^i_0) - v_0 = 0, \ \forall i \tag{C.20}$$

$$\frac{d\mathcal{L}}{dX^{i,\theta}_1} = -v^\theta_1 + \alpha^i \beta V^{i,\theta}_n + \alpha^i \kappa^i_1 \Phi^i_{1x} + \beta \sum_j \alpha_j V^{j,\theta}_{N^i}, \ \forall i, \theta \tag{C.21}$$

$$\frac{d\mathcal{L}}{dK^i_1} = -v_0 h'^i(K^i_1) + \alpha^i \beta \mathbb{E}_0[V^{i,\theta}_k] + \alpha^i \kappa^i_1 \Phi^i_{1k} + \beta \sum_j \alpha^j \mathbb{E}_0[V^{j,\theta}_{K^i}] = 0, \ \forall i \tag{C.22}$$

Note that there are no expectation terms in the second first-order condition since $X^{i,\theta}_1$ is chosen for each θ.

The first first-order condition in the decentralized equilibrium implies $v_0 = \alpha^i \lambda^i_0$, so $\alpha^b/\alpha^l = \lambda^l_0/\lambda^b_0$. We divide the second FOC by α^i, and use $\alpha^i = v_0/\lambda^i_0$ as well as the envelope condition in the decentralized equilibrium $V_n^{i,\theta} = \lambda_1^{i,\theta}$. This gives us

$$\frac{v_1^\theta}{v_0}\lambda_0^i = \beta_i \lambda_1^{i,\theta} + \kappa_1^i \Phi_{1x^\theta}^i + \beta \sum_j \frac{\alpha^j}{\alpha^i} V_{N^i}^{j,\theta},\ \forall i, \theta \tag{C.23}$$

We then use the third first-order condition and the envelope condition to get

$$h^{i\prime}(K_1^i)\lambda_0^i = \beta \mathbb{E}_0[\lambda_1^{i,\theta}(F_{1k}^{i,\theta}(K_1^i, l_{1d}^{i,\theta}) + q^\theta)] + \kappa_1^i \Phi_{1k}^i + \beta \sum_j \frac{\alpha^j}{\alpha^i} \mathbb{E}_0[V_{K^i}^{j,\theta}],\ \forall i, \tag{C.24}$$

Equations (C.23) and (C.24), together with the constraints of the social planner's problem describe the constrained efficient allocation. Note that variables in $t \geq 1$ are optimal choices by the agents. Lemma 1 gives more detailed expressions being $V_{N^i}^{j,\theta}$ and $V_{K^i}^{j,\theta}$.

Implementation of constrained efficiency. These derivations correspond to Proposition 1 (b) and the associated proof in DK18. The constrained efficient allocation can be implemented by setting taxes on Arrow-Debreu security purchases and capital investment that satisfy

$$\tau_x^{i,\theta} = -\sum_j MRS_{01}^{j,\theta} \mathcal{D}_{1N^i}^{j,\theta} - \sum_j MRS_{02}^{j,\theta} \mathcal{D}_{2N^i}^{j,\theta} - \sum_j \tilde{\kappa}_2^{j,\theta} \mathcal{C}_{N^i}^{j,\theta},\ \forall i, \theta \tag{C.25}$$

$$\tau_k^i = -\sum_j \mathbb{E}_0[MRS_{01}^{j,\theta} \mathcal{D}_{1K^i}^{j,\theta}] - \sum_j \mathbb{E}_0[MRS_{02}^{j,\theta} \mathcal{D}_{2K^i}^{j,\theta}] - \sum_j \mathbb{E}_0[\tilde{\kappa}_2^{j,\theta} \mathcal{C}_{K^i}^{j,\theta}],\ \forall i \tag{C.26}$$

where $MRS_{01}^{j,\theta} \equiv \beta \lambda_1^{j,\theta}/\lambda_0^j$, $MRS_{02}^{j,\theta} \equiv \beta \lambda_2^{j,\theta}/\lambda_0^j$ and $\tilde{\kappa}_2^{j,\theta} \equiv \beta \kappa_2^{j,\theta}/\lambda_0^j$. This can be shown as follows. Re-write the period-0 first-order conditions (C.8) and (C.9) by including tax wedges for security purchases ($\tau_x^{i,\theta}$) and capital investment (τ_k^i). This gives

$$(m_1^\theta + \tau_x^{i,\theta})\lambda_0^i = \beta \lambda_1^{i,\theta} + \kappa_1^i \Phi_{1x^\theta}^i \tag{C.27}$$

$$(h^{\prime i}(k_1^i) + \tau_k^i)\lambda_0^i = \beta \mathbb{E}_0[\lambda_1^{i,\theta}(F_{1k}^{i,\theta}(k_1^i, l_{d1}^{i,\theta}) + q^\theta)] + \kappa_1^i \Phi_{1k}^i\ \forall i \tag{C.28}$$

Substituting the above tax rates into these optimality conditions replicates the planner's optimality conditions (C.23) and (C.24). Note that $m_1^\theta = \frac{v_1^\theta}{v_0}$ in the replicated allocations, i.e., Arrow-Debreu price in the decentralized equilibrium should equal the value of state contingent commodity in the social planner's problem measured by the shadow prices. Importantly, note also that the expressions for the tax rates contain additional terms relative to DK18 due to the presence of labor markets and the more general financial constraint formulation.

Combining equations (C.25) and (C.26) with equation (C.18) and (C.19) gives a set of tax rates

$$\tau_x^{i,\theta} = -\Delta MRS_{01}^{ij,\theta}\mathcal{D}_{1N^i}^{i,\theta} - \Delta MRS_{02}^{ij,\theta}\mathcal{D}_{2N^i}^{i,\theta} - \tilde{\kappa}_2^{b,\theta}\mathcal{C}_{N^i}^{b,\theta},\ \forall i,\theta \tag{C.29}$$

$$\tau_k^{i} = -\mathbb{E}_0[\Delta MRS_{01}^{ij,\theta}\mathcal{D}_{1K^i}^{i,\theta}] - \mathbb{E}_0[\Delta MRS_{02}^{ij,\theta}\mathcal{D}_{2K^i}^{i,\theta}] - \mathbb{E}_0[\tilde{\kappa}_2^{b,\theta}\mathcal{C}_{K^i}^{b,\theta}],\ \forall i \tag{C.30}$$

$\Delta MRS_{0t}^{ij,\theta} \equiv MRS_{0t}^{i,\theta} - MRS_{0t}^{j,\theta}$ for $t = 1,2$ denotes the difference between agents in the marginal rate of substitution (MRS) across time, $MRS_{01}^{j,\theta} \equiv \beta\lambda_1^{j,\theta}/\lambda_0^j$, $MRS_{02}^{j,\theta} \equiv \beta\lambda_2^{j,\theta}/\lambda_0^j$. We define $\tilde{\kappa}_2^{b,\theta} \equiv \beta\kappa_2^{b,\theta}/\lambda_0^b$ as the relative shadow price. A positive $\tau_x^{i,\theta}$ implies that agent i saves too much (borrows too little) in the market outcome. The planner thus wants to impose a tax on savings (remember that $x_1^i > 0$ implies saving, $x_1^i < 0$ borrowing). A positive τ_k^i means that agent i invests too much in capital relative to the constrained efficient allocation, so the planner imposes a tax on investment. In our formal welfare analysis, we focus on over-/under-borrowing since over-/under-investment effects cannot be signed in the DK18 framework. In the numerical application of the model, we do allow for both forces.

Nature of externalities and sufficient statistics. The optimal tax wedges, in combination with the distributive effects $\mathcal{D}$ and the constraint effects $\mathcal{C}$ derived in Lemma 1, allow us to characterize the externalities in this economy. In essence, by analyzing and interpreting the different terms in (C.29) and (C.30), we can understand how outcomes in the market economy deviate from the constrained efficient allocation and how such distortions could be corrected. Building on the earlier terminology we distinguish *distributive externalities* and *constraint externalities*.

The sign and magnitude of *distributive externalities* are determined by the product of:

(i) The difference in MRS of agents in periods 1 and 2, $\Delta MRS_{01}^{ij,\theta}$ and $\Delta MRS_{02}^{ij,\theta}$

(ii) The net trading positions on capital $\Delta K_2^{i,\theta}$, financial assets $X_2^{i,\theta}$, labor supply in periods 1 and 2 $\ell_{s1}^{i,\theta}, \ell_{s2}^{i,\theta}$, and labor demand in periods 1 and 2 $\ell_{d1}^{i,\theta}, \ell_{d2}^{i,\theta}$

(iii) The sensitivity of equilibrium prices to changes in aggregate state variables $\frac{\partial q^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial m_2^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}$, $\frac{\partial q^\theta}{\partial K_1^j}$, $\frac{\partial m_2^\theta}{\partial K_1^j}$, $\frac{\partial w_1^\theta}{\partial K_1^j}$

The sign and magnitude of *constraint externalities* are determined by the product of:

(i) The relative shadow price of the financial constraint $\tilde{\kappa}_2^{i,\theta}$

(ii) The sensitivity of the financial constraint to the price of capital, asset price and wages for period 1 and 2 $\partial\Phi_2^{i,\theta}/\partial q^\theta$, $\partial\Phi_2^{i,\theta}/\partial m_2^\theta$, $\partial\Phi_2^{i,\theta}/\partial w_1^\theta$, $\partial\Phi_2^{i,\theta}/\partial w_2^\theta$

(iii) The sensitivity of the equilibrium capital price, asset price and wages in periods 1 and 2 to changes in aggregate states $\frac{\partial q^\theta}{\partial N_1^{j,\theta}}, \frac{\partial m_2^\theta}{\partial N_1^{j,\theta}}, \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}, \frac{\partial w_2^\theta}{\partial N_1^{j,\theta}}, \frac{\partial q^\theta}{\partial K_1^j}, \frac{\partial m_2^\theta}{\partial K_1^j}, \frac{\partial w_1^\theta}{\partial K_1^j}, \frac{\partial w_2^\theta}{\partial K_1^j}$

Remarks on the externalities. The lists above reveal how distortions in the model can be parsed into a compact list of sufficient statistics. Distributive externalities, those driven by effects which are "zero sum," depend on the difference in marginal rates of substitution in combination with the positions that agents take in quantities of capital, labor and financial assets in equilibrium. If these externalities were fully corrected, these quantities would be such that marginal rates of substitution equalize across agents. Logically, constraint externalities depend on the shadow price on the financial constraint, in combination with how the constraint moves with prices changes. Finally, both types of externalities depend on how prices react to changes in the aggregate states, making clear any externalities ultimately operate through price changes.

Appendix C.5. Insensitivity to re-definition of net worth

In our model, we do not include production output as part of the definition of net worth. This is because output is not predetermined at the beginning of the period due to labor markets clearing during the period. It therefore cannot be a state variable of the model. To ensure that this definitional change does not affect the results, we show in this Appendix that a re-definition of net worth along the same lines gives identical results in the original Dávila and Korinek (2018) (DK18) framework. This is also useful to interpret our Lemma 1 in relation to its analogue in DK18: in our model, we obtain extra terms that contain additional economically meaningful effects.

We proceed by re-defining net worth in DK18 by excluding production output and prove that the *distributive effects* and *collateral effects* in DK18's version of Lemma 1 are identical. We denote net worth as defined by DK18 as $N_{DK}^{i,\theta} \equiv e_1^{i,\theta} + X_1^{i,\theta} + F_1^{i,\theta}(K_1^i)$. The resulting equilibrium capital and debt price are denoted by $q_{DK}^\theta(N_{DK}^\theta, K_1)$ and $m_{2,DK}^\theta(N_{DK}^\theta, K_1)$. We define net worth without production output as $N_{WP}^{i,\theta} \equiv e_1^{i,\theta} + X_1^{i,\theta}$ and the resulting equilibrium capital and debt price are denoted by $q_{WP}^\theta(N_{WP}^\theta, K_1)$ and $m_{2,WP}^\theta(N_{WP}^\theta, K_1)$. A simple re-definition of the model's state variables cannot change the prices in equilibrium, so that we can set

$$q_{WP}^\theta(N_{WP}^\theta, K_1) = q_{DK}^\theta(N_{DK}^\theta, K_1) \tag{C.31}$$

$$m_{2,WP}^\theta(N_{WP}^\theta, K_1) = m_{2,DK}^\theta(N_{DK}^\theta, K_1) \tag{C.32}$$

Noting that $N_{DK}^{i,\theta} = N_{WP}^{i,\theta} + F_1^{i,\theta}(K_1^i)$, we differentiate both sides of (C.31) and (C.32) with respect to $N^{i,\theta}_{(\cdot)}$ and K^i_1, in order to determine how the derivatives of prices with respect to net worth and capital are related across models. This gives us

$$\frac{\partial q^\theta_{WP}}{\partial N^{i,\theta}_{WP}} = \frac{\partial q^\theta_{DK}}{\partial N^{i,\theta}_{DK}} \tag{C.33}$$

$$\frac{\partial m^\theta_{2,WP}}{\partial N^{i,\theta}_{WP}} = \frac{\partial m^\theta_{2,DK}}{\partial N^{i,\theta}_{DK}} \tag{C.34}$$

$$\frac{\partial q^\theta_{WP}}{\partial K^i_1} = \frac{\partial q^\theta_{DK}}{\partial N^{i,\theta}_{DK}}\frac{\partial N^{i,\theta}_{DK}}{\partial K^i_1} + \frac{\partial q^\theta_{DK}}{\partial K^i_1} = \frac{\partial q^\theta_{DK}}{\partial N^{i,\theta}_{DK}}F'(K^i_1) + \frac{\partial q^\theta_{DK}}{\partial K^i_1} \tag{C.35}$$

$$\frac{\partial m^\theta_{2,WP}}{\partial K^i_1} = \frac{\partial m^\theta_{2,DK}}{\partial N^{i,\theta}_{DK}}\frac{\partial N^{i,\theta}_{DK}}{\partial K^i_1} + \frac{\partial m^\theta_{2,DK}}{\partial K^i_1} = \frac{\partial m^\theta_{2,DK}}{\partial N^{i,\theta}_{DK}}F'(K^i_1) + \frac{\partial m^\theta_{2,DK}}{\partial K^i_1} \tag{C.36}$$

where we used the chain rule for the differentiation with respect to capital. (C.35) and (C.36) make clear that the derivatives of prices with respect to capital after the re-definition of net worth "contain" the partial derivatives of $F(\cdot)$ that appear in DK18's Lemma 1. The *distributive effects* in DK18 are the following:

$$\mathcal{D}^{DK,i,\theta}_{N^{j,\theta}_{DK}} = -\left[\frac{\partial q^\theta_{DK}}{\partial N^{j,\theta}_{DK}}\Delta K^{i,\theta}_2 + \frac{\partial m^\theta_{2,DK}}{\partial N^{j,\theta}_{DK}}X^{i,\theta}_2\right] \tag{C.37}$$

$$\mathcal{D}^{DK,i,\theta}_{K^j_1} = F'(K^i_1)\mathcal{D}^{DK,i,\theta}_{N^{j,\theta}_{DK}} - \left[\frac{\partial q^\theta_{DK}}{\partial K^j_1}\Delta K^{i,\theta}_2 + \frac{\partial m^\theta_{2,DK}}{\partial K^j_1}X^{i,\theta}_2\right] \tag{C.38}$$

The *distributive effects* with the re-definition of net-worth can be derived as

$$\mathcal{D}^{WP,i,\theta}_{N^{j,\theta}_{WP}} = -\left[\frac{\partial q^\theta_{WP}}{\partial N^{j,\theta}_{WP}}\Delta K^{i,\theta}_2 + \frac{\partial m^\theta_{2,WP}}{\partial N^{j,\theta}_{WP}}X^{i,\theta}_2\right] \tag{C.39}$$

$$\mathcal{D}^{WP,i,\theta}_{K^j_1} = -\left[\frac{\partial q^\theta_{WP}}{\partial K^j_1}\Delta K^{i,\theta}_2 + \frac{\partial m^\theta_{2,WP}}{\partial K^j_1}X^{i,\theta}_2\right] \tag{C.40}$$

Using (C.33) - (C.36), we obtain

$$\mathcal{D}^{DK,i,\theta}_{N^{j,\theta}_{DK}} = \mathcal{D}^{WP,i,\theta}_{N^{j,\theta}_{WP}} \tag{C.41}$$

$$\mathcal{D}^{DK,i,\theta}_{K^j_1} = \mathcal{D}^{WP,i,\theta}_{K^j_1} \tag{C.42}$$

Similarly, it can be shown that

$$\mathcal{C}^{DK,i,\theta}_{N^{j,\theta}_{DK}} = \mathcal{C}^{WP,i,\theta}_{N^{j,\theta}_{WP}} \tag{C.43}$$

$$\mathcal{C}^{DK,i,\theta}_{K^{j}_{1}} = \mathcal{C}^{WP,i,\theta}_{K^{j}_{1}} \tag{C.44}$$

This shows that a re-definition of net worth in the original DK18 model gives identical results. Furthermore, these derivations show that Lemma 1 in our model would be identical to Lemma 1 to its counterpart in DK18 if we did not include labor markets and did not have a more general definition of the financial constraint.

Appendix D. More details on model results

Appendix D.1. Intuition for Proposition 1

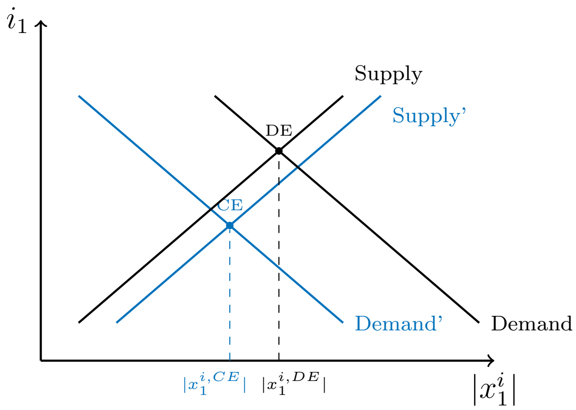
Proposition 1 confirms one of the main insights of DK18 and the existing literature more generally. The borrower's decisions exert an externality through the market price of capital. As borrowers increase their debt position in period $t = 0$, they reduce aggregate net worth in the borrowing sector in period $t = 1$. Since the price of capital positively depends on sector-wide net worth by condition (43), it falls in $t = 1$.[1] Through the collateral constraint, the lower price of capital limits the ability to borrow between $t = 1$ and $t = 2$. As borrowers in $t = 0$ do not internalize this negative effect on future borrowing capability, the amount of debt taken on in $t = 0$ is suboptimally high, that is, there is over-borrowing. The social planner internalizes this relation, and thus discourages borrowing in $t = 0$ through subsidies on saving (for any given level of distributive externalities).

Graphical representation. Figure Appendix D.1 provides the intuition behind Proposition 1 graphically. This graphical analysis will be especially helpful as a benchmark for the results with the earnings-based constraint below. It shows the period-0 credit market, period-1 capital market, and period-1 credit market. In each panel, points CE and DE represent the constrained efficient allocation and the decentralized equilibrium, respectively. The figure conveys how externalities emerge from borrowing decisions in $t = 0$, which through changes in the price of capital affect credit constraints in $t = 1$.

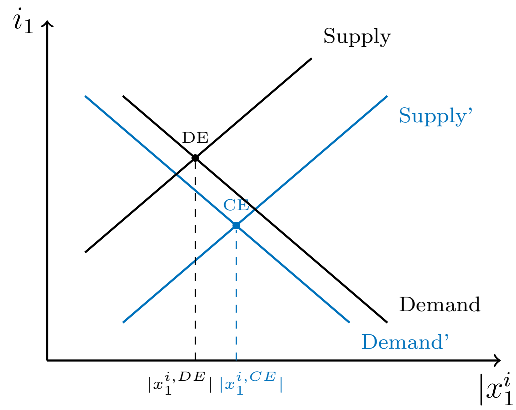
To explain Figure Appendix D.1, we focus first on the decentralized equilibrium, point DE across Panels (a)-(d). The difference between Panels (a) and (b) only becomes relevant

[1] While borrowing more reduces future aggregate net worth in the borrowing sector, it also increases future net worth in the lending sector. By condition (43), the latter effect actually puts upward pressure on the price of capital. However, the net effect of changes in borrower and lender net worth leads to a fall in the price of capital. We highlight this in the graphical illustration we provide further below.

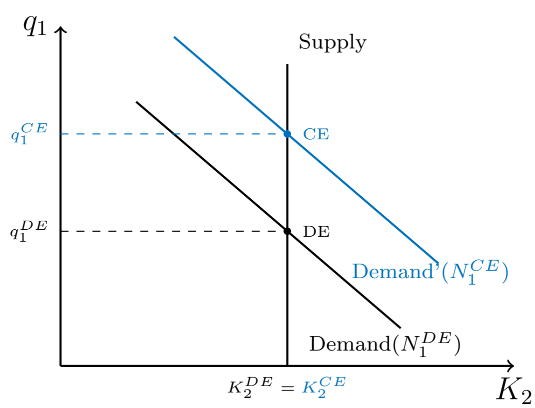
Figure Appendix D.1: MARKET VS. PLANNER ALLOCATIONS: COLLATERAL CONSTRAINT



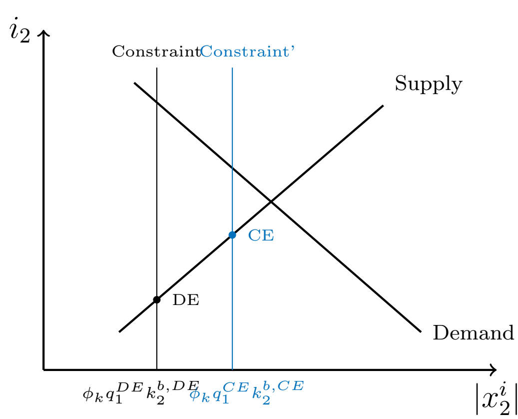
(a) Period-0 credit market (case 1: $|\tau_x^b| > |\tau_x^l|$)



(b) Period-0 credit market (case 2: $|\tau_x^b| < |\tau_x^l|$)



(c) Period-1 capital market (both cases)



(d) Period-1 credit market (both cases)

Notes. Decentralized equilibrium (DE) and constrained efficient equilibrium (CE) in the period-0 credit market, period-1 capital market and period-1 credit market of the model. State θ is omitted from the notation in the labeling. The figure distinguishes case 1 ($\partial q_1^\theta / \partial N_1^{b,\theta} > \partial q_1^\theta / \partial N_1^{l,\theta} \Leftrightarrow |\tau_x^{b,\theta}| > |\tau_x^{l,\theta}|$) and case 2 ($\partial q_1^\theta / \partial N_1^{b,\theta} < \partial q_1^\theta / \partial N_1^{l,\theta} \Leftrightarrow |\tau_x^{b,\theta}| < |\tau_x^{l,\theta}|$) as described in the text. In both cases, the social planner internalizes that period-0 borrowing decisions reduce equilibrium prices in the market for physical capital in period 1, which tightens the collateral constraint. The constrained efficient allocation features higher capital prices and more credit in period 1, as more saving (less borrowing) is incentivized through taxes/subsidies in period 0.

for implementing constrained efficiency, so for now consider Panel (a) to understand the period-0 credit market. The horizontal axis depicts the financial asset position of each agent in absolute value, that is, borrowing or credit demand $-x_1^{b,\theta}$, and saving or credit supply $x_1^{l,\theta}$. The vertical axis captures the interest rate between periods 0 and 1, $i_1^\theta = 1/m_1^\theta - 1$. Due to market clearing, saving and borrowing positions net out to 0, so $x_1^{b,\theta,DE} + x_1^{l,\theta,DE} = 0 \Rightarrow |x_1^{b,\theta,DE}| = |x_1^{l,\theta,DE}|$. Decisions on the credit market in $t = 0$ impact future net worth and thereby affect investment decisions in period $t = 1$. This is visible in Panel (c), which plots the capital supply curve (given by the vertical line indicating K_1) and the capital demand curve (given by the downward sloping relation between K_2^θ and q_1^θ). Capital supply is in general governed by an upward sloping relationship between K_1 and $q_1^\theta, \forall\theta$. However, since the analysis in the figure traces out the effects of period-0 borrowing externalities, and how these result from changes in period-1 net worth, capital supply is effectively predetermined at the beginning of period $t = 1$.[2] The location of the demand curve does depend on the realization of aggregate net worth. Finally, the capital market equilibrium is linked to the period-1 credit market through the collateral constraint. Panel (d) shows credit supply and credit demand in period 1, by plotting $-x_2^{b,\theta}$ and $x_2^{l,\theta}$ in absolute value against the interest rate i_2^θ. The collateral constraint (44) puts a cap $\phi_k q_1^{\theta,DE} k_2^{\theta,DE}$ on the amount of credit, represented by a vertical line. Importantly, its location is determined by the market clearing price of capital $q_1^{\theta,DE}$. The decentralized equilibrium in the period-1 credit market is given by the intersection of the constraint and the credit supply curve.

By Proposition 1, the decentralized equilibrium is not efficient: the social planner distorts borrowing decisions in period 0 to drive up capital prices and thereby relax borrowing constraints in period 1. Under condition (43), sector-wide net worth of both borrowers and lenders positively impacts the price of capital. For the graphical analysis of the constrained efficient allocation, point CE across Panels (a)-(d), two finer cases can be distinguished: in case 1 the impact of the borrower sector net worth on wages is stronger than that of net worth in the lender sector ($\partial q_1^\theta / \partial N_1^{b,\theta} > \partial q_1^\theta / \partial N_1^{l,\theta}$) and in case 2, the opposite is true ($\partial q_1^\theta / \partial N_1^{b,\theta} < \partial q_1^\theta / \partial N_1^{l,\theta}$). In both cases, the social planner alters borrower and lender equilibrium net worth such that capital prices increase in $t = 1$. However, depending on the relative impact of net worth in the different sectors on the price of capital, the planner will tax borrowing (subsidize saving) more heavily for either the borrower or the lender to achieve the desired increase in the price of capital: in case 1,

[2]This would be different in a graphical analysis of pecuniary externalities that result from over- and under-investment between $t = 0$ and $t = 1$.

$|\tau_x^{b,\theta}| > |\tau_x^{l,\theta}|$, while in case 2, $|\tau_x^{b,\theta}| < |\tau_x^{l,\theta}|$. In other words, the planner reverts the over-borrowing of that agent more heavily whose decisions have a stronger impact on capital prices, making capital prices in period 1 rise in either case.[3] This is visible in Panels (a) and (b) which show the constrained efficient equilibrium for cases 1 and 2. In both cases, the planner incentivizes lenders to save more and borrowers to borrow less, to counteract the *over-borrowing* motive of both agents.[4] As a result, the credit supply curve is located to the right, and the credit demand curve to the left relative to their counterparts in the decentralized case. However, in Panel (a) (case 1), $|\tau_x^{b,\theta}| > |\tau_x^{l,\theta}|$, so the decrease in demand from the borrower is larger than the increase in supply from the lender, and the equilibrium quantity of credit is below that of the decentralized equilibrium. With a smaller amount of equilibrium borrowing, borrower net worth in period 1 will be higher while lender net worth will be lower relative to the decentralized equilibrium. Since $\partial q_1^\theta / \partial N_1^{b,\theta} > \partial q_1^\theta / \partial N_1^{l,\theta}$, capital prices are higher. In Panel (b) (case 2), $|\tau_x^{b,\theta}| < |\tau_x^{l,\theta}|$ so there is a greater amount of equilibrium borrowing, and borrower net worth in period 1 will be lower while lender net worth will be higher. Since $\partial q_1^\theta / \partial N_1^{b,\theta} < \partial q_1^\theta / \partial N_1^{l,\theta}$, capital prices are higher, as in case 1. This makes clear that while the collateral constraint induces over-borrowing motives (borrowers want to borrow too much, savers want to save too little), a corrective policy may actually increase or decrease equilibrium credit.

In both cases 1 and 2, the corrective wedges introduced by the planner lead capital demand to shift upward, while changes the net worth induced by the planner do not move the capital supply curve, all else equal. These effects, shown in Panel (c), are the graphical counterpart to our discussion of condition (43) above.[5] As a result, capital prices in the constrained efficient equilibrium in period $t = 1$ are higher relative to the decentralized equilibrium. As in the decentralized case, the period-1 credit market, shown in Panel (d), is connected to the capital market through the price of capital. An increase in the price of

[3]This can be seen as follows. According to Proposition 1, the constraint externality from the collateral constraint is non-negative, meaning that through equation (C.29) the planner desires a negative $\tau_x^{i,\theta}$ for $i \in \{b, l\}$. By equation (C.29), the size of the tax rate the planner chooses to implement the constrained efficient equilibrium is proportional to the size of the derivative of capital prices to sector wide net worth, that is, $\tilde{\kappa}_2^{b,\theta} \mathcal{C}_{N^i}^{b,\theta} \propto \partial q_1^\theta / \partial N_1^{i,\theta}$. As a result, when constraint externalities are corrected by the planner, the relative magnitude of $\partial q_1^\theta / \partial N_1^{b,\theta}$ and $\partial q_1^\theta / \partial N_1^{l,\theta}$ determines the relative magnitude of $\tau_x^{b,\theta}$ and $\tau_x^{l,\theta}$.

[4]This explanation highlights that in principle, in the case of the lender one could alternatively call the over-borrowing force an 'under-saving' effect.

[5]Recall that in the formal welfare analysis we focus on pecuniary externalities that operate through changes in net worth, and do not characterize over- or under-investment effects. In the graphical depiction, we therefore abstract from any difference in investment in $t = 0$ that may occur between the decentralized equilibrium and the constrained efficient allocation that the planner implements. In the numerical application of the model in Section 4.3, we also allow for over- und under-investment.

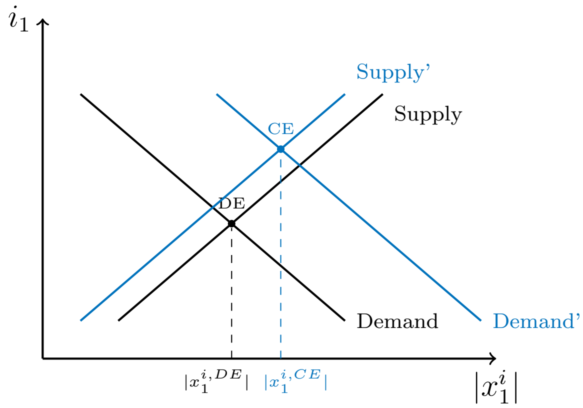
capital loosens the collateral constraint, moving the intersection of the vertical line with the credit supply curve in Panel (d) to the right relative to the decentralized equilibrium. The planner internalizes the effect of period-0 borrowing decisions on future prices, and in turn on future borrowing space. The over-borrowing force in $t = 0$ is corrected through a tax wedge so that borrowers can obtain more credit between period 1 and 2 in the constrained efficient economy.

Appendix D.2. Intuition for Proposition 2

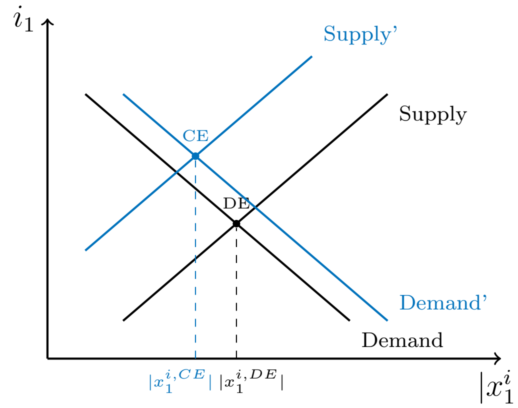
Proposition 2 delivers one of our main theoretical insights. An earnings-based borrowing constraint implies that the borrower takes a debt position that is too small relative to the social optimum. The mechanics of the model are similar to our explanation of Proposition 1, but operate through the real wage rate rather than the price of capital. A larger debt position in $t = 0$ reduces net worth in the borrowing sector in $t = 1$, which in turn reduces wages due to condition (42) (recall the discussion around labor demand and labor supply). Borrowers in $t = 0$ do not internalize that lower wages increase earnings and provide slack in the borrowing limit in $t = 1$. Therefore, in the market economy, agents under-borrow. The social planner internalizes the positive effect of borrowing in $t = 0$ on debt capacity in $t = 1$ through wages, and subsidizes (lowers the tax on) borrowing in period $t = 0$ (for a given level of distributive externalities).

Graphical representation. Figure Appendix D.2 presents a graphical analysis for the case of the earnings-based borrowing constraint. As in Figure Appendix D.1, points CE and DE represent the constrained efficient allocation and the decentralized equilibrium. The figure conveys how externalities emerge from borrowing decisions in $t = 0$, which through wage determination in the labor market affect credit constraints in $t = 1$. Relative to the case of the collateral constraint, Panel (c) now depicts the labor market in $t = 1$ rather than the market for physical capital. The earnings-based constraint (45) is represented by a vertical line in Panel (d), putting a cap $\phi_\pi \pi(w_1^\theta) = \phi_\pi(F^b(k_1^b, \ell_{d1}^{b,\theta}) - w_1^\theta \ell_{d1}^{b,\theta})$ on the amount of credit. Its location is affected by the market clearing wage. Similar to the collateral constraint and Figure Appendix D.1, there is a refinement of condition (42) on the response of wages to changes in net worth. In both cases, according to Proposition 2, the decentralized equilibrium features under-borrowing and the social planner subsidizes borrowing (taxes saving) in $t = 0$. In period $t = 0$ agents do not internalize that by reducing net worth in period 1 wages are reduced and this relaxes future borrowing constraints. To lower wages and thus create space for the constrained optimal amount of period-1 credit, the planner induces more debt in period 0 through corrective tax wedges.

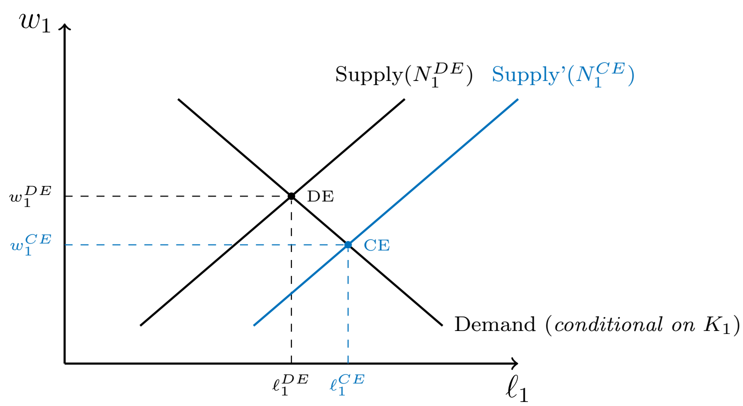
Figure Appendix D.2: MARKET VS. PLANNER ALLOCATIONS: EARNINGS-BASED BORROWING CONSTRAINT



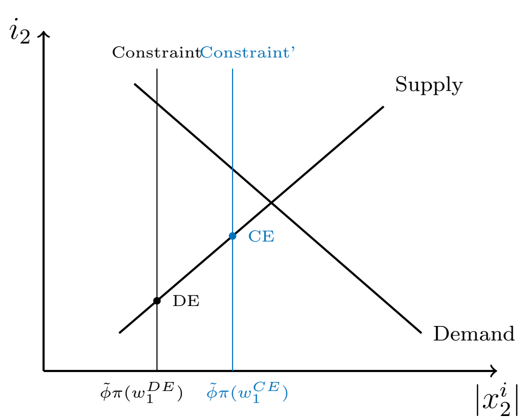
(a) Period-0 credit market (case 1: $|\tau_x^b| > |\tau_x^l|$)



(b) Period-0 credit market (case 2: $|\tau_x^b| < |\tau_x^l|$)



(c) Period-1 labor market (both cases)



(d) Period-1 credit market (both cases)

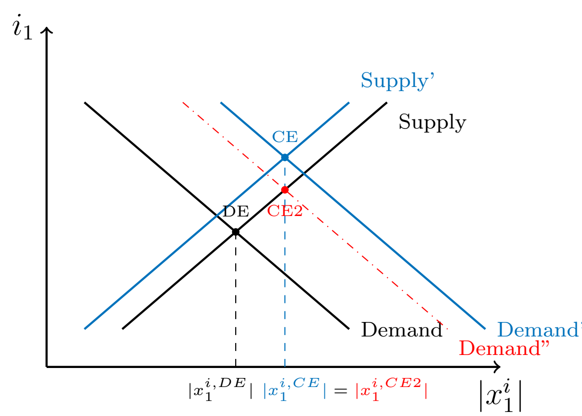
Notes. Decentralized equilibrium (DE) and constrained efficient equilibrium (CE) in the period-0 credit market, period-1 labor market and period-1 credit market of the model. State θ is omitted from the notation in the labeling. The figure distinguishes case 1 ($\partial w_1^\theta/\partial N_1^{b,\theta} > \partial w_1^\theta/\partial N_1^{l,\theta} \Leftrightarrow |\tau_x^{b,\theta}| > |\tau_x^{l,\theta}|$) and case 2 ($\partial w_1^\theta/\partial N_1^{b,\theta} < \partial w_1^\theta/\partial N_1^{l,\theta} \Leftrightarrow |\tau_x^{b,\theta}| < |\tau_x^{l,\theta}|$) as described in the text. In both cases, the social planner internalizes that period-0 borrowing decisions reduce equilibrium wages in period 1, which relaxes the earnings-based borrowing constraint. The constrained efficient allocation features lower wages and more credit in period 1, as less saving (more borrowing) is incentivized through taxes/subsidies in period 0.

The graphical representation of the economy with earnings-based borrowing constraint highlights the new insights that come with signing pecuniary externalities in our model with labor markets. The condition that wages increase with sector wide net worth in $t = 1$ requires understanding the response of labor demand as well as labor supply. Given that the capital available for production (K_1) is predetermined at the beginning of the period, labor demand is already pinned down, while labor supply responds to changes in sector-wide net worth (see Panel (c) of Figure Appendix D.2). This is different in the market of capital relevant for the collateral constraint case, where the supply of capital is fixed, but the demand for new capital (K_2) increases with net worth (compare Panel (c) of Figure Appendix D.1). In the presence of earnings-based constraints the planner can therefore induce more borrowing in the initial period, and thereby reduce borrower net worth in $t = 1$ to increase labor supply. This leads wages to fall.

Take-aways from graphical analysis of both constraints. In conclusion to the graphical analysis, the differences between Figures Appendix D.1 and Appendix D.2 reveal the sharp contrast between the normative consequences of the earnings-based and the collateral constraint. In the earnings-based constraint an *input price* (through the wage bill) enters with the opposite sign to how an *asset price* (the value of capital) enters the collateral constraint. Since wages and the price of capital respond with the same sign to changes in borrower net worth, all else equal, the implications in terms of whether agents borrow to much or too little in period $t = 0$ from a normative standpoint are the opposite for the two constraint types.

Alternative implementations of constrained efficiency. The set of tax rates τ_x^i, $i \in \{b, l\}$ that implements the constrained efficient equilibrium is not unique. There is an infinite number of combination of τ_x^b and τ_x^l that will alter $N_1^{b,\theta}$ and $N_1^{l,\theta}$ such that the same changes in period-1 prices and credit access are achieved. For the case of the earnings-based borrowing constraint we illustrate this in Figure Appendix D.3, which is constructed as Panel (a) of Figure Appendix D.2 but also plots an alternative implementation of the constrained efficient equilibrium (denoted $CE2$). This equilibrium represents the polar case in which only the borrower's financial asset position is taxed (borrowing is subsidized), while the lender is not taxed, $\tau_x^l = 0$. As the graph conveys, there is a choice for τ_x^b that achieves the identical equilibrium credit amount as point CE. As a result, the labor and credit market outcomes in period 1 would be the same as in Figure Appendix D.2. A similar argument can be made for case 2 in Figure Appendix D.2 and for both cases of the collateral constraint analyzed in Figure Appendix D.1.

Figure Appendix D.3: NON-UNIQUENESS OF IMPLEMENTATION



Period 0 Credit Market ($|\tau_x^b| > |\tau_x^l|$)

Notes. This figure repeats Panel (a) of Figure Appendix D.2 but also plots an alternative implementation of the constrained efficient equilibrium (denoted $CE2$). Constrained efficiency can be achieved with different sets of tax rates $\tau_x^{i,\theta}$, $i \in \{b, l\}$, which give rise to the same change in aggregate net worth (and resulting wage reduction) in the constrained efficient relative to the decentralized equilibrium. In this case, only the borrowers' savings decisions are taxes (borrowing is subsidized), while $\tau_x^{l,\theta} = 0$. State θ is omitted from the notation in the labeling of the graph.

Appendix E. Robustness of numerical model experiments

To explore robustness of our model parameterization, we construct variations of Tables 2 and 3 from the main text in which we change the values of key parameters and then report the resulting optimal tax rates and welfare losses. We focus on the capital share α and the labor supply elasticity ψ. These parameters are of particular interest, since the sufficient condition we can derive for our main assumption to hold (see Section 2, case (ii) of the main text) depends on these two parameters. For each parameter, we solve the model for a 20% larger and a 20% smaller value relative to the baseline calibration, which sets $\alpha = 0.33$ and $\psi = 2$. In the case of α we can do this for the model version with inelastic labor supply as well as the one with endogenous labor supply. The variation of ψ only applies in the model version where labor supply is chosen by the agents.

Table Appendix E.1 reports the resulting optimal tax rates. The table is constructed in the same way as Table 2 in the main text, but each panel corresponds to a different parameter variation. The important take-away from this table is that our main assumption holds also for variations in the parameter values. In particular, the signs of $\tau_x^{b,c.e.}$ $\tau_x^{l,c.e.}$ are the same as in the analysis in the main text, indicating that our assumptions on the derivatives of the price of capital and wage with respect to changes in net worth are also satisfied for a higher and lower capital share and labor supply elasticity.

Table Appendix E.1: Optimal corrective taxes in different economies (in %)

Economy ($\alpha = 0.33 \times 1.2$)	τ_x^b	τ_x^l	τ_k^b	τ_k^l	$\tau_x^{b,c.e.}$	$\tau_x^{l,c.e.}$
Collateral constraints, inelastic labor	-21.6	4.0	-33.3	-30.6	-0.7	-0.3
Earnings-based constraints, inelastic labor	-9.7	-2.2	-31.9	-10.5	0.0	0.0
Collateral constraints, endogenous labor	-2.1	-3.5	-1.5	0.8	-2.3	-3.4
Earnings-based constraints, endogenous labor	0.2	0.3	-3.2	-5.9	0.9	0.3
Economy ($\alpha = 0.33 \times 0.8$)						
Collateral constraints, inelastic labor	-17.6	3.5	-22.1	-23.6	-0.0	-0.0
Earnings-based constraints, inelastic labor	-5.7	-0.6	-20.1	-12.5	0.0	0.0
Collateral constraints, endogenous labor	-1.3	-3.2	-0.4	0.7	-1.5	-3.1
Earnings-based constraints, endogenous labor	0.4	0.4	-1.5	-8.0	0.9	0.3
Economy ($\psi = 2 \times 1.2$)						
Collateral constraints, endogenous labor	-1.9	-3.4	-1.4	0.6	-2.1	-3.3
Earnings-based constraints, endogenous labor	0.1	0.3	-2.9	-7.1	0.8	0.3
Economy ($\psi = 2 \times 0.8$)						
Collateral constraints, endogenous labor	-1.4	-3.3	-0.6	0.6	-1.7	-3.2
Earnings-based constraints, endogenous labor	0.5	0.5	-2.1	-7.1	1.0	0.3

Table Appendix E.2 presents the results of our experiment of rolling out the wrong policy. It reveals that we find significant welfare losses across the parameter variations we introduce. A higher capital share makes the welfare even larger than in the main text, reaching up to over 3% in consumption equivalents for the model with inelastic labor supply. When the capital share is decreased, the welfare losses are smaller but still substantial with more than 1% welfare loss. For the labor supply elasticity, it is visible that a lower parameter value increases the strength of the negative welfare consequences. With a higher labor supply elasticity, the effect is still strong, again around 1% in consumption equivalents, so not very different for the effect in the main text when labor supply is endogenous. Finally, As in the experiment in main text, the welfare losses coming from the constraint externality by itself are smaller. This highlights again that distributive externalities are important in the general model.

Table Appendix E.2: Consumption equivalent welfare change in different counterfactuals

Panel (a): all types of externalities

Economy ($\alpha = 0.33 \times 1.2$)	**Right policy, $\boldsymbol{\lambda}$(%)**	**Wrong policy, $\boldsymbol{\lambda}$(%)**	**$\boldsymbol{\Delta}$(%)**
Earnings-based constraints, inelastic labor	0.89	-2.28	3.16
Earnings-based constraints, endogenous labor	0.60	-0.54	1.14
Economy ($\alpha = 0.33 \times 0.8$)			
Earnings-based constraints, inelastic labor	0.39	-0.97	1.36
Earnings-based constraints, endogenous labor	0.61	-0.51	1.12
Economy ($\psi = 2 \times 1.2$)			
Earnings-based constraints, endogenous labor	0.49	-0.50	0.99
Economy ($\psi = 2 \times 0.8$)			
Earnings-based constraints, endogenous labor	0.77	-0.55	1.32

Panel (b): constraint externalities only

Economy ($\alpha = 0.33 \times 1.2$)	**Right policy, $\boldsymbol{\lambda}$(%)**	**Wrong policy, $\boldsymbol{\lambda}$(%)**	**$\boldsymbol{\Delta}$(%)**
Earnings-based constraints, inelastic labor	0.00	-0.04	0.04
Earnings-based constraints, endogenous labor	0.06	-0.50	0.56
Economy ($\alpha = 0.33 \times 0.8$)			
Earnings-based constraints, inelastic labor	0.00	-0.00	0.00
Earnings-based constraints, endogenous labor	0.05	-0.45	0.51
Economy ($\psi = 2 \times 1.2$)			
Earnings-based constraints, endogenous labor	0.04	-0.45	0.50
Economy ($\psi = 2 \times 0.8$)			
Earnings-based constraints, endogenous labor	0.08	-0.50	0.58

Notes. The table shows the welfare impact of policies carried out in the 'true' economy, which features earnings-based constraints. The right policy is the solution to the social planner's problem in that economy. It moves the allocation in the decentralized equilibrium to the constrained efficient allocation. The wrong policy is calculated under the incorrect assumption that agents face asset-based borrowing constraints. It moves the allocation in the decentralized equilibrium to allocation that arises from the wrong policy.