EARNINGS-BASED BORROWING CONSTRAINTS
AND PECUNIARY EXTERNALITIES

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Advances in Macro Finance
Tepper-LAEF Conference
April 8, 2022
US firms face two types of credit constraints: asset-based and earnings-based

- Liquidation value of physical assets vs. borrower’s current EBITDA limit debt access
- Direct micro evidence: 80% of corporate debt is earnings-based (Lian and Ma, 2020)
- Consequences for business cycle dynamics (Drechsel, 2020)

Limited understanding of normative implications of earnings-based constraints

- This paper provides a theoretical treatment
- Structural model with formal welfare characterization
- Implications for optimal macroprudential policy
Asset-based constraint: firms over-borrow in decentralized equilibrium

- Echoes existing insights of the literature, e.g. Dávila and Korinek (2018)
- Higher asset price relaxes constraint → not internalized

Earnings-based constraint: firms under-borrow in decentralized equilibrium

- Higher input price (wage) tightens constraint → not internalized
- Depending on labor market structure, can also lead to constrained efficiency
RELATD LITERATURE

▶ Pecuniary externalities with financial frictions:

  ▶ Subtleties in the policy implications of different types of credit constraints

▶ Insights on the specific nature of credit constraints:

  ▶ Normative implications of asset-based and earnings-based constraints
PLAN FOR THE TALK

1. Main intuition

2. Empirical evidence

3. The model
   A. Setting
   B. Efficiency analysis
   C. Model restrictions and main results

4. Extensions
   - Working capital, open economy, input vs. output prices

5. Conclusion
MAIN INTUITION
Consider a generic financial constraint:

\[ \Phi(x', z, \tilde{z}) \geq 0 \]

- \( x' \): financial asset position (\( x' < 0 \): borrowing)
- \( z \): endogenous variables chosen by the agent
- \( \tilde{z} \): endogenous or exogenous variables taken as given by the agent (e.g., prices)

Agents’ choices move prices in \( \tilde{z} \) → pecuniary externality

The direction of price changes matters for normative implications
MAIN INTUITION: ASSET-BASED CONSTRAINT

- Asset-based collateral constraint:
  - $z = k', \tilde{z} = q$, and $\Phi(x', z, \tilde{z}) = x' + \phi q k' \geq 0 \Rightarrow -x' \leq \phi q k'$
  - $q = q(X, K)$: market price of capital as a function of the aggregate state variables
  - Aggregate states are net worth and capital

- Suppose $q$ depends positively on net worth
  - If more borrowing today:
    - $\Rightarrow$ Future aggregate borrower net worth ↓
    - $\Rightarrow$ Future price of capital ↓ – through lower demand for capital
    - $\Rightarrow$ Tightening of future borrowing limit

- Agents do not internalize this effect, **over-borrow** relative to the social optimum
Earnings-based constraint:
\[ z = [y, \ell], \tilde{z} = w, \text{ and } \Phi(x', z, \tilde{z}) = x' + \tilde{\phi}(y - w\ell) \geq 0 \Rightarrow -x' \leq \tilde{\phi}(y - w\ell) \]

\[ w = w(X, K): \text{ market wage as a function of the aggregate state variables} \]

Suppose \( w \) increases with net worth

If more borrowing today:
\[ \Rightarrow \text{Future aggregate borrower net worth } \downarrow \]
\[ \Rightarrow \text{Future wage } \downarrow - \text{through lower supply of labor} \]
\[ \Rightarrow \text{Loosening of future borrowing limit} \]

Agents do not internalize this effect, under-borrow relative to the social optimum
EMPIRICAL EVIDENCE
EMPIRICAL EVIDENCE

- Mounting microeconomic evidence in favor of $-x' \leq \tilde{\phi}(y - w\ell)$

- Earnings-based borrowing constraints can arise through:
  - Debt covenants: legal provisions in loan contracts
  - Credit ratings, bankruptcy procedures, …

- Lian and Ma (2020): 80% of corporate debt earnings-based

- Drechsel (2020): earnings-based constraints matter for business cycle dynamics

- Caglio, Darst, and Kalemli-Özcan (2021) shows that earnings-based are prevalent for private small and medium-sized companies (SMEs)
EMPIRICAL EVIDENCE (DRECHSEL, 2020)

<table>
<thead>
<tr>
<th>Covenant type</th>
<th>Median</th>
<th>Mean</th>
<th>Freq. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Max Debt to <strong>EBITDA</strong></td>
<td>3.75</td>
<td>4.60</td>
<td>60.5</td>
</tr>
<tr>
<td>2 Min <strong>EBITDA</strong> to Interest</td>
<td>2.50</td>
<td>2.56</td>
<td>46.7</td>
</tr>
<tr>
<td>3 Min <strong>EBITDA</strong> to Fixed Charge</td>
<td>1.25</td>
<td>1.42</td>
<td>22.1</td>
</tr>
<tr>
<td>4 Max. Leverage ratio</td>
<td>0.60</td>
<td>0.64</td>
<td>21.3</td>
</tr>
<tr>
<td>5 Max. Capex</td>
<td>20M</td>
<td>194M</td>
<td>15.1</td>
</tr>
<tr>
<td>6 Net Worth</td>
<td>126M</td>
<td>3.2B</td>
<td>11.5</td>
</tr>
</tbody>
</table>

**EBITDA** is *earnings before interest, taxes, depreciation and amortization*

- ▶ Covenants based on earnings very prevalent
- ▶ Covenants bind frequently with large economic effects
  (see e.g. Chodorow-Reich and Falato, 2021)
BUSINESS CYCLE CONSEQUENCES (DRECHSEL, 2020)

Model IRFs of debt to investment shock

SVAR IRF of debt to investment shock

Aggregate debt response consistent with earnings-based constraint, not with collateral constraint
BUSINESS CYCLE CONSEQUENCES (DRECHSEL, 2020)

Split of debt response across borrower types consistent with model prediction across alternative constraints
THE MODEL
SETTING

- Build on structure Dávila and Korinek (2018) + labor market
- Three period model \( t = 0, 1, 2 \)
- The state of nature, \( \theta \in \Omega \), is revealed at date 1
- Two types of agents: borrowers \( b \) and lenders \( l \)
- Both agents produce, consume and supply labor
- Borrowers face credit constraints
AGENTS’ PROBLEM

Agent $i \in \{b, l\}$ maximizes

$$U^i = \mathbb{E}_0 \left[ \sum_{t=0}^{2} \beta^t u^i(c^i_t, l^i_{st}) \right]$$

subject to budget constraints

$$c^i_0 + h^i(k^i_1) + \int_{\theta \in \Theta} m^\theta x^i_1 d\theta = e^i_0$$

$$c^i_1 + q^\theta \Delta k^i_2 + m^\theta x^i_2 = e^i_1 + x^i_1 + F^i(k^i_1, l^i_{d1}) - w^\theta l^i_{d1} + w^\theta l^i_{s1}, \quad \forall \theta$$

$$c^i_2 = e^i_2 + x^i_2 + F^i(k^i_2, l^i_{d2}) - w^\theta l^i_{d2} + w^\theta l^i_{s2}, \quad \forall \theta$$

and financial constraints

$$\Phi^b_1(x^b_1, k^b_1) \geq 0$$

$$\Phi^b_2(x^b_2, k^b_1, k^b_2, \{l^b_{dt}, l^b_{st}\}_{i=1}^2; q^\theta, w^\theta_1, w^\theta_2, m^\theta_2) \geq 0, \quad \forall \theta$$
Main constraint of interest: period-1 financial constraint

\[ \Phi_{2,b,\theta}(x_{2,b,\theta}, k_{1,b}, k_{2,b,\theta}, \{\ell_{dt}^{b,\theta}, \ell_{st}^{b,\theta}\}_{t=1}^2; q^{\theta}, w_1^{\theta}, w_2^{\theta}, m_2^{\theta}) \geq 0, \forall \theta \]

General formulation in which all model variables can enter

Includes:

- Asset-based constraint: \[ -x_{2,b,\theta} \leq \phi q^{\theta} k_{2,b,\theta} \]
- Earnings-based constraint: \[ -x_{2,b,\theta} \leq \tilde{\phi} (F^{b}(k_{1,b}, \ell_{d1}^{b,\theta}) - w_1^{\theta} \ell_{d1}^{b,\theta}) \]
- Interest coverage constraint: \[ -x_{2,b,\theta} \leq \hat{\phi} F^{b}(k_{2,b,\theta}, \ell_{d2}^{b,\theta} - w_2^{\theta} \ell_{d2}^{b,\theta}) \]
SOLVING THE MODEL

- Decentralized equilibrium (backward induction)
  - Date 2: purely intra-temporal consumption, labor supply and demand
  - Date 1: express welfare as a function of state variables

\[
V^{i,\theta}(n_1^{i,\theta}, k_1^{i}; N_1^{\theta}, K_1) = \max_{\{c_1^{i,\theta}, c_2^{i,\theta}, k_2^{i,\theta}, x_2^{i,\theta}, \ell_1^{i,\theta}, \ell_2^{i,\theta}\}} \left\{ U_i(c_1^{i,\theta}, \ell_1^{i,\theta}) + \beta U_i(c_2^{i,\theta}, \ell_2^{i,\theta}) \right\}
\]

s.t. period 1 and 2 budget constraint and period 1 financial constraint

- net worth: \( n_1^{i,\theta} \equiv c_1^{i,\theta} + x_1^{i,\theta} \)
- Prices are functions of only aggregate states
- In equilibrium, \( n_1^{i,\theta} = N_1^{i,\theta}, k_1^{i} = K_1^{i} \)

The effect of changes in $N_{j}^{i,\theta}$ on $V_{i,\theta}^{i,\theta}$:

$$V_{i,\theta}^{i,\theta} \equiv \frac{dV_{i,\theta}^{i,\theta}(\cdot)}{dN_{1}^{i,\theta}} = \lambda_{1}^{i,\theta} D_{1N_{j}}^{i,\theta} + \lambda_{2}^{i,\theta} D_{2N_{j}}^{i,\theta} + \kappa_{2}^{i,\theta} C_{N_{j}}^{i,\theta}$$

Welfare changes that are not internalized by the agents, work through prices.

Distinguish between distributive effects ($D$) and constraint effects ($C$).
SUFFICIENT STATISTICS

\[ V_{N_1}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dN_1^{i,\theta}} = \lambda_1^{i,\theta} D_1^{i,\theta} + \lambda_2^{i,\theta} D_2^{i,\theta} + \kappa_2^{i,\theta} C_{N_1}^{i,\theta} \]

- **Distributive effects:**
  - Changes in prices that benefit one agent, make other agent worse off
  - Not our focus
**SUFFICIENT STATISTICS**

\[ V_{N_1}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dN_{1,j}^{i,\theta}} = \lambda_1^{i,\theta} D_{1N_j}^{i,\theta} + \lambda_2^{i,\theta} D_{2N_j}^{i,\theta} + \kappa_2^{i,\theta} C_{N_j}^{i,\theta} \]

- **Constraint effects:** changes in prices that affect tightness of credit constraints

\[
C_{N_j}^{b,\theta} \equiv \frac{\partial \Phi_2^{b,\theta}}{\partial q^{\theta}} \frac{\partial q^{\theta}}{\partial N_{1,j}^{i,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial m_2^{\theta}} \frac{\partial m_2^{\theta}}{\partial N_{1,j}^{i,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_1^{\theta}} \frac{\partial w_1^{\theta}}{\partial N_{1,j}^{i,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_2^{\theta}} \frac{\partial w_2^{\theta}}{\partial N_{1,j}^{i,\theta}}
\]

\[
C_{N_j}^{l,\theta} = 0
\]

\( (\kappa_2^{i,\theta} \) is Lagrange multiplier on the financial constraint)
In the same vein, can study effects coming from \( \frac{dV^{i,\theta}(\cdot)}{dK^{j,\theta}_1} \).

We focus on over-/under-borrowing rather than over-/under-investing.

Bound by “anything goes” result of Dávila and Korinek (2018).
EFFICIENCY ANALYSIS
CONSTRANGED EFFICIENT ALLOCATION

- Planner internalizes distributive and constraint effects of borrowing decision
- Chooses allocations in $t = 0$ subject to:
  1. The same $t = 0$ constraints as the private agents
  2. The optimal behavior of private agents in periods $t = 1, 2$
- Corresponds to problem of constrained Ramsey planner who can levy $t = 0$ taxes
**Proposition:** A decentralized equilibrium with the following corrective taxes replicates the constrained efficient allocation

\[ \tau_{i,\theta} = -\Delta MR_S^{i,j,\theta} D_{1Ni}^{i,\theta} - \Delta MR_S^{i,j,\theta} D_{2Ni}^{i,\theta} - \tilde{\kappa}^{b,\theta} C_{N_i}^{b,\theta}, \forall i, \theta \]

- \( \tau_{i,\theta} > 0 \): taxes on saving \( \Rightarrow \) under-borrowing in decentralized equilibrium
- \( \Delta MR_S^{i,j,\theta} \equiv MR_S^{i,\theta} - MR_S^{j,\theta} \)
- \( \tilde{\kappa}^{b,\theta} \): shadow price on credit constraint
HOW TO PROCEED WITH EFFICIENCY ANALYSIS

- For specific financial constraints $\Phi_{2}^{b,\theta}$, find $C_{N_{i}}^{b,\theta}$

- Given sign of $C_{N_{i}}^{b,\theta}$, determine sign of $\tau_{x}^{i,\theta}$
  - If $\tau_{x}^{i,\theta} < 0$: planner corrects ‘over-borrowing’
  - If $\tau_{x}^{i,\theta} > 0$: planner corrects ‘under-borrowing’

- To pin down signs, need to specialize model further
ADDITIONAL MODEL RESTRICTIONS AND MAIN RESULTS
Condition for collateral constraints:

$$\frac{\partial q^\theta}{\partial N_{i,\theta}^1} \geq 0, \forall i$$

Explanation:

- Supply of capital is predetermined
- An increase in net worth raises the demand for capital
  ⇒ upward pressure on capital price
**ADDITIONAL MODEL RESTRICTIONS**

- **Condition for earnings-based constraints:**

\[
\frac{\partial w_i^\theta}{\partial N_i^{1,\theta}} \geq 0, \ \forall i
\]

- **Argument:**
  - Demand for labor is pinned down conditional on capital
  - Higher net worth increases consumption ⇒ (under standard preference) demand for leisure ↑, so decrease in labor supply
    ⇒ upward pressure on wage
**ADDITIONAL MODEL RESTRICTIONS**

- **Condition for interest coverage constraints:**

\[
\frac{\partial m_{2}^{\theta}}{\partial N_{1}^{i,\theta}} \geq 0, \; \forall i
\]

\[
\frac{\partial w_{2}^{\theta}}{\partial N_{1}^{i,\theta}} \geq 0, \; \forall i
\]

- **Argument:**
  - Higher net worth increases incentive to save more to smooth consumption
  - \( \Rightarrow \) Price of debt (= inverse of interest rate) increase (tends to move in the same way with the price of capital due to no-arbitrage restriction)
  - Direct analogy to the argument for the period 1 wage
MAIN RESULTS

- **Collateral constraint:**

\[
\Phi_{2}^{b,\theta}(\cdot) = x_{2}^{b,\theta} + \phi_{q}^{\theta} k_{2}^{b,\theta} \geq 0
\]

- **Proposition:** There is an **over-borrowing** effect through constraint externalities

- **Proof:**

\[
- \tilde{\kappa}_{2}^{b,\theta} c_{N_{i}}^{b,\theta} = - \tilde{\kappa}_{2}^{b,\theta} \frac{\partial \Phi_{2}^{b,\theta}}{\partial q^\theta} \frac{\partial q^\theta}{\partial N_{1,i}^\theta} \leq 0 \Rightarrow \text{subsidize saving (}= \text{penalize borrowing})
\]
MAIN RESULTS

▶ Earnings-based constraint:

\[ \Phi_{2}^{b,\theta}(\cdot) = x_{2}^{b,\theta} + \tilde{\phi}(F^{b}(k_{1}^{b}, \ell_{d1}^{b,\theta}) - w_{1}^{\theta} \ell_{d1}^{b,\theta}) \geq 0 \]

▶ Proposition: There is an under-borrowing effect through constraint externalities

▶ Proof:

\[ -\tilde{\kappa}_{2}^{b,\theta} C_{N_{i}}^{b,\theta} = -\tilde{\kappa}_{2}^{b,\theta} \frac{\partial \Phi_{2}^{b,\theta}}{\partial w_{1}^{\theta}} \frac{\partial w_{1}^{\theta}}{\partial N_{i}^{\theta}} \geq 0 \Rightarrow \text{penalize saving (\(=\) subsidize borrowing)} \]

▶ Note: if labor supply inelastic, \(\partial w/\partial N\) term drops out \(\Rightarrow\) constrained efficiency
Interest coverage constraint:

\[
\Phi^{b,\theta}_2(\cdot) = x^{b,\theta}_2 + \phi \frac{F^b(k^{b,\theta}_2, l^{b,\theta}_2) - w^{\theta}_2 l^{b,\theta}_2}{i^{\theta}_2} \geq 0
\]

Proposition: There is an ambiguous effect through constraint externalities

Proof:

\[
-\tilde{\kappa}^{b,\theta}_2 C^{b,\theta}_N = -\tilde{\kappa}^{b,\theta}_2 \left( \frac{\partial \Phi^{b,\theta}_2}{\partial w^{\theta}_2} \frac{\partial w^{\theta}_2}{\partial N^{i,\theta}_1} + \frac{\partial \Phi^{b,\theta}_2}{\partial i^{\theta}_2} \frac{\partial i^{\theta}_2}{\partial N^{i,\theta}_1} \right) \ll 0
\]

This constraint is “mixture” of earnings-based and asset-based constraint

Why? 1/i co-moves with q through no-arbitrage condition
SUMMARY OF FINDINGS

- **Asset-based constraint:** agents *over-borrow* in decentralized equilibrium
  - Echoes existing insights of the literature, e.g. Dávila and Korinek (2018)
  - Higher asset price relaxes constraint $\rightarrow$ not internalized

- **Earnings-based constraint:** agents *under-borrow* in decentralized equilibrium
  - Higher input price (wage) tightens constraint $\rightarrow$ not internalized

- **Interest coverage constraint:** ‘mixture’ of earnings- and asset-based constraint
EXTENSIONS
Several authors propose models with working capital and collateral constraints

- See e.g. Bianchi and Mendoza (2010), Jermann and Quadrini (2012), Bianchi (2016)

Suppose wage bill financed with an intraperiod loan $x_{wc} = -\psi w\ell$

\[-(x' - \psi w\ell) \leq \tilde{\phi}(F(k, \ell) - w\ell) \Rightarrow -x' \leq -(\tilde{\phi}F(k, \ell) - (\tilde{\phi} + \psi)w\ell)\]

- $\tilde{\phi} + \psi > \tilde{\phi}$: more pronounced under-borrowing effect
Several papers on welfare consequences of borrowing constraints in small open economies (see e.g. Bianchi, 2011)

We focus on an endogenous interest rate because the background on earnings-based constraints is largely provided for the U.S.

Microeconomic evidence on the specific forms of constraints is thinner for emerging economies, but would be very welcome

Note that a fixed interest rates would make interest coverage constraint inherit the consequences of the earnings-based constraint (no ‘mixture’ result)
In our real model, \( w \) denotes relative price.

But what if final goods price is not equal to 1?

Need multi-good environment to think about meaningful output price variation.

1. Monopolistically competitive firms environment
   - Prices are choice variables, so firms internalize how price affects the constraint
   - However, firms would not internalize how their individual choices affect aggregate inflation, which could affect nominal debt limits

2. Perfectly competitive firms environment
   - Effects on relative prices between different goods not internalized? (Fazio (2021))
CONCLUSION
CONCLUSION

- Whether debt is backed by collateral or linked to firms’ earnings has sharply different implications for macroprudential policy.

- The pecuniary externality through wages in earnings-based constraints prescribes that a regulatory authority should, if anything, encourage firms to borrow.

- Our analysis highlights the importance of a proper understanding of the microeconomic details behind which constraints matter in which markets.
  - Asset-based borrowing: mortgage markets, repo markets, ...
  - Earnings-based borrowing: corporate credit markets.
REFERENCES


FORMAL SOCIAL PLANNER PROBLEM

$$\max \sum_i \alpha^i \left\{ u^i(C^i_0) + \beta \mathbb{E}_0 [V^{i,\theta}(N^i_1, K^i_1; N^\theta, K_1)] \right\}$$

subject to \( t = 0 \) resource and credit constraints

$$\sum_i [C^i_0 + h^i(K^i_1) - e^i_0] \leq 0$$

$$\sum_i X^i,\theta = 0, \ \forall \theta$$

$$\Phi_1(X^i_1, K^i_1) \geq 0, \ \forall i$$