

# ECON 747 - Assignment 1

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- Work in groups of 3-4 people
- Please hand in by Friday 19 Feb 2021 via email to drechsel@umd.edu
- Solutions (including model output) should be presented in one single pdf file, with the corresponding Matlab/Dynare codes in one single zip file per group (except for Question 5 which is to be submitted individually)

## Question 1

Consider the business cycle model discussed in Lectures 2 and 3.

(a) Derive the policy rules for consumption, capital, output and investment for the case  $\sigma = \delta = 1$  using the ‘guess and verify’ approach.

(b) Look at the policy rules from part (a) and think of this model as a data-generating process. What do these policies imply for business cycle fluctuations and comovement, when:

- i Only TFP shocks hit the economy
- ii Only IST shocks hit the economy
- iii Both shocks hit the economy

(no derivations or programs are required for this question)

## Question 2

For the business cycle model and calibration solved on the computer in class, we obtained the following policy rules using Dynare:

## POLICY AND TRANSITION FUNCTIONS

	c	k	z	v	y	i
Constant	1.972191	21.436072	1.000000	1.000000	2.508093	0.535902
k(-1)	0.032524	0.977577	0	0	0.035101	0.002577
z(-1)	0.424665	1.832618	0.900000	0	2.257283	1.832618
v(-1)	-0.462397	0.944708	0	0.900000	0	0.944708
ez	0.471850	2.036243	1.000000	0	2.508093	2.036243
ev	-0.513774	1.049676	0	1.000000	0	1.049676

(a) In Matlab (you don't need Dynare), simulate a data sample of size  $T = 100$  for output, consumption, investment and the capital stock using the policy rules, assuming that

- i Both TFP and IST shocks hit the economy
- ii Only TFP shocks hit the economy
- iii Only IST shocks hit the economy

*Hint: you need to first simulate the  $\varepsilon_z$  and  $\varepsilon_v$  time series using the 'randn' command in Matlab and then calculate the resulting time series for  $Z$  and  $V$ . You then initialize the variables at their steady state level and use a loop over policy rules. Be careful about the timing of the state variables!*

(b) Present the simulated data using the 'figure' and 'plot' functions in Matlab (you may also find the 'subplot' function helpful). You should plot four time series for three different economies.

(c) What is the correlation between consumption and output in the simulated data for the case i, ii and iii? Given what you know about business cycle correlations in real-world data, which type of shock makes more sense as a source of business cycles?

### Question 3

Consider the following business cycle model with a stochastic endowment income  $W_t$ :

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

subject to

$$C_t = W_t$$

where

$$W_t = (1 - \phi)\bar{W} + \phi W_{t-1} + \eta_t$$
$$\eta_t \sim N(0, \sigma_\eta^2)$$

and  $0 < \beta \leq 1$ ,  $\sigma \geq 0$ ,  $\bar{W} > 0$ ,  $0 < \phi < 1$ . The economy is closed, so that the resource constraint always holds.

(a) Which equations characterize the solution of this model? What can this model say about business cycle (consumption) fluctuations?

(b) Suppose the household has access to a bond in which she can save/borrow. The price of this bond in period  $t$  is  $P_{B,t}$ : saving  $P_{B,t}B_t$  units in period  $t$  pays  $B_t$  unit of the consumption good in period  $t + 1$ . Ponzi schemes are ruled out.

i Introducing the bond gives you two extra variables, the price  $P_B$  and the quantity of bonds bought by the household,  $B$ . State the three extra conditions you need to characterize the solution of the model (one of them is a transversality condition).

ii Look at these extra conditions: what type of fluctuations does this model imply for the asset and its price when  $\sigma = 1$ ?

iii How do these fluctuations affect fluctuations in consumption? Do financial variables “matter” in this economy?

iv How is the interest rate related to the bond price? What can you say about interest rate fluctuations in this economy?

(c) Collect all equations that characterize the solution of the model. Calculate the steady state of the model.

(d) Choose some sensible parameter values and solve for the policy function of this model in Dynare. Generate impulse response functions and check whether these confirm your intuition for the answers above.

## Question 4

Let's consider once more the business cycle model discussed in class, but we now include endogenous labour supply. That is, we add an endogenous variable  $N_t$  and modify the production function to be

$$Y_t = Z_t K_t^\alpha N_t^{1-\alpha}.$$

Let's specify the objective function as

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ \log C_t - \theta \log(1 - N_t) \}$$

- (a) Derive the system of equations that characterizes the solution of this model.
- (b) Calculate the steady state analytically. Finding an analytical solution for the steady state is a little tedious but possible here. (This facilitated by the log-log utility specification and the Cobb-Douglas production technology.)
- (c) Choose sensible parameters values and solve for the policy rules using Dynare.
- (d) What do you note about the impulse response functions to TFP and IST shocks?
- (e) Report the relative standard deviation of output, employment and wages. Explain why the model does not generate realistic fluctuations in hours worked and wages (remember that the wage in this economy is the marginal product of labor). What solutions has the economic literature proposed to solve this shortcoming of RBC models?

## Questions 5

Write a short and concise essay (1 page) that answers the following questions:

1. Why do economists use models?
2. Why are models useful in this course?

*This exercise is to be submitted individually.*