ECON747 - Assignment 2

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- Work in groups of around 3 people; work with different people on each assignment
- Please hand in by Sunday, March 2, 2025 via email to drechsel@umd.edu
- Solutions (including model output) should be presented in one single pdf file, with the corresponding Matlab/Dynare codes in one single zip file per group

Question 1 - Complete markets models

Ljungqvist and Sargent, 2nd edition, **Exercise 8.3**. Hint for this question: guess and verify an allocation in part (b).

Question 2 - Complete markets models

Ljungqvist and Sargent, 2nd edition, **Exercise 8.8**. For subquestion (e) you only need to state the Arrow price in a given period t (use the answers to the previous subquestions for this). Hint for this question: notice that one of the agent is risk-neutral and use this to guess an allocation in part (b).

Question 3 - Pricing productive capital

Consider the following dynamic investment model with quadratic capital adjustment costs. A firm maximizes the discounted stream of profits

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_t \Pi_t$$

subject to

$$\begin{aligned} \Pi_t &= Z_t K_t^{\alpha} - I_t - \frac{\gamma}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t \\ K_{t+1} &= (1-\delta)K_t + I_t \\ K_0 &\quad \text{given,} \end{aligned}$$

with $0 < \alpha \leq 1$, $0 < \delta \leq 1$, and $\gamma > 0$. Λ_t captures the discount factor that the owner of the firm applies to the payoff stream coming from the firm. The term $\frac{\gamma}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$ is a resource cost that the firm needs to pay when making adjustments to its capital stock.

(a) Set up the dynamic Lagrangian to this problem. Substitute out Π_t so that your problem has only one constraint, the capital accumulation equation. Denote the Lagrange multiplier on this constraint as Q_t. Find the optimality conditions.

(b) Give an economic interpretation to both of your optimality conditions, focusing on what Q_t captures.

(c) If the owner of the firm was a representative household with a discount factor β and a utility function $u(\cdot)$ that satisfies standard assumptions, what would Λ_t be? What is $\frac{\Lambda_{t+1}}{\Lambda_t}$ sometimes called in finance? What would Λ_t be if the household is risk-neutral?

For the remaining questions, you can assume that $\Lambda_t = 1$.

(d) Suppose the firm also has access to a risk-free bond with gross return R_t. Derive the corresponding Euler equation. Derive a no-arbitrage restriction between the bond return and the return on capital.

(e) Choose a sensible calibration for the parameters and solve for the policy rules of the model using Dynare. Show how Q_t responds to a TFP shock and how Q_t correlates with other variables in the model.

(f) Can we measure Q in the data? What are the challenges? Your answer should contain an explanation of the difference between marginal Q and average Q.

(g) Browse the empirical literature on the Q-theory of investment. What is a typical regression specification to test this the predictions from a model such as the one above? Does the Q-theory hold empirically? Why/why not?

(h) Why is the specific functional form of the adjustment costs convenient for us?

(You have probably noticed this when taking the model to Dynare)

(i) In the New-Keynesian DSGE literature, it is quite frequently assumed that there are costs in adjusting $\frac{I_t}{I_{t-1}}$ rather than in adjusting $\frac{I_t}{K_t}$. See for example in Christiano, Eichenbaum and Evans (2005). Why is this assumption made?