ECON747 - Assignment 3

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- Work in groups of around 3 people; work with different people on each assignment
- Please hand in by Sunday, March 16, 2025 via email to drechsel@umd.edu
- Solutions (including model output) should be presented in one single pdf file, with the corresponding Matlab/Dynare codes in one single zip file per group

Question 1

Take the original baseline model of Kiyotaki and Moore (Section II of their 1997 JPE paper) and solve it using Dynare. Instead of characterizing a perfect foresight equilibrium analytically, assume that there is a stochastic process that governs the farmer's tradable production. You will need to come up with a calibration that satisfies the assumptions made in the paper.

(a) Can you replicate the key insights of the paper by characterizing impulse response functions and moments?

(b) Is there a way you can disentangle static from dynamic multipliers using your model solution? (Hint: you can solve an alternative model in which you keep some variables of interest at their steady state value.)

(c) Add some other shocks to the model and demonstrate how amplification plays out in response to these shocks. Be creative!

In answering a, b, c above, present your solutions in a concise manner by deciding what are the most insightful results to show and by linking them back to what you learned about the Kiyotaki and Moore paper in class.

Question 2

Suppose you want to set up a model with a representative household and a representative firm. You want the household to be the owner of the firm and receive dividends as a lump-sum rebate. The firm produces with labor, which it hires from the household, and capital, which it owns and accumulates itself. You want to characterize an equilibrium in which the firm borrows from the household using a risk-free bond subject to a collateral constraint in which capital serves as collateral, and in which this constraint is binding.

(a) Explain why such an equilibrium cannot arise without further assumptions. (Hint: this has to do with discount factors.) Illustrate your answer in a concise way with some equations.

(b) What would be two possible additional model assumptions you can make for the desired equilibrium to arise? For each of those, give a paper as an example.

(c) For one of these two assumptions, show how they affect the relevant optimality conditions of the household and the firm so that the constraint becomes binding.

Question 3

We introduce a collateral constraint in the spirit of Kiyotaki and Moore (1997) into a two-agent business cycle model with standard preferences and technology. In particular, consider two households indexed by $i = F, G$ that face the following maximization problem

$$\max\ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \frac{C_{i,t}^{1-\sigma_i}}{1-\sigma_i}$$

subject to

$$
\begin{aligned}
K_{i,t+1} &= (1-\delta_i)K_{i,t} + I_{i,t} \\
Y_{i,t} &= Z_{i,t}K_{i,t}^{\alpha_i} \\
\frac{B_{i,t+1}}{R_t} + Y_{i,t} &= C_{i,t} + I_{i,t} + B_{i,t} \\
Z_{i,t} &= (1-\rho_i) + \rho_i Z_{i,t-1} + \varepsilon_{i,t} \\
K_{i,0} &\quad \text{given}
\end{aligned}
$$

were $B_{i,t}$ captures agent i's issuance of a one period risk-free bond.

Also assume that

$$\beta_F < \beta_G$$

and that agent F faces the following borrowing constraint

$$\frac{B_{F,t+1}}{R_t} \leq \theta K_{F,t+1},$$

where $0 < \theta < 1$ is an exogenous parameter.

Denote total output, consumption, investment and capital by Y_t, C_t, I_t, K_t.

(a) Which types of market incompleteness are reflected in the borrowing constraint?

(b) What features of the real world could the parameter θ capture?

(c) Assume from now on that $0 < \alpha_G < \alpha_F < 1$. What will the assumption of decreasing returns in both sectors ensure in this model?

(d) State the market clearing condition in the debt market.

(e) Without solving the maximization problem explicitly: which agent will price the bond in equilibrium and why? Is the interest rate constant as in Kiyotaki-Moore, Section II? Why/why not?

(f) Solve the agents' maximization problems and list the equations that characterize the equilibrium in this model.

(g) Solve this model in Dynare. Is the model's equilibrium well defined throughout the parameter space? Explore the degree to which the borrowing constraint delivers amplification. Try to vary the parameters to see which parameters contribute to amplification. (It may be helpful to loop over different parameter values using one Dynare .mod file the way you saw it in class.) What are your conclusions?

Question 4

Consider the following household populating a small open economy with an endowment income Y_t:

$$\max \ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

subject to

$$\begin{aligned} R^* B_t + Y_t &= C_t + B_{t+1} \\ Y_t &= (1-\rho) + \rho Y_{t-1} + \varepsilon_t \\ B_0 & \quad \text{given} \end{aligned}$$

The economy is open, which means that the household can borrow/save from abroad and consume more/less than the endowment income in a given period. The interest rate R^* is determined on world markets. B_{t+1} denotes the economy's bond purchases. Suppose that

$$\beta R^* < 1$$

and that the rest of the world supplies any amount of debt/savings inelastically.

(a) Explain why the solution to this model is not well defined without further assumptions.

(b) Add the following borrowing constraint

$$R^* B_{t+1} \geq -\theta Y_t$$

Will this constraint be binding? Why/why not? Does the model now have a well defined solution?

(c) Solve analytically for the steady state of the economy. Compare steady state consumption with its counterpart a closed economy version of the model (in which $B_t = 0 \ \forall t$). In which model is steady state consumption higher? In which economy is the household better off?

(d) Characterize the responses of the model to an endowment shock using Dynare. Relative to the closed economy version, does the model feature endogenous amplification in the response of consumption to the endowment shock?

(e) Suppose there is no borrowing constraint. Another way to ensure a solution in small open economies is to assume that the interest is given on world markets but there is a debt elastic premium (see for example Schmitt-Grohe and Uribe, 2003 JIE). Formally, the interest rate on the bond is

$$R_t = R^* - \psi(\widetilde{B}_{t+1} - B^*),$$

where B^* is steady state borrowing, $\widetilde{B}_{t+1} = B_{t+1}$ in equilibrium, but the agent does not realize that her choice of B affects $\widetilde{B}$. Explain why this ensures a solution.