

ECON 747 – LECTURE 2:
BUSINESS CYCLE MODEL REFRESHER

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THEMES COVERED IN THIS PART OF THE COURSE

- ▶ DSGE models as a core framework for this course
- ▶ What is a *solution* to a DSGE model?
- ▶ How to get to a solution?
- ▶ Using Dynare

DSGE MODELS

- ▶ DSGE:
 - ▶ **D**ynamic
 - ▶ **S**tochastic
 - ▶ **G**eneral **E**quilibrium

- ▶ Why are DSGE models complex?
 - ▶ Typically feature rational expectations
 - ⇒ agents are forward-looking
 - ⇒ decisions depend on expectations of behavior in all future states of the world

- ▶ DSGEs are the framework within which we characterize financial frictions

COMPONENTS OF A DSGE

- ▶ Preferences
- ▶ Technology
- ▶ Market structure

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- ▶ Technology
- ▶ Market structure
 - ▶ Understanding the market structure will be key for what we study in this course
 - ▶ We will start with **complete asset markets**
 - ▶ We will see that one ingredient we need for financial variable to matter is **asset market incompleteness**
 - ▶ The other one is **heterogeneity** between agents, e.g. one agent wants to save and another one wants to borrow

A TINY BIT OF HISTORY

- ▶ The first DSGE models were real business cycle (RBC) models, and they originate in the [Lucas \(1976\)](#) critique
- ▶ They were a response to the (old) Keynesian tradition, in which structural relationships were assumed in an ad-hoc way to do econometric policy evaluation
- ▶ First RBC models developed by [Kydland and Prescott \(1982\)](#) & [Long and Plosser \(1983\)](#)
- ▶ Keynesian elements – such as imperfect competition and nominal rigidities – were blended into the RBC framework through “New Neoclassical Synthesis” (see e.g. [Goodfriend and King, 1997](#))
- ▶ Modern New-Keynesian DSGEs feature many shocks and frictions and are used as quantitative tools, e.g. [Smets and Wouters \(2007\)](#)
- ▶ Nice historical retrospective is offered by [Kehoe, Midrigan, and Pastorino \(2018\)](#)

NEOCLASSICAL CORE AND FRICTIONS

- ▶ DSGE models feature a neoclassical (RBC) core
- ▶ This core is usually composed of optimizing agents with rational expectations
- ▶ (Financial) frictions are added around this core
- ▶ Sometimes these frictions are very specific, derived from microfounded behavior, while sometimes they are more ad-hoc (reduced form)

FRICTIONS AND WEDGES

- ▶ One way to “detect” frictions is by adding “wedges” to the core neoclassical model
- ▶ Taking a model with “wedges” to the data, and then studying their properties can guide us on where frictions distort behavior
- ▶ See in particular [Chari, Kehoe, and McGrattan \(2007\)](#)
- ▶ Example: we know that the “labor wedge” (the deviation of MPL from MRS) is usually an important wedge
- ▶ Therefore financial frictions that affect the “labor wedge” are likely to play an important role
 - ▶ Explained well by [Quadrini \(2011\)](#)

Let's look at an example of a simple DSGE model ...

A SIMPLE BUSINESS CYCLE MODEL

Consider an RBC model with shocks to TFP (Z_t) and IST (V_t)

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

subject to

$$K_{t+1} = (1 - \delta)K_t + V_t I_t$$

$$Y_t = Z_t K_t^\alpha$$

$$Y_t = C_t + I_t$$

$$K_0 \text{ given}$$

with $0 < \alpha < 1$, $0 < \beta \leq 1$, $0 < \delta \leq 1$, $\sigma \geq 0$

and stochastic processes for Z_t and V_t .

REMARKS

- ▶ The two exogenous variables are both technological
 - ▶ Total factor productivity (TFP): *how much output can be produced for given inputs*
 - ▶ Investment-specific technology (IST): *how much capital can be created from given level of investment*
- ▶ There could also be shocks to preferences (e.g. to discount factor β), or other technological shocks (e.g. to the depreciation rate)
- ▶ Note: this model can be thought of as a model with inelastic labor supply in which

$$Y_t = \tilde{Z}_t K_t^\alpha \bar{N}^{1-\alpha} \quad (1)$$

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- ▶ We typically write down models that admit a **recursive** formulation
- ▶ What is a recursive problem?
 - ▶ **Same realization of the state variables**
⇒ **same choice of the control variable**
- ▶ Many problems are not recursive

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- ▶ In this model, the state variables are:
 - ▶ K_t, Z_t, V_t

THE BELLMAN EQUATION

$$V(K_t, Z_t, V_t) = \max_{C_t, K_{t+1}} \frac{C_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t V(K_{t+1}, Z_{t+1}, V_{t+1})$$

subject to

$$C_t + \frac{K_{t+1}}{V_t} = Z_t K_t^\alpha + (1 - \delta) \frac{K_t}{V_t}$$

- ▶ Note that I have substituted out Y_t and I_t from the problem; if I want to I can also substitute out C_t
- ▶ $V(\cdot)$ is time-invariant because the problem is **recursive!**

A REMARK ON NOTATION

- ▶ The notational formulation of time subscripts is not unique: what matters is what are choices and what are predetermined variables
- ▶ An equivalent problem would be

$$V(K_{\mathbf{t}-1}, Z_t, V_t) = \max_{C_t, K_t} \frac{C_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t V(K_t, Z_{t+1}, V_{t+1})$$

subject to

$$C_t + \frac{K_t}{V_t} = Z_t K_{\mathbf{t}-1}^\alpha + (1 - \delta) \frac{K_{\mathbf{t}-1}}{V_t}$$

- ▶ Important when we go to Dynare

OPTIMALITY CONDITIONS

- ▶ What **characterizes** the solution to a DSGE model?
 - ▶ Optimality conditions
 - ▶ Constraints
 - ▶ Stochastic processes
- ▶ The optimality conditions can be derived using a Lagrangian or using the Bellman equation (and envelope conditions)
- ▶ If you list the optimally conditions, constraints and the stochastic processes you should get $\#equations = \#variables$
- ▶ If you drop time scripts on this system, this collection of equations characterizes the (nonstochastic) steady state

OPTIMALITY CONDITIONS

- ▶ If we assume that Z_t and V_t follow AR(1) processes, the solution to our model is characterized by

$$\begin{aligned}C_t^{-\sigma} \frac{1}{V_t} &= \beta \mathbb{E}_t \left(C_{t+1}^{-\sigma} \left[\alpha Z_{t+1} K_{t+1}^{\alpha-1} + (1 - \delta) \frac{1}{V_{t+1}} \right] \right) \\C_t + \frac{K_{t+1}}{V_t} &= Z_t K_t^\alpha + (1 - \delta) \frac{K_t}{V_t} \\Z_t &= 1 - \rho_z + \rho_z Z_{t-1} + \varepsilon_{z,t} \\V_t &= 1 - \rho_v + \rho_v V_{t-1} + \varepsilon_{v,t}\end{aligned}$$

- ▶ 4 equations, 4 variables

OPTIMALITY CONDITIONS

- ▶ Substituting out C_t and C_{t+1} , gives you a second-order difference equation in K_t
- ▶ K_0 given & transversality condition needed
 - ▶ K_0 is a primitive of the model
 - ▶ Transversality condition is part of the solution

$$\lim_{t \rightarrow \infty} \beta^t MPK_t K_t = 0$$

- ▶ A transversality condition is conceptually not the same as a no-Ponzi condition (we will come back to this in Lecture 4)

SOLUTION

- ▶ What **is** the solution to a DSGE model?
 - ▶ It is a set **policy functions**
- ▶ The policy functions map state variables into control variable. For our example they are of the form:

$$\begin{aligned}C_t &= g_c(K_t, Z_t, V_t) \\ K_{t+1} &= g_k(K_t, Z_t, V_t)\end{aligned}$$

- ▶ The $g(\cdot)$ functions are time-invariant, because it is a recursive problem: remember **same states** \Rightarrow **same controls**

SOLUTION

- ▶ There are also policy functions to the variables we have substituted out above:

$$Y_t = g_y(K_t, Z_t, V_t)$$

$$I_t = g_i(K_t, Z_t, V_t)$$

- ▶ These can be easily calculated once we found g_c and g_k
- ▶ Finding the policy can be a very difficult problem: there is rarely an analytical solution and we therefore use **numerical techniques**.

A SIMPLE CASE

- ▶ For $\sigma = 1$ and $\delta = 1$, we can derive policy rules analytically, from guessing and verifying $C_t = (1 - s)Y_t$, $\frac{K_{t+1}}{V_t} = sY_t$
- ▶ This is the Brock and Mirman (1972) model, but I augmented it with IST shocks

A SIMPLE CASE

- ▶ The solution is:

$$\begin{aligned}K_{t+1} &= \alpha\beta Z_t V_t K_t^\alpha \\C_t &= (1 - \alpha\beta) Z_t K_t^\alpha \\Y_t &= Z_t K_t^\alpha \\I_t &= K_{t+1} = \alpha\beta Z_t V_t K_t^\alpha\end{aligned}$$

- ▶ You will show this with pen and paper in your first assignment

SOLVING DSGE MODELS

- ▶ Let's go back to the general case for any σ and δ
- ▶ Finding the policy rule analytically is not possible in this case
- ▶ We need to numerically approximate the policy function
- ▶ For example, find $\hat{g}_c(K_t, Z_t, V_t) \approx g_c(K_t, Z_t, V_t)$ where

$$\hat{g}_c(K_t, Z_t, V_t) = a_c + a_{ck}K_t + a_{cz}Z_t + a_{cv}V_t$$

- ▶ In this case $\hat{g}_c(K_t, Z_t, V_t)$ approximates the policy rule with a 1st order polynomial
- ▶ As you will see, Dynare finds $\hat{g}_c(K_t, Z_t, V_t)$ using a Taylor expansion around the nonstochastic steady state of the model $(\bar{K}, \bar{Z}, \bar{V})$

A SIDE NOTE

- ▶ Some of the policy functions may not need to be approximated
- ▶ For example, in an RBC model with only TFP, we would not need to find an approximation $\hat{g}_y(K_t, Z_t)$, since we already know that

$$Y_t = Z_t K_t^\alpha$$

is the true nonlinear relation between output and the two state variables

- ▶ If we give the model to Dynare and include Y_t as a variable, Dynare will (unnecessarily) linearize this equation

SOLUTION METHODS

- ▶ There are different methods to approximate $g(K_t, Z_t, V_t)$
- ▶ Some methods focus on approximating the first order conditions (for example perturbation or projection methods), others are based on iterating on the Bellman equation (e.g. value function iteration)
- ▶ There are important tradeoffs when choosing a solution method
 - ▶ Perturbation methods can deal with many state variables, but the problem needs to be “smooth” (for example no discrete choices such as *default vs. don't default*)
 - ▶ Value function iteration can deal with discrete choices, but only a few state variables
- ▶ New solution methods, to old and new problems, are constantly being developed in macroeconomics

DYNARE

- ▶ You already learned a lot about solution methods in the class with Boragan
- ▶ In my course, we will focus on understanding financial frictions
- ▶ To solve models we will mostly use *Dynare*, which does a lot of the job for us when it comes to finding the solution to the models
 - ▶ I have sometimes 'cooked' the examples to that they work in Dynare
 - ▶ I want you to be conscious of this
- ▶ In Lecture 3, I will explain some of the basics of what Dynare actually does for us in the background
- ▶ In general, be mindful that DSGE models are complex and in your own research things are typically not that easy

DSGES AS DATA-GENERATING PROCESSES

- ▶ When we think of DSGE model as a system that generates data, this system consists of
 - ▶ Policy rules
 - ▶ Stochastic processes
- ▶ In your first assignment you generate data from the DSGE model above and think about whether this simulated data matches patterns in real-world data

INSIGHTS TO TAKE HOME

- ▶ What **characterizes** a solution to a DSGE model?
 - ▶ Optimality conditions, constraints and stochastic processes
- ▶ What **is** a solution?
 - ▶ Policy functions
- ▶ How do we **get to** a solution?
 - ▶ Different ways... this is an art!

PREVIEW

- ▶ In Lecture 3 we will look in detail at Dynare
- ▶ I will give you a “live programming” demonstration

BIBLIOGRAPHY

- BROCK, W. A. AND L. J. MIRMAN (1972): "Optimal economic growth and uncertainty: The discounted case," *Journal of Economic Theory*, 4, 479 – 513.
- CHARI, V. V., P. J. KEHOE, AND E. R. MCGRATTAN (2007): "Business Cycle Accounting," *Econometrica*, 75, 781–836.
- GOODFRIEND, M. AND R. G. KING (1997): "The new neoclassical synthesis and the role of monetary policy," *NBER macroeconomics annual*, 12, 231–283.
- KEHOE, P. J., V. MIDRIGAN, AND E. PASTORINO (2018): "Evolution of Modern Business Cycle Models: Accounting for the Great Recession," *Journal of Economic Perspectives*, 32, 141–66.
- KYDLAND, F. E. AND E. C. PRESCOTT (1982): "Time to Build and Aggregate Fluctuations," *Econometrica*, 50, 1345–1370.
- LONG, J. B. AND C. I. PLOSSER (1983): "Real Business Cycles," *Journal of Political Economy*, 91, 39–69.
- LUCAS, R. E. J. (1976): "Econometric policy evaluation: A critique," in *Carnegie-Rochester conference series on public policy*, North-Holland, vol. 1, 19–46.
- QUADRINI, V. (2011): "Financial frictions in macroeconomic fluctuations," *Economic Quarterly, Federal Reserve Bank of Richmond*, 97, 209–254.
- SMETS, F. AND R. WOUTERS (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97, 586–606.