ECON 747 – LECTURE 2: BUSINESS CYCLE MODEL REFRESHER

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THEMES COVERED IN THIS PART OF THE COURSE

- DSGE models as a core framework for this course
- What is a *solution* to a DSGE model?
- How to get to a solution?
- Using Dynare

DSGE MODELS

- DSGE:
  - **D**ynamic
  - **S**tochastic
  - **G**eneral **E**quilibrium
- Why are DSGE models complex?
  - Typically feature rational expectations

    $\Rightarrow$ agents are forward-looking
    $\Rightarrow$ decisions depend on expectations of behavior in all future states of the world
- DSGEs are the framework within which we characterize financial frictions

COMPONENTS OF A DSGE

- Preferences
- Technology
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- Market structure
  - Understanding the market structure will be key for what we study in this course
  - We will start with **complete asset markets**
  - We will see that one ingredient we need for financial variable to matter is **asset market incompleteness**
  - The other one is **heterogeneity** between agents, e.g. one agent wants to save and another one wants to borrow

A TINY BIT OF HISTORY

- The first DSGE models were real business cycle (RBC) models, and they originate in the Lucas (1976) critique
- They were a response to the (old) Keynesian tradition, in which structural relationships were assumed in an ad-hoc way to do econometric policy evaluation
- First RBC models developed by Kydland and Prescott (1982) & Long and Plosser (1983)
- Keynesian elements – such as imperfect competition and nominal rigidities – were blended into the RBC framework through "New Neoclassical Synthesis" (see e.g. Goodfriend and King, 1997)
- Modern New-Keynesian DSGEs feature many shocks and frictions and are used as quantitative tools, e.g. Smets and Wouters (2007)
- Nice historical retrospective is offered by Kehoe, Midrigan, and Pastorino (2018)

NEOCLASSICAL CORE AND FRICTIONS

- DSGE models feature a neoclassical (RBC) core
- This core is usually composed of optimizing agents with rational expectations
- (Financial) frictions are added around this core
- Sometimes these frictions are very specific, derived from microfounded behavior, while sometimes they are more ad-hoc (reduced form)

FRICTIONS AND WEDGES

- One way to "detect" frictions is by adding "wedges" to the core neoclassical model
- Taking a model with "wegdes" to the data, and then studying their properties can guide us on where frictions distort behavior
- See in particular Chari, Kehoe, and McGrattan (2007)
- Example: we know that the "labor wedge" (the deviation of MPL from MRS) is usually an important wedge
- Therefore financial frictions that affect the "labor wedge" are likely to play an important role
    - Explained well by Quadrini (2011)

Let's look at an example of a simple DSGE model ...

A SIMPLE BUSINESS CYCLE MODEL

Consider an RBC model with shocks to TFP (Z_t) and IST (V_t)

$$\max \ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

subject to

$$\begin{aligned} K_{t+1} &= (1-\delta)K_t + V_t I_t \\ Y_t &= Z_t K_t^{\alpha} \\ Y_t &= C_t + I_t \\ K_0 & \ \text{given} \end{aligned}$$

with $0 < \alpha < 1$, $0 < \beta \leq 1$, $0 < \delta \leq 1$, $\sigma \geq 0$

and stochastic processes for Z_t and V_t.

REMARKS

- The two exogenous variables are both technological
  - Total factor productivity (TFP): *how much output can be produced for given inputs*
  - Investment-specific technology (IST): *how much capital can be created from given level of investment*

- There could also be shocks to preferences (e.g. to discount factor β), or other technological shocks (e.g. to the depreciation rate)

- Note: this model can be thought of as a model with inelastic labor supply in which

$$Y_t = \widetilde{Z}_t K_t^{\alpha} \bar{N}^{1-\alpha} \tag{1}$$

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- We typically write down models that admit a **recursive** formulation
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- We typically write down models that admit a **recursive** formulation
- What is a recursive problem?
  - **Same realization of the state variables**
    $\Rightarrow$ **same choice of the control variable**
- Many problems are not recursive

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  - **Predetermined at the beginning of the period**
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- In this model, the state variables are:
  - K_t, Z_t, V_t

THE BELLMAN EQUATION

$$V(K_t, Z_t, V_t) = \max_{C_t, K_{t+1}} \frac{C_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t V(K_{t+1}, Z_{t+1}, V_{t+1})$$

subject to

$$C_t + \frac{K_{t+1}}{V_t} = Z_t K_t^{\alpha} + (1-\delta)\frac{K_t}{V_t}$$

- Note that I have substituted out Y_t and I_t from the problem;
  If I want to I can also substitute out C_t
- $V(\cdot)$ is time-invariant because the problem is **recursive**!

A REMARK ON NOTATION

- The notational formulation of time subscripts is not unique: what matters is what are choices and what are predetermined variables
- An equivalent problem would be

$$V(K_{\mathbf{t\text{-}1}}, Z_t, V_t) = \max_{C_t, K_{\mathbf{t}}} \frac{C_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t V(K_{\mathbf{t}}, Z_{t+1}, V_{t+1})$$

subject to

$$C_t + \frac{K_{\mathbf{t}}}{V_t} = Z_t K_{\mathbf{t\text{-}1}}^{\alpha} + (1-\delta)\frac{K_{\mathbf{t\text{-}1}}}{V_t}$$

- Important when we go to Dynare

OPTIMALITY CONDITIONS

- What **characterizes** the solution to a DSGE model?
  - Optimality conditions
  - Constraints
  - Stochastic processes

- The optimality conditions can be derived using a Lagrangian or using the Bellman equation (and envelope conditions)

- If you list the optimally conditions, constraints and the stochastic processes you should get #equations = #variables

- If you drop time scripts on this system, this collection of equations characterizes the (nonstochastic) steady state

OPTIMALITY CONDITIONS

- If we assume that Z_t and V_t follow AR(1) processes, the solution to our model is characterized by

$$
\begin{aligned}
C_t^{-\sigma}\frac{1}{V_t} &= \beta\mathbb{E}_t\left(C_{t+1}^{-\sigma}\left[\alpha Z_{t+1}K_{t+1}^{\alpha-1}+(1-\delta)\frac{1}{V_{t+1}}\right]\right)\\
C_t+\frac{K_{t+1}}{V_t} &= Z_tK_t^{\alpha}+(1-\delta)\frac{K_t}{V_t}\\
Z_t &= 1-\rho_z+\rho_z Z_{t-1}+\varepsilon_{z,t}\\
V_t &= 1-\rho_v+\rho_v V_{t-1}+\varepsilon_{v,t}
\end{aligned}
$$

- 4 equations, 4 variables

OPTIMALITY CONDITIONS

- Substituting out C_t and C_{t+1}, gives you a second-order difference equation in K_t
- K_0 given & transversality condition needed
  - K_0 is a primitive of the model
  - Transversality condition is part of the solution

$$\lim_{t \to \infty} \beta^t \lambda_t K_t = 0$$

    where λ_t is the Lagrange multiplier on the budget constraint
  - A transversality condition is conceptually not the same as a no-Ponzi condition (we will come back to this in Lecture 4)

SOLUTION

- What **is** the solution to a DSGE model?
  - It is a set policy functions
- The policy functions map state variables into control variable. For our example they are of the form:

$$\begin{aligned} C_t &= g_c(K_t, Z_t, V_t) \\ K_{t+1} &= g_k(K_t, Z_t, V_t) \end{aligned}$$

- The $g(\cdot)$ functions are time-invariant, because it is a recursive problem: remember **same states $\Rightarrow$ same controls**

SOLUTION

- There are also policy functions to the variables we have substituted out above:

$$\begin{aligned} Y_t &= g_y(K_t, Z_t, V_t) \\ I_t &= g_i(K_t, Z_t, V_t) \end{aligned}$$

- These can be easily calculated once we found g_c and g_k
- Finding the policy can be a very difficult problem: there is rarely an analytical solution and we therefore use **numerical techniques**.

A SIMPLE CASE

- For $\sigma = 1$ and $\delta = 1$, we can derive policy rules analytically, from guessing and verifying $C_t = (1 - s)Y_t$, $\frac{K_{t+1}}{V_t} = sY_t$
- This is the Brock and Mirman (1972) model, but I augmented it with IST shocks

A SIMPLE CASE

- The solution is:

$$
\begin{aligned}
K_{t+1} &= \alpha\beta Z_t V_t K_t^{\alpha} \\
C_t &= (1-\alpha\beta) Z_t K_t^{\alpha} \\
Y_t &= Z_t K_t^{\alpha} \\
I_t &= K_{t+1} = \alpha\beta Z_t V_t K_t^{\alpha}
\end{aligned}
$$

- You will show this with pen and paper in your first assignment

- Let's go back to the general case for any σ and δ
- Finding the policy rule analytically is not possible in this case
- We need to numerically approximate the policy function
- For example, find $\hat{g}_c(K_t, Z_t, V_t) \approx g_c(K_t, Z_t, V_t)$ where

$$\hat{g}_c(K_t, Z_t, V_t) = a_c + a_{ck}K_t + a_{cz}Z_t + a_{cv}V_t$$

- In this case $\hat{g}_c(K_t, Z_t, V_t)$ approximates the policy rule with a 1st order polynomial
- As you will see, Dynare finds $\hat{g}_c(K_t, Z_t, V_t)$ using a Taylor expansion around the nonstochastic steady state of the model $(\bar{K}, \bar{Z}, \bar{V})$

A SIDE NOTE

- Some of the policy functions may not need to be approximated
- For example, in an RBC model with only TFP, we would not need to find an approximation $\hat{g}_y(K_t, Z_t)$, since we already know that

$$Y_t = Z_t K_t^{\alpha}$$

  is the true nonlinear relation between output and the two state variables
- If we give the model to Dynare and include Y_t as a variable, Dynare will (unnecessarily) linearize this equation

SOLUTION METHODS

- There are different methods to approximate $g(K_t, Z_t, V_t)$
- Some methods focus on approximating the first order conditions (for example perturbation or projection methods), others are based on iterating on the Bellman equation (e.g. value function iteration)
- There are important tradeoffs when choosing a solution method
  - Perturbation methods can deal with many state variables, but the problem needs to be "smooth" (for example no discrete choices such as *default* vs. *don't default*)
  - Value function iteration can deal with discrete choices, but only a few state variables
- New solution methods, to old and new problems, are constantly being developed in macroeconomics

DYNARE

- You already learned a lot about solution methods in the class with Boragan
- In my course, we will focus on understanding financial frictions
- To solve models we will mostly use *Dynare*, which does a lot of the job for us when it comes to finding the solution to the models
  - I have sometimes ‘cooked’ the examples to that they work in Dynare
  - I want you to be conscious of this
- In Lecture 3, I will explain some of the basics of what Dynare actually does for us in the background
- In general, be mindful that DSGE models are complex and in your own research things are typically not that easy

DSGES AS DATA-GENERATING PROCESSES

- When we think of DSGE model as a system that generates data, this system consists of
  - Policy rules
  - Stochastic processes
- In your first assignment you generate data from the DSGE model above and think about whether this simulated data matches patterns in real-world data

INSIGHTS TO TAKE HOME

- What **characterizes** a solution to a DSGE model?
  - Optimality conditions, constraints and stochastic processes
- What **is** a solution?
  - Policy functions
- How do we **get to** a solution?
  - Different ways... this is an art!

PREVIEW

- In Lecture 3 we will look in detail at Dynare
- I will give you a "live programming" demonstration

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