ECON 747 – LECTURE 5: INCOMPLETE MARKETS, HETEROGENEOUS AGENTS AND PRECAUTIONARY SAVINGS

Thomas Drechsel

University of Maryland

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RECAP FROM PREVIOUS LECTURE

- We built a benchmark complete markets model
- Heterogeneity (idiosyncratic risk) could be insured away

THIS LECTURE

- Introduce **heterogeneity** and **incomplete markets** in combination
  - Recall that we need both for financial frictions to matter
- In today's lecture, the heterogeneity will consist of **idiosyncratic income risk**, as in the complete markets case; the market incompleteness will be such that agents only have **non-state contingent bonds** and face an **exogenous borrowing limit**
  - In future lectures, we will have simpler heterogeneity (e.g. two agents) but deal with borrowing limits that are endogenous
- We will learn today how **precautionary savings** are a consequence of market incompleteness

REFERENCES

- A good textbook reference for this lecture is the Ljungqvist and Sargent text book *Recursive Macroeconomic Theory* (2nd edition), Chapters 16 and 17
- As usual, I will provide additional references throughout

OVERVIEW

1. Precautionary savings in partial equilibrium

2. Incomplete markets models: baseline setting

3. Models with incomplete markets, heterogeneous agents and precautionary savings
   3.1 Pure credit economy: Huggett (1993)
   3.2 Adding capital: Aiyagari (1994)
   3.3 Adding aggregate risk: Krusell and Smith (1998)

4. The latest generation of heterogeneous agent models ("HANK")

PRECAUTIONARY SAVINGS IN PARTIAL EQUILIBRIUM

WHY START IN PARTIAL EQUILIBRIUM?

- We want to build intuition for the basic decision margins of individual agents
- Recall Lucas tree model: take consumption patterns as given, price assets (GE)
- Here: take asset prices as given, study agents' consumption behavior (PE)

A SIMPLE PE SETTING

- Assume single household, endowment income, no uncertainty
- Budget constraint and no-Ponzi condition

$$\begin{aligned} c_t + b_{t+1} &= Rb_t + y_t \\ \lim_{t\to\infty} \frac{b_{t+1}}{R_t} &= 0 \end{aligned}$$

- Euler equation for risk-free bond is

$$u'(c_t) = \beta R u'(c_{t+1})$$

- Consumption path will be:
  - Constant if $\beta R = 1$
  - Increasing if $\beta R > 1$
  - Decreasing if $\beta R < 1$

- In this world, current income has no effect on consumption path, only lifetime income (permanent income) does
- How to see this formally? Consolidate the budget constraint and impose no-Ponzi condition:

$$
\begin{aligned}
Rb_t &= c_t - y_t + b_{t+1} \\
Rb_t &= c_t - y_t + \frac{c_{t+1} - y_{t+1}}{R} + \frac{b_{t+2}}{R} \\
&\cdots \\
Rb_t &= \sum_{j=0}^{\infty} \frac{c_{t+j} - y_{t+j}}{R^j}
\end{aligned}
$$

THE PERMANENT INCOME HYPOTHESIS

- Suppose we have log utility and $\beta R = 1$ so that $c_{t+1} = c_t$

- In that case we get

$$
\begin{aligned}
Rb_t &= c_t \sum_{j=0}^{\infty} \frac{1}{R^j} + \sum_{j=0}^{\infty} \frac{y_{t+j}}{R^j} \\
c_t &= \frac{R}{R-1} \left[Rb_t + \sum_{j=0}^{\infty} \frac{y_{t+j}}{R^j} \right]
\end{aligned}
$$

- Current consumption depends only on lifetime income (asset wealth and all future endowment income)

- Current income changes (that leave permanent income the same) do not change consumption

TESTING THE PIH EMPIRICALLY

- **Macro data**
  - Hall (1978): assume quadratic utility $\Rightarrow \mathbb{E}_t c_{t+1} = c_t$; no current information (other than current consumption) should predict future consumption; confirmed for disposable income, rejected for stock prices
  - Wilcox (1989): look at pre-announced increases in social security benefit; do not rise consumption immediately, so PIH rejected; possible explanation: liquidity constraints

- **Micro data**
  - Zeldes (1989a): tests for liquidity constraint explanation; PIH holds for "rich" households
  - Shea (1995): consider households with long-term union contracts; consumption responds to predictable wage movements; responds asymetrically to increase/decreases

- With uncertainty, current consumption depends only on *expected* lifetime income
- "Precautionary savings" $=$ difference between consumption
  1. when income is certain
  2. when income is uncertain but has same mean as in 1.

PRECAUTIONARY SAVINGS

- Look at case $\beta R = 1$ and and compare certainty to uncertainty case:

$$\begin{aligned} u'(c_t) &= u'(c_{t+1}) \\ u'(c_t^*) &= \mathbb{E}_t\left[u'(c_{t+1}^*)\right] \end{aligned}$$

- Suppose $\mathbb{E}_t\left[c_{t+1}^*\right] = c_{t+1}$
- Precautionary savings would mean $c_t^* < c_t$. Is this the case?

PRECAUTIONARY SAVINGS

- Use result $\mathbb{E}_t\left[u'(c^*_{t+1})\right] > u'\left(\mathbb{E}_t\left[c^*_{t+1}\right]\right)$
  - This holds iff $u''' > 0$ (based on Jensen's inequality)
  - $u''' > 0$ means *marginal utility is convex*
  - See Zeldes (1989b)

- Substitute assumption $\mathbb{E}_t\left[c^*_{t+1}\right] = c_{t+1}$ into the right hand side of the above inequality

$$\begin{aligned} \mathbb{E}_t\left[u'(c^*_{t+1})\right] &> u'(c_{t+1}) \\ u'(c^*_t) &> u'(c_t) \\ c^*_t &< c_t \end{aligned}$$

(if $u'' < 0$ i.e. function u' is decreasing)

INCOMPLETE MARKETS MODELS: A BASELINE SETTING

ENVIRONMENT

- Similar to previous lecture, with some changes in notation
- Households $i = 1, ..., I$, ex ante identical, but ex-post heterogeneous
- Preferences:

$$U_{i,t} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_{i,\tau})$$

- **Incomplete markets**: only risk free bonds

$$c_{i,t} + a_{i,t+1} \leq (1+r)a_{i,t} + s_{i,t}w$$

ENVIRONMENT

- $s_{i,t}$ is employment status or productivity
- $s_{i,t}$ follows m state Markov process with transition matrix $\mathcal{P}_{m\times m}$ which gives transition probabilities between states k and ℓ as $\mathcal{P}(s_{i,t} = s^k, s_{i,t+1} = s^\ell)$
- Suppose assets can be chosen from grid $\mathcal{A} = \{a^1, ..., a^n\}$

  (Careful: here the superscript denotes the realization of the state, not the history)
- No aggregate uncertainty
- Assume for now that $\beta(1 + r) < 1$, treat r and w as parameters

REMARKS

- Defining s and a over discrete spaces will be helpful when we take about solution methods for these models further below
- Important note: in this setting, there are actual individual savings (no "illusion of choice") because there are incomplete markets <u>and</u> heterogeneity

BELLMAN EQUATION OF AGENT i

$$V(a, s^k) = \max_{c, a' \in \mathcal{A}} \left\{ u(c) + \beta \sum_{\ell=1}^{m} \mathcal{P}(s^k, s^\ell) V(a', s^\ell) \right\}$$

subject to

$$c + a' = (1 + r)a + s^k w$$

(I have dropped subscript i for ease of notation)

WHAT'S THE SOLUTION

- What are the policy functions in this economy?

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$$
\begin{aligned}
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c &= g_c(a, s)
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$$

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$$
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\end{aligned}
$$

- With a discrete state space, the policy functions are also transition matrices
- They are the same for each agent, because agents are ex-ante identical

WEALTH-EMPLOYMENT DISTRIBUTION

- In this economy, we can define the unconditional distribution of states for a given agent recursively

- Define $\lambda_t(a, s) = Prob(a_t = a, s_t = s)$. We get

$$\lambda_{t+1}(a', s') = \sum_{s} \sum_{\{a:a'=g(a,s)\}} \lambda_t(a, s)\mathcal{P}(s, s')$$

- A *stationary* distribution satisfies

$$\lambda(a, s) = \lambda_{t+1}(a, s) = \lambda_t(a, s)$$

INTERPRETATION OF STATIONARY DISTRIBUTION

- The fraction of time that an infinitely lived agent spends in state (a, s)
- Fraction of households in state (a, s) in a given period in a stationary equilibrium
- In the models we study below, the initial distribution of agents over individual state variables (a, s) remains constant over time even though the state of the individual household is a stochastic process

INTRODUCING INCOMPLETE MARKETS AND HETEROGENEITY

- I will run you through some specific models that build on the above framework
- In this lecture, the market incompleteness we look at is relatively ad-hoc (in addition to only a non-state contingent bond being available to agents, we introduce a simple exogenous borrowing limit)
- In the future lectures, we will have more elaborate frictions and but will then typically make the heterogeneity aspect simpler
  - e.g. one saver and one borrower

PURE CREDIT ECONOMY: HUGGETT (1993)

HUGGET ENVIRONEMNT

- Two changes to framework above
  1. We determine r in equilibrium
  2. We add a borrowing limit

- Borrowing limit is exogenously assumed to be

$$a > -\phi$$

- Could be natural debt limit ($\phi = \frac{ws^1}{r}$) or exogenous parameter that is more restrictive (more on this later)

COMPETITIVE EQUILIBRIUM

- Given ϕ and $\mathcal{P}$, a stationary competitive equilibrium is an interest rate r, a policy function $a' = g(a, s)$ and a stationary distribution $\lambda(a, s)$, such that

  1. $g(a, s)$ solves households' maximization problem, taking r as given
  2. $\lambda(a, s)$ is implied by $\mathcal{P}$ and $g(a, s)$
  3. Asset markets clear

$$\sum_{a,s} \lambda(a, s) g(a, s) = 0$$

COMPUTATIONAL SOLUTION ALGORITHM

1. Guess an interest rate r^j
2. Given r^j, solve household's problem for $g^j(a,s)$
3. Use $g^j(a,s)$ and $\mathcal{P}$ to compute $\lambda^j(a,s)$
4. Compute excess demand of savings

$$e^j \equiv \sum_{a,s} \lambda^j(a,s) g^j(a,s)$$

5. If $|e^j| < tol$, stop
   Otherwise, if $e^j > 0$, set $r^{j+1} < r^j$; if $e^j < 0$, set $r^{j+1} > r^j$

adding capital: Aiyagari (1994)

ENVIRONMENT: PRODUCTION ECONOMY

- Households:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t + k_{t+1} = (r_t + 1 - \delta)k_t + w_t s_t$$

(again, I have omitted subscript i for the households' problem)

- Representative firm:

$$\max AF(K_t, N_t) - r_t K_t - w_t N_t$$

ENVIRONMENT: PRODUCTION ECONOMY

- Firm FOCs:

$$\begin{aligned} w_t &= A\frac{\partial F}{\partial N_t} \\ r_t &= A\frac{\partial F}{\partial K_t} \end{aligned}$$

- Household policy function

$$k_{t+1} = g(s_t, k_t)$$

- A constant, so no aggregate risk

COMPETITIVE EQUILIBRIUM

- Given $\mathcal{P}$, a stationary competitive equilibrium is a set of prices (r, w), a policy function $k' = g(k, s)$, a stationary distribution $\lambda(k, s)$ and an aggregate allocation (K, N) such that

1. $g(k, s)$ solves households' maximization problem, taking prices as given
2. $\lambda(k, s)$ is implied by $\mathcal{P}$ and $g(k, s)$
3. Firms maximize profits
4. Labor markets clear
$$N_t = \xi^{\infty} \bar{s}$$
5. Capital markets clear
$$\sum_{k,s} \lambda(k, s) g(k, s) = K$$

COMPUTATIONAL SOLUTION ALGORITHM

1. Guess a capital stock K^j
2. Given K^j, solve firm's problem for r^j, w^j
3. Given r^j, w^j solve households problem for $g^j(k,s)$
4. Use $g^j(k,s)$ and $\mathcal{P}$ to compute $\lambda^j(k,s)$
5. Calculate implied aggregate capital stock

$$\tilde{K}^j \equiv \sum_{k,s} \lambda^j(k,s) g^j(k,s)$$

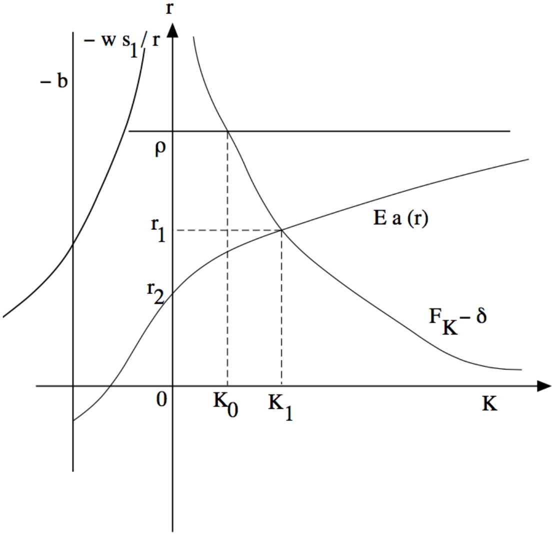
6. If $|\tilde{K}^j - K^j| < tol$, stop

   Otherwise, if $\tilde{K}^j - K^j > 0$, set $K^{j+1} > K^j$; if $\tilde{K}^j - K^j < 0$, set $K^{j+1} < K^j$

PUTTING THINGS TOGETHER

- In Bewley setting, there is a precautionary savings motive for each individual agent. However, aggregate savings are still zero in equilibrium. Tightening borrowing limit will be reflected in lower interest rates.
- The Aiyagari setting, there is a positive supply of aggregate savings, because savings are in capital. In this setting, precautionary motives will lead to lower interest rates (returns) and a higher capital stock.
- Can put things together in one diagram ...

AGGREGATE PRECAUTIONARY SAVINGS



adding aggregate risk: Krusell and Smith (1998)

AGGREGATE RISK

- Suppose A_t subject to shocks (aggregate risk)
- In this case, no invariant distribution $\lambda(k, s)$
- Since the distribution in a given period matters for prices, each agent will want to know the entire distribution
- Insight of Krusell and Smith: can approximate distribution with a finite set of moments (the mean)

INSIGHTS OF KRUSELL AND SMITH

- Not that much lost with a representative agent model
- Model with uninsurable idiosyncratic risk displays far less dispersion and skewness in wealth than we see in US data
- The framework remains a very influential baseline in macroeconomics!

THE LATEST GENERATION OF HETEROGENEOUS AGENT MACRO MODELS

THE WORLD ACCORDING TO HANK

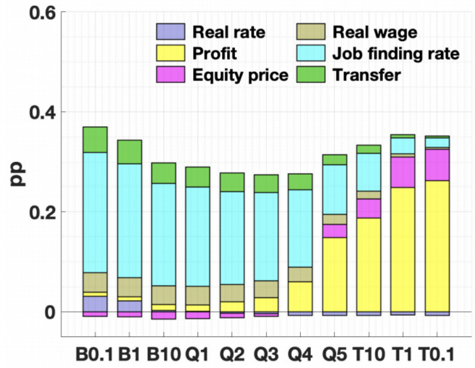
- As reviewed in Lecture 2, modern quantitative DSGE models usually feature New Keynesian elements (e.g. nominal rigidities) around a neoclassical core
- The models studied above add substantial complexity to the neoclassical core, through heterogeneity (idiosyncratic risk) and incomplete markets
- The very latest generation of macro models bridges this more complex core again with New Keynesian features
    - Heterogeneous Agent New Keynesian ("HANK") models
    - See e.g. Kaplan, Moll, and Violante (2018)

- HANK models allow to study empirically realistic wealth and income distributions
- A key element is the marginal propensity to consume out of transitory income (the PIH does not hold)
- Distributional features of the economy:
  - Are an object of interest
  - Matter meaningfully for aggregate dynamics

FROM MICRO TO MACRO

- In the first HANK models, emphasis mostly on matching micro moments
- More recently, emphasis on also quantitatively matching/explaining aggregate dynamics, e.g. responses to different types of shocks
  - E.g. Auclert, Rognlie, and Straub (2020) and Bayer, Born, and Luetticke (2020)

A PICTURE FROM A FORMER UMD STUDENT



- From Lee (2020): "Quantitative Easing and Inequality"
- The graph shows the welfare effects of QE in terms of consumption equivalents for different income groups and decomposed into different sources

REMARKS AND FURTHER READING

- In this course, we focus in more detail on financial frictions and will typically have relatively simple layers of heterogeneity
- Important building blocks of HANK models are related to what we study
  - Different types of market incompleteness
  - E.g. often multiple assets with different degrees of liquidity and different returns
- Highly recommended further reading on the HANK literature:
  - Ben Moll's teaching material (available on his website)

WRAPPING UP

WRAPPING UP

- We have introduced **idiosyncratic risk** and **incomplete markets** in combination
- We got to know some canonical heterogeneous agent models
- From now on, we will start going deeper into the formulation of financial frictions, and study how they matter for macroeconomic dynamics

Bibliography

Aiyagari, S. R. (1994): "Uninsured idiosyncratic risk and aggregate saving," *The Quarterly Journal of Economics*, 109, 659–684.

Auclert, A., M. Rognlie, and L. Straub (2020): "Micro jumps, macro humps: Monetary policy and business cycles in an estimated HANK model," Tech. rep., National Bureau of Economic Research.

Bayer, C., B. Born, and R. Luetticke (2020): "Shocks, frictions, and inequality in US business cycles," .

Hall, R. E. (1978): "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, 86, 971–987.

Huggett, M. (1993): "The risk-free rate in heterogeneous-agent incomplete-insurance economies," *Journal of economic Dynamics and Control*, 17, 953–969.

Kaplan, G., B. Moll, and G. L. Violante (2018): "Monetary policy according to HANK," *American Economic Review*, 108, 697–743.

Krusell, P. and A. A. Smith, Jr (1998): "Income and wealth heterogeneity in the macroeconomy," *Journal of political Economy*, 106, 867–896.

Lee, D. (2020): "Quantitative Easing and Inequality," *Job Market Paper*.

Shea, J. (1995): "Union Contracts and the Life-Cycle/Permanent-Income Hypothesis," *The American Economic Review*, 85, 186–200.

Wilcox, D. W. (1989): "Social security benefits, consumption expenditure, and the life cycle hypothesis," *Journal of Political Economy*, 97, 288–304.

Zeldes, S. P. (1989a): "Consumption and Liquidity Constraints: An Empirical Investigation," *Journal of Political Economy*, 97, 305–346.

——— (1989b): "Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence," *The Quarterly Journal of Economics*, 104, 275–298.