We built a benchmark complete markets model

Heterogeneity (idiosyncratic risk) could be insured away

We started looking at some form of market incompleteness

So far, no meaningful interaction between market incompleteness and heterogeneity of agents
THIS LECTURE

- Introduce **idiosyncratic risk** and **incomplete markets** in combination
  - Recall that we need both for financial frictions to matter

- In today’s lecture, the focus will be primarily on heterogeneity, the market incompleteness (financial friction) will be ad-hoc

- We will learn today how **precautionary savings** are a consequence of market incompleteness
A good textbook reference for this lecture is the Ljungqvist and Sargent textbook *Recursive Macroeconomic Theory* (2nd edition), Chapters 16 and 17.

As usual, I will provide additional references throughout.
1. Precautionary savings in partial equilibrium

2. Incomplete markets models: baseline setting

3. Models with incomplete markets, heterogeneous agents and precautionary savings
   3.1 Pure credit economy: Huggett (1993)
   3.2 Adding capital: Aiyagari (1994)
   3.3 Adding aggregate risk: Krusell and Smith (1998)

4. The latest generation of heterogeneous agent models ("HANK")
PRECAUTIONARY SAVINGS IN PARTIAL EQUILIBRIUM
WHY START IN PARTIAL EQUILIBRIUM?

- We want to build intuition for the basic decision margins of individual agents

- Recall Lucas tree model: take consumption patterns as given, price assets (GE)

- Here: take asset prices as given, study agents’ consumption behavior (PE)
A SIMPLE PE SETTING

- Assume single household, endowment income, no uncertainty

- Budget constraint and no-Ponzi condition

\[ c_t + b_{t+1} = Rb_t + y_t \]

\[ \lim_{t \to \infty} \frac{b_{t+1}}{R_t} = 0 \]

- Euler equation for risk-free bond is

\[ u'(c_t) = \beta Ru'(c_{t+1}) \]

- Consumption path will be:
  - Constant if $\beta R = 1$
  - Increasing if $\beta R > 1$
  - Decreasing if $\beta R < 1$
THE PERMANENT INCOME HYPOTHESIS

- In this world, current income has no effect on consumption path, only lifetime income (permanent income) does.

- How to see this formally? Consolidate the budget constraint and impose no-Ponzi condition:

\[ Rb_t = c_t - y_t + b_{t+1} \]

\[ Rb_t = c_t - y_t + \frac{c_{t+1} - y_{t+1}}{R} + \frac{b_{t+2}}{R} \]

\[ \ldots \]

\[ Rb_t = \sum_{j=0}^{\infty} \frac{c_{t+j} - y_{t+j}}{R^j} \]
THE PERMANENT INCOME HYPOTHESIS

- Suppose we have log utility and $\beta R = 1$ so that $c_{t+1} = c_t$

- In that case we get

\[
Rb_t = c_t \sum_{j=0}^{\infty} \frac{1}{R^j} + \sum_{j=0}^{\infty} \frac{y_{t+j}}{R^j}
\]

\[
c_t = \frac{R}{R - 1} \left[ Rb_t + \sum_{j=0}^{\infty} \frac{y_{t+j}}{R^j} \right]
\]

- Current consumption depends only on lifetime income (asset wealth and all future endowment income)

- Current income changes (that leave permanent income the same) do not change consumption
TESTING THE PIH EMPIRICALLY

- **Macro data**
  - Hall (1978): assume quadratic utility ⇒ $E_t c_{t+1} = c_t$; no current information (other than current consumption) should predict future consumption; confirmed for disposable income, rejected for stock prices
  - Wilcox (1989): look at pre-announced increases in social security benefit; do not rise consumption immediately, so PIH rejected; possible explanation: liquidity constraints

- **Micro data**
  - Zeldes (1989a): tests for liquidity constraint explanation; PIH holds for “rich” households
  - Shea (1995): consider households with long-term union contracts; consumption responds to predictable wage movements; responds asymmetrically to increase/decreases
Precautionary savings

- With uncertainty, current consumption depends only on expected lifetime income.

- "Precautionary savings" = difference between consumption
  1. when income is certain
  2. when income is uncertain but has same mean as in 1.
PRECAUTIONARY SAVINGS

- Look at case $\beta R = 1$ and compare certainty to uncertainty case:

  \[
  u'(c_t) = u'(c_{t+1}) \quad u'(c^*_t) = E_t[u'(c^*_{t+1})]
  \]

- Suppose $E_t[c^*_{t+1}] = c_{t+1}$

- Precautionary savings would mean $c^*_t < c_t$. Is this the case?
PRECAUTIONARY SAVINGS

▶ Use result $\mathbb{E}_t \left[ u'(c^*_{t+1}) \right] > u' \left( \mathbb{E}_t \left[ c^*_{t+1} \right] \right)$

▶ This holds iff $u''' > 0$ (based on Jensen’s inequality)
▶ $u''' > 0$ means *marginal utility is convex*
▶ See Zeldes (1989b)

▶ Substitute assumption $\mathbb{E}_t \left[ c^*_{t+1} \right] = c_{t+1}$ into the right hand side of the above inequality

$$
\mathbb{E}_t \left[ u'(c^*_{t+1}) \right] > u'(c_{t+1})
$$

$$
   u'(c^*) > u'(c_t)
$$

$$
   c^*_t < c_t
$$

(if $u'' < 0$ i.e. function $u'$ is decreasing)
INCOMPLETE MARKETS MODELS: A BASELINE SETTING
ENVIRONMENT

- Similar to previous lecture, with some changes in notation
- Households $i = 1, ..., I$, ex ante identical, but ex-post heterogeneous
- Preferences:

$$U_{i,t} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{t-\tau} u(c_{i,\tau})$$

- Incomplete markets: only risk free bonds

$$c_{i,t} + a_{i,t+1} \leq (1 + r)a_{i,t} + s_{i,t}w$$
- $s_{i,t}$ is employment status or productivity

- $s_{i,t}$ follows $m$ state Markov process with transition matrix $P_{m \times m}$ which gives transition probabilities between states $k$ and $\ell$ as $\mathcal{P}(s_{i,t} = s^k, s_{i,t+1} = s^\ell)$

- Suppose assets can be chosen from grid $A = \{a^1, \ldots, a^n\}$
  (Careful: here the superscript denotes the realization of the state, not the history)

- No aggregate uncertainty

- Assume for now that $\beta(1 + r) < 1$, treat $r$ and $w$ as parameters
Defining $s$ and $a$ over discrete spaces will be helpful when we take about solution methods for these models further below.

Important note: in this setting, there are actual individual savings (no “illusion of choice”) because there are incomplete markets and heterogeneity.
BELLMAN EQUATION OF AGENT $i$

\[ V(a, s^k) = \max_{c, a' \in \mathcal{A}} \left\{ u(c) + \beta \sum_{\ell=1}^{m} \mathcal{P}(s^k, s^\ell) V(a', s^\ell) \right\} \]

subject to

\[ c + a' = (1 + r)a + s^k w \]

(I have dropped subscript $i$ for ease of notation)
WHAT’S THE SOLUTION

- What are the policy functions in this economy?
What’s the solution

What are the policy functions in this economy?

\[
a' = g_a(a, s) \\
c = g_c(a, s)
\]
WHAT’S THE SOLUTION

What are the policy functions in this economy?

\[ a' = g_a(a, s) \]
\[ c = g_c(a, s) \]

With a discrete state space, the policy functions are also transition matrices.

They are the same for each agent, because agents are ex-ante identical.
In this economy, we can define the unconditional distribution of states for a given agent recursively:

Define \( \lambda_t(a, s) = \text{Prob}(a_t = a, s_t = s) \). We get

\[
\lambda_{t+1}(a', s') = \sum_s \sum_{\{a: a' = g(a, s)\}} \lambda_t(a, s) \mathcal{P}(s, s')
\]

A stationary distribution satisfies

\[
\lambda(a, s) = \lambda_{t+1}(a, s) = \lambda_t(a, s)
\]
INTERPRETATION OF STATIONARY DISTRIBUTION

- The fraction of time that an infinitely lived agent spends in state \((a, s)\)

- Fraction of households in state \((a, s)\) in a given period in a stationary equilibrium

- In the models we study below, the initial distribution of agents over individual state variables \((a, s)\) remains constant over time even though the state of the individual household is a stochastic process
I will run you through some specific models that build on the above framework

In this lecture, the market incompleteness we look at is relatively ad-hoc (a simple exogenous borrowing limit) and the focus is on the heterogeneity aspect.

In the future lectures, we will have more elaborate frictions and but will then typically make the heterogeneity aspect simpler:
  - e.g. one saver and one borrower
PURE CREDIT ECONOMY: HUGGETT (1993)
Two changes to framework above

1. We determine $r$ in equilibrium
2. We add a borrowing limit

Borrowing limit is exogenously assumed to be

$$a > -\phi$$

Could be natural debt limit ($\phi = \frac{ws^1}{r}$) or exogenous parameter that is more restrictive (more on this later)
COMPETITIVE EQUILIBRIUM

- Given $\phi$ and $P$, a stationary competitive equilibrium is an interest rate $r$, a policy function $a' = g(a, s)$ and a stationary distribution $\lambda(a, s)$, such that

1. $g(a, s)$ solves households’ maximization problem, taking $r$ as given
2. $\lambda(a, s)$ is implied by $P$ and $g(a, s)$
3. Asset markets clear

\[ \sum_{a,s} \lambda(a, s)g(a, s) = 0 \]
COMPUTATIONAL SOLUTION ALGORITHM

1. Guess an interest rate $r^j$
2. Given $r^j$, solve household’s problem for $g^j(a, s)$
3. Use $g^j(a, s)$ and $\mathcal{P}$ to compute $\lambda^j(a, s)$
4. Compute excess demand of savings

\[ e^j \equiv \sum_{a,s} \lambda^j(a, s) g^j(a, s) \]

5. If $|e^j| < tol$, stop
   Otherwise, if $e^j > 0$, set $r^{j+1} < r^j$; if $e^j < 0$, set $r^{j+1} > r^j$
adding capital: Aiyagari (1994)
ENVIROMENT: PRODUCTION ECONOMY

- Households:

\[
\max \sum_{t=0}^{\infty} \beta^t u(c_t)
\]

subject to

\[
c_t + k_{t+1} = (r_t + 1 - \delta)k_t + w_t s_t
\]

(again, I have omitted subscript \(i\) for the households’ problem)

- Representative firm:

\[
\max AF(K_t, N_t) - r_t K_t - w_t N_t
\]
ENVIRONMENT: PRODUCTION ECONOMY

- Firm FOCs:

\[ w_t = A \frac{\partial F}{\partial N_t} \]
\[ r_t = A \frac{\partial F}{\partial K_t} \]

- Household policy function

\[ k_{t+1} = g(s_t, k_t) \]

- A constant, so no aggregate risk
Given $\mathcal{P}$, a stationary competitive equilibrium is a set of prices $(r, w)$, a policy function $k' = g(k, s)$, a stationary distribution $\lambda(k, s)$ and an aggregate allocation $(K, N)$ such that

1. $g(k, s)$ solves households’ maximization problem, taking prices as given
2. $\lambda(k, s)$ is implied by $\mathcal{P}$ and $g(k, s)$
3. Firms maximize profits
4. Labor markets clear
   \[ N_t = \xi^\infty \bar{s} \]
5. Capital markets clear
   \[ \sum_{k,s} \lambda(k,s)g(k,s) = K \]
1. Guess a capital stock $K^j$

2. Given $K^j$, solve firm’s problem for $r^j, w^j$

3. Given $r^j, w^j$ solve households problem for $g^j(k, s)$

4. Use $g^j(k, s)$ and $P$ to compute $\lambda^j(k, s)$

5. Calculate implied aggregate capital stock

\[ \tilde{K}^j \equiv \sum_{{k, s}} \lambda^j(k, s)g^j(K, s) \]

6. If $|\tilde{K}^j - K^j| < tol$, stop

   Otherwise, if $\tilde{K}^j - K^j > 0$, set $r^{j+1} < r^j$; if $\tilde{K}^j - K^j < 0$, set $r^{j+1} > r^j$
In Bewley setting, there is a precautionary savings motive for each individual agent. However, aggregate savings are still zero in equilibrium. Tightening borrowing limit will be reflected in lower interest rates.

The Aiyagari setting, there is a positive supply of aggregate savings, because savings are in capital. In this setting, precautionary motives will lead to lower interest rates (returns) and a higher capital stock.

Can put things together in one diagram ...
AGGREGATE PRECAUTIONARY SAVINGS
ADDING AGGREGATE RISK: KRUSELL AND SMITH (1998)
AGGREGATE RISK

- Suppose $A_t$ subject to shocks (aggregate risk)

- In this case, no invariant distribution $\lambda(k, s)$

- Since the distribution in a given period matters for prices, each agent will want to know the entire distribution

- Insight of Krusell and Smith: can approximate distribution with a finite set of moments (the mean)
INSIGHTS OF KRUSELL AND SMITH

- Not that much lost with a representative agent model
- Model with uninsurable idiosyncratic risk displays far less dispersion and skewness in wealth than we see in US data
- The framework remains a very influential baseline in macroeconomics!
THE LATEST GENERATION OF HETEROGENEOUS AGENT MACRO MODELS
THE WORLD ACCORDING TO HANK

- As reviewed in Lecture 2, modern quantitative DSGE models usually feature New Keynesian elements (e.g. nominal rigidities) around a neoclassical core.

- The models studied above add substantial complexity to the neoclassical core, through heterogeneity (idiosyncratic risk) and incomplete markets.

- The very latest generation of macro models bridges this more complex core again with New Keynesian features:
  - Heterogeneous Agent New Keynesian ("HANK") models
  - See e.g. Kaplan, Moll, and Violante (2018)
FROM MICRO TO MACRO

- HANK models allow to study empirically realistic wealth and income distributions
- A key element is the marginal propensity to consume out of transitory income (the PIH does not hold)
- Distributional features of the economy:
  - Are an object of interest
  - Matter meaningfully for aggregate dynamics
• In the first HANK models, strong emphasis on matching micro moments

• More recently, emphasis on also quantitatively matching/explaining aggregate dynamics, e.g. responses to different types of shocks
  • E.g. Auclert, Rognlie, and Straub (2020) and Bayer, Born, and Luetticke (2020)
A PICTURE FROM A FORMER UMD STUDENT

From Lee (2020): “Quantitative Easing and Inequality”

The graph shows the welfare effects of QE in terms of consumption equivalents for different income groups and decomposed into different sources.
In this course, we focus in more detail on financial frictions and will typically have relatively simple layers of heterogeneity.

Important building blocks of HANK models are related to what we study:

- Different types of market incompleteness
- E.g. often multiple assets with different degrees of liquidity and different returns

Highly recommended further reading on the HANK literature:

- Ben Moll’s teaching material (available on his website)
WRAPPING UP
WRAPPING UP

- We have introduced **idiosyncratic risk** and **incomplete markets** in combination.
- We got to know some canonical heterogeneous agent models.
- From now on, we will start going deeper into the formulation of financial frictions, and study how they matter for macroeconomic dynamics.


