

ECON 747 – LECTURE 8:  
MODELS WITH CONSTRAINTS ON RISK-FREE DEBT:  
ISSUES, LIMITATIONS, APPLICATIONS

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## WHERE WE STAND

- ▶ Financial variables comove with the business cycle; we have experienced massive financial crises; we observe interesting trends in financial markets; financial variables (sometimes) help predicting recessions; ...
- ▶ We therefore study whether and how financial markets cause and/or amplify macroeconomic fluctuations
- ▶ Ingredients to meaningfully think about this:
  1. Market incompleteness
  2. Heterogeneity between agents

## WHERE WE STAND

- ▶ Kiyotaki and Moore (1997): framework to understand how a financial contract can deliver strong and persistent propagation of a small and transitory shock
- ▶ Key to the amplification:
  - ▶ Leverage (and net worth) of constrained agent matters
  - ▶ Price movements relax/tighten constraint
  - ▶ Reallocation of resources between (marginally) productive and unproductive use

# PLAN FOR TODAY

## 1. Issues and limitations: how seriously should we take Kiyotaki and Moore?

- ▶ Limited amplification with standard preferences & technology
- ▶ Debt constraint vs. non-state contingency of debt
- ▶ Studying risk

## 2. Applications of constraints on risk-free debt

- ▶ Household debt and firm debt
- ▶ Financial shocks
- ▶ Working capital constraints
- ▶ Firm dynamics and the firm life cycle
- ▶ International macro: sudden stops and deleveraging
- ▶ Occasionally binding constraints
- ▶ Normative implications
- ▶ ...

# 1. ISSUES AND LIMITATIONS

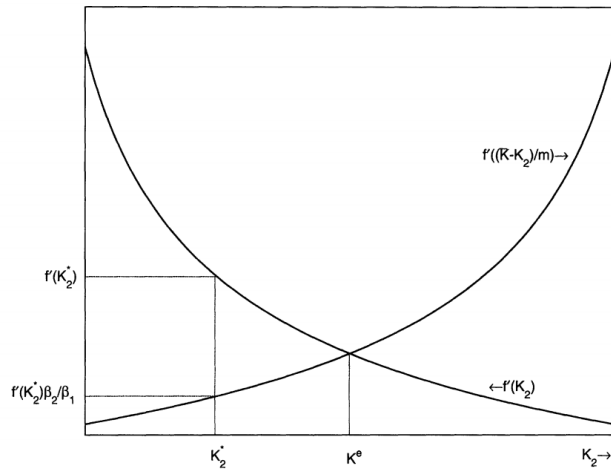
## LIMITED AMPLIFICATION

- ▶ [Kiyotaki and Moore \(1997\)](#): risk-neutrality; one agent CRS, the other agent DRS; fixed supply of land; perfect foresight; ...
- ▶ How would things play out if some of these assumptions are generalized?
- ▶ Answers to this question are provided by [Cordoba and Ripoll \(2004\)](#)
- ▶ You will also look into this question in your assignment

## LIMITED AMPLIFICATION

- ▶ Cordoba and Ripoll (2004) systematically assess the *quantitative* significance of collateral constraints as an amplification mechanism of shocks
- ▶ Set up Kiyotaki-Moore model in which both agents have concave utility functions and both agents have concave production functions
- ▶ Explore how model ingredients and parameter values affect amplification

# LIMITED AMPLIFICATION





## FINDINGS OF CORDOBA AND RIPOLL (2004)

- ▶ For standard parameter values (e.g. capital share of  $1/3$  and EIS of  $0.2$ ) the amplification is very small
- ▶ To open up the quantitative mechanism, characterize formula:  
output response = (productivity gap)  
                           $\times$  (collateral share in production)  
                           $\times$  (production share constrained agents)  
                           $\times$  (redistribution of collateral)
- ▶ For example, if agents are twice more productive (productivity gap is  $1/2$ ) produce  $1/2$  of the total output, and the collateral share is  $1/2$ , then agents must increase their holdings of collateral by 800% for output to end up increasing by 1%

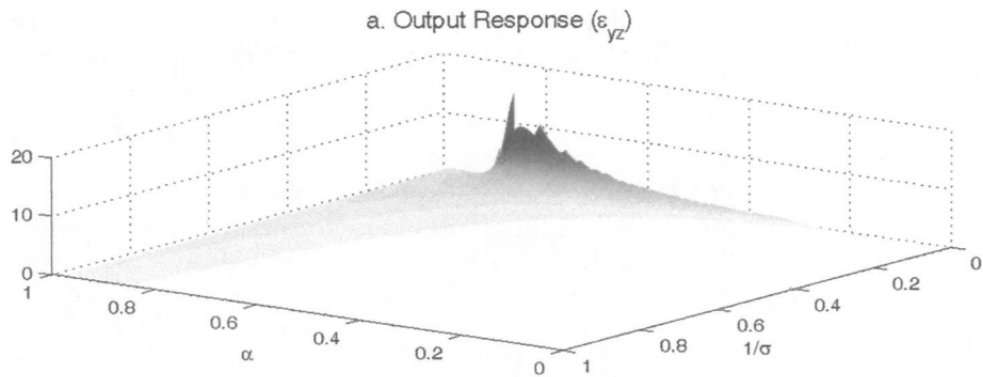
## LIMITED AMPLIFICATION

- ▶ Focus on  $\epsilon_{YZ}$ , the elasticity of output w.r.t the shock
- ▶ The formula they derive is

$$\epsilon_{YZ} = \frac{f'_2 - f'_1}{f'_2} \cdot \alpha \cdot \frac{Y_2}{Y} \cdot \epsilon_{K_2Z}$$

- ▶ The examine the magnitude of  $\epsilon_{YZ}$  throughout the parameter space, for the key parameters  $\alpha$  and  $\frac{1}{\sigma}$

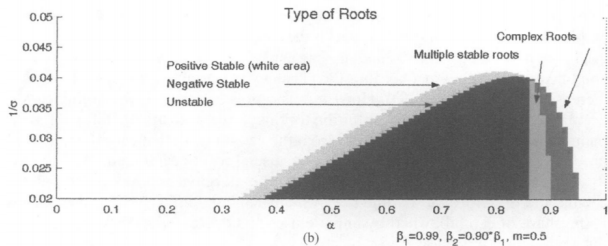
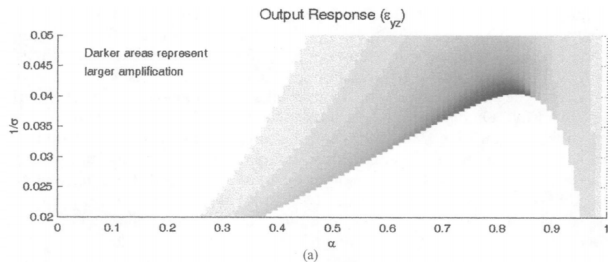
## LIMITED AMPLIFICATION



## LIMITED AMPLIFICATION

- ▶ In addition to very limited amplification, those regions that do deliver it are close to parts of the parameter space in which the model is not well defined

# LIMITED AMPLIFICATION



## LIMITED AMPLIFICATION

- ▶ Similar conclusions about the limited quantitative role of collateral constraints in amplifying shocks are drawn by [Kocherlakota \(2000\)](#)
  - ▶ Uses representative agent framework where borrowing comes from abroad
  - ▶ Interest rate is fixed
  - ▶ Focus is mainly on role of capital share
  - ▶ Some nice analytical derivations

## CONSTRAINT VS. STATE-CONTINGENCY

- ▶ Another interesting aspect of Kiyotaki and Moore (1997) is highlighted by Cao and Nie (2017)
- ▶ These authors point out that models in the spirit of Kiyotaki and Moore (1997) typically assume two types of markets incompleteness
  1. Non-state contingent debt
  2. Debt limited by collateral
- ▶ Question of the paper: which one matters more?

## CONSTRAINT VS. STATE-CONTINGENCY

- ▶ Cao and Nie (2017) build a heterogeneous agents model with aggregate shocks
- ▶ Solve different versions of this model:
  1. Non-state contingent debt, collateral constraint
  2. Non-state contingent debt, no collateral constraint
  3. State contingent debt, collateral constraint
- ▶ Findings:
  - ▶ Model 2 generates 2/3 of the amplification of model 1
  - ▶ Model 3 generates virtually no amplification



## CONSTRAINT VS. STATE-CONTINGENCY

- ▶ Note that [Cao and Nie \(2017\)](#) not only focus on amplification but also on the asymmetry in the responses to shocks
- ▶ Their model characterization requires a global solution technique
- ▶ The paper is a nice read given the material we have covered in this course already

## CONSTRAINT VS. STATE-CONTINGENCY

TABLE 3—AVERAGE LAND PRICE AND OUTPUT CHANGES AFTER A 3 PERCENT INCREASE OR 3 PERCENT DECREASE IN AGGREGATE PRODUCTIVITY

Type of friction	Land price		Output	
	Expansion	Recession	Expansion	Recession
Complete markets (Model 0)	3.00%	−3.00%	3.00%	−3.00%
Collateral constraint (Model 1a)	3.60%	−4.22%	3.25%	−3.65%
Collateral constraint (alt, Model 1b)	3.62%	−4.30%	3.26%	−3.70%
Incomplete markets (Model 2)	3.46%	−3.81%	3.21%	−3.43%
Complete hedging (Model 3)	3.00%	−3.00%	3.00%	−3.00%
Partial hedging (Model 4)	3.71%	−2.92%	3.20%	−2.95%

*Notes:* This table shows the long-run average responses of land price and output after productivity changes from normal to either high (Expansion) or low (Recession). Productivity can take on one of three realizations, (normal, high, and low), where low and high realizations are 3 percent away from normal.

## CONSTRAINT VS. STATE-CONTINGENCY

- ▶ In general, non-state contingency of bonds probably not a bad assumption
- ▶ Also empirical evidence for the use of collateral  
(more on this later on in the course)
- ▶ We want to properly understand the theoretical mechanism to think about how to interpret macro data, so the [Cao and Nie \(2017\)](#) paper should make us think ...

## STUDYING RISK

- ▶ Constraints in the spirit of [Kiyotaki and Moore \(1997\)](#) typically operate on risk-free debt contracts
- ▶ Remember that they are actually derived from an underlying contracting setting in which there is an off-equilibrium threat of default (withdrawal of human capital)
- ▶ However, the threat of default (risk) does not matter for aggregate dynamics

## STUDYING RISK

- ▶ Empirical observation: there is important cyclical variation in interest rate *premia*
- ▶ Remember also from lecture 1: these are the financial variables that (sometimes) help us to detect recessions early
- ▶ The model of [Kiyotaki and Moore \(1997\)](#) does not generate a risk premium or variation therein

## STUDYING RISK

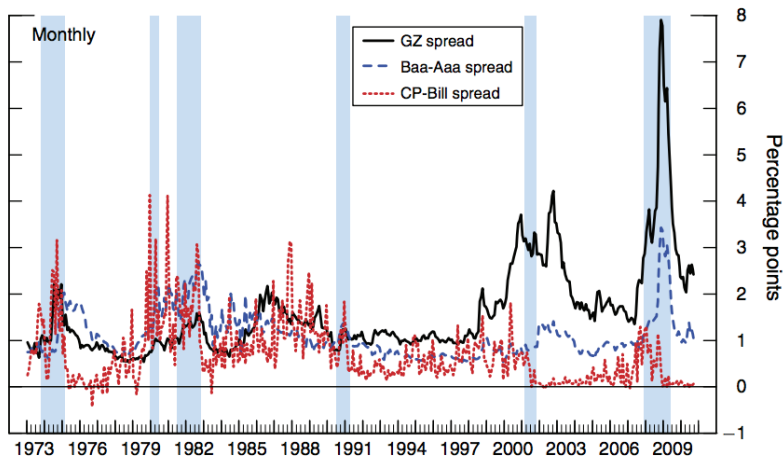


FIGURE 1. SELECTED CORPORATE CREDIT SPREADS

## STUDYING RISK

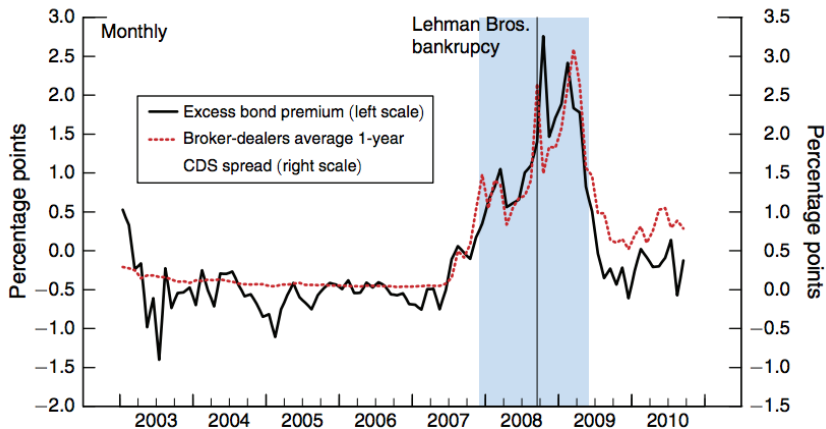


FIGURE 8. THE EXCESS BOND PREMIUM AND FINANCIAL INTERMEDIARY CDS SPREADS

## STUDYING RISK

- ▶ The above two figures are taken from [Gilchrist and Zakrajsek \(2012\)](#)
- ▶ Later in this course, we will study models that can generate cyclicalities in credit spreads and in which credit spreads matter for macro dynamics
- ▶ We will see how this is captured in the *external finance premium* in CSV models



## WHAT TO MAKE OF IT?

Does all of this mean Kiyotaki and Moore is a useless model?

## WHAT TO MAKE OF IT?

Does all of this mean Kiyotaki and Moore is a useless model?

Not at all ...

- ▶ The collateral mechanism may be relevant for individual markets, instead of the aggregate economy
- ▶ The collateral mechanism interacts with other mechanisms
- ▶ The financial sector is not only an amplifier but also a *source* of shocks

In any case, the Kiyotaki-Moore model is extremely useful in **guiding our thinking!**

## 2. APPLICATIONS

# APPLICATIONS

- ▶ Lots of applications of models with endogenous constraints on debt
- ▶ I will throw a few ideas at you in the following slides ...
  - ▶ This is by no means a complete overview

## MORTGAGE DEBT

- ▶ Suppose  $b_t$  is mortgage debt,  $p_{h,t}h_t$  is the value of a house:

$$b_t \leq \theta p_{h,t}h_t$$

- ▶ Can this help explain the 2008-09 financial crisis?
- ▶ Do mortgage contracts actually reflect such a constraint?
- ▶ See for example [Iacoviello \(2005\)](#) or [Greenwald \(2017\)](#)

## FIRM DEBT

- ▶ Plenty of applications where collateral constraint is on firm debt
  - ▶ E.g. [Khan and Thomas \(2013\)](#) in a heterogeneous firm model
- ▶ If  $k_t$  is firm capital, is  $b_t \leq \theta p_{k,t} k_t$  a relevant constraint?
  - ▶ Maybe not if  $k_t$  is intangible capital ...  
→ [Falato, Kadyrzhanova, and Sim \(2013\)](#)
  - ▶ Maybe it should be  $b_t \leq \tilde{\theta} \pi_t$  ...  
→ [Drechsel \(2020\)](#)

## LIFE CYCLE AND OTHER FIRM CHARACTERISTICS

- ▶ What is the relevant constraint for firms that are
  - ▶ Young vs. old?
  - ▶ Big vs. small?
  - ▶ Private vs. public?
- ▶ See work by [Dinlersoz, Kalemli-Ozcan, Hyatt, and Penciakova \(2018\)](#) and [Caglio, Darst, and Kalemli-Ozcan \(2021\)](#)

## WHAT IF THE ECONOMY IS OPEN?

- ▶ Suppose  $b_t$  represents the debt position of a small open economy
- ▶ Can borrowing constraint dynamics explain capital flows, sudden stops, strong volatility in emerging markets, ...?
- ▶ See for example [Mendoza \(2006\)](#)
- ▶ Interesting: fixed interest usually contributes to amplification



## NORMATIVE IMPLICATIONS

- ▶ Borrowing constraints lead to *pecuniary externalities*:
  - ▶ Agents do not internalize how their actions affect prices that appear in the constraints (e.g.  $q_t$  in Kiyotaki-Moore)
  - ▶ Welfare can be improved, e.g. by taxing borrowing
- ▶ Dávila and Korinek (2018) develop a general framework to study such pecuniary externalities, and optimal macroprudential regulation
- ▶ Bianchi (2011) studies 'over-borrowing' and welfare in a small open economy
- ▶ Drechsel and Kim (2021) investigate pecuniary externalities arising from earnings-based borrowing constraints

## SHOCKS TO THE CONSTRAINT ITSELF

- ▶ What about:

$$b_t \leq \theta_t p_{k,t} k_t,$$

where  $\theta_t$  is a stochastic disturbance subject to “financial shocks”

- ▶ Such shocks to the constraint could capture that the financial sector is a *source* rather than an *amplifier* of shocks
- ▶ See for example [Jermann and Quadrini \(2012\)](#) and [Khan and Thomas \(2013\)](#)

## WORKING CAPITAL

- ▶ What about

$$\ell_t \leq \theta p_{k,t} k_t$$

where instead of intertemporal debt,  $\ell_t$  is needed to pre-finance the wage bill:

$$w_t n_t \leq \ell_t$$

- ▶ This makes the constraint interact with the “labor wedge”
- ▶ This is explained intuitively in [Quadrini \(2011\)](#). See also [Bianchi and Mendoza \(2010\)](#) in a small open economy context.

## OCCASIONALLY BINDING CONSTRAINTS

- ▶ What if the constraint only binds sometimes?
- ▶ This is a source of nonlinearity
- ▶ Need method to solve such a model, for example [Guerrieri and Iacoviello \(2015\)](#)
- ▶ Could be applied for example to explain asymmetric business cycles: [Jensen, Petrella, Ravn, and Santoro \(2020\)](#)
- ▶ Recent work on *estimating* models with occasionally binding constraints [Aruoba, Cuba-Borda, Higa-Flores, Schorfheide, and Villalvazo \(2021\)](#)

- ▶ Much more work that can in one way or another be connected to [Kiyotaki and Moore \(1997\)](#) ...

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