ECON 747 – LECTURE 10: MODELS WITH COSTLY STATE VERIFICATION AND RISKY DEBT

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PLAN

- This lecture: models with CSV frictions
  - Focus on classic paper by Bernanke and Gertler (1989)
- Following lectures:
  - Quantitative version: Bernanke, Gertler, and Gilchrist (1999)
  - Applications: Gertler and Karadi (2011), ...
  - Issues, limitations, alternatives, ...

OVERVIEW

- We already explored one canonical financial friction: borrowing constraints that apply to risk-free debt, in the spirit of Kiyotaki and Moore (1997)
- The underlying rationalization for these constraints is limited enforcement
- There is another prominent way of modeling financial frictions, based on different class of agency problems

  $\rightarrow$ asymmetric information
- "Costly state verification" (CSV) gives rise risky financing arrangements

OVERVIEW (CONTINUED)

- CSV models go back to the work of Townsend (1979, 1988)
- The main idea is that the outcome of an investment project can be observed by an entrepreneur but not by the financier
- This issue is solved with an optimal "risky debt" contract
- Typical features of CSV models
  - Cost of 'external' finance $>$ cost of 'internal' finance
  - More borrower net worth reduces severity of agency problem
  - Shocks to net worth give persistent real fluctuations $\rightarrow$ financial accelerator

BERNANKE AND GERTLER (1989) ['BG1989']

APPROACH

- Similar to Kiyotaki and Moore (1997), BG1989 characterize a model that is tractable, but not meant to be serious from a quantitative point of you
- An *iid* shock is shown to give rise to large an persistent movements in output and investment
- Different from Kiyotaki and Moore (1997): the underlying contractual problem plays a bigger role in the exposition of the model

SETTING

- Overlapping generations (OLG) model
- Time is discrete, runs infinitely
- Agents live for two periods (young, old)
- Within each generation:
  - Fraction η of entrepreneurs
  - Fraction $1 - \eta$ of lenders
- Entrepreneurs and lenders differ in preferences, endowments and access to technology

WHY OLG?

- Introducing a finite life span is a way of introducing a motive for borrowing
  - Similar to different discount factors, tax advantage for debt, and so on
- Since young agents are born without assets, they are financially constrained
- If they lived infinitely, agents could accumulate enough assets over time to become unconstrained
- A closely related assumption is that entrepreneurs “die” with a constant probability every period, e.g. in Bernanke, Gertler, and Gilchrist (1999)

PRODUCTION TECHNOLOGIES

- There are two goods: output and capital
- Output is produced from capital and labor
- Output can be:
  - Consumed by agents
  - Stored for next period at exogenous return r
  - Used to produce capital for next period
- Capital is used to produce output, fully depreciates after use

OUTPUT PRODUCTION

- Output is produced with capital and labor based on a CRS technology
- Labor is supplied inelastically
- We can therefore write production in per-capita terms

$$y_t = \tilde{\theta}_t f(k_t)$$

- $\tilde{\theta}_t$ is an iid aggregate technology shock
- It is assumed that $f(0) > 0$

CAPITAL PRODUCTION

- Entrepreneurs can produce capital using output through investment projects
- Each entrepreneur is endowed with one project
- Entrepreneurs are heterogeneous:
  - Type $\omega \sim u[0, 1]$
  - Determines the cost of undertaking a project $x(\omega)$, $x' > 0$
  - Entrepreneurs with a low ω are more efficient

CAPITAL PRODUCTION

- The amount of resulting capital when a project is undertaken is random:
  - n possible discrete outcomes $\kappa_1, ..., \kappa_n$
  - $\kappa_j \geq \kappa_k$ for $j > k$
  - Expected outcome is κ
- The outcome does not depend on the entrepreneur's type, only the cost $x(\omega)$ does

INFORMATION STRUCTURE

- Only an individual entrepreneur can costlessly observe the outcome after undertaking the investment project
- Entrepreneur can under-report and enjoy extra consumption
- Other agents have access to an auditing technology
  - Need to pay γ units of the capital good to observe the outcome
- Random auditing is possible
  - Lenders can commit to audit with a chosen probability p

TIMING

- Project outcomes realize, announcements are made, and auditing takes place before $\tilde{\theta}$ realized
- This means that the decisions in t depend on the expected value of $\tilde{\theta}_{t+1}$, which is denoted θ

AGGREGATE INVESTMENT

- Investment project are independents across entrepreneurs so no uncertainty about the aggregate quantity of capital produced
- In other words, expected and actual aggregate capital are equal
- Denote i_t the (per-capita) number of investment projects undertaken and h_t the fraction of projects that are audited
- This gives

$$k_{t+1} = (\kappa - h_t \gamma) i_t$$

SOME FURTHER ASSUMPTIONS

- It is assumed that

$$\theta f'(0)\kappa - \gamma > rx(0)$$
$$\theta f'(\kappa\eta) < rx(1)$$

- These restrictions ensure that investing is profitable for some but not all entrepreneurs
  - $rx(0)$ is the opportunity cost of investing for the entrepreneur with the lowest cost
  - $rx(1)$ that of the one with the highest cost

- Every individual has a fixed labor endowment
  - Entrepreneurs: L^e
  - Lenders: L
- Normalization
  - $\eta L^e + (1-\eta)L = 1$
  - This means per-capita and per-labor variables are the same

PREFERENCES

- Entrepreneurs:
  - Risk-neutral, only consume when old
- Lenders:
  - A lender born in t has utility function

$$U(z_t^y) + \beta \mathbb{E}_t(z_{t+1}^o)$$

  - z denotes consumption of the output good
- The fact that entrepreneurs only consume when old is not essential
- The fact that both agents are risk neutral in $t+1$ is important
  - We can abstract from additional risk sharing considerations

SAVINGS AND RETURNS

- An assumption will ensure that total savings exceed total capital formation (***)
  - Will be formalized below, after introducing the entrepreneurs' perfect information equilibrium productivity cutoff
- This means that some savings always go into storage and that the *marginal* return in the economy will be r
- The average return in the economy will be higher than r
- This assumption helps us in characterizing the savings choice of lenders

AGENTS' CHOICES

- Relatively easy to characterize choices in this setting, focus on total savings
- Entrepreneurs save their entire income because no utility from consuming in t:

$$S_t^e = w_t L^e$$

- Lenders save whatever they do not consume

$$S_t = w_t L - z_t^{y*}(r)$$

  - The fact that consumption is a function of only r relies on the fact that r is the marginal return (see previous slide)

IMPORTANT NOTE

- Savings are related to wages, therefore depend on marginal productivity and thus on the aggregate state of the economy
- Aggregate shocks move savings (net worth), which will give variation in the severeness of agency problems, which will in turn affect aggregate outcomes

EQUILIBRIUM WITH PERFECT INFORMATION

EQUILIBRIUM WITH PERFECT INFORMATION

- First characterize benchmark case, the equilibrium with perfect information: $\gamma = 0$
- We solve for an equilibrium at time t, in which k_t is predetermined
- This equilibrium will serve as a benchmark case

EQUILIBRIUM WITH PERFECT INFORMATION

- Let $\hat{q}_{t+1}$ be the expected price of capital in period t
- Then $\kappa\hat{q}_{t+1}$ is the expected gross return from each individual investment project
  - Recall that entrepreneurs only differ in cost
- Opportunity cost of investing for a type ω:
  - Not incurring cost $x(\omega)$ and instead getting return r
- Calculate $\bar{\omega}$ such that entrepreneur indifferent between undertaking project or not:

$$\kappa\hat{q}_{t+1} = rx(\bar{\omega})$$

EQUILIBRIUM WITH PERFECT INFORMATION

- Entrepreneurs with $\omega \leq \bar{\omega}$ produce an expected surplus relative to storage
- Entrepreneurs with $\omega > \bar{\omega}$ are better of using storage
- Note that $\bar{\omega}$ is endogenous, a function of $\hat{q}_{t+1}$

KEY ASSUMPTION

- Introduce following assumption:

$$\eta S^e + (1-\eta)S > \int_0^{\bar{\omega}} x(\omega)d\omega$$

  for any k_t, θ_t, $\bar{\omega}$

- This means that total savings exceed the cost of all investment projects that are undertaken, and that some savings always go to storage in equilibrium
- For the assumption to be plausible, the entrepreneurial sector needs to be relatively small
- This assumption was already used above when characterizing the savings choice of lenders and its dependence on r (***)

EQUILIBRIUM WITH PERFECT INFORMATION

- We now turn to the joint determination of $\hat{q}_{t+1}$ and k_{t+1}
- Investment, i.e. the number of projects undertaken is

$$i_t = \bar{\omega}\eta$$

- With $\gamma = 0$ we also have that

$$k_{t+1} = \kappa i_t$$

- Combine indifference condition and equations on the previous slide to eliminate $\bar{\omega}$

$$\hat{q}_{t+1} = x\left(\frac{k_{t+1}}{\kappa\eta}\right)\frac{r}{\kappa}$$

- This is an increasing function in k_{t+1}, an upward-sloping supply function for aggregate capital
- A higher expected price of capital increases the number of entrepreneurs who can profitably invest

CAPITAL DEMAND FUNCTION

- The capital demand function is determined by the fact that the expected price and marginal products must be equal

$$\hat{q}_{t+1} = \theta f'(k_{t+1})$$

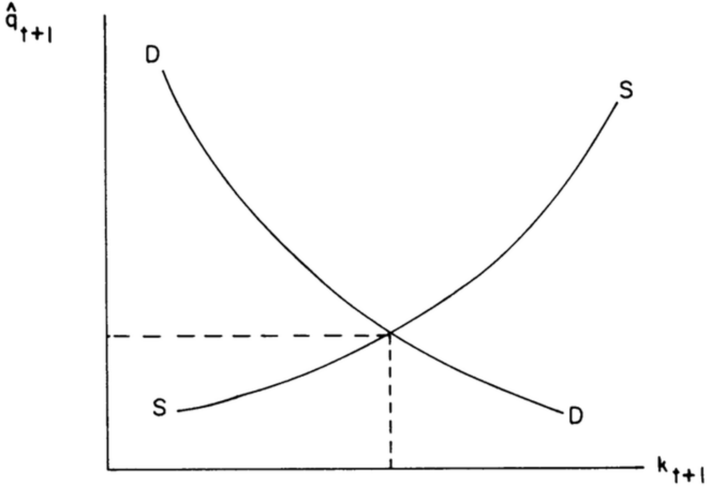
- Recall that timing is such that the MPK depends on the expected value θ

EQUILIBRIUM

- In period t, price $\hat{q}_{t+1}$ and quantity k_{t+1} are determined by the solution to the following system

$$
\begin{aligned}
\hat{q}_{t+1} &= x\left(\frac{k_{t+1}}{\kappa\eta}\right)\frac{r}{\kappa} \\
\hat{q}_{t+1} &= \theta f'(k_{t+1})
\end{aligned}
$$

EQUILIBRIUM



DYNAMICS

- The dynamics in this system are extremely simple
- Both supply and demand are independent of period-t state variables, so that $\hat{q}$ and k are constant over time!
- Output will fluctuate with the iid variation in θ_t, while consumption will be serially correlated (as it can be stored)
  - You will look at this in more detail in Assignment 4
- The setting is designed so that any dynamics in investment will come from the information friction!

EQUILIBRIUM WITH ASYMMETRIC INFORMATION

EQUILIBRIUM WITH ASYMMETRIC INFORMATION

- Now focus on the case $\gamma > 0$
- Strategy to characterize the equilibrium
  1. Focus on situation of an entrepreneur who undertakes project with certainty, but for whom own savings (net worth) cannot cover costs $S^e < x(\omega)$
     - Optimal financial contract
  2. Analyze the decision to invest or not
     - Find ω cutoffs based on indifference conditions
  3. Characterize the within-period equilibrium
  4. Investigate the dynamics following shocks
- Step 1 was not necessary in the perfect information case, steps 2-4 are similar

THE OPTIMAL FINANCIAL CONTRACT

- Consider entrepreneur of type ω
- Suppose $S^e < x(\omega)$, that is, the entrepreneur needs outside financing
- Borrow from a lender with opportunity cost of funds r
- For now we take S^e, $\hat{q}$, r as given

THE OPTIMAL FINANCIAL CONTRACT

- Asymmetric information now becomes an issue: lender could provide funds, but entrepreneur can under-report the return
- The optimal contract is found by maximizing the entrepreneur's next period consumption, subject to the following constraints:
  1. Lenders receive an expected return of at least r
  2. The entrepreneur has no incentive to lie
  3. Consumption and auditing probabilities are feasible

THE OPTIMAL FINANCIAL CONTRACT

- Focus on the special case $n = 2$
- Two states, good and bad
  - Bad: outcome is κ_1 with probability π_1
  - Good: outcome is κ_2 with probability π_2
- The appendix of the paper analyzes the general case $n > 2$
  - In Assignment 4 you consider the case $n = 3$
- It can be formally shown that no auditing occurs when the good state is reported
  - Three consumption levels: c_1, c_a for state 1, c_2 for state 2

THE OPTIMAL FINANCIAL CONTRACT

$$\max_{\{p,c_1,c_a,c_2\}} \pi_1 \left(pc_a + (1-p)c_1\right) + \pi_2 c_2$$

subject to

$$\begin{aligned}
\pi_1 \left[\hat{q}\kappa_1 - p(c_a + \hat{q}\gamma) - (1-p)c_1\right] + \pi_2 \left[\hat{q}\kappa_2 - c_2\right] &\geq r(x(\omega) - S^e) \\
c_2 &\geq (1-p)\left[\hat{q}(\kappa_2 - \kappa_1) + c_1\right] \\
c_1 &\geq 0 \\
c_a &\geq 0 \\
0 \leq p &\leq 1
\end{aligned}$$

THE OPTIMAL FINANCIAL CONTRACT

- Objective is to maximize expected consumption of entrepreneur (when old)
  - Remember that the entrepreneur is risk neutral
- Let's consider the constraints one by one ...

$$\pi_1 \left[\hat{q}\kappa_1 - p(c_a + \hat{q}\gamma) - (1-p)c_1\right] + \pi_2 \left[\hat{q}\kappa_2 - c_2\right] \geq r(x(\omega) - S^e)$$

- This constraint implies that lender receive an expected return of at least r
- If the project goes ahead, the lender gets the expected return on capital minus the expected consumption of the entrepreneur and possible auditing costs
- This constraint will always bind

THE OPTIMAL FINANCIAL CONTRACT

$$c_2 \geq (1-p)\left[\hat{q}(\kappa_2 - \kappa_1) + c_1\right]$$

- This constraint implies truth-telling constraint on the part of the entrepreneur
- If this constraint is satisfied, she will not misreport the good state as the bad state
- c_1 is the agreed consumption in the bad state, and $\hat{q}(\kappa_2 - \kappa_1)$ is the extra consumption achieved from under-reporting
- This constraint binds if $p > 0$

THE OPTIMAL FINANCIAL CONTRACT

$$c_1 \geq 0, \; c_a \geq 0$$

- Requirement that the entrepreneur's consumption is non-negative in the bad state
- This is also required for c_2 but is already implied by the previous constraint
- These are "limited liability" conditions: restrict entrepreneur's ability to repay and render net worth important

THE OPTIMAL FINANCIAL CONTRACT

$$0 \leq p \leq 1$$

- This simply means that the auditing probability must be feasible

THE OPTIMAL FINANCIAL CONTRACT

- Note: the optimal contract is not a debt contract strictly speaking
  - Payments to the lender depend on the project payoff
  - This is a consequence of allowing random auditing
- Important in this contract is distinction between internal and external financing
- The CSV setting in Bernanke, Gertler, and Gilchrist (1999) (and other papers) gives rise to a debt contract in the stricter sense
  - Difference between external and internal funds is an interest rate spread

THE OPTIMAL FINANCIAL CONTRACT

- The solution to the optimal contract can be characterized for two regimes (cases)

  1. Net worth large enough that lender can repay even in bad state

  $$\hat{q}\kappa_1 \geq r\left(x(\omega) - S^e\right)$$

  2. Entrepreneur's savings S^e are insufficient, so that

  $$\hat{q}\kappa_1 < r\left(x(\omega) - S^e\right)$$

- The regimes will depend on $\hat{q}$ and S^e in equilibrium

THE OPTIMAL FINANCIAL CONTRACT

- In the first case:
  - There is no agency problem
  - $p = 0$
  - Lender's payoff independent of project outcome
  - BG1989 call this the "full collateralization" case
  - The expected level of consumption is the expected project return minus the required return for lenders

$$\hat{c}_{fc} = \hat{q}\kappa - r\left(x(\omega) - S^e\right)$$

where $\kappa = \pi_1\kappa_1 + \pi_2\kappa_2$

THE OPTIMAL FINANCIAL CONTRACT

- In the second case:
  - $p > 0$
  - Truth-telling constraint and limited liability constraint become binding
  - Can calculate the optimal choice for the auditing probability as

$$p = \frac{r\left(x(\omega) - S^e\right) - \hat{q}\kappa_1}{\pi_2 \hat{q}(\kappa_2 - \kappa_1) - \pi_1 \hat{q}\gamma}$$

  - Under the assumptions made, $0 < p < 1$
  - The expected level of consumption is now given by

$$\hat{c}_{ic} = \alpha\left(\hat{q}\kappa - r\left(x(\omega) - S^e\right) - \pi_1 \hat{q}\gamma\right)$$

where $\alpha = \frac{\pi_2 \hat{q}(\kappa_2 - \kappa_1)}{\pi_2 \hat{q}(\kappa_2 - \kappa_1) - \pi_1 \hat{q}\gamma} > 1$

FEATURES OF THE OPTIMAL CONTRACT

- Expected auditing costs are given by

$$\pi_1 p \hat{q} \gamma$$

- They are decreasing in the entrepreneur's contribution to the project, S^e
- Importantly, we have that

$$\frac{\partial \hat{c}_{ic}}{\partial S^e} = \alpha r > r$$

- This means that the return to inside funds exceeds the return to outside funds

THE ENTREPRENEURIAL INVESTMENT DECISION

- So far we have focused on an entrepreneur who undergoes investment with certainty (given ω)
- The next step to constructing the equilibrium is to characterize the decision margin to invest or not
- Recall that in full information world there were two types:
  - $\omega \leq \bar{\omega}$: invest
  - $\omega > \bar{\omega}$: store at rate r
- We now have three cases

THE ENTREPRENEURIAL INVESTMENT DECISION

- With asymmetric information, we have two cutoffs, $\bar{\omega}$ and $\underline{\omega}$, which are given by

$$\hat{q}\kappa - rx(\underline{\omega}) - \hat{q}\pi_1\gamma = 0$$

and

$$\hat{q}\kappa - rx(\bar{\omega}) = 0$$

- Three types of entrepreneurs
  - "Good" ($\omega \leq \underline{\omega}$): Positive expected net return even if $p = 1$
  - "Fair" ($\underline{\omega} < \omega \leq \bar{\omega}$): Positive expected net return only without auditing
  - "Poor" ($\omega > \bar{\omega}$): Negative expected net returns even without agency costs
    $\rightarrow$ indifference condition is the same as in perfect information case

THE ENTREPRENEURIAL INVESTMENT DECISION

- As in the perfect information case, the entrepreneurs decision (cutoff rule) is endogenous as it depends on $\hat{q}$
- Define $S^*(\omega)$ as the amount of entrepreneurial savings that will make her able to repay even in the worst state (the first regime desired for the optimal contract)

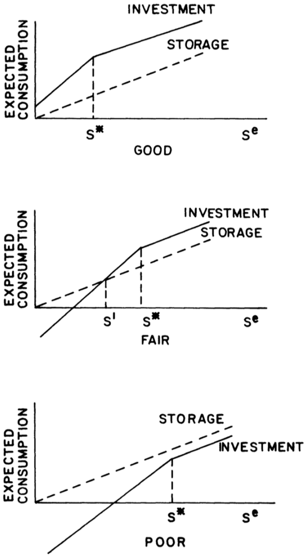
$$S^*(\omega) = x(\omega) - \frac{\hat{q}}{r}\kappa_1$$

- An entrepreneur with net worth higher than $S^*(\omega)$ will be able to invest without agency costs, zero probability of auditing
- $S^*(\omega)$ is type dependent and decreasing in $\hat{q}$

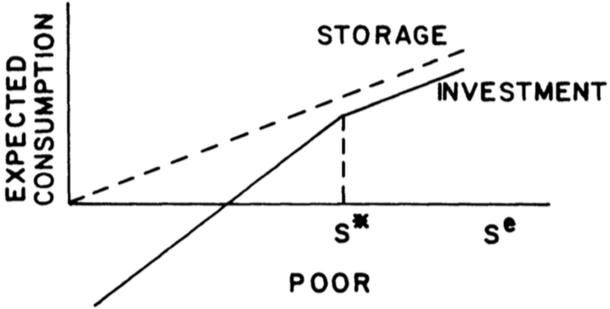
THE ENTREPRENEURIAL INVESTMENT DECISION

- Now characterize the opportunity sets of the three different types graphically
- This is still a partial equilibrium analysis, for a given $\hat{q}$ and therefore given $S^*(\omega)$
- We plot the consumption achieved by type good/fair/poor as a function of S^e
  - If $S^e(\omega) \leq S^*(\omega)$:
$$\hat{c}_{ic} = \alpha\left(\hat{q}\kappa - r\left(x(\omega) - S^e\right) - \pi_1 \hat{q}\gamma\right)$$
  - If $S^e(\omega) > S^*(\omega)$:
$$\hat{c}_{fc} = \hat{q}\kappa - r\left(x(\omega) - S^e\right)$$

THE ENTREPRENEURIAL INVESTMENT DECISION



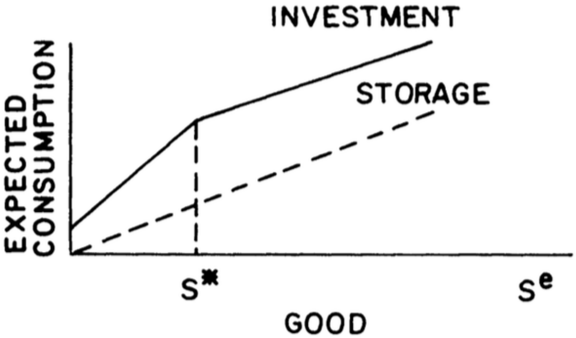
“POOR” TYPE



"POOR" TYPE

- The return to storage exceeds the project return for any level of savings
- Entrepreneurs of this type do not undertake investment projects

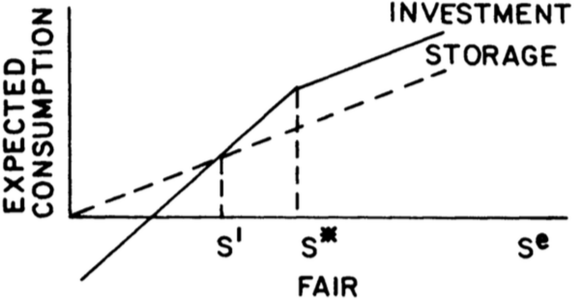
"GOOD" TYPE



“GOOD” TYPE

- If the quantity of savings contributed by the entrepreneur is below $S^*(\omega)$ the marginal return to investment is greater than the return to storage
- The entrepreneur will put all savings into her own project up to the point where they equal $S^*(\omega)$ beyond which she is indifferent
- If $S^e \leq S^*(\omega)$, auditing occurs with positive probability

“FAIR” TYPE



"FAIR" TYPE

- Prefers storage when $S^e < S'(\omega)$
- Above S' the decision is similar to the good type
- However there is a complication: convex opportunity set between 0 and $S^*(\omega)$
  - Entrepreneurs prefer fair lottery that pays $S^*(\omega)$ with prob. $\frac{S^e}{S^*(\omega)}$ and 0 otherwise
  - This means that a fraction $g(\omega)$ become fully collateralized investors (win lottery) and the rest invests zero
  - Note: one could arbitrarily rule out such lotteries or introduce concavity in the relation between wealth and returns

THE ENTREPRENEURIAL INVESTMENT DECISION

- Different outcomes of good and fair entrepreneurs highlight the crucial role of borrower net worth
- All entrepreneurs with $\omega < \bar{\omega}$ invest in a world without information frictions
- With asymmetric information, two groups emerge within this set entrepreneurs
  1. A group out of which a fraction is fully self-financing and does not face agency costs
  2. A group that is more efficient and can borrow externally, but with higher costs of external finance than internal finance
- The sizes of these groups are endogenous to the business cycle

Within-period equilibrium

- Now we focus on the determination of k_{t+1} and $\hat{q}_{t+1}$
- Similar to the full information case above, bear in mind that capital is predetermined (and thus are wages and savings)
- Derive the supply and demand curves for aggregate capital

COLLECTING TERMS

- The auditing probability as a function of ω is given by

$$p(\omega) = \max\left\{\frac{r\left(x(\omega) - S^e\right) - \hat{q}\kappa_1}{\pi_2\hat{q}(\kappa_2 - \kappa_1) - \pi_1\hat{q}\gamma}, 0\right\}$$

  for $\omega \leq \underline{\omega}$

- For $\underline{\omega} < \omega \leq \bar{\omega}$, a fraction $g(\omega) = \frac{S^e}{S^*\omega}$ of entrepreneurs invests
- Since $S^*(\omega) = x(\omega) - \frac{\hat{q}}{r}\kappa_1$, we get

$$g(\omega) = \min\left\{\frac{rS^e}{rx(\omega) - \hat{q}\kappa_1}, 1\right\}$$

- No investment for $\omega > \bar{\omega}$

- We can write total capital formation as

$$k_{t+1} = \left[\kappa\underline{\omega} - \pi_1\gamma \int_0^{\underline{\omega}} p(\omega)d\omega\right]\eta + \left[\kappa \int_{\underline{\omega}}^{\bar{\omega}} g(\omega)d\omega\right]\eta$$

  - First term: investment by good entrepreneurs
  - Second term: investment by fair entrepreneurs

- This is the capital supply function for $\gamma > 0$

- Compare full information case: $k_{t+1} = \kappa\bar{\omega}\eta$

WITHIN-PERIOD EQUILIBRIUM: CAPITAL SUPPLY

- The supply function is increasing in $\hat{q}_{t+1}$
  - Can be shown from differentiating k_{t+1} above w.r.t. $\hat{q}_{t+1}$
  - Need partial derivatives of $p(\omega)$, $g(\omega)$, $\bar{\omega}$ and $\underline{\omega}$
- The supply function lies to the left of the full information supply curve
- It approaches the full information supply curve for $\hat{q}_{t+1} \to \infty$

WITHIN-PERIOD EQUILIBRIUM: CAPITAL SUPPLY

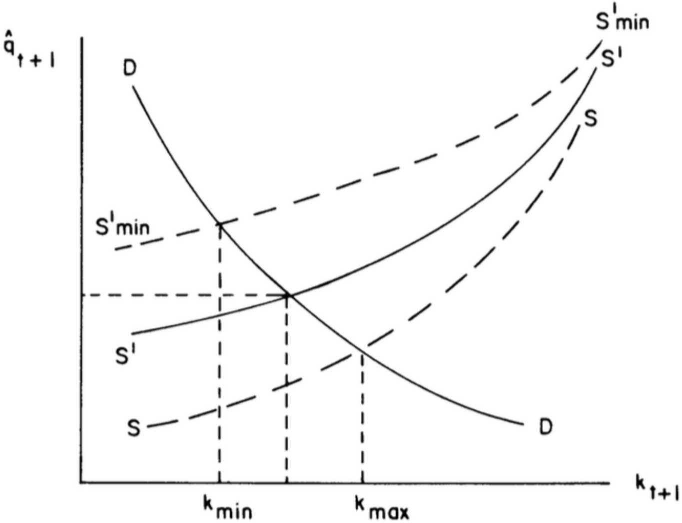
- Important: the supply curve depends on period-t state variable S^e (which enters $p(\omega)$ and $g(\omega)$)
- High values of S^e push the supply curve to the full information supply curve, low values away from it
- There is a minimum supply of capital for the minimum

WITHIN-PERIOD EQUILIBRIUM: CAPITAL DEMAND

- Same as in full information case

$$\hat{q}_{t+1} = \theta f'(k_{t+1})$$

EQUILIBRIUM



DYNAMICS

- Recall $\gamma = 0$ case:
  - q and k constant
  - Output varies directly with $\tilde{\theta}_t$
- Very different for $\gamma > 0$ case
- A transitory rise in $\tilde{\theta}_t$
  - Stimulates investment by increasing entrepreneurial net worth
  - Capital supply curves shifts out
  - Expansion persists because rise in future capital makes investment higher in subsequent periods

DYNAMICS

- The dynamics of the model captures the intuition that in good times, when balance sheets are healthy, it is easier for firms to obtain outside funds
- This stimulates investment and propagates the good times
- The analogous reasoning applies for bad times
- Crucial: countercyclical agency costs

WRAPPING UP

SOME THOUGHTS

- What do you think about the notion of business cycles formalized in this model?
- Is leverage countercyclical?
- Maybe in booms lenders are more willing to lend even conditional on fundamentals? Financial shocks?
- Maybe cyclical changes in fundamental risk?
- Quantitative relevance?

Bibliography

Bernanke, B. and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79, 14–31.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999): "The financial accelerator in a quantitative business cycle framework," *Handbook of Macroeconomics*, 1, 1341 – 1393.

Gertler, M. and P. Karadi (2011): "A model of unconventional monetary policy," *Journal of monetary Economics*, 58, 17–34.

Kiyotaki, N. and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211–248.

Townsend, R. M. (1979): "Optimal contracts and competitive markets with costly state verification," *Journal of Economic Theory*, 21, 265 – 293.

——— (1988): "Information constrained insurance: The revelation principle extended," *Journal of Monetary Economics*, 21, 411 – 450.