ECON 747 – LECTURE 11:
THE FINANCIAL ACCELERATOR IN A QUANTITATIVE BUSINESS CYCLE MODEL

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MOTIVATION

- Previous lecture: Bernanke and Gertler (1989) ["BG"] as a classic CSV model
  - Countercyclical strength of agency frictions gives rise to endogenous amplification
  - Transmission through borrower net worth
- This lecture: Bernanke, Gertler, and Gilchrist (1999) ["BGG"]
  - Take a *quantitative* standpoint
  - Add nominal rigidities and decision lags in investment
- Also highlight a number of other contributions that stand in the CSV tradition

BGG 1999

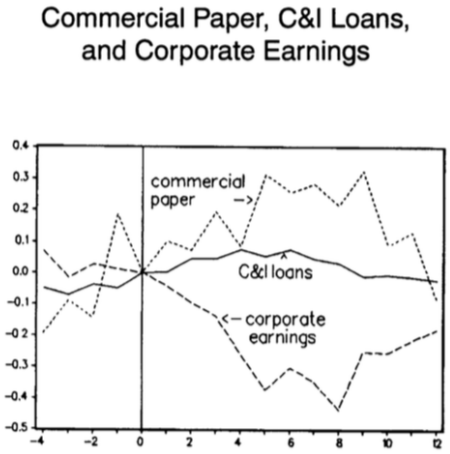
MOTIVATION OF BGG

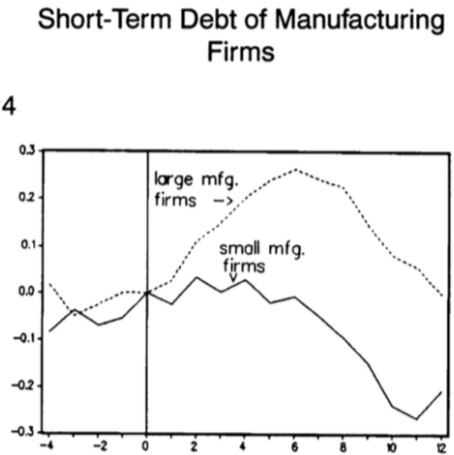
- BGG essentially motivate their work along two lines:
  1. Credit market frictions could help explaining "garden variety" cyclical fluctuations
     - Endogenous amplification is possible, as shown e.g. in BG
  2. Mounting empirical evidence that credit frictions are relevant
     - Investment literature, e.g. Fazzari, Hubbard, and Petersen (1988)
     - Empirical macro literature, e.g. earlier work by the same authors: Bernanke, Gertler, and Gilchrist (1996)
- From our standpoint, we should bear in mind that this is work from well before the financial crisis of 2008/09 ...

BGG (1996, RESTAT): EMPIRICAL EVIDENCE

- Look at episodes of monetary tightening (as defined by Romer and Romer)
- Consider as proxies for credit constraints the difference between:
  1. Bank loans vs. commercial paper
  2. Credit to large firms vs. small firms

BGG (1996, RESTAT): EMPIRICAL EVIDENCE





BGG (1996, RESTAT): EMPIRICAL EVIDENCE

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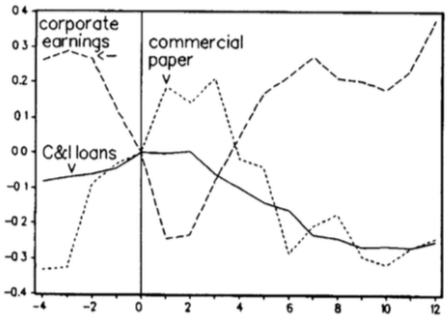
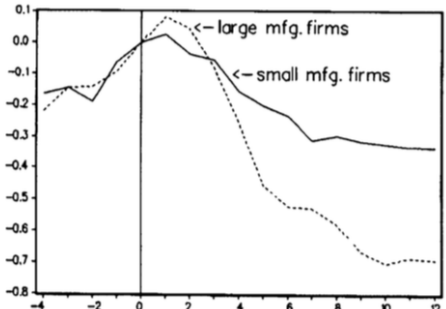
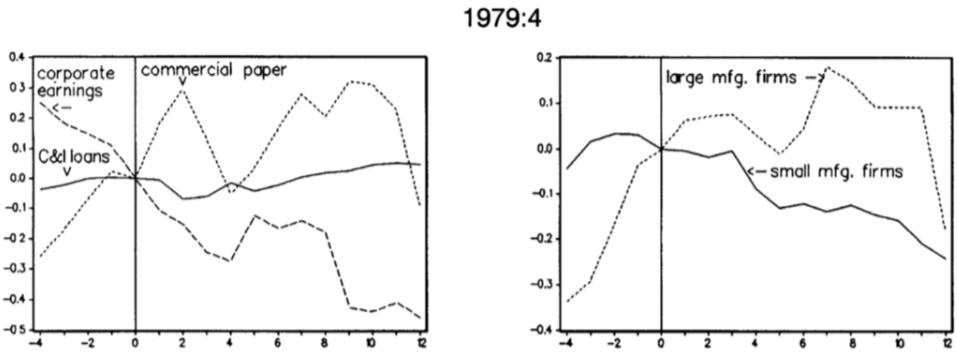




BGG (1996, RESTAT): EMPIRICAL EVIDENCE



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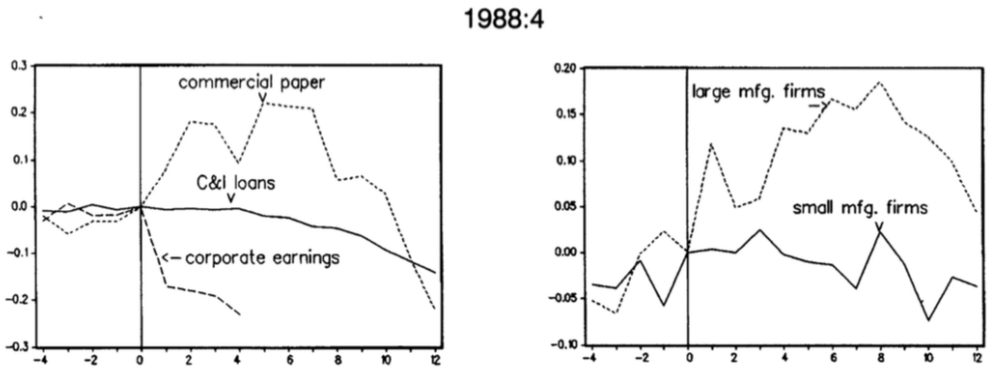


IDEA OF BGG 1999 MODEL

- Embed CSV friction into a New Keynesian framework
- Why New Keynesian?
  - Has become a widely accepted framework for quantitative business cycle questions
  - Can be used to study monetary policy
- Remember the key New Keynesian ingredients
  - Monopolistic competition
  - Sticky prices

IDEA OF BGG 1999 MODEL

- BGG explain two challenges of adding CSV into New Keynesian environment:
  1. Need lending and borrowing to occur, so need to introduce some heterogeneity
  2. Derive financial contracts from first principals, lay out the market incompleteness
- These are the very ingredients necessary for financial frictions to matter, as highlighted throughout this course

THE MODEL: OUTLINE

- Proceed in three steps
  1. Overview: agents and objectives, highlight some comparison with BG
  2. Describe agency friction: contracting problem in partial equilibrium
  3. Embed friction into New Keynesian framework, general equilibrium analysis

THE MODEL: OVERVIEW

- Discrete time, runs infinitely
- Three agents: households, entrepreneurs and retailers
- Households and entrepreneurs have similar roles as in BG
- Retailers are present to introduce nominal rigidities without complicating the entrepreneur's problem
  - Entrepreneurs become wholesalers who sell inputs to retailers
- There is a government which runs fiscal and monetary policy

THE MODEL: OVERVIEW

- Households work, consume and save
  - Relative to BG, they live infinitely and make a labor supply decision
- Entrepreneurs are risk-neutral
- Entrepreneurs die with probability γ, which gives expected live span $\frac{1}{1-\gamma}$
  - "capture phenomenon of ongoing births and deaths of firms"
  - "preclude possibility that the entrepreneurial sector will ultimately accumulate enough wealth to be fully self financing"
- Entrepreneurs acquire physical capital in t, produce with capital and labor in $t+1$, supply labor inelastically and consume

THE MODEL: OVERVIEW

- Acquisition of capital by entrepreneurs is financed from net worth and borrowing
- Net worth comes from two sources:
  1. Supplying labor inelastically (as in BG)
  2. Profits, incl. capital gains accumulated from previous capital investment (not in BG)
- Higher net worth mitigates agency problems associated with external finance and reduces the external finance premium faced by entrepreneurs in equilibrium
  - The agency problem will be discussed below

THE MODEL: OVERVIEW

- Retailers buy the goods that entrepreneurs produce, differentiate them and sell them to households
- They face monopolistic competition, so that they become price setters (face downward sloping demand curves)
- Their price setting is subject to stickiness

THE MODEL: AGENCY FRICTION

- Begin with entrepreneurs' investment & contracting problem in partial equilibrium
  - Price of capital and the marginal return on capital are taken as given for now
  - Similar way of proceeding as in BG
- Assume only one-period contracts are possible
  - E.g. because markets are sufficiently anonymous
  - Is this a good assumption?

THE MODEL: MORE DETAILS ON ENTREPRENEUR

- At time t, entrepreneur j purchases a homogeneous capital good at price Q_t
- Return between t and $t+1$ is $\omega^j R^k_{t+1}$, features idiosyncratic and aggregate risk
  - Idiosyncratic risk ω^j is iid across time and entrepreneurs
  - Distributed with cdf $F(\omega)$ and mean $\mathbb{E}(\omega^j) = 1$
- Important: setting differs from BG
  - Cost of undertaking is the same across entrepreneurs, because capital is homogeneous, all entrepreneurs face price Q_t
  - But return is different across j

- Entrepreneurs finance purchases of capital goods with net worth N and by borrowing the amount B:

$$Q_t K_{t+1}^j = N_{t+1}^j + B_{t+1}^j$$

- Here $t+1$ denotes end-of-period t quantities
- B_{t+1} is provided by households, who face an opportunity of funds given by the riskless rate of return R_{t+1}

THE MODEL: AGENCY FRICTION

- CSV problem: only entrepreneur j can observe ω^j
- Lenders must pay an auditing cost
- In BG this is a constant expressed in units of capital (γ in the BG notation)
- Here it is a share μ of the realized gross return:

$$\mu\omega^j R^k_{t+1} Q_t K^j_{t+1}$$

THE MODEL: AGENCY FRICTION

- We start by abstracting from aggregate risk
  - Not only are the return and price of capital given, for now we assume they are certain
- The setting described above gives rise to an optimal contract
- This optimal contract is a **risky debt contract**
  - Features a constant repayment, independent of the project outcome (in case of no default)
  - It is risky contract as it features default

THE MODEL: AGENCY FRICTION

- Take as given $Q_t K^j_{t+1}$, B^j_{t+1} and R^k_{t+1}
- The contract is characterized by:
  - A gross non-default loan rate Z^j_{t+1}
  - A threshold value for idiosyncratic risk $\bar{\omega}^j$ above which the loan is repaid
- The threshold value is given by

$$\bar{\omega}^j R^k_{t+1} Q_t K^j_{t+1} = Z^j_{t+1} B^j_{t+1}$$

THE MODEL: AGENCY FRICTION

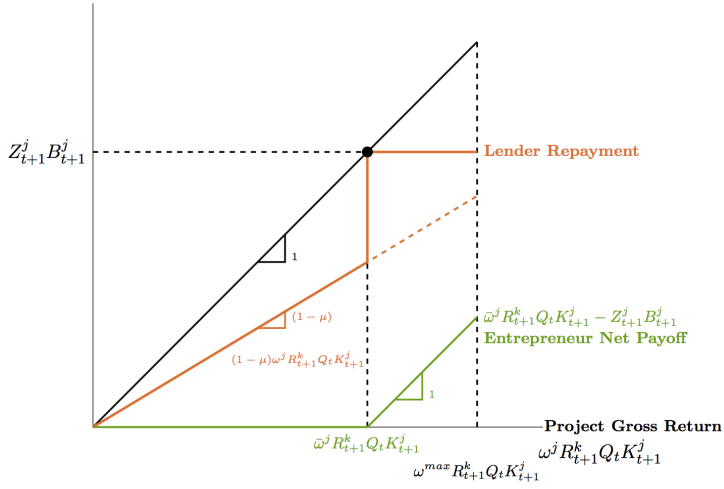
- For $\omega^j \geq \bar{\omega}^j$:
  - Lender gets $Z^j_{t+1} B^j_{t+1}$
  - Entrepreneur gets $\omega^j R^k_{t+1} Q_t K^j_{t+1} - Z^j_{t+1} B^j_{t+1}$
- For $\omega^j < \bar{\omega}^j$, default:
  - Lender gets $(1-\mu)\omega^j R^k_{t+1} Q_t K^j_{t+1}$
  - Entrepreneur gets 0

- The loan contract must satisfy lender participation **(***)**

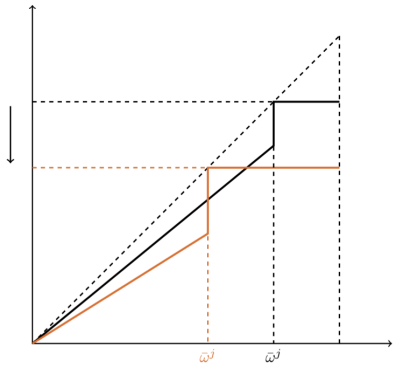
$$\left[1 - F(\bar{\omega}^j)\right] Z_{t+1}^j B_{t+1}^j + (1-\mu) \int_0^{\bar{\omega}^j} \omega R_{t+1}^k Q_t K_{t+1}^j dF(\omega)$$
$$= R_{t+1} B_{t+1}^j$$

- Tradeoff: the higher $\bar{\omega}^j$ ...
  - The higher the non-default payoff
  - The larger the default region
- Under assumptions on $F(\omega)$ and right model parameterization there is a unique interior $\bar{\omega}^j$ which maximizes expected return

RISKY DEBT CONTRACT



PREVIEW OF MECHANICS



- If net worth $\uparrow$
- Less borrowing necessary: $B^j_{t+1} \downarrow = Q_t K^j_{t+1} - N^j_{t+1} \uparrow$
- Lowers default probability, decreases lender's non-default payoff

- So far: contract in partial equilibrium, abstracting from aggregate risk
- We now stay in partial equilibrium but add aggregate risk
  - This means the price and return of capital are taken as given, but are uncertain
- Still take as given $Q_t K^j_{t+1}$ and B^j_{t+1}

THE MODEL: AGENCY FRICTION

- With aggregate risk the contract generalizes easily, due to risk neutrality of the entrepreneur
- $\bar{\omega}^j$ will now generally depend on the ex-post realization R^k_{t+1}
- Since entrepreneur is risk neutral, she only cares about the mean return and is willing to bear all aggregate risk
- Agents contract contingent on aggregate state, gives a state-contingent non-default repayment
- In other words, there is a set of of contracts, one for each realization of R^k_{t+1}

NET WORTH AND OPTIMAL CHOICE OF CAPITAL

- Given the contracting setting we now turn to the entrepreneur's general problem of choosing K_{t+1}^j (and B_{t+1}^j)
- The entrepreneur's payoff is

$$\mathbb{E}\left\{\int_{\bar{\omega}^j}^{\infty} \omega R_{t+1}^k Q_t K_{t+1}^j dF(\omega) - (1 - F(\bar{\omega}^j)) Z_{t+1} B_{t+1}\right\}$$

- Using the relation $\bar{\omega}^j R_{t+1}^k Q_t K_{t+1}^j = Z_{t+1}^j B_{t+1}^j$, we get

$$\mathbb{E}\left\{\int_{\bar{\omega}^j}^{\infty} \omega R_{t+1}^k Q_t K_{t+1}^j dF(\omega) - (1 - F(\bar{\omega}^j)) \bar{\omega}^j R_{t+1}^k Q_t K_{t+1}^j\right\}$$

- Problem reduces to choosing K_{t+1}^j and $\bar{\omega}^j$ to maximize the objective on the previous slide, subject to the constraint that the optimal contract satisfies **(***)**
- The optimality condition for this problem can be shown to be of the following form

$$Q_t K_{t+1}^j = \psi(s_t) N_{t+1}^j$$

with $s_t \equiv \mathbb{E}(R_{t+1}^k / R_{t+1})$, $\psi(1) = 1$, $\psi' > 0$

NET WORTH AND OPTIMAL CHOICE OF CAPITAL

$$Q_t K_{t+1}^j = \psi(s_t) N_{t+1}^j$$

- Key relationship in the model
- Capital expenditures are proportional to the net worth of the entrepreneur
- Proportionality factor is increasing in the expected discounted return to capital s_t
  - All else equal, if $s_t \uparrow$, the default probability is reduced, so entrepreneur can take on more debt and buy more capital
  - The increase in capital will be constrained by the fact that expected default costs also rise as the ratio of borrowing to net worth increases
- The linear relation facilitates aggregation across entrepreneurs

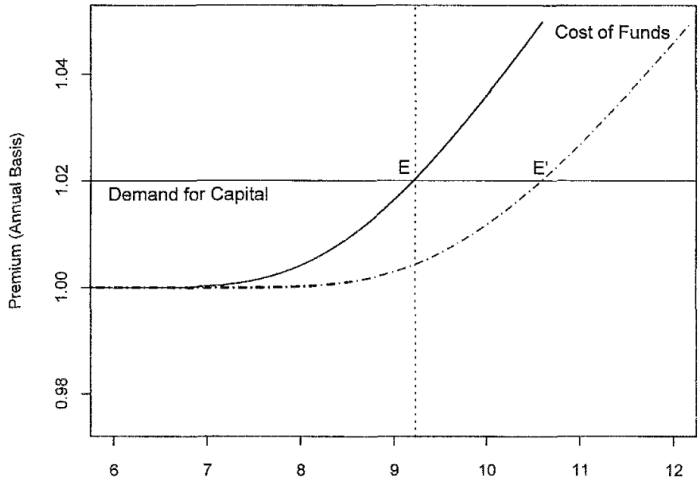
- An equivalent way of expressing the condition is

$$\mathbb{E}(R^k_{t+1}) = s\left(\frac{N^j_{t+1}}{Q_t K^j_{t+1}}\right) R_{t+1}$$

  with $s' < 0$

- Equilibrium marginal return on capital must equal the marginal cost of external finance
- External finance is given by the safe rate, scaled up by a premium term $s\left(N^j_{t+1}/Q_t K^j_{t+1}\right)$
- Premium depends inversely on the share financed by entrepreneur's own capital

NET WORTH AND OPTIMAL CHOICE OF CAPITAL



NET WORTH AND OPTIMAL CHOICE OF CAPITAL

- The chart shows the cost of funds curve $s(\cdot)$: the return relative to the risk-free rate as a function of K
- This is plotted for a given Q and given N
- If N is increased, the curve shifts out: the cost required to finance a given amount of capital falls
- We are still characterizing a partial equilibrium in which the return on capital is given (in this case by 1.02 relative to the risk-free rate)
- Therefore for a given return more capital can be financed when net worth rises

- We now embed the partial equilibrium contracting problem into a New Keynesian (NK) structure
- R_{t+1}, R^k_{t+1} and Q_t will be endogenous
- Focus on describing the entrepreneur problem, since the household and retailer problems will be standard

FULL NK MODEL: ENTREPRENEURS

- The return on capital results from the fact that capital can be used in production

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

- Here $L_t = H_t^{\Omega}(H_t^e)^{1-\Omega}$, where
  - H_t is labor endogenously supplied by households
  - H_t^e is labor inelastically supplied by entrepreneurs (will be normalized to 1)
- Like in BG, this links the available resources to the state of the economy via wages
  - As capital does not fully depreciate, resources also affected by previous return

- Capital is accumulated as follows

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) K_t + (1-\delta)K_t$$

- The presence of adjustment costs gives the familiar variation in the price of capital in equilibrium (Tobin's Q)

$$Q_t = \left[\Phi'\left(\frac{I_t}{K_t}\right)\right]^{-1}$$

- Entrepreneurs sell their output to retailers, at the relative price $1/X_t$, which they take as given
- This means that the rent paid to one unit capital is given by

$$\frac{1}{X_t}\alpha\frac{Y_{t+1}}{K_{t+1}}$$

- The capital Euler equation gives is the expected gross return on capital

$$\mathbb{E}_t(R^k_{t+1}) = \mathbb{E}_t\left[\frac{\frac{1}{X_t}\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)Q_{t+1}}{Q_t}\right]$$

- We can derive the aggregate supply of funds from the solution to the contracting problem for entrepreneur j
- Linearity in $Q_t K_{t+1}^j = \psi(s_t) N_{t+1}^j$ allows to sum over j to get

$$Q_t K_{t+1} = \psi(s_t) N_{t+1}$$

which we can invert to the relation

$$\mathbb{E}(R_{t+1}^k) = s\left(\frac{N_{t+1}}{Q_t K_{t+1}}\right) R_{t+1}$$

that we have studied above

- Aggregate entrepreneurial net worth is given by wage income as well as by equity holdings V_t (from past returns on capital) of entrepreneurs that do not die

$$N_{t+1} = \gamma V_t + W_t^e$$

- Entrepreneurs who die just consume their equity share

$$C_T^e = (1 - \gamma) V_t$$

- Equity holdings V_t are given by the gross return on capital, net of repayments to the lender
- This is the aggregation of the objective function of an individual entrepreneur in the contracting problem above (combined with lender break even condition)
- V_t is equal to

$$R_t^k Q_{t-1} K_t - \left(R_t + \frac{\mu \int_0^{\bar{\omega}} \omega R_t^k Q_{t-1} K_t dF(\omega)}{Q_{t-1} K_t - N_{t-1}} \right) (Q_{t-1} K_t - N_t)$$

  - You will derive this expression in Assignment 4

- In the above expression, the term

$$\frac{\mu \int_0^{\bar{\omega}} \omega R_t^k Q_{t-1} K_t dF(\omega)}{Q_{t-1} K_t - N_{t-1}}$$

  captures the premium for external finance

- Spread between the total rate per unit of external funds and the risk free rate R_t

- From $N_{t+1} = \gamma V_t + W_t^e$, the expression of V_t and the equilibrium expression for W_t, we can derive a law of motion for net worth:

$$N_{t+1} =$$
$$\gamma \left\{ R_t^k Q_{t-1} K_t - \left(R_t + \frac{\mu \int_0^{\bar{\omega}} \omega R_t^k Q_{t-1} K_t dF(\omega)}{Q_{t-1} K_t - N_{t-1}} \right) (Q_{t-1} K_t - N_t) \right\}$$
$$+(1-\alpha)(1-\Omega) A_t K_t^{\alpha} H_t^{(1-\alpha)\Omega}$$

FULL NK MODEL: ENTREPRENEURS

- Two key equilibrium equations
  1. The supply of funds, derive further above

$$\mathbb{E}(R^k_{t+1}) = s\left(\frac{N_{t+1}}{Q_t K_{t+1}}\right) R_{t+1}$$

  2. The evolution of net worth, shown on the previous slide

- One equation tells us how variation in net worth allows more capital to be finance, the other one how net worth endogenously varies over the business cycle

FULL NK MODEL: HOUSEHOLD AND RETAILERS

- Households work, consume and save
- Retailers buy the goods that entrepreneurs produce, differentiate them and sell them to households
  - Face monopolistic competition and price stickiness (Calvo)
- The details are omitted here

FULL NK MODEL: DYNAMICS

- The model is log-linearized
- Parameters are calibrated
- Consider the responses to the following shocks
  1. Monetary policy shock
  2. Technology shock
  3. Government expenditure shock
  4. Wealth shock
- In the case of the monetary policy shock BGG shows some VAR responses as a benchmark

MONETARY POLICY SHOCK IN SVAR

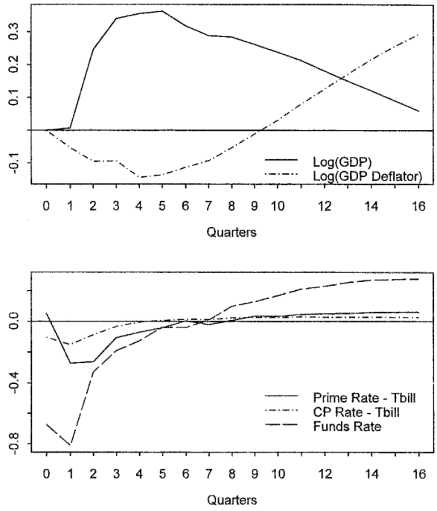


Fig. 2. Impulse response to a funds rate shock.

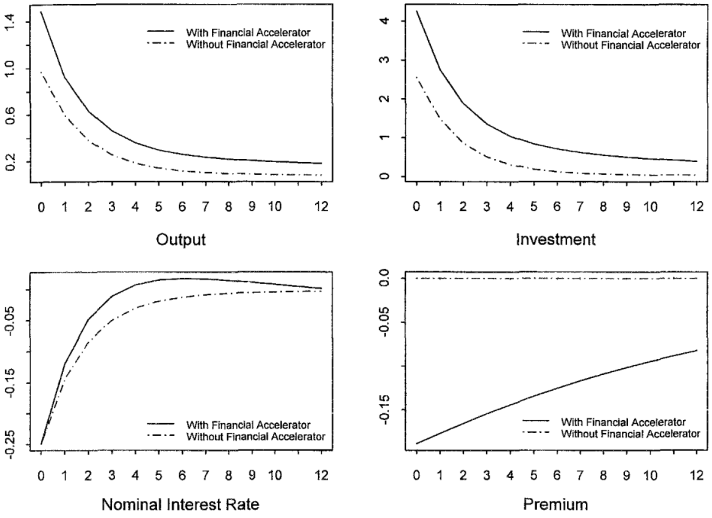
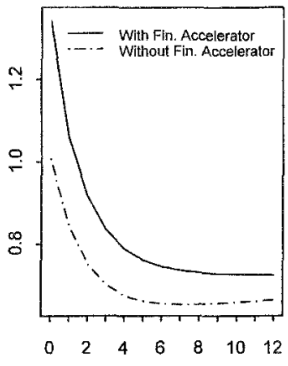
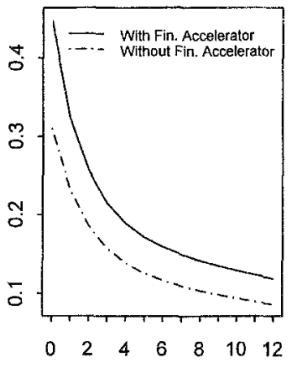


Fig. 3. Monetary shock – no investment delay. All panels: time horizon in quarters.

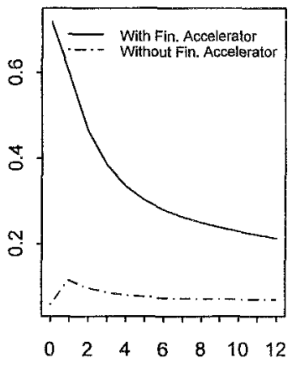
MODEL DYNAMICS



Technology Shock



Demand Shock



Wealth Shock

THE MODEL: EXTENSION WITH INVESTMENT DELAYS

- Suppose investment is chosen j periods in advance
- This generates hump shaped responses for output and investment

MODEL DYNAMICS

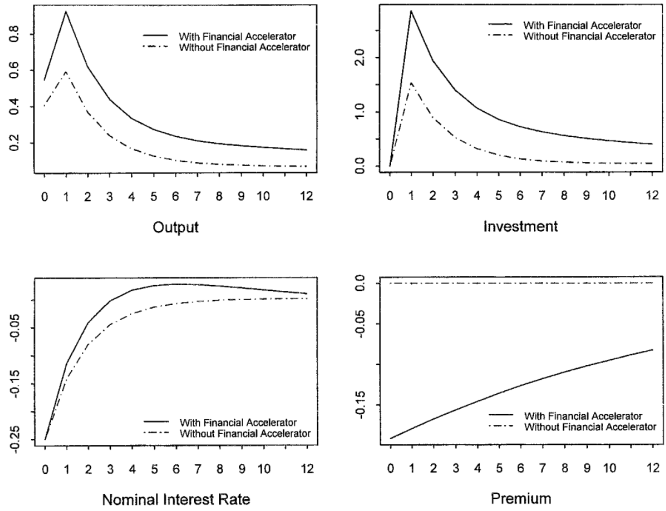


Fig. 5. Monetary shock – one period investment delay. All panels: time horizon in quarters.

THE MODEL: EXTENSION WITH SEVERAL SECTORS

- Introduce two types of firms, one with easy and one with difficult access to credit (different average premia)

MODEL DYNAMICS

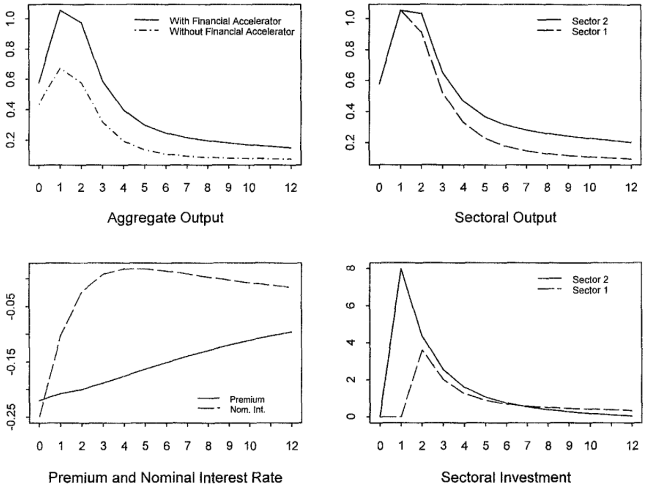


Fig. 6. Monetary shock – multisector model with investment delays. All panels: time horizon in quarters. Aggregate output: models with and without financial accelerator; other panels: model with financial accelerator.

SUMMARY OF INSIGHTS

- Financial accelerator has significant quantitative influence on business cycles
- Two-way feedback between credit and real activity at the heart of the model
- Countercyclical premia in line with what we see in the data
- Financial friction is relevant in the transmission of various shocks

SUMMARY & TAKING STOCK

A BGG SOLUTION RECIPE: 4 STEPS

1. Characterize optimal contract in partial equilibrium, no risk:
   - Take as given K^j_{t+1}, B^j_{t+1}, N^j_{t+1}, Q_t, R^k_{t+1}, R_{t+1}
   - Find Z^j_{t+1} and $\bar{\omega}^j$ (risky debt contract)
   - Solution is given by:
     (1) Threshold condition for $\bar{\omega}^j$
     (2) Lender participation **(***)**

2. Add risk to partial equilibrium contract:
   - Allow R^k_{t+1} to vary, but not endogenously
   - Can write contract contingent on realization of R^k_{t+1}

A BGG SOLUTION RECIPE: 4 STEPS

3. Determine choice of how much to invest:
   - Find K_{t+1}^j, B_{t+1}^j (together with Z_{t+1}^j and $\bar{\omega}^j$)
   - Take as given N_{t+1}^j and Q_t, R_{t+1}^k, R_{t+1}
   - Can substitute out $B_{t+1}^j Z_{t+1}^j$ from problem
   - Get optimality condition: K_{t+1}^j choice is linear in N_{t+1}^j, scale is increasing in $s_t \equiv \mathbb{E}(R_{t+1}^k / R_{t+1})$

4. Characterize general equilibrium:
   - Aggregate up condition from step 3 (linearity!) to get aggregate cost of funds curve as first key equation
   - Q_t, R_{t+1}^k, R_{t+1} determined from production, capital and safe asset choice
   - Obtain aggregate law of motion for N_{t+1} as second key equation

BG1989 vs. BGG1999: setup

- Agents:
  - BG: HH, entrepreneurs
  - BGG: HH, entrepreneurs, retailers
- Borrowing motive:
  - BG: OLG structure
  - BGG: entrepreneurs die with constant probability
- Labor supply:
  - BG: inelastic for both agents
  - BGG: choice for HH, inelastic for entrepreneur

BG1989 vs. BGG1999: friction and contract

- Heterogeneity across entrepreneurs:
  - BG: cost $x(\omega)$
  - BGG: return $\omega^j R^k_{t+1}$
- Auditing cost:
  - BG: γ units of capital
  - BGG: share μ of project outcome
- Opportunity cost of funds:
  - BG: constant r
  - BGG: endogenous R_{t+1}
- Optimal contract features:
  - BG: not a debt contract
  - BGG: debt contract

BG1989 vs. BGG1999: analysis

- General focus:
  - BG: amplification in stylized model
  - BGG: role of friction in quantitative model
- Shocks:
  - BG: productivity
  - BGG: seceral shocks, including monetary policy

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