Cross-Country Differences in Productivity: The Role of Allocation and Selection

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This paper investigates the effect of idiosyncratic (firm-level) policy distortions on aggregate outcomes. Exploiting harmonized firm-level data for a number of countries, we show that there is substantial and systematic cross-country variation in the within-industry covariance between size and productivity. We develop a model in which heterogeneous firms face adjustment frictions (overhead labor and quasi-fixed capital) and distortions. The model can be readily calibrated so that variations in the distribution of distortions allow matching the observed cross-country moments. We show that the differences in the distortions that account for the size-productivity covariance imply substantial differences in aggregate performance. (JEL D24, L25, O47)

A vast theoretical and empirical literature has been devoted to identify the sources of the large and persistent differences in productivity across countries. At the same time, a parallel strand of research has emerged over the past decade suggesting large and persistent heterogeneity in firm-level productivity, even in narrowly defined industries, in a variety of countries (e.g., Bartelsman, Haltiwanger, and Scarpetta 2004).

A few recent papers have tried to combine these two strands of research by shedding light on how cross-country differences in economic outcomes relate to differences in the within-industry productivity dispersion across firms. For example, recent papers explore the interplay between the heterogeneity in firm-level productivity, the business environment, and aggregate economic performance (see, e.g., Restuccia and Rogerson 2008; Hsieh and Klenow 2009; Alfaro, Charlton, and Kanczuk 2008; and Midrigan and Yi Xu 2010). A common element of this emerging literature is that heterogeneity in firm-level productivity performance may indicate misallocation of resources across firms with negative effects at the aggregate level.

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However, an open question is which measures of firm-level heterogeneity are most instructive for detecting possible misallocation. Hsieh and Klenow (2009) argue, for example, that higher dispersion of productivity across firms within a given industry in China and India relative to the United States reflects greater misallocation of resources in China and India.

Our paper provides a contribution to this line of research along two broad lines. First, we argue, both theoretically and empirically, that the within-industry covariance between size and productivity is a robust measure to assess the impact of misallocation. Second, we extend recent theoretical models to allow for distortions to play a role not only in the allocation of resources amongst existing firms but also in the selection of firms in the market and in the degree of firm churning.

The motivation for our approach is that the empirical evidence from firm-level data has shown that the widespread heterogeneity in firm-level performance is accompanied by substantial heterogeneity in the size of firms, even within narrowly defined industries. Moreover, consistent with core models of the size distribution of firms (e.g., Lucas 1978 and Melitz 2003), there is evidence in the firm-level data that the distributions of productivity and size exhibit a positive correlation—that is to say, more productive firms tend to be larger than less productive ones. However, our cross-country data suggest that there is considerable variation in the strength of the link between productivity and size across countries and industries and over time. The working hypothesis of this paper is that policy-induced distortions may be the source of the observed variation in this covariance across countries and that this contributes to explaining observed differences in aggregate performance.

This misallocation hypothesis is not new (see, e.g., the handbook paper by Banerjee and Duflo 2005 for a review) but the development of firm-level databases in a variety of countries now permits exploring this hypothesis more directly. In this paper, we explore the misallocation hypothesis using a variety of moments drawn from a harmonized firm-level database for the United States and a number of European countries, including transition economies of Central and Eastern Europe. Our focus is to assess the extent to which distortions can account for the observed differences across countries and over time in the within-industry productivity dispersion, as discussed by Hsieh and Klenow (2009), and in our preferred measure of the covariance between productivity and size. Further, we emphasize that distortions affect not only the allocation of resources across firms, but also the selection of firms producing in each market.

To quantify the within-industry covariance between size and productivity, we use an established empirical decomposition of the level of industry productivity as proposed by Olley and Pakes (1996)—henceforth, OP. The OP decomposition splits an index of industry-level productivity, defined as the weighted average of firm-level (log-) productivity, into an unweighted firm-level average and a covariance term. The covariance term is a summary measure of the within-industry cross sectional covariance between size and productivity. In our analysis, we find that the OP covariance term for labor productivity averages about 50 log points within US manufacturing industries. In an accounting sense, this implies that the industry index of labor productivity in the average US manufacturing industry is 50 percent higher than it would be if employment shares were randomly allocated within industries. However, the OP covariance term reaches only 20–30 log points in Western Europe,
and it was close to zero, if not negative, in Central and Eastern European countries at the beginning of their transition to a market economy. Quite remarkably, however, in these latter countries the covariance term increased substantially in the 1990s as their transition to a market economy progressed.

The use of the OP covariance term to explore the role of market distortions is, of course, not new. In their seminal contribution, OP found that the covariance term (using a decomposition of industry Total Factor Productivity, TFP) increased substantially in the US telecommunications equipment industry following the deregulation of the sector in the early 1980s. They argued that this was because the deregulation permitted outputs and inputs to be reallocated more readily from less productive to more productive US firms.

We focus our analysis on three moments of the firm-level distributions, namely the within-industry standard deviations of labor productivity and of total factor productivity, and the within-industry covariance between labor productivity and employment shares. The individual moments, as well as the relationship between them, prove instructive in assessing the role of misallocation and this class of models. We find that the within-industry dispersion of labor productivity is larger than the within-industry dispersion of total factor productivity.\footnote{Syverson (2004b) reports that, within narrowly defined industries, the difference in the United States between the ninetieth and the tenth percentiles of the firm-level productivity distributions is about 99 log points for total factor productivity (TFP) and about 140 log points for labor productivity.} Our database of harmonized moments shows that this finding is robust across the countries, industries, and time periods studied.

This finding is difficult to reconcile with many of the standard models in the literature, insofar as they have specific features that do not allow for any dispersion in labor productivity. In particular, the production function is often assumed to be Cobb-Douglas or, more generally, has the property that the \textit{average} product of labor is proportional to the \textit{marginal} product of labor. Moreover, many models make assumptions so that profit-maximizing firms equate the marginal revenue product of labor to the market wage. These two assumptions together imply that there should be no dispersion in labor productivity within industries (in the absence of distortions), even if there is significant dispersion in physical TFP. Our model includes frictions such as overhead labor that, interacted with quasi-fixed capital, allow for the dispersion in labor productivity to be larger than the dispersion in TFP, even in the absence of distortions.

In accounting for these features, we are careful to distinguish between physical and revenue based measures of productivity in the model and the data, as emphasized in the recent literature (Foster, Haltiwanger, and Syverson 2008 and Hsieh and Klenow 2009). In particular, the frictions in our model imply that even in the absence of distortions there will be substantial dispersion in revenue based measures of labor productivity and total factor productivity and that each of these alternative measures of productivity will be positively correlated with each other. A further implication is that the core prediction from models of firm heterogeneity of a positive covariance between physical output and physical productivity extends to predictions of a positive covariance between measures of size and productivity using the revenue based measures of productivity.
Furthermore, our simple model allows comparing and contrasting the effect of distortions on different moments in the data. For example, Hsieh and Klenow (2009) present evidence that the dispersion in revenue based measures of TFP is higher in China and India than in the United States. They use the quantitative variation in such measures of dispersion to back out the implied distortions that can account for these patterns. In our setting, we have similar measures of dispersion in revenue based TFP for eight countries, but we also have measures of the dispersion of revenue based labor productivity as well as the covariance between firm size and revenue labor productivity for these countries.

To preview our main results, we find that our model can be readily calibrated to match the cross-country, within-industry patterns of the covariance of productivity and size, while it is more difficult to match the cross-country, within-industry productivity dispersion patterns observed in the data. As will become clear, the reason is that the cross-country productivity dispersion patterns in the data and the model are less systematic than the cross-country covariance patterns.

The paper proceeds as follows. Section I describes the harmonized firm-level database used in our empirical analysis and presents some basic facts about the within-industry productivity dispersion (TFP and labor productivity) as well as the empirical OP decompositions of productivity. In Section II, we develop our model of allocative efficiency with idiosyncratic distortions in the selection and allocation process. Section III calibrates the model numerically to explore its implications in light of the empirical patterns discussed in Section I. Section IV reports the results from the numerical simulations of the impact of distortions on the key moments and explores the extent to which we can match the cross-country patterns of key moments observed in the data. Section V briefly summarizes the extensive sensitivity analysis we have conducted to assess the robustness of our results, while Section VI compares our results with those presented in recent literature. Section VII presents our concluding remarks.

I. The Harmonized Indicators of Dispersion and Covariance

For the empirical analysis in this paper, we use a database of harmonized industry-level moments drawn from firm-level data for five industrial economies and three transition economies of Central and Eastern Europe. The database was constructed using firm-level data from business registers, social security and corporate tax rolls, and enterprise surveys. In the construction of our database, particular
attention was devoted to harmonizing key concepts (e.g., the definition of the unit of measurement) as well as to using common methods to compute the moments. A detailed technical description of the dataset and the measures of outputs, inputs, deflators, and productivity measures can be found in Bartelsman, Haltiwanger, and Scarpetta (2009), but we provide a brief overview here.

The firm-level data of the eight countries underlying our analysis cover the manufacturing sector and include variables permitting consistent measurement of output, employment, materials, and capital inputs. The measure of gross output is based on sales data at the firm level deflated with an industry-level deflator (two- or three-digit industries). The measure of labor input is based on the number of employees, while the measure of capital is based on book values, with further adjustments described in detail in Bartelsman, Haltiwanger, and Scarpetta (2009). The measure of materials is based on nominal materials expenditures deflated with an industry-level materials deflator. Total factor productivity calculations use expenditure shares for labor, capital, and materials. Since all gross output and materials are deflated with industry-level deflators, the measures of real labor productivity (LPR) and real multi-factor productivity (TFPR) are revenue based measures.

Three basic moments from these harmonized data are used in this paper: the within-industry standard deviation of log revenue labor productivity STD(LPR); the within-industry standard deviation of log revenue total factor productivity STD(TFPR); and a measure of within-industry covariance between size and productivity (OP). To compute the covariance measure we exploit a cross-sectional decomposition of industry-level productivity developed by OP (1996). They note that an index of productivity for an industry, defined as the weighted average of firm-level productivity, can be decomposed as follows:

$$\Omega_t = \sum_i \theta_{it} \omega_{it} = \bar{\omega}_t + \sum_i (\theta_{it} - \bar{\theta}_i)(\omega_{it} - \bar{\omega}_t),$$

where $\Omega_t$ is the industry index, $\omega_{it}$ is firm-level productivity, $\theta_{it}$ is the share of activity for the firm, and a “bar” over a variable represents the unweighted industry average of the firm-level measure. The industry index of productivity includes two terms: the unweighted average of firm-level productivity and a covariance term that reflects the extent to which firms with higher than average productivity have a higher than average share of activity. This second term is the OP covariance measure used in our analysis, with log labor productivity at the firm level for $\omega_{it}$ and the firm’s labor share in the industry for $\theta_{it}$.

We acknowledge that the three moments used in this paper are quite simple. First, revenue labor productivity, used for computing STD(LPR) and OP, has the advantage of being based on measures that are widely available in firm-level datasets.
across countries. While it is computed as revenue per worker and, thus, does not control for hours or quality of labor, we find that LPR captures useful and systematic variation.\(^5\) Next, the revenue based productivity measures, LPR and TFPR, do not control for within-industry variation in firm-level prices. In our theoretical model, firm-level heterogeneity occurs in physical productivity (labelled TFPQ, as in Foster, Haltiwanger, and Syverson 2008 and Hsieh and Klenow 2009). In the model, frictions will yield a high, positive correlation between LPR, TFPR, and TFPQ, across firms in the same industry. The empirical evidence for the United States supports such high correlations. For example, Foster, Haltiwanger, and Syverson (2008) find a correlation of about 0.75 for TFPR and TFPQ and 0.6 for TFPQ and LPR in the United States.\(^6\)

Further, measurement (e.g., index number) problems generally affect cross-country comparisons of productivity. We avoid some of these problems by focusing on within-industry measures of dispersion and covariance. Moreover, in order to facilitate cross-country comparisons and, in particular, to remove the possible influence of differences in the industry composition of the manufacturing sector in the cross-country comparisons, we construct manufacturing average indexes for each country aggregating industry-level indicators using a common set of industry weights. In particular, for all of the data analysis in this paper, we use time-invariant US industry labor shares as weights to aggregate over industries.\(^7\)

Table 1 shows the three moments of interest for the sample of eight countries used in this analysis. Several observations emerge from the table. First, the within-industry dispersion of both revenue labor productivity and revenue total factor productivity is large in all countries. Second, the within-industry dispersion of revenue labor productivity always exceeds the within-industry dispersion of revenue total factor productivity.\(^8\) Thus, the finding by Syverson (2004b) is pervasive across countries and industries. Third, the covariance term is positive for almost all countries but exhibits systematic and notable cross-country patterns: in particular, the covariance term is the highest in the US manufacturing; it is much lower on average in the Western European countries and even lower in the transition economies of Central and Eastern Europe.\(^9\)

\(^5\)As a cross check on this latter point, we have used US firm-level data to conduct some robustness analysis. In particular, we considered an alternative measure of revenue labor productivity defined as revenue per unit of payroll. The latter has the advantage that payroll incorporates hours worked and, to the extent that wages reflect worker quality, it also controls for labor quality. We found that the correlation between the measure of LPR we use in this paper and this alternative is 0.80, across plants within the industries in US manufacturing. We also find that the correlation between based on real value added per worker and real value added per unit of payroll is 0.82. Moreover, when we computed the within-industry OP covariance measure for each of the more than 450 industries in US manufacturing (on a four-digit SIC basis), we found the correlation between the OP measure we use in this paper and the alternative based on revenue per unit payroll (using payroll weights) to be 0.76 (using real value added the analogous correlation is 0.82). We also find that the ratio of the OP measure we use in this paper and the payroll-based measure is about one for the average industry. The latter implies that the magnitude of the US OP measure is about the same for the employment and payroll based measures at the industry level.

\(^6\)Eslava et al. (2011) report that, in Colombia, the correlation between TFPR and TFPQ within industries is 0.70.

\(^7\)The patterns are robust to using country-average industry or country-specific industry weights. See the online Appendix for details.

\(^8\)Of the nearly 1,000 industry and year observations for the set of countries under study, the dispersion in revenue labor productivity exceeds the dispersion in revenue total factor productivity in all but 15 cases.

\(^9\)A slight exception to this pattern is that Hungary and the United Kingdom have about the same covariance term. For the United Kingdom, this may be associated with the rather weak performance of aggregate productivity performance in the early 1990s (see, e.g., Scarpetta 2003). However, a new round of data collection for EU
It is also instructive to explore the within-country variation over time of these moments. These moments are available for a number of years spanning the period 1992–2001 (except for France, where data are available through 1995). Table 2 presents the within-country changes in the moments comparing the average for the period 1997–2001 and the average for the period 1992–1996.

A number of observations emerge. First, the covariance term increased substantially in the transition economies over time, while the increases were generally less marked in Western Europe and, in particular, in the United States. This is consistent with the view that the transition to a market-based system has allowed Central and Eastern European countries to improve their allocation of resources. Over this period, the United States had more stable market structures and economic institutions. Second, the dispersion measures are relatively stable over time in all countries. Overall, the covariance term exhibits systematic cross-country variation in terms of both levels and changes, while the within-industry dispersion measures are all large but relatively stable over time. In addition, there is a systematic pattern that the dispersion of labor productivity exceeds the dispersion of total factor productivity in all countries. These are the key features of the firm-level data that we confront with the model analysis below.

II. A Model of Idiosyncratic Distortions, Selection and Allocation

To guide our analysis of distortions and allocative efficiency, we develop a model that shares some common features with Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). In common with both models, we have production units with heterogeneous productivity that face idiosyncratic distortions. Further, the curvature in the profit function, necessary for an equilibrium of firms with heterogeneous productivity, combines decreasing returns to scale and a downward sloping demand curve associated with a differentiated product environment. However, the model differs from the recent literature in a number of key dimensions. In particular, the countries for the period 1998–2005 shows that the OP covariance term in the United Kingdom reached about 0.35 in the most recent period.
model is developed to capture an important fact observed in the data in all countries: the dispersion in revenue labor productivity within industries exceeds the dispersion in revenue total factor productivity, even in economies with few or no distortions. To match this feature of the data, we include quasi-fixed capital in the presence of transitory productivity shocks and overhead labor.

### A. The Model

Starting with the behavior of firms, we assume that firms produce according to a production function given by

$$Y_{it} = A_i \varepsilon_{it}(n_{it} - f)^{\gamma-\alpha} k_{it}^\alpha, \quad \gamma < 1,$$

where $Y_{it}$ is output for firm $i$ in period $t$, $A_i$ is the firm-specific, time-invariant productivity component for firm $i$, $k_{it}$ is the amount of capital input of firm $i$ at time $t$, $n_{it}$ is employment at time $t$, $f$ is overhead labor, and $\varepsilon_{it}$ is an i.i.d. shock drawn from a time-invariant distribution and observed each period after $k$ is chosen and the decision to produce has been made. We also allow for decreasing returns to scale, possibly related to some unobserved fixed factor—such as managerial ability, as in Lucas (1978). The decreasing returns hypothesis is one of the factors that insure that the most productive firm/manager does not take over the market. The overhead labor implies that the distribution of labor productivity is not degenerate even in an economy without distortions (i.e., while the marginal revenue product of labor

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**Table 2—Changes in Productivity Dispersion and OP Covariance Term**

<table>
<thead>
<tr>
<th></th>
<th>STD in revenue labor productivity</th>
<th>STD in revenue total factor productivity</th>
<th>OP covariance term</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>0.02</td>
<td>0.00</td>
<td>0.09</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.04</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>Germany</td>
<td>0.06</td>
<td>NA</td>
<td>0.14</td>
</tr>
<tr>
<td>France</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.01</td>
<td>0.00</td>
<td>0.11</td>
</tr>
<tr>
<td>Hungary</td>
<td>-0.02</td>
<td>-0.03</td>
<td>0.18</td>
</tr>
<tr>
<td>Romania</td>
<td>0.03</td>
<td>-0.03</td>
<td>0.25</td>
</tr>
<tr>
<td>Slovenia</td>
<td>-0.06</td>
<td>-0.02</td>
<td>0.16</td>
</tr>
</tbody>
</table>


*Source: Firm-level database; see Bartelsman, Haltiwanger, and Scarpetta (2009).
will be set equal to the wage rate, the average product of labor will vary with scale, given overhead labor. Moreover, since capital is quasi-fixed, only labor will absorb the transitory shocks which, in turns, yields heterogeneity in the marginal revenue product of capital.

Firms face a downward-sloping demand schedule that arises from a differentiated products environment. The final good is assumed to be a CES aggregator of intermediate goods produced by the individual firms. The final goods sector is assumed to be perfectly competitive with the only inputs coming from intermediate goods. In particular,

\[ Y_t = N_t^{(\rho-1)/\rho} \left( \sum_i Y_{it}^\rho \right)^{1/\rho}, \]

where \( \rho < 1 \). This implementation of the CES aggregator includes an adjustment factor to make the degree of substitution scale free, as in Alessandria and Choi (2007) where \( N \) is the number of intermediate firms in operation.\(^{12}\) This implies that the inverse demand for good \( i \) is given by

\[ P_{it} = \left( \frac{\bar{Y}_t}{Y_{it}} \right)^{1-\rho}, \]

where \( P_t \) is the aggregate price for the final good and \( \bar{Y}_t \) is average output measured as final output divided by \( N \).

Firms producing the intermediate goods maximize profits, within an environment with distortions to nominal output, in each period given by

\[ \pi_{it} = (1 - \tau_i - \kappa_{it}) P_t \bar{Y}_t^{1-\rho} \left[ A_i x_{it}(n_{it} - f)^{\gamma-\alpha} k_{it}^\alpha \right]^{\rho} - w_i n_{it} - R_t k_{it}, \]

where \( w_{it} \) is the wage paid to homogenous workers, and \( R_t = r_t + \delta_t \) is the user cost of capital which equals the real interest rate plus the rate of depreciation. The firm-specific and time-invariant disturbance to revenue, \( \tau_i \), and the firm-specific time-varying disturbance, \( \kappa_{it} \), can be interpreted broadly to include any distortion that impacts the scale of a business. The inclusion of idiosyncratic components of distortion is consistent with evidence that certain regulations apply de jure differently to firms of different size,\(^{13}\) whereas others are enforced unevenly across firms. Evidence from enterprise surveys (e.g., the World Bank Investment Climate Survey, Investments Climate Assessment, see World Bank 2004) indicates that there is a

\(^{12}\)As first noted by Benassy (1996) and discussed by Alessandria and Choi (2007), including this adjustment factor permits distinguishing between the love of variety effect and the impact of market power. In our modeling, we chose not to have the “selection effect” change the degree of market power for the remaining firms. The welfare losses of distortions would be larger if utility were reduced through this channel. This is not critical for the main conclusions of the paper.

\(^{13}\)For example, regulations affecting the hiring and firing of workers apply only to firms above a certain threshold in a number of countries (see, e.g., Venn 2009).
substantial idiosyncratic component in the market and policy-driven obstacles, such as start-up costs, or enforcement of regulations and taxes, faced by different firms.\footnote{In many developing and emerging economies, labor and other regulations are de facto enforced to a different degree across firms of different size, sectoral affiliation, etc. (see, e.g., Pierre and Scarpetta 2006; World Bank 2004; Aterido, Hallward-Driemeier, and Pagés 2007).}

To make the model and analysis tractable, we assume a simple ex ante and ex post timing of information and decisions. Ex ante, before new firms enter, we assume that they do not know their productivity and distortion draws but they know the distribution of these idiosyncratic variables. New firms face a fixed cost of entry, $c_e$, that they have to pay to enter the market and to learn their draws from the joint ex ante distribution of productivity and distortions, $G(A, \tau)$. Once a firm learns its draws of $A$ and $\tau$, their values remain constant over time. In each period, the firm is subject to a further transitory idiosyncratic productivity shock from an ex ante, known distribution, and a transitory idiosyncratic distortion shock that it learns after deciding whether to produce and choosing $k$.\footnote{The fixed entry costs are denominated in output, which is reasonable since firms face these costs before starting production. They likely reflect a variety of costs including the time and effort of the entrepreneur but also the bureaucratic and other transactions costs (attorney fees, licensing and filing fees, and the like) for setting up a business. In terms of measurement of national accounts, costs that are paid in the market (attorney, licensing, filing fees) are captured as expenditures but are unlikely to be properly allocated to the correct sector for industry measures of productivity. Moreover, the time and effort of entrepreneurs in the nascent entrepreneur phase are unlikely to be well captured in national accounts and associated productivity statistics. The approach we take in the article is consistent with much of the literature including Hopenhayn (1992), Hopenhayn and Rogerson (1993), and Restuccia and Rogerson (2008). In Hopenhayn (1992) there is a further discussion of this issue and possible alternatives.}

Firms discount the future at rate $1/(1 + r)$ and face an exogenous probability of exiting in each period given by $\lambda$. Given free entry and the assumptions about the arrival of information, new firms enter up to the point where the expected discounted value of profits is equal to the entry fee. Moreover, given that the draws are time invariant in the steady state, the present discounted value for an incumbent firm $i$ ex post is given simply by

$$W(A_i, \tau_i) = \frac{E[\pi(A_i, \tau_i, \kappa_{it}, \epsilon_{it})]}{(1 - \chi)}, \quad \text{where} \quad \chi = (1 - \lambda)/(1 + r).$$

In turn, the free entry condition is given by

$$W^e = \int_{A, \tau} \max(0, W(A, \tau)) \ dG(A, \tau) - c_e,$$

where $c_e$ is the sunk entry cost. In evaluating (3), firms know their permanent draws and take expectations based on the distributions of the transitory shocks. In contrast, for (4) the firms don’t know their permanent or temporary draws and take expectations over the distribution of all the disturbances.

New firms with a low productivity and/or a high-scale distortion draw will exit immediately upon learning their draws, if they cannot cover their operating costs. In what follows, we find that the share of firms that survive upon learning their productivity and distortion draws is an important factor for assessing the overall effects of distortions. The surviving subset, $M(A, \tau)$, comprises the firms that entered based on their knowledge of the distribution $G(A, \tau)$ and continued to operate owing to
a positive present discounted value of operating profits, \( W(A_i, \tau_i) \). Distortions thus influence the pace of churning of firms, which is captured in our model by the pace of entry (the number of firms deciding to pay the entry fee) and exit (the number of firms that exit upon learning their draws plus those hit by the exogenous exit shock). More importantly, the distortions and their potential correlation with idiosyncratic productivity affect which firms survive.

Conditional on survival, the distribution \( M(A, \tau) \) and the equilibrium input prices will determine optimal firm-level capital input. In addition, given the optimal amount of capital, the labor choice will depend upon the realization of the transitory shocks. It is useful to start backwards within a period considering optimal employment for a given capital stock, which must satisfy

\[
\gamma - \alpha \rho (1 - \tau_i - \kappa_it) \left( P_t \bar{Y}_t^{1-\rho} [A_i \varepsilon_{it} k^g_{it}]^\rho (n_{it} - f)^{\gamma - \alpha \rho - 1} \right) = w_t. 
\]

In turn, the optimal capital stock must satisfy

\[
k_{it} = \left[ \frac{\alpha \rho A_i \rho P_t E(\bar{Y}_t^{1-\rho} (1 - \tau_i - \kappa_{it}) (\varepsilon_i (n_{it} - f)^{\gamma - \alpha})^\rho)}{R_t} \right]^{1/(1-\alpha \rho)}.
\]

Output and profits for the operating firm are given by (1) and (2). Even though the firm is subject to transitory productivity and distortion shocks each period, the expected profits of the firm are the same every period, and the optimal capital stock is the same every period. The firm adjusts to the ex post transitory productivity and distortion draws, \( \varepsilon_{it}, \kappa_{it} \), by adjusting employment. The firm is, of course, deciding whether to produce and with how much capital, conditional on the distribution of these transitory disturbances.

To close the model we must describe labor supply and the behavior of households and workers. A fixed number of households are assumed to supply labor inelastically so that aggregate labor supply is equal to \( N^S \). Aggregate labor demand is given by the sum of labor demands for operating firms from (5). In equilibrium, the number of firms and wages must satisfy both the free entry condition and the equality of labor demand and aggregate labor supply.

\[
W^e = 0, \quad N^d_t = N^s_t.
\]

An aggregate resource constraint ensures that aggregate consumption plus resources spent on entry and depreciation will equal aggregate output in the stochastic steady state:

\[
C_t + E_t c_e + \delta K_t = Y_t,
\]

\[^{16}\text{In steady state, gross investment is equal to replacement investment and net investment is zero.}\]
where $K_t$ is the aggregate of capital of ex post operating firms, and $E_t$ is the mass of entry.

The interest rate is pinned down by the production technology and utility maximization by the households. We assume a representative household that supplies labor inelastically and chooses consumption to maximize

$$
\sum_{t=0}^{\infty} \beta^t U(C_t).
$$

Subject to the budget constraint

$$
\sum_{t=0}^{\infty} p_t (C_t + K_{t+1} - (1 - \delta)K_t) = \sum_{t=0}^{\infty} p_t (w_t N_t + R_t K_t + \pi_t),
$$

where $p_t$ is the time zero price of period $t$ consumption, $w_t$ and $R_t$ are the period $t$ rental prices of labor and capital measured relative to period $t$ output, and $\pi_t$ is the total profit from the operations of all plants. A standard result emerges from the first-order conditions of this problem given by

$$
r_t = R_t - \delta = \left(\frac{1}{\beta}\right) - 1.
$$

So the real interest rate and rental cost of capital are pinned down by the discount factor for utility and the capital depreciation rate.

The model is deliberately simple so that steady-state inferences can be easily made. Model features that are at odds with the data include the capital stock being fixed over the firm’s life, exit being exogenous after the decision to produce has been made, and the absence of business cycles or growth.

**B. Allocation and Selection in our Heterogeneous Firm Model**

The model provides two different channels through which distortions affect output and welfare, namely, resource allocation and firm selection. The first channel, resource allocation, is present in the models of Restuccia and Rogerson (2008), Hsieh and Klenow (2009) or Midrigan and Xu (2010). In these models, allocation is distorted if the marginal revenue product of a production factor does not equal its marginal cost. Assuming wage rates to be constant across firms, Cobb-Douglas technology, and isoelastic demand, this would, for example, imply that in the absence of distortions the marginal revenue product of labor is equalized. In this environment, the average revenue product is proportional to the marginal revenue product, so this also implies equalization of the average revenue product of labor. With an idiosyncratic revenue distortion added to this environment, the dispersion in the measured marginal and average revenue products will be directly proportional to the dispersion in the idiosyncratic tax/subsidy. In Hsieh and Klenow (2010), STD(LPR) and STD(TFPR) are both proportional to the dispersion in the idiosyncratic distortion.
In our model, LPR and TFPR both exhibit dispersion and are correlated with TFPQ even without distortions. Overhead labor, quasi-fixed capital, and selection effects all play a role in influencing these patterns. With overhead labor, there is a wedge between marginal and average revenue products even in the absence of distortions. Further, overhead labor interacts with productivity heterogeneity and distortions to generate an endogenous exit threshold for firms.\(^{17}\)

To help highlight the intuition for the role of overhead labor, it is useful to consider briefly a simplified version of our model without transitory productivity shocks (and, thus, no quasi-fixed capital). Specifically, consider the production function in equation (1), without the transitory productivity term, \(\varepsilon_{it}\). In turn, consider the profit function of equation (2), again without the transitory productivity shock, and isolate the “profit shock” that is a product of the idiosyncratic tax and the productivity shock: \(S_i = (1 - \tau_i)\alpha_i^\rho\). The capital and labor allocations that satisfy the FOCs for profit maximization can be expressed as (where time subscripts have been suppressed for ease of exposition)

\[
k_i^* = S_i^{1/(1-\rho\gamma)}K_x(w, \bar{Y}) \quad \text{and} \quad \left( n_i^* - f \right) = S_i^{1/(1-\rho\gamma)}\frac{R}{w} \frac{\gamma - \alpha}{\alpha} K_x(w, \bar{Y}),
\]

where \(K_x\) is a function of the equilibrium wage rate, average output, and parameters of the model that do not vary across firms. The equilibrium wage rate, average output, and the cutoff of \(S\) below which firms do not produce, \(S_l\), can be solved for simultaneously. The equilibrium conditions imply that the flow profit at the cutoff \(S_l\) equals zero and the expected discounted profits of operating firms equal the entry fee.

The simple expressions for capital and labor yield a number of basic properties. First, factor inputs will be increasing functions of the profit shock. The implication is that in the absence of distortions, the firms with the highest TFPQ will have the highest output and inputs. It is this property that yields a positive OP covariance term if measured in terms of TFPQ and physical output or TFPQ and a composite input. The latter prediction is robust to a wide range of models. In our setting with overhead labor, such key predictions for TFPQ carry over to alternative measures of productivity such as LPR as we discuss now.

For the firms with idiosyncratic distortions and productivity above \(S_l\), the following relation must hold:

\[
\frac{P_iY_i}{n_i^* - f} = \frac{1}{1 - \tau_i} \bar{Y}^{1-\rho} \left( \frac{R}{w} \frac{\gamma - \alpha}{\alpha} \right)^{\rho(\gamma - \alpha) - 1} K_x^{\rho\gamma - 1} = \frac{1}{(1 - \tau_i)} C \propto \frac{1}{(1 - \tau_i)}.
\]

Without idiosyncratic distortions, revenue per effective unit of labor is a constant, \(C\), across firms. However, the moment of interest, LPR, will vary across firms, even without idiosyncratic distortions:

\[
\text{LPR}_i = \frac{P_iY_i}{n_i^*} = \frac{1}{(1 - \tau_i)} C - \frac{Cf}{(1 - \tau_i)n_i^*}.
\]

\(^{17}\)Overhead capital or other per-period fixed operating expenses generate similar endogenous exit thresholds.
Once the model incorporates overhead labor, average revenue products of labor will vary across firms even without distortions. It is also evident that LPR will be increasing in firm size as measured by employment in the absence of distortions. This relationship has a number of related implications. First, in the absence of distortions, the positive relationship between LPR and size will yield a positive OP covariance using LPR and employment to measure shares. Second, there will be a positive correlation between TFPQ and LPR. It is also evident that distortions will have an impact on these relationships.

Because of overhead labor, a firm facing a low value of $S$ will exit rather than pay the wage associated with $f$ and incur negative flow profits. Without idiosyncratic distortions, the cutoff will occur only for firms with productivity below a given threshold. With idiosyncratic distortion, some highly productive firms with a bad distortion draw will exit, while some low productivity firms will be able to operate (leading to a misallocation of inputs). In other words, $\tau$ distorts selection as well. The threshold $S_l$ is given by

$$S_l = \frac{\nabla^\rho \rho^{\rho \alpha} W^{1-\rho \alpha} f^{1-\rho \gamma}}{\alpha^\rho \alpha (\gamma - \alpha) \rho^{\rho \gamma} (1 - \rho \gamma)^{1-\rho \gamma}}.$$ 

The threshold depends on the equilibrium wage rate and average output, which are simultaneously determined. A higher dispersion in distortions will worsen selection based on productivity and will lower the overall efficiency of allocation of the operating firms.

We note that adding quasi-fixed capital with transitory shocks enhances the effects discussed in this section. The quasi-fixed nature of capital implies that only labor can absorb the impact of transitory shocks. This, in turn, implies that the variation in labor will yield the type of variation in LPR discussed above.

### III. Calibration of Benchmark “US” Model

We set a number of key parameters of our model drawing from the empirical evidence in the literature and choose the remaining benchmark parameters to match the US moments from Table 1. For the purpose of this “benchmark” calibration, we make the assumption that the United States is a nondistorted economy—i.e., no idiosyncratic distortions—and choose the parameters to match US moments. In the analysis in the next sections, we assume that all countries face the same technology, the same distribution of technology shocks, the same curvature parameters of the profit function and then seek to account for the cross-country variations observed in Tables 1 and 2 using variation in the distortion parameters alone.

In exploring the model simulations, it is useful to note that there are a number of possible measures of firm-level productivity that are interesting to examine. The measure of physical TFP, TFPQ, is given in the model by the product $A_i \varepsilon_i$ with the permanent component of physical productivity given by $A_i$. The measure of revenue TFP, TFPR, and the associated measure of revenue labor productivity, LPR, are also
of interest. It is important to note that in our numerical analysis, the moments and decompositions we report are based on log(TFPQ), log(TFPR), and log(LPR).\footnote{In the discussion that follows in the text, when we refer to TFPQ, TFPR, and LPR we typically omit the reference to logs for expositional convenience, but in all cases log-based measures are used. In the online Appendix, we include additional details about relating the model to the data moments.}

We also note that in our numerical analysis of the theoretical model we consider an OP decomposition of revenue labor productivity that corresponds to what we measure in the data. That is, one of the OP decompositions we consider is based on the employment-share weighted average of firm-level log revenue labor productivity. Using the simulated moment that matches what we measure in the data permits us to benchmark to the US patterns and, in turn, seek to account for differences across countries on this moment. However, theoretically, it is also interesting to consider alternative moments capturing the covariance between size and productivity. In the simulations, we consider the OP covariance using TPPQ and physical output as well as the OP covariance using TFPR and an input weight.\footnote{In the online Appendix, we show and discuss results for alternatives. As we noted in Section IIB, the most robust prediction is that in the absence of distortions, higher TFPQ firms will have more physical output. In our model, this prediction carries over to other correlations/covariances such as those between LPR and employment, TFPR and inputs, TFPR and revenue.}

For our calibration of the nondistorted economy, we select the key parameters drawing from empirical evidence. In particular,\footnote{We have conducted robustness analysis on each of these parameters, and the findings reported in the paper are robust to modest variations in them.}

\begin{itemize}
  \item $\gamma = 0.95$ (returns to scale—much of the evidence points towards close to constant returns to scale—see the discussion in Baily, Hulten, and Campbell 1992; Syverson 2004a; Foster, Haltiwanger, and Syverson 2008). We include some modest decreasing returns to scale since this has been used as an important source of curvature in the profit function in the literature (e.g., Restuccia and Rogerson 2008 and Cooper and Haltiwanger 2006). However, as will become apparent below, we also include curvature in the profit function through the demand structure. We need curvature from one source or the other but not necessarily from both.
  \item $\alpha = 0.3$ (capital output elasticity), so the implied labor elasticity is 0.65 (these are in the range of standard values for these parameters).
  \item $\lambda = 0.10$; this is consistent with evidence of exit rates in the United States (see the Business Dynamic Statistics from the US Census Bureau\footnote{See http://www/ces.census.gov/index.php/bds/bds_home.} and other OECD countries for businesses more than five years old (Bartelsman, Haltiwanger, and Scarpetta 2004), and $r = 0.02$, and $\delta = 0.10$, consistent with long-run real interest rates in OECD countries and typical depreciation rates from national accounts.
  \item $\rho = 0.8$; this is in the Broda and Weinstein (2006) range and implies a markup of 25 percent (which is broadly consistent with evidence from the cross-country empirical literature, e.g., Oliveira Martins and Scarpetta 2002).
\end{itemize}

The remaining parameters include the overhead labor parameter $f$, the entry cost $c_e$ and the variances of the permanent and transitory productivity shocks. To pin these...
parameters down for the benchmark model, we use the information in Table 1 for the United States along with auxiliary information from the harmonized database on survival rates. Our model is not well suited to explore the full dynamics of young businesses, with rich selection and learning dynamics over the first part of their life (see, e.g., Foster, Haltiwanger, and Krizan 2001, 2006). However, our model has a simple form of endogenous selection of young businesses, as entrants pay the entry fee, learn their productivity and distortion draws, and decide whether or not to produce. To approximate this process, we use information on the exit rates of young businesses in the United States. In our harmonized data, about 55 percent of businesses survive after five years in the United States. This is about the same as is found from other US sources, such as the Business Dynamic Statistics from the US Census Bureau. We match this moment in our calibration so that in the benchmark nondistorted economy, 55 percent of the businesses that pay the sunk entry cost survive. We then seek to match the US moments in Table 1. Not surprisingly, the survival, dispersion, and covariance moments are all closely connected. For example, setting a high overhead labor parameter yields lower survival rates as fewer firms can cover their fixed operating costs, and, as we shall see, this selection effect impacts the dispersion and covariance moments.

In practice, we have found that we can match exactly the survival rate and one of the moments in Table 1 for the United States, while we can respect the qualitative properties of the other remaining moments in Table 1 for the United States. The reason, as discussed in more detail below, is that the moments are nonlinear functions of key parameters. For our analysis, we have chosen to match exactly the OP covariance term so that in our benchmark calibration we have a survival rate of 55 percent and an OP covariance term of 0.51 (the value in Table 1 for the United States). For the standard deviation of revenue total factor productivity (TFPR) the benchmark calibration has 0.47 compared to 0.39 for the United States in Table 1. For the standard deviation of revenue labor productivity the benchmark calibration has 0.75 compared to 0.58 for the United States in Table 1. Thus, while we do not obtain exact matches for the two moments of productivity dispersion, they have the same qualitative properties as those observed in the data, i.e., the standard deviation of revenue labor productivity exceeds the standard deviation of revenue total factor productivity as observed in the data.22 We show below that measurement error may account for the difficulty of matching dispersion moments.

Some other properties of the benchmark model are worth noting since they are relevant for the analysis of distortions. In the calibrated model, we also compute the OP covariance term for revenue total factor productivity (using a composite input weight with factor elasticities as weights) and obtain an OP covariance term for TFPR of 0.40. Likewise, we compute the OP covariance term using physical productivity in the model and obtain an OP covariance term of TFPQ of 0.91 (using physical output weights). Thus, the calibrated model has the property that more productive businesses are larger on a number of different dimensions. This is not

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22 We have also considered benchmark calibrations in which we used the US standard deviation of TFPR as a benchmark instead of calibration to the OP covariance. The impact of distortions is quite similar both qualitatively and quantitatively to what we present here. We focus on the benchmark calibrated to the OP covariance term in the United States since we can match exactly the OP covariance term patterns in Table 1 by permitting only the distribution of distortions to vary.
surprising, since the source of heterogeneity in the benchmark model is idiosyncratic productivity variation, and more productive businesses employ more workers, use more capital, and produce more output. However, it is important to emphasize that the frictions yield the dispersion in TFPR and LPR that would otherwise not be present even with dispersion in TFPQ. A related implication is that the frictions also lead to high positive correlations between TFPQ, TFPR, and LPR in the benchmark calibration.

Two other properties of the benchmark model are worth noting. First, in matching the moments, we obtain that overhead labor accounts for 14 percent of total employment. To put this in perspective, we note that in the US manufacturing, nonproduction workers account for roughly 30 percent of total employment, and managers account for about 10 percent of the total. Classifying all nonproduction workers as overhead labor is probably too strong an assumption, while focusing only on managers does not fully account for overhead labor. Ramey (1991) reviews the evidence on the range of estimates of overhead labor and concludes that 0.20 is a reasonable consensus estimate. Interestingly, our estimate is close to these estimates and based on matching evidence on dispersion in LPR as well as survival rates.

Another instructive statistic from the calibrated model is the share of overall output accounted for by entry costs. Recall that output is used for consumption, capital accumulation (replacing depreciated capital in equilibrium), and entry costs. In the nondistorted economy, we find that about 15 percent of output goes to entry costs. This reflects the costs incurred via the ongoing churning process with some businesses exiting on an ongoing base and other businesses paying sunk entry costs to learn their draws and deciding to exit conditional on bad draws. As will become clear, this statistic is sensitive to the distortions and will be one of the factors that influence the relationship between distortions and consumption.

Before proceeding to the effects of the idiosyncratic distortions, it is worth highlighting the role that overhead labor plays in our exercise of matching the key moments in the US data. Figure 1 shows the patterns for a number of key moments and outcomes as we change the overhead labor parameter $f$, keeping all other parameters constant.\(^{23}\) The vertical line in Figure 1 shows the benchmark calibration for the United States. Lowering overhead costs relative to the benchmark leads to a lower OP covariance term, a lower standard deviation of LPR, and a much higher firm survival rate.\(^{24}\) It is clear from Figure 1 that overhead labor plays a critical role in matching a number of key features of the US data. First, it is critical for matching the finding that there is greater dispersion of revenue labor productivity relative to revenue total factor productivity. Second, overhead labor is critical for matching the magnitude of the OP covariance term. Third, overhead labor is critical for capturing the high pace of exit of recent entrants.

Figure 1 also helps illustrate the difficulty of matching all of the US moments exactly. The figure shows that, as overhead labor rises, the covariance between size and productivity first rises sharply but then becomes flat and eventually declines with further increases in overhead labor. The underlying reason for the nonmonotonicity...
is suggested by another key pattern in Figure 1—that is, the share of entering firms that survive declines monotonically with overhead labor costs. Eventually overhead labor costs become sufficiently high that much of the distribution of firms is cut off by selection, and this yields a decline in the covariance. Similarly, the standard deviation of LPR, while also initially rising sharply with overhead labor, eventually becomes flat with respect to further increases. This pattern also reflects the selection effects.

IV. The Impact of Distortions

We now turn to assess the potential role of distortions for the allocation and selection processes. Our objective is twofold. First, we explore the implications of such distortions for key outcomes of the economy. Second, we explore whether the patterns of dispersion and covariance displayed in Tables 1 and 2 can be accounted for and understood in terms of differences in the distribution of distortions across countries.

A. Uncorrelated Distortions

We begin with a relatively simple version of distortions, that is to say, permanent distortions that are uncorrelated with any fundamental. Figure 2 shows the effect of increasing the dispersion of distortions on different moments. The metric of the horizontal axis is the standard deviation of the ex ante distortions (permanent plus transitory)—we discuss how to interpret the magnitudes of this metric below. Note that the nondistorted (US benchmark) case is on the left of the figures (with zero distortions). We find that an increase in the dispersion of distortions yields modest and nonmonotonic effects on the standard deviation of LPR and TFPR, a decrease in the OP covariance terms (for LPR, TFPR, and TFPQ) and in the fraction of surviving
firms, with an accompanying increase in the cost of entry as a share of output, an increase in the capital-labor ratio, and a decline in consumption. Both the capital-labor ratio and consumption are indexed to the nondistorted economy, so an index above (below) one indicates the variable has increased (decreased) relative to the nondistorted economy.\footnote{For consumption, the log of aggregate consumption is computed and then converted to index form. For the capital-labor ratio, the log of the aggregate capital-labor ratio is computed and then converted into index form. Indices here are more interpretable given that level (even log level) units in the calibration are not meaningful. The remaining moments are all unit free.}

It is clear from Figure 2 that increasing the dispersion of the distortions has non-trivial effects on a number of indicators. One of them is the effect on the selection margin. As the “noise” from distortions increases, so does the churning of businesses that pay the sunk cost, and this has significant effects, for example, on consumption. In turn, the strong sensitivity of the selection margin implies that, while the dispersion in TFPR rises with the dispersion in distortions, as expected the quantitative increase is relatively small. Similarly, the quantitative decline in the OP covariance terms is relatively small. The reason for these outcomes is that the selection effect truncates the lower end of the distribution of firms, which mitigates the impact on the other key moments.

To interpret the magnitude of the dispersion in idiosyncratic distortions, it is useful to consider this in the context of the dispersion of profit shocks facing firms. Recall that profit shocks are given by the product of (one minus) distortions and productivity shocks (raised to the power of $\rho$). We calculate the ratio of the standard deviation of the distribution of profit shocks with idiosyncratic distortions to the standard deviation of the distribution of profit shocks without idiosyncratic distortions. This ratio provides a perspective on the magnitude of the distortion dispersion

Figure 2. The Relationship between Key Moments and Outcomes with Distortion Dispersion
(Uncorrelated case)
reported in Figure 2. Over the range of the horizontal axis in Figure 2, the ratio of dispersion of distorted to nondistorted profit shocks ranges from 1 to 2.

B. Correlated Distortions

A much stronger effect of distortions on the economy can be obtained by allowing their distribution to be positively correlated with the idiosyncratic productivity draw. This hypothesis is consistent with empirical evidence (see, e.g., the World Bank Investment Climate Survey, Aterido, Hallward-Driemeier, and Pagés 2007) suggesting that when regulations are very strict and enforcement more pronounced among large businesses and/or when corruption and rent seeking loom large, it may be better for productive firms to stay small and “fly beneath the radar screen.”

Likewise, there is evidence, as best exemplified by the experience of the centrally planned economies, suggesting that in countries characterized by widespread market- and policy-induced distortions, the largest businesses are not necessarily the most productive ones but rather those that receive preferential treatment.

Figure 3 shows the relationship between key moments and distortion dispersions when we allow the idiosyncratic productivity and the idiosyncratic distortion to be positively correlated. We set the positive correlation very high (about 0.95) and vary the dispersion of the distortions for this high degree of correlation. The relevant range of dispersion in distortions is greater in Figure 2 relative to Figure 3, because

Restuccia and Rogerson (2008) also consider correlated distortions and find that they have a much more sizeable effect on misallocation. In their exercise, they calibrate the benchmark model (nondistorted) to the US size distribution and then consider the impact of misallocation distortions. They do not seek to account for cross-country differences in observed moments.
with the correlated distortion, an increase in dispersion does not impact selection as readily. The intuition here is straightforward. Increases in dispersion of distortion tend to increase selection, whether uncorrelated or not. But in the case of correlated distortions, low productivity producers are more likely to obtain a favorable distortion draw which mitigates the impact of dispersion on selection.

We observe similar qualitative patterns to those in Figure 2, but the quantitative variation is much greater for specific moments. In particular, we find that increasing the dispersion of distortions yields: (i) an increase in TFPR dispersion; (ii) an initial decline and then an increase in LPR dispersion; (iii) substantial reductions in the OP covariance terms for TFPQ, TFPR, and LPR; (iv) a decline in survival with an accompanying increase in the entry cost as a share of aggregate output; (v) an increase in the capital-labor ratio index; and (vi) a decline in the consumption index. Even though the correlated case yields substantially larger quantitative variation in the covariance moments, it is interesting that the impact on the consumption index in the correlated case is roughly similar to what we obtain in the uncorrelated case.

There are opposite effects of distortions on the dispersion of profit shocks in this context. On the one hand, as the dispersion of distortions increases, this tends to increase the dispersion of profit shocks. On the other hand, the positive correlation between distortions and productivity shocks lowers the dispersion in profit shocks. A high productivity firm is more likely to draw a high distortion which lowers profits, while a low productivity firm is more likely to draw a low distortion (or positive subsidy) which raises profits. Over the range depicted in Figure 3, the ratio of the dispersion of total profit shocks including distortions to dispersion in profit shocks without distortions begins at 1 at the far left of Figure 3, it falls to about 0.5 when the dispersion in distortions itself is around 1.5, and then rises to about 0.7 at the far right of Figure 3. These patterns, along with Figure 3 itself, highlight that both the dispersion of distortions and the correlation of distortions with productivity are important in this context. These patterns also suggest some caution in comparing the magnitudes of the dispersion of distortions in Figures 2 and 3, since, in terms of the metric of the ratio of distorted to undistorted profit shocks, Figure 2 exhibits a range of 1 to 2, while Figure 3 exhibits a range from 1 to 0.5.

What lessons can we draw from these calibrations? First, consistent with the intuition from OP (1996), we find that economies characterized by sizable scale distortions have distorted size-productivity relationships, as measured by the OP covariance terms for TFPQ, TFPR, and LPR. We think it is striking that the impact of distortions on the covariances is similar across these alternative indicators of productivity. The ideal measure is the OP covariance using TFPQ and physical output since the canonical prediction of firm heterogeneity models is that, in the absence of distortions, this covariance should be positive. Moreover, it makes intuitive sense that scale distortions will reduce this covariance. Given the frictions we have included in our model, these patterns carry over to the covariance measures using TFPR (with input weights) and LPR (and employment weights).

Second, we find that, other than the size-productivity relationship, many other margins are affected by distortion, including the productivity dispersion, the selection margin, and the capital-labor ratio. Third, increases in the dispersion of distortions across firms reduce consumption. Fourth, the impact of the increase in the dispersion of distortions on the OP covariance terms and on productivity dispersion
is much greater if the distortions are positively correlated with productivity. This makes intuitive sense as distortions in this case have a greater bite insofar as they not only add noise but actively induce the most productive firms to stay smaller and the least productive firms to survive and be larger than would have been the case in a distortion-free environment.

C. Matching the Observed Moments

We now turn to the second objective of our analysis where we try to match the observed cross-country differences in our key moments shown in Tables 1 and 2 by using differences in distortions alone. Figures 2 and 3 already suggest that it will not be feasible to match all of the patterns shown in Tables 1 and 2. Matching the OP covariance term patterns is feasible since there is strong monotonic relationship between increasing distortions and reducing the size-productivity relationship, and Tables 1 and 2 show systematic patterns in the OP covariance terms across countries. However, Figures 2 and 3 also show that increasing distortion dispersion systematically yields some increase in the dispersion of TFPR, but they do not show systematic patterns for the dispersion of LPR. Thus, matching these patterns with the distribution of distortions alone is not feasible. Given this obvious challenge, we think it is instructive to show the patterns consistent with exactly matching the OP covariance terms in Table 1. Table 3 shows the model and data moments in this case along with the implied model implications for consumption. The implied cross-country variation in the standard deviation for LPR in the model only roughly matches the patterns in the data. In particular, the model manages to replicate the similar LPR dispersions observed in the data for the Western European economies but does not come close to matching the very high standard deviation of LPR in the transition economies. The model is also not effective in accounting for the cross-country patterns in the standard deviation of TFPR.

Taken at face value, the model yields interesting implications for the consumption patterns across countries. The general pattern is that the implied consumption index for most Western European countries is about 0.95 of that in the United States, while in some of the transition economies the consumption index is only 0.85 of that in the United States.

<table>
<thead>
<tr>
<th>Country</th>
<th>COV_LPR (Data)</th>
<th>COV_LPR (Model)</th>
<th>STD_LPR (Data)</th>
<th>STD_LPR (Model)</th>
<th>STD_TFPR (Data)</th>
<th>STD_TFPR (Model)</th>
<th>Consumption Index (Model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>0.51</td>
<td>0.51</td>
<td>0.58</td>
<td>0.75</td>
<td>0.39</td>
<td>0.47</td>
<td>1.00</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.15</td>
<td>0.15</td>
<td>0.59</td>
<td>0.66</td>
<td>0.42</td>
<td>0.69</td>
<td>0.93</td>
</tr>
<tr>
<td>Germany</td>
<td>0.28</td>
<td>0.28</td>
<td>0.71</td>
<td>0.59</td>
<td>NA</td>
<td>0.64</td>
<td>0.97</td>
</tr>
<tr>
<td>France</td>
<td>0.24</td>
<td>0.24</td>
<td>0.53</td>
<td>0.60</td>
<td>0.23</td>
<td>0.66</td>
<td>0.96</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.30</td>
<td>0.30</td>
<td>0.55</td>
<td>0.59</td>
<td>0.15</td>
<td>0.63</td>
<td>0.97</td>
</tr>
<tr>
<td>Hungary</td>
<td>0.16</td>
<td>0.16</td>
<td>1.04</td>
<td>0.65</td>
<td>0.92</td>
<td>0.69</td>
<td>0.93</td>
</tr>
<tr>
<td>Romania</td>
<td>−0.03</td>
<td>−0.03</td>
<td>1.05</td>
<td>0.72</td>
<td>0.55</td>
<td>0.70</td>
<td>0.88</td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.04</td>
<td>0.04</td>
<td>0.80</td>
<td>0.70</td>
<td>0.22</td>
<td>0.70</td>
<td>0.89</td>
</tr>
</tbody>
</table>
The calibrated model is also capable of capturing the changes in covariance terms in Table 2 for the Central and Eastern European countries during the transition. Using Figure 3 and Table 3 together, an increase in the covariance term from around 0 to 0.20 is associated with an increase in the consumption index of about 0.05 to 0.10. So again taken at face value, our results suggest that the improved size-productivity relationship in the transition economies can be associated with a substantial increase in consumption as a result of an improved allocative efficiency.

Our approach to matching moments is a form of indirect inference in the sense that we identify key parameters by matching data and model moments. That is, we choose key parameters to first match the US data moments to model moments in Table 1 as our benchmark and then in turn choose the distribution of distortions to match the cross-country variation in data moments in Table 1. However, we readily acknowledge that our approach for parameter estimation is not a formal estimation approach to indirect inference such as in Cooper and Haltiwanger (2006). In the latter approach, parameters are also chosen to minimize the difference between actual and simulated moments, but the estimation of the parameters is through the method of simulated moments. We leave such formal estimation for future work but would also note that this would likely require dealing with some of the other specification issues discussed in more detail below.

V. Robustness

This section summarizes the main results of our extensive sensitivity analysis aimed at assessing the robustness of our results to changes in the key assumptions and parameterization of our model. In particular, we consider: (i) lower levels of overhead labor; (ii) measurement errors that may affect differently the key moments used in the analysis; (iii) heterogeneity in factor elasticity of labor; and (iv) a negative, instead of positive, correlation between distortions and productivity draws. We provide a brief overview of our findings here and provide more details in the online Appendix.

A. Lower Overhead Labor

A distinguishing feature of our model and analysis is to allow for the effect of overhead labor on endogenous selection. To assess the implications for our model simulations of a lower overhead labor, we set it sufficiently low so that all firms from the ex ante productivity distribution produce in the absence of distortions. From Figure 1 we know that this implies that we cannot come close to matching the US moments for labor productivity dispersion or selection. Accordingly, we benchmark the nondistorted economy to match the standard deviation of TFPR for the United States and consider the correlated case. A number of interesting results emerge as seen in Figure 4. First, with very low overhead labor there is a substantial range over which increasing distortions has no impact on selection. However, for sufficiently large distortion dispersion, selection begins to bite. Second, productivity dispersion in terms of both TFPR and LPR rises much more rapidly with distortion dispersion when selection is not playing a role. Third, the general pattern of the covariance between size and productivity declining with distortion dispersion is confirmed even
with low overhead labor, although the pattern is somewhat more complex for LPR. For TFPQ, we find that the covariance between size and productivity falls monotonically with distortion dispersion. For LPR, the covariance first increases over some range and then declines. The increasing portion reflects the fact that in the absence of distortions and overhead labor, there is relatively low LPR dispersion. The magnitude of the covariance depends on the magnitude of the dispersion and, thus, over some range dispersion in LPR is so low that the covariance with LPR is also low. It is not surprising that the patterns for LPR are less systematic in a low overhead labor environment with low distortion dispersion. With low overhead labor, we cannot match the observed pattern that LPR has greater dispersion than TFPR. Again, recall that this is a robust finding for all countries in Table 1 (and also a robust finding in Syverson 2004b). With low overhead labor, this pattern can be met only with sufficiently large dispersion in idiosyncratic distortions.

B. Measurement Error

Measurement error in firm-level data is a ubiquitous challenge for empirical analyses and also affects significantly the recent misallocation literature that relies on within-industry second moments to identify distortions. Measurement error is likely to vary not only across countries but also across specific variables, within a country. Sales or revenue are likely to suffer from greater measurement errors than employment, while capital is likely to suffer from greater measurement error than both sales and employment. To illustrate the potential effect of measurement error, we have considered a multiplicative measurement error in revenue (where the multiplicative factor is centered at one) and assessed its impact on two key moments: the OP covariance term, using revenue labor productivity, and the standard deviation of TFPR. We find (see Figure 5) that the OP covariance term is much less sensitive to revenue measurement error than is the standard deviation of TFPR. The reason is straightforward. Multiplicative measurement error in revenue yields increased dispersion in measured revenue relative to actual revenue translating directly into the standard deviation of measured TFPR. However, multiplicative measurement error...
in revenue that is classical does not change the measured covariance between LPR and employment. This follows from the property that the expectation of the product of uncorrelated variables is the product of the expectations. However, in unreported results we show that multiplicative measurement error in employment has a substantial impact on both dispersion and the covariance. The effect on the covariance results from the fact that measurement error influences the denominator of LPR and the numerator of the employment weight, yielding a negative contribution to the covariance. However, as noted above, we think it is more plausible that revenue is measured with error than is employment.

While this simulation of the role of measurement error within the context of our model is not definitive, it also sheds light on why in the data we observe much less systematic variation across countries in dispersion moments relative to the OP covariance moment.

C. Dispersion in Factor Elasticity for Labor

In our model, we have relied on overhead labor to obtain a higher dispersion in revenue labor productivity relative to TFPR as observed in the data. There are, however, other factors that could yield this pattern between the dispersions in labor productivity and TFPR. An obvious one is to allow for heterogeneity in factor elasticities with respect to labor.

We considered a parameterization of the model with low overhead labor, no distortions, and no dispersion in factor elasticities (see Figure 6). With no dispersion in factor elasticity of labor, dispersion in TFPR matches that in the United States but is much larger than that of LPR. Moreover, the OP covariance term is relatively small and much smaller than that observed in Table 1 for the United States. As we increase the dispersion in the factor elasticity of labor (along with that of capital since in this experiment we keep the return to scale constant), dispersion in LPR rises rapidly. But the OP covariance falls rather than rises. The intuition for the latter is straightforward. Firms with high factor elasticity of labor have a lower capital/labor ratio for a given level of TFPQ which, in turn, yields a lower LPR. Hence, matching the

Figure 5. The Impact of Measurement Error on Dispersion of TFPR and Covariance (LPR)
key US moments from dispersion in factor elasticities is not feasible. We also note that other features of the data work against this assumption. We find, for example, that the persistence in labor shares in the United States over a five-year horizon is only about 0.18. Presumably dispersion in factor elasticities would yield more persistence in labor shares at the plant level.

D. Negative Correlation between Distortions and Productivity

In our framework, matching the cross-country patterns in the covariance between productivity and size requires a positive correlation between distortions and productivity. Uncorrelated distortions lower the size/productivity covariance but not by a sufficient amount. Given that the nondistorted economy is not a first-best in the presence of market power, an interesting question is whether it would be possible to increase allocative efficiency and consumption by introducing distortions that are negatively correlated with productivity. Model simulations suggest that allowing for a negative correlation between distortions and productivity can yield (at least modest) increases in the productivity/size covariance and consumption. However, the impact of changing either the degree of dispersion or the negative correlation of distortions and productivity yields nonmonotonic results. It is beyond the scope of this paper to consider optimal policy in this context, but the lack of monotonicity is intuitively the interaction of opposing forces. On the other hand, distortions by their very nature affect incentives. On the other hand, the nondistorted economy has its own distortions from market power such that businesses are not producing as much as they should given their productivity and the given level of market prices.

VI. Discussion

What are our contributions relative to the results obtained using similar models of misallocation in the recent literature? First, we are the first to consider the firm-level size-productivity relationship in this class of models. We regard this relationship as
critical to the canonical models of the size distribution of activity that all of these recent papers build upon. That is, in the absence of distortions, these models have the property that the largest firms are the most productive ones. We think it is intuitive that distortions will have a nontrivial impact on the size-productivity relationship. We present novel empirical evidence on the size-productivity relationship using a sample of countries and explore the implications in the context of these models. We also show some suggestive evidence that the size-productivity relationship is more robust to measurement error than dispersion moments.

Second, we pay particular attention to one key empirical finding from our analysis (consistent with results obtained in the recent empirical literature), namely that the within-industry dispersion in labor productivity is larger than that of total factor productivity. We regard this basic feature of the firm-level data as important since it highlights possible frictions that need to be taken into account even in the absence of distortions.

Third, and in a closely related manner, we consider the role of distortions where there is a nontrivial role for endogenous selection. None of the recent papers calibrate their models using information on survival and exit. In our setting, overhead labor plays a critical role in matching key patterns in the nondistorted benchmark and also, in turn, serves as a key factor in permitting endogenous selection.

Finally, we limit our discussion of welfare implications of distortions to the impact on consumption (per capita), through the changes in allocation and selection. By construction, closely analogous implications hold for GDP (per capita). That is, an increase in distortions reduces GDP per capita. It is potentially interesting to consider also the implications of allocation and selection for aggregate measures of productivity. There are, however, both measurement and conceptual issues in considering indices of aggregate TFP in our framework. Conceptually, there is no simple aggregate production relationship in our framework relating aggregate output to aggregate inputs. Instead, allocation and selection interact in complex ways with firm heterogeneity, frictions, and distortions to yield aggregates. Still, a number of alternative aggregate indices of productivity can be produced. For example, input-weighted indices of micro-level TFPR and TFPQ can be readily constructed within our model and, in turn, could be compared to the empirical analogs of such measures. We leave such empirical analysis for future work, as it requires appropriately harmonized indicators developed from cross-country micro data. We show in the online Appendix that, within our model framework, such indices are highly correlated with consumption and GDP per capita that we illustrate in Figure 3 as the dispersion of distortions varies. Measures of aggregate productivity relating outputs to resources required for production can also be constructed from the aggregates that emerge from our framework. While the theoretical interpretation of such an index is an open question, we show (details discussed in the online Appendix) that such an index is also highly correlated with the consumption and GDP (per capita) measures we can construct in our framework. We note that in constructing such measures it is important to take into account the resource costs of entry.

VII. Concluding Remarks

In this paper, we shed light on the impact of idiosyncratic distortions at the firm level on the allocation of resources and on aggregate outcomes. We show that the
within-industry distributions of productivity and size are closely related to each other and that this relationship varies significantly across countries. The evidence presented suggests that the size/productivity relationship is stronger in the more advanced economies, although there are large differences even within this group of countries, and became stronger in Central and Eastern European countries as they progressed in the transition to a market economy.

We confront these interesting patterns in the data with a model that seeks to account for variation in outcomes due to distortions. Our simple model with heterogeneous firms facing frictions (overhead labor and quasi-fixed capital) and idiosyncratic productivity shocks provides a reasonable match to key patterns in the US indicators of productivity and size built up from firm-level data. In particular, the model captures a number of features observed in the data, namely: the high dispersion in revenue based measures of total factor and labor productivity; the greater dispersion in labor productivity than in total factor productivity; the large positive covariance between size and productivity; and finally the high failure rate among new entrants in the early years of activity.

Using this model, we seek to match the cross-country variation in a number of key moments using differences in the distribution of distortions alone. We are successful in matching the significant within-industry differences in the size-productivity relationship across countries, with the United States experiencing a much higher covariance than countries in Western Europe and especially those in Central and Eastern Europe, but also the significant increase in the covariance observed in the latter group as they progressed in the transition to a market economy. In matching these patterns, our calibrated model implies a relatively large negative impact on consumption resulting from these distortions.

While we are quite successful in matching the size-productivity covariance patterns, we are less successful in matching other patterns observed in the data. In particular, the theoretical model implies that dispersion in revenue based measures of total factor productivity should monotonically increase with an increase in the dispersion of idiosyncratic distortions to businesses in the correlated dispersion case. While this pattern makes sense theoretically, there are not systematic patterns in the dispersion of revenue based measures of total factor productivity in the data. One possible explanation is that this moment is less robust to measurement error. However, it is equally likely that the model is not capturing other factors that influence the dispersion of productivity.

In this latter respect, we think our analysis helps to shed further light on the strengths and weaknesses of this class of models of misallocation to track key firm-based productivity moments observed in the data. While we believe these models offer rich new insights, they are quite simple relative to the theoretical and empirical models of firm behavior in the literature. For one, they are steady-state models with no meaningful dynamics. A large literature models and empirically analyzes firm dynamics, including the role of selection and learning effects for young firms as well as the adjustment dynamics of capital and labor. The adjustment dynamics of capital and labor (as well as other adjustment frictions) are undoubtedly also related to distortions (e.g., firing taxes). A more refined or different approach will be needed to capture the potential distortions to these multiple margins. But there are undoubtedly other missing pieces as well. Imposing
the same distribution of shocks and market structure across countries and in turn attributing all of the differences in cross-country firm-level moments to differences in the distribution of distortions is obviously a heroic assumption. This is well understood in this literature, and one can view the approach used as a way to assess how far distortions can go in accounting for key cross-country difference. Our strategy using multiple moments for many countries highlights some of the challenges of this approach.

REFERENCES


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