Estimation of a Labour Supply Model with Censoring Due to Unemployment and Underemployment

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This study proposes and implements a method of labour supply estimation which is appropriate when the sample contains unemployed and underemployed workers. The estimation method consists of excluding unemployed and underemployed workers from the sample and then using (to avoid selection bias) an extension of Heckman’s approach to the case where two correlated selection rules generate the sample. Hausman’s specification test is then used to determine whether ignoring constrained workers has led to biases in traditional labour supply estimates, and the empirical results suggest that previous estimates of several important parameters are biased. Since the biases go in the direction that would be predicted by the hypothesis that the unemployed and underemployed are constrained, the results support this hypothesis.

1. INTRODUCTION

In recent years the estimation of labour supply functions has been one of the most active areas of research in labour economics, and this estimation, along with most policy applications stemming from it, is based on the crucial assumption that individuals can work as many hours as they desire. However, two related developments have made this assumption less tenable. First, the number of unemployed workers has grown over the past decade. Second, a substantial body of theoretical and empirical work has developed in which unemployment does not simply represent leisure. For example, Benassy (1976), Dreze (1975), Hahn (1978) and Malinvaud (1977) present Keynesian disequilibrium models where unemployment represents a constraint on labour supply behaviour. Alternatively, Grossman and Hart (1981) develop a theoretical implicit contract model where the unemployed are willing to work at the value of their marginal product, while Abowd and Ashenfelter (1979) estimate a model of compensating wage differentials due to unemployment. Here the unemployment also represents a constraint on labour supply behaviour but now the workers are compensated for the constraint. Finally, Kiefer and Neumann (1979) and Nickell (1979) estimate models where unemployment reflects search behaviour on the part of workers.

Moreover, the unemployed are not the only potentially constrained workers in the labour market, since hours reductions or the existence of an institutionally fixed work-week may lead to workers being underemployed in the sense that they are unable to work as many hours as they want, even if they are not unemployed. If firms substitute work-sharing and reduced hours for layoffs underemployment can be caused by the same factors which lead to unemployment.

If the unemployed or underemployed are truly constrained, the standard approach to estimating labour supply functions is inappropriate. As a result, in recent years several
authors have proposed methods for handling this problem. However, all of these procedures are based on fairly restrictive assumptions (noted below); in particular they assume that these workers are constrained and/or that the constraints are exogenous and not a result of previous choice.

The purpose of this paper is to provide a new method of labour supply estimation in the presence of potentially constrained workers. This new method of estimation possesses two desirable features. First, it provides consistent estimates under much more general conditions than previous approaches; most importantly the estimates are consistent independent of whether the unemployed and underemployed are truly constrained. Second, the results of this method may be used to test the hypothesis that the unemployed and underemployed are truly constrained.

The estimation procedure is an extension of Heckman's sample selection technique to the case where two correlated selection rules generate the sample. The procedure first estimates, by bivariate probit analysis, the factors affecting the probability of unemployment and underemployment. It then estimates labour supply behaviour by excluding constrained workers from the sample and using the probit coefficients to correct for selectivity bias in the estimation. Finally, the study employs a result from Hausman (1978) to test whether the use of the standard least squares approach to labour supply estimation seriously biases the parameter estimates.

The outline of the paper is as follows. Section 2 critically examines previous approaches to the problem of estimating labour supply functions when some workers are underemployed or unemployed. Section 3 discusses the extension of Heckman's procedure and illustrates how it may be used to obtain consistent estimates of the labour supply parameters. Section 4 provides estimates of the factors affecting the probability of being unemployed or underemployed.

Section 5 uses the results of Sections 3 and 4 to estimate the labour supply function. Significant evidence of selection bias from excluding unemployed and underemployed workers is found. The results from comparing the new estimates with those obtained from the traditional approach of ignoring constrained workers support the hypothesis that these workers are constrained. First, the new estimates differ from the traditional estimates in the direction that one would predict if the unemployed and underemployed are constrained. Second, the traditional method of least squares produces several coefficients which appear to be significantly biased; this is especially true for demographic variables such as education and race. Least squares estimates are found to compound the effect of a variable on desired labour supply with the effect of the variable on unemployment or underemployment. For example, least squares overestimates the effect of education on desired labour supply because increased education significantly reduces unemployment and underemployment.

2. PREVIOUS APPROACHES TO UNEMPLOYED AND UNDEREMPLOYED WORKERS IN LABOUR SUPPLY ANALYSIS

In the past the problem of underemployed and unemployed workers in labour supply analysis has been handled in several ways. Most researchers have assumed that these workers are not constrained, while others have assumed the opposite. Some economists have taken a more agnostic approach and simply excluded the unemployed and underemployed from the sample. Each of these approaches has problems of its own and each will be discussed below.

The most common approach to unemployed and underemployed workers has been to ignore constraints and treat these workers as if they are not constrained. To see the kind of econometric problems that can arise in this situation consider the case of an individual who is truly unemployed or underemployed, and write the desired hours of
work (labour supply) equation for all workers as

$$H_i^e = X_i' \beta + \epsilon_i. \tag{1}$$

In (1), $X_i$ is a vector of standard labour supply variables, $\beta$ is a vector of coefficients and $\epsilon_i$ is a homoskedastic normally distributed error term. Equation (1) can of course be estimated by standard regression techniques given the necessary data. However, for the unemployed and underemployed only actual hours $H_i$ are available. Now by definition, $H_i < H_i^e$, which implies that $H_i - H_i^e < 0$. Equation (1) may be rewritten as

$$H_i = X_i' \beta + \epsilon_i + (H_i - H_i^e) = X_i' \beta + v_i. \tag{2}$$

In the unlikely event that $H_i - H_i^e$ is independent of $X_i$, using $H_i$ instead of $H_i^e$ as the dependent variable will simply lower the constant term. On the other hand, if the unemployed or underemployed are more likely to be young or black, or to have relatively less schooling, (as is shown to be the case below), then the independent variables will be correlated with the new error term $v_i$ and the resulting parameter estimates will be inconsistent. To see the direction of the inconsistency differentiate (2) with respect to one of the $X_i$ to obtain

$$\frac{\partial H_i}{\partial X_{ij}} = \beta_j - \frac{\partial (H_i^e - H_i)}{\partial X_{ij}}. \tag{3}$$

Now $\partial H_i/\partial X_{ij}$ will be the least squares coefficient from a regression on the full sample, while $\partial (H_i^e - H_i)/\partial X_{ij}$ is the effect of the $j$-th variable on a worker’s unemployment or underemployment, holding the other labour supply variables constant. Thus, if these workers are truly constrained, and a variable decreases (increases) unemployment or underemployment, the least squares coefficient will overstate (understate) the true effect of this variable on desired labour supply.

An alternative approach to the problem is to remove the unemployed and underemployed from the sample. Wales and Woodland (1976), (1977) removed underemployed workers from the sample while DaVanzo, De Tray and Greenberg (1976) removed the unemployed from the sample. However, censoring (removing) these workers from the sample will lead to inconsistent parameter estimates (Heckman (1979)) if the probability of being underemployed or unemployed is correlated with the error term and the independent variables. Moreover, under certain plausible assumptions the bias in the censored sample estimates will go in the same direction as the bias in the full sample estimates; thus it is incorrect to argue that if the two sets of coefficients look the same, no problems arise from ignoring or censoring constrained workers.

Finally, Ham (1980, Chapter 1) avoided some of the problems of previous studies by using a Tobit-like estimation scheme where workers faced an upper-bound equation on the hours they could work. However this model is only valid if the underemployed and unemployed are truly constrained; otherwise it will produce inconsistent parameter estimates.

There are several possible objections to this treatment of the traditional method of ignoring constraints. First, one may simply maintain that unemployment does indeed represent leisure. While some economists hold this view, others would strongly dispute it. Second, one could argue that if workers receive compensating differentials for their expected unemployment, they cannot be truly constrained since they voluntarily entered the job anticipating the constraints. While this argument may have important implications for policy, it certainly does not eliminate the problem for labour supply estimation that the unemployed are restricted in their work–leisure tradeoff, since if they were not restricted, they would not receive compensating differentials.

Third, one could argue that constrained workers comprise too small a fraction of the labour force to be of any practical importance. To examine this objection, consider
Table I which summarizes the employment experience of a sample of prime aged males taken from the University of Michigan’s Panel Study of Income Dynamics (PSID) for the period 1967–1974. For this sample, the fraction of workers either underemployed or unemployed in a given year ranged from 0.192 in 1973 to 0.282 in 1970, and thus accounting for constrained workers may indeed be important in labour supply estimation. (This study does not consider individuals wanting to work fewer hours on their job.)

It should be noted, however, that the classification scheme may overestimate the number of constrained workers. Each year the PSID contained retrospective questions on whether there was more work available (in the previous year) on a worker’s job (any of his jobs) so that he could have worked more if he had wanted to, and if not, whether he had wanted to work more. A worker who answered no to the first question and yes to the second question was classified as underemployed. Since the wage is not specified, a worker classified as underemployed may want to work more hours only at an overtime premium. A much better question would ask a worker what his marginal wage was last year, and at what marginal wage would he have been willing to work more hours. Moreover, in the PSID a worker was classified as unemployed if he lost one or more days of work through unemployment or strikes, and there is no straight-forward means of separating the two types of workers.

Thus economists are likely to differ on the existence and the importance of constraints in the labour market. As a result, it is important to develop an estimation method which is compatible with alternative views and allows the data to determine the validity of the competing hypotheses. The estimation method presented in Section 3 meets this requirement.

3. A SAMPLE SELECTION APPROACH TO ESTIMATING LABOUR SUPPLY FUNCTIONS

There are at least two ways of consistently estimating the labour supply function and testing the hypothesis that the unemployed and underemployed are constrained. Consider a general model of hours determination where there are four groups of workers: workers experiencing neither unemployment nor underemployment (Group 1); those experiencing unemployment but not underemployment (Group 2); those experiencing underemployment but not unemployment (Group 3); and those experiencing both unemployment and underemployment (Group 4). All workers have the same desired hours equation

\[ y_{ii} = X'_{ii} \beta_{1} + \epsilon_{1i} \]  

(4)
but if the unemployed and underemployed are truly constrained, desired hours are observed only for workers in Group 1. (In the remainder of the main text, the subscript \( i \) will be dropped for notational convenience.) Allowing for this possibility, define the following unconditional actual hours equations for workers in Groups 2, 3 and 4:

\[
y_2 = X'_1\beta_2 + \varepsilon_2, \quad \text{(Group 2)}
\]

\[
y_3 = X'_3\beta_3 + \varepsilon_3, \quad \text{(Group 3)}
\]

\[
y_4 = X'_4\beta_4 + \varepsilon_4, \quad \text{(Group 4)}.
\]

Assume that workers experience unemployment if

\[
y_5 = X'_5\gamma_5 + \varepsilon_5 < 0,
\]

while workers experience underemployment if

\[
y_6 = X'_6\gamma_6 + \varepsilon_6 < 0.
\]

In equations (4) through (9), \( X_1 \) through \( X_6 \) are vectors of independent variables, \( \beta_1 \) through \( \beta_4 \), \( \gamma_5 \) and \( \gamma_6 \) are vectors of parameters, and \( \varepsilon_1 \) through \( \varepsilon_6 \) are error terms.

To form the likelihood function, define

\[
Z_k = (Y_k - X'_k\beta_k)/\sigma_k, \quad k = 1, \ldots, 4
\]

and

\[
Z_k = X'_k\gamma_k, \quad k = 5, 6,
\]

where \( \sigma_k \) is the standard deviation of \( \varepsilon_k \) and \( \sigma_5 = \sigma_6 = 1 \).

Then, assuming that the vector \( \varepsilon^* = (\varepsilon_1, \ldots, \varepsilon_6) \) is independently and identically distributed across the sample with a joint normal distribution, the likelihood function is

\[
L = \prod_1 G_1(Z_1, Z_5, Z_6) \prod_2 G_2(Z_2, -Z_5, Z_6) \prod_3 G_3(Z_3, Z_5, -Z_6) \prod_4 G_4(Z_4, -Z_5, -Z_6)
\]

where the number beside the multiplication sign references the group in which the worker falls. Further, the functions \( G_k(\cdot) \) are defined by the expression

\[
G_k(r_1, r_2, r_3) = \int_{-\infty}^{r_2} \int_{-\infty}^{r_3} g_k(r_1, w_2, w_3)dw_3dw_2,
\]

where \( g_k(\cdot) \) is a standard trivariate normal density function with an appropriately defined correlation matrix. For example, the correlation matrix associated with \( g_1(\cdot) \) is

\[
C_1 = \begin{bmatrix}
1 & -\rho_{15} & -\rho_{16} \\
-\rho_{51} & 1 & \rho_{56} \\
-\rho_{61} & -\rho_{65} & 1
\end{bmatrix}.
\]

To examine the hypothesis of no constraints, one can test whether the hours equation is the same across the four groups. However, maximizing the full likelihood function to carry out this test involves estimating a prohibitively large number of parameters by non-linear methods. Further, such a test will be hampered by the relatively small number of observations in Groups 2, 3 and especially in Group 4.

In order to avoid these difficulties, an alternative and more tractable approach is followed. The labour supply function is estimated by first excluding unemployed and underemployed workers from the sample and then adjusting the estimation method to avoid sample selection bias. Separate selection rules are estimated for both unemployment and underemployment, (rather than estimating one combined rule), since the equation describing the probability of unemployment may differ from that describing underemployment (this is confirmed below) and Poirier (1980) has argued persuasively
that combining two selection rules into one will produce a hybrid model and inconsistent parameter estimates. (Indeed, many economists appear to believe that unemployed, but not underemployed, workers are potentially constrained and therefore believe the two groups are made up of significantly different types of workers.)

This selection rule approach is valid under much more general conditions than previous approaches. In particular, if the selection rules are correctly specified this method will produce consistent, albeit inefficient, estimates of the labour supply function even if none of the unemployed and underemployed are constrained. Thus, the specification test from Hausman (1978) can be used to determine whether the labour supply results are affected by moving from the full sample least squares estimates to the selection rule estimates. If a significant difference is found between the selection rule and full sample least squares estimates, this provides evidence supporting the hypothesis that the unemployed and underemployed are constrained, since if they were not constrained the labour supply estimates should not be affected by moving to the selection rule procedure.

There are two possible method of eliminating selection bias. First, one can estimate (4), (8) and (9) by maximum likelihood. The likelihood function is now

\[ L = \prod_1 G_1(Z_1, Z_5, Z_6) \prod_2 F(-Z_5, Z_6, -\rho) \prod_3 F(Z_5, -Z_6, -\rho) \prod_4 F(-Z_5, -Z_6, \rho) \]

where \( F(\cdot) \) is the bivariate standard normal distribution function.

However, maximizing (11) still involves jointly estimating a large number of parameters. Heckman (1979) proposed a two-stage estimator for the one selection rule case, and this has been extended to the two selection rule problem by both Ham (1980) and Poirier (1980). This study uses the extension of the Heckman procedure to estimate the labour supply function.

The crucial issue in correcting for bias with two selection rules is the expectation of \( y_1 \) (desired hours) conditional on \( y_5 > 0, y_6 > 0 \) (no unemployment or underemployment). Using Tallis (1961) or Lee (1979b), this conditional expectation can be expressed as

\[ E(y_1 | y_5 > 0, y_6 > 0) = X_1' \beta_1 + \sigma_{15} \lambda_5 + \sigma_{16} \lambda_6 \]

where

\[ \lambda_5 = \phi(Z_5)\Phi(Z_5^*)/F(Z_5, Z_6, \rho) \]
\[ \lambda_6 = \phi(Z_6)\Phi(Z_6^*)/F(Z_5, Z_6, \rho) \]
\[ Z_5^* = (Z_6 - \rho Z_5)/(1 - \rho^2)^{1/2} \]
\[ Z_6^* = (Z_5 - \rho Z_6)/(1 - \rho^2)^{1/2} \]
\[ \rho = \rho_{56} \]

and \( \phi(\cdot) \) and \( \Phi(\cdot) \) are the univariate standard normal density and distribution functions respectively.

If \( \sigma_{15} \) and \( \sigma_{16} \) are not both equal to zero, the expectation in (12) is not equal to \( X_1' \beta_1 \) and least squares estimation of (4) on the censored sample will lead to exactly the same sort of specification error biases that Heckman described in the one selection rule case. Of course, given Heckman's results, the solution to the problem is immediate. Estimate the parameters of the selection rule equations by bivariate probit analysis and then use these parameter estimates to form consistent estimates \( \hat{\lambda}_5 \) and \( \hat{\lambda}_6 \) of \( \lambda_5 \) and \( \lambda_6 \). Then least squares can be used to obtain consistent estimates of \( \beta, \sigma_{15} \) and \( \sigma_{16} \) by estimating

\[ y_1 = X_1' \beta_1 + \sigma_{15} \hat{\lambda}_5 + \sigma_{16} \hat{\lambda}_6 + e_1, \]

(13)
where

\[ e_1 = e_1 + \sigma_{15}(\lambda_5 - \hat{\lambda}_5) + \sigma_{16}(\lambda_6 - \hat{\lambda}_6). \]

However, the least squares estimates of \( \sigma_1^2 \) and the standard errors will be inconsistent. Consistent estimators for these parameters are given in Appendix 1.

Given either the full maximum likelihood model based on (10) or the selection rule procedure, one can see intuitively the difference between least squares on the full sample and the selection rule estimates, and why the latter will be consistent independent of whether these workers are constrained. Least squares on the full sample restricts the hours equation to be the same for both constrained and unconstrained workers while the selection rule estimator imposes no such restriction. Moreover, the selection rule approach is consistent with several different theories of unemployment. For example, if a search-theoretic explanation of unemployment (e.g. Mortenson (1970)) is accepted, the unemployment incidence equation should be interpreted as a reduced-form search equation. Alternatively, if one believes in a model where workers voluntarily sort themselves into jobs with different probabilities of layoffs and underemployment and different compensating wage differentials, the probit equations should be viewed as reduced-form sorting equations. Finally, the approach taken here is clearly consistent with a Keynesian theory where workers do not anticipate their unemployment but differ in their unemployment probabilities.

It is also possible to estimate duration equations for unemployed and underemployed workers, where a worker’s duration is defined as the difference between his desired and actual hours. Unfortunately, the current data set is too small for precise estimation of these equations. Therefore the details of this procedure and the results obtained from using it are contained in Appendix 2.

Thus the approach taken here provides consistent estimates of the labour supply function as well as a test for constraints. The first step in implementing this approach is to estimate the probit equations (8) and (9), and the paper now turns to this issue.

4. ESTIMATING THE INCIDENCE OF UNEMPLOYMENT AND UNDEREMPLOYMENT

The data used to estimate the incidence equations consisted of a cross-section of 835 prime-aged males chosen from the PSID for 1971. An individual was included in the sample if:

1. he was either black or white and between the ages of 25 and 50 in 1967;
2. he was not a member of the non-random poverty sub-sample in 1967;
3. he stayed in the sample for all the eight years 1967–1974;
4. he had not retired from the labour force.

Criterion 4 was adopted out of expediency and eliminated 40 individuals from the sample. It is clearly an example of censoring on the basis of a dependent variable, although it involves a loss of less than six per cent of the sample.8

All variables to be included in the labour supply function are also included in the probit equations for unemployment and underemployment. On the basis of previous studies and the relevant theory, the gross wage, unearned income (including a seven percent return on housing equity but excluding unemployment insurance payments), and several standard demographic variables (listed in Table III below) are included in the labour supply equation.9 The probit equations also include several other independent variables which were expected to influence the probability of experiencing unemployment or underemployment:

- U1. tenure and tenure squared;
- U2. local and occupational unemployment rates;
- U3. regional dummies;
U4. unemployment insurance benefits relative to the lagged wage;
U5. a dummy variable coded 1 if the individual belonged to a union.

Tenure, defined as the length of time a worker has been employed in his job, (measured in the previous year), and its square are included to reflect the important role that seniority can play in determining which workers face layoffs. The inclusion of the local and occupational unemployment rates is intended to capture differing demand conditions among workers. In the Michigan data the tenure and local unemployment rate variables are measured in interval form. To keep the number of parameters within reason (especially in the bivariate probit estimation), these variables were assigned the mid-point of their respective interval.\(^\text{10}\)

Since the local unemployment rate may not reflect all of the relevant regional differences in demand conditions for unemployment and especially underemployment, regional dummies are also used as independent variables. The union dummy is included for two reasons. First, using a different data set than the PSID, Medoff (1979) found that union members were more likely to experience unemployment, and this study investigates whether his result also applies to underemployment. The use of the union variable should also partially alleviate the problem of including striking workers in the unemployment data.

As in most labour supply studies, the wage is treated as endogenous. (Note that since the gross wage is being used, taxes are ignored here.) The choice of instruments for the wage is ambiguous since there are two possible causes of the endogeneity. First, unmeasured permanent characteristics may affect both an individual’s wage and desired hours. Second, the wage is formed from dividing earnings by hours worked, and any measurement error in hours will create a spurious negative correlation between hours and wages. Borjas (1980) considered this problem and concluded that if an alternative measure of hours worked is available, (in addition to the measure used as the dependent variable), the alternative measure should be used to calculate the wage. The Michigan data does not provide an alternative measure of hours worked in the same year, but a natural substitute is to use the lagged wage as an instrument for the current wage. (Borjas found that substituting a lagged wage for the alternative measure did not affect the results.) This should eliminate the measurement error problem but will not deal with endogeneity due to unmeasured characteristics. To deal with the latter problem, one must consider additional variables for instruments (such as variables U1, U2, U3 and U5 above). Unfortunately, Borjas found that when this was done the labour supply results could be extremely sensitive to the choice of instruments and exclusion restrictions. As a result, this study uses the lagged wage as an instrument for the wage (rather than simply substituting in for it), but also examines the sensitivity of the labour supply results to using Borjas’ procedure.

Treating the wage as endogenous also necessitates modifying the selection rule approach. One can eliminate the simultaneous nature of the selection rules by following Lee (1979a) and substituting in the wage equation for the wage in the probit equations, estimating reduced-form probit coefficients, and using these coefficients to form the \(\lambda\) terms. To deal with the endogeneity of the wage in the structural hours equation, Lee, Maddala and Trost (1980) have shown that the proper procedure is to first regress the wage on all of the instruments and the \(\lambda\) terms and use the coefficients to form an estimated wage for each individual. Then one replaces the wage by its estimated value in the hours equation and proceeds in the usual selection rule fashion. (This approach is outlined in more detail in Appendix 1.)

In order to implement the selection rule method, reduced-form unemployment and underemployment equations were estimated by bivariate probit analysis and the results have been placed in the first two columns of Table II.\(^\text{11}\) To obtain structural incidence equations, individual reduced-form probit equations were estimated. Then the structural
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unemployment and underemployment probit coefficients were derived from the reduced-
form estimates and placed in the third and fourth columns of Table II. Since both

<table>
<thead>
<tr>
<th>Variable</th>
<th>Bivariate probit estimates of reduced-form parameters</th>
<th>Univariate probit estimates of structural parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unemployed$^2$</td>
<td>Underemployed$^3$</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.0948</td>
<td>-0.4250</td>
</tr>
<tr>
<td></td>
<td>(0.8465)</td>
<td>(0.8351)</td>
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<tr>
<td>Lagged wage ($)</td>
<td>-0.0754</td>
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<tr>
<td></td>
<td>(0.0676)</td>
<td>(0.0538)</td>
</tr>
<tr>
<td>Wage ($)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unearned income (000's of $)</td>
<td>0.1408</td>
<td>-0.0702</td>
</tr>
<tr>
<td></td>
<td>(0.2670)</td>
<td>(0.2216)</td>
</tr>
<tr>
<td>Married$^4$</td>
<td>-0.4609</td>
<td>0.1160</td>
</tr>
<tr>
<td></td>
<td>(0.2896)</td>
<td>(0.2880)</td>
</tr>
<tr>
<td>Number of children</td>
<td>0.0826*</td>
<td>0.0218</td>
</tr>
<tr>
<td></td>
<td>(0.0422)</td>
<td>(0.0400)</td>
</tr>
<tr>
<td>Health limitation$^4$</td>
<td>0.0113</td>
<td>0.0393</td>
</tr>
<tr>
<td></td>
<td>(0.2767)</td>
<td>(0.2154)</td>
</tr>
<tr>
<td>Age (years $\times 10^{-1}$)</td>
<td>-0.0729</td>
<td>-0.0468</td>
</tr>
<tr>
<td></td>
<td>(0.1002)</td>
<td>(0.0909)</td>
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<tr>
<td>Education (years)</td>
<td>-0.0473**</td>
<td>-0.0750*</td>
</tr>
<tr>
<td></td>
<td>(0.0271)</td>
<td>(0.0252)</td>
</tr>
<tr>
<td>Black$^4$</td>
<td>0.0145</td>
<td>0.2653</td>
</tr>
<tr>
<td></td>
<td>(0.2390)</td>
<td>(0.2162)</td>
</tr>
<tr>
<td>Tenure (years $\times 10^{-1}$)</td>
<td>-0.9189*</td>
<td>-0.0433</td>
</tr>
<tr>
<td></td>
<td>(0.2920)</td>
<td>(0.2430)</td>
</tr>
<tr>
<td>Tenure-squared</td>
<td>0.3383*</td>
<td>-0.0373</td>
</tr>
<tr>
<td>(years $\times 10^{-2}$)</td>
<td>(0.1112)</td>
<td>(0.0975)</td>
</tr>
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<td>Union$^4$</td>
<td>0.6816*</td>
<td>0.3276*</td>
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<tr>
<td></td>
<td>(0.1532)</td>
<td>(0.1253)</td>
</tr>
<tr>
<td>Local unem. Rate (%)</td>
<td>0.0899*</td>
<td>0.0239</td>
</tr>
<tr>
<td></td>
<td>(0.0297)</td>
<td>(0.0263)</td>
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<tr>
<td>Occupational unemployment</td>
<td>0.0365</td>
<td>0.0524**</td>
</tr>
<tr>
<td>Rate (%)</td>
<td>(0.0377)</td>
<td>(0.0323)</td>
</tr>
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<td>North east$^4$</td>
<td>-0.1341</td>
<td>0.8219*</td>
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<tr>
<td></td>
<td>(0.2102)</td>
<td>(0.2064)</td>
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<td>North Central$^4$</td>
<td>-0.1130</td>
<td>0.4773*</td>
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<tr>
<td></td>
<td>(0.2154)</td>
<td>(0.1943)</td>
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<tr>
<td>South$^4$</td>
<td>0.1072</td>
<td>0.5064*</td>
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<tr>
<td></td>
<td>(0.2194)</td>
<td>(0.2105)</td>
</tr>
<tr>
<td>Unemployment insurance$^5 \times 10^{-1}$</td>
<td>-0.1526</td>
<td>-0.1799</td>
</tr>
<tr>
<td></td>
<td>(0.1715)</td>
<td>(0.1602)</td>
</tr>
</tbody>
</table>

$^p$                         

Log L                          
-612                           
-262                           
-353

Notes:
2. Dependent variable coded 1 if unemployed, 0 otherwise.
3. Dependent variable coded 1 if underemployed, 0 otherwise.
4. All dummy variables are coded 1 if individual is in category.
5. Calculated as ratio of potential weekly unemployment benefits to lagged wage.
6. *Significant at the 5% level.
7. **Significant at the 10% level.
equations are exactly identified, the structural standard errors are calculated in the usual fashion, using results from Amemiya (1978).

The structural parameter estimates in Table II suggest that increased schooling or being married reduces the probability of unemployment while union membership or more children increases this probability. The result for children is surprising and there appears to be no obvious explanation for it. Increased tenure up to 14 years lowers the probability of unemployment after which the probability of unemployment begins to increase with tenure. Since workers with between 10 and 20 years of tenure are assigned a value of 15 years, this result may reflect workers with more than 20 years of tenure taking early retirement with a pension and then searching for a new job. As one would expect, increases in both the local and occupational unemployment rates increase the probability of unemployment, although only the effect of the former rate would be judged significant at conventional test levels. The unemployment insurance coefficient is perversely signed but is clearly insignificant. Since most striking workers are not eligible for unemployment insurance (Kennan (1979)), this result may reflect in part the mixing of the strike and unemployment data.

Increased schooling also significantly reduces the probability of underemployment while increases in the occupational unemployment rate and union membership raise this probability. Thus, union members are more likely also to experience underemployment. The regional coefficients indicate that the probability of underemployment may differ with respect to location, even holding the local unemployment rate constant. (A likelihood ratio test for the joint significance of the regional coefficients was not carried out.) Finally, the unemployment insurance variable is again insignificant, but now plausibly signed, since if firms face imperfect experience rating they would be more likely to substitute layoffs for work-sharing as unemployment benefits increase.

In Section II it was shown that if the underemployed and unemployed are truly constrained, then least squares on the full sample would overestimate (underestimate) the coefficient of a variable which reduced (increased) unemployment and underemployment. Thus, for the labour supply results in the next section to provide evidence of constraints, not only must the full sample and selection rule estimates differ, but they must differ in a particular direction. In order to predict the direction of the potential bias, the impact of a variable on unemployment or underemployment, holding only the other labour supply variables constant, is needed. To carry out the prediction analysis, the structural incidence equations were re-estimated using only the independent variables from the labour supply equation, and the results have been placed in Table III.12

<table>
<thead>
<tr>
<th>TABLE III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probit coefficients for prediction of bias in labour supply estimates</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unemployment Coefficient/Standard error</th>
<th>Underemployment Coefficient/Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.4820 (0.5167)</td>
<td>0.4849 (0.4859)</td>
</tr>
<tr>
<td>Wage</td>
<td>-0.0227 (0.0314)</td>
<td>0.0198 (0.0260)</td>
</tr>
<tr>
<td>Unearned income</td>
<td>0.0624 (0.2177)</td>
<td>-0.0402 (0.1961)</td>
</tr>
<tr>
<td>Married</td>
<td>-0.3524 (0.2604)</td>
<td>0.0577 (0.2610)</td>
</tr>
<tr>
<td>Number of children</td>
<td>0.0654** (0.0388)</td>
<td>0.0345 (0.0362)</td>
</tr>
<tr>
<td>Health limitation</td>
<td>-0.0039 (0.2443)</td>
<td>0.1285 (0.2131)</td>
</tr>
<tr>
<td>Age</td>
<td>-0.0986 (0.0846)</td>
<td>-0.1223 (0.0763)</td>
</tr>
<tr>
<td>Education</td>
<td>-0.0810* (0.0216)</td>
<td>-0.0969* (0.0197)</td>
</tr>
<tr>
<td>Black</td>
<td>0.0482 (0.2241)</td>
<td>0.3824** (0.1975)</td>
</tr>
<tr>
<td>Log L</td>
<td>-2.93</td>
<td>-3.75</td>
</tr>
</tbody>
</table>

*Note: Units and notes the same as in Table II.*
Assuming that the unemployed and underemployed are truly constrained, and considering only those coefficients which are significant at least at the 10% level, then least squares on the full sample should overestimate the coefficients for age and education while underestimating the race and number of children coefficients. (The age coefficient has an asymptotic normal statistic of $-1.6$ in the underemployment equation.) These estimates are based on the assumption that the gross wage is a function only of the lagged wage and the other labour supply variables. If one assumes that variables U1 through U5 above (tenure, etc.) also belong in the wage equation, then Amemiya’s principle (Amemiya (1978)) should be used to estimate the structural probit equations. Implementing Amemiya’s principle does not change any of the predictions.

5. ALTERNATIVE ESTIMATES OF THE LABOUR SUPPLY FUNCTION

In this section the labour supply function is estimated by least squares on the full and censored samples as well as by the selection rule method on the censored sample. (I assumed that the lagged wage is the only identifying instrument so that the labour supply function is exactly identified; adding variables U1 through U5 above to the wage equation does not affect the estimates.)

Consider first the full sample least squares estimates in column one of Table IV. The wage coefficient implies an uncompensated elasticity of $-0.16$ (quite close to Lewis’

**TABLE IV**

*Alternative estimates of the labour supply function*

<table>
<thead>
<tr>
<th>Variable</th>
<th>(1) 2SLS Full sample</th>
<th>(2) 2SLS Censored sample</th>
<th>(3) 2SLS 2 Selection rule censored sample</th>
<th>Difference (3) − (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>2.2565** (0.1863)</td>
<td>2.4525* (0.2098)</td>
<td>3.0753* (0.2914)</td>
<td>0.8188* (0.2240)</td>
</tr>
<tr>
<td>Wage ($)</td>
<td>−0.0697* (0.0094)</td>
<td>−0.0698* (0.0100)</td>
<td>−0.0684* (0.0123)</td>
<td>0.0012 (0.0079)</td>
</tr>
<tr>
<td>Unearned Income (0000's of $)</td>
<td>−0.2087* (0.0743)</td>
<td>−0.1488** (0.0803)</td>
<td>−0.1477 (0.0907)</td>
<td>0.0610 (0.0520)</td>
</tr>
<tr>
<td>Married</td>
<td>0.1215 (0.1000)</td>
<td>0.1627 (0.1117)</td>
<td>0.1443 (0.1279)</td>
<td>0.0228 (0.0798)</td>
</tr>
<tr>
<td>Number of children</td>
<td>0.0001 (0.0136)</td>
<td>0.0177 (0.0151)</td>
<td>0.0307** (0.0174)</td>
<td>0.0306* (0.0108)</td>
</tr>
<tr>
<td>Health limitation</td>
<td>−0.3009* (0.0867)</td>
<td>−0.3046* (0.0967)</td>
<td>−0.2573* (0.1053)</td>
<td>0.0435 (0.0597)</td>
</tr>
<tr>
<td>Age (years $\times 10^{-1}$)</td>
<td>−0.0557** (0.0307)</td>
<td>−0.0641** (0.0341)</td>
<td>−0.0897* (0.0391)</td>
<td>−0.0341 (0.0243)</td>
</tr>
<tr>
<td>Education (years)</td>
<td>0.0546* (0.0076)</td>
<td>0.0416* (0.0089)</td>
<td>0.0159 (0.0124)</td>
<td>−0.0388* (0.0098)</td>
</tr>
<tr>
<td>Black</td>
<td>−0.1534** (0.0866)</td>
<td>−0.1297 (0.1067)</td>
<td>−0.0101 (0.1190)</td>
<td>0.1452** (0.0815)</td>
</tr>
<tr>
<td>$\lambda_5$ (unemployed)</td>
<td>—</td>
<td>—</td>
<td>−0.3188</td>
<td>—</td>
</tr>
<tr>
<td>$\lambda_6$ (underemployed)</td>
<td>—</td>
<td>—</td>
<td>−0.6142* (0.2286)</td>
<td>—</td>
</tr>
<tr>
<td>Chi-square statistic</td>
<td>—</td>
<td>—</td>
<td>31.8</td>
<td>—</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.179</td>
<td>0.192</td>
<td>0.215</td>
<td>—</td>
</tr>
<tr>
<td>S.E.E.</td>
<td>0.583</td>
<td>0.556</td>
<td>0.664</td>
<td>—</td>
</tr>
<tr>
<td>Sample size</td>
<td>835</td>
<td>835</td>
<td>617</td>
<td>—</td>
</tr>
</tbody>
</table>

*Note:* Dependent Variable: Annual Hours Worked in 000's in 1971.
ballpark estimate of $-0.15$), while the unearned income coefficient is plausibly signed and implies a small elasticity of $-0.023$. However, the unearned income coefficient is too small to produce a positive compensated wage elasticity, since it is estimated to be significant, but quite small, at $-0.05$. (All of the elasticities are within the range of previous results summarized by Heckman, Killingsworth and MaCurdy (1981, p. 80), with the substitution elasticity lying right on the boundary.)

Examining the demographic variables, the full sample coefficient on children is essentially zero while the education coefficient is large and quite significant. Moreover, the race and age coefficients are significantly negative at the 10% level.

Next consider the selection rule estimates in column 3. A test of the joint significance of the $\lambda$ terms yields an $F$ statistic of $9.2$, considerably larger than the critical $F$ value of $4.6$ at the 1% level. The results suggest that both the unemployed and underemployed possess below-average tastes for leisure since the respective correlation coefficients between the labour supply equations and the reduced-form probit equations (8) and (9) are $-0.48$ and $-0.92$. (Although the underemployment correlation seems somewhat stronger, the unemployment $\lambda$ coefficient becomes quite significant if the underemployed $\lambda$ term is dropped.) The selection rule method produces wage and unearned income elasticities of $-0.140$ and $-0.014$ respectively, which are reasonably close to the full sample estimates. The compensated wage elasticity is estimated to be significantly different from zero at $-0.06$, so that the selection rule method does not eliminate the unsatisfactory negative estimate of the substitution elasticity.

The full sample least squares and selection rule estimates do differ for several other coefficients. For example, the constant term rises by over 800 hours when the selection rule method is used. While this parameter is of little economic interest, its increase when the least squares estimates are replaced by the selection rule estimates supports the conjecture of Section 2 that ignoring constrained workers will lower the constant. Further, race differences are no longer significant and the role of education is reduced considerably using the selection rule method. Alternatively, the selection rule method increases the absolute value and the significance of the coefficients for age, and especially for children. Use of the selection method implies a net increase in desired hours (at the mean) of 319 hours, which reflects the change in the coefficients for age and education which lowers desired hours by 617 hours, while the change in the remaining coefficients raises hours by 936 hours.

From column 4 it is clear that the overall change in the coefficients is significantly different from zero, since the test statistic of 31.8 is greater than the critical value of 21.7 for the Chi-square distribution at the 1% level. The coefficients for education, children, race and the constant are all significantly different when the selection rule method is used. (The change in the age coefficient has an asymptotic normal statistic of $-1.9$.)

It is important to note that the changes in the coefficients, both in terms of significance and direction, are exactly those one would predict using the probit coefficients under the hypothesis that the unemployed and underemployed are truly constrained. Since the labour supply results should not be affected by moving to the selection rule estimates if these workers are not constrained, these results provide strong support for the hypothesis that the unemployed and underemployed are constrained.

The constrained hypothesis can also provide an intuitive explanation of the results. For example, consider the education coefficients and assume that more schooling decreases unemployment and increases desired hours. The full sample least squares estimates cannot determine whether increased education raises actual hours because more schooling reduces unemployment or because it increases desired hours. Since the selection rule method measures only the impact on desired hours, this method produces a much smaller and less significant coefficient.
Of course, there are alternatives to the specification of the labour supply function chosen here, and changing the specification may affect the differences between the least squares and selection rule estimates. For example, the labour supply function may be non-linear and the λ terms may simply be picking up these non-linearities. While distinguishing between a very general non-linear supply function and the selection rule estimates will be quite difficult, it is possible to follow Ashenfelter and Heckman (1973) and examine the effect of including a quadratic term in age to capture life-cycle effects. Alternatively, one could argue that if unemployment simply represents leisure, unemployment insurance payments should be included in unearned income, and then this variable should be treated as endogenous using the previous definition of unearned income and variables U1 and U5 as additional instruments. Examination of columns 1 and 2 of Table V indicates that neither of these modifications affects the differences between the least squares and selection rule estimates.

Finally, one could argue that using the lagged wage as an instrument is inappropriate, and that this may be creating spurious differences between the estimates. To examine

**Table V**

*Additional labour supply estimates*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Column 4</th>
<th>Column 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.5732*</td>
<td>2.2538*</td>
<td>1.7551*</td>
<td>2.9929*</td>
<td></td>
</tr>
<tr>
<td>Wage ($)</td>
<td>0.0094</td>
<td>0.0094</td>
<td>0.0542</td>
<td>0.0476</td>
<td></td>
</tr>
<tr>
<td>Unearned income (0000's of $)</td>
<td>0.02126*</td>
<td>0.02098*</td>
<td>0.1619**</td>
<td>-0.1484**</td>
<td></td>
</tr>
<tr>
<td>Married</td>
<td>0.1004</td>
<td>0.0998</td>
<td>0.0968</td>
<td>0.0897</td>
<td></td>
</tr>
<tr>
<td>Number of children</td>
<td>0.0035</td>
<td>0.0032</td>
<td>0.0294**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age limitation (years × 10⁻¹)</td>
<td>0.0298</td>
<td>0.0639</td>
<td>0.0244</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Education (years)</td>
<td>0.0547*</td>
<td>0.0553*</td>
<td>0.1044*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Black</td>
<td>-0.1601**</td>
<td>-0.1574**</td>
<td>-0.3040*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age-squared (years × 10⁻³)</td>
<td>0.4203</td>
<td>-0.4311</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>λ₁ (unemployed)</td>
<td>-0.4034**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>λ₂ (underemployed)</td>
<td>-0.3865**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.182</td>
<td>0.180</td>
<td>-0.341</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.E.E.</td>
<td>0.583</td>
<td>0.582</td>
<td>0.746</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample size</td>
<td>835</td>
<td>835</td>
<td>617</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
1. Dependent Variable: Annual Hours Worked in 000's in 1971
2. Effect of adding age-squared to full sample hours regression. (Adding this variable does not change the selection rule results.)
3. Effect of adding unemployment insurance payments to unearned income and treating this variable as endogenous.
4. Full sample least squares estimates using variables U1, U2, U3, U5 and experience squared as instruments for the wage.
5. Selection rule estimates using variables U1, U2, U3, U5 and experience squared as instruments for the wage.
this conjecture, the labour supply function was re-estimated (by both least squares and the selection rule method) using variables U1, U2, U3, and U5 above and potential experience squared as instruments. (The unemployment insurance variable was dropped from the probit equations since it depends on the lagged wage.) The results have been placed in columns 3 and 4 of Table V and indicate that the selection rule estimates are insensitive to the choice of instruments. (The only real difference is the fall in the coefficient for \( \lambda_6 \).) On the other hand, the full sample labour supply estimates do exhibit some sensitivity; in particular the wage coefficient almost triples and the substitution effect becomes correspondingly more negative. Further, the differences in the demographic coefficients remain strong and in fact increase. Thus, if changing the instruments has any effect on the coefficients, it is to increase the differences between the selection rule and least squares estimates.

In summary, the full sample and selection rule coefficients differ in exactly the direction predicted by the constrained hypothesis. Moreover, these differences are robust to simple, but potentially important, changes in specification.

6. CONCLUSION

This paper has presented and implemented a new method of labour supply estimation which is valid under much more general assumptions than previous approaches. This method is based on a generalization of Heckman's sample selection procedure, and estimates labour supply behaviour for workers who are neither unemployed nor underemployed while correcting the estimates for selection bias. The empirical results suggest that selection bias is an important problem when excluding underemployed and unemployed workers. The empirical results are also favourable to the hypothesis that the unemployed and underemployed are constrained, since the difference between the full sample least squares estimates and the selection rule estimates are in the direction that one would predict under the hypothesis that the unemployed and underemployed are constrained. Moreover, it appears that using least squares on the full sample and ignoring constrained workers leads to significant biases in several labour supply parameters.

APPENDIX 1

Consistent estimation of \( \sigma_1^2 \)

The censoring causes the error terms in the censored sample to be heteroskedastic. Using Tallis (1961)

\[
\sigma_1^2 = E(\varepsilon_1^2 | y_{5i} > 0, y_{6i} > 0) = \sigma_1^2 - \sigma_{15}^2 Z_{5i} \lambda_{5i} - \sigma_{16}^2 Z_{6i} \lambda_{6i} + \mu_i [2\sigma_{15} \sigma_{16} - \rho (\sigma_{15}^2 + \sigma_{16}^2)] - (\sigma_{15} \lambda_{5i} + \sigma_{16} \lambda_{6i})^2
\]

\[
= \sigma_1^2 + \nu_i \quad (A.1)
\]

where \( \mu_i = f(Z_{5i}, Z_{6i}, \rho) / F(Z_{5i}, Z_{6i}, \rho) \), \( \rho \) is the correlation between \( \varepsilon_{5i} \) and \( \varepsilon_{6i} \) and \( f(\cdot) \) is the bivariate standard normal density function.

As a result, the least squares estimator of \( \sigma_1^2 \) will be inconsistent. However, one may use the following generalization of Heckman's estimator for \( \sigma_1^2 \)

\[
\hat{\sigma}_1^2 = (S - \sum \hat{\nu}_i) / N, \quad (A.2)
\]

where \( S \) is the sum of squared residuals, \( N \) is the size of the censored sample, and \( \hat{\nu}_i \) equals \( \nu_i \) after parameter estimates have been substituted for their true values.
Consistent estimation of the variance covariance matrix

Calculating the correct variance-covariance matrix of the parameter estimates is a simple generalization of Heckman or Lee, Maddala and Trost (1980). Following the latter, write the residual of (13) as

\[ e_{1i} = \varepsilon_{1i} + \sigma_{15}(\lambda_{5i} - \hat{\lambda}_{5i}) + \sigma_{16}(\lambda_{6i} - \hat{\lambda}_{6i}). \]  

(A.3)

Then define a 1 x 1 vector \( \delta' = (\gamma'_5, \gamma'_6, \rho) \) and take first-order approximations of \( \lambda_{5i} \) and \( \lambda_{6i} \)

\[ (\lambda_{5i} - \hat{\lambda}_{5i}) = G_{5i}(\delta - \hat{\delta}) \]  

(A.4a)

and

\[ (\lambda_{6i} - \hat{\lambda}_{6i}) = G_{6i}(\delta - \hat{\delta}), \]  

(A.4b)

where \( G_{5i} = \partial \lambda_{5i}/\partial \delta \) and \( G_{6i} = \partial \lambda_{6i}/\partial \delta \) are 1 x 1 vectors.

Now let \( X'_i = (X'_1, \hat{\lambda}_{5i}, \hat{\lambda}_{6i}) \) and \( \beta'^* = (\beta'_1, \sigma_{15}, \sigma_{16}) \) be 1 x K vectors and \( X'^* = (X'_1, \ldots, X'_K) \) be the K x N matrix of observations on the independent variables. Further, define the 1 x N vectors \( e'_i = (e_{11}, \ldots, e_{1N}) \) and \( \varepsilon'_i = (\varepsilon_{11}, \ldots, \varepsilon_{1N}) \) as well as the 1 x 1 vector \( C_i = (\sigma_{15}G_{5i} + \sigma_{16}G_{6i}) \) and the K x N matrix \( C' = (C_1, \ldots, C_N) \). Then

\[ (\hat{\beta}^* - \beta^*) = (X'^*X'^*)^{-1}X'^*e_1 \]

\[ = (X'^*X'^*)^{-1}X'^*[\varepsilon_1 + C(\delta - \hat{\delta})]. \]  

(A.5)

Ignoring covariance terms (Lee et al. (1980), p. 500) yields

\[ \text{var}(\hat{\beta}^*) = (X'^*X'^*)^{-1}X'^*[\text{var}(\varepsilon_i) + C \text{var}(\hat{\delta})C']X'^*(X'^*X'^*)^{-1} \]

\[ = (X'^*X'^*)^{-1}[X'^* \text{var}(\varepsilon_i)X'^* + X'^*C \text{var}(\hat{\delta})C'C][X'^*X'^*]^{-1} \]

\[ = (X'^*X'^*)^{-1}[B_1 + B_2 \text{var}(\hat{\delta})B_2'][(X'^*X'^*)^{-1}] \]  

(A.6)

where

\[ B_1 = \sum_{i=1}^{N} \sigma_{15}^2 X'_i X'^*_i \]

and

\[ B_2 = \sum_{i=1}^{N} X'_i C'_i. \]

Equation (A.6) is a computationally efficient expression for calculating the variance-covariance matrix, since it only requires one extra pass through the data, (just as Greene’s (1981b) estimator does in the one selection rule case), and the largest matrix it involves storing in the computer has dimension l x K.

Adjustments for an endogenous wage

Considering the endogeneity of the probit equations first, use Lee’s (1979a) suggestion and write the wage equation as

\[ w = R'_1 \Pi_1 + u, \]  

(A.7)

where \( R_1 \) and \( \Pi_1 \) are vectors of independent variables and coefficients respectively, \( u \) is a normally distributed error term, and again the subscript \( i \) has been dropped for notational convenience. Next rewrite the probit equations (8) and (9) as

\[ y_5 = \gamma_{51} w + X'_5 \gamma_{52} + \varepsilon_5 \]  

(A.8a)
and
\[ y_6 = y_{61} w + X'_6 \gamma_{62} + \varepsilon_{6}, \]  
(A.8b)
where \( X_5 \) and \( X_6 \) no longer contain \( w \). Then substitute (A.7) into (A.8a) and (A.8b) to obtain
\[ y_5 = R'_1 (\gamma_{51} \Pi_1) + X'_5 \gamma_{52} + \varepsilon_5 + \gamma_{51} u \]  
(A.9a)
\[ = R'_5 \Pi_5 + \varepsilon'_5 \]  
(A.9b)
\[ y_6 = R'_1 (\gamma_{61} \Pi_1) + X'_6 \gamma_{62} + \varepsilon_6 + \gamma_{61} u \]  
(A.10a)
\[ = R'_6 \Pi_6 + \varepsilon'_6. \]  
(A.10b)

In (A.9b) and (A.10b), \( R_5 \) and \( R_6 \) are vectors of independent variables, \( \Pi_5 \) and \( \Pi_6 \) are vectors of coefficients and \( \varepsilon'_5 \) and \( \varepsilon'_6 \) are error terms implicitly defined by (A.9a) and (A.10a). Next, replace \( Z_5 \) and \( Z_6 \) by \( Z'_5 = R'_1 \Pi_5 \) and \( Z'_6 = R'_1 \Pi_6 \) in the definitions of \( \lambda_5 \) and \( \lambda_6 \) under (12) in the text. Finally estimate (A.9b) and (A.10b) by bivariate probit analysis and form estimates of \( \lambda_5 \) and \( \lambda_6 \).

Given the new definitions of the \( \lambda \) terms, one can simply follow the suggestions of Lee, Maddala and Trost (1980). First, estimate the following wage equation on the censored sample
\[ w = R'_1 \Pi_{11} + \Pi_{15} \hat{\lambda}_5 + \Pi_{16} \hat{\lambda}_6 + u' \]  
(A.11)
and use the coefficients to form an estimated value of the wage, \( \hat{w} \). Then replace \( w \) by \( \hat{w} \) in \( X_1 \) to obtain a new vector of independent variables \( \hat{X}_1 \), and estimate (13) in the text using the new \( \hat{X}_1 \) vector. To calculate the consistent estimate of the variance covariance matrix, use the formulas given above with \( X_1 \) replaced by \( \hat{X}_1 \).

**APPENDIX 2**

Estimating the duration of unemployment and underemployment

Define a worker's duration of unemployment (underemployment) as the difference between his desired and actual hours. Next estimate the actual hours equation by running the following regression on the subsample of unemployed workers
\[ E(H | y_5 < 0) = X'_7 \theta_1 + \alpha_5 \hat{\lambda}_5 \]  
(A.12)
where \( \hat{\lambda}_5 = \phi(Z_5)/\Phi(-Z_5) \), and \( X_7 \) and \( \theta_1 \) are vectors of independent variables and coefficients respectively. Given the labour supply estimates from (13), the unconditional (or potential) duration of unemployment equation can then be expressed as
\[ E(U) = E(H' - H) = X'_1 \beta_1 - X'_7 \theta_1 \]  
(A.13)
\[ = X'_8 \theta_5, \]  
(A.14)
where \( X_8 \) and \( \theta_5 \) are vectors of independent variables and coefficients implicitly defined by (A.13). On the other hand, the conditional duration of unemployment may be expressed as
\[ E(U | y_5 < 0) = X'_8 \theta_5 - (\sigma_{15} + \alpha_5) \hat{\lambda}_5, \]  
(A.15)
where
\[ \frac{\partial E(U | y_5 < 0)}{\partial X_{8i}} = \theta_2 f(\sigma_{15} + \alpha_5) \frac{\partial \hat{\lambda}_5}{\partial X_{8i}}. \]  
(A.16)

Estimates from (A.14) will differ from a traditional duration of unemployment equation. The latter measures the effect of the independent variables on the length of
an unemployment spell, while (A.14) measures the effect of the independent variables on the unemployed hours that a worker could not compensate for by working longer hours after his unemployment spell ended (or before it began, if the spell was anticipated).\textsuperscript{16} Clearly, both methods of estimating the duration of unemployment convey useful information.

A similar equation can be estimated for the duration of underemployment. To the best of my knowledge, the equation presented below is the first attempt to measure the duration of underemployment.

There are two potential problems in estimating (A.12) and thus (A.14). First, there are relatively few observations on unemployed or underemployed workers and thus the estimates are likely to be quite imprecise. (Indeed, it would be preferable to estimate separate duration equations for workers unemployed but not underemployed, underemployed but not unemployed, and underemployed and unemployed. However, given the current sample sizes this is totally impractical.) Second, the variables affecting the incidence of unemployment are likely to be the same as those affecting the hours worked by the unemployed, and the latter equation will be identified only by the normality assumption and the resulting non-linearity of $\lambda$. Goldberger (1980) has shown that in this situation the estimates can be quite unstable and should be viewed with caution. On the other hand, it is important to note that this problem also occurs when the duration of unemployment is estimated in the traditional manner.

In fact both of these problems arose when the actual hours equations were estimated. The coefficient for the $\lambda$ term was quite insignificant in the equation for unemployed workers; in the equation for underemployed workers it was almost significant but implied a correlation coefficient well outside the unit interval. This result probably reflects the inherent difficulty of investigating selection issues when identification is based solely on functional form.

\begin{table}[h]
\centering
\caption{Estimates of the duration of unemployment and underemployment}
\begin{tabular}{lcc}
\hline
Variable & Unemployment Coefficient/standard error & Underemployment Coefficient/standard error \\
\hline
Constant & -0.0261 (0.8886) & 1.1820 (0.7599) \\
Wage & 0.0157 (0.0590) & 0.0267 (0.0618) \\
Unearned income & 0.5820** (0.3161) & 0.3285** (0.1966) \\
Married & 0.5327 (0.3739) & -0.2961 (0.2894) \\
Number of children & 0.0100 (0.0516) & 0.0489 (0.0366) \\
Health limitation & 0.2048 (0.3750) & -0.1486 (0.2213) \\
Age & -0.0307 (0.1527) & 0.0438 (0.0881) \\
Education & -0.0396 (0.0297) & -0.0371 (0.0235) \\
Black & 0.2556 (0.2917) & 0.0417 (0.2007) \\
Tenure & 0.2874 (0.3481) & -0.0115 (0.2058) \\
Tenure-squared & -0.1139 (0.1242) & 0.0142 (0.0838) \\
Union & -0.3203 (0.2022) & 0.0370 (0.1049) \\
Local unemployment & -0.0090 (0.0305) & 0.0148 (0.0227) \\
Occup. unemployment & 0.0109 (0.0401) & 0.0252 (0.0265) \\
North East & 0.3061 (0.2944) & -0.0982 (0.2032) \\
North Central & 0.3775 (0.2661) & 0.0538 (0.1962) \\
South dummy & 0.0540 (0.2991) & 0.0618 (0.2081) \\
Unem. insurance & 0.2421 (0.1554) & -0.1103 (0.1434) \\
$\lambda$ & 0.3187 & 0.2268 \\
$\lambda$ & - & - \\
\hline
\end{tabular}
\end{table}

\textit{Notes:}
1. Duration measured in 000's of hours.
2. Units and notes the same as in Table II.
As a result, the actual hours equations were estimated by least squares (ignoring the sample selection issue) and these estimates were used to calculate the potential duration of unemployment and underemployment equations. Columns 1 and 3 of Table VI contain the estimated coefficients for these equations while columns 2 and 4 contain the respective standard errors. As one would expect given the sample sizes, the results are generally imprecise although they do contain some useful information. Considering variables with an asymptotic normal statistic in the neighbourhood of 1.6 (in absolute value) or greater, they indicate that the duration of unemployment rises as unearned income rises (fifty-eight hours for every $1000) and with more generous unemployment insurance benefits. On the other hand, union membership decreases this duration. (Education has a normal statistic of approximately 1.4.) The sign and especially the size of the union coefficient are somewhat surprising; because of the mixing of unemployment and strike data this result may reflect strikes ending more quickly than unemployment spells. The results for the underemployment equation suggest that raising unearned income increases this duration while more schooling lowers it. Ignoring taste differences and using equation (A.14), the estimated annual duration for unemployed and underemployed workers are 745 and 621 hours respectively. These estimates seem quite plausible, especially given the overlap between the two groups. On the other hand, when taste differences are considered and equation (A.15) is used, the durations are estimated somewhat less plausibly at 1199 and 1401 hours respectively.

First version received December 1980, final version accepted March 1982 (Eds.).

An earlier version of the paper was presented at the Fourth World Congress of the Econometric Society in Aix-en-Provence, France, 1980. The paper was jointly released as Working Paper 8108, Institute for Policy Analysis, University of Toronto and Working Paper 141, Industrial Relations Section, Princeton University. This paper represents an extensive revision of Chapter 2 of my dissertation in the Department of Economics, Princeton University. I am indebted to my thesis committee, O. Ashenfelter, R. Quandt and J. Brown, for excellent comments and suggestions. I would also like to thank R. Blundell, G. Chamberlain, J. Hausman, J. Heckman, S. Nickell, D. Poirier, P. Ruud, S. Rosen, and A. Yatchew for very helpful discussions or correspondence, as well as J. Altonji, R. Gordon, M. Gunderson, Jane Ham, C. Hsiao, M. King, G. MacDonald, A. Melino, S. Rea, S. Tychsen, R. Winter and two anonymous referees for very useful comments on earlier drafts of this paper. I am responsible for any remaining errors. I also benefitted from comments received during seminars at Bell Laboratories, Columbia, Cornell, McMaster, Princeton, Queen's and Toronto. Generous financial support was provided by the Industrial Relations Section, Princeton University, the Institute for Policy Analysis, University of Toronto, and the Social Sciences and Humanities Research Council of Canada.

NOTES

1. In certain situations one could estimate the true labour supply parameters (but not their standard errors) directly from the full sample least squares estimates. To do so, one would proceed in an analogous fashion to Greene (1981a), who shows that under appropriate assumptions the coefficients of a Tobit model can be easily computed from the full sample least squares estimates.

2. These workers are numerically much less important than the unemployed and underemployed, and are ignored here to keep the computations within the feasible range. Since the test of constraints is carried out under the null hypothesis of no constraints, ignoring these workers will not affect the validity of the test.

3. A referee pointed out that mandatory overtime provisions may cause underemployment, since employers would have to pay time and a half for hours in excess of a standard work-week. In this case both the employer and employee may want to increase hours of work at the normal wage, but the employer is unwilling to do so at the overtime wage.

4. I am viewing the underemployed as being constrained in terms of their hours of work per week and the unemployed as being constrained in terms of their weeks of work per year, so that it makes sense for a worker to be unemployed but not underemployed or both unemployed and underemployed. The estimation method proposed in Section 3 remains valid whether or not this interpretation is correct.

5. One could reduce these problems somewhat by restricting the hours equation to be the same for workers in Groups 2, 3, 4; since the test is carried out under the null hypothesis of no constraints this would still be a valid test. However, restricting the hours equation in this way still leaves a large number of parameters to be estimated.

6. For further results on this specification test, see Hausman and Taylor (1980).
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7. Catsiapis and Robinson (1978) and Behrman, Wolfe and Tunali (1980) estimate models with two independent selection rules. Poirier also suggests a situation where it is appropriate to use two correlated selection rules.

8. Ham (1980, Chapter 1) accounted for this truncation and found that it changed the labour supply estimates very little.

9. There remains one problem for correction in future work. By not including the wife's wage in the supply equation, I am ignoring any family labour supply considerations. Handling this problem correctly is quite difficult. Whether or not the wife works is endogenous, and the labour supply function will differ between husbands with working and nonworking wives. Olsen (1977) contains a complete discussion of this problem and a means of handling it.

10. The occupational unemployment rate was created by entering the estimated unemployment rate in the individual's occupation. I am very grateful to Kevin Barry, head librarian, Industrial Relations Section, Princeton University for extremely valuable help in creating this unemployment rate. Two individuals with missing values for tenure were excluded from the sample, while a small number of missing observations (less than 3% of the sample) for the occupational and local unemployment rates were replaced by mean values.

11. The bivariate probit likelihood was maximized using the GRADX algorithm of Goldfeld and Quandt (1972). The likelihood function was evaluated using a bivariate normal integration program written by P. Ruud which is accurate to eight decimal places. I first attempted to maximize this likelihood function using numerical derivatives but could not achieve convergence. Convergence was obtained using analytical first derivatives and the Berndt et al. (1972) approximation to the second derivative matrix. Amemiya (1974) contains several identities which were helpful in calculating the derivatives.

12. Nickell (1980) obtained the same result and contains an extensive discussion of why more children may lead to a higher probability of unemployment. He concluded that this result was something of a puzzle.

13. This, of course, assumes that a variable's effect on the duration of unemployment or underemployment is in the same direction as its effect on the incidence. This would be satisfied, for example, if unemployment and underemployment followed a Tobit structure.

14. If one considers those variables in columns 3 and 4 of Table II which have an asymptotic normal statistic greater than 1 in absolute value (as I did in an earlier version of the paper), then least squares should also overestimate the marital coefficient. However, the difference in this coefficient is essentially zero.

15. The expression \( Y = X \) means that \( Y \) has the same asymptotic distribution as \( X \).

16. A simple example may clarify this distinction. Consider a worker who, at the beginning of the year, intends to work 200 hours per month or 2400 hours per year. He is unemployed for the first month and a half of the year, but works 2200 hours during the rest of the year to compensate for the unemployment. Then the traditional measure of the duration of unemployment is 300 hours while the measure used here is only 200 hours.

17. Alternatively, one could view the estimated actual hours equations as conditional, and then form conditional unemployment duration equations by also differentiating the \( \lambda \) term from the labour supply equation along the lines of (A.16). Doing so does not change the sign of any of the coefficients discussed here. For clear discussion of the difference between conditional and unconditional equations, see Poirier and Ruud (1981).

REFERENCES


