Technical Appendix for Defensive Innovation and Firm Growth in the U.S.: Impact of International Trade

Karam Jo

November 22, 2019

TA1 Technology-Gaps Evolution

Denote $\overline{x}^H$ as total external innovation intensity by domestic firms (both incumbents and startups), $\overline{x}^F$ as that of foreign counterparts, and $\overline{x} \equiv \overline{x}^H + \overline{x}^F$ as a total external innovation intensity in the economy.

TA1.1 (H H) case

For any q_j, the followings hold, where the last column specifies ownership change in domestic and foreign market.

$$(H\,H),\,(\Delta^1\,\Delta^1),\,\infty \rightarrow \begin{cases} \text{a. no ext. innov.} & \Delta^{G\prime} = \infty & & \\ (H\,H),\,(\Delta^1\,\Delta^1) & & \text{w/ } (1-z^H)(1-\overline{x}) & \text{x x} \\ (H\,H),\,(\Delta^2\,\Delta^2) & & \text{w/ } z^H\,(1-\overline{x}) & \text{x x} \\ \text{b. H ext. innov.} & \Delta^{G\prime} = \infty & & \\ (H\,H),\,(\Delta^3\,\Delta^3) & & \text{w/ } (1-z^H)\,\overline{x}^H & \text{o o} \\ (H\,H),\,(\Delta^3\,\Delta^3) & & \text{w/ } z^H\,\overline{x}^H & \text{o o} \\ \text{c-i. F ext. innov.} & \Delta^{G\prime} = 1/\eta & \text{no int. innov.} & \\ (F\,F),\,(\Delta^3\,\Delta^3) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\ (H\,F),\,(\Delta^1\,\Delta^3) & \text{if } \Delta^{G\prime} \in [\underline{\Omega},\,\overline{\Omega}] & \text{w/ } (1-z^H)\,\overline{x}^F & \text{x o} \\ (H\,H),\,(\Delta^1\,\Delta^1) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x} \\ \text{c-ii. F ext. innov.} & \Delta^{G\prime} = \lambda/\eta & \text{H int. innov.} & \\ (F\,F),\,(\Delta^3\,\Delta^3) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\ (H\,F),\,(\Delta^2\,\Delta^3) & \text{if } \Delta^{G\prime} \in [\underline{\Omega},\,\overline{\Omega}] & \text{w/ } z^H\,\overline{x}^F & \text{x o} \\ (H\,H),\,(\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x} \end{cases} \tag{TA1.1}$$

$$
(H\ H),\ (\Delta^2\ \Delta^2),\ \infty \rightarrow \begin{cases}
\text{a. no ext. innov.} & \Delta^{G\prime} = \infty & & \\
(H\ H),\ (\Delta^1\ \Delta^1) & & \text{w/ } (1-z^H)(1-\overline{x}) & \text{x x} \\
(H\ H),\ (\Delta^2\ \Delta^2) & & \text{w/ } z^H\ (1-\overline{x}) & \text{x x} \\
\text{b. H ext. innov.} & \Delta^{G\prime} = \infty & & \\
(H\ H),\ (\Delta^4\ \Delta^4) & & \text{w/ } (1-z^H)\ \overline{x}^H & \text{o o} \\
(H\ H),\ (\Delta^2\ \Delta^2) & & \text{w/ } z^H\ \overline{x}^H & \text{x x} \\
\text{c-i. F ext. innov.} & \Delta^{G\prime} = \lambda/\eta & \text{no int. innov.} & \\
(F\ F),\ (\Delta^4\ \Delta^4) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\
(H\ F),\ (\Delta^1\ \Delta^4) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)\ \overline{x}^F & \text{x o} \\
(H\ H),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x} \\
\text{c-ii. F ext. innov.} & \Delta^{G\prime} = \lambda^2/\eta & \text{H int. innov.} & \\
(F\ F),\ (\Delta^4\ \Delta^4) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\
(H\ F),\ (\Delta^2\ \Delta^4) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\ \overline{x}^F & \text{x o} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x}
\end{cases} \tag{TA1.2}
$$

$$
(H\ H),\ (\Delta^3\ \Delta^3),\ \infty \rightarrow \begin{cases}
\text{a. no ext. innov.} & \Delta^{G\prime} = \infty & & \\
(H\ H),\ (\Delta^1\ \Delta^1) & & \text{w/ } (1-z^H)(1-\overline{x}) & \text{x x} \\
(H\ H),\ (\Delta^2\ \Delta^2) & & \text{w/ } z^H\ (1-\overline{x}) & \text{x x} \\
\text{b. H ext. innov.} & \Delta^{G\prime} = \infty & & \\
(H\ H),\ (\Delta^1\ \Delta^1) & & \text{w/ } (1-z^H)\ \overline{x}^H & \tfrac{1}{2}\ \tfrac{1}{2} \\
(H\ H),\ (\Delta^2\ \Delta^2) & & \text{w/ } z^H\ \overline{x}^H & \text{x x} \\
\text{c-i. F ext. innov.} & \Delta^{G\prime} = 1 & \text{no int. innov.} & \\
(F\ F),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\
(H\ F),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)\ \overline{x}^F & \text{x o} \\
(H\ H),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x} \\
\text{c-ii. F ext. innov.} & \Delta^{G\prime} = \lambda & \text{H int. innov.} & \\
(F\ F),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\
(H\ F),\ (\Delta^2\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\ \overline{x}^F & \text{x o} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x}
\end{cases} \tag{TA1.3}
$$

$$
(H\,H),\ (\Delta^4\,\Delta^4),\ \infty \rightarrow
\begin{cases}
\text{a. no ext. innov.} & \Delta^{G\prime} = \infty & & \\
(H\,H),\ (\Delta^1\,\Delta^1) & & \text{w/ } (1-z^H)(1-\overline{x}) & \text{x x} \\
(H\,H),\ (\Delta^2\,\Delta^2) & & \text{w/ } z^H\,(1-\overline{x}) & \text{x x} \\
\text{b. H ext. innov.} & \Delta^{G\prime} = \infty & & \\
(H\,H),\ (\Delta^2\,\Delta^2) & & \text{w/ } (1-z^H)\,\overline{x}^H & \text{o o} \\
(H\,H),\ (\Delta^2\,\Delta^2) & & \text{w/ } z^H\,\overline{x}^H & \tfrac{1}{2}\ \tfrac{1}{2} \\
\text{c-i. F ext. innov.} & \Delta^{G\prime} = 1/\lambda & \text{no int. innov.} & \\
(F\,F),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\
(H\,F),\ (\Delta^1\,\Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)\,\overline{x}^F & \text{x o} \\
(H\,H),\ (\Delta^1\,\Delta^1) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x} \\
\text{c-ii. F ext. innov.} & \Delta^{G\prime} = 1 & \text{H int. innov.} & \\
(F\,F),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\
(H\,F),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,\overline{x}^F & \text{x o} \\
(H\,H),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x}
\end{cases}
\tag{TA1.4}
$$

TA1.2 (F F) case

For any q_j, the followings hold.

$$(F\ F),\ (\Delta^1\ \Delta^1),\ -\infty \rightarrow \begin{cases}
\text{a. no ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\ F),\ (\Delta^1\ \Delta^1) & & \text{w/ } (1-z^F)(1-\overline{x}) & \text{x x} \\
(F\ F),\ (\Delta^2\ \Delta^2) & & \text{w/ } z^F\ (1-\overline{x}) & \text{x x} \\
\text{b-i. H ext. innov.} & \Delta^{G\prime} = \eta & \text{no int. innov.} & \\
(F\ F),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\ F),\ (\Delta^3\ \Delta^1) & \text{if } \Delta^{G\prime} \in [\underline{\Omega},\ \overline{\Omega}] & \text{w/ } (1-z^F)\ \overline{x}^H & \text{o x} \\
(H\ H),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{b-ii. H ext. innov.} & \Delta^{G\prime} = \eta/\lambda & \text{F int. innov.} & \\
(F\ F),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\ F),\ (\Delta^3\ \Delta^2) & \text{if } \Delta^{G\prime} \in [\underline{\Omega},\ \overline{\Omega}] & \text{w/ } z^F\ \overline{x}^H & \text{o x} \\
(H\ H),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{c. F ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\ F),\ (\Delta^3\ \Delta^3) & & \text{w/ } (1-z^F)\ \overline{x}^F & \text{o o} \\
(F\ F),\ (\Delta^3\ \Delta^3) & & \text{w/ } z^F\ \overline{x}^F & \text{o o}
\end{cases} \tag{TA1.5}$$

$$(F\ F),\ (\Delta^2\ \Delta^2),\ -\infty \rightarrow \begin{cases}
\text{a. no ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\ F),\ (\Delta^1\ \Delta^1) & & \text{w/ } (1-z^F)(1-\overline{x}) & \text{x x} \\
(F\ F),\ (\Delta^2\ \Delta^2) & & \text{w/ } z^F\ (1-\overline{x}) & \text{x x} \\
\text{b-i. H ext. innov.} & \Delta^{G\prime} = \eta/\lambda & \text{no int. innov.} & \\
(F\ F),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\ F),\ (\Delta^4\ \Delta^1) & \text{if } \Delta^{G\prime} \in [\underline{\Omega},\ \overline{\Omega}] & \text{w/ } (1-z^F)\ \overline{x}^H & \text{o x} \\
(H\ H),\ (\Delta^4\ \Delta^4) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{b-ii. H ext. innov.} & \Delta^{G\prime} = \eta/\lambda^2 & \text{F int. innov.} & \\
(F\ F),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\ F),\ (\Delta^4\ \Delta^2) & \text{if } \Delta^{G\prime} \in [\underline{\Omega},\ \overline{\Omega}] & \text{w/ } z^F\ \overline{x}^H & \text{o x} \\
(H\ H),\ (\Delta^4\ \Delta^4) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{c. F ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\ F),\ (\Delta^4\ \Delta^4) & & \text{w/ } (1-z^F)\ \overline{x}^F & \text{o o} \\
(F\ F),\ (\Delta^2\ \Delta^2) & & \text{w/ } z^F\ \overline{x}^F & \text{x x}
\end{cases} \tag{TA1.6}$$

$$
(F\,F),\ (\Delta^3\,\Delta^3),\ -\infty \rightarrow \left\{ \begin{array}{llll}
\text{a. no ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\,F),\ (\Delta^1\,\Delta^1) & & \text{w/ } (1-z^F)(1-\overline{x}) & \text{x x} \\
(F\,F),\ (\Delta^2\,\Delta^2) & & \text{w/ } z^F\,(1-\overline{x}) & \text{x x} \\
\text{b-i. H ext. innov.} & \Delta^{G\prime} = 1 & \text{no int. innov.} & \\
(F\,F),\ (\Delta^1\,\Delta^1) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\,F),\ (\Delta^1\,\Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^F)\,\overline{x}^H & \text{o x} \\
(H\,H),\ (\Delta^1\,\Delta^1) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{b-ii. H ext. innov.} & \Delta^{G\prime} = 1/\lambda & \text{F int. innov.} & \\
(F\,F),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\,F),\ (\Delta^1\,\Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^F\,\overline{x}^H & \text{o x} \\
(H\,H),\ (\Delta^1\,\Delta^1) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{c. F ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\,F),\ (\Delta^1\,\Delta^1) & & \text{w/ } (1-z^F)\,\overline{x}^F & \frac{1}{2}\ \frac{1}{2} \\
(F\,F),\ (\Delta^2\,\Delta^2) & & \text{w/ } z^F\,\overline{x}^F & \text{x x}
\end{array} \right. \tag{TA1.7}
$$

$$
(F\,F),\ (\Delta^4\,\Delta^4),\ -\infty \rightarrow \left\{ \begin{array}{llll}
\text{a. no ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\,F),\ (\Delta^1\,\Delta^1) & & \text{w/ } (1-z^F)(1-\overline{x}) & \text{x x} \\
(F\,F),\ (\Delta^2\,\Delta^2) & & \text{w/ } z^F\,(1-\overline{x}) & \text{x x} \\
\text{b-i. H ext. innov.} & \Delta^{G\prime} = \lambda & \text{no int. innov.} & \\
(F\,F),\ (\Delta^1\,\Delta^1) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\,F),\ (\Delta^2\,\Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^F)\,\overline{x}^H & \text{o x} \\
(H\,H),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{b-ii. H ext. innov.} & \Delta^{G\prime} = 1 & \text{F int. innov.} & \\
(F\,F),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\,F),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^F\,\overline{x}^H & \text{o x} \\
(H\,H),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{c. F ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\,F),\ (\Delta^2\,\Delta^2) & & \text{w/ } (1-z^F)\,\overline{x}^F & \text{o o} \\
(F\,F),\ (\Delta^2\,\Delta^2) & & \text{w/ } z^F\,\overline{x}^F & \frac{1}{2}\ \frac{1}{2}
\end{array} \right. \tag{TA1.8}
$$

TA1.3 (H F) case

For any q_j, $\Delta^{D\ell}$, $\Delta^{F\ell}$, and Δ^G, the followings hold. Note that in all of below cases, $\Delta^G \in [\underline{\Omega}, \overline{\Omega}]$. Thus, if $\Delta^{G\prime} < \Delta^G$, then $(H\, H)$ cannot be realized next period, and if $\Delta^{G\prime} > \Delta^G$, then $(F\, F)$ cannot be realized.

TA1.3.1 No External Innovation

$$
(H\,F), (\Delta^{D\ell}\,\Delta^{F\ell}),\ \Delta^G \rightarrow
\begin{cases}
\text{a.} & \Delta^{G\prime} = \Delta^G & \text{no int. innov.} & \\
(H\,F),\ (\Delta^1\,\Delta^1) & \Delta^{G\prime} \in [\underline{\Omega}, \overline{\Omega}] & \text{w/ } (1-z^H)(1-z^F)(1-\overline{x}) & \text{x x} \\
\text{b.} & \Delta^{G\prime} = \lambda\Delta^G & \text{H int. innov.} & \\
(H\,F),\ (\Delta^2\,\Delta^1) & \text{if } \Delta^{G\prime} \in [\underline{\Omega}, \overline{\Omega}] & \text{w/ } z^H\,(1-z^F)(1-\overline{x}) & \text{x x} \\
(H\,H),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x o} \\
\text{c.} & \Delta^{G\prime} = \frac{1}{\lambda}\Delta^G & \text{F int. innov.} & \\
(F\,F),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)\,z^F\,(1-\overline{x}) & \text{o x} \\
(H\,F),\ (\Delta^1\,\Delta^2) & \text{if } \Delta^{G\prime} \in [\underline{\Omega}, \overline{\Omega}] & & \text{x x} \\
\text{d.} & \Delta^{G\prime} = \Delta^G & \text{all int. innov.} & \\
(H\,F),\ (\Delta^2\,\Delta^2) & \Delta^{G\prime} \in [\underline{\Omega}, \overline{\Omega}] & \text{w/ } z^H\,z^F\,(1-\overline{x}) & \text{x x}
\end{cases}
\tag{TA1.9}
$$

TA1.3.2 External Innovation by a Domestic Firm

Since domestic firm builds its external innovation on the past-period domestic technology, technology gap in foreign market does not matter in terms of realized outcomes in this case.

$$
(H\,F), (\Delta^1\,\Delta^{F\ell}),\ \Delta^G \rightarrow
\begin{cases}
\text{a.} & \Delta^{G\prime} = \eta\Delta^G & \text{no int. innov.} & \\
(H\,F),\ (\Delta^3\,\Delta^1) & \text{if } \Delta^{G\prime} \in [\underline{\Omega}, \overline{\Omega}] & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^H & \text{o x} \\
(H\,H),\ (\Delta^3\,\Delta^3) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{b.} & \Delta^{G\prime} = \eta\Delta^G & \text{H int. innov.} & \\
(H\,F),\ (\Delta^3\,\Delta^1) & \text{if } \Delta^{G\prime} \in [\underline{\Omega}, \overline{\Omega}] & \text{w/ } z^H\,(1-z^F)\,\overline{x}^H & \text{o x} \\
(H\,H),\ (\Delta^3\,\Delta^3) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{c.} & \Delta^{G\prime} = \frac{\eta}{\lambda}\Delta^G & \text{F int. innov.} & \\
(H\,F),\ (\Delta^3\,\Delta^2) & \text{if } \Delta^{G\prime} \in [\underline{\Omega}, \overline{\Omega}] & \text{w/ } (1-z^H)\,z^F\,\overline{x}^H & \text{o x} \\
(H\,H),\ (\Delta^3\,\Delta^3) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{d.} & \Delta^{G\prime} = \frac{\eta}{\lambda}\Delta^G & \text{all int. innov.} & \\
(H\,F),\ (\Delta^3\,\Delta^2) & \text{if } \Delta^{G\prime} \in [\underline{\Omega}, \overline{\Omega}] & \text{w/ } z^H\,z^F\,\overline{x}^H & \text{o x} \\
(H\,H),\ (\Delta^3\,\Delta^3) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o}
\end{cases}
\tag{TA1.10}
$$

$$(H\ F), (\Delta^2\ \Delta^{F\ell}),\ \Delta^G \rightarrow$$

$$\begin{cases} \text{a.} & \Delta^{G\prime} = \frac{\eta}{\lambda}\Delta^G & \text{no int. innov.} & \\ (H\ F),\ (\Delta^4\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^H & \text{o x} \\ (H\ H),\ (\Delta^4\ \Delta^4) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\ \text{b.} & \Delta^{G\prime} = \lambda\Delta^G & \text{H int. innov.} & \\ (H\ F),\ (\Delta^2\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,(1-z^F)\,\overline{x}^H & \text{x x} \\ (H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x o} \\ \text{c.} & \Delta^{G\prime} = \frac{\eta}{\lambda^2}\Delta^G & \text{F int. innov.} & \\ (F\ F),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)\,z^F\,\overline{x}^H & \text{o x} \\ (H\ F),\ (\Delta^4\ \Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{o x} \\ \text{d.} & \Delta^{G\prime} = \Delta^G & \text{all int. innov.} & \\ (H\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,z^F\,\overline{x}^H & \text{x x} \end{cases} \tag{TA1.11}$$

$$(H\ F), (\Delta^3\ \Delta^{F\ell}),\ \Delta^G \rightarrow$$

$$\begin{cases} \text{a.} & \Delta^{G\prime} = \Delta^G & \text{no int. innov.} & \\ (H\ F),\ (\Delta^1\ \Delta^1) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^H & \frac{1}{2}\ \text{x} \\ \text{b.} & \Delta^{G\prime} = \lambda\Delta^G & \text{H int. innov.} & \\ (H\ F),\ (\Delta^2\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,(1-z^F)\,\overline{x}^H & \text{x x} \\ (H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x o} \\ \text{c.} & \Delta^{G\prime} = \frac{1}{\lambda}\Delta^G & \text{F int. innov.} & \\ (F\ F),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)\,z^F\,\overline{x}^H & \text{o x} \\ (H\ F),\ (\Delta^1\ \Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \frac{1}{2}\ \text{x} \\ \text{d.} & \Delta^{G\prime} = \Delta^G & \text{all int. innov.} & \\ (H\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,z^F\,\overline{x}^H & \text{x x} \end{cases} \tag{TA1.12}$$

$$(H\ F), (\Delta^4\ \Delta^{F\ell}),\ \Delta^G \rightarrow$$

$$
\begin{cases}
\text{a.} & \Delta^{G\prime} = \lambda \Delta^{G} & \text{no int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^H & \text{o x} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{b.} & \Delta^{G\prime} = \lambda \Delta^{G} & \text{H int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,(1-z^F)\,\overline{x}^H & \tfrac{1}{2}\ \text{x} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \tfrac{1}{2}\ \text{o} \\
\text{c.} & \Delta^{G\prime} = \Delta^{G} & \text{F int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)\,z^F\,\overline{x}^H & \text{o x} \\
\text{d.} & \Delta^{G\prime} = \Delta^{G} & \text{all int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,z^F\,\overline{x}^H & \tfrac{1}{2}\ \text{x}
\end{cases}
\qquad \text{(TA1.13)}
$$

TA1.3.3 External Innovation by a Foreign Firm

$$
(H\ F), (\Delta^{D\ell}\ \Delta^1),\ \Delta^G \rightarrow
$$

$$
\left\{
\begin{array}{llll}
\text{a.} & \Delta^{G\prime} = \frac{1}{\eta}\Delta^G & \text{no int. innov.} & \\
(F\ F),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^F & \text{o o} \\
(H\ F),\ (\Delta^1\ \Delta^3) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x o} \\
\text{b.} & \Delta^{G\prime} = \frac{\lambda}{\eta}\Delta^G & \text{H int. innov.} & \\
(F\ F),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } z^H\,(1-z^F)\,\overline{x}^F & \text{o o} \\
(H\ F),\ (\Delta^2\ \Delta^3) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x o} \\
\text{c.} & \Delta^{G\prime} = \frac{1}{\eta}\Delta^G & \text{F int. innov.} & \\
(F\ F),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)\,z^F\,\overline{x}^F & \text{o o} \\
(H\ F),\ (\Delta^1\ \Delta^3) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x o} \\
\text{d.} & \Delta^{G\prime} = \frac{\lambda}{\eta}\Delta^G & \text{all int. innov.} & \\
(F\ F),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } z^H\,z^F\,\overline{x}^F & \text{o o} \\
(H\ F),\ (\Delta^2\ \Delta^3) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x o}
\end{array}
\right. \tag{TA1.14}
$$

$$
(H\ F), (\Delta^{D\ell}\ \Delta^2),\ \Delta^G \rightarrow
$$

$$
\left\{
\begin{array}{llll}
\text{a.} & \Delta^{G\prime} = \frac{\lambda}{\eta}\Delta^G & \text{no int. innov.} & \\
(F\ F),\ (\Delta^4\ \Delta^4) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^F & \text{o o} \\
(H\ F),\ (\Delta^1\ \Delta^4) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x o} \\
\text{b.} & \Delta^{G\prime} = \frac{\lambda^2}{\eta}\Delta^G & \text{H int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^4) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,(1-z^F)\,\overline{x}^F & \text{x o} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x o} \\
\text{c.} & \Delta^{G\prime} = \frac{1}{\lambda}\Delta^G & \text{F int. innov.} & \\
(F\ F),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)\,z^F\,\overline{x}^F & \text{o x} \\
(H\ F),\ (\Delta^1\ \Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x x} \\
\text{d.} & \Delta^{G\prime} = \Delta^G & \text{all int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,z^F\,\overline{x}^F & \text{x x}
\end{array}
\right. \tag{TA1.15}
$$

$$
(H\ F), (\Delta^{D\ell}\ \Delta^3),\ \Delta^G \rightarrow
$$

$$\left\{\begin{array}{lllll}
\text{a.} & \Delta^{G\prime} = \Delta^{G} & \text{no int. innov.} & \\
(H\ F),\ (\Delta^1\ \Delta^1) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^F & \text{x } \tfrac{1}{2} \\
\text{b.} & \Delta^{G\prime} = \lambda\Delta^{G} & \text{H int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,(1-z^F)\,\overline{x}^F & \text{x } \tfrac{1}{2} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x o} \\
\text{c.} & \Delta^{G\prime} = \frac{1}{\lambda}\Delta^{G} & \text{F int. innov.} & \\
(F\ F),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)\,z^F\,\overline{x}^F & \text{o x} \\
(H\ F),\ (\Delta^1\ \Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x x} \\
\text{d.} & \Delta^{G\prime} = \Delta^{G} & \text{all int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,z^F\,\overline{x}^F & \text{x x}
\end{array}\right. \tag{TA1.16}$$

$$(H\ F), (\Delta^{D\ell}\ \Delta^4),\ \Delta^G \rightarrow$$

$$\left\{\begin{array}{lllll}
\text{a.} & \Delta^{G\prime} = \frac{1}{\lambda}\Delta^{G} & \text{no int. innov.} & \\
(F\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^F & \text{o o} \\
(H\ F),\ (\Delta^1\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x o} \\
\text{b.} & \Delta^{G\prime} = \Delta^{G} & \text{H int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,(1-z^F)\,\overline{x}^F & \text{x o} \\
\text{c.} & \Delta^{G\prime} = \frac{1}{\lambda}\Delta^{G} & \text{F int. innov.} & \\
(F\ F),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)\,z^F\,\overline{x}^F & \text{o } \tfrac{1}{2} \\
(H\ F),\ (\Delta^1\ \Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x } \tfrac{1}{2} \\
\text{d.} & \Delta^{G\prime} = \Delta^{G} & \text{all int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,z^F\,\overline{x}^F & \text{x } \tfrac{1}{2}
\end{array}\right. \tag{TA1.17}$$

TA1.4 Inflows and Outflows

$\left(\Delta^{\ell^H}\ \Delta^{\ell^F}\ \ \Delta^G\right)$ for internal innovation intensity terms are omitted for notational simplicity. They should match to that of $\mu(\cdot)$. Here, $\sum_{\Delta^G}$ is sum over $\Delta^G \in [\underline{\Omega}, \overline{\Omega}]$.

$H\ H\ \Delta^1\ \Delta^1\ \infty$ case:

$$\begin{aligned} Outflow\left(\Delta^1\ \Delta^1\ \ \infty\right) &= \Big[z^H(1-\overline{x}) + (1-z^H)\overline{x}^H + z^H\overline{x}^H \\ &\quad + \left(\mathcal{I}_1\left(\frac{1}{\eta}\right) + \mathcal{I}_2\left(\frac{1}{\eta}\right)\right)(1-z^H)\overline{x}^F + z^H\overline{x}^F\Big]\mu\left(\Delta^1\ \Delta^1\ \ \infty\right) \\ &= \left[1 - \left[(1-z^H)(1-\overline{x}) + \mathcal{I}_3\left(\frac{1}{\eta}\right)(1-z^H)\overline{x}^F\right]\right]\mu\left(\Delta^1\ \Delta^1\ \ \infty\right) \end{aligned}$$

$$\begin{aligned} Inflow\left(\Delta^1\ \Delta^1\ \ \infty\right) &= \left[(1-z^H)(1-\overline{x}) + \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)(1-z^H)\overline{x}^F\right]\mu\left(\Delta^2\ \Delta^2\ \ \infty\right) \\ &\quad + \left[(1-z^H)(1-\overline{x}) + (1-z^H)\overline{x}^H + \mathcal{I}_3(1)(1-z^H)\overline{x}^F\right]\mu\left(\Delta^3\ \Delta^3\ \ \infty\right) \\ &\quad + \left[(1-z^H)(1-\overline{x}) + \mathcal{I}_3\left(\frac{1}{\lambda}\right)(1-z^H)\overline{x}^F\right]\mu\left(\Delta^4\ \Delta^4\ \ \infty\right) \\ &\quad + \left[\mathcal{I}_3(1)(1-z^F)\overline{x}^H + \mathcal{I}_3\left(\frac{1}{\lambda}\right)z^F\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^3\ \ -\infty\right) \end{aligned} \tag{TA1.18}$$

$H\ H\ \Delta^2\ \Delta^2\ \infty$ case:

$$Outflow\left(\Delta^2\ \Delta^2\ \ \infty\right) = \left[1 - \left[z^H(1-\overline{x}) + z^H\overline{x}^H + \mathcal{I}_3\left(\frac{\lambda^2}{\eta}\right)z^H\overline{x}^F\right]\right]\mu\left(\Delta^2\ \Delta^2\ \ \infty\right)$$

$$\begin{aligned} Inflow\left(\Delta^2\ \Delta^2\ \ \infty\right) &= \left[z^H(1-\overline{x}) + \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)z^H\overline{x}^F\right]\mu\left(\Delta^1\ \Delta^1\ \ \infty\right) \\ &\quad + \left[z^H(1-\overline{x}) + z^H\overline{x}^H + \mathcal{I}_3(\lambda)\,z^H\overline{x}^F\right]\mu\left(\Delta^3\ \Delta^3\ \ \infty\right) \\ &\quad + \left[z^H(1-\overline{x}) + z^H\overline{x}^H + \mathcal{I}_3(1)\,z^H\overline{x}^F\right]\mu\left(\Delta^4\ \Delta^4\ \ \infty\right) \\ &\quad + \left[\mathcal{I}_3(\lambda)(1-z^F)\overline{x}^H + \mathcal{I}_3(1)\,z^F\overline{x}^H\right]\mu\left(\Delta^4\ \Delta^4\ \ -\infty\right) \\ &\quad + \sum_{\Delta^H}\sum_{\Delta^F}\sum_{\Delta^G}\mathcal{I}_3\left(\lambda\Delta^G\right)z^H(1-z^F)(1-\overline{x})\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right) \\ &\quad + \sum_{\Delta^H=\Delta^2}^{\Delta^3}\sum_{\Delta^F}\sum_{\Delta^G}\mathcal{I}_3\left(\lambda\Delta^G\right)z^H(1-z^F)\overline{x}^H\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right) \\ &\quad + \sum_{\Delta^F}\sum_{\Delta^G}\mathcal{I}_3\left(\lambda\Delta^G\right)(1-z^F)\overline{x}^H\mu\left(\Delta^4\ \Delta^F\ \ \Delta^G\right) \\ &\quad + \sum_{\Delta^H}\sum_{\Delta^G}\mathcal{I}_3\left(\frac{\lambda^2}{\eta}\Delta^G\right)z^H(1-z^F)\overline{x}^H\mu\left(\Delta^H\ \Delta^2\ \ \Delta^G\right) \\ &\quad + \sum_{\Delta^H}\sum_{\Delta^G}\mathcal{I}_3\left(\lambda\Delta^G\right)z^H(1-z^F)\overline{x}^H\mu\left(\Delta^H\ \Delta^3\ \ \Delta^G\right) \end{aligned} \tag{TA1.19}$$

$H\ H\ \Delta^3\ \Delta^3\ \infty$ case:

$$Outflow\left(\Delta^3\ \Delta^3\ \ \infty\right) = \mu\left(\Delta^3\ \Delta^3\ \ \infty\right)$$

$$\begin{aligned} Inflow\left(\Delta^3\ \Delta^3\ \ \infty\right) = \overline{x}^H \mu\left(\Delta^1\ \Delta^1\ \ \infty\right) + \left[\mathcal{I}_3\left(\eta\right)(1-z^F)\overline{x}^H + \mathcal{I}_3\left(\frac{\eta}{\lambda}\right) z^F \overline{x}^H\right]\mu\left(\Delta^1\ \Delta^1\ \ -\infty\right) \\ + \sum_{\Delta^F}\sum_{\Delta^G}\left[\mathcal{I}_3\left(\eta\Delta^G\right)(1-z^F)\overline{x}^H + \mathcal{I}_3\left(\frac{\eta}{\lambda}\Delta^G\right) z^F\overline{x}^H\right]\mu\left(\Delta^1\ \Delta^F\ \ \Delta^G\right) \end{aligned} \tag{TA1.20}$$

$H\ H\ \Delta^4\ \Delta^4\ \infty$ case:

$$Outflow\left(\Delta^4\ \Delta^4\ \ \infty\right) = \mu\left(\Delta^4\ \Delta^4\ \ \infty\right)$$

$$\begin{aligned} Inflow\left(\Delta^4\ \Delta^4\ \ \infty\right) = (1-z^H)\overline{x}^H\mu\left(\Delta^2\ \Delta^2\ \ \infty\right) \\ + \left[\mathcal{I}_3\left(\frac{\eta}{\lambda}\right)(1-z^F)\overline{x}^H + \mathcal{I}_3\left(\frac{\eta}{\lambda^2}\right) z^F\overline{x}^H\right]\mu\left(\Delta^2\ \Delta^2\ \ -\infty\right) \\ + \sum_{\Delta^F}\sum_{\Delta^G}\left[\mathcal{I}_3\left(\frac{\eta}{\lambda}\Delta^G\right)(1-z^H)(1-z^F)\overline{x}^H\right]\mu\left(\Delta^2\ \Delta^F\ \ \Delta^G\right) \end{aligned} \tag{TA1.21}$$

$F\ F\ \Delta^1\ \Delta^1\ \ -\infty$ case:

$$Outflow\left(\Delta^1\ \Delta^1\ \ -\infty\right) = \left[1-\left[(1-z^F)(1-\overline{x}) + \mathcal{I}_1\left(\eta\right)(1-z^F)\overline{x}^H\right]\right]\mu\left(\Delta^1\ \Delta^1\ \ -\infty\right)$$

$$\begin{aligned} Inflow\left(\Delta^1\ \Delta^1\ \ -\infty\right) = \left[\mathcal{I}_1\left(1\right)(1-z^H)\overline{x}^F + \mathcal{I}_1\left(\lambda\right) z^H\overline{x}^F\right]\mu\left(\Delta^3\ \Delta^3\ \ \infty\right) \\ + \left[(1-z^F)(1-\overline{x}) + \mathcal{I}_1\left(\frac{\eta}{\lambda}\right)(1-z^F)\overline{x}^H\right]\mu\left(\Delta^2\ \Delta^2\ \ -\infty\right) \\ + \left[(1-z^F)(1-\overline{x}) + \mathcal{I}_1\left(1\right)(1-z^F)\overline{x}^H + (1-z^F)\overline{x}^F\right]\mu\left(\Delta^3\ \Delta^3\ \ -\infty\right) \\ + \left[(1-z^F)(1-\overline{x}) + \mathcal{I}_1\left(\lambda\right)(1-z^F)\overline{x}^H\right]\mu\left(\Delta^4\ \Delta^4\ \ -\infty\right) \end{aligned} \tag{TA1.22}$$

$F\ F\ \Delta^2\ \Delta^2\ \ -\infty$ case:

$$Outflow\left(\Delta^2\ \Delta^2\ \ -\infty\right) = \left[1-\left[z^F(1-\overline{x}) + \mathcal{I}_1\left(\frac{\eta}{\lambda^2}\right) z^F\overline{x}^H + z^F\overline{x}^F\right]\right]\mu\left(\Delta^2\ \Delta^2\ \ -\infty\right)$$

$$\begin{aligned} Inflow\left(\Delta^2\ \Delta^2\ \ -\infty\right) = \left[\mathcal{I}_1\left(\frac{1}{\lambda}\right)(1-z^H)\overline{x}^F + \mathcal{I}_1\left(1\right) z^H\overline{x}^F\right]\mu\left(\Delta^4\ \Delta^4\ \ \infty\right) \\ + \left[z^F(1-\overline{x}) + \mathcal{I}_1\left(\frac{\eta}{\lambda}\right) z^F\overline{x}^H\right]\mu\left(\Delta^1\ \Delta^1\ \ -\infty\right) \\ + \left[z^F(1-\overline{x}) + \mathcal{I}_1\left(\frac{1}{\lambda}\right) z^F\overline{x}^H + z^F\overline{x}^F\right]\mu\left(\Delta^3\ \Delta^3\ \ -\infty\right) \\ + \left[z^F(1-\overline{x}) + \mathcal{I}_1\left(1\right) z^F\overline{x}^H + \overline{x}^F\right]\mu\left(\Delta^4\ \Delta^4\ \ -\infty\right) \\ + \sum_{\Delta^H}\sum_{\Delta^F}\sum_{\Delta^G}\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)(1-z^H)z^F(1-\overline{x})\right]\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right) \end{aligned}$$

$$+\sum_{\Delta^F}\sum_{\Delta^G}\left[\mathcal{I}_1\left(\frac{\eta}{\lambda^2}\Delta^G\right)(1-z^H)z^F\overline{x}^H\right]\mu\left(\Delta^2\ \Delta^F\ \ \Delta^G\right)$$

$$+\sum_{\Delta^F}\sum_{\Delta^G}\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)(1-z^H)z^F\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^F\ \ \Delta^G\right)$$

$$+\sum_{\Delta^F=\Delta^2}^{\Delta^3}\sum_{\Delta^H}\sum_{\Delta^G}\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)(1-z^H)z^F\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right)$$

$$+\sum_{\Delta^H}\sum_{\Delta^G}\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)(1-z^H)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^4\ \ \Delta^G\right) \tag{TA1.23}$$

$F\ F\ \Delta^3\ \Delta^3\ -\infty$ case:

$$Outflow\left(\Delta^3\ \Delta^3\ \ -\infty\right)=\mu\left(\Delta^3\ \Delta^3\ \ -\infty\right)$$

$$Inflow\left(\Delta^3\ \Delta^3\ \ -\infty\right)=\left[\mathcal{I}_1\left(\frac{1}{\eta}\right)(1-z^H)\overline{x}^F+\mathcal{I}_1\left(\frac{\lambda}{\eta}\right)z^H\overline{x}^F\right]\mu\left(\Delta^1\ \Delta^1\ \ \infty\right)$$

$$+\overline{x}^F\mu\left(\Delta^1\ \Delta^1\ \ -\infty\right)$$

$$+\sum_{\Delta^H}\sum_{\Delta^G}\left[\mathcal{I}_1\left(\frac{1}{\eta}\Delta^G\right)(1-z^H)\overline{x}^F+\mathcal{I}_1\left(\frac{\lambda}{\eta}\Delta^G\right)z^H\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^1\ \ \Delta^G\right) \tag{TA1.24}$$

$F\ F\ \Delta^4\ \Delta^4\ -\infty$ case:

$$Outflow\left(\Delta^4\ \Delta^4\ \ -\infty\right)=\mu\left(\Delta^4\ \Delta^4\ \ -\infty\right)$$

$$Inflow\left(\Delta^4\ \Delta^4\ \ -\infty\right)=\left[\mathcal{I}_1\left(\frac{\lambda}{\eta}\right)(1-z^H)\overline{x}^F+\mathcal{I}_1\left(\frac{\lambda^2}{\eta}\right)z^H\overline{x}^F\right]\mu\left(\Delta^2\ \Delta^2\ \ \infty\right)$$

$$+(1-z^F)\overline{x}^F\mu\left(\Delta^2\ \Delta^2\ \ -\infty\right)$$

$$+\sum_{\Delta^H}\sum_{\Delta^G}\left[\mathcal{I}_1\left(\frac{\lambda}{\eta}\Delta^G\right)(1-z^H)(1-z^F)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^2\ \ \Delta^G\right) \tag{TA1.25}$$

$H\ F\ \Delta^1\ \Delta^1\ \Delta^G$ case:

$$Outflow\left(\Delta^1\ \Delta^1\ \ \Delta^G\right)=\left[1-(1-z^H)(1-z^F)(1-\overline{x})\right]\mu\left(\Delta^1\ \Delta^1\ \ \Delta^G\right)$$

$$Inflow\left(\Delta^1\ \Delta^1\ \ \Delta^G\right)=\mathcal{I}_{\{\Delta^G=1\}}\left[(1-z^H)\overline{x}^F\right]\mu\left(\Delta^3\ \Delta^3\ \ \infty\right)$$

$$+\mathcal{I}_{\{\Delta^G=1\}}\left[(1-z^F)\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^3\ \ -\infty\right)$$

$$+\sum_{(\Delta^H\ \Delta^F)\neq(\Delta^1\ \Delta^1)}\left[(1-z^H)(1-z^F)(1-\overline{x})\right]\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right)$$

$$+\sum_{\Delta^F}\left[(1-z^H)(1-z^F)\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^F\ \ \Delta^G\right)$$

$$+\sum_{\Delta^H}\left[(1-z^H)(1-z^F)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^3\ \ \Delta^G\right) \tag{TA1.26}$$

$H\ F\ \Delta^1\ \Delta^2\ \Delta^G$ case:

$$Outflow\left(\Delta^1\ \Delta^2\ \ \Delta^G\right) = \mu\left(\Delta^1\ \Delta^2\ \ \Delta^G\right)$$

$$\begin{aligned} Inflow\left(\Delta^1\ \Delta^2\ \ \Delta^G\right) = & \mathcal{I}_{\left\{\Delta^G=\frac{1}{\lambda}\right\}}\left[(1-z^H)\overline{x}^F\right]\mu\left(\Delta^4\ \Delta^4\ \ \infty\right) \\ & + \mathcal{I}_{\left\{\Delta^G=\frac{1}{\lambda}\right\}}\left[z^F\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^3\ \ -\infty\right) \\ & + \sum_{\Delta^H}\sum_{\Delta^F}\left[(1-z^H)z^F(1-\overline{x})\right]\mu\left(\Delta^H\ \Delta^F\ \ \lambda\Delta^G\right) \\ & + \sum_{\Delta^F}\left[(1-z^H)z^F\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^F\ \ \lambda\Delta^G\right) \\ & + \sum_{\Delta^H}\left[(1-z^H)z^F\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^2\ \ \lambda\Delta^G\right) \\ & + \sum_{\Delta^H}\left[(1-z^H)z^F\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^3\ \ \lambda\Delta^G\right) \\ & + \sum_{\Delta^H}\left[(1-z^H)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^4\ \ \lambda\Delta^G\right) \end{aligned} \quad \text{(TA1.27)}$$

$H\ F\ \Delta^1\ \Delta^3\ \Delta^G$ case:

$$Outflow\left(\Delta^1\ \Delta^3\ \ \Delta^G\right) = \mu\left(\Delta^1\ \Delta^3\ \ \Delta^G\right)$$

$$\begin{aligned} Inflow\left(\Delta^1\ \Delta^3\ \ \Delta^G\right) = & \mathcal{I}_{\left\{\Delta^G=\frac{1}{\eta}\right\}}\left[(1-z^H)\overline{x}^F\right]\mu\left(\Delta^1\ \Delta^1\ \ \infty\right) \\ & + \sum_{\Delta^H}\left[(1-z^H)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^1\ \ \eta\Delta^G\right) \end{aligned} \quad \text{(TA1.28)}$$

$H\ F\ \Delta^1\ \Delta^4\ \Delta^G$ case:

$$Outflow\left(\Delta^1\ \Delta^4\ \ \Delta^G\right) = \mu\left(\Delta^1\ \Delta^4\ \ \Delta^G\right)$$

$$\begin{aligned} Inflow\left(\Delta^1\ \Delta^4\ \ \Delta^G\right) = & \mathcal{I}_{\left\{\Delta^G=\frac{\lambda}{\eta}\right\}}\left[(1-z^H)\overline{x}^F\right]\mu\left(\Delta^2\ \Delta^2\ \ \infty\right) \\ & + \sum_{\Delta^H}\left[(1-z^H)(1-z^F)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^2\ \ \frac{\eta}{\lambda}\Delta^G\right) \end{aligned} \quad \text{(TA1.29)}$$

$H\ F\ \Delta^2\ \Delta^1\ \Delta^G$ case:

$$Outflow\left(\Delta^2\ \Delta^1\ \ \Delta^G\right) = \mu\left(\Delta^2\ \Delta^1\ \ \Delta^G\right)$$

$$\begin{aligned} Inflow\left(\Delta^2\ \Delta^1\ \ \Delta^G\right) = & \mathcal{I}_{\{\Delta^G=\lambda\}}\left[z^H\overline{x}^F\right]\mu\left(\Delta^3\ \Delta^3\ \ \infty\right) \\ & + \mathcal{I}_{\{\Delta^G=\lambda\}}\left[(1-z^F)\overline{x}^H\right]\mu\left(\Delta^4\ \Delta^4\ \ -\infty\right) \end{aligned}$$

$$
\begin{aligned}
&+\sum_{\Delta^H}\sum_{\Delta^F}\left[z^H(1-z^F)(1-\overline{x})\right]\mu\left(\Delta^H\ \Delta^F\ \ \frac{1}{\lambda}\Delta^G\right)\\
&+\sum_{\Delta^F}\left[z^H(1-z^F)\overline{x}^H\right]\mu\left(\Delta^2\ \Delta^F\ \ \frac{1}{\lambda}\Delta^G\right)\\
&+\sum_{\Delta^F}\left[z^H(1-z^F)\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^F\ \ \frac{1}{\lambda}\Delta^G\right)\\
&+\sum_{\Delta^F}\left[(1-z^F)\overline{x}^H\right]\mu\left(\Delta^4\ \Delta^F\ \ \frac{1}{\lambda}\Delta^G\right)\\
&+\sum_{\Delta^H}\left[z^H(1-z^F)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^3\ \ \frac{1}{\lambda}\Delta^G\right)
\end{aligned}
\tag{TA1.30}
$$

$H\ F\ \Delta^2\ \Delta^2\ \Delta^G$ case:

$$
Outflow\left(\Delta^2\ \Delta^2\ \ \Delta^G\right)=\left[1-\left[z^Hz^F(1-\overline{x})+z^Hz^F\overline{x}^H+z^Hz^F\overline{x}^F\right]\right]\mu\left(\Delta^2\ \Delta^2\ \ \Delta^G\right)
$$

$$
\begin{aligned}
Inflow\left(\Delta^2\ \Delta^2\ \ \Delta^G\right)=&\ \mathcal{I}_{\{\Delta^G=1\}}\left[z^H\overline{x}^F\right]\mu\left(\Delta^4\ \Delta^4\ \ \infty\right)+\mathcal{I}_{\{\Delta^G=1\}}\left[z^F\overline{x}^H\right]\mu\left(\Delta^4\ \Delta^4\ \ -\infty\right)\\
&+\sum_{(\Delta^H\ \Delta^F)\neq(\Delta^2\ \Delta^2)}\left[z^Hz^F(1-\overline{x})\right]\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right)\\
&+\sum_{\Delta^F\neq\Delta^2}\left[z^Hz^F\overline{x}^H\right]\mu\left(\Delta^2\ \Delta^F\ \ \Delta^G\right)\\
&+\sum_{\Delta^F}\left[z^Hz^F\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^F\ \ \Delta^G\right)\\
&+\sum_{\Delta^F}\left[z^F\overline{x}^H\right]\mu\left(\Delta^4\ \Delta^F\ \ \Delta^G\right)\\
&+\sum_{\Delta^H\neq\Delta^2}\left[z^Hz^F\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^2\ \ \Delta^G\right)\\
&+\sum_{\Delta^H}\left[z^Hz^F\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^3\ \ \Delta^G\right)\\
&+\sum_{\Delta^H}\left[z^H\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^4\ \ \Delta^G\right)
\end{aligned}
\tag{TA1.31}
$$

$H\ F\ \Delta^2\ \Delta^3\ \Delta^G$ case:

$$
Outflow\left(\Delta^2\ \Delta^3\ \ \Delta^G\right)=\mu\left(\Delta^2\ \Delta^3\ \ \Delta^G\right)
$$

$$
\begin{aligned}
Inflow\left(\Delta^2\ \Delta^3\ \ \Delta^G\right)=&\ \mathcal{I}_{\left\{\Delta^G=\frac{\lambda}{\eta}\right\}}\left[z^H\overline{x}^F\right]\mu\left(\Delta^1\ \Delta^1\ \ \infty\right)\\
&+\sum_{\Delta^H}\left[z^H\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^1\ \ \frac{\eta}{\lambda}\Delta^G\right)
\end{aligned}
\tag{TA1.32}
$$

$H\ F\ \Delta^2\ \Delta^4\ \Delta^G$ case:

$$
Outflow\left(\Delta^2\ \Delta^4\ \ \Delta^G\right)=\mu\left(\Delta^2\ \Delta^4\ \ \Delta^G\right)
$$

$$Inflow\left(\Delta^2\ \Delta^4\ \ \Delta^G\right) = \mathcal{I}_{\left\{\Delta^G = \frac{\lambda^2}{\eta}\right\}}\left[z^H \overline{x}^F\right]\mu\left(\Delta^2\ \Delta^2\ \ \infty\right) + \sum_{\Delta^H}\left[z^H(1-z^F)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^2\ \ \frac{\eta}{\lambda^2}\Delta^G\right) \tag{TA1.33}$$

H F Δ^3 Δ^1 Δ^G case:

$$Outflow\left(\Delta^3\ \Delta^1\ \ \Delta^G\right) = \mu\left(\Delta^3\ \Delta^1\ \ \Delta^G\right)$$

$$Inflow\left(\Delta^3\ \Delta^1\ \ \Delta^G\right) = \mathcal{I}_{\{\Delta^G = \eta\}}\left[(1-z^F)\overline{x}^H\right]\mu\left(\Delta^1\ \Delta^1\ \ -\infty\right) + \sum_{\Delta^F}\left[(1-z^F)\overline{x}^H\right]\mu\left(\Delta^1\ \Delta^F\ \ \frac{1}{\eta}\Delta^G\right) \tag{TA1.34}$$

H F Δ^3 Δ^2 Δ^G case:

$$Outflow\left(\Delta^3\ \Delta^2\ \ \Delta^G\right) = \mu\left(\Delta^3\ \Delta^2\ \ \Delta^G\right)$$

$$Inflow\left(\Delta^3\ \Delta^2\ \ \Delta^G\right) = \mathcal{I}_{\left\{\Delta^G = \frac{\eta}{\lambda}\right\}}\left[z^F \overline{x}^H\right]\mu\left(\Delta^1\ \Delta^1\ \ -\infty\right) + \sum_{\Delta^F}\left[z^F \overline{x}^H\right]\mu\left(\Delta^1\ \Delta^F\ \ \frac{\lambda}{\eta}\Delta^G\right) \tag{TA1.35}$$

H F Δ^3 Δ^3 Δ^G case:

$$Outflow\left(\Delta^3\ \Delta^3\ \ \Delta^G\right) = \mu\left(\Delta^3\ \Delta^3\ \ \Delta^G\right)$$

$$Inflow\left(\Delta^3\ \Delta^3\ \ \Delta^G\right) = 0 \tag{TA1.36}$$

H F Δ^3 Δ^4 Δ^G case:

$$Outflow\left(\Delta^3\ \Delta^4\ \ \Delta^G\right) = \mu\left(\Delta^3\ \Delta^4\ \ \Delta^G\right)$$

$$Inflow\left(\Delta^3\ \Delta^4\ \ \Delta^G\right) = 0 \tag{TA1.37}$$

H F Δ^4 Δ^1 Δ^G case:

$$Outflow\left(\Delta^4\ \Delta^1\ \ \Delta^G\right) = \mu\left(\Delta^4\ \Delta^1\ \ \Delta^G\right)$$

$$Inflow\left(\Delta^4\ \Delta^1\ \ \Delta^G\right) = \mathcal{I}_{\left\{\Delta^G = \frac{\eta}{\lambda}\right\}}\left[(1-z^F)\overline{x}^H\right]\mu\left(\Delta^2\ \Delta^2\ \ -\infty\right) + \sum_{\Delta^F}\left[(1-z^H)(1-z^F)\overline{x}^H\right]\mu\left(\Delta^2\ \Delta^F\ \ \frac{\lambda}{\eta}\Delta^G\right) \tag{TA1.38}$$

$H\ F\ \Delta^4\ \Delta^2\ \Delta^G$ case:

$$Outflow\left(\Delta^4\ \Delta^2\ \ \Delta^G\right) = \mu\left(\Delta^4\ \Delta^2\ \ \Delta^G\right)$$

$$\begin{aligned} Inflow\left(\Delta^4\ \Delta^2\ \ \Delta^G\right) &= \mathcal{I}_{\left\{\Delta^G = \frac{\eta}{\lambda^2}\right\}}\left[z^F \overline{x}^H\right]\mu\left(\Delta^2\ \Delta^2\ \ -\infty\right) \\ &+ \sum_{\Delta^F}\left[(1-z^H)z^F\overline{x}^H\right]\mu\left(\Delta^2\ \Delta^F\ \ \frac{\lambda^2}{\eta}\Delta^G\right) \end{aligned} \tag{TA1.39}$$

$H\ F\ \Delta^4\ \Delta^3\ \Delta^G$ case:

$$Outflow\left(\Delta^4\ \Delta^3\ \ \Delta^G\right) = \mu\left(\Delta^4\ \Delta^3\ \ \Delta^G\right)$$

$$Inflow\left(\Delta^4\ \Delta^3\ \ \Delta^G\right) = 0 \tag{TA1.40}$$

$H\ F\ \Delta^4\ \Delta^4\ \Delta^G$ case:

$$Outflow\left(\Delta^4\ \Delta^4\ \ \Delta^G\right) = \mu\left(\Delta^4\ \Delta^4\ \ \Delta^G\right)$$

$$Inflow\left(\Delta^4\ \Delta^4\ \ \Delta^G\right) = 0 \tag{TA1.41}$$

TA2 External Innovation Outcomes

TA2.1 Outcomes from a Successful External Innovation, Home Firm

$$\left(q\ H\ H\ \Delta^1\ \Delta^1\ \infty \right) \rightarrow \begin{cases} \left(\eta q\ H\ H\ \Delta^3\ \Delta^3\ \infty \right) & \text{w/ } 1 \end{cases} \tag{TA2.42}$$

$$\left(q\ H\ H\ \Delta^2\ \Delta^2\ \infty \right) \rightarrow \begin{cases} \left(\frac{\eta}{\lambda} q\ H\ H\ \Delta^4\ \Delta^4\ \infty \right) & \text{w/ } 1 - z^H \\ \mathbf{0} & \text{w/ } z^H \end{cases} \tag{TA2.43}$$

$$\left(q\ H\ H\ \Delta^3\ \Delta^3\ \infty \right) \rightarrow \begin{cases} \left(q\ H\ H\ \Delta^1\ \Delta^1\ \infty \right) & \text{w/ } \frac{1}{2}(1 - z^H) \\ \mathbf{0} & \text{w/ } \frac{1}{2}(1 + z^H) \end{cases} \tag{TA2.44}$$

$$\left(q\ H\ H\ \Delta^4\ \Delta^4\ \infty \right) \rightarrow \begin{cases} \left(\lambda q\ H\ H\ \Delta^2\ \Delta^2\ \infty \right) & \text{w/ } (1 - z^H) \\ \left(\lambda q\ H\ H\ \Delta^2\ \Delta^2\ \infty \right) & \text{w/ } \frac{1}{2} z^H \\ \mathbf{0} & \text{w/ } \frac{1}{2} z^H \end{cases} \tag{TA2.45}$$

$$\left(q\ F\ F\ \Delta^1\ \Delta^1\ -\infty \right) \rightarrow \begin{cases} \begin{bmatrix} \left(\eta q\ H\ F\ \Delta^3\ \Delta^1\ \ \eta \right) \mathcal{I}_2\left(\eta\right) \\ +\left(\eta q\ H\ H\ \Delta^3\ \Delta^3\ \ \infty \right) \mathcal{I}_3\left(\eta\right) \end{bmatrix} & \text{w/ } 1 - z^F \\ \begin{bmatrix} \left(\eta q\ H\ F\ \Delta^3\ \Delta^2\ \ \frac{\eta}{\lambda} \right) \mathcal{I}_2\left(\frac{\eta}{\lambda}\right) \\ +\left(\eta q\ H\ H\ \Delta^3\ \Delta^3\ \ \infty \right) \mathcal{I}_3\left(\frac{\eta}{\lambda}\right) \end{bmatrix} & \text{w/ } z^F \end{cases} \tag{TA2.46}$$

$$\left(q\ F\ F\ \Delta^2\ \Delta^2\ -\infty \right) \rightarrow \begin{cases} \begin{bmatrix} \left(\frac{\eta}{\lambda} q\ H\ F\ \Delta^4\ \Delta^1\ \ \frac{\eta}{\lambda} \right) \mathcal{I}_2\left(\frac{\eta}{\lambda}\right) \\ +\left(\frac{\eta}{\lambda} q\ H\ H\ \Delta^4\ \Delta^4\ \ \infty \right) \mathcal{I}_3\left(\frac{\eta}{\lambda}\right) \end{bmatrix} & \text{w/ } 1 - z^F \\ \begin{bmatrix} \left(\frac{\eta}{\lambda} q\ H\ F\ \Delta^4\ \Delta^2\ \ \frac{\eta}{\lambda^2} \right) \mathcal{I}_2\left(\frac{\eta}{\lambda^2}\right) \\ +\left(\frac{\eta}{\lambda} q\ H\ H\ \Delta^4\ \Delta^4\ \ \infty \right) \mathcal{I}_3\left(\frac{\eta}{\lambda^2}\right) \end{bmatrix} & \text{w/ } z^F \end{cases} \tag{TA2.47}$$

$$\left(q\ F\ F\ \Delta^3\ \Delta^3\ -\infty \right) \rightarrow \begin{cases} \begin{bmatrix} \left(q\ H\ F\ \Delta^1\ \Delta^1\ \ 1 \right) \mathcal{I}_2\left(1\right) \\ +\left(q\ H\ H\ \Delta^1\ \Delta^1\ \ \infty \right) \mathcal{I}_3\left(1\right) \end{bmatrix} & \text{w/ } 1 - z^F \\ \begin{bmatrix} \left(q\ H\ F\ \Delta^1\ \Delta^2\ \ \frac{1}{\lambda} \right) \mathcal{I}_2\left(\frac{1}{\lambda}\right) \\ +\left(q\ H\ H\ \Delta^1\ \Delta^1\ \ \infty \right) \mathcal{I}_3\left(\frac{1}{\lambda}\right) \end{bmatrix} & \text{w/ } z^F \end{cases} \tag{TA2.48}$$

$$\left(q\ F\ F\ \Delta^4\ \Delta^4\ -\infty \right) \rightarrow \begin{cases} \begin{bmatrix} \left(\lambda q\ H\ F\ \Delta^2\ \Delta^1\ \ \lambda \right) \mathcal{I}_2\left(\lambda\right) \\ +\left(\lambda q\ H\ H\ \Delta^2\ \Delta^2\ \ \infty \right) \mathcal{I}_3\left(\lambda\right) \end{bmatrix} & \text{w/ } 1 - z^F \\ \begin{bmatrix} \left(\lambda q\ H\ F\ \Delta^2\ \Delta^2\ \ 1 \right) \mathcal{I}_2\left(1\right) \\ +\left(\lambda q\ H\ H\ \Delta^2\ \Delta^2\ \ \infty \right) \mathcal{I}_3\left(1\right) \end{bmatrix} & \text{w/ } z^F \end{cases} \tag{TA2.49}$$

$$\left(\, q\; H\; F\; \Delta^1\; \Delta^{F\ell}\; \Delta^G \,\right) \to \begin{cases} \left[\begin{array}{l} \left(\, \eta q\; H\; F\; \Delta^3\; \Delta^1\;\; \eta\Delta^G \,\right) \mathcal{I}_2\left(\eta\Delta^G\right) \\ +\left(\, \eta q\; H\; H\; \Delta^3\; \Delta^3\;\; \infty \,\right) \mathcal{I}_3\left(\eta\Delta^G\right) \end{array}\right] & \text{w/ } (1-z^H)(1-z^F) \\ \left[\begin{array}{l} \left(\, \eta q\; H\; F\; \Delta^3\; \Delta^1\;\; \eta\Delta^G \,\right) \mathcal{I}_2\left(\eta\Delta^G\right) \\ +\left(\, \eta q\; H\; H\; \Delta^3\; \Delta^3\;\; \infty \,\right) \mathcal{I}_3\left(\eta\Delta^G\right) \end{array}\right] & \text{w/ } z^H(1-z^F) \\ \left[\begin{array}{l} \left(\, \eta q\; H\; F\; \Delta^3\; \Delta^2\;\; \tfrac{\eta}{\lambda}\Delta^G \,\right) \mathcal{I}_2\left(\tfrac{\eta}{\lambda}\Delta^G\right) \\ +\left(\, \eta q\; H\; H\; \Delta^3\; \Delta^3\;\; \infty \,\right) \mathcal{I}_3\left(\tfrac{\eta}{\lambda}\Delta^G\right) \end{array}\right] & \text{w/ } (1-z^H)z^F \\ \left[\begin{array}{l} \left(\, \eta q\; H\; F\; \Delta^3\; \Delta^2\;\; \tfrac{\eta}{\lambda}\Delta^G \,\right) \mathcal{I}_2\left(\tfrac{\eta}{\lambda}\Delta^G\right) \\ +\left(\, \eta q\; H\; H\; \Delta^3\; \Delta^3\;\; \infty \,\right) \mathcal{I}_3\left(\tfrac{\eta}{\lambda}\Delta^G\right) \end{array}\right] & \text{w/ } z^H z^F \end{cases} \tag{TA2.50}$$

$$\left(\, q\; H\; F\; \Delta^2\; \Delta^{F\ell}\; \Delta^G \,\right) \to \begin{cases} \left[\begin{array}{l} \left(\, \tfrac{\eta}{\lambda} q\; H\; F\; \Delta^4\; \Delta^1\;\; \tfrac{\eta}{\lambda}\Delta^G \,\right) \mathcal{I}_2\left(\tfrac{\eta}{\lambda}\Delta^G\right) \\ +\left(\, \tfrac{\eta}{\lambda} q\; H\; H\; \Delta^4\; \Delta^4\;\; \infty \,\right) \mathcal{I}_3\left(\tfrac{\eta}{\lambda}\Delta^G\right) \end{array}\right] & \text{w/ } (1-z^H)(1-z^F) \\ \left(\, \tfrac{\eta}{\lambda} q\; H\; F\; \Delta^4\; \Delta^2\;\; \tfrac{\eta}{\lambda^2}\Delta^G \,\right) \mathcal{I}_2\left(\tfrac{\eta}{\lambda^2}\Delta^G\right) & \text{w/ } (1-z^H)z^F \\ \mathbf{0} & \text{w/ } z^H \end{cases} \tag{TA2.51}$$

$$\left(\, q\; H\; F\; \Delta^3\; \Delta^{F\ell}\; \Delta^G \,\right) \to \begin{cases} \left(\, q\; H\; F\; \Delta^1\; \Delta^1\;\; \Delta^G \,\right) & \text{w/ } \tfrac{1}{2}(1-z^H)(1-z^F) \\ \left(\, q\; H\; F\; \Delta^1\; \Delta^2\;\; \tfrac{1}{\lambda}\Delta^G \,\right) \mathcal{I}_2\left(\tfrac{1}{\lambda}\Delta^G\right) & \text{w/ } \tfrac{1}{2}(1-z^H)z^F \\ \mathbf{0} & \text{w/ } \tfrac{1}{2}(1+z^H) \end{cases} \tag{TA2.52}$$

$$\left(\, q\; H\; F\; \Delta^4\; \Delta^{F\ell}\; \Delta^G \,\right) \to \begin{cases} \left[\begin{array}{l} \left(\, \lambda q\; H\; F\; \Delta^2\; \Delta^1\;\; \lambda\Delta^G \,\right) \mathcal{I}_2\left(\lambda\Delta^G\right) \\ +\left(\, \lambda q\; H\; H\; \Delta^2\; \Delta^2\;\; \infty \,\right) \mathcal{I}_3\left(\lambda\Delta^G\right) \end{array}\right] & \text{w/ } (1-z^H)(1-z^F) \\ \left[\begin{array}{l} \left(\, \lambda q\; H\; F\; \Delta^2\; \Delta^1\;\; \lambda\Delta^G \,\right) \mathcal{I}_2\left(\lambda\Delta^G\right) \\ +\left(\, \lambda q\; H\; H\; \Delta^2\; \Delta^2\;\; \infty \,\right) \mathcal{I}_3\left(\lambda\Delta^G\right) \end{array}\right] & \text{w/ } \tfrac{1}{2}z^H(1-z^F) \\ \left(\, \lambda q\; H\; F\; \Delta^2\; \Delta^2\;\; \Delta^G \,\right) & \text{w/ } (1-z^H)z^F \\ \left(\, \lambda q\; H\; F\; \Delta^2\; \Delta^2\;\; \Delta^G \,\right) & \text{w/ } \tfrac{1}{2}z^H z^F \\ \mathbf{0} & \text{w/ } \tfrac{1}{2}z^H \end{cases} \tag{TA2.53}$$

TA2.2 Outcomes from a Successful External Innovation, Foreign Firm

$$\left(q\; H\; H\; \Delta^1\; \Delta^1\; \infty \right) \rightarrow \begin{cases} \begin{bmatrix} \left(\eta q\; F\; F\; \Delta^3\; \Delta^3\; -\infty \right) \mathcal{I}_1\left(\frac{1}{\eta}\right) \\ +\left(\eta q\; H\; F\; \Delta^1\; \Delta^3\; \frac{1}{\eta} \right) \mathcal{I}_2\left(\frac{1}{\eta}\right) \end{bmatrix} & \text{w/ } 1-z^H \\ \begin{bmatrix} \left(\eta q\; F\; F\; \Delta^3\; \Delta^3\; -\infty \right) \mathcal{I}_1\left(\frac{\lambda}{\eta}\right) \\ +\left(\eta q\; H\; F\; \Delta^2\; \Delta^3\; \frac{\lambda}{\eta} \right) \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \end{bmatrix} & \text{w/ } z^H \end{cases} \tag{TA2.54}$$

$$\left(q\; H\; H\; \Delta^2\; \Delta^2\; \infty \right) \rightarrow \begin{cases} \begin{bmatrix} \left(\frac{\eta}{\lambda} q\; F\; F\; \Delta^4\; \Delta^4\; -\infty \right) \mathcal{I}_1\left(\frac{\lambda}{\eta}\right) \\ +\left(\frac{\eta}{\lambda} q\; H\; F\; \Delta^1\; \Delta^4\; \frac{\lambda}{\eta} \right) \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \end{bmatrix} & \text{w/ } 1-z^H \\ \begin{bmatrix} \left(\frac{\eta}{\lambda} q\; F\; F\; \Delta^4\; \Delta^4\; -\infty \right) \mathcal{I}_1\left(\frac{\lambda^2}{\eta}\right) \\ +\left(\frac{\eta}{\lambda} q\; H\; F\; \Delta^2\; \Delta^4\; \frac{\lambda^2}{\eta} \right) \mathcal{I}_2\left(\frac{\lambda^2}{\eta}\right) \end{bmatrix} & \text{w/ } z^H \end{cases} \tag{TA2.55}$$

$$\left(q\; H\; H\; \Delta^3\; \Delta^3\; \infty \right) \rightarrow \begin{cases} \begin{bmatrix} \left(q\; F\; F\; \Delta^1\; \Delta^1\; -\infty \right) \mathcal{I}_1(1) \\ +\left(q\; H\; F\; \Delta^1\; \Delta^1\; 1 \right) \mathcal{I}_2(1) \end{bmatrix} & \text{w/ } 1-z^H \\ \begin{bmatrix} \left(q\; F\; F\; \Delta^1\; \Delta^1\; -\infty \right) \mathcal{I}_1(\lambda) \\ +\left(q\; H\; F\; \Delta^2\; \Delta^1\; \lambda \right) \mathcal{I}_2(\lambda) \end{bmatrix} & \text{w/ } z^H \end{cases} \tag{TA2.56}$$

$$\left(q\; H\; H\; \Delta^4\; \Delta^4\; \infty \right) \rightarrow \begin{cases} \begin{bmatrix} \left(\lambda q\; F\; F\; \Delta^2\; \Delta^2\; -\infty \right) \mathcal{I}_1\left(\frac{1}{\lambda}\right) \\ +\left(\lambda q\; H\; F\; \Delta^1\; \Delta^2\; \frac{1}{\lambda} \right) \mathcal{I}_2\left(\frac{1}{\lambda}\right) \end{bmatrix} & \text{w/ } 1-z^H \\ \begin{bmatrix} \left(\lambda q\; F\; F\; \Delta^2\; \Delta^2\; -\infty \right) \mathcal{I}_1(1) \\ +\left(\lambda q\; H\; F\; \Delta^2\; \Delta^2\; 1 \right) \mathcal{I}_2(1) \end{bmatrix} & \text{w/ } z^H \end{cases} \tag{TA2.57}$$

$$\left(q\; F\; F\; \Delta^1\; \Delta^1\; -\infty \right) \rightarrow \begin{cases} \left(\eta q\; F\; F\; \Delta^3\; \Delta^3\; -\infty \right) & \text{w/ } 1 \end{cases} \tag{TA2.58}$$

$$\left(q\; F\; F\; \Delta^2\; \Delta^2\; -\infty \right) \rightarrow \begin{cases} \left(\frac{\eta}{\lambda} q\; F\; F\; \Delta^4\; \Delta^4\; -\infty \right) & \text{w/ } 1-z^F \\ \mathbf{0} & \text{w/ } z^F \end{cases} \tag{TA2.59}$$

$$\left(q\; F\; F\; \Delta^3\; \Delta^3\; -\infty \right) \rightarrow \begin{cases} \left(q\; F\; F\; \Delta^1\; \Delta^1\; -\infty \right) & \text{w/ } \frac{1}{2}(1-z^F) \\ \mathbf{0} & \text{w/ } \frac{1}{2}(1+z^F) \end{cases} \tag{TA2.60}$$

$$\left(q\; F\; F\; \Delta^4\; \Delta^4\; -\infty \right) \rightarrow \begin{cases} \left(\lambda q\; F\; F\; \Delta^2\; \Delta^2\; -\infty \right) & \text{w/ } (1-\frac{1}{2}z^F) \\ \mathbf{0} & \text{w/ } \frac{1}{2}z^F \end{cases} \tag{TA2.61}$$

$$\left(\, q\; H\; F\; \Delta^{\ell^H}\; \Delta^1\;\; \Delta^G \,\right) \to \begin{cases} \left[\begin{array}{r} \left(\, \eta q\; F\; F\; \Delta^3\; \Delta^3\;\; -\infty \,\right) \mathcal{I}_1\left(\frac{1}{\eta}\Delta^G\right) \\ +\left(\, \eta q\; H\; F\; \Delta^1\; \Delta^3\;\; \frac{1}{\eta}\Delta^G \,\right) \mathcal{I}_2\left(\frac{1}{\eta}\Delta^G\right) \end{array}\right] & \text{w/ } 1 - z^H \\ \left[\begin{array}{r} \left(\, \eta q\; F\; F\; \Delta^3\; \Delta^3\;\; -\infty \,\right) \mathcal{I}_1\left(\frac{\lambda}{\eta}\Delta^G\right) \\ +\left(\, \eta q\; H\; F\; \Delta^3\; \Delta^3\;\; \frac{\lambda}{\eta}\Delta^G \,\right) \mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right) \end{array}\right] & \text{w/ } z^H \end{cases} \tag{TA2.62}$$

$$\left(\, q\; H\; F\; \Delta^{\ell^H}\; \Delta^2\;\; \Delta^G \,\right) \to \begin{cases} \left[\begin{array}{r} \left(\, \frac{\eta}{\lambda} q\; F\; F\; \Delta^4\; \Delta^4\;\; -\infty \,\right) \mathcal{I}_1\left(\frac{\lambda}{\eta}\Delta^G\right) \\ +\left(\, \frac{\eta}{\lambda} q\; H\; F\; \Delta^1\; \Delta^4\;\; \frac{\lambda}{\eta}\Delta^G \,\right) \mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right) \end{array}\right] & \text{w/ } (1 - z^H)(1 - z^F) \\ \left(\, \frac{\eta}{\lambda} q\; H\; F\; \Delta^2\; \Delta^4\;\; \frac{\lambda^2}{\eta}\Delta^G \,\right) \mathcal{I}_2\left(\frac{\lambda^2}{\eta}\Delta^G\right) & \text{w/ } z^H(1 - z^F) \\ \mathbf{0} & \text{w/ } z^F \end{cases} \tag{TA2.63}$$

$$\left(\, q\; H\; F\; \Delta^{\ell^H}\; \Delta^3\;\; \Delta^G \,\right) \to \begin{cases} \left(\, q\; H\; F\; \Delta^1\; \Delta^1\;\; \Delta^G \,\right) & \text{w/ } \frac{1}{2}(1 - z^H)(1 - z^F) \\ \left(\, q\; H\; F\; \Delta^2\; \Delta^1\;\; \lambda\Delta^G \,\right) \mathcal{I}_2\left(\lambda\Delta^G\right) & \text{w/ } \frac{1}{2} z^H(1 - z^F) \\ \mathbf{0} & \text{w/ } \frac{1}{2}(1 + z^F) \end{cases} \tag{TA2.64}$$

$$\left(\, q\; H\; F\; \Delta^{\ell^H}\; \Delta^4\;\; \Delta^G \,\right) \to \begin{cases} \left[\begin{array}{r} \left(\, \lambda q\; F\; F\; \Delta^2\; \Delta^2\;\; -\infty \,\right) \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) \\ +\left(\, \lambda q\; H\; F\; \Delta^1\; \Delta^2\;\; \frac{1}{\lambda}\Delta^G \,\right) \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) \end{array}\right] & \text{w/ } (1 - z^H)(1 - \frac{1}{2} z^F) \\ \left(\, \lambda q\; H\; F\; \Delta^2\; \Delta^2\;\; \Delta^G \,\right) & \text{w/ } z^H(1 - \frac{1}{2} z^F) \\ \mathbf{0} & \text{w/ } \frac{1}{2} z^F \end{cases} \tag{TA2.65}$$

TA3 Complete Description of $\mathcal{Q}$ Evolution

$$
\begin{aligned}
\mathcal{Q}'_{HH} = & \left\{ \begin{array}{l}
\left[\begin{array}{l}
\left[\,(1-z^H)+\lambda z^H\,\right](1-\overline{x})+\eta\overline{x}^H \\
+\left[\mathcal{I}_2\left(\frac{1}{\eta}\right)+\mathcal{I}_3\left(\frac{1}{\eta}\right)\right](1-z^H)\overline{x}^F \\
+\left[\mathcal{I}_2\left(\frac{\lambda}{\eta}\right)+\mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\right]\lambda\, z^H\,\overline{x}^F
\end{array} \right] \mu\left(\Delta^1\ \Delta^1\ \ \infty\right) \\
+ \left[\begin{array}{l}
\left[\,(1-z^H)+\lambda z^H\,\right](1-\overline{x}) \\
+\left[\,\frac{\eta}{\lambda}(1-z^H)+\lambda z^H\,\right]\overline{x}^H \\
+\left[\mathcal{I}_2\left(\frac{\lambda}{\eta}\right)+\mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\right](1-z^H)\overline{x}^F \\
+\left[\mathcal{I}_2\left(\frac{\lambda^2}{\eta}\right)+\mathcal{I}_3\left(\frac{\lambda^2}{\eta}\right)\right]\lambda\, z^H\,\overline{x}^F
\end{array} \right] \mu\left(\Delta^2\ \Delta^2\ \ \infty\right) \\
+ \left[\begin{array}{l}
\left[\,(1-z^H)+\lambda z^H\,\right](1-\overline{x}) \\
+\left[\,(1-z^H)+\lambda z^H\,\right]\overline{x}^H \\
+\left[\,\mathcal{I}_2\,(1)+\mathcal{I}_3\,(1)\,\right](1-z^H)\overline{x}^F \\
+\left[\,\mathcal{I}_2\,(\lambda)+\mathcal{I}_3\,(\lambda)\,\right]\lambda\, z^H\,\overline{x}^F
\end{array} \right] \mu\left(\Delta^3\ \Delta^3\ \ \infty\right) \\
+ \left[\begin{array}{l}
\left[\,(1-z^H)+\lambda z^H\,\right](1-\overline{x})+\lambda\overline{x}^H \\
+\left[\,\mathcal{I}_2\left(\frac{1}{\lambda}\right)+\mathcal{I}_3\left(\frac{1}{\lambda}\right)\right](1-z^H)\overline{x}^F \\
+\left[\,\mathcal{I}_2\,(1)+\mathcal{I}_3\,(1)\,\right]\lambda\, z^H\,\overline{x}^F
\end{array} \right] \mu\left(\Delta^4\ \Delta^4\ \ \infty\right)
\end{array} \right\} \times \mathcal{Q}_{HH} \\
& + \left\{ \sum_{\Delta^H}\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]} \left[\begin{array}{l}
(1-z^H)(1-z^F) \\
+\left[\,\mathcal{I}_2\left(\lambda\Delta^G\right)+\mathcal{I}_3\left(\lambda\Delta^G\right)\,\right]\lambda\, z^H(1-z^F) \\
+\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)(1-z^H)z^F+\lambda\, z^H z^F
\end{array} \right] (1-\overline{x})\,\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right) \right\} \times \mathcal{Q}_{HH} \\
& + \left\{ \begin{array}{l}
\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]} \left[\begin{array}{l}
\left[\,\mathcal{I}_2\left(\eta\Delta^G\right)+\mathcal{I}_3\left(\eta\Delta^G\right)\,\right]\eta\,(1-z^F) \\
+\left[\,\mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right)+\mathcal{I}_3\left(\frac{\eta}{\lambda}\Delta^G\right)\,\right]\eta\, z^F
\end{array} \right] \overline{x}^H\,\mu\left(\Delta^1\ \Delta^F\ \ \Delta^G\right) \\
+\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]} \left[\begin{array}{l}
\left[\,\mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right)+\mathcal{I}_3\left(\frac{\eta}{\lambda}\Delta^G\right)\,\right]\frac{\eta}{\lambda}\,(1-z^H)(1-z^F) \\
+\left[\,\mathcal{I}_2\left(\lambda\Delta^G\right)+\mathcal{I}_3\left(\lambda\Delta^G\right)\,\right]\lambda\, z^H(1-z^F) \\
+\mathcal{I}_2\left(\frac{\eta}{\lambda^2}\Delta^G\right)\ \frac{\eta}{\lambda}\,(1-z^H)z^F+\lambda\, z^H z^F
\end{array} \right] \overline{x}^H\,\mu\left(\Delta^2\ \Delta^F\ \ \Delta^G\right) \\
+\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]} \left[\begin{array}{l}
(1-z^H)(1-z^F) \\
+\left[\,\mathcal{I}_2\left(\lambda\Delta^G\right)+\mathcal{I}_3\left(\lambda\Delta^G\right)\,\right]\lambda\, z^H(1-z^F) \\
+\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\ (1-z^H)z^F+\lambda\, z^H z^F
\end{array} \right] \overline{x}^H\,\mu\left(\Delta^3\ \Delta^F\ \ \Delta^G\right) \\
+\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]} \left[\ \left[\,\mathcal{I}_2\left(\lambda\Delta^G\right)+\mathcal{I}_3\left(\lambda\Delta^G\right)\,\right]\lambda\,(1-z^F)+\ \lambda\, z^F\ \right] \overline{x}^H\,\mu\left(\Delta^4\ \Delta^F\ \ \Delta^G\right)
\end{array} \right\} \times \mathcal{Q}_{HH}
\end{aligned}
$$

$$
+\left\{\begin{array}{l}
\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\ \mathcal{I}_2\left(\frac{1}{\eta}\Delta^G\right)\ (1-z^H)+\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\ \lambda\, z^H\ \right]\overline{x}^F\,\mu\left(\Delta^H\ \Delta^1\ \ \Delta^G\right)\\
+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\ (1-z^H)(1-z^F)\\ +\left[\ \mathcal{I}_2\left(\frac{\lambda^2}{\eta}\Delta^G\right)+\mathcal{I}_3\left(\frac{\lambda^2}{\eta}\Delta^G\right)\ \right]\ \lambda\, z^H(1-z^F)\\ +\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\ (1-z^H)z^F+\lambda\, z^H z^F\end{array}\right]\overline{x}^F\,\mu\left(\Delta^H\ \Delta^2\ \ \Delta^G\right)\\
+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}(1-z^H)(1-z^F)\\ +\left[\ \mathcal{I}_2\left(\lambda\Delta^G\right)+\mathcal{I}_3\left(\lambda\Delta^G\right)\ \right]\ \lambda\, z^H(1-z^F)\\ +\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\ (1-z^H)z^F+\lambda\, z^H z^F\end{array}\right]\overline{x}^F\,\mu\left(\Delta^H\ \Delta^3\ \ \Delta^G\right)\\
+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\ \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\ (1-z^H)+\lambda\, z^H\ \right]\overline{x}^F\,\mu\left(\Delta^H\ \Delta^4\ \ \Delta^G\right)
\end{array}\right\}\times\mathcal{Q}_{HH}
$$

$$
+\left\{\begin{array}{l}
\left[\begin{array}{l}\left[\,\mathcal{I}_2\left(\eta\right)+\mathcal{I}_3\left(\eta\right)\,\right]\ \eta\,(1-z^F)\overline{x}^H\\ +\left[\,\mathcal{I}_2\left(\frac{\eta}{\lambda}\right)+\mathcal{I}_3\left(\frac{\eta}{\lambda}\right)\,\right]\ \eta\, z^F\,\overline{x}^H\end{array}\right]\mu\left(\Delta^1\ \Delta^1\ \ -\infty\right)\\
+\left[\begin{array}{l}\left[\,\mathcal{I}_2\left(\frac{\eta}{\lambda}\right)+\mathcal{I}_3\left(\frac{\eta}{\lambda}\right)\,\right]\ \frac{\eta}{\lambda}\,(1-z^F)\overline{x}^H\\ +\left[\,\mathcal{I}_2\left(\frac{\eta}{\lambda^2}\right)+\mathcal{I}_3\left(\frac{\eta}{\lambda^2}\right)\,\right]\ \frac{\eta}{\lambda}\, z^F\,\overline{x}^H\end{array}\right]\mu\left(\Delta^2\ \Delta^2\ \ -\infty\right)\\
+\left[\begin{array}{l}\left[\,\mathcal{I}_2\left(1\right)+\mathcal{I}_3\left(1\right)\,\right]\ (1-z^F)\overline{x}^H\\ +\left[\,\mathcal{I}_2\left(\frac{1}{\lambda}\right)+\mathcal{I}_3\left(\frac{1}{\lambda}\right)\,\right]\ z^F\,\overline{x}^H\end{array}\right]\mu\left(\Delta^3\ \Delta^3\ \ -\infty\right)\\
+\left[\begin{array}{l}\left[\,\mathcal{I}_2\left(\lambda\right)+\mathcal{I}_3\left(\lambda\right)\,\right]\ \lambda\,(1-z^F)\overline{x}^H\\ +\left[\,\mathcal{I}_2\left(1\right)+\mathcal{I}_3\left(1\right)\,\right]\ \lambda\, z^F\,\overline{x}^H\end{array}\right]\mu\left(\Delta^4\ \Delta^4\ \ -\infty\right)
\end{array}\right\}\times\mathcal{Q}_{FH}
$$

$$
\mathcal{Q}'_{FH}=\left\{\begin{array}{l}
\left[\ \left[\mathcal{I}_1\left(\frac{1}{\eta}\right)\ (1-z^H)+\mathcal{I}_1\left(\frac{\lambda}{\eta}\right)\ z^H\right]\eta\,\overline{x}^F\ \right]\mu\left(\Delta^1\ \Delta^1\ \ \infty\right)\\
+\left[\ \left[\mathcal{I}_1\left(\frac{\lambda}{\eta}\right)\ (1-z^H)+\mathcal{I}_1\left(\frac{\lambda^2}{\eta}\right)\ z^H\right]\frac{\eta}{\lambda}\,\overline{x}^F\ \right]\mu\left(\Delta^2\ \Delta^2\ \ \infty\right)\\
+\left[\ \left[\,\mathcal{I}_1\left(1\right)\ (1-z^H)+\mathcal{I}_1\left(\lambda\right)\ z^H\,\right]\overline{x}^F\ \right]\mu\left(\Delta^3\ \Delta^3\ \ \infty\right)\\
+\left[\ \left[\,\mathcal{I}_1\left(\frac{1}{\lambda}\right)\ (1-z^H)+\mathcal{I}_1\left(1\right)\ z^H\,\right]\lambda\,\overline{x}^F\ \right]\mu\left(\Delta^4\ \Delta^4\ \ \infty\right)
\end{array}\right\}\times\mathcal{Q}_{HH}
$$

$$
+\left\{\ \sum_{\Delta^H}\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\ \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)\ \lambda\,(1-z^H)\,z^F\,(1-\overline{x})\ \right]\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right)\ \right\}\times\mathcal{Q}_{HH}
$$

$$
+\left\{\begin{array}{l}
\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\ \mathcal{I}_1\left(\frac{\eta}{\lambda^2}\Delta^G\right)\ \lambda\,(1-z^H)\,z^F\,\overline{x}^H\ \right]\mu\left(\Delta^2\ \Delta^F\ \ \Delta^G\right)\\
+\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\ \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)\ \lambda\,(1-z^H)\,z^F\,\overline{x}^H\ \right]\mu\left(\Delta^3\ \Delta^F\ \ \Delta^G\right)
\end{array}\right\}\times\mathcal{Q}_{HH}
$$

$$
+\left\{
\begin{array}{l}
\sum_{\Delta^H} \sum_{\Delta^G \in [\underline{\Omega}, \overline{\Omega}]} \left[\ \mathcal{I}_1 \left(\frac{1}{\eta} \Delta^G \right) \ (1 - z^H) + \mathcal{I}_1 \left(\frac{\lambda}{\eta} \Delta^G \right) \ z^H \ \right] \eta \, \overline{x}^F \, \mu \left(\Delta^H \ \Delta^1 \ \ \Delta^G \right) \\
+ \sum_{\Delta^H} \sum_{\Delta^G \in [\underline{\Omega}, \overline{\Omega}]} \left[\begin{array}{l} \mathcal{I}_1 \left(\frac{\lambda}{\eta} \Delta^G \right) \ \frac{\eta}{\lambda} \, (1 - z^H)(1 - z^F) \\ + \mathcal{I}_1 \left(\frac{1}{\lambda} \Delta^G \right) \ \lambda \, (1 - z^H) z^F \end{array} \right] \overline{x}^F \, \mu \left(\Delta^H \ \Delta^2 \ \ \Delta^G \right) \\
+ \sum_{\Delta^H} \sum_{\Delta^G \in [\underline{\Omega}, \overline{\Omega}]} \left[\ \mathcal{I}_1 \left(\frac{1}{\lambda} \Delta^G \right) \ \lambda \, (1 - z^H) z^F \ \right] \overline{x}^F \, \mu \left(\Delta^H \ \Delta^3 \ \ \Delta^G \right) \\
+ \sum_{\Delta^H} \sum_{\Delta^G \in [\underline{\Omega}, \overline{\Omega}]} \left[\ \mathcal{I}_1 \left(\frac{1}{\lambda} \Delta^G \right) \ \lambda \, (1 - z^H) \ \right] \overline{x}^F \, \mu \left(\Delta^H \ \Delta^4 \ \ \Delta^G \right)
\end{array}
\right\} \times \mathcal{Q}_{HH}
$$

$$
+\left\{
\begin{array}{l}
\left[\begin{array}{l} \left[\, (1 - z^F) + \lambda \, z^F \, \right] (1 - \overline{x}) \\ + \left[\, \mathcal{I}_1 \left(\eta \right) (1 - z^F) + \mathcal{I}_1 \left(\frac{\eta}{\lambda} \right) \ \lambda \, z^F \, \right] \, \overline{x}^H + \eta \, \overline{x}^F \end{array} \right] \mu \left(\Delta^1 \, \Delta^1 \ \ - \infty \right) \\
+ \left[\begin{array}{l} \left[\, (1 - z^F) + \lambda \, z^F \, \right] (1 - \overline{x}) \\ + \left[\, \mathcal{I}_1 \left(\frac{\eta}{\lambda} \right) (1 - z^F) + \mathcal{I}_1 \left(\frac{\eta}{\lambda^2} \right) \ \lambda \, z^F \, \right] \, \overline{x}^H \\ + \left[\, \frac{\eta}{\lambda} (1 - z^F) + \lambda \, z^F \, \right] \, \overline{x}^F \end{array} \right] \mu \left(\Delta^2 \, \Delta^2 \ \ - \infty \right) \\
+ \left[\begin{array}{l} \left[\, (1 - z^F) + \lambda \, z^F \, \right] (1 - \overline{x}) \\ + \left[\, \mathcal{I}_1 \left(1 \right) (1 - z^F) + \mathcal{I}_1 \left(\frac{1}{\lambda} \right) \ \lambda \, z^F \, \right] \, \overline{x}^H \\ + \left[\, (1 - z^F) + \lambda \, z^F \, \right] \, \overline{x}^F \end{array} \right] \mu \left(\Delta^3 \, \Delta^3 \ \ - \infty \right) \\
+ \left[\begin{array}{l} \left[\, (1 - z^F) + \lambda \, z^F \, \right] (1 - \overline{x}) \\ + \left[\, \mathcal{I}_1 \left(\lambda \right) (1 - z^F) + \mathcal{I}_1 \left(1 \right) \ \lambda \, z^F \, \right] \, \overline{x}^H + \lambda \, \overline{x}^F \end{array} \right] \mu \left(\Delta^4 \, \Delta^4 \ \ - \infty \right)
\end{array}
\right\} \times \mathcal{Q}_{FH}
$$

$$
\mathcal{Q}'_{FF} = \left\{ \begin{array}{l}
\left[\begin{array}{l} \left[\,(1-z^F) + \lambda z^F\,\right](1-\overline{x}) \\ + \left[\,\mathcal{I}_1(\eta) + \mathcal{I}_2(\eta)\,\right](1-z^F)\overline{x}^H \\ + \left[\,\mathcal{I}_1\left(\frac{\eta}{\lambda}\right) + \mathcal{I}_2\left(\frac{\eta}{\lambda}\right)\,\right]\,\lambda\, z^F\,\overline{x}^H + \eta\,\overline{x}^F \end{array} \right] \mu\left(\Delta^1\,\Delta^1\ \ -\infty\right) \\
+ \left[\begin{array}{l} \left[\,(1-z^F) + \lambda z^F\,\right](1-\overline{x}) \\ + \left[\,\mathcal{I}_1\left(\frac{\eta}{\lambda}\right) + \mathcal{I}_2\left(\frac{\eta}{\lambda}\right)\,\right](1-z^F)\overline{x}^H \\ + \left[\,\mathcal{I}_1\left(\frac{\eta}{\lambda^2}\right) + \mathcal{I}_2\left(\frac{\eta}{\lambda^2}\right)\,\right]\,\lambda\, z^F\,\overline{x}^H \\ + \frac{\eta}{\lambda}\,(1-z^F)\overline{x}^F + \lambda\, z^F\,\overline{x}^F \end{array} \right] \mu\left(\Delta^2\,\Delta^2\ \ -\infty\right) \\
+ \left[\begin{array}{l} \left[\,(1-z^F) + \lambda z^F\,\right](1-\overline{x}) \\ + \left[\,\mathcal{I}_1(1) + \mathcal{I}_2(1)\,\right](1-z^F)\overline{x}^H \\ + \left[\,\mathcal{I}_1\left(\frac{1}{\lambda}\right) + \mathcal{I}_2\left(\frac{1}{\lambda}\right)\,\right]\,\lambda\, z^F\,\overline{x}^H \\ +(1-z^F)\overline{x}^F + \lambda\, z^F\,\overline{x}^F \end{array} \right] \mu\left(\Delta^3\,\Delta^3\ \ -\infty\right) \\
+ \left[\begin{array}{l} \left[\,(1-z^F) + \lambda z^F\,\right](1-\overline{x}) \\ + \left[\,\mathcal{I}_1(\lambda) + \mathcal{I}_2(\lambda)\,\right](1-z^F)\overline{x}^H \\ + \left[\,\mathcal{I}_1(1) + \mathcal{I}_2(1)\,\right]\,\lambda\, z^F\,\overline{x}^H + \lambda\,\overline{x}^F \end{array} \right] \mu\left(\Delta^4\,\Delta^4\ \ -\infty\right)
\end{array} \right\} \times \mathcal{Q}_{FF}
$$

$$
+ \left\{ \sum_{\Delta^H} \sum_{\Delta^F} \sum_{\Delta^G \in [\underline{\Omega},\overline{\Omega}]} \left[\begin{array}{l} (1-z^H)(1-z^F) \\ + \mathcal{I}_2\left(\lambda\Delta^G\right)\ z^H(1-z^F) \\ + \left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) + \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]\,\lambda\,(1-z^H)z^F \\ +\lambda\, z^H z^F \end{array} \right] (1-\overline{x})\,\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right) \right\} \times \mathcal{Q}_{FF}
$$

$$
+ \left\{ \begin{array}{l}
\sum_{\Delta^F} \sum_{\Delta^G \in [\underline{\Omega},\overline{\Omega}]} \left[\ \mathcal{I}_2\left(\eta\Delta^G\right)\ (1-z^F) + \mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right)\ \lambda\, z^F\ \right] \overline{x}^H\,\mu\left(\Delta^1\ \Delta^F\ \ \Delta^G\right) \\
+ \sum_{\Delta^F} \sum_{\Delta^G \in [\underline{\Omega},\overline{\Omega}]} \left[\begin{array}{l} \mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right)\ (1-z^H)(1-z^F) \\ +\mathcal{I}_2\left(\lambda\Delta^G\right)\ z^H(1-z^F) \\ + \left[\mathcal{I}_1\left(\frac{\eta}{\lambda^2}\Delta^G\right) + \mathcal{I}_2\left(\frac{\eta}{\lambda^2}\Delta^G\right)\right]\,\lambda\,(1-z^H)\,z^F \\ +\lambda\, z^H\, z^F \end{array} \right] \overline{x}^H\,\mu\left(\Delta^2\ \Delta^F\ \ \Delta^G\right) \\
+ \sum_{\Delta^F} \sum_{\Delta^G \in [\underline{\Omega},\overline{\Omega}]} \left[\begin{array}{l} (1-z^H)(1-z^F) \\ +\mathcal{I}_2\left(\lambda\Delta^G\right)\ z^H(1-z^F) \\ + \left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) + \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]\,\lambda\,(1-z^H)\,z^F \\ +\lambda\, z^H\, z^F \end{array} \right] \overline{x}^H\,\mu\left(\Delta^3\ \Delta^F\ \ \Delta^G\right) \\
+ \sum_{\Delta^F} \sum_{\Delta^G \in [\underline{\Omega},\overline{\Omega}]} \left[\ \mathcal{I}_2\left(\lambda\Delta^G\right)(1-z^F) + \lambda\, z^F\ \right] \overline{x}^H\,\mu\left(\Delta^4\ \Delta^F\ \ \Delta^G\right)
\end{array} \right\} \times \mathcal{Q}_{FF}
$$

$$
+\left\{\begin{array}{l}
\displaystyle\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}\left[\mathcal{I}_1\left(\frac{1}{\eta}\Delta^G\right)+\mathcal{I}_2\left(\frac{1}{\eta}\Delta^G\right)\right]\,\eta\,(1-z^H)\\ +\left[\mathcal{I}_1\left(\frac{\lambda}{\eta}\Delta^G\right)+\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\right]\,\eta\,z^H\end{array}\right]\overline{x}^F\,\mu\left(\Delta^H\,\Delta^1\;\;\Delta^G\right)\\
\displaystyle+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}\left[\mathcal{I}_1\left(\frac{\lambda}{\eta}\Delta^G\right)+\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\right]\,\frac{\eta}{\lambda}\,(1-z^H)(1-z^F)\\ +\mathcal{I}_2\left(\frac{\lambda^2}{\eta}\Delta^G\right)\,\frac{\eta}{\lambda}\,z^H(1-z^F)\\ +\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)+\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]\,\lambda\,(1-z^H)\,z^F\\ +\lambda\,z^H\,z^F\end{array}\right]\overline{x}^F\,\mu\left(\Delta^H\,\Delta^2\;\;\Delta^G\right)\\
\displaystyle+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}(1-z^H)(1-z^F)+\mathcal{I}_2\left(\lambda\Delta^G\right)\,z^H(1-z^F)\\ +\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)+\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]\,\lambda\,(1-z^H)\,z^F\\ +\lambda\,z^H\,z^F\end{array}\right]\overline{x}^F\,\mu\left(\Delta^H\,\Delta^3\;\;\Delta^G\right)\\
\displaystyle+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)+\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]\,\lambda\,(1-z^H)+\lambda\,z^H\right]\overline{x}^F\,\mu\left(\Delta^H\,\Delta^4\;\;\Delta^G\right)
\end{array}\right\}\times\mathcal{Q}_{FF}
$$

$$
+\left\{\begin{array}{l}
\left[\begin{array}{l}\left[\mathcal{I}_1\left(\frac{1}{\eta}\right)+\mathcal{I}_2\left(\frac{1}{\eta}\right)\right]\,\eta\,(1-z^H)\overline{x}^F\\ +\left[\mathcal{I}_1\left(\frac{\lambda}{\eta}\right)+\mathcal{I}_2\left(\frac{\lambda}{\eta}\right)\right]\,\eta\,z^H\,\overline{x}^F\end{array}\right]\mu\left(\Delta^1\,\Delta^1\;\;\infty\right)\\
+\left[\begin{array}{l}\left[\mathcal{I}_1\left(\frac{\lambda}{\eta}\right)+\mathcal{I}_2\left(\frac{\lambda}{\eta}\right)\right]\,\frac{\eta}{\lambda}\,(1-z^H)\overline{x}^F\\ +\left[\mathcal{I}_1\left(\frac{\lambda^2}{\eta}\right)+\mathcal{I}_2\left(\frac{\lambda^2}{\eta}\right)\right]\,\frac{\eta}{\lambda}\,z^H\,\overline{x}^F\end{array}\right]\mu\left(\Delta^2\,\Delta^2\;\;\infty\right)\\
+\left[\begin{array}{l}\left[\mathcal{I}_1(1)+\mathcal{I}_2(1)\right]\,(1-z^H)\overline{x}^F\\ +\left[\mathcal{I}_1(\lambda)+\mathcal{I}_2(\lambda)\right]\,z^H\,\overline{x}^F\end{array}\right]\mu\left(\Delta^3\,\Delta^3\;\;\infty\right)\\
+\left[\begin{array}{l}\left[\mathcal{I}_1\left(\frac{1}{\lambda}\right)+\mathcal{I}_2\left(\frac{1}{\lambda}\right)\right]\,\lambda\,(1-z^H)\overline{x}^F\\ +\left[\mathcal{I}_1(1)+\mathcal{I}_2(1)\right]\,\lambda\,z^H\,\overline{x}^F\end{array}\right]\mu\left(\Delta^4\,\Delta^4\;\;\infty\right)
\end{array}\right\}\times\mathcal{Q}_{HF}
$$

$$
\mathcal{Q}'_{HF}=\left\{\begin{array}{l}
\left[\mathcal{I}_3(\eta)\,\eta\,(1-z^F)\overline{x}^H+\mathcal{I}_3\left(\frac{\eta}{\lambda}\right)\,\eta\,z^F\,\overline{x}^H\right]\mu\left(\Delta^1\,\Delta^1\;\;-\infty\right)\\
+\left[\mathcal{I}_3\left(\frac{\eta}{\lambda}\right)\,\frac{\eta}{\lambda}\,(1-z^F)\overline{x}^H+\mathcal{I}_3\left(\frac{\eta}{\lambda^2}\right)\,\frac{\eta}{\lambda}\,z^F\,\overline{x}^H\right]\mu\left(\Delta^2\,\Delta^2\;\;-\infty\right)\\
+\left[\mathcal{I}_3(1)\,(1-z^F)\overline{x}^H+\mathcal{I}_3\left(\frac{1}{\lambda}\right)\,z^F\,\overline{x}^H\right]\mu\left(\Delta^3\,\Delta^3\;\;-\infty\right)\\
+\left[\mathcal{I}_3(\lambda)\,\lambda\,(1-z^F)\overline{x}^H+\mathcal{I}_3(1)\,\lambda\,z^F\,\overline{x}^H\right]\mu\left(\Delta^4\,\Delta^4\;\;-\infty\right)
\end{array}\right\}\times\mathcal{Q}_{FF}
$$

$$
+\left\{\sum_{\Delta^H}\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\mathcal{I}_3\left(\lambda\Delta^G\right)\,\lambda\,z^H\,(1-z^F)\right](1-\overline{x})\,\mu\left(\Delta^H\,\Delta^F\;\;\Delta^G\right)\right\}\times\mathcal{Q}_{FF}
$$

$$
+\left\{\begin{array}{l}
\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\;\mathcal{I}_3\left(\eta\Delta^G\right)\;\eta\,(1-z^F)+\mathcal{I}_3\left(\frac{\eta}{\lambda}\Delta^G\right)\;\eta\,z^F\;\right]\overline{x}^H\,\mu\left(\Delta^1\,\Delta^F\;\;\Delta^G\right)\\
+\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}\mathcal{I}_3\left(\frac{\eta}{\lambda}\Delta^G\right)\;\frac{\eta}{\lambda}\,(1-z^H)(1-z^F)\\ +\mathcal{I}_3\left(\lambda\Delta^G\right)\;\lambda\,z^H(1-z^F)\end{array}\right]\overline{x}^H\,\mu\left(\Delta^2\,\Delta^F\;\;\Delta^G\right)\\
+\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\;\mathcal{I}_3\left(\lambda\Delta^G\right)\;\lambda\,z^H(1-z^F)\;\right]\overline{x}^H\,\mu\left(\Delta^3\,\Delta^F\;\;\Delta^G\right)\\
+\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\;\mathcal{I}_3\left(\lambda\Delta^G\right)\;\lambda\,(1-z^F)\;\right]\overline{x}^H\,\mu\left(\Delta^4\,\Delta^F\;\;\Delta^G\right)
\end{array}\right\}\times\mathcal{Q}_{FF}
$$

$$
+\left\{\begin{array}{l}
\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\;\mathcal{I}_3\left(\frac{\lambda^2}{\eta}\Delta^G\right)\;\lambda\,z^H(1-z^F)\;\right]\overline{x}^F\,\mu\left(\Delta^H\,\Delta^2\;\;\Delta^G\right)\\
+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\;\mathcal{I}_3\left(\lambda\Delta^G\right)\;\lambda\,z^H(1-z^F)\;\right]\overline{x}^F\,\mu\left(\Delta^H\,\Delta^3\;\;\Delta^G\right)
\end{array}\right\}\times\mathcal{Q}_{FF}
$$

$$
+\left\{\begin{array}{l}
\left[\begin{array}{l}(1-z^H)(1-\overline{x})+\lambda\,z^H(1-\overline{x})+\eta\overline{x}^H\\ +\mathcal{I}_3\left(\frac{1}{\eta}\right)\;(1-z^H)\overline{x}^F+\mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\;\lambda\,z^H\,\overline{x}^F\end{array}\right]\mu\left(\Delta^1\,\Delta^1\;\;\infty\right)\\
+\left[\begin{array}{l}(1-z^H)(1-\overline{x})+\lambda\,z^H(1-\overline{x})\\ +\frac{\eta}{\lambda}\,(1-z^H)\overline{x}^H+\lambda\,z^H\,\overline{x}^H\\ +\mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\;(1-z^H)\overline{x}^F+\mathcal{I}_3\left(\frac{\lambda^2}{\eta}\right)\;\lambda\,z^H\,\overline{x}^F\end{array}\right]\mu\left(\Delta^2\,\Delta^2\;\;\infty\right)\\
+\left[\begin{array}{l}(1-z^H)(1-\overline{x})+\lambda\,z^H(1-\overline{x})\\ +(1-z^H)\overline{x}^H+\lambda\,z^H\,\overline{x}^H\\ +\mathcal{I}_3\left(1\right)\;(1-z^H)\overline{x}^F+\mathcal{I}_3\left(\lambda\right)\;\lambda\,z^H\,\overline{x}^F\end{array}\right]\mu\left(\Delta^3\,\Delta^3\;\;\infty\right)\\
+\left[\begin{array}{l}(1-z^H)(1-\overline{x})+\lambda\,z^H(1-\overline{x})+\lambda\,\overline{x}^H\\ +\mathcal{I}_3\left(\frac{1}{\lambda}\right)\;(1-z^H)\overline{x}^F+\mathcal{I}_3\left(1\right)\;\lambda\,z^H\,\overline{x}^F\end{array}\right]\mu\left(\Delta^4\,\Delta^4\;\;\infty\right)
\end{array}\right\}\times\mathcal{Q}_{HF}
$$

TA3.1 Proof of Proposition 1

TA4 Value Function

TA4.1 One Product Case Example

For $\Phi^f = \big\{ (\, q \; H \; H \; \Delta^1 \; \Delta^1 \; \infty \,) \big\}$,

$$
\begin{aligned}
V\big(\, q\, \Delta^1\, \Delta^1\, \infty\,\big) = \; & (\pi^H + \pi^F) q - \widehat{\chi} z^{\widehat{\psi}} q - \overline{q}\widetilde{\chi} x^{\widetilde{\psi}} \\
& + \widetilde{\beta} \times \left[\begin{array}{lll}
V\left(\left\{ \left(q\, \Delta^1\, \Delta^1\, \infty \right) \right\} \right) & & \times\; (1-z)(1-\overline{x}) \\
+\, V\left(\left\{ \left(\lambda q\, \Delta^2\, \Delta^2\, \infty \right) \right\} \right) & & \times\; z\,(1-\overline{x}) \\
+\, V(\varnothing) & & \times\; (1-z)\,\overline{x}^H \\
+\, V(\varnothing) & & \times\; z\,\overline{x}^H \\
+ \left[\begin{array}{ll}
V(\varnothing) & \mathcal{I}_1\left(\frac{1}{\eta}\right) \\
+\, V\left(\left\{ \left(q\, \Delta^1\, \Delta^3\;\, \frac{1}{\eta} \right) \right\} \right) & \mathcal{I}_2\left(\frac{1}{\eta}\right) \\
+\, V\left(\left\{ \left(q\, \Delta^1\, \Delta^1\;\, \infty \right) \right\} \right) & \mathcal{I}_3\left(\frac{1}{\eta}\right)
\end{array} \right] & & \times\; (1-z)\,\overline{x}^F \\
+ \left[\begin{array}{ll}
V(\varnothing) & \mathcal{I}_1\left(\frac{\lambda}{\eta}\right) \\
+\, V\left(\left\{ \left(\lambda\, q\, \Delta^2\, \Delta^3\;\, \frac{\lambda}{\eta} \right) \right\} \right) & \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \\
+\, V\left(\left\{ \left(\lambda q\, \Delta^2\, \Delta^2\;\, \infty \right) \right\} \right) & \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)
\end{array} \right] & & \times\; z\,\overline{x}^F
\end{array} \right] \times (1-x)
\end{aligned}
$$

$$
+\, \widetilde{\beta} \times \int_{\Phi_{-j}} \sum_{\mathcal{I}^{ZH}_{-j}=0}^{1} \; \sum_{\mathcal{I}^{ZF}_{-j}=0}^{1} \; \sum_{c-t_{-j}=\text{win}}^{\text{lose}} \left(z^H\right)^{\mathcal{I}^{ZH}_{-j}} \left(1-z^H\right)^{1-\mathcal{I}^{ZH}_{-j}} \left(z^F\right)^{\mathcal{I}^{ZF}_{-j}} \left(1-z^F\right)^{1-\mathcal{I}^{ZF}_{-j}} \; \frac{1}{2}
$$

$$
\times \left[\begin{array}{ll}
V\left(\left\{ \left(q\, \Delta^1\, \Delta^1\, \infty \right)\,,\; \Phi'_{-j} \,|\, \Phi_{-j}\,,\; \mathcal{I}^{ZH}_{-j}\,, \mathcal{I}^{ZF}_{-j}\,,\; c-t_{-j} \right\} \right) & \times\; (1-z)(1-\overline{x}) \\
+\, V\left(\left\{ \left(\lambda q\, \Delta^2\, \Delta^2\, \infty \right)\,,\; \Phi'_{-j} \,|\, \Phi_{-j}\,,\; \mathcal{I}^{ZH}_{-j}\,, \mathcal{I}^{ZF}_{-j}\,,\; c-t_{-j} \right\} \right) & \times\; z\,(1-\overline{x}) \\
+\, V\left(\left\{ \Phi'_{-j} \,|\, \Phi_{-j}\,,\; \mathcal{I}^{ZH}_{-j}\,, \mathcal{I}^{ZF}_{-j}\,,\; c-t_{-j} \right\} \right) & \times\; (1-z)\,\overline{x}^H \\
+\, V\left(\left\{ \Phi'_{-j} \,|\, \Phi_{-j}\,,\; \mathcal{I}^{ZH}_{-j}\,, \mathcal{I}^{ZF}_{-j}\,,\; c-t_{-j} \right\} \right) & \times\; z\,\overline{x}^H \\
+ \left[\begin{array}{ll}
V\left(\left\{ \Phi'_{-j} \,|\, \Phi_{-j}\,,\; \mathcal{I}^{ZH}_{-j}\,, \mathcal{I}^{ZF}_{-j}\,,\; c-t_{-j} \right\} \right) & \mathcal{I}_1\left(\frac{1}{\eta}\right) \\
+\, V\left(\left\{ \left(q\, \Delta^1\, \Delta^3\;\, \frac{1}{\eta} \right)\,,\; \Phi'_{-j} \,|\, \Phi_{-j}\,,\; \mathcal{I}^{ZH}_{-j}\,, \mathcal{I}^{ZF}_{-j}\,,\; c-t_{-j} \right\} \right) & \mathcal{I}_2\left(\frac{1}{\eta}\right) \\
+\, V\left(\left\{ \left(q\, \Delta^1\, \Delta^1\;\, \infty \right),\; \Phi'_{-j} \,|\, \Phi_{-j}\,,\; \mathcal{I}^{ZH}_{-j}\,, \mathcal{I}^{ZF}_{-j}\,,\; c-t_{-j} \right\} \right) & \mathcal{I}_3\left(\frac{1}{\eta}\right)
\end{array} \right] & \times\; (1-z)\,\overline{x}^F \\
+ \left[\begin{array}{ll}
V\left(\left\{ \Phi'_{-j} \,|\, \Phi_{-j}\,,\; \mathcal{I}^{ZH}_{-j}\,, \mathcal{I}^{ZF}_{-j}\,,\; c-t_{-j} \right\} \right) & \mathcal{I}_1\left(\frac{\lambda}{\eta}\right) \\
+\, V\left(\left\{ \left(\lambda\, q\, \Delta^2\, \Delta^3\;\, \frac{\lambda}{\eta} \right),\; \Phi'_{-j} \,|\, \Phi_{-j}\,,\; \mathcal{I}^{ZH}_{-j}\,, \mathcal{I}^{ZF}_{-j}\,,\; c-t_{-j} \right\} \right) & \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \\
+\, V\left(\left\{ \left(\lambda q\, \Delta^2\, \Delta^2\;\, \infty \right)\,,\; \Phi'_{-j} \,|\, \Phi_{-j}\,,\; \mathcal{I}^{ZH}_{-j}\,, \mathcal{I}^{ZF}_{-j}\,,\; c-t_{-j} \right\} \right) & \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)
\end{array} \right] & \times\; z\,\overline{x}^F
\end{array} \right]
$$

$$
\times\, \mu\left(\Phi_{-j}\right)\; \mathbf{d}(\Phi_{-j})\, x\,,
$$

where $\mathcal{I}_1(\Delta^G)$ is an indicator equal to one if $\Delta^G < \underline{\Omega}$, $\mathcal{I}_2(\Delta^G)$ is an indicator equal to one if $\Delta^G \in \left[\, \underline{\Omega}\, , \, \overline{\Omega} \,\right]$, and $\mathcal{I}_3(\Delta^G)$ is an indicator equal to one if $\Delta^G > \overline{\Omega}$. Then, with the guessed value function, the above becomes

$$
\begin{aligned}
A\big(\, \Delta^1\, \Delta^1\, \infty \,\big) q + B\overline{q} = \; & (\pi^H + \pi^F) q - \widehat{\chi} z^{\widehat{\psi}} q - \overline{q} \widetilde{\chi} x^{\widetilde{\psi}} \\
& + \widetilde{\beta} \times \left[\begin{array}{ll}
A\,\big(\, \Delta^1\, \Delta^1\, \infty\, \big)\, q & \times\; (1-z)(1-\overline{x}) \\
+\, A\,\big(\, \Delta^2\, \Delta^2\, \infty\, \big)\, \lambda\, q & \times\; z\,(1-\overline{x}) \\
+\, 0 & \times\; (1-z)\, \overline{x}^H \\
+\, 0 & \times\; z\, \overline{x}^H \\
+ \left[\begin{array}{ll} 0 & \mathcal{I}_1\left(\frac{1}{\eta}\right) \\ +\, A\left(\, \Delta^1\, \Delta^3\, \frac{1}{\eta} \,\right) q & \mathcal{I}_2\left(\frac{1}{\eta}\right) \\ +\, A\,\big(\, \Delta^1\, \Delta^1\, \infty\, \big)\, q & \mathcal{I}_3\left(\frac{1}{\eta}\right) \end{array} \right] & \times\; (1-z)\, \overline{x}^F \\
+ \left[\begin{array}{ll} 0 & \mathcal{I}_1\left(\frac{\lambda}{\eta}\right) \\ +\, A\left(\, \Delta^2\, \Delta^3\, \frac{\lambda}{\eta} \,\right) \lambda\, q & \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \\ +\, A\,\big(\, \Delta^2\, \Delta^2\, \infty\, \big)\, \lambda\, q & \mathcal{I}_3\left(\frac{\lambda}{\eta}\right) \end{array} \right] & \times\; z\, \overline{x}^F
\end{array} \right] \\
& + \widetilde{\beta} B (1+g) \overline{q} \\
& + \widetilde{\beta}\, \overline{q} \Bigg[\sum_{\mathcal{I}^{ZH}_{-j}=0}^{1} \; \sum_{\mathcal{I}^{ZF}_{-j}=0}^{1} \; \sum_{c-t_{-j}=\text{win}}^{\text{lose}} \; \left(z^H\right)^{\mathcal{I}^{ZH}_{-j}} \left(1-z^H\right)^{1-\mathcal{I}^{ZH}_{-j}} \; \left(z^F\right)^{\mathcal{I}^{ZF}_{-j}} \left(1-z^F\right)^{1-\mathcal{I}^{ZF}_{-j}} \; \frac{1}{2} \\
& \times \int_{(\, \Delta^H_{-j}\;\, \Delta^F_{-j}\;\, \Delta^G_{-j}\,)} A\Big(\, \Delta^{H\prime}_{-j}\;\, \Delta^{F\prime}_{-j}\;\, \Delta^{G\prime}_{-j} \;\big|\; \Delta^H_{-j}\, ,\, \Delta^F_{-j}\, ,\, \Delta^G_{-j}\, ,\, \mathcal{I}^{ZH}_{-j}\, ,\mathcal{I}^{ZF}_{-j}\, ,\, c - t_{-j} \Big)\, \Delta^{H\prime}_{-j} \\
& \qquad \times \mu\,\big(\Delta^H_{-j}\;\, \Delta^F_{-j}\;\, \Delta^G_{-j}\,\big)\; \mathbf{d}\,\big(\Delta^H_{-j}\;\, \Delta^F_{-j}\;\, \Delta^G_{-j}\,\big) \Bigg]\, x\, ,
\end{aligned}
$$

where $1 + g = \frac{\overline{q}'}{\overline{q}}$. Define $A_{takeover}$ as the terms in the last brackets. Then from FONCs, we get

$$
\begin{aligned}
z^H(\, \Delta^1\;\, \Delta^1\;\, \infty\,) = \left(\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}} \right)^{\frac{1}{\widehat{\psi}-1}} \Bigg[& \Big[A\,(\, \Delta^2\;\, \Delta^2\;\, \infty\,)\; \lambda - A\,(\, \Delta^1\;\, \Delta^1\;\, \infty\,) \Big] (1-\overline{x}) \\
& + \Bigg[\left(A\left(\, \Delta^2\;\, \Delta^3\;\, \frac{\lambda}{\eta} \,\right) \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) + A\,(\, \Delta^2\;\, \Delta^2\;\, \infty\,)\; \mathcal{I}_3\left(\frac{\lambda}{\eta}\right) \right) \lambda \\
& \quad - \left(A\left(\, \Delta^1\;\, \Delta^3\;\, \frac{1}{\eta} \,\right) \mathcal{I}_2\left(\frac{1}{\eta}\right) + A\,(\, \Delta^1\;\, \Delta^1\;\, \infty\,)\; \mathcal{I}_3\left(\frac{1}{\eta}\right) \right) \Bigg] \overline{x}^F \Bigg]^{\frac{1}{\widehat{\psi}-1}}
\end{aligned}
$$

and

$$
x = \left(\frac{\widetilde{\beta}}{\widetilde{\psi}\widetilde{\chi}} \right)^{\frac{1}{\widetilde{\psi}-1}} \left(A_{takeover}\right)^{\frac{1}{\widetilde{\psi}-1}} .
$$

Thus,

$$A\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)=\pi^H+\pi^F-\widehat{\chi}\,z\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)^{\widehat{\psi}}
+\widetilde{\beta}\times\left[\begin{array}{ll}
\quad A\left(\,\Delta^1\,\Delta^1\,\infty\,\right) & \times\ \left(1-z\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)\right)(1-\overline{x}) \\
+\,A\left(\,\Delta^2\,\Delta^2\,\infty\,\right)\lambda & \times\ z\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)\ (1-\overline{x}) \\
+\left[\begin{array}{l} A\left(\,\Delta^1\,\Delta^3\ \frac{1}{\eta}\,\right)\ \ \mathcal{I}_2\left(\frac{1}{\eta}\right) \\ +\,A\left(\,\Delta^1\,\Delta^1\ \infty\,\right)\ \ \mathcal{I}_3\left(\frac{1}{\eta}\right)\end{array}\right] & \times\ \left(1-z\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)\right)\overline{x}^F \\
+\left[\begin{array}{l} A\left(\,\Delta^2\,\Delta^3\ \frac{\lambda}{\eta}\,\right)\lambda\ \ \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \\ +\,A\left(\,\Delta^2\,\Delta^2\ \infty\,\right)\lambda\ \ \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\end{array}\right] & \times\ z\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)\,\overline{x}^F
\end{array}\right],$$

and

$$B=\frac{\left(\widetilde{\psi}-1\right)\,\widetilde{\chi}}{1-\widetilde{\beta}(1+g)}\left(\frac{\widetilde{\beta}}{\widetilde{\psi}\,\widetilde{\chi}}\right)^{\frac{\widetilde{\psi}}{\widetilde{\psi}-1}}\ \left(A_{takeover}\right)^{\frac{\widetilde{\psi}}{\widetilde{\psi}-1}}.$$

Value function and internal innovation intensity for other cases can be derived symmetrically.

TA4.2 Proof of Proposition 2

$$x=\left(\frac{\widetilde{\beta}}{\widetilde{\psi}\widetilde{\chi}}\right)^{\frac{1}{\widetilde{\psi}-1}}\ \left(A_{takeover}\right)^{\frac{1}{\widetilde{\psi}-1}} \tag{TA4.66}$$

and

$$B=\frac{\left(\widetilde{\psi}-1\right)\,\widetilde{\chi}}{1-\widetilde{\beta}(1+g)}\left(\frac{\widetilde{\beta}}{\widetilde{\psi}\,\widetilde{\chi}}\right)^{\frac{\widetilde{\psi}}{\widetilde{\psi}-1}}\ \left(A_{takeover}\right)^{\frac{\widetilde{\psi}}{\widetilde{\psi}-1}},$$

where $A_{takeover}$ is defined below.

TA4.2.0.1 Optimal Internal Innovation Intensity, Home Firm

H superscripts from all the $A(\cdot)$ and $z(\cdot)$ are dropped for notational simplicity.

$$z^H(\,\Delta^1\ \ \Delta^1\ \ \infty\,)=\left(\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}}\Bigg[\ \Big[A\left(\,\Delta^2\ \ \Delta^2\ \ \infty\,\right)\ \lambda-A\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)\Big](1-\overline{x})
+\Bigg[\left(A\left(\ \Delta^2\ \ \Delta^3\ \frac{\lambda}{\eta}\right)\ \mathcal{I}_2\left(\frac{\lambda}{\eta}\right)+A\left(\,\Delta^2\ \ \Delta^2\ \ \infty\,\right)\ \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\right)\lambda
-\left(A\left(\ \Delta^1\ \ \Delta^3\ \frac{1}{\eta}\right)\ \mathcal{I}_2\left(\frac{1}{\eta}\right)+A\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)\ \mathcal{I}_3\left(\frac{1}{\eta}\right)\right)\Bigg]\,\overline{x}^F\ \Bigg]^{\frac{1}{\widehat{\psi}-1}} \tag{TA4.67}$$

$$z^H(\,\Delta^2\;\;\Delta^2\;\;\infty\,) = \left(\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}} \Bigg[\; \Big[A\,(\,\Delta^2\;\;\Delta^2\;\;\infty\,)\;\lambda - A\,(\,\Delta^1\;\;\Delta^1\;\;\infty\,)\Big](1-\overline{x}) \;+\; A\,(\,\Delta^2\;\;\Delta^2\;\;\infty\,)\;\lambda\,\overline{x}^H$$

$$+\Bigg[\Bigg(A\left(\,\Delta^2\;\;\Delta^4\;\;\frac{\lambda^2}{\eta}\,\right)\mathcal{I}_2\left(\frac{\lambda^2}{\eta}\right) + A\,(\,\Delta^2\;\;\Delta^2\;\;\infty\,)\;\mathcal{I}_3\left(\frac{\lambda^2}{\eta}\right)\Bigg)\lambda$$

$$-\Bigg(A\left(\,\Delta^1\;\;\Delta^4\;\;\frac{\lambda}{\eta}\,\right)\mathcal{I}_2\left(\frac{\lambda}{\eta}\right) + A\,(\,\Delta^1\;\;\Delta^1\;\;\infty\,)\;\mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\Bigg)\Bigg]\,\overline{x}^F\;\;\Bigg]^{\frac{1}{\widehat{\psi}-1}} \tag{TA4.68}$$

$$z^H(\,\Delta^3\;\;\Delta^3\;\;\infty\,) = \left(\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}} \Bigg[\; \Big[A\,(\,\Delta^2\;\;\Delta^2\;\;\infty\,)\;\lambda - A\,(\,\Delta^1\;\;\Delta^1\;\;\infty\,)\Big](1-\overline{x})$$

$$+\;\Big[A\,(\,\Delta^2\;\;\Delta^2\;\;\infty\,)\;\lambda\;-\;\frac{1}{2}A\,(\,\Delta^1\;\;\Delta^1\;\;\infty\,)\Big]\,\overline{x}^H$$

$$+\Big[\Big(A\,(\,\Delta^2\;\;\Delta^1\;\;\lambda\,)\;\mathcal{I}_2\,(\lambda) + A\,(\,\Delta^2\;\;\Delta^2\;\;\infty\,)\;\mathcal{I}_3\,(\lambda)\Big)\,\lambda$$

$$-\Big(A\,(\,\Delta^1\;\;\Delta^1\;\;1\,)\;\mathcal{I}_2\,(1) + A\,(\,\Delta^1\;\;\Delta^1\;\;\infty\,)\;\mathcal{I}_3\,(1)\Big)\Big]\,\overline{x}^F\;\;\Bigg]^{\frac{1}{\widehat{\psi}-1}} \tag{TA4.69}$$

$$z^H(\,\Delta^4\;\;\Delta^4\;\;\infty\,) = \left(\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}} \Bigg[\; \Big[A\,(\,\Delta^2\;\;\Delta^2\;\;\infty\,)\;\lambda - A\,(\,\Delta^1\;\;\Delta^1\;\;\infty\,)\Big](1-\overline{x}) + \;A\,(\,\Delta^2\;\;\Delta^2\;\;\infty\,)\;\lambda\,\frac{1}{2}\,\overline{x}^H$$

$$+\Big[\Big(A\,(\,\Delta^2\;\;\Delta^2\;\;1\,)\;\mathcal{I}_2\,(1) + A\,(\,\Delta^2\;\;\Delta^2\;\;\infty\,)\;\mathcal{I}_3\,(1)\Big)\,\lambda$$

$$-\Bigg(A\left(\,\Delta^1\;\;\Delta^2\;\;\frac{1}{\lambda}\,\right)\mathcal{I}_2\left(\frac{1}{\lambda}\right) + A\,(\,\Delta^1\;\;\Delta^1\;\;\infty\,)\;\mathcal{I}_3\left(\frac{1}{\lambda}\right)\Bigg)\Bigg]\,\overline{x}^F\;\;\Bigg]^{\frac{1}{\widehat{\psi}-1}} \tag{TA4.70}$$

Here, z^F corresponds to the internal innovation intensity under the same state variable we consider for z^H and the state-variable notations are removed for notational simplicity.

For $(H\;F)$ cases, the terms corresponding to no external innovation by either home firm or foreign firm are identical up to Δ^G and are defined as

$$A^{zH}_{NE}\left(\Delta^G\right) \equiv \Big[\Big(A\,(\,\Delta^2\;\;\Delta^1\;\;\lambda\Delta^G\,)\,\mathcal{I}_2\left(\lambda\Delta^G\right) + A\,(\,\Delta^2\;\;\Delta^2\;\;\infty\,)\,\mathcal{I}_3\left(\lambda\Delta^G\right)\Big)\,\lambda - A\,(\,\Delta^1\;\;\Delta^1\;\;\Delta^G\,)\Big](1-z^F)(1-\overline{x})$$
$$+\;\Bigg[A\,(\,\Delta^2\;\;\Delta^2\;\;\Delta^G\,)\;\lambda - A\left(\,\Delta^1\;\;\Delta^2\;\;\frac{1}{\lambda}\Delta^G\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\Bigg]\,z^F\,(1-\overline{x})\,,$$

where z^F corresponds to technology gaps corresponding to the z^H that $A^{zH}_{NE}\left(\Delta^G\right)$ is used. For any Δ^H, terms corresponding to foreign external innovation depend only on foreign technology gap Δ^F and global technology gap Δ^G. Thus, for $A^{zH}_{H\ell}\left(\Delta^G\right)$

corresponding to $(\Delta^H\ \Delta^\ell\ \ \Delta^G)$,

$$A^{zH}_{H1}\left(\Delta^G\right) \equiv \left[\ A\left(\ \Delta^2\ \ \Delta^3\ \ \frac{\lambda}{\eta}\Delta^G\ \right)\ \mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\ \lambda - A\left(\ \Delta^1\ \ \Delta^3\ \ \frac{1}{\eta}\Delta^G\ \right)\ \mathcal{I}_2\left(\frac{1}{\eta}\Delta^G\right)\ \right]\ \overline{x}^F$$

$$\begin{aligned}A^{zH}_{H2}\left(\Delta^G\right) \equiv &\left[\left(A\left(\ \Delta^2\ \ \Delta^4\ \ \frac{\lambda^2}{\eta}\Delta^G\ \right)\mathcal{I}_2\left(\frac{\lambda^2}{\eta}\Delta^G\right) + A\left(\ \Delta^2\ \ \Delta^2\ \ \infty\ \right)\mathcal{I}_3\left(\frac{\lambda^2}{\eta}\Delta^G\right)\right)\ \lambda\right.\\ &\left. - A\left(\ \Delta^1\ \ \Delta^4\ \ \frac{\lambda}{\eta}\Delta^G\ \right)\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\right]\left(1 - z^F\right)\overline{x}^F\\ &+ \left[A\left(\ \Delta^2\ \ \Delta^2\ \ \Delta^G\ \right)\ \lambda - A\left(\ \Delta^1\ \ \Delta^2\ \ \frac{1}{\lambda}\Delta^G\ \right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]\ z^F\ \overline{x}^F\end{aligned}$$

$$\begin{aligned}A^{zH}_{H3}\left(\Delta^G\right) \equiv &\left[\left(A\left(\ \Delta^2\ \ \Delta^1\ \ \lambda\Delta^G\ \right)\mathcal{I}_2\left(\lambda\Delta^G\right) + A\left(\ \Delta^2\ \ \Delta^2\ \ \infty\ \right)\mathcal{I}_3\left(\lambda\Delta^G\right)\right)\ \lambda - A\left(\ \Delta^1\ \ \Delta^1\ \ \Delta^G\ \right)\right]\left(1 - z^F\right)\overline{x}^F\\ &+ \left[A\left(\ \Delta^2\ \ \Delta^2\ \ \Delta^G\ \right)\ \lambda - A\left(\ \Delta^1\ \ \Delta^2\ \ \frac{1}{\lambda}\Delta^G\ \right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]\ z^F\ \overline{x}^F\end{aligned}$$

$$A^{zH}_{H4}\left(\Delta^G\right) \equiv \left[A\left(\ \Delta^2\ \ \Delta^2\ \ \Delta^G\ \right)\ \lambda\ -\ A\left(\ \Delta^1\ \ \Delta^2\ \ \frac{1}{\lambda}\Delta^G\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]\ \overline{x}^F$$

Similarly for any Δ^F, terms corresponding to domestic external innovation depend only on domestic technology gap Δ^H and global technology gap Δ^G. Thus, for $A^{zH}_{\ell F}\left(\Delta^G\right)$ corresponding to $(\Delta^\ell\ \Delta^F\ \ \Delta^G)$,

$$\begin{aligned}A^{zH}_{2F}\left(\Delta^G\right) \equiv &\left(A\left(\ \Delta^2\ \ \Delta^1\ \ \lambda\Delta^G\ \right)\mathcal{I}_2\left(\lambda\Delta^G\right) + A\left(\ \Delta^2\ \ \Delta^2\ \ \infty\ \right)\mathcal{I}_3\left(\lambda\Delta^G\right)\right)\ \lambda\left(1 - z^F\right)\overline{x}^H\\ &+\ A\left(\ \Delta^2\ \ \Delta^2\ \ \Delta^G\ \right)\ \lambda\, z^F\ \overline{x}^H\end{aligned}$$

$$\begin{aligned}A^{zH}_{3F}\left(\Delta^G\right) \equiv &\left[\left(A\left(\ \Delta^2\ \ \Delta^1\ \ \lambda\Delta^G\ \right)\mathcal{I}_2\left(\lambda\Delta^G\right) + A\left(\ \Delta^2\ \ \Delta^2\ \ \infty\ \right)\mathcal{I}_3\left(\lambda\Delta^G\right)\right)\ \lambda - \frac{1}{2}A\left(\ \Delta^1\ \ \Delta^1\ \ \Delta^G\ \right)\right]\left(1 - z^F\right)\overline{x}^H\\ &+ \left[A\left(\ \Delta^2\ \ \Delta^2\ \ \Delta^G\ \right)\ \lambda - \frac{1}{2}A\left(\ \Delta^1\ \ \Delta^2\ \ \frac{1}{\lambda}\Delta^G\ \right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]\ z^F\ \overline{x}^H\end{aligned}$$

$$\begin{aligned}A^{zH}_{4F}\left(\Delta^G\right) \equiv &\left(A\left(\ \Delta^2\ \ \Delta^1\ \ \lambda\Delta^G\ \right)\mathcal{I}_2\left(\lambda\Delta^G\right) + A\left(\ \Delta^2\ \ \Delta^2\ \ \infty\ \right)\mathcal{I}_3\left(\lambda\Delta^G\right)\right)\ \lambda\,\frac{1}{2}\left(1 - z^F\right)\overline{x}^H\\ &+\ A\left(\ \Delta^2\ \ \Delta^2\ \ \Delta^G\ \right)\ \lambda\,\frac{1}{2}\, z^F\ \overline{x}^H\end{aligned}$$

For any $\ell^F \in \{1\ ,\ 2\ ,\ 3\ ,\ 4\}$, the following holds:

$$z^H(\ \Delta^1\ \ \Delta^{\ell^F}\ \ \Delta^G\ ) = \left(\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}}\left[\ A^{zH}_{NE}\left(\Delta^G\right) +\ A^{zH}_{H\ell^F}\left(\Delta^G\right)\ \right]^{\frac{1}{\widehat{\psi}-1}}, \tag{TA4.71}$$

and for any $\ell^H > 1$ and ℓ^F, the following holds:

$$z^H(\ \Delta^{\ell^H}\ \ \Delta^{\ell^F}\ \ \Delta^G\ ) = \left(\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}}\left[\ A^{zH}_{NE}\left(\Delta^G\right) +\ A^{zH}_{\ell^H F}\left(\Delta^G\right) + A^{zH}_{H\ell^F}\left(\Delta^G\right)\ \right]^{\frac{1}{\widehat{\psi}-1}}. \tag{TA4.72}$$

TA4.2.1 Value from Existing Product, Home Firm

H superscripts from all the $A(\cdot)$ and $z(\cdot)$ are dropped for notational simplicity.

$$A\left(\Delta^1\ \Delta^1\ \infty\right) = \pi^{HH} + \pi^{HF} - \widehat{\chi}\, z\left(\Delta^1\ \Delta^1\ \infty\right)^{\widehat{\psi}}$$
$$+\widetilde{\beta}\times\left[\begin{array}{ll} A\left(\Delta^1\,\Delta^1\,\infty\right) & \times\ \left(1-z\left(\Delta^1\ \Delta^1\ \infty\right)\right)(1-\overline{x}) \\ +A\left(\Delta^2\,\Delta^2\,\infty\right)\lambda & \times\ z\left(\Delta^1\ \Delta^1\ \infty\right)\ (1-\overline{x}) \\ +\left[\begin{array}{ll} A\left(\Delta^1\,\Delta^3\ \frac{1}{\eta}\right) & \mathcal{I}_2\left(\frac{1}{\eta}\right) \\ +A\left(\Delta^1\,\Delta^1\ \infty\right) & \mathcal{I}_3\left(\frac{1}{\eta}\right)\end{array}\right] & \times\ \left(1-z\left(\Delta^1\ \Delta^1\ \infty\right)\right)\overline{x}^F \\ +\left[\begin{array}{ll} A\left(\Delta^2\,\Delta^3\ \frac{\lambda}{\eta}\right) & \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \\ +A\left(\Delta^2\,\Delta^2\ \infty\right) & \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\end{array}\right]\lambda & \times\ z\left(\Delta^1\ \Delta^1\ \infty\right)\ \overline{x}^F \end{array}\right]$$

$$A\left(\Delta^2\ \Delta^2\ \infty\right) = \pi^{HH} + \pi^{HF} - \widehat{\chi}\, z\left(\Delta^2\ \Delta^2\ \infty\right)^{\widehat{\psi}}$$
$$+\widetilde{\beta}\times\left[\begin{array}{ll} A\left(\Delta^1\,\Delta^1\,\infty\right) & \times\ \left(1-z\left(\Delta^2\ \Delta^2\ \infty\right)\right)(1-\overline{x}) \\ +A\left(\Delta^2\,\Delta^2\,\infty\right)\lambda & \times\ z\left(\Delta^2\ \Delta^2\ \infty\right)\ (1-\overline{x}) \\ +A\left(\Delta^2\,\Delta^2\,\infty\right)\lambda & \times\ z\left(\Delta^2\ \Delta^2\ \infty\right)\ \overline{x}^H \\ +\left[\begin{array}{ll} A\left(\Delta^1\,\Delta^4\ \frac{\lambda}{\eta}\right) & \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \\ +A\left(\Delta^1\,\Delta^1\ \infty\right) & \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\end{array}\right] & \times\ \left(1-z\left(\Delta^2\ \Delta^2\ \infty\right)\right)\overline{x}^F \\ +\left[\begin{array}{ll} A\left(\Delta^2\,\Delta^4\ \frac{\lambda^2}{\eta}\right) & \mathcal{I}_2\left(\frac{\lambda^2}{\eta}\right) \\ +A\left(\Delta^2\,\Delta^2\ \infty\right) & \mathcal{I}_3\left(\frac{\lambda^2}{\eta}\right)\end{array}\right]\lambda & \times\ z\left(\Delta^2\ \Delta^2\ \infty\right)\ \overline{x}^F \end{array}\right]$$

$$A\left(\Delta^3\ \Delta^3\ \infty\right) = \pi^{HH} + \pi^{HF} - \widehat{\chi}\, z\left(\Delta^3\ \Delta^3\ \infty\right)^{\widehat{\psi}}$$
$$+\widetilde{\beta}\times\left[\begin{array}{ll} A\left(\Delta^1\,\Delta^1\,\infty\right) & \times\ \left(1-z\left(\Delta^3\ \Delta^3\ \infty\right)\right)(1-\overline{x}) \\ +A\left(\Delta^2\,\Delta^2\,\infty\right)\lambda & \times\ z\left(\Delta^3\ \Delta^3\ \infty\right)\ (1-\overline{x}) \\ +A\left(\Delta^1\,\Delta^1\,\infty\right) & \times\ \frac{1}{2}\left(1-z\left(\Delta^3\ \Delta^3\ \infty\right)\right)\overline{x}^H \\ +A\left(\Delta^2\,\Delta^2\,\infty\right)\lambda & \times\ z\left(\Delta^3\ \Delta^3\ \infty\right)\ \overline{x}^H \\ +\left[\begin{array}{ll} A\left(\Delta^1\,\Delta^1\ 1\right) & \mathcal{I}_2\left(1\right) \\ +A\left(\Delta^1\,\Delta^1\ \infty\right) & \mathcal{I}_3\left(1\right)\end{array}\right] & \times\ \left(1-z\left(\Delta^3\ \Delta^3\ \infty\right)\right)\overline{x}^F \\ +\left[\begin{array}{ll} A\left(\Delta^2\,\Delta^1\ \lambda\right) & \mathcal{I}_2\left(\lambda\right) \\ +A\left(\Delta^2\,\Delta^2\ \infty\right) & \mathcal{I}_3\left(\lambda\right)\end{array}\right]\lambda & \times\ z\left(\Delta^3\ \Delta^3\ \infty\right)\ \overline{x}^F \end{array}\right]$$

$$A\left(\Delta^4\ \Delta^4\ \infty\right) = \pi^{HH} + \pi^{HF} - \widehat{\chi}\, z\left(\Delta^4\ \Delta^4\ \infty\right)^{\widehat{\psi}} + \widetilde{\beta} \times \left[\begin{array}{ll} A\left(\Delta^1\,\Delta^1\,\infty\right) & \times\ \left(1 - z\left(\Delta^4\ \Delta^4\ \infty\right)\right)(1-\overline{x}) \\ + A\left(\Delta^2\,\Delta^2\,\infty\right)\lambda & \times\ z\left(\Delta^4\ \Delta^4\ \infty\right)\ (1-\overline{x}) \\ + A\left(\Delta^2\,\Delta^2\,\infty\right)\lambda & \times\ \frac{1}{2} z\left(\Delta^4\ \Delta^4\ \infty\right)\ \overline{x}^H \\ + \left[\begin{array}{ll} A\left(\Delta^1\,\Delta^2\ \frac{1}{\lambda}\right) & \mathcal{I}_2\left(\frac{1}{\lambda}\right) \\ + A\left(\Delta^1\,\Delta^1\ \infty\right) & \mathcal{I}_3\left(\frac{1}{\lambda}\right) \end{array}\right] & \times\ \left(1 - z\left(\Delta^4\ \Delta^4\ \infty\right)\right)\overline{x}^F \\ + \left[\begin{array}{ll} A\left(\Delta^2\,\Delta^2\ 1\right) & \mathcal{I}_2(1) \\ + A\left(\Delta^2\,\Delta^2\ \infty\right) & \mathcal{I}_3(1) \end{array}\right]\lambda & \times\ z\left(\Delta^4\ \Delta^4\ \infty\right)\overline{x}^F \end{array}\right]$$

Both z^H and z^F are the ones corresponding to the state-variable of interest. The state-variable is removed for the notational simplicity.

$$A^{AH}_{NE} \equiv \left[\begin{array}{ll} A\left(\Delta^1\,\Delta^1\,\Delta^G\right) & \times\ (1-z^H)(1-z^F)(1-\overline{x}) \\ + \left[\begin{array}{ll} A\left(\Delta^2\,\Delta^1\ \lambda\Delta^G\right) & \mathcal{I}_2\left(\lambda\Delta^G\right) \\ + A\left(\Delta^2\,\Delta^2\ \infty\right) & \mathcal{I}_3\left(\lambda\Delta^G\right) \end{array}\right]\lambda & \times\ z^H\,(1-z^F)(1-\overline{x}) \\ + A\left(\Delta^1\,\Delta^2\ \frac{1}{\lambda}\Delta^G\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) & \times\ (1-z^H)\,z^F\,(1-\overline{x}) \\ + A\left(\Delta^2\,\Delta^2\,\Delta^G\right)\lambda & \times\ z^H\,z^F\,(1-\overline{x}) \end{array}\right]$$

$$A^{AH}_{H1} \equiv A\left(\Delta^1\,\Delta^3\ \frac{1}{\eta}\Delta^G\right)\mathcal{I}_2\left(\frac{1}{\eta}\Delta^G\right) \times\ (1-z^H)\,\overline{x}^F + A\left(\Delta^2\,\Delta^3\ \frac{\lambda}{\eta}\Delta^G\right)\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\lambda \times\ z^H\,\overline{x}^F$$

$$A^{AH}_{H2} \equiv \left[\begin{array}{ll} A\left(\Delta^1\,\Delta^4\ \frac{\lambda}{\eta}\Delta^G\right)\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right) & \times\ (1-z^H)(1-z^F)\overline{x}^F \\ + \left[\begin{array}{ll} A\left(\Delta^2\,\Delta^4\ \frac{\lambda^2}{\eta}\Delta^G\right) & \mathcal{I}_2\left(\frac{\lambda^2}{\eta}\Delta^G\right) \\ + A\left(\Delta^2\,\Delta^2\ \infty\right) & \mathcal{I}_3\left(\frac{\lambda^2}{\eta}\Delta^G\right) \end{array}\right]\lambda & \times\ z^H\,(1-z^F)\overline{x}^F \\ + A\left(\Delta^1\,\Delta^2\ \frac{1}{\lambda}\Delta^G\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) & \times\ (1-z^H)\,z^F\,\overline{x}^F \\ + A\left(\Delta^2\,\Delta^2\,\Delta^G\right)\lambda & \times\ z^H\,z^F\,\overline{x}^F \end{array}\right]$$

$$A^{AH}_{H3} \equiv \left[\begin{array}{ll} A\left(\Delta^1\,\Delta^1\ \Delta^G\right) & \times\ (1-z^H)(1-z^F)\overline{x}^F \\ + \left[\begin{array}{ll} A\left(\Delta^2\,\Delta^1\ \lambda\Delta^G\right) & \mathcal{I}_2\left(\lambda\Delta^G\right) \\ + A\left(\Delta^2\,\Delta^2\ \infty\right) & \mathcal{I}_3\left(\lambda\Delta^G\right) \end{array}\right]\lambda & \times\ z^H\,(1-z^F)\overline{x}^F \\ + A\left(\Delta^1\,\Delta^2\ \frac{1}{\lambda}\Delta^G\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) & \times\ (1-z^H)\,z^F\,\overline{x}^F \\ + A\left(\Delta^2\,\Delta^2\,\Delta^G\right)\lambda & \times\ z^H\,z^F\,\overline{x}^F \end{array}\right]$$

$$A^{AH}_{H4} \equiv A\left(\Delta^1\, \Delta^2\ \frac{1}{\lambda}\Delta^G \right) \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) \times\ (1-z^H)\,\overline{x}^F\ +\ A\left(\Delta^2\,\Delta^2\ \Delta^G\right)\lambda \times\ z^H\,\overline{x}^F$$

$$A^{AH}_{2F} \equiv \left[\begin{array}{ll} \left[\begin{array}{ll} A\left(\Delta^2\,\Delta^1\ \lambda\Delta^G\right) & \mathcal{I}_2\left(\lambda\Delta^G\right) \\ +A\left(\Delta^2\,\Delta^2\ \infty\right) & \mathcal{I}_3\left(\lambda\Delta^G\right) \end{array}\right]\lambda & \times\ z^H\,(1-z^F)\overline{x}^H \\ +A\left(\Delta^2\,\Delta^2\,\Delta^G\right)\lambda & \times\ z^H\,z^F\,\overline{x}^H \end{array}\right]$$

$$A^{AH}_{3F} \equiv \left[\begin{array}{ll} A\left(\Delta^1\,\Delta^1\ \Delta^G\right) & \times\ \frac{1}{2}\,(1-z^H)(1-z^F)\overline{x}^H \\ +\left[\begin{array}{ll} A\left(\Delta^2\,\Delta^1\ \lambda\Delta^G\right) & \mathcal{I}_2\left(\lambda\Delta^G\right) \\ +A\left(\Delta^2\,\Delta^2\ \infty\right) & \mathcal{I}_3\left(\lambda\Delta^G\right) \end{array}\right]\lambda & \times\ z^H\,(1-z^F)\overline{x}^H \\ +A\left(\Delta^1\,\Delta^2\,\frac{1}{\lambda}\Delta^G\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) & \times\ \frac{1}{2}\,(1-z^H)\,z^F\,\overline{x}^H \\ +A\left(\Delta^2\,\Delta^2\,\Delta^G\right)\lambda & \times\ z^H\,z^F\,\overline{x}^H \end{array}\right]$$

$$A^{AH}_{4F} \equiv \left[\begin{array}{ll} \left[\begin{array}{ll} A\left(\Delta^2\,\Delta^1\ \lambda\Delta^G\right) & \mathcal{I}_2\left(\lambda\Delta^G\right) \\ +A\left(\Delta^2\,\Delta^2\ \infty\right) & \mathcal{I}_3\left(\lambda\Delta^G\right) \end{array}\right]\lambda & \times\ \frac{1}{2}\,z^H\,(1-z^F)\overline{x}^H \\ +A\left(\Delta^2\,\Delta^2\ \Delta^G\right)\lambda & \times\ \frac{1}{2}\,z^H\,z^F\,\overline{x}^H \end{array}\right]$$

For any $\ell^F \in \{1\,,\,2\,,\,3\,,\,4\}$, the following holds:

$$A\left(\Delta^1\ \ \Delta^{\ell^F}\ \ \Delta^G\right) = \pi^{HH} - \widehat{\chi}\ \left(z^H\right)^{\widehat{\psi}} + \widetilde{\beta}\times\left[\,A^{AH}_{NE}\ +\ A^{AH}_{H\ell^F}\,\right],$$

and for any $\ell^H > 1$ and ℓ^F, the following holds:

$$A\left(\Delta^{\ell^H}\ \ \Delta^{\ell^F}\ \ \Delta^G\right) = \pi^{HH} - \widehat{\chi}\ \left(z^H\right)^{\widehat{\psi}} + \widetilde{\beta}\times\left[\,A^{AH}_{NE}\ +\ A^{AH}_{\ell^H F}\ +\ A^{AH}_{H\ell^F}\,\right].$$

TA4.2.2 Value from a New Product Line $A^H_{takeover}$ for Home Firm

$$A^H_{takeover} \equiv \Bigg[\sum_{\mathcal{I}^{ZH}_{-j}=0}^{1}\ \sum_{\mathcal{I}^{ZF}_{-j}=0}^{1}\ \sum_{c-t_{-j}=\text{win}}^{\text{lose}}\ \left(z^H\right)^{\mathcal{I}^{ZH}_{-j}}\left(1-z^H\right)^{1-\mathcal{I}^{ZH}_{-j}}\ \left(z^F\right)^{\mathcal{I}^{ZF}_{-j}}\left(1-z^F\right)^{1-\mathcal{I}^{ZF}_{-j}}\ \frac{1}{2}$$

$$\times\int_{(\ \Delta^H_{-j}\ \ \Delta^F_{-j}\ \ \Delta^G_{-j}\ )} A^H\Big(\Delta^{H\prime}_{-j}\ \ \Delta^{F\prime}_{-j}\ \ \Delta^{G\prime}_{-j}\ \mid\ \Delta^H_{-j}\,,\,\Delta^F_{-j}\,,\,\Delta^G_{-j}\,,\,\mathcal{I}^{ZH}_{-j}\ ,\mathcal{I}^{ZF}_{-j}\ ,\,c-t_{-j}\Big)\,\Delta^{H\prime}_{-j}$$

$$\times\,\mu\left(\Delta^H_{-j}\ \ \Delta^F_{-j}\ \ \Delta^G_{-j}\ \right)\,\mathbf{d}\left(\Delta^H_{-j}\ \ \Delta^F_{-j}\ \ \Delta^G_{-j}\ \right)\Bigg],$$

where the RHS is equal to

$$
\begin{aligned}
& A\left(\,\Delta^3\,\Delta^3\;\infty\,\right)\,\eta\,\mu\left(\Delta^1\,\Delta^1\;\infty\right)\;+\;A\left(\,\Delta^4\,\Delta^4\;\infty\,\right)\,\frac{\eta}{\lambda}\,\left(1-z^H\left(\Delta^2\,\Delta^2\;\infty\right)\right)\,\mu\left(\Delta^2\,\Delta^2\;\infty\right)\\
& +\;A\left(\,\Delta^1\,\Delta^1\;\infty\,\right)\,\frac{1}{2}\,\left(1-z^H\left(\Delta^3\,\Delta^3\;\infty\right)\right)\,\mu\left(\Delta^3\,\Delta^3\;\infty\right)\\
& +\;A\left(\,\Delta^2\,\Delta^2\;\infty\,\right)\,\lambda\,\left(1-\frac{1}{2}\,z^H\left(\Delta^4\,\Delta^4\;\infty\right)\right)\,\mu\left(\Delta^4\,\Delta^4\;\infty\right)\\
& +\left[\begin{array}{ll}\left[A\left(\,\Delta^3\,\Delta^1\;\eta\,\right)\mathcal{I}_2\left(\eta\right)+A\left(\,\Delta^3\,\Delta^3\;\infty\,\right)\mathcal{I}_3\left(\eta\right)\right] & \left(1-z^F\left(\Delta^1\,\Delta^1\;-\infty\right)\right)\\ +\left[A\left(\,\Delta^3\,\Delta^2\;\tfrac{\eta}{\lambda}\,\right)\mathcal{I}_2\left(\tfrac{\eta}{\lambda}\right)+A\left(\,\Delta^3\,\Delta^3\;\infty\,\right)\mathcal{I}_3\left(\tfrac{\eta}{\lambda}\right)\right] & z^F\left(\Delta^1\,\Delta^1\;-\infty\right)\end{array}\right]\eta\,\mu\left(\Delta^1\,\Delta^1\;-\infty\right)\\
& +\left[\begin{array}{ll}\left[A\left(\,\Delta^4\,\Delta^1\;\tfrac{\eta}{\lambda}\,\right)\mathcal{I}_2\left(\tfrac{\eta}{\lambda}\right)+A\left(\,\Delta^4\,\Delta^4\;\infty\,\right)\mathcal{I}_3\left(\tfrac{\eta}{\lambda}\right)\right] & \left(1-z^F\left(\Delta^2\,\Delta^2\;-\infty\right)\right)\\ +\left[A\left(\,\Delta^4\,\Delta^2\;\tfrac{\eta}{\lambda^2}\,\right)\mathcal{I}_2\left(\tfrac{\eta}{\lambda^2}\right)+A\left(\,\Delta^4\,\Delta^4\;\infty\,\right)\mathcal{I}_3\left(\tfrac{\eta}{\lambda^2}\right)\right] & z^F\left(\Delta^2\,\Delta^2\;-\infty\right)\end{array}\right]\frac{\eta}{\lambda}\,\mu\left(\Delta^2\,\Delta^2\;-\infty\right)\\
& +\left[\begin{array}{ll}\left[A\left(\,\Delta^1\,\Delta^1\;1\,\right)\mathcal{I}_2\left(1\right)+A\left(\,\Delta^1\,\Delta^1\;\infty\,\right)\mathcal{I}_3\left(1\right)\right] & \left(1-z^F\left(\Delta^3\,\Delta^3\;-\infty\right)\right)\\ +\left[A\left(\,\Delta^1\,\Delta^2\;\tfrac{1}{\lambda}\,\right)\mathcal{I}_2\left(\tfrac{1}{\lambda}\right)+A\left(\,\Delta^1\,\Delta^1\;\infty\,\right)\mathcal{I}_3\left(\tfrac{1}{\lambda}\right)\right] & z^F\left(\Delta^3\,\Delta^3\;-\infty\right)\end{array}\right]\mu\left(\Delta^3\,\Delta^3\;-\infty\right)\\
& +\left[\begin{array}{ll}\left[A\left(\,\Delta^2\,\Delta^1\;\lambda\,\right)\mathcal{I}_2\left(\lambda\right)+A\left(\,\Delta^2\,\Delta^2\;\infty\,\right)\mathcal{I}_3\left(\lambda\right)\right] & \left(1-z^F\left(\Delta^4\,\Delta^4\;-\infty\right)\right)\\ +\left[A\left(\,\Delta^2\,\Delta^2\;1\,\right)\mathcal{I}_2\left(1\right)+A\left(\,\Delta^2\,\Delta^2\;\infty\,\right)\mathcal{I}_3\left(1\right)\right] & z^F\left(\Delta^4\,\Delta^4\;-\infty\right)\end{array}\right]\lambda\,\mu\left(\Delta^4\,\Delta^4\;-\infty\right)\\
& +\sum_{\Delta^G}\sum_{\ell^F=1}^{4}\left[\begin{array}{ll}
\left[\begin{array}{ll}\left[A\left(\,\Delta^3\,\Delta^1\;\eta\Delta^G\,\right)\mathcal{I}_2\left(\eta\Delta^G\right)+A\left(\,\Delta^3\,\Delta^3\;\infty\,\right)\mathcal{I}_3\left(\eta\Delta^G\right)\right] & \left(1-z^F\left(\Delta^1\,\Delta^{\ell^F}\;\Delta^G\right)\right)\\ +\left[A\left(\,\Delta^3\,\Delta^2\;\tfrac{\eta}{\lambda}\Delta^G\,\right)\mathcal{I}_2\left(\tfrac{\eta}{\lambda}\Delta^G\right)+A\left(\,\Delta^3\,\Delta^3\;\infty\,\right)\mathcal{I}_3\left(\tfrac{\eta}{\lambda}\Delta^G\right)\right] & z^F\left(\Delta^1\,\Delta^{\ell^F}\;\Delta^G\right)\end{array}\right] & \eta\mu\left(\Delta^1\;\ell^F\;\Delta^G\right)\\
+\left[\begin{array}{ll}\left[A\left(\,\Delta^4\,\Delta^1\;\tfrac{\eta}{\lambda}\Delta^G\,\right)\mathcal{I}_2\left(\tfrac{\eta}{\lambda}\Delta^G\right)+A\left(\,\Delta^4\,\Delta^4\;\infty\,\right)\mathcal{I}_3\left(\tfrac{\eta}{\lambda}\Delta^G\right)\right] & \left(1-z^H\left(\Delta^2\,\Delta^{\ell^F}\;\Delta^G\right)\right)\\ & \times\left(1-z^F\left(\Delta^2\,\Delta^{\ell^F}\;\Delta^G\right)\right)\\ +A\left(\,\Delta^4\,\Delta^2\;\tfrac{\eta}{\lambda^2}\Delta^G\,\right)\mathcal{I}_2\left(\tfrac{\eta}{\lambda^2}\Delta^G\right) & \left(1-z^H\left(\Delta^2\,\Delta^{\ell^F}\;\Delta^G\right)\right)\\ & \times z^F\left(\Delta^2\,\Delta^{\ell^F}\;\Delta^G\right)\end{array}\right] & \tfrac{\eta}{\lambda}\mu\left(\Delta^2\;\ell^F\;\Delta^G\right)\\
+\left[\begin{array}{ll}A\left(\,\Delta^1\,\Delta^1\;\Delta^G\,\right) & \tfrac{1}{2}\left(1-z^H\left(\Delta^3\,\Delta^{\ell^F}\;\Delta^G\right)\right)\\ & \times\left(1-z^F\left(\Delta^3\,\Delta^{\ell^F}\;\Delta^G\right)\right)\\ +A\left(\,\Delta^1\,\Delta^2\;\tfrac{1}{\lambda}\Delta^G\,\right)\mathcal{I}_2\left(\tfrac{1}{\lambda}\Delta^G\right) & \tfrac{1}{2}\left(1-z^H\left(\Delta^3\,\Delta^{\ell^F}\;\Delta^G\right)\right)\\ & \times z^F\left(\Delta^3\,\Delta^{\ell^F}\;\Delta^G\right)\end{array}\right] & \mu\left(\Delta^3\;\ell^F\;\Delta^G\right)\\
+\left[\begin{array}{ll}\left[A\left(\,\Delta^2\,\Delta^1\;\lambda\Delta^G\,\right)\mathcal{I}_2\left(\lambda\Delta^G\right)+A\left(\,\Delta^2\,\Delta^2\;\infty\,\right)\mathcal{I}_3\left(\lambda\Delta^G\right)\right] & \left(1-\tfrac{1}{2}z^H\left(\Delta^4\,\Delta^{\ell^F}\;\Delta^G\right)\right)\\ & \times\left(1-z^F\left(\Delta^4\,\Delta^{\ell^F}\;\Delta^G\right)\right)\\ +A\left(\,\Delta^2\,\Delta^2\;\Delta^G\,\right) & \left(1-\tfrac{1}{2}z^H\left(\Delta^4\,\Delta^{\ell^F}\;\Delta^G\right)\right)\\ & \times z^F\left(\Delta^4\,\Delta^{\ell^F}\;\Delta^G\right)\end{array}\right] & \lambda\mu\left(\Delta^4\;\ell^F\;\Delta^G\right)
\end{array}\right]
\end{aligned}
$$

TA4.2.3 Optimal Internal Innovation Intensity, Foreign Firm

F superscripts from all the $A(\cdot)$ and $z(\cdot)$ are dropped for notational simplicity.

$$
z^F\left(\,\Delta^1\;\;\Delta^1\;\;-\infty\,\right)=\left(\frac{\widetilde{\beta}^F}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}}\left[\;\left[A\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)\,\lambda-A\left(\,\Delta^1\;\;\Delta^1\;\;-\infty\,\right)\right]\left(1-\overline{x}\right)\right.
$$

$$+\left[\left(A\left(\Delta^2\ \Delta^2\ \ -\infty\right)\mathcal{I}_1\left(\frac{\eta}{\lambda}\right)+A\left(\Delta^3\ \Delta^2\ \frac{\eta}{\lambda}\right)\mathcal{I}_2\left(\frac{\eta}{\lambda}\right)\right)\lambda\right.$$

$$\left.\left.-\left(A\left(\Delta^1\ \Delta^1\ \ -\infty\right)\mathcal{I}_1(\eta)+A\left(\Delta^3\ \Delta^1\ \eta\right)\mathcal{I}_2(\eta)\right)\right]\overline{x}^H\ \right]^{\frac{1}{\hat{\psi}-1}} \tag{TA4.73}$$

$$z^F\left(\Delta^2\ \Delta^2\ \ -\infty\right)=\left(\frac{\widetilde{\beta}^F}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\hat{\psi}-1}}\left[\ \left[A\left(\Delta^2\ \Delta^2\ \ -\infty\right)\lambda-A\left(\Delta^1\ \Delta^1\ \ -\infty\right)\right](1-\overline{x})\right.$$

$$+\left[\left(A\left(\Delta^2\ \Delta^2\ \ -\infty\right)\mathcal{I}_1\left(\frac{\eta}{\lambda^2}\right)+A\left(\Delta^4\ \Delta^2\ \frac{\eta}{\lambda^2}\right)\mathcal{I}_2\left(\frac{\eta}{\lambda^2}\right)\right)\lambda\right.$$

$$\left.-\left(A\left(\Delta^1\ \Delta^1\ \ -\infty\right)\mathcal{I}_1\left(\frac{\eta}{\lambda}\right)+A\left(\Delta^4\ \Delta^1\ \frac{\eta}{\lambda}\right)\mathcal{I}_2\left(\frac{\eta}{\lambda}\right)\right)\right]\overline{x}^H$$

$$\left.+\ A\left(\Delta^2\ \Delta^2\ \ -\infty\right)\lambda\,\overline{x}^F\ \right]^{\frac{1}{\hat{\psi}-1}} \tag{TA4.74}$$

$$z^F\left(\Delta^3\ \Delta^3\ \ -\infty\right)=\left(\frac{\widetilde{\beta}^F}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\hat{\psi}-1}}\left[\ \left[A\left(\Delta^2\ \Delta^2\ \ -\infty\right)\lambda-A\left(\Delta^1\ \Delta^1\ \ -\infty\right)\right](1-\overline{x})\right.$$

$$+\left[\left(A\left(\Delta^2\ \Delta^2\ \ -\infty\right)\mathcal{I}_1\left(\frac{1}{\lambda}\right)+A\left(\Delta^1\ \Delta^2\ \frac{1}{\lambda}\right)\mathcal{I}_2\left(\frac{1}{\lambda}\right)\right)\lambda\right.$$

$$\left.-\left(A\left(\Delta^1\ \Delta^1\ \ -\infty\right)\mathcal{I}_1(1)+A\left(\Delta^1\ \Delta^1\ 1\right)\mathcal{I}_2(1)\right)\right]\overline{x}^H$$

$$\left.+\ \left[A\left(\Delta^2\ \Delta^2\ \ -\infty\right)\lambda\ -\ \frac{1}{2}A\left(\Delta^1\ \Delta^1\ \ -\infty\right)\right]\overline{x}^F\ \right]^{\frac{1}{\hat{\psi}-1}} \tag{TA4.75}$$

$$z^F\left(\Delta^4\ \Delta^4\ \ -\infty\right)=\left(\frac{\widetilde{\beta}^F}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\hat{\psi}-1}}\left[\ \left[A\left(\Delta^2\ \Delta^2\ \ -\infty\right)\lambda-A\left(\Delta^1\ \Delta^1\ \ -\infty\right)\right](1-\overline{x})\right.$$

$$+\left[\left(A\left(\Delta^2\ \Delta^2\ \ -\infty\right)\mathcal{I}_1(1)+A\left(\Delta^2\ \Delta^2\ 1\right)\mathcal{I}_2(1)\right)\lambda\right.$$

$$\left.-\left(A\left(\Delta^1\ \Delta^1\ \ -\infty\right)\mathcal{I}_1(\lambda)+A\left(\Delta^2\ \Delta^1\ \lambda\right)\mathcal{I}_2(\lambda)\right)\right]\overline{x}^H$$

$$\left.+\ A\left(\Delta^2\ \Delta^2\ \ -\infty\right)\lambda\,\frac{1}{2}\,\overline{x}^F\ \right]^{\frac{1}{\hat{\psi}-1}} \tag{TA4.76}$$

Here, z^F corresponds to the internal innovation intensity under the same state variable we consider for z^H and the state-variable notations are removed for notational simplicity.

For $(H\ F)$ cases, the terms corresponding to no external innovation by either home firm or foreign firm are identical up tp Δ^G

and are defined as

$$A^{zF}_{NE}\left(\Delta^G\right) \equiv \Bigg[\left(A\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) + A\left(\;\Delta^1\;\;\Delta^2\;\;\frac{1}{\lambda}\Delta^G\;\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right)\lambda$$
$$- A\left(\,\Delta^1\;\;\Delta^1\;\;\Delta^G\,\right)\Bigg](1-z^H)(1-\overline{x})$$
$$+\left[A\left(\,\Delta^2\;\;\Delta^2\;\;\Delta^G\,\right)\,\lambda - A\left(\,\Delta^2\;\;\Delta^1\;\;\lambda\Delta^G\,\right)\mathcal{I}_2\left(\lambda\Delta^G\right)\right]z^H\,(1-\overline{x})\,.$$

For any Δ^F, terms corresponding to domestic external innovation depend only on domestic technology gap Δ^H and global technology gap Δ^G. Thus, for $A^{zF}_{\ell F}\left(\Delta^G\right)$ corresponding to $(\Delta^\ell\;\Delta^F\;\;\Delta^G)$,

$$A^{zF}_{1F}\left(\Delta^G\right) \equiv \left[\;A\left(\;\Delta^3\;\;\Delta^2\;\;\frac{\eta}{\lambda}\Delta^G\;\right)\lambda\,\mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right) - A\left(\,\Delta^3\;\;\Delta^1\;\;\eta\Delta^G\,\right)\,\mathcal{I}_2\left(\eta\Delta^G\right)\right]\overline{x}^H$$

$$A^{zF}_{2F}\left(\Delta^G\right) \equiv \Bigg[\left(A\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)\mathcal{I}_1\left(\frac{\eta}{\lambda^2}\Delta^G\right) + A\left(\;\Delta^4\;\;\Delta^2\;\;\frac{\eta}{\lambda^2}\Delta^G\;\right)\mathcal{I}_2\left(\frac{\eta}{\lambda^2}\Delta^G\right)\right)\lambda$$
$$- A\left(\;\Delta^4\;\;\Delta^1\;\;\frac{\eta}{\lambda}\Delta^G\;\right)\mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right)\Bigg](1-z^H)\,\overline{x}^H$$
$$+\left[A\left(\,\Delta^2\;\;\Delta^2\;\;\Delta^G\,\right)\,\lambda - A\left(\,\Delta^2\;\;\Delta^1\;\;\lambda\Delta^G\,\right)\mathcal{I}_2\left(\lambda\Delta^G\right)\right]z^H\,\overline{x}^H$$

$$A^{zF}_{3F}\left(\Delta^G\right) \equiv \Bigg[\left(A\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) + A\left(\;\Delta^1\;\;\Delta^2\;\;\frac{1}{\lambda}\Delta^G\;\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right)\lambda$$
$$- A\left(\,\Delta^1\;\;\Delta^1\;\;\Delta^G\,\right)\Bigg](1-z^H)\,\overline{x}^H$$
$$+\left[A\left(\,\Delta^2\;\;\Delta^2\;\;\Delta^G\,\right)\,\lambda - A\left(\,\Delta^2\;\;\Delta^1\;\;\lambda\Delta^G\,\right)\mathcal{I}_2\left(\lambda\Delta^G\right)\right]z^H\,\overline{x}^H$$

$$A^{zF}_{4F}\left(\Delta^G\right) \equiv \left[A\left(\,\Delta^2\;\;\Delta^2\;\;\Delta^G\,\right)\,\lambda\;-\;A\left(\,\Delta^2\;\;\Delta^1\;\;\lambda\Delta^G\right)\mathcal{I}_2\left(\lambda\Delta^G\right)\right]\overline{x}^H$$

Similarly for any Δ^H, terms corresponding to foreign external innovation depend only on foreign technology gap Δ^F and global technology gap Δ^G. Thus, for $A^{zH}_{H\ell}$ corresponding to $(\Delta^H\;\Delta^\ell\;\;\Delta^G)$,

$$A^{zF}_{H2}\left(\Delta^G\right) \equiv \left(A\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) + A\left(\;\Delta^1\;\;\Delta^2\;\;\frac{1}{\lambda}\Delta^G\;\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right)\lambda\,(1-z^H)\,\overline{x}^F$$
$$+\;A\left(\,\Delta^2\;\;\Delta^2\;\;\Delta^G\,\right)\,\lambda\,z^H\,\overline{x}^F$$

$$A^{zF}_{H3}\left(\Delta^G\right) \equiv \Bigg[\left(A\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) + A\left(\;\Delta^1\;\;\Delta^2\;\;\frac{1}{\lambda}\Delta^G\;\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right)\lambda$$
$$-\frac{1}{2}A\left(\,\Delta^1\;\;\Delta^1\;\;\Delta^G\,\right)\Bigg](1-z^H)\,\overline{x}^F$$
$$+\left[A\left(\,\Delta^2\;\;\Delta^2\;\;\Delta^G\,\right)\,\lambda - \frac{1}{2}A\left(\,\Delta^2\;\;\Delta^1\;\;\lambda\Delta^G\,\right)\mathcal{I}_2\left(\lambda\Delta^G\right)\right]z^H\,\overline{x}^F$$

$$A^{zF}_{H4}\left(\Delta^G\right) \equiv \left(A\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) + A\left(\;\Delta^1\;\;\Delta^2\;\;\frac{1}{\lambda}\Delta^G\;\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right)\lambda\,\frac{1}{2}\left(1-z^H\right)\overline{x}^F$$
$$+\;A\left(\,\Delta^2\;\;\Delta^2\;\;\Delta^G\,\right)\;\lambda\,\frac{1}{2}\,z^H\,\overline{x}^F$$

For any $\ell^H \in \{1\,,\,2\,,\,3\,,\,4\}$, the following holds:

$$z^F\left(\,\Delta^{\ell^H}\;\;\Delta^1\;\;\Delta^G\,\right) = \left(\frac{\widetilde{\beta}^F}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}}\left[\;A^{zF}_{NE}\left(\Delta^G\right) +\;A^{zF}_{\ell^H F}\left(\Delta^G\right)\;\right]^{\frac{1}{\widehat{\psi}-1}}, \tag{TA4.77}$$

and for any $\ell^F > 1$ and ℓ^H, the following holds:

$$z^F\left(\,\Delta^{\ell^H}\;\;\Delta^{\ell^F}\;\;\Delta^G\,\right) = \left(\frac{\widetilde{\beta}^F}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}}\left[\;A^{zF}_{NE}\left(\Delta^G\right) +\;A^{zF}_{\ell^H F}\left(\Delta^G\right) + A^{zF}_{H\ell^F}\left(\Delta^G\right)\;\right]^{\frac{1}{\widehat{\psi}-1}} \tag{TA4.78}$$

TA4.2.4 Value from Existing Product, Foreign Firm

All $A(\cdot)$ and $z(\cdot)$ without superscript are for foreign firms.

$$A\left(\,\Delta^1\;\;\Delta^1\;\;-\infty\,\right) = \pi^{FF} + \pi^{FH} - \widehat{\chi}\,z\left(\,\Delta^1\;\;\Delta^1\;\;-\infty\,\right)^{\widehat{\psi}}$$
$$+\,\widetilde{\beta}^F\times\left[\begin{array}{ll}
A\left(\,\Delta^1\,\Delta^1\;-\infty\,\right) & \times\;\left(1-z\left(\,\Delta^1\;\;\Delta^1\;\;-\infty\,\right)\right)(1-\overline{x}) \\
+\,A\left(\,\Delta^2\,\Delta^2\;-\infty\,\right)\lambda & \times\;z\left(\,\Delta^1\;\;\Delta^1\;\;-\infty\,\right)\;(1-\overline{x}) \\
+\left[\begin{array}{ll} A\left(\,\Delta^1\,\Delta^1\;\;-\infty\,\right) & \mathcal{I}_1\left(\eta\right) \\ +\,A\left(\,\Delta^3\,\Delta^1\;\;\eta\,\right) & \mathcal{I}_2\left(\eta\right)\end{array}\right] & \times\;\left(1-z\left(\,\Delta^1\;\;\Delta^1\;\;-\infty\,\right)\right)\overline{x}^H \\
+\left[\begin{array}{ll} A\left(\,\Delta^2\,\Delta^2\;\;-\infty\,\right) & \mathcal{I}_1\left(\frac{\eta}{\lambda}\right) \\ +\,A\left(\,\Delta^3\,\Delta^2\;\;\frac{\eta}{\lambda}\,\right) & \mathcal{I}_2\left(\frac{\eta}{\lambda}\right)\end{array}\right]\lambda & \times\;z\left(\,\Delta^1\;\;\Delta^1\;\;-\infty\,\right)\;\overline{x}^H
\end{array}\right]$$

$$A\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right) = \pi^{FF} + \pi^{FH} - \widehat{\chi}\,z\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)^{\widehat{\psi}}$$
$$+\,\widetilde{\beta}^F\times\left[\begin{array}{ll}
A\left(\,\Delta^1\,\Delta^1\;-\infty\,\right) & \times\;\left(1-z\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)\right)(1-\overline{x}) \\
+\,A\left(\,\Delta^2\,\Delta^2\;-\infty\,\right)\lambda & \times\;z\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)\;(1-\overline{x}) \\
+\left[\begin{array}{ll} A\left(\,\Delta^1\,\Delta^1\;\;-\infty\,\right) & \mathcal{I}_1\left(\frac{\eta}{\lambda}\right) \\ +\,A\left(\,\Delta^4\,\Delta^1\;\;\frac{\eta}{\lambda}\,\right) & \mathcal{I}_2\left(\frac{\eta}{\lambda}\right)\end{array}\right] & \times\;\left(1-z\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)\right)\overline{x}^H \\
+\left[\begin{array}{ll} A\left(\,\Delta^2\,\Delta^2\;\;-\infty\,\right) & \mathcal{I}_1\left(\frac{\eta}{\lambda^2}\right) \\ +\,A\left(\,\Delta^4\,\Delta^2\;\;\frac{\eta}{\lambda^2}\,\right) & \mathcal{I}_2\left(\frac{\eta}{\lambda^2}\right)\end{array}\right]\lambda & \times\;z\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)\;\overline{x}^H \\
+\,A\left(\,\Delta^2\,\Delta^2\;-\infty\,\right)\lambda & \times\;z\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)\;\overline{x}^F
\end{array}\right]$$

$$A\left(\Delta^3\ \Delta^3\ \ -\infty\right) = \pi^{FF} + \pi^{FH} - \widehat{\chi}\, z\left(\Delta^3\ \Delta^3\ \ -\infty\right)^{\widehat{\psi}}$$

$$+\,\widetilde{\beta}^F \times \left[\begin{array}{ll}
A\left(\Delta^1\,\Delta^1\ -\infty\right) & \times\ \big(1 - z\left(\Delta^3\ \Delta^3\ \ -\infty\right)\big)(1-\overline{x}) \\
+\,A\left(\Delta^2\,\Delta^2\ -\infty\right)\lambda & \times\ z\left(\Delta^3\ \Delta^3\ \ -\infty\right)\ (1-\overline{x}) \\
+\left[\begin{array}{ll} A\left(\Delta^1\,\Delta^1\ \ -\infty\right) & \mathcal{I}_1\left(1\right) \\ +\,A\left(\Delta^1\,\Delta^1\ \ 1\right) & \mathcal{I}_2\left(1\right)\end{array}\right] & \times\ \big(1 - z\left(\Delta^3\ \Delta^3\ \ -\infty\right)\big)\,\overline{x}^H \\
+\left[\begin{array}{ll} A\left(\Delta^2\,\Delta^2\ \ -\infty\right) & \mathcal{I}_1\left(\frac{1}{\lambda}\right) \\ +\,A\left(\Delta^1\,\Delta^2\ \ \frac{1}{\lambda}\right) & \mathcal{I}_2\left(\frac{1}{\lambda}\right)\end{array}\right]\lambda & \times\ z\left(\Delta^3\ \Delta^3\ \ -\infty\right)\ \overline{x}^H \\
+\,A\left(\Delta^1\,\Delta^1\ -\infty\right) & \times\ \frac{1}{2}\big(1 - z\left(\Delta^3\ \Delta^3\ \ -\infty\right)\big)\,\overline{x}^F \\
+\,A\left(\Delta^2\,\Delta^2\ -\infty\right)\lambda & \times\ z\left(\Delta^3\ \Delta^3\ \ -\infty\right)\ \overline{x}^F
\end{array}\right]$$

$$A\left(\Delta^4\ \Delta^4\ \ -\infty\right) = \pi^{FF} + \pi^{FH} - \widehat{\chi}\, z\left(\Delta^4\ \Delta^4\ \ -\infty\right)^{\widehat{\psi}}$$

$$+\,\widetilde{\beta}^F \times \left[\begin{array}{ll}
A\left(\Delta^1\,\Delta^1\ -\infty\right) & \times\ \big(1 - z\left(\Delta^4\ \Delta^4\ \ -\infty\right)\big)(1-\overline{x}) \\
+\,A\left(\Delta^2\,\Delta^2\ -\infty\right)\lambda & \times\ z\left(\Delta^4\ \Delta^4\ \ -\infty\right)\ (1-\overline{x}) \\
+\left[\begin{array}{ll} A\left(\Delta^1\,\Delta^1\ \ -\infty\right) & \mathcal{I}_1\left(\lambda\right) \\ +\,A\left(\Delta^2\,\Delta^1\ \ \lambda\right) & \mathcal{I}_2\left(\lambda\right)\end{array}\right] & \times\ \big(1 - z\left(\Delta^4\ \Delta^4\ \ -\infty\right)\big)\,\overline{x}^H \\
+\left[\begin{array}{ll} A\left(\Delta^2\,\Delta^2\ \ -\infty\right) & \mathcal{I}_1\left(1\right) \\ +\,A\left(\Delta^2\,\Delta^2\ \ 1\right) & \mathcal{I}_2\left(1\right)\end{array}\right]\lambda & \times\ z\left(\Delta^4\ \Delta^4\ \ -\infty\right)\ \overline{x}^H \\
+\,A\left(\Delta^2\,\Delta^2\ -\infty\right)\lambda & \times\ \frac{1}{2}z\left(\Delta^4\ \Delta^4\ \ -\infty\right)\ \overline{x}^F
\end{array}\right]$$

Both z^H and z^F are the ones corresponding to the state-variable of interest. The state-variable is removed for the notational simplicity.

$$A^{AF}_{NE} \equiv \left[\begin{array}{ll}
A\left(\Delta^1\,\Delta^1\,\Delta^G\right) & \times\ (1-z^H)(1-z^F)(1-\overline{x}) \\
+\left[\begin{array}{ll} A\left(\Delta^2\,\Delta^2\ \ -\infty\right) & \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) \\ +\,A\left(\Delta^1\ \Delta^2\ \ \frac{1}{\lambda}\Delta^G\right) & \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\end{array}\right]\lambda & \times\ (1-z^H)\,z^F\,(1-\overline{x}) \\
+\,A\left(\Delta^2\,\Delta^1\ \ \lambda\Delta^G\right)\mathcal{I}_2\left(\lambda\Delta^G\right) & \times\ z^H\,(1-z^F)\,(1-\overline{x}) \\
+\,A\left(\Delta^2\,\Delta^2\ \ \Delta^G\right)\lambda & \times\ z^H\,z^F\,(1-\overline{x})
\end{array}\right]$$

$$A^{AF}_{1F} \equiv A\left(\Delta^3\,\Delta^1\ \ \eta\Delta^G\right)\mathcal{I}_2\left(\eta\Delta^G\right) \times\ \ (1-z^F)\,\overline{x}^H\ +\ A\left(\Delta^3\,\Delta^2\ \ \frac{\eta}{\lambda}\Delta^G\right)\mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right)\lambda \times\ \ z^F\,\overline{x}^H$$

$$A^{AF}_{2F} \equiv \left[\begin{array}{ll} A\left(\Delta^4\, \Delta^1\, \; \frac{\eta}{\lambda}\Delta^G \right) \mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right) & \times\; (1-z^H)(1-z^F)\overline{x}^H \\ + \left[\begin{array}{ll} A\left(\Delta^2\, \Delta^2\; -\infty \right) & \mathcal{I}_1\left(\frac{\eta}{\lambda^2}\Delta^G\right) \\ + A\left(\Delta^4\, \Delta^2\; \frac{\eta}{\lambda^2}\Delta^G \right) & \mathcal{I}_2\left(\frac{\eta}{\lambda^2}\Delta^G\right) \end{array} \right] \lambda & \times\; (1-z^H)\, z^F\, \overline{x}^H \\ + A\left(\Delta^2\, \Delta^1\; \lambda\Delta^G \right) \mathcal{I}_2\left(\lambda\Delta^G\right) & \times\; z^H\, (1-z^F)\, \overline{x}^H \\ + A\left(\Delta^2\, \Delta^2\, \Delta^G \right) \lambda & \times\; z^H\, z^F\, \overline{x}^H \end{array} \right]$$

$$A^{AF}_{3F} \equiv \left[\begin{array}{ll} A\left(\Delta^1\, \Delta^1\; \Delta^G \right) & \times\; (1-z^H)(1-z^F)\overline{x}^H \\ + \left[\begin{array}{ll} A\left(\Delta^2\, \Delta^2\; -\infty \right) & \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) \\ + A\left(\Delta^1\, \Delta^2\; \frac{1}{\lambda}\Delta^G \right) & \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) \end{array} \right] \lambda & \times\; (1-z^H)\, z^F\, \overline{x}^H \\ + A\left(\Delta^2\, \Delta^1\, \lambda\Delta^G \right) \mathcal{I}_2\left(\lambda\Delta^G\right) & \times\; z^H\, (1-z^F)\, \overline{x}^H \\ + A\left(\Delta^2\, \Delta^2\, \Delta^G \right) \lambda & \times\; z^H\, z^F\, \overline{x}^H \end{array} \right]$$

$$A^{AF}_{4F} \equiv A\left(\Delta^2\, \Delta^1\; \lambda\Delta^G \right) \mathcal{I}_2\left(\lambda\Delta^G\right) \times\; (1-z^F)\, \overline{x}^H \;+\; A\left(\Delta^2\, \Delta^2\; \Delta^G \right) \lambda \times\; z^F\, \overline{x}^H$$

$$A^{AF}_{H2} \equiv \left[\begin{array}{ll} \left[\begin{array}{ll} A\left(\Delta^2\, \Delta^2\; -\infty \right) & \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) \\ + A\left(\Delta^1\, \Delta^2\; \frac{1}{\lambda}\Delta^G \right) & \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) \end{array} \right] \lambda & \times\; (1-z^H)\, z^F\, \overline{x}^F \\ + A\left(\Delta^2\, \Delta^2\; \Delta^G \right) \lambda & \times\; z^H\, z^F\, \overline{x}^F \end{array} \right]$$

$$A^{AF}_{H3} \equiv \left[\begin{array}{ll} A\left(\Delta^1\, \Delta^1\; \Delta^G \right) & \times\; \frac{1}{2}\,(1-z^H)(1-z^F)\overline{x}^F \\ + \left[\begin{array}{ll} A\left(\Delta^2\, \Delta^2\; -\infty \right) & \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) \\ + A\left(\Delta^1\, \Delta^2\; \frac{1}{\lambda}\Delta^G \right) & \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) \end{array} \right] \lambda & \times\; (1-z^H)\, z^F\, \overline{x}^F \\ + A\left(\Delta^2\, \Delta^1\; \lambda\Delta^G \right) \mathcal{I}_2\left(\lambda\Delta^G\right) & \times\; \frac{1}{2}\, z^H\, (1-z^F)\, \overline{x}^F \\ + A\left(\Delta^2\, \Delta^2\; \Delta^G \right) \lambda & \times\; z^H\, z^F\, \overline{x}^F \end{array} \right]$$

$$A^{AF}_{H4} \equiv \left[\begin{array}{ll} \left[\begin{array}{ll} A\left(\Delta^2\, \Delta^2\; -\infty \right) & \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) \\ + A\left(\Delta^1\, \Delta^2\; \frac{1}{\lambda}\Delta^G \right) & \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) \end{array} \right] \lambda & \times\; \frac{1}{2}\,(1-z^H)\, z^F\, \overline{x}^F \\ + A\left(\Delta^2\, \Delta^2\; \Delta^G \right) \lambda & \times\; \frac{1}{2}\, z^H\, z^F\, \overline{x}^F \end{array} \right]$$

For any $\ell^F \in \{1\,,\, 2\,,\, 3\,,\, 4\}$, the following holds:

$$A^F\left(\Delta^{\ell^H}\; \Delta^1\; \Delta^G \right) = \pi^{FF} - \widehat{\chi}\, \left(z^F\right)^{\widehat{\psi}} + \widetilde{\beta}^F \times \left[\, A^{AF}_{NE} \;+\; A^{AF}_{\ell^H F} \,\right],$$

and for any $\ell^F > 1$ and ℓ^H, the following holds:

$$A^F\left(\Delta^{\ell^F}\ \Delta^{\ell^F}\ \Delta^G\right) = \pi^{FF} - \widehat{\chi}\ \left(z^F\right)^{\widehat{\psi}} + \widetilde{\beta}^F \times \left[A^{AF}_{NE} + A^{AF}_{\ell^H F} + A^{AF}_{H\ell^F}\right].$$

TA4.2.5 Value from a New Product Line $A^F_{takeover}$ for Foreign Firm

$$\begin{aligned} A^F_{takeover} \equiv \Bigg[& \sum_{\mathcal{I}^{ZH}_{-j}=0}^{1} \sum_{\mathcal{I}^{ZF}_{-j}=0}^{1} \sum_{c-t_{-j}=\text{win}}^{\text{lose}} \left(z^H\right)^{\mathcal{I}^{ZH}_{-j}} \left(1-z^H\right)^{1-\mathcal{I}^{ZH}_{-j}} \left(z^F\right)^{\mathcal{I}^{ZF}_{-j}} \left(1-z^F\right)^{1-\mathcal{I}^{ZF}_{-j}} \frac{1}{2} \\ & \times \int_{(\Delta^H_{-j}\ \Delta^F_{-j}\ \Delta^G_{-j})} A^F\left(\Delta^{H\prime}_{-j}\ \Delta^{F\prime}_{-j}\ \Delta^{G\prime}_{-j} \mid \Delta^H_{-j}, \Delta^F_{-j}, \Delta^G_{-j}, \mathcal{I}^{ZH}_{-j}, \mathcal{I}^{ZF}_{-j}, c-t_{-j}\right) \Delta^{H\prime}_{-j} \\ & \times \mu\left(\Delta^H_{-j}\ \Delta^F_{-j}\ \Delta^G_{-j}\right) \mathbf{d}\left(\Delta^H_{-j}\ \Delta^F_{-j}\ \Delta^G_{-j}\right)\Bigg], \end{aligned}$$

where the RHS is equal to

$$\begin{aligned} & \left[\begin{array}{ll} \left[A\left(\Delta^3\,\Delta^3\ -\infty\right)\mathcal{I}_1\left(\frac{1}{\eta}\right) + A\left(\Delta^1\,\Delta^3\ \frac{1}{\eta}\right)\mathcal{I}_2\left(\frac{1}{\eta}\right)\right] & \left(1-z^H\left(\Delta^1\,\Delta^1\ \infty\right)\right) \\ +\left[A\left(\Delta^3\,\Delta^3\ -\infty\right)\mathcal{I}_1\left(\frac{\lambda}{\eta}\right) + A\left(\Delta^2\,\Delta^3\ \frac{\lambda}{\eta}\right)\mathcal{I}_2\left(\frac{\lambda}{\eta}\right)\right] & z^H\left(\Delta^1\,\Delta^1\ \infty\right) \end{array}\right] \eta\,\mu\left(\Delta^1\,\Delta^1\ \infty\right) \\ + & \left[\begin{array}{ll} \left[A\left(\Delta^4\,\Delta^4\ -\infty\right)\mathcal{I}_1\left(\frac{\lambda}{\eta}\right) + A\left(\Delta^1\,\Delta^4\ \frac{\lambda}{\eta}\right)\mathcal{I}_2\left(\frac{\lambda}{\eta}\right)\right] & \left(1-z^H\left(\Delta^2\,\Delta^2\ \infty\right)\right) \\ +\left[A\left(\Delta^4\,\Delta^4\ -\infty\right)\mathcal{I}_1\left(\frac{\lambda^2}{\eta}\right) + A\left(\Delta^2\,\Delta^4\ \frac{\lambda^2}{\eta}\right)\mathcal{I}_2\left(\frac{\lambda^2}{\eta}\right)\right] & z^H\left(\Delta^2\,\Delta^2\ \infty\right) \end{array}\right] \frac{\eta}{\lambda}\,\mu\left(\Delta^2\,\Delta^2\ \infty\right) \\ + & \left[\begin{array}{ll} \left[A\left(\Delta^1\,\Delta^1\ -\infty\right)\mathcal{I}_1(1) + A\left(\Delta^1\,\Delta^1\ 1\right)\mathcal{I}_2(1)\right] & \left(1-z^H\left(\Delta^3\,\Delta^3\ \infty\right)\right) \\ +\left[A\left(\Delta^1\,\Delta^1\ -\infty\right)\mathcal{I}_1(\lambda) + A\left(\Delta^2\,\Delta^1\ \lambda\right)\mathcal{I}_2(\lambda)\right] & z^H\left(\Delta^3\,\Delta^3\ \infty\right) \end{array}\right] \mu\left(\Delta^3\,\Delta^3\ \infty\right) \\ + & \left[\begin{array}{ll} \left[A\left(\Delta^2\,\Delta^2\ -\infty\right)\mathcal{I}_1\left(\frac{1}{\lambda}\right) + A\left(\Delta^1\,\Delta^2\ \frac{1}{\lambda}\right)\mathcal{I}_2\left(\frac{1}{\lambda}\right)\right] & \left(1-z^H\left(\Delta^4\,\Delta^4\ \infty\right)\right) \\ +\left[A\left(\Delta^2\,\Delta^2\ -\infty\right)\mathcal{I}_1(1) + A\left(\Delta^2\,\Delta^2\ 1\right)\mathcal{I}_2(1)\right] & z^H\left(\Delta^4\,\Delta^4\ \infty\right) \end{array}\right] \lambda\,\mu\left(\Delta^4\,\Delta^4\ \infty\right) \\ + & \ A\left(\Delta^3\,\Delta^3\ -\infty\right)\,\eta\,\mu\left(\Delta^1\,\Delta^1\ -\infty\right) + A\left(\Delta^4\,\Delta^4\ -\infty\right)\,\frac{\eta}{\lambda}\,\left(1-z^F\left(\Delta^2\,\Delta^2\ -\infty\right)\right)\,\mu\left(\Delta^2\,\Delta^2\ -\infty\right) \\ + & \ A\left(\Delta^1\,\Delta^1\ -\infty\right)\,\frac{1}{2}\,\left(1-z^F\left(\Delta^3\,\Delta^3\ -\infty\right)\right)\,\mu\left(\Delta^3\,\Delta^3\ -\infty\right) \\ + & \ A\left(\Delta^2\,\Delta^2\ -\infty\right)\,\lambda\,\left(1-\frac{1}{2}\,z^F\left(\Delta^4\,\Delta^4\ -\infty\right)\right)\,\mu\left(\Delta^4\,\Delta^4\ -\infty\right) \end{aligned}$$

$$
+\sum_{\Delta^G}\sum_{\ell^H=1}^{4}\left[\begin{array}{ll}
+\left[\begin{array}{ll}
\left[A\left(\,\Delta^3\,\Delta^3\;-\infty\,\right)\mathcal{I}_1\left(\frac{1}{\eta}\Delta^G\right)+A\left(\,\Delta^1\,\Delta^3\,\frac{1}{\eta}\Delta^G\,\right)\mathcal{I}_2\left(\frac{1}{\eta}\Delta^G\right)\right] & \left(1-z^H\left(\Delta^{\ell^H}\;\Delta^1\;\Delta^G\right)\right)\\
+\left[A\left(\,\Delta^3\,\Delta^3\;-\infty\,\right)\mathcal{I}_1\left(\frac{\lambda}{\eta}\Delta^G\right)+A\left(\,\Delta^2\,\Delta^3\,\frac{\lambda}{\eta}\Delta^G\,\right)\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\right] & z^H\left(\Delta^{\ell^H}\;\Delta^1\;\Delta^G\right)
\end{array}\right] & \eta\mu\left(\ell^H\;\Delta^1\;\Delta^G\right)\\
+\left[\begin{array}{ll}
\left[A\left(\,\Delta^4\,\Delta^4\;-\infty\,\right)\mathcal{I}_1\left(\frac{\lambda}{\eta}\Delta^G\right)+A\left(\,\Delta^1\,\Delta^4\,\frac{\lambda}{\eta}\Delta^G\,\right)\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\right] & \left(1-z^H\left(\Delta^{\ell^H}\;\Delta^2\;\Delta^G\right)\right)\\
 & \times\left(1-z^F\left(\Delta^{\ell^H}\;\Delta^2\;\Delta^G\right)\right)\\
+A\left(\,\Delta^2\,\Delta^4\,\frac{\lambda^2}{\eta}\Delta^G\,\right)\mathcal{I}_2\left(\frac{\lambda^2}{\eta}\Delta^G\right) & z^H\left(\Delta^{\ell^H}\;\Delta^2\;\Delta^G\right)\\
 & \times\left(1-z^F\left(\Delta^{\ell^H}\;\Delta^2\;\Delta^G\right)\right)
\end{array}\right] & \frac{\eta}{\lambda}\mu\left(\ell^H\;\Delta^2\;\Delta^G\right)\\
+\left[\begin{array}{ll}
A\left(\,\Delta^1\,\Delta^1\,\Delta^G\,\right) & \frac{1}{2}\left(1-z^H\left(\Delta^{\ell^H}\;\Delta^3\;\Delta^G\right)\right)\\
 & \times\left(1-z^F\left(\Delta^{\ell^H}\;\Delta^3\;\Delta^G\right)\right)\\
+A\left(\,\Delta^2\,\Delta^1\,\lambda\Delta^G\,\right)\mathcal{I}_2\left(\lambda\Delta^G\right) & \frac{1}{2}z^H\left(\Delta^{\ell^H}\;\Delta^3\;\Delta^G\right)\\
 & \times\left(1-z^F\left(\Delta^{\ell^H}\;\Delta^3\;\Delta^G\right)\right)
\end{array}\right] & \mu\left(\ell^H\;\Delta^3\;\Delta^G\right)\\
+\left[\begin{array}{ll}
\left[A\left(\,\Delta^2\,\Delta^2\;-\infty\,\right)\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)+A\left(\,\Delta^1\,\Delta^2\,\frac{1}{\lambda}\Delta^G\,\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right] & \left(1-z^H\left(\Delta^{\ell^H}\;\Delta^4\;\Delta^G\right)\right)\\
 & \times\left(1-\frac{1}{2}z^F\left(\Delta^{\ell^H}\;\Delta^4\;\Delta^G\right)\right)\\
+A\left(\,\Delta^2\,\Delta^2\,\Delta^G\,\right) & z^H\left(\Delta^{\ell^H}\;\Delta^4\;\Delta^G\right)\\
 & \times\left(1-\frac{1}{2}z^F\left(\Delta^{\ell^H}\;\Delta^4\;\Delta^G\right)\right)
\end{array}\right] & \lambda\mu\left(\ell^H\;\Delta^4\;\Delta^G\right)
\end{array}\right]
$$

TA4.3 Potential Startup's Problem

Potential startup's ex-ante entry expected profit can be written as

$$
\begin{aligned}
\Pi^e_c &= x^c_e\,\widetilde{\beta}_c\,\mathbb{E}\Big[\,V^c\left(\left\{q'_j\;\Delta^{H\prime}_j\;\Delta^{F\prime}_j\;\;\Delta^{G\prime}_j\right\}\right)\,\Big]\;-\;\widetilde{\chi}^e\left(x^c_e\right)^{\widetilde{\psi}^e}\;\overline{q}_c\\
&= x^c_e\,\widetilde{\beta}_c\,\mathbb{E}\Big[\,A^c\left(\Delta^{H\prime}_j\;\Delta^{F\prime}_j\;\;\Delta^{G\prime}_j\right)q'_j+B^c\,\overline{q}'_c\,\Big]\;-\;\widetilde{\chi}^e\left(x^c_e\right)^{\widetilde{\psi}^e}\;\overline{q}_c\\
&= x^c_e\,\widetilde{\beta}_c\,\Big[\,A^c_{takeover}\,\overline{q}_c+B^c\,\overline{q}'_c\,\Big]\;-\;\widetilde{\chi}^e\left(x^c_e\right)^{\widetilde{\psi}^e}\;\overline{q}_c\,.
\end{aligned}
$$

Thus, the FOC gives us an optimal level of external innovation intensity for potential startup equals to

$$
\begin{aligned}
\frac{\partial\Pi^e_c}{\partial x^c_e}\;&:\widetilde{\psi}^e\,\widetilde{\chi}^e\left(x^c_e\right)^{\widetilde{\psi}^e-1}\;\overline{q}_c=\widetilde{\beta}_c\,\Big[\,A^c_{takeover}\,\overline{q}_c+B^c\,\overline{q}'_c\,\Big]\\
\Rightarrow\;x^c_e&=\left(\frac{\widetilde{\beta}_c}{\widetilde{\psi}^e\;\widetilde{\chi}^e}\right)^{\frac{1}{\widetilde{\psi}^e-1}}\left(\,A^c_{takeover}\;+B^c\,\frac{\overline{q}'_c}{\overline{q}_c}\,\right)^{\frac{1}{\widetilde{\psi}^e-1}}
\end{aligned}
\tag{TA4.79}
$$

TA5 Complete List of Equations

TA5.1 Labor Market

$$\frac{w_H}{\overline{q}_H} = \theta(1-\theta)^{\frac{1-2\theta}{\theta}} \left[\left(\frac{w_H}{\overline{q}_H}\right)^{-\frac{1-\theta}{\theta}} \frac{\mathcal{Q}_{HH}}{\overline{q}_H} + \left(\tau_{FH}\frac{w_F}{\overline{q}_F}\right)^{-\frac{1-\theta}{\theta}} \frac{\mathcal{Q}_{FH}}{\overline{q}_H} \right] (P_H)^{\frac{1}{\theta}}$$

$$\frac{w_F}{\overline{q}_F} = \theta(1-\theta)^{\frac{1-2\theta}{\theta}} \left[\left(\frac{w_F}{\overline{q}_F}\right)^{-\frac{1-\theta}{\theta}} \frac{\mathcal{Q}_{FF}}{\overline{q}_F} + \left(\tau_{HF}\frac{w_H}{\overline{q}_H}\right)^{-\frac{1-\theta}{\theta}} \frac{\mathcal{Q}_{HF}}{\overline{q}_F} \right] (P_F)^{\frac{1}{\theta}}$$

$$\widetilde{L}_H = (1-\theta)^{\frac{1}{\theta}} \left(\frac{w_H}{\overline{q}_H}\right)^{-\frac{1}{\theta}} \left[L_H (P_H)^{\frac{1}{\theta}} \frac{\mathcal{Q}_{HH}}{\overline{q}_H} + L_F (P_F)^{\frac{1}{\theta}} (\tau_{HF})^{-\frac{1}{\theta}} \frac{\mathcal{Q}_{HF}}{\overline{q}_H} \right]$$

$$\widetilde{L}_F = (1-\theta)^{\frac{1}{\theta}} \left(\frac{w_F}{\overline{q}_F}\right)^{-\frac{1}{\theta}} \left[L_F (P_F)^{\frac{1}{\theta}} \frac{\mathcal{Q}_{FF}}{\overline{q}_F} + L_H (P_H)^{\frac{1}{\theta}} (\tau_{FH})^{-\frac{1}{\theta}} \frac{\mathcal{Q}_{FH}}{\overline{q}_F} \right]$$

$$L_H = \overline{L}_H - \widetilde{L}_H$$

$$L_F = \overline{L}_F - \widetilde{L}_F$$

TA5.2 Prices and Quantities

$$p_j^H = \begin{cases} \frac{1}{1-\theta} \frac{w_H}{\overline{q}_H} & \text{for domestic absorption} \\ \frac{1}{1-\theta} \tau_{FH} \frac{w_F}{\overline{q}_F} & \text{for imports} \end{cases}$$

$$p_j^F = \begin{cases} \frac{1}{1-\theta} \frac{w_F}{\overline{q}_F} & \text{for domestic absorption} \\ \frac{1}{1-\theta} \tau_{HF} \frac{w_H}{\overline{q}_H} & \text{for imports} \end{cases}$$

$$y_j^{HH}(q_j) = (1-\theta)^{\frac{1}{\theta}} (P_H)^{\frac{1}{\theta}} L_H \left(\frac{w_H}{\overline{q}_H}\right)^{-\frac{1}{\theta}} q_j$$

$$y_j^{HF}(q_j) = (1-\theta)^{\frac{1}{\theta}} (P_F)^{\frac{1}{\theta}} L_F \left(\tau_{HF}\frac{w_H}{\overline{q}_H}\right)^{-\frac{1}{\theta}} q_j$$

$$y_j^{FF}(q_j) = (1-\theta)^{\frac{1}{\theta}} (P_F)^{\frac{1}{\theta}} L_F \left(\frac{w_F}{\overline{q}_F}\right)^{-\frac{1}{\theta}} q_j$$

$$y_j^{FH}(q_j) = (1-\theta)^{\frac{1}{\theta}} (P_H)^{\frac{1}{\theta}} L_H \left(\tau_{FH}\frac{w_F}{\overline{q}_F}\right)^{-\frac{1}{\theta}} q_j$$

$$\ell_j^{HH}(q_j) = (1-\theta)^{\frac{1}{\theta}} (P_H)^{\frac{1}{\theta}} L_H \left(\frac{w_H}{\overline{q}_H}\right)^{-\frac{1}{\theta}} \frac{q_j}{\overline{q}_H}$$

$$\ell_j^{HF}(q_j) = (1-\theta)^{\frac{1}{\theta}}\,(P_F)^{\frac{1}{\theta}}\,L_F\left(\tau_{HF}\frac{w_H}{\overline{q}_H}\right)^{-\frac{1}{\theta}}\;\frac{q_j}{\overline{q}_H}$$

$$\ell_j^{FF}(q_j) = (1-\theta)^{\frac{1}{\theta}}\,(P_F)^{\frac{1}{\theta}}\,L_F\left(\frac{w_F}{\overline{q}_F}\right)^{-\frac{1}{\theta}}\;\frac{q_j}{\overline{q}_F}$$

$$\ell_j^{FH}(q_j) = (1-\theta)^{\frac{1}{\theta}}\,(P_H)^{\frac{1}{\theta}}\,L_H\left(\tau_{FH}\frac{w_F}{\overline{q}_F}\right)^{-\frac{1}{\theta}}\;\frac{q_j}{\overline{q}_F}$$

$$\pi^{HH}(q_j) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}}L_H\left(\frac{w_H}{\overline{q}_H}\right)^{-\frac{1-\theta}{\theta}}\;(P_H)^{\frac{1}{\theta}}}_{\equiv\pi^{HH}}\;q_j$$

$$\pi^{HF}(q_j) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}}L_F\left(\tau_{HF}\;\frac{w_H}{\overline{q}_H}\right)^{-\frac{1-\theta}{\theta}}\;(P_F)^{\frac{1}{\theta}}}_{\equiv\pi^{HF}}\;q_j$$

$$\pi^{FF}(q_j) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}}L_F\left(\frac{w_F}{\overline{q}_F}\right)^{-\frac{1-\theta}{\theta}}\;(P_F)^{\frac{1}{\theta}}}_{\equiv\pi^{FF}}\;q_j$$

$$\pi^{FH}(q_j) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}}L_H\left(\tau_{FH}\;\frac{w_F}{\overline{q}_F}\right)^{-\frac{1-\theta}{\theta}}\;(P_H)^{\frac{1}{\theta}}}_{\equiv\pi^{FH}}\;q_j$$

$$Y_H = (1-\theta)^{\frac{1-2\theta}{\theta}}\,(P_H)^{\frac{1-\theta}{\theta}}\,L_H\left[\left(\frac{w_H}{\overline{q}_H}\right)^{-\frac{1-\theta}{\theta}}\;\mathcal{Q}_{HH} + \left(\tau_{FH}\frac{w_F}{\overline{q}_F}\right)^{-\frac{1-\theta}{\theta}}\;\mathcal{Q}_{FH}\right]$$

$$Y_F = (1-\theta)^{\frac{1-2\theta}{\theta}}\,(P_F)^{\frac{1-\theta}{\theta}}\,L_F\left[\left(\frac{w_F}{\overline{q}_F}\right)^{-\frac{1-\theta}{\theta}}\;\mathcal{Q}_{FF} + \left(\tau_{HF}\frac{w_H}{\overline{q}_H}\right)^{-\frac{1-\theta}{\theta}}\;\mathcal{Q}_{HF}\right]$$

$$P_H = P_F = 1 \quad \text{(under Assumption 1)}$$

TA5.3 Aggregate External Innovation Intensity

For $c \in \{H, F\}$,

$$\overline{x}^c = x^c\,\mathcal{F}_c + x_e^c\,\mathcal{E}_c\,,$$

and

$$\overline{x} = \overline{x}^H + \overline{x}^F \ .$$

TA5.4 International Trade

TA5.4.1 Value of Trade

Value of differentiated goods imported from foreign country to home country:

$$\int_{j \in \mathcal{J}_{FH}} p_j^H y_j^{FH} dj = (1-\theta)^{\frac{1-\theta}{\theta}} (P_H)^{\frac{1}{\theta}} L_H \left(\tau_{FH} \frac{w_F}{\overline{q}_F} \right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{FH}$$

Value of differentiated goods exported from home country to foreign country:

$$\frac{P_H}{P_F} \int_{j \in \mathcal{J}_{HF}} p_j^F y_j^{HF} dj = \frac{P_H}{P_F} (1-\theta)^{\frac{1-\theta}{\theta}} (P_F)^{\frac{1}{\theta}} L_F \left(\tau_{HF} \frac{w_H}{\overline{q}_H} \right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{HF}$$

Value of final good traded (in a perspective of home country):

$$P_H X_H = (1-\theta)^{\frac{1-\theta}{\theta}} \left[(P_H)^{\frac{1}{\theta}} L_H \left(\tau_{FH} \frac{w_F}{\overline{q}_F} \right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{FH} - \frac{P_H}{P_F} (P_F)^{\frac{1}{\theta}} L_F \left(\tau_{HF} \frac{w_H}{\overline{q}_H} \right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{HF} \right]$$

$$\Leftrightarrow X_H = (1-\theta)^{\frac{1-\theta}{\theta}} \left[(P_H)^{\frac{1-\theta}{\theta}} L_H \left(\tau_{FH} \frac{w_F}{\overline{q}_F} \right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{FH} - (P_F)^{\frac{1-\theta}{\theta}} L_F \left(\tau_{HF} \frac{w_H}{\overline{q}_H} \right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{HF} \right],$$

$$X_F = -X_H$$

Recall: $\overline{q}_H = \mathcal{Q}_{HH} + \mathcal{Q}_{FH}$, and $\overline{q}_F = \mathcal{Q}_{FF} + \mathcal{Q}_{HF}$. In a BGP, $\mathcal{Q}_{cc}$ and $\mathcal{Q}_{c'c}$ should grow at the same rate as $\overline{q}_c$ or one should completely dominate another. We can see this from the analytic expression for equilibrium wages w_H and w_F.

TA5.4.2 Trade Cutoffs

$$\underline{\Omega} \equiv \left(\frac{1}{\tau_{FH}} \right)^{\frac{1-\theta}{\theta}} \left[\frac{w_H}{\overline{q}_H} \left(\frac{w_F}{\overline{q}_F} \right)^{-1} \right]^{\frac{1-\theta}{\theta}}$$

$$\overline{\Omega} \equiv (\tau_{HF})^{\frac{1-\theta}{\theta}} \left[\frac{w_H}{\overline{q}_H} \left(\frac{w_F}{\overline{q}_F} \right)^{-1} \right]^{\frac{1-\theta}{\theta}}$$

TA5.5 Other Macroeconomic Variables

$$C_H = w_H \overline{L}_H + \Pi_H + \widetilde{\Pi}_H + G_H$$

$$C_F = w_F \overline{L}_F + \Pi_F + \widetilde{\Pi}_F + G_F$$