How Big are the Gains from International Financial Integration?†

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Abstract

The literature has shown that the implied welfare gains from international financial integration are very small. We revisit the existing findings and document that welfare gains can be substantial if capital goods are not perfect substitutes. We use a model of optimal savings that includes a production function where the elasticity of substitution between capital varieties is less then infinity, but more than the value that would generate endogenous growth. This production structure is consistent with empirical estimates of the actual elasticity of substitution between capital types, as well as with the relatively slow speed of convergence documented in the growth literature. Calibrating the model, our results are that welfare gains from financial integration are equivalent to a 9% increase in consumption for the median developing country, and up to 14% for the most capital-scarce. These gains rise substantially if capital’s share in output increases even modestly above the baseline value of 0.3, and remain large even if inflows of foreign capital after integration are limited to a fraction of the existing capital stock.

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1 Introduction

How big are the gains from international financial integration? Given the crises of the emerging market countries after their liberalizations, and the current global financial meltdown, one wonders if there are any gains at all from the trade in international assets. Indeed, the literature so far has shown that the implied welfare gains from financial integration are very small. We reexamine the potential benefits of integration by incorporating the tools of endogenous growth theory into an otherwise standard neo-classical model of consumption and savings.

The essential idea of our paper is that the welfare gains increase when capital varieties within a country are not perfect substitutes, as is the case in the typical production functions used so far in this literature. Once we allow for an elasticity of substitution between capital varieties of less than infinity, the potential gains of financial integration become quite large. Our simple model of capital varieties lies between the neo-classical model (with infinite substitution of capital varieties) and the endogenous growth literature (which makes a knife-edge assumption implying that substitution is very low). In our more realistic intermediate setting, financial integration benefits a country by providing access to scarce capital, even though it has no effect on the long-run rate of technological progress.

Financial integration generates a welfare gain, relative to autarky, because capital flows in immediately to bring the rate of return down to the world rate, which allows for a permanently higher level of consumption. This benefit is larger the longer it would have taken in autarky to reach the world rate. Once capital types are not perfect substitutes, the marginal product of any single type is less sensitive to the size of the aggregate stock of capital. As capital accumulates, the rate of return falls more slowly than in the standard neo-classical model, and the time it takes in autarky to reach the world rate is extended. Hence, the welfare gain of integrating is larger once capital types are imperfect substitutes.

Several recent empirical studies indicate the elasticity of substitution between capital types cluster around values of 3–4. In our calibrations, elasticities in this range imply a welfare gain of integration equivalent to a 9% permanent increase in consumption, on average, for developing countries with the median capital/output ratio. For those countries that are very capital scarce, with capital/output ratios of one or lower, the welfare gains are equivalent to a 14% permanent increase in consumption on average. If we allow for capital’s share in output to be as high as 0.40, which is well within the observed range of values, then the gains are as large as 23% for the median country and 34% for the most capital-scarce.

Allowing for imperfect substitution between types of capital or intermediate goods is commonly used in studies of both economic growth and trade. Romer (1990) builds his original endogenous growth model with varieties of intermediate goods, and explicitly mentions that these can be thought of as different capital types. Later endogenous growth models dealing with scale effects, such as Young (1998), Peretto (1998), and Dinopoulos and Thompson (1998), all rely on varieties of intermediate goods. Voigtländer and Voth (2006) use the idea of different capital varieties explicitly in their model of long-run growth, while Acemoglu and Guerrieri (2008) study structural change and accumulation using different varieties of intermediate goods produced directly using capital.

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1 See Goolsbee (2004), Goolsbee and Gross (2000), and Chun and Mun (2003).
Different varieties of capital and intermediate goods have also been used to study the relationship of trade and growth, as in Grossman and Helpman (1989, 1990), Rivera-Batiz and Romer (1991), and Young (1991). More recently, Broda, Greenfield, and Weinstein (2006) use a similar production structure to ours to actually estimate the elasticity of substitution between intermediate good varieties, which includes capital goods. They find elasticities of 3–4, similar to the more direct estimates, from a sample that includes 73 countries.

Our contribution is to bring the concept of imperfect substitution between capital varieties to the study of welfare gains from financial integration, which has typically relied on standard neo-classical production functions with infinite substitutability. There have been two approaches in the literature to quantify the welfare gains from financial integration. The first approach focuses on the risk sharing mechanism. International asset trades allow agents to pool idiosyncratic risk and smooth consumption. Starting with Lucas’s work (1987), there is an extensive literature that shows, in a representative agent framework with transitory shocks, the welfare gains from consumption smoothing upon integration are very small. Lucas himself finds a welfare cost of fluctuations that is around 0.042% of average consumption. Although subsequent work showed that with permanent shocks and/or a feedback effect on industrial structure welfare gains via the risk sharing mechanism of integration can be as big as 20%, once calibrated to different countries the gains stay around 1% on average (Obstfeld, 1994; Kalemli-Ozcan, Sorensen and Yosha, 2001).

The second approach, and the one more closely related to our paper, focuses on capital scarcity. This channel will work through reducing the cost of capital, accelerating capital accumulation and raising consumption due to an influx of foreign capital. Gourinchas and Jeanne (2006) (GJ hereafter) were the first ones to have investigated the implied welfare gains of this channel. They find very small gains, equivalent to a permanent increase in consumption of 1.74% for the median of their sample of developing countries. They find that, although foreign integration provides an influx of capital to a country, the gain from this is small due to the fact that countries would have converged to the world rate of return very quickly in autarky. Their results arise, in part, from their standard assumption that capital is perfectly substitutable. In GJ, the marginal product of capital falls very quickly to the assumed world rate, even in autarky, as a country accumulates capital. Hence integration provides little benefit. Allowing for even a small departure from perfect substitution of capital types generates much larger implied welfare gains.

We explore the plausibility of our results by looking at the actual marginal product of capital over the period 1960–2000. Across a sample of 102 countries, marginal products of capital decline very slowly, so that by 2000 most of the original differences across countries from 1960 remain. Compared to this data, the neo-classical model of perfect capital substitution predicts that marginal products of capital would have converged within only 10–15 years (i.e. by 1975) to a common world rate of approximately 5.42%, far too fast compared to the data. On the other hand, our model that allows for imperfect substitution is consistent with the general pattern of slow declines in the marginal product of capital over time.

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2There is an enormous literature that tries to quantify the effect of financial integration on growth. See Kose, Prasad, Rogoff, and Wei (2009) for an extensive survey.

3It will still be the case that gains are relatively larger for the countries that are further away from their steady states.

4This result holds even after we apply the relative price adjustment to marginal product of capital as suggested by Caselli and Feyrer (2007).
The observations on rates of return link our work to the work on convergence in output per capita. As has been commonly noted, the estimated speed of output convergence is approximately 2–2.5% per year (see Barro and Sala-i-Martin, 1992). This slow convergence holds across countries, U.S. states and the OECD (Barro and Sala-i-Martin, 1992), Japanese prefectures, European regions, and Canadian provinces (Sala-i-Martin, 1996a,b), Indian provinces (Cashin and Sahay, 1996), and Swedish counties (Persson, 1997).

The standard neo-classical model predicts convergence speeds that are far faster, but allowing for imperfect substitution of capital varieties generates speeds of output convergence consistent with the rates of 2–2.5% per year.

To proceed we first lay out a simple production structure involving capital varieties and relate it to the existing work. Following that, we put this production structure into a standard model of intertemporal utility maximization, and use that model to evaluate the welfare gains of financial integration, similar to GJ. Calibrating that model, we find much larger welfare gains than are implied by existing work, and we then discuss the plausibility of these results in light of existing data on marginal products of capital, and under several alternative assumptions.

2 Capital Varieties and Financial Integration

To build towards a model in which we can evaluate the welfare effects of financial integration, we begin with the following production function, which is also used by Broda, Greenfield, and Weinstein (2006) in the context of imperfect substitutability of consumption goods where they estimate the role of product variety in trade and growth.

\[ Y_t = (A_t L_t)^{1-\alpha} \left( \sum_{i=0}^{M_t} x_{it}^\varepsilon \right)^{\alpha/\varepsilon} \] (1)

in which various varieties of capital, \( x_i \), with a constant elasticity of substitution of \( 1/(1 - \varepsilon) \), are combined with labor, \( L \), to produce the final good \( Y \). Note that, given this production function, labor will earn \( 1 - \alpha \) of output, while the various capital varieties earn \( \alpha \).

This production function nests versions used across both the endogenous growth and financial integration literatures. If we allow \( \varepsilon = 1 \), then the elasticity of substitution between capital varieties is infinite, and the production function reduces to the standard neo-classical model with the capital stock equal to the sum of the various varieties. Alternatively, if we allow \( \varepsilon = \alpha \), for an elasticity of substitution equal to roughly 1.4 given a value of \( \alpha = 0.3 \), then we have the standard expanding product-variety model found in Romer (1990) or Grossman and Helpman (1991).

Similar to those authors, we consider a case where each capital variety is produced by a single monop-
olistically competitive firm. The firms produce these varieties by using units of the final good, so that the aggregate capital stock can be considered as the amount of foregone consumption in the economy. More precisely,

\[ K_t = \sum_i x_{it}. \]  

We will, as is common, make the assumption that the firms are identical, and therefore that each firm’s \( x_{it} = x_t \). This implies that \( K_t = M_t x_t \). Putting this together with the production function above yields the following

\[ Y_t = K_t^{\alpha/\epsilon} (A_t L_t)^{1-\alpha} x_t^{\alpha-\alpha/\epsilon} \]  

which shows the relationship of aggregate output to the total capital stock.

The value of \( x_t \), however, is still to be determined. Here, the assumptions one makes will have various effects on the presence of endogenous growth and/or scale effects in the economy. The original endogenous growth models of Romer (1990) and Grossman and Helpman (1991) not only used the assumption that \( \epsilon = \alpha \), but that the nature of firms is such that \( x_t \) is constant over time. This leads to scale effects, which is at odds with the data presented by Jones (1995). Next generation models by Young (1998), Peretto (1998), and Dinopoulous and Thompson (1998) all provided a way of eliminating the scale effect. In general, these models relied on making the cost of innovation (for our purposes, the cost of entry) increasing with the the scale of the economy, so that \( x_t \) was also increasing in the scale of the economy.

To avoid scale effects and their implications we adopt a similar strategy to that used by Voigtländer and Voth (2006) in addressing this issue. Details can be found in the appendix. Basically, by setting the fixed cost of entry for a capital-variety producer to be proportional to \( A_t L_t \), and with an appropriate choice of units, we have that the optimal firm decision will be to produce \( x_t = A_t L_t \) for each variety. This implies that the amount of each variety per efficiency unit is constant, but the capital employed of each type grows along with efficiency and population.

As mentioned, assuming that the fixed costs work in this manner is done to prevent any scale effects from arising, which we want to eliminate for several reasons. First there is little empirical evidence consistent with scale effects (see Jones, 1995). Secondly, we are trying here to establish the role that financial integration can have in alleviating capital scarcity, and to focus on this alone we want to eliminate the possibility that financial integration will contribute directly to the long-run growth rate. Notice that, if it did raise the growth rate, welfare gains would be even bigger and hence we are essentially calculating a lower bound to the benefits of integration. Mechanically speaking, our assumption that fixed costs are proportional to \( A_t L_t \) ensures that there is a well-defined balanced growth path.

With \( x_t = A_t L_t \) due to the firm decision, the aggregate production function can be written as

\[ Y_t = K_t^{\alpha/\epsilon} (A_t L_t)^{1-\alpha/\epsilon} \]  

which features an elasticity of output with respect to capital of \( \alpha/\epsilon \), even though capital’s share of output is equal to \( \alpha \). If we assume that \( \epsilon = 1 \), this reduces to the standard neo-classical production function, in which
the share paid to capital is equal to the elasticity of output with respect to capital.

Alternatively, if we assume that $\epsilon = \alpha$, then we reach an “AK” style production function, which will allow for endogenous growth. Again, in examining financial integration, we will not be interested in such an extreme assumption. Rather, we will simply consider values such that $\alpha < \epsilon \leq 1$ and their implications for the potential welfare gain of integration. So while we are using the framework of an endogenous growth model, we do not need to incorporate the starkest assumptions of these models for our findings to hold.

An important implication of the production function in (4) is that the convergence speed of output per capita towards steady state will depend on $\alpha/\epsilon$. Given this production function, the convergence speed can be approximated in a Solow model of constant savings rates, as

$$\left(1 - \frac{\alpha}{\epsilon}\right) (g + n + \delta)$$

where $g$ is the exogenous rate of technological growth, $n$ is population growth, and $\delta$ is the depreciation rate.

In a neo-classical model with $\epsilon = 1$ (implying infinite substitutability between capital types) and $\alpha = 0.3$, the convergence rate is predicted to be about 5.56% per year, assuming values of $g = 0.012$, $n = 0.0074$, and $\delta = 0.06$, as in GJ. This is nearly triple the empirical estimates of the convergence rate, which generally cluster in the 2-2.5% range.

However, note that once $\epsilon$ is not constrained to be equal to one, it is possible to rationalize the observed convergence speeds. In particular, if the value of $\epsilon$ is as low as 0.45, implying an elasticity of substitution between capital varieties of about 1.8, then the convergence speed would be 2.5% per year, matching the results documented by Barro and Sala-i-Martin (1992), Sala-i-Martin (1996a, 1996b), and Durlauf, Johnson, and Temple (2005).

The rate of convergence towards steady state in an economy will turn out to be a crucial determinant of the implied welfare gains of financial integration. By incorporating capital varieties into our model of production, we will be able to more accurately capture the empirical convergence speeds while also maintaining the assumption that capital earns a reasonable share of output.

### 2.1 Households and Maximization

We have a flexible production structure in place, but for welfare calculations we must specify utility and the behavior of households. Here we follow very closely to the work of GJ mentioned in the introduction. Their work implicitly assumed that the elasticity of substitution for capital varieties was infinite, or $\epsilon = 1$. We use their optimization framework combined with an exogenously varying value of $\epsilon$ so that we can provide comparable welfare estimates under different assumptions regarding the elasticity of substitution between capital varieties.

Households are infinitely-lived and have a utility function of

$$V = \sum_{t=0}^{\infty} \beta^t (1 + n)^t u(c_t)$$

(6)
where $\beta \in (0, 1)$ is the time discount factor, $n$ is the growth rate of the population and $u(c_t)$ is the utility of consumption in period $t$. For our purposes, we will assume that $u(c_t) = c_t^{1-\sigma}/(1-\sigma)$, a constant relative risk aversion utility function with $\sigma > 0$, or alternatively that $u(c, t) = \ln(c_t)$.

Production takes place as described above, but we add exogenous growth in productivity and population. The exogenous growth in productivity implies that steady-state growth is similar in the long-run across economies. We make this assumption, as do GJ, so that we can focus exclusively on the role that integration plays in addressing capital scarcity, as opposed to any effect of financial integration on productivity directly.

Specifically, $L_{t+1} = (1+n)L_t$ and $A_{t+1} = (1+g)A_t$. If we denote productivity and population normalized variables with a hat, $\hat{x}_t = x_t/(A_tL_t)$, then we can write the dynamic budget constraint for each economy as

$$\hat{k}_{t+1}(1+n)(1+g) = \hat{k}_t(1-\delta) + \hat{y}_t - \hat{c}_t \tag{7}$$

where $\delta$ is the depreciation rate of capital. Note that with firm profits equal to zero, all of output ($\hat{y}$) is paid out as either wages or returns to capital. Hence, a household’s dynamic budget constraint will be identical to the above.

Drawing on the production function specified in equation (4), we have that output per efficiency unit is described by

$$\hat{y}_t = \hat{k}_t^{\alpha/\epsilon}. \tag{8}$$

The return on a unit capital is determined as follows,

$$R_{t+1} = \frac{\alpha Y_t}{K_t} = \alpha \hat{k}_t^{\alpha/\epsilon-1} + 1 - \delta. \tag{9}$$

Note that households only account for the share of output, $\alpha$, that they earn, but the relationship of $R_{t+1}$ to the capital stock per efficiency unit depends upon the overall elasticity, $\alpha/\epsilon$. If we considered the optimization problem from the perspective of a central planner, then they would internalize the role of capital varieties, and would perceive the social rate of return to capital as being equal to $\alpha/\epsilon$ times $Y_t/K_t$. This would imply that the central planner’s optimal solution would be to save more than in the decentralized case.

Utility maximization delivers the Euler equation relating consumption over adjacent periods as

$$\hat{c}_{t+1} = \hat{c}_t \left(\beta R_{t+1}\right)^{1/\sigma}. \tag{10}$$

At a steady state $\hat{c}_{t+1} = \hat{c}_t$ and thus the steady state return to capital is

$$R^* = \frac{(1+g)^{\sigma}}{\beta} \tag{11}$$

and note that this does not depend on either the capital share or the capital elasticity. The long-run return on capital depends only upon the patience parameter and the long-run growth rate of technology. Incorporating some kind of capital externality does not alter the long-run interest rate, it simply slows down the rate at
which \( R_{t+1} \) converges to \( R^* \).

Given this return, the long-run steady state level of capital per efficiency unit is

\[
\dot{k}^* = \left( \frac{\alpha}{R^* + \delta - 1} \right)^{1/(1-\alpha/\epsilon)}
\]

and the capital/output ratio is

\[
k^* \frac{y^*}{y^*} = \frac{\alpha}{R^* + \delta - 1}.
\]

Note that the capital/output ratio is also not affected by the size of \( \epsilon \), and allowing for capital varieties does not imply the existence of inconceivable capital/output ratios in the long run.

### 2.2 Welfare from Financial Integration

We follow the work of GJ, and presume that all countries share an identical value of \( R^* \), which is the presumed world rate of return on capital. This is done to focus on the role of capital scarcity, as opposed to differences in fundamental growth rates \( (g) \) or patience \( (\beta) \). Thus any welfare gains we find will be due only to the ability of financial integration to alleviate capital scarcity, and not through any change in fundamental parameters in the economy. Given this, we describe how the economy will work in two distinct situations: integration and autarky.

Under integration, capital flows into the country immediately to assure that \( R_{t+1} = R^* \). Given this and the Euler equation, this implies that consumption per capita grows at exactly the rate \( g \) each period. Combining this fact with the countries inter-temporal budget constraint leads the following solution for initial consumption

\[
c^\text{int}_0 = \left[ R^* - (1+n)(1+g) \right] A_0 \hat{k}_0 + (1-\alpha) A_0 \hat{k}^* \frac{\alpha}{\epsilon}
\]

which states that initial consumption depends not only on the country’s initial stock of capital, \( \hat{k}_0 \), but on the present discount value of future wages, which depend upon the value \( \hat{k}^* \).

With initial consumption and knowing the growth rate of consumption, utility can be directly calculated as

\[
U^\text{int} = \frac{R^*}{R^* - (1+n)(1+g)} \left( \frac{c^\text{int}_0}{\epsilon} \right)^{1-\sigma}
\]

where we’ve assumed that initial population is normalized to one.

Turning to the autarky case, we do not have such simple analytical answers. In autarky, the household takes initial capital, \( \hat{k}_0 \), as given, and then plans the optimal path of consumption and saving. We can solve for consumption numerically, and then calculate the value of \( U^\text{aut} \) directly from this.

Again, to match previous work we look at the Hicksian equivalent variation, \( \mu \), the percentage increase in a country’s consumption that brings welfare under autarky up to the level of welfare under integration. This is

\[
\mu = \left( \frac{U^\text{int}}{U^\text{aut}} \right)^{1/(1-\sigma)} - 1
\]
when $\sigma > 1$ and $\mu = \exp((1 - (1 + n)\beta)(U^{int} - U^{aut})) - 1$ if we have log utility.

2.3 Calculating Welfare Gains

To actually calculate $\mu$, we need to specify several parameters and initial values. We will begin with the assumption that $\epsilon = 1$, or that capital is perfectly substitutable. This provides the baseline against which we can compare alternatives, and will match the original results of GJ.

To proceed, we require values for several other parameters. Again, to stay consistent with prior research, we follow GJ in choosing these. The time preference rate is set to $\beta = 0.96$, $\sigma = 1$ for log utility, depreciation is set to $\delta = 0.06$, and the initial value of $A_0$ is set to one. The growth rates of population and TFP are set to the long-run U.S. values, so that $n = 0.0074$ and $g = 0.012$. Finally, capital’s share in output is set to $\alpha = 0.3$.

The initial value of $\hat{k}_0$ is found by specifying the initial capital/output ratio, $\hat{k}_0/\hat{y}_0$ and solving for $\hat{k}_0$. GJ, in their examination of a set of developing countries, find that the median capital-output ratio in 1995 was 1.4, with a value of 1.0 at the 10th percentile and 2.1 at the 90th. We will focus on these three initial values, as they allow us to explore the welfare gains for countries that are at various levels of capital scarcity. For reference purposes, the chosen parameter values imply that the steady-state capital/output ratio is equal to 2.63, and the steady-state return to capital is $R^* = 1.054$.

Table 1 shows the value of $\mu$, in percents, as we vary the value of $\epsilon$ and the initial capital/output ratio in Panel A. The first line, with $\epsilon = 1$ (implying an infinite elasticity of substitution between capital types), replicates the results of GJ. In particular, for a country with an initial capital/output ratio of 1.4, the implied welfare gain of financial integration is 1.74%, the number they report. For a country with the lower initial ratio of 1, the welfare gain is larger, but is still only 3.31 percent. For a country at the 90th percentile of the capital/output ratio, at 2.1, the gain is negligible at 0.26 percent.

As we move down the rows of table 1, we have decreased the value of $\epsilon$ while keeping capital’s share of output equal to 0.3. For $\epsilon = 0.75$, for example, the implied elasticity of substitution between capital types is 4.0, and the elasticity of output with respect to capital ($\alpha/\epsilon$) is 0.4.

As can be seen, once we increase $\epsilon$ to 0.75, the welfare gains for each of the three initial capital/output situations rise. The increase is substantial. For the most capital-scarce countries, the welfare gain of integration is now over 9 percent, while even for the median countries the welfare gain is approximately 5.74%. For those countries with initial capital/output ratios of 2.1, the welfare gain is still small, but is nearly six times larger than it was in the standard neo-classical setting.

The welfare gains continue to increase rapidly as the value of $\epsilon$ falls. For the case where $\epsilon = 0.67$, the elasticity of substitution between capital varieties is 3.0. In this case the welfare gains are approximately 14% for the most capital-scarce countries, almost 9% for the median countries, and about 2.5% for those that are capital-abundant. With $\epsilon = 0.6$, the gains are 21%, 13%, and 4%, respectively, for the three initial conditions. Raising $\epsilon$ to 0.5, for an implied elasticity of substitution between capital varieties of 2.0, leads to even larger gains of 48%, 30%, and 9% for the three initial situations.

Panel B of table 1 repeats these calculations, only now assuming that capital’s share in output, $\alpha$, is
actually equal to 0.4. The work of Gollin (2002), suggests that there is not any systematic relationship between labor shares and GDP per capita once adjustments for self-employed labor are made, but that does not necessarily imply that labor shares are identical across countries. From his data on employee/workforce ratios, the implied capital share (i.e. one minus the labor share) is as large as 0.52 for Bolivia and Botswana, 0.51 for Mauritius, 0.50 for Ecuador, 0.44 for Norway, and 0.40 for Portugal. It appears quite plausible that labor shares could be larger than the typical 0.3 assumed in the literature.

Once we make this adjustment to capital’s share, the overall gains to integration rise. Simply adjusting the elasticity of substitution between capital varieties to be 5.0 rather than infinity implies welfare gains on the order of 26% for the most capital-scarce countries, nearly 18% for the median situation, and even 8% for those countries that have a relatively high capital/output ratio of 2.1. As the value of \( \epsilon \) is set lower in the following rows, the implied welfare gains increase dramatically. For \( \epsilon = 0.75 \), so that the elasticity of substitution between capital types is equal to 4.0, the welfare gains are about 34%, 23%, and 11% for the three initial conditions, each significant gains based solely on integration.

What panel B shows is that as capital’s share rises, the gains to integration become substantial, even in the case of the neo-classical model with perfect capital substitution (\( \epsilon = 1 \)). Once we allow for even a slight imperfection in substitutability, such as with \( \epsilon = 0.8 \) for an elasticity of substitution of 5.0, the welfare gains are on the order of 26% for the most capital-scarce countries. Given Gollin’s (2002) data, it’s quite possible that capital shares in developing countries are well above the 0.3 typically assumed in this literature, and this alone increases substantially the implied welfare gains of integration.

2.4 How Large is the Elasticity of Substitution?

From the results in table 1, it is clear that the potential gains from financial integration can be large, provided that the elasticity of substitution between capital varieties is less than infinity. There is evidence that capital is, in fact, not perfectly substitutable across varieties. Chun and Mun (2003) use data from 41 U.S. industries, and find that the (Allen) elasticity of substitution between IT equipment and non-IT equipment is roughly 4.7, on average. For durable goods producers, the elasticity is 4.4, and in services the elasticity is 4.5. For “IT-intensive” industries, they document an even lower elasticity of 1.2. These values are slightly higher than the ones found by Goolsbee (2004), who examines the elasticity across types of mining, construction, and farm equipment varieties. He finds an elasticity of 4.1 when using all types, and an elasticity of 3.0 for mining equipment alone. Drilling down even further, Goolsbee and Gross (2000) document that within airlines, the elasticity of substitution between types of capital (i.e. aircraft) is only 1.5.

Broda, Greenfield, and Weinstein (2006) estimate elasticities of substitution across all intermediate goods. While not a direct measure of capital good substitutability, they find values as low as 2.3 for the U.S., and as high as 5.0 for Canada. Breaking down by types of intermediates, the median elasticity of substitution for commodity goods is 3.8, but 3.0 for goods with a U.S. reference price, and 3.3 for the differentiated goods that are not in the first two categories. These values are all lower than the previously mentioned estimates, but remain in the same range of elasticities between 3–4. In this range, the welfare gains of financial integration
are large, as table 1 documents. With a capital share of 0.3, and an elasticity of substitution of 4.0, the gains are three times larger than prior work would suggest, and are equivalent to a nearly 10% permanent increase in consumption for the most capital scarce countries. For an elasticity of substitution between capital types of 3.0, the gains are on the order of 14%. If the capital share in output is closer to 0.4, as the data indicate is plausible, then the gains are likely to be equivalent to a nearly 25% permanent increase in consumption.

Once we have broken the link between capital’s share and the capital elasticity, a key insight of endogenous growth theory, it becomes quite plausible that financial integration can deliver impressive gains in welfare to capital-scarce countries. The mechanism at work is that as the elasticity of output with respect to capital rises, the loss of consumption necessary in autarky to reach steady state becomes very large. Integration allows a country to avoid this large sacrifice of consumption by providing the necessary capital stock immediately. Even the long-run loss of consumption necessitated by having to make interest payments on that foreign capital is not enough to offset the immediate gains to consumption.

2.5 What is Driving the Gains?

The implied welfare gains of integration are dictated by assumptions about how a country would operate in autarky. The basic intuition is that the faster that a country reaches the steady state rate of return to capital in autarky, the lower the implied gain to integration.

\[ R \]

\[ R_0 \]

\[ R^w \]

Figure 1: Rates of Return under Different Scenarios

Notes: This figure plots stylized paths of the rate of return under different scenarios. From an initial value of $R_0$, a financially integrated country would have a path of the domestic rate exactly equal to $R^w$. If that country were in autarky, though, the baseline neo-classical model would predict a path as in curve A. Using an elasticity of substitution between capital varieties would slow down convergence, as in curve B. Finally, if one allowed $\epsilon = \alpha$, then we have an endogenous growth model, as in curve C, and the rate of return never reaches the world rate.

The size of the welfare gain from integration can be inferred from figure 1 as the distance between the
autarky path of rates of return and the world rate, denoted $R^w$. Imagine a country that has an initial rate of return of $R_0$, and borrows an amount of money on the world market at $R^w$. They gain $R_0$ in marginal output, but will only have to pay $R^w$ in interest, gaining higher consumption and increased utility in the transaction. The country could continue to do this each period, but the gains will decline as the rate of return naturally converges towards $R^w$ over time due to their own domestic savings.

For the situation depicted by curve A, the neo-classical case, the convergence happens very quickly, and the gains of borrowing internationally dwindle very quickly. The welfare gain of integration is small, as found by GJ. However, if $\epsilon < 1$, and capital varieties are not infinitely substitutable, then the implied rate of return in autarky declines more slowly, as in curve B. This implies that the gain of having access to foreign capital remains large for a longer time. A limiting case is when $\epsilon = \alpha$, and we have an “AK” model, where the rate of return never declines, and conceptually the welfare gains of integration are infinite, as in curve C. Regardless, as capital gets less substitutable, and $\epsilon$ falls, convergence slows down and the welfare gains increase, something seen in table 1.

We can provide some evidence that, in fact, a situation like curve B in figure 1 seems appropriate. To do this, we examine the observed marginal product of capital for a sample of 102 countries starting in 1960. The idea, starting at this point, is that we can observe how $R_t$ evolves over time for a set of countries, and see whether the neo-classical model, $\epsilon = 1$, matches the data.

Using the same sample of countries as GJ, we calculate an initial value for the rate of return in 1960 as

$$R_{1960} = 1 - \delta + MPK_{1960} = 1 - \delta + \frac{Y_{1960}}{K_{1960}}$$

(17)

where the depreciation rate is set to 6% per year and $\alpha$ is set to 0.3. The value of $Y_{1960}$ is taken from the Penn World Tables 6.1, and the value of $K_{1960}$ is calculated by a perpetual inventory method using investment data from the Penn World Tables as well. Note that for this calculation we do not need any information on $\epsilon$. All we are using is the assumption that capital earns 30% of output.\(^6\) While this measure of the return to capital actually available on the open market is crude, it has the value of being directly comparable to the marginal product of capital that comes out of our calibrated model. In that sense, then, it will allow us to compare the ability of the model to match the observed data.

Now, we look at the time series data on the rate of return. We plot the values of $R_{t+1}$ from 1961 to the year 2000, calculated in exactly the same manner as was $R_{1960}$ as given in equation (17), using data from the Penn World Tables for output and investment to derive $Y_t$ and $K_t$ in every year, again assuming that $\alpha = 0.3$, and not requiring any assumption regarding $\epsilon$.

Figure 2 plots these calculated rates of return for each of the 102 countries in our sample, by year. As can be seen, the initial spread of returns in 1960 declines only slightly over the forty years of data we have. Countries appear to be converging, but only very slowly, towards a long-run steady state. Note that we

\(^6\)Specifically, the initial size of $K_{1960}$ is calculated as $K_{1960} = I_{1960}(1 - \delta)/(\delta + g) + I_{1960}$ where $g$ is the growth rate of aggregate output from 1960-1970. This method will tend to under-state the size of the capital stock in $K_{1960}$, meaning that initial rates of return are relatively large. This bias will work against us finding large welfare gains, as it will appear that rates of return converge to long-run levels very quickly. Estimating $g$ using growth rates from 1950-1960 and starting our capital series with $K_{1950}$ for the 45 countries for which this data is available only strengthens our findings.
Notes: The figure displays the observed rates of return on capital for all 102 countries in the sample, by year. The return is the marginal product of capital calculated from observed capital stock and output data, as described in the text. The assumed world rate, equal to 1.0542, is denoted by the horizontal line.

have not restricted the sample to countries that were closed to financial flows. Thus figure 2 likely overstates the speed at which rates of return would fall in autarky, as some of what we see in the figure is driven by cross-border capital flows that reduce the variation between countries over time.\footnote{The figure looks similar after adjusting the MPK’s using the relative price of investment goods, as in Caselli and Feyrer (2007). This result is available upon request.}

This is useful because We can compare the actual data in figure 2 to the predictions of the neo-classical model, setting $\epsilon = 1$ and using the observed capital stocks in 1960 as the initial conditions. Again, $\alpha = 0.3$, and we use the model described in the previous section to extract the optimal path of consumption, and hence the rate of return, by year, for each country. The predicted values of $R_{t+1}$ are plotted in figure 3.

As can be seen in the figure, using the assumption that $\epsilon = 1$ implies very fast convergence of rates of return to the assumed steady state value of 1.054. In 1960 there is a wide spread of rates of return, running from around 1 to nearly 2 (an implied return of 100%). Very quickly, though, the predicted autarky rates of return converge towards the long-run world rate of 1.0542. By 1975, there is essentially no variance remaining in the predicted rate of return, as every country is assumed to be very close to this long-run steady state. Given figure 1, we see that there is little scope for welfare gains, regardless of how large is the initial autarky rate of return. $\mu$ will be small even for countries that begin with a very large value of $R_{1960}$.

The implication of figure 3 is that assuming $\epsilon = 1$ - as in the neo-classical model - leads to predictions inconsistent with the observed path of rates of return from 1960 to 2000. In particular, the standard neo-classical model assumes that countries reach their steady states in an unbelievably short amount of time.
Figure 3: Neo-classical Predicted Autarky Rates of Return, 1960-2000

Notes: The figure displays the predicted rate of return on capital for all 102 countries in the sample, by year, assuming that \( \alpha = 0.3 \) and \( \epsilon = 1 \). The return is the marginal product of capital calculated from the neo-classical model of optimal savings assuming the country is in autarky, as described in the text. The assumed world rate, equal to 1.0542, is denoted by the horizontal line.

The message of figure 2, however, is that country’s rates of return only converge very slowly.

By setting \( \epsilon < 1 \) in our production structure, we raise output’s elasticity with respect to capital, and make the rate of return decline more slowly as countries save. Thus the predictions of the optimal savings model will be closer to the observed rates of return. In short, as \( \epsilon \) gets smaller, the elasticity of substitution between capital varieties gets smaller, and the convergence speed of the rate of return to the world rate slows down. Allowing \( \epsilon < 1 \) within our model of optimal savings would lead to a predicted path of returns closer to those actually observed in figure 2. The mechanism is that as \( \epsilon \) goes down, the marginal product of any particular variety of capital falls slowly with the addition of new capital to the economy. Thus the aggregate rate of return on capital falls more slowly, similar to what is seen in figure 2.

### 2.6 Alternative Rates of Return

In addition to the speed at which rates converge, the welfare gain of integration will also depend upon the size of the steady state rate of return. In our original calculations, we presumed that each country would reach a common steady state with a rate of return of \( R^* = 1.054 \). If the actual steady state value is higher, then the potential gains of integration become smaller. In figure 1, this would be captured by a shift upward of the \( R^* \) line, which shrinks the gap between it and the other curves.

To address the impact of this, we consider an alternative set of calculations in which the steady state return is higher, implying a lower steady state capital stock. To parameterize this, we modify the original
calibrations, now setting $\beta = 0.93$, making countries more impatient. This leads to a steady state rate of return of 1.088. Note that we are assuming that the world will only invest in a country at this higher rate, perhaps reflecting structural differences. Hence, even if the country integrates, the long-run rate of return it has access to is 1.088. The welfare gains are thus driven by capital scarcity alone, and not through a permanent ability to borrow internationally at a low rate.

In panel B of table 2, the results for $\mu$ are reported under the same combination of values for $\epsilon$ and the initial capital/output ratio, only changing the steady state rate to 1.088. For the countries that are most capital-scarce, with capital/output ratios of 1, the welfare gains of integration are smaller than in table 1, but still remain quite large. With $\epsilon = 0.6$ and an elasticity of substitution of 2.5, for example, the gain of integrating is equivalent to a 21% increase in consumption. For the median case, the gains are also smaller than before, but if the elasticity of substitution is less than four, there still are meaningful welfare benefits to integration.

For the most capital-abundant countries, one can see that the welfare gains are not applicable in most cases. That is because the rate of return to capital with a $k/y$ ratio of 2.1 is already below the steady state rate of 1.088, and we presume that they have a steady state rate of return already below this level. In other words, these are likely to be economies that function well enough to access world markets closer to the developed rate of 1.0542.

### 2.7 Incremental Integration

Our results suggest the potential for very large welfare gains from full financial integration. That is, we compare autarky to the alternative where foreign capital flows in immediately to a country in amounts sufficient to drive down the rate of return to the world rate. However, most real experiences with financial integration have proceeded more cautiously, allowing foreign capital to flow in only limited amounts over time.

As an alternative way of understanding the welfare gains of integration, we consider here the welfare gain of an incremental integration in which foreign capital is allowed to enter a country, but only as a constant fraction of the installed capital base. In each period, a country allows foreign capital in the amount equal to $\phi \times \hat{k}_t$, where $0 \leq \phi \leq 1$. The inflow of foreign capital is thus limited, and we can examine the welfare gain of different values of $\phi$.

To implement this, consider the dynamic budget constraint of the model economy, which will now be modified to be as follows:

$$\hat{k}_{t+1} (1+n)(1+g) + \hat{c}_t + \phi\hat{k}_{t+1}(1+n)(1+g)R^w = \hat{k}_t(1-\delta) + ((1+\phi)\hat{k}_t)^{\alpha/\epsilon} + \phi\hat{k}_t. \quad (18)$$

The economy has access to $\phi \hat{k}_t$ in additional capital from foreign inflows. This foreign capital will produce additional output (which depends on the current MPK), and offsets a portion of the depreciation that occurs naturally. In the following period, the economy must pay the foreign investors at the rate $R^w$ (the world rate) for the use of the capital. The additional foreign capital loosens the dynamic budget constraint so long
as the marginal product of capital is higher than $R^w$. As the economy approaches the steady state, there will be no net gain from adding foreign capital.

We can evaluate the welfare gain from incremental integration by computing the optimal path of consumption, allowing for $\phi k_t$ inflows, and using this to find $U^{int}$. Then we can compute the value of $\mu$ as in equation (16). In table 3, we show the value of $\mu$ across values of $\epsilon$, assuming that $\alpha = 0.3$ and that the economy is capital-scarce initially, with $k_0/y_0 = 1.0$. The world rate is presumed, again, to be identical to the steady state value of 1.0542.

The welfare gain was calculated under three different values of $\phi$: 1%, 5%, 10%, and 20% of the current capital stock. These values provide useful benchmarks for examining financial integration, as they are similar magnitudes to observed values for private foreign capital flows relative to capital stocks. Lane and Milesi-Ferretti’s (2007) report the value of private foreign capital flows across countries. Comparing these flows to capital stocks, in constant dollars from the Penn World Tables, we can infer reasonable values of $\phi$. From the early 1980’s to the late 1990’s, Argentina had private foreign capital flows equal to 2.6% of their total capital stock on average, per year. These flows reached a maximum of 8.6% in 1993. In Botswana during a similar time period, the average flow of foreign capital per year was equal to 2.6% of the capital stock, peaking at a value of 11.9% in 1980. For Brazil, the average was 2.3% with a maximum of 4.7%. Several Asian economies experienced similar patterns of inflows. China had a an average value of 1.7% per year, with a maximum of 4.6%. Malaysia averaged inflows equal to 3.3% of their capital stock, peaking at 7.3%, and Singapore was similar with an average of 2.8% and a maximum of 4.2%.

There are several examples of much larger inflows, however. Hong Kong average inflows equal to 16% of the capital stock in the late 1990’s, peaking at 26% in 2000. Ireland averaged 4.3%, but peaked at inflows equal to 45% of capital in 1998. Very large inflows are certainly within the realm of possibility.

Table 3 shows the potential gains to allowing in these limited flows of capital. In the case where capital is perfectly substitutable, with $\epsilon = 1$, the gains range from 0.17% to 3.01%, depending on how large are the foreign capital flows. As can be seen, allowing in capital equivalent to 20% of the existing capital stock is very close to the welfare gain of full integration (3.31%) found in table 1 with $\epsilon = 1$.

Again, as we allow for imperfect substitution between capital types, the implied welfare gains rise. For elasticities of substitution of 3.0 or 4.0 ($\epsilon$ equal to 0.67 and 0.75, respectively), the welfare gains are as high as 5.88% or 7.35% when allowing foreign capital flows equivalent to 20% of the current capital stock. Even with capital flows of only 10% of current capital, the gains are on the order of 3.07% to 3.85%. These are obviously smaller than those from the full integration investigated in table 1, but the gains are larger than even a full integration when capital is assumed to be completely substitutable. The results suggest that even marginal adjustments to the ability of foreign capital to enter a country can have distinct welfare advantages.

3 Conclusion

We show that welfare gains from international financial integration can be as large as a 14% permanent increase in consumption for the most capital scarce countries, and 9% for a developing country with the
median capital/output ratio. With a reasonable increase in the share of income accruing to capital, these
gains can be as high as 34% for the capital scarce countries and 24% for the median. These estimate
are an order of magnitude larger than the gains calculated by previous works. Our paper departs from
this literature by allowing capital goods to be imperfect substitutes, as is commonly assumed in models of
endogenous growth and trade following the work of Romer (1990) and Grossman and Helpman (1991), and
as supported by empirical estimates from several sources.

By allowing for imperfect substitution of capital types, the production structure allows us to separate
capital’s share of output from the elasticity of output with respect to capital, which are assumed to be
identical in the standard neo-classical model. Our model allows for capital’s share to be 0.3–0.4, while also
being consistent with existing empirical evidence on countries slow speed of convergence to steady state.

Although we use the production structure of endogenous growth models, we do not incorporate the
additional assumptions necessary in those models that create endogenous growth in steady state. Rather,
our work focuses only on the gains from integration in alleviating capital scarcity, disregarding the endogenous
productivity benefits such as knowledge spill-overs and increased know-how that foreign capital may bring.
As such, our estimates likely represent a lower bound for the gains of integration.

Our work show that developing nations have much to gain from the process of financial integration, in
the sense that they will be able to support higher levels of consumption now and in the immediate future,
as compared to the decades they may take to reach those levels by remaining in autarky.
4 Appendix A: Firm Behavior

In the model of imperfect capital substitution, each variety is provided by a single monopolistically competitive producer. Given the original production function in equation (1), firms face the inverse demand curve for variety $i$ of

$$p_{it} = \alpha (A_{it}L_{it})^{1-\alpha} \left( \sum_{i} x_{it}^{\epsilon} \right)^{\alpha/\epsilon - 1} x_{it}^{\epsilon-1}. \quad (19)$$

Firm profits are

$$\pi_{it} = p_{it}x_{it} - R_{it}x_{it} - F_{it} \quad (20)$$

which shows that firms earn $p_{it}x_{it}$ in revenue, and must pay $R_{it}$ to produce a unit of capital, while also being required to pay a fixed cost of $F_{it}$ to produce. For simplicity, we presume that the fixed cost must be paid each period, so that one can think of firms as being one-period lived.

Taking into account the inverse demand curve, firms will optimize over the amount that they will produce, which results in the optimal decision of

$$x_{it}^* = \left( \frac{\alpha \epsilon}{R_{it}} (A_{it}L_{it})^{1-\alpha} \left( \sum_{i} x_{it}^{\epsilon} \right)^{\alpha/\epsilon - 1} \right)^{1/(1-\epsilon)}. \quad (21)$$

This incorporates the assumption that individual firms take the aggregate supply of capital goods (the summation term) as a given. Note that, given that each firm is identical, each variety $i$ is produced in precisely the same amount.

Free entry implies that firms will enter until profits are driven to zero. This will occur when the fixed costs exactly offset the profits made due to the markup over marginal cost producing $x_{it}^*$. As mentioned in the text, we make an assumption regarding fixed costs that ensures the model will not display scale effects. In particular, let

$$F_{it} = \alpha (1 - \epsilon) \frac{A_{it}L_{it}}{k^{1-\alpha/\epsilon}} \quad (22)$$

which shows that fixed costs are proportional to effective labor, $A_{it}L_{it}$, which is the mechanism that prevents scale effects from arising. As the economy grows extensively, fixed costs are higher, and this will counteract the increased profits available to firms due to increased demand for their variety. Scaling the fixed cost by capital per efficiency unit is necessary to ensure the presence of a balanced growth path, and simply implies that fixed costs are rising with $A_{it}L_{it}$ once the economy is in steady state.
References


Table 1: Welfare Gains of Integration, by Capital/Output Ratio and Capital’s Share.

Assumed Steady State Return: $R^* = 1.0542$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Elas. of subs.</th>
<th>Production function</th>
<th>$\mu$ (in percent)</th>
<th>Panel A: $\alpha = 0.3$</th>
<th>Panel B: $\alpha = 0.4$</th>
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</thead>
<tbody>
<tr>
<td>$\epsilon$</td>
<td>$1/(1-\epsilon)$</td>
<td>$\alpha/\epsilon$</td>
<td>$k/y = 1.0$</td>
<td>$k/y = 1.4$</td>
<td>$k/y = 2.1$</td>
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<td>0.60</td>
<td>48.16</td>
<td>30.04</td>
<td>8.90</td>
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</table>

Notes: The table reports the welfare gain, $\mu$, of financial integration relative to autarky. The assumed steady state return to capital in a country is 1.0542, which is also the assumed rate of return that the country is assumed to face in world markets. The welfare gain is measured as the percent permanent increase in consumption that is equivalent to allowing in foreign capital in the limited amounts. $\epsilon$ is the coefficient on the CES function for capital types in the production function in equation (1). The value $\alpha/\epsilon$ is the coefficient on capital in the aggregate production function in (4).
Table 2: Welfare Gains of Integration, Alternative Steady State Return

Assumed Steady State Return: \( R^* = 1.088 \)

| Parameter Elas. of subs. function for initial condition: | \( \epsilon \) \( 1/(1 - \epsilon) \) \( \alpha/\epsilon \) \( k/y = 1.0 \) \( k/y = 1.4 \) \( k/y = 2.1 \) |
|------------------------------------------------------|------------------|------------------|------------------|------------------|------------------|
| Capital varieties: Production \( \mu \) (in percent) | \( \epsilon \) \( 1/(1 - \epsilon) \) \( \alpha/\epsilon \) \( k/y = 1.0 \) \( k/y = 1.4 \) \( k/y = 2.1 \) |
| 1.00 \( \infty \) 0.30 2.86 0.99 n/a |
| 0.75 4.0 0.40 8.96 3.88 n/a |
| 0.60 2.5 0.50 20.63 9.91 n/a |
| 0.50 2.0 0.60 45.76 21.65 n/a |
| 0.42 1.7 0.70 114.05 49.66 n/a |

Notes: The table reports the welfare gain, \( \mu \), of financial integration relative to autarky. The assumed steady state return to capital in a country is 1.088, which is also the assumed rate of return that the country is assumed to face in world markets. The welfare gain is measured as the percent permanent increase in consumption that is equivalent to allowing in foreign capital in the limited amounts. \( \epsilon \) is the coefficient on the CES function for capital types in the production function in equation (1). The value \( \alpha/\epsilon \) is the coefficient on capital in the aggregate production function in (4).
Table 3: Welfare Gains of Integration, by Size of Foreign Capital Inflows

Assumed Steady State Return: $R^* = 1.0542$
Initial capital/output ratio: $k/y = 1.0$

<table>
<thead>
<tr>
<th>Capital varieties:</th>
<th>Production function for foreign capital inflows as percent of $\tilde{k}$</th>
</tr>
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<tbody>
<tr>
<td>Parameter $\epsilon$</td>
<td>$\alpha/\epsilon$</td>
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Notes: The table reports the welfare gain, $\mu$, of financial integration relative to autarky. The assumed steady state return to capital in a country is 1.0542, which is also the assumed rate of return that the country is assumed to face in world markets. The inflow of foreign capital under financial integration is limited to a constant proportion of the existing capital stock (the parameter $\phi$ in the paper), so that the rate of return does not immediately fall to the world rate, $R^w$. The calculation is described more fully in section 2.7. The welfare gain is measured as the percent permanent increase in consumption that is equivalent to allowing in foreign capital in the limited amounts. $\epsilon$ is the coefficient on the CES function for capital types in the production function in equation (1). The value $\alpha/\epsilon$ is the coefficient on capital in the aggregate production function in (4).