ELSEVIER

Journal of Development Economics 70 (2003) 103–118

JOURNAL OF Development ECONOMICS

www.elsevier.com/locate/econbase

A stochastic model of mortality, fertility, and human capital investment

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Abstract

This paper examines the relationship between fertility and human capital investment and its implications for economic growth, focusing on the effects of declining mortality. Unlike the existing literature, this paper stresses the role of uncertainty about the number of surviving children. If the marginal utility of a surviving child is convex, then there will be a precautionary demand for children. As the mortality rate and thus, uncertainty fall, this demand decreases. Furthermore, lower mortality encourages educational investment in children. The key result is that this empirically observed quality–quantity trade-off is realized only if uncertainty is incorporated into individual's optimization problem.

JEL classification: O40; I12; J11; J13
Keywords: Uncertainty; Precautionary demand; Quality–quantity trade-off; Growth

1. Introduction

Three major developments of the past century are the declines in mortality and fertility rates and the growth of educational investment. These events have occurred both

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PII: S0304-3878(02)00089-5

in the developed countries and in the developing countries.[1] The relationship between these three phenomena has nontrivial implications for economic growth.

Since increased life span implies a higher rate of return, declining child and youth mortality provides an important incentive to increase investment in the education of each child. Numerous researchers have emphasized that human capital accumulation is the prime engine for economic growth. They have not, however, rigorously investigated this particular mechanism through which increased survival chances promote growth by raising the human capital investment.

Historical and contemporary data show that mortality decline preceded the fertility decline in general.[2] As a result, countries have experienced a phase of increasing population growth rates followed by a phase of declining rates. This whole phenomenon is known as "demographic transition" and has important implications for the process of economic growth. Lucas (2002) argues that demographic transition and industrial revolution are linked events and what is new about 1800 is not technological change by itself but the fact that fertility increases no longer translated improvements in technology into increases in population. Therefore, understanding the causes of the fertility transition is crucial in terms of past, present, and future economic growth.

One explanation of the fertility transition is the reduction in infant and child mortality.[3] Previous literature models causality running from mortality to fertility as follows: if

[1] Although the likelihood of survival for all ages increased tremendously between 1780 and 1990, the most significant reduction in mortality was realized at infancy and childhood. In 1780, in Sweden, a newborn child had a 60% chance of living to age 20. By 1930, this figure had risen to 90%. Infant and child mortality fell approximately 60% between 1950 and 1990 in less developed regions (LDCs) of the world. In developed countries, fertility decline, which began by the end of the 19th century, was completed by World War II. During this period, the total fertility rate (TFR) declined from 5 children to 2.5 children. In the developing world, the fertility transition started around the 1950s, and over the past 40 years, TFR declined from 6 children to 3 children. The average number of years of schooling in England rose from 2.3 for the cohorts born between 1801 and 1805 to 9.1 for the cohorts born between 1897 and 1906. It rose even further to 14 for the 1974–1992 cohorts. In LDCs, gross secondary school enrollment increased from 17.1% in 1960 to 46.9% in 1990. See Livi-Bacci (1997), United Nations (1999), and Matthews et al. (1982).

[2] France is an exception, where fertility decline began early in the 19th century before the mortality had declined. The US also has declining fertility early in the 19th century.

[3] The other explanations of the fertility transition include the following. Becker (1981) proposes that parents decrease their fertility because of the increased opportunity cost of children due to higher wages given the assumption that substitution effect dominates at higher levels of income. Caldwell (1976) claims the decline for the need of old-age support from children as a result of development and modernization caused fertility to decline. Galor and Weil (1999, 2000) argue that as a result of increased technological progress, the returns to education increases, causing a quality–quantity trade-off and hence, a fertility transition. In another paper, Galor and Weil (1996) argue that higher wages for women raise the cost of children relatively more than they raise household income and lead to a reduction in the number of children that couples choose to have. Becker and Barro (1988) say the decline in fertility is a result of the aggregate consumption growth, and Becker et al. (1990) claim fertility declined due to the increase in the aggregate level of human capital. Azariadis and Drazen (1991), Brezis (2001), and Dahan and Tsiddon (1998) use the change in the structure of the economy, the role of the social classes, and the effect of income inequality as explanations for the fertility transition, respectively. For the developing countries, the dissemination of birth control methods is also proposed as an explanation for the decline in fertility. However, studies found that family planning programs explain only 10–40% of the decline in fertility in developing countries, and the rest of the decline is explained by the changes in desired fertility, i.e., number of children families wanted to have (Weil, 2001). Which of these explanations, including mortality decline, can explain a bigger fraction of the decline in fertility is still an open empirical question.

households are concerned about the number of surviving children, and if they desire a specific number of survivors, then a reduction in mortality may lead to a corresponding reduction in fertility. While this direct channel accounts for the significant portion of the reduction in fertility, it is an incomplete explanation since it cannot explain the reduction in the "net rate of reproduction" (NRR) and hence, the reduction in the population growth rate.[4]

Existing models of endogenous fertility, if they allow for mortality at all, do so in the context of certainty.[5] High mortality is modeled as reducing the fraction of children who survive to adulthood. Thus, these models ignore the uncertainty about the number of surviving children that is present in a high-mortality environment. Uncertainty has been found to be important in a number of other areas of economics, and this paper illustrates that it is also critical in the context of survival of children. Indeed, it is shown that a model without uncertainty produces results at variance with the data.

The model here is built on the effects of mortality decline on the fertility and education decision of parents. If the marginal utility of a surviving child is convex in the number of survivors, then there will be a precautionary demand for children. Thus, parents are prudent in the sense of Kimball (1990). Since parents choose the number of births before they know how many children will survive, in a high mortality environment, they will increase the number of births beyond the number required to produce the desired number of survivors in expectation. As the mortality rate and thus, uncertainty fall, this precautionary demand decreases, and so does the NRR. Second, lower mortality increases a child's expected life span, which encourages investment in the child's human capital. The resources allocated to additional investment in education are freed by the reduction in fertility. Thus, parents find it optimal to move along a quality–quantity frontier, having fewer children and investing more resources in each one. The key result is that this quality–quantity trade-off is realized only if uncertainty is incorporated into individual's optimization problem.

As a result, this paper makes two novel contributions. The first is to establish the positive link between increased survival and human capital investment in an endogenous fertility framework. Some of the previous literature found that changes in mortality had no effect on the optimal amount of education. The second contribution is a proposed mechanism under which the relationship between the mortality decline and the fertility decline can be causal. Uncertainty about child survival gives rise to a precautionary demand for children. Thus, exogenous reductions in mortality lead to a decline in fertility and eventually in population growth. These two contributions indicate that relying on the causality from mortality to fertility to explain the fertility transition is not at odds with the view that the fertility decline is due to a quality–quantity trade-off. Increased child survival and the quality–quantity trade-off are complementary explanations, and together they tell a more complete story of the fertility decline.

[4] The NRR is the number of daughters that each girl can be expected to produce, i.e., it is the factor by which the number of girls in each generation will increase. Thus, NRR shows by what factor population will grow over a generation.

[5] See Section 2 for the review of the literature.

The rest of the paper is structured as follows. Section 2 briefly presents the related literature. Section 3 introduces and solves the model under both certainty and uncertainty. Section 4 discusses the conclusion and extensions.

2. Empirical evidence and theoretical literature

Eckstein et al. (1998), in a study of Swedish fertility dynamics, show that the reduction in infant and child mortality is the most important factor explaining the fertility decline, while increases in the real wages can explain less than one-third of the fertility decline.[6]

There may be two different strategies at work that generate the fertility response.[7] First, the "replacement strategy," is the response of fertility to experienced deaths, where parents replace deceased children. Second, the "insurance strategy," or hoarding, is the response of fertility to expected deaths, where parents bear more children than their optimal number of survivors. If parents follow a replacement strategy, they can produce their target number of survivors with no error and a change in child mortality will have no effect on NRR. In the empirical studies using micro data, the estimated replacement effect is always smaller than 0.5 and generally, it is around 0.2. But only a replacement effect of 1 means a fully working replacement strategy.[8] To the extent that parents do not engage in a full-fledged replacement strategy, they also follow a partial insurance strategy; that is, they have a precautionary demand for children.

The earliest theoretical formulations is based on the "target fertility model," which provides the intuition for hoarding mechanism. This model implies that in order to have N surviving children, N/q children must be borne if the survival rate is q. Thus, an increase in q always reduces the number of births, whereas the number of surviving children would not change, since the decline in the number of births is exactly compensatory. However, this framework ignores the fact that children are economic goods and hence, they are costly. O'Hara (1975) and Ben-Porath (1976) tried to incorporate a budget constraint into their analysis, where an increase in the survival rate reduces the number of births only if demand for children is inelastic with respect to its price (the cost of a child). Nevertheless, the number of surviving children must increase given the fact that children are not Giffen goods. Thus, it could be optimal to have fewer births at a positive mortality rate than at a zero mortality rate, which is the opposite of hoarding. Indeed, Becker and Barro (1988) include mortality in their basic model of fertility and show that the decline in mortality lowers the cost of raising a survivor and thus, increases the demand for surviving children. This implies that births rise in response to a decline in mortality, which is not consistent with the data. Sah (1991) develops a stochastic discrete time model where he shows that the number of children produced by a couple declines as the mortality rate declines. This is the first theoretical paper that investigates this causal relationship in an uncertain environment.

[6] See also Galloway et al. (1998), Coale (1986), and Preston (1978).

[7] In general, fertility responds to the mortality decline with a lag. The main reason for this lag is that it takes parents some time to recognize that mortality has fallen.

[8] See Schultz (1997).

There has not been much work linking mortality to investments in education. Some researchers investigate the direct effect of mortality on education. Meltzer (1992) shows that mortality decline in Mexico from 1920 to 1965 has resulted in a 9.2% increase in the rate of return, which in turn implies a 20% increase in the enrollment rates. Ram and Schultz (1979) argue that improvements in mortality have been an important incentive to increase investment in education, and the post-war experience of India is consistent with this incentive. In a recent paper, Kalemli-Ozcan et al. (2000), calibrating their model by using returns to schooling estimates, show that mortality decline produces significant increases in schooling.[9]

3. The model

Consider an OLG model, where individuals within a generation have identical preferences. Members of generation t live for two periods: in the first period of life $(t-1)$, individuals consume a fraction of their parent's unit time endowment. In the beginning of the second period of life (t), individuals make a one-time fertility decision.

The preferences of the altruistic member of generation t are defined over second period's consumption, C_t, and the future income of the survivors, $N_t w_{t+1} h_{t+1}$, where N_t is the number of survivors, w_{t+1} is the future wage of a survivor per unit of human capital, and h_{t+1} is the human capital of a survivor. E_t denotes expectation as of time (t). The utility function for a member of generation t can be written as,[10]

$$U_t = \gamma \ln[C_t] + (1-\gamma) E_t\{\ln[N_t w_{t+1} h_{t+1}]\}. \tag{1}$$

Human capital production is given by

$$h_{t+1} = e_t^{\beta} h_t, \qquad 0 < \beta < 1, \tag{2}$$

where e_t is the education level of a child and h_t is the level of parental human capital.

Households choose the number of children, n_t, and the optimal amount of education to give to each child, e_t, where each child's survival is uncertain. These choices are subject to a constraint on the total amount of time, which is unity. Assuming a fixed time cost, $v \in (0,1)$, for every child, the time left for the household after the child-

[9] Some papers have investigated the effects of changes in life expectancy and/or child mortality both on fertility and human capital investment (see Ehrlich and Lui, 1991; Meltzer, 1992; Jones, 1999; Tamura, 2002). Uncertainty regarding the survival of children, however, is not part of any of these models, with the exception of Tamura (2002). Like this paper, Tamura (2002) adopts the expected utility maximization problem and the methodology to solve this problem from Kalemli-Ozcan (2000).

[10] This formulation, where parents get utility from the future income of survivors instead of the future utility, is adopted from Galor and Weil (2000). In that paper, the authors do not consider mortality, and hence, parents get utility from the future income of their children.

bearing cost is incurred is $1-vn_t$. This remaining time is divided between work to earn a wage income and educational investment.[11,12] Therefore, the budget constraint is

$$w_t h_t(1-(v+e_t)n_t) = C_t. \tag{3}$$

3.1. Benchmark case: optimization without uncertainty

To show the importance of uncertainty, I first examine a benchmark case with certainty. In this case, the random nature of the number of surviving children is completely ignored. Hence, the number of survivors are given by the expected number of survivors, $N_t = E_t(N_t) = n_t q$, where $q \in (0,1)$ is the survival probability of each child. Substituting the budget constraint into the utility function, the optimization problem of a member of generation t is

$$\{n_t, e_t\} = \text{argmax}\{\gamma \ln[w_t h_t(1-(v+e_t)n_t)] + (1-\gamma)\ln[n_t q w_{t+1} h_{t+1}]\},$$
$$\text{subject to: } (n_t, e_t) \geq 0. \tag{4}$$

The optimization with respect to n_t implies that the total time spent on children is a fixed fraction of the total time endowment, $1-\gamma$, and the remaining fraction, γ, is devoted to labor force participation,

$$n_t = \frac{1-\gamma}{v+e_t}. \tag{5}$$

The optimization with respect to e_t implies that the time spent educating children is negatively related to the number of children and positively related to the return to human capital investment,

$$e_t = \frac{\beta(1-\gamma)}{(\beta(1-\gamma)+\gamma)} \frac{(1-vn_t)}{n_t}. \tag{6}$$

[11] Notice that there can be two different scenarios regarding the educational investment. Education may be provided before or after the uncertainty about mortality is realized. This paper investigates the ex-ante case. The ex-post case, where education is provided after the uncertainty is resolved, is considered in Kalemli-Ozcan (2000), which yields results similar to those of this paper. This is important since some researchers have argued that most of the mortality decline has occurred in infancy, and therefore, a decline in mortality should not matter for the human capital investment decision, which comes later in life.

[12] Tamura and Sadler (2001) show that child mortality can effect human capital investment via a different channel, namely, differential treatment of children. Human capital investment can be specialized in a few children, not in all children. This would also work to decrease investments in the face of child mortality. However, uncertainty about child survival, in the sense of this paper, is not part of their model.

Solving two first-order conditions simultaneously implies

$$n_t = \frac{(1-\gamma)(1-\beta)}{v},$$

$$e_t = \frac{\beta v}{(1-\beta)}. \tag{7}$$

Proposition 1. *Under certainty, an exogenous increase in the survival probability has no effect on either fertility or human capital investment.*

Proof. Follows directly from Eq. (7). □

As a result, if human capital investment is the engine for growth, mortality decline will not be conducive to economic growth. Why does mortality decline have no effect on the optimal choices of the endogenous variables? Technically, the result follows from the fact that log utility implies unitary elastic demand for children with respect to their cost. The intuition is straightforward. With increased survival, the price of a surviving birth, v/q, declines, thus making the quantity of survivors relatively more desirable given the fact that children are normal goods. This price change has income and substitution effects. The income effect indicates that both consumption and the expected number of survivors will increase, which in turn implies that the number of children will rise. The substitution effect works in the opposite direction, meaning consumption will decline and the expected number of survivors will increase. With log utility, the income and substitution effects balance out, so consumption is not affected by the increase in q. Thus, the budget constraint in Eq. (3) implies the optimum number of children does not change, given the fact that the optimum amount of education does not change with q.[13]

Why doesn't the optimum amount of education change with q in a world with no uncertainty? Due to the decreased cost of survivors, parents want to increase the expected number of survivors by producing more children. However, the relative price of an educated survivor doesn't change. With high mortality, like 50%, in order to have one educated survivor parents must have two children and provide the same amount of education to each. However, if there is no mortality, then parents can have one child and educate him or her, and hence, they can have one educated survivor. Thus, there is no point in changing the optimal amount of education with an increase

[13] One can also perform a similar exercise with a CRRA utility function. With high elasticity of substitution, consumption decreases with an increase in q, which in turn implies $\mathrm{d}n_t/\mathrm{d}q > 0$ through Eq. (3), again given the fact that the optimum amount of education does not change with q. Note that with a CRRA utility function, such as $(C_t^{1-\sigma})/(1-\sigma)$, the parameter σ is the inverse value of intertemporal elasticity of substitution in consumption. Thus, the result $\mathrm{d}C_t/\mathrm{d}q < 0$ requires $\sigma < 1$. This parameter restriction also implies that children have an elastic demand with respect to their cost.

in q.[14] When uncertainty is allowed, there will be an additional effect (risk effect) which alters the results of Proposition 1, as shown in the next section.

What about the effect of increased survival on the population growth rate? The population growth rate can be written as

$$\frac{L_{t+1}}{L_t} - 1 = E_t(N_t) - 1 = n_t q - 1, \tag{8}$$

where L_t is the size of the population at time t.[15]

Proposition 2. *Under certainty, there is a positive effect of an increase in the survival probability on the expected number of survivors and hence, on the population growth rate.*

Proof

$$\frac{\mathrm{d}(L_{t+1}/L_t)}{\mathrm{d}q} = n_t = \frac{(1-\gamma)(1-\beta)}{v} > 0. \tag{9}$$

Eq. (9) simply follows from the fact that the optimum number of children does not change with q. This is what I call the "mean effect." □

3.2. Optimization with uncertainty

In this section, I incorporate uncertainty into the previous setup. As in the benchmark case, let q be the survival probability of each child, which is fixed over time. With uncertainty, N_t, the number of survivors, will be a random variable drawn from a binomial distribution. Thus, the probability that N_t out of n_t children will survive is

$$f(N_t; n_t, q) = \binom{n_t}{N_t} q^{N_t}(1-q)^{n_t - N_t} \qquad N_t = 0, 1, \ldots, n_t, \forall t. \tag{10}$$

Members of generation t choose the number of children, and the optimal amount of education to provide in order to maximize their expected utility as of time (t),

$$E_t(U_t) = \sum_{N_t=0}^{n_t} \{\gamma \ln[C_t] + (1-\gamma)\ln[N_t w_{t+1} h_{t+1}]\} f(N_t; n_t, q). \tag{11}$$

[14] In technical terms, for any type of utility, the change in q affects marginal rate of substitution between consumption and the number of children but does not affect the marginal rate of substitution between the number of children and the amount of education that is given to each child. This is due to the combined effect of the following two assumptions: the separability of the utility function and the multiplicative formulation of the second argument of the utility function.

[15] Note that due to the law of large numbers, there is no aggregate uncertainty even though there is individual uncertainty, and hence, the population growth rate is $E_t(N_t) - 1$.

This formulation implies that the number of children born and the number of surviving children are represented as nonnegative integers, which is a discrete representation.[16]

I use the Delta Method to approximate the utility around the mean of the binomial distribution. This approach allows me to incorporate the variance, which is nothing but the risk effect, in a tractable way. By using the Delta Method and taking expectations, I can rewrite the maximization of expected utility as,[17]

$$\{n_t, e_t\} = \operatorname{argmax}\left\{\gamma \ln[w_t h_t(1-(v+e_t)n_t)] + (1-\gamma)\ln[n_t q w_{t+1} h_{t+1}] - \frac{(1-\gamma)(1-q)}{2n_t q}\right\}, \quad \text{subject to}: \ (n_t, e_t) \geq 0. \tag{12}$$

The first-order condition with respect to e_t is same as in the certainty case,

$$e_t = \frac{\beta(1-\gamma)}{(\beta(1-\gamma)+\gamma)} \frac{(1-v n_t)}{n_t}. \tag{13}$$

The main difference between the certainty case and the uncertainty case is the optimization with respect to n_t. Here, the first-order condition with respect to n_t is nonlinear due to the expected utility maximization,

$$\frac{-\gamma(v+e_t)}{1-(v+e_t)n_t} + \frac{(1-\gamma)}{n_t} + \frac{(1-\gamma)(1-q)}{2qn_t^2} = 0. \tag{14}$$

Due to uncertainty, parents engage in a self-insurance strategy and overshoot their desired fertility. This "insurance effect" is nothing but the risk effect that is incorporated through the variance of the binomial distribution, which affects the optimization with respect to n_t, and hence the comparative statics. Thus,

Proposition 3. *When uncertainty is incorporated into the parents' optimization problem, an exogenous increase in the survival probability (a decline in mortality) causes them to engage in a quality–quantity trade-off.*

$$\frac{dn_t}{dq} < 0, \ \forall q,$$

$$\frac{de_t}{dq} < 0, \ \forall q. \tag{15}$$

[16] See Sah (1991).

[17] See Appendix A for details.

Proof. See Appendix B.[18] □

Comparing Proposition 3 to Proposition 1 reveals how significant the role of uncertainty is. The model also generates the stylized fact of the demographic transition, that is, population growth is a hump-shaped function of the survival probability.

Proposition 4. *Under uncertainty, at low levels of survival, an increase in the survival probability unambiguously raises the population growth rate, while at high levels of survival, an increase in the survival probability causes the population growth rate to decline if the returns to education are high enough.*

$$\frac{\mathrm{d}(L_{t+1}/L_t)}{\mathrm{d}q} > 0 \text{ if } q \cong 0,$$

$$\frac{\mathrm{d}(L_{t+1}/L_t)}{\mathrm{d}q} < 0 \quad \text{if } q \cong 1 \text{ and } \beta \cong 1, \quad \frac{\mathrm{d}^2(L_{t+1}/L_t)}{\mathrm{d}q^2} < 0, \ \forall q. \tag{16}$$

Proof. See Appendix B. □

When the survival probability is low, the mean effect (see Proposition 2) dominates, and therefore, the population growth rate increases with the increased survival rate. When the survival probability is high, although the population growth rate may increase due to the increased number of survivors, the negative response of fertility can offset this mean effect. Most of the quality–quantity trade-off literature assumed a linear human capital production function, meaning $\beta = 1$. Therefore, as evidence suggests, population growth is a concave function of the survival probability for any q, and if β is close to 1, it is a hump-shaped function.

As a result, this setup establishes the link from mortality to fertility and to human capital investment, and hence, it is enough to show the positive effect of mortality decline on economic growth. The higher human capital investment and the lower population growth will enhance economic growth. These forces arise as a result of quality–quantity trade-off decision of parents. However, this quality–quantity trade-off is realized only if uncertainty is incorporated into individual's optimization problem.

[18] Note that $\mathrm{d}n/\mathrm{d}q = \partial n/\partial$ uncertainty ∂ uncertainty$/\partial q$. Uncertainty comes from the variance, but high mortality does not necessarily mean high variance. Thus, the second partial does not have to be negative for all values of the survival probability, q. For a given number of children, the expected number of survivors (mean of the binomial distribution) always increases with a rise in the survival probability, but the variance of the binomial rises or falls according to the value of the survival probability. If the survival probability is bigger than 1/2, the variance falls with a rise in the survival probability. However, due to the Delta Method approximation, the second partial is negative for all values of q. This is consistent with the data. We don't see mortality rates that are higher than 50% in the data for historical populations. Thus, the survival probability we observe in the data is always higher than 1/2. This implies that an increase in the survival probability will always lower the variance in the data. Therefore, showing the negativeness of the total derivative in Proposition 3 implies that the first partial is positive as it should be, since this represents the precautionary demand.

4. Conclusion and extensions

Empirical studies show that there is a significant positive effect of human capital investment and a negative effect of fertility on economic development. Furthermore, researchers have found a positive correlation between life expectancy (or health proxies) and the rate of economic growth (Barro, 1997). However, there is not any study that provides direct evidence regarding the mechanism through which higher life expectancy promotes growth.

Based on these empirical findings, one such mechanism could be mortality decline working through the channels of education and fertility to enhance economic growth. Economic models will miss this point unless the role of uncertainty about the number of surviving children is fully incorporated. This omission is the primary reason that some of the previous literature has found results that are at variance with the data. This paper resolves this inconsistency between theory and the data by incorporating uncertainty into the individual's optimization problem. Here, individuals are prudent in the face of uncertainty about child survival, which causes a precautionary demand for children. As the mortality rate and thus, uncertainty fall, precautionary demand decreases, and hence, parents choose to move along a quality–quantity frontier.

In this paper, the survival probability has been assumed exogenous and fixed over time. However, both time-series and cross-sectional empirical studies have found that as income per capita in a country rises, mortality rates tend to fall. Based on this evidence, future research will involve endogenizing the survival probability. This can be done by considering the survival probability to be a concave function of income per capita. Then, this concave relation between the survival probability and income per capita results in a hump-shaped relation between the population growth rate and income per capita, since the population growth rate is a hump-shaped function of the survival probability as shown in this paper. This hump-shaped pattern of the population growth rate as a function of income per capita will have dynamical implications. In a stochastic world, depending on the levels of income and hence, the nature of improvements in mortality, a country can be trapped around a development trap type steady state or can grow forever. One can further include endogenous technological progress. In that case, an exogenous decline in mortality can serve as the basis for a unified growth model that describes the complete transition from an underdeveloped world to the modern growth era.[19]

Acknowledgements

I thank the referee, Oded Galor, Herschel Grossman, Harl Ryder, David Weil, and participants at numerous seminars and conferences for valuable comments and suggestions.

[19] A discussion of this is given in Galor and Weil (1999).

Appendix A

The Delta Method is a statistical method that has been used extensively in statistical contexts to solve similar problems. The first step of the Delta Method is a Taylor series approximation of the utility around the mean of the distribution. I am going to do a third-degree approximation around the mean $N_t = n_t q$, since higher order terms can be disregarded for simplicity.

$$U_t(N_t) \cong U(n_t q) + (N_t - n_t q)U_N(n_t q) + \frac{(N_t - n_t q)^2}{2!}U_{NN}(n_t q) + \frac{(N_t - n_t q)^3}{3!}U_{NNN}(n_t q). \tag{17}$$

From log utility, the above partial derivatives are

$$U_N = \frac{(1-\gamma)}{N_t}, \quad U_{NN} = -\frac{(1-\gamma)}{N_t^2}, \quad U_{NNN} = \frac{2(1-\gamma)}{N^3}. \tag{18}$$

Substituting Eq. (18) in Eq. (17) and taking expectations implies

$$E_t(U_t) = U_t(n_t q) + 0 - \frac{(1-\gamma)}{2(n_t q)^2} n_t q(1-q) + 0. \tag{19}$$

The first term is the utility function evaluated at the mean. The second term is zero since $E(N - nq) = 0$. The third term is the variance (second central moment) of the binomial, which is $E(N - nq)^2 = \sigma^2 = nq(1-q)$. The fourth term is zero since it is the third central moment, $E(N - \mu)^3 = 0$.[20]

Appendix B

Proof of Proposition 3. Multiplying Eq. (14) by n_t^2 and substituting e_t from Eq. (13) gives

$$G(n_t, q) = \frac{-n_t(vn_t\gamma + \beta(1-\gamma))}{1 - vn_t} + (1-\gamma)n_t + \frac{(1-\gamma)(1-q)}{2q} = 0, \tag{20}$$

which defines n_t implicitly.

Suppressing t subscript and using subscripts for partial derivatives

$$\frac{dn}{dq} = -\frac{G_g}{G_n}. \tag{21}$$

[20] See Casella and Berger (1990).

Eq. (20) can also be written as

$$\frac{(1-\gamma)(1-q)}{2q} = \frac{n(vn\gamma + \beta(1-\gamma))}{1-vn} - (1-\gamma)n. \tag{22}$$

Thus, LHS of Eq. (22) is only a function of q, and RHS of it is only a function of n.

$$\mathrm{LHS}_q(q) = G_q,$$

$$\mathrm{RHS}_n(n) = -G_n. \tag{23}$$

Given $0 < q \leq 1$, it is easy to show that LHS(q) is always negative

$$\mathrm{LHS}_q(q) = -\frac{(1-\gamma)}{2q^2} < \ 0 \ \forall q,$$

$$\mathrm{LHS}_{qq}(q) = \frac{(1-\gamma)}{q^3} > \ 0 \ \forall q,$$

$$\lim_{q \to 0} \mathrm{LHS}(q) = +\infty,$$

$$\lim_{q \to 1} \mathrm{LHS}(q) = 0. \tag{24}$$

Thus, Eq. (24) implies

$$G_q < 0 \ \forall q. \tag{25}$$

The budget constraint in Eq. (3) implies that $0 \leq n \leq (1/v)$. Then, it is easy to show that RHS(n) is always positive for the range of n that is relevant for finding an optimum. RHS(n) can be written as

$$\mathrm{RHS}(n) = \frac{n(vn - (1-\gamma)(1-\beta))}{(1-vn)}. \tag{26}$$

Taking the derivative with respect to n gives

$$\mathrm{RHS}_n(n) = \frac{vn(2-vn) - (1-\gamma)(1-\beta)}{(1-vn)^2}. \tag{27}$$

To determine the sign of Eq. (27), one has to evaluate the following:

$$\lim_{n \to 0} \mathrm{RHS}_n(n) = -(1-\gamma)(1-\beta) < \ 0,$$

$$\lim_{n \to 1/v} \mathrm{RHS}_n(n) = +\infty,$$

$$\mathrm{RHS}_{nn}(n) = \frac{2v(\gamma + \beta(1-\gamma))}{(1 - vn^3)} > \ 0 \ \forall n,$$

$$\lim_{n \to 0} \mathrm{RHS}(n) = 0,$$

$$\lim_{n \to 1/v} \mathrm{RHS}(n) = +\infty. \tag{28}$$

Thus Eq. (28) implies

$$-G_n > 0 \ \forall n. \tag{29}$$

Therefore Eqs. (25) and (29) together with Eq. (21) imply

$$\frac{\mathrm{d}n}{\mathrm{d}q} < 0. \tag{30}$$

Thus,

$$\frac{\mathrm{d}e}{\mathrm{d}q} = -\frac{\beta(1-\gamma)}{(\beta(1-\gamma)+\gamma)(n)^2}\frac{\mathrm{d}n}{\mathrm{d}q} > \ 0. \tag{31}$$

□

Proof of Proposition 4. Substituting $n_t = L_{t+1}/L_t/q$ and the optimal e_t from Eq. (13) into Eq. (14) and using the notation $P = P_{t+1} = L_{t+1}/L_t$ gives

$$\tilde{G}(P,q) = -2vP^2 + 2q(1-\beta)(1-\gamma)P + (1-\gamma)(1-q)(q-vP) = 0. \tag{32}$$

Evaluating this with implicit function theorem gives

$$\frac{\mathrm{d}P}{\mathrm{d}q} = -\frac{\tilde{G}_q}{\tilde{G}_P}, \tag{33}$$

Evaluating this gives

$$\frac{\mathrm{d}P}{\mathrm{d}q} = \frac{(1-\gamma)[2(1-\beta)P + vP + 1 - 2q]}{4vP - (1-\gamma)[2(1-\beta)q - (1-q)v]}. \tag{34}$$

When $q=0$, $P=0$ and $n=1/v$. Thus, Eq. (34) is unambiguously positive,

$$\frac{\mathrm{d}P}{\mathrm{d}q} = \frac{1}{v} > 0 \text{ if } q = 0. \tag{35}$$

When $q=1$, $P=n$ and $n=(1-\gamma)(1-\beta)/v$. Thus, Eq. (34) is ambiguous in sign,

$$\frac{\mathrm{d}P}{\mathrm{d}q} = \frac{(1-\gamma)[2(1-\beta)n + vn - 1]}{4vn - 2(1-\gamma)(1-\beta)} < \ 0 \text{ if } q = 1 \text{ and } \beta = 1. \tag{36}$$

Substituting $n=(1-\gamma)(1-\beta)/v$ (since $q=1$) gives the necessary condition for Eq. (36),

$$\frac{(1-\gamma)(1-\beta)}{v} < \frac{1}{2(1-\beta)+v}. \tag{37}$$

If $\beta=1$, or $v>(1-\gamma)(1-\beta)$ and $2(1-\beta)+v<1$, this equation is satisfied. This condition also implies having the maximum point of P as a function of q bigger than 1. □

Proof of Concavity. $(\mathrm{d}^2P)/(\mathrm{d}q^2)$ can be written as

$$\frac{\mathrm{d}^2P}{\mathrm{d}q^2} = \frac{G_{nn}(G_q)^2 q}{(-G_n)^3} + \frac{qG_{qq}}{-G_n} + \frac{2G_q}{-G_n}. \tag{38}$$

However, $qG_{qq}=-2G_q$, and $-G_n>0$ and $-G_{nn}>0$; thus,

$$\frac{\mathrm{d}^2P}{\mathrm{d}q^2} = \frac{G_{nn}(G_q)^2 q}{(-G_n)^3} < 0 \ \forall q. \tag{39}$$

□

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