International Macroeconomics: Lecture 1

Ethan Kaplan

Current Account Definition

- One definition: the net exports of goods and services during a year
  - What are goods?
  - What are services?

- Another definition: the change in the value of the net claims of a country on the rest of the world during a year
  - Why are the first and second definitions equal?
  - Are they equal bilaterally?
  - What about valuation effects?

Current Account: US Department of Commerce Definition

- Current Account (goods and services)
- Financial Account (assets)
- Capital Account (rare transfers of wealth: i.e. from migration, debt cancellation)

CA+FA+KA=0

Current Account: Missing Money Puzzle

- It turns out that the world is on net in deficit... Why?
  - Shipping
  - Interest Payments (Evidence?)

Current Account Determination: 2 Period Closed (I)

- Real Side Model, Country=Individual

- Utility Function:

$$U_i\left(C_1\right)+\beta U_i\left(C_2\right);\frac{\partial \mathrm{U_i}}{\partial \mathrm{C_t}}>0$$

- Income Endowments:

$$Y_1, Y_2$$

Current Account Determination: 2 Period Closed (II)

- Budget Constraint:

$$C_2 - Y_2 = (1+r)\left(Y_1 - C_1\right)$$

- or:

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

Current Account Determination: 2 Period Closed (III)

- Maximization problem:

$$\max_{C_1, C_2} U_i(C_1) + \beta U_i(C_2)$$

- subject to: $$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

- Closed Economy Equilibrium Condition (no storage):

$$C_1 \leq Y_1, C_2 \leq Y_2$$

Current Account Determination: 2 Period Closed (IV)

- Maximization problem:

$$\max_{C_1,C_2} U_i(C_1)+\beta U_i(C_2)-\lambda\left[C_1+\frac{C_2}{1+r}-Y_1-\frac{Y_2}{1+r}\right]$$

- FOCs: $\dfrac{\partial U_i(C_1)}{\partial C_1}=\lambda=(1+r)\beta\dfrac{\partial U_i(C_2)}{\partial C_2}$

- Equilibrium Condition: $\dfrac{\;\dfrac{\partial U_i(Y_1)}{\partial C_1}\;}{\;\dfrac{\partial U_i(Y_2)}{\partial C_2}\;}=(1+r)\beta$

Current Account Determination: 2 Period Closed (V)

- Solution of r: $$\frac{\dfrac{\partial U_i(Y_1)}{\partial C_1}}{\beta\dfrac{\partial U_i(Y_2)}{\partial C_2}} - 1 = r$$

- Current Account?

- Interpretation: $$\frac{\partial U_i(C_1^*)}{\partial C_1} = (1+r)\beta\frac{\partial U_i(C_2^*)}{\partial C_2}$$

Current Account Determination: Small Open (I)

- Maximization problem:

$$\max_{C_1,C_2} U_i(C_1)+\beta U_i(C_2)$$

- subject to: $$C_1+\frac{C_2}{1+r}=Y_1+\frac{Y_2}{1+r}$$

- Now r is taken as a parameter (and no restrictions on consumption being less than production in each period)

Current Account Determination: Small Open (II)

- For ease of exposition, assume that:

$$\beta = \frac{1}{1+r}$$

- subject to:

$$\frac{\partial U(C_1)}{\partial C_1} = \frac{\partial U(C_2)}{\partial C_2} \Rightarrow (\text{given } \frac{\partial U^2(C_1)}{\partial C_1^2} < 0) \;\; C_1^* = C_2^*$$

- Now we look at the definition of the current account and bonds (denoted by B):

$$CA_t = B_{t+1} - B_t$$

$$B_{t+1} = Y_t + (1+r)B_t - C_t$$

Current Account Determination: Small Open (III)

- Thus:

$$CA_2 = B_3 - B_2 = Y_2 + r\left(Y_1 - C_1\right) - C_2$$

- And:

$$Y_1 + \frac{Y_2}{1+r} - C_1 + \frac{C_2}{1+r} = 0 \Rightarrow Y_2 + r\left(Y_1 - C_1\right) - C_2 = -\left(Y_1 - C_1\right)$$

- Which implies:

$$CA_2 = -(Y_1 - C_1) = -B_2 = -CA_1$$

Current Account Determination: Small Open (IV)

- Going back to the budget constraint, knowing that consumption is equated across the two periods:

$$\frac{C(2+r)}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

- Thus:

$$C = \frac{(1+r)Y_1}{2+r} + \frac{Y_2}{2+r}$$

- Which implies:

$$CA_1 = Y_1 - C_1 = \frac{Y_1 - Y_2}{2+r}$$

Current Account Determination: Small Open (V)

- Comparative Statics:
  - Current Temporary Shock:

$$\frac{\partial CA_1}{\partial Y_1} = \frac{1}{2+r} > 0, \frac{\partial CA_2}{\partial Y_1} = -\frac{1}{2+r} < 0$$

  - Future Temporary Shock:

$$\frac{\partial CA_1}{\partial Y_2} = -\frac{1}{2+r} < 0, \frac{\partial CA_2}{\partial Y_2} = \frac{1}{2+r} > 0$$

Current Account Determination: Small Open (VI)

- Comparative Statics (Permanent Shock):
  - Express income as having permanent and temporary components:

$$Y_1 = Y_1^{'} + Y, Y_2 = Y_2^{'} + Y$$

  - Then, there is no effect of a permanent increase in wealth on the current account:

$$\frac{\partial CA_1}{\partial Y} = 0, \frac{\partial CA_2}{\partial Y} = 0$$

Current Account Determination: Small Open (VII)

- More generally (and with respect to r):
  - Take logs of Euler Equation:

$$\log\frac{dU(C_1)}{dC_1}-\log\frac{dU(C_2)}{dC_2}=\log(1+r)+\log\beta$$

  - Now totally differentiate:

$$\frac{U''(C_1)}{U'(C_1)}dC_1-\frac{U''(C_2)}{U'(C_2)}dC_2=d\log(1+r)$$

  - Now define the intertemporal elasticity of substitution:

$$\sigma(C)=-\frac{U'(C)}{CU''(C)}$$

Current Account Determination: Small Open (VIII)

- Now Remember:

$$d\log C = \frac{1}{C}dC \Rightarrow dc = Cd\log C$$

- Thus:

$$\frac{C_1 U''(C_1)}{U'(C_1)} d\log C_1 - \frac{C_2 U''(C_2)}{U'(C_2)} d\log C_2 = d\log(1+r)$$

- And:

$$\frac{1}{\sigma}\left(d\log C_2 - d\log C_1\right) = d\log(1+r)$$

$$d\log\frac{C_2}{C_1} = \sigma d\log(1+r)$$

Current Account Determination: Small Open (IX)

- Now we will look at the impact of an increase in the interest rate on first period consumption. We start by expressing the Euler equation in terms of first period consumption using the budget constraint:

$$U'(C_1) = \beta(1+r)U'\left[(1+r)(Y_1 - C_1) + Y\right]$$

- Now totally differentiate:

$$U''(C_1)dC_1 = \beta U'\left[(1+r)(Y_1 - C_1) + Y_2\right]dr +$$
$$(1+r)\beta(Y_1 - C_1)U''\left[(1+r)(Y_1 - C_1) + Y_2\right]dr - (1+r)^2\beta U''\left[(1+r)(Y_1 - C_1) + Y_2\right]dC_1$$

Current Account Determination: Small Open (X)

- Combine like differentials:

$$\left[U''(C_1)+(1+r)^2\beta U''(C_2)\right]dC_1=\left[\beta U'(C_2)+(1+r)\beta U''(C_2)(Y_1-C_1)\right]dr$$

- Divide differentials:

$$\frac{dC_1}{dr}=\frac{\beta U'(C_2)+(1+r)\beta U''(C_2)(Y_1-C_1)}{U''(C_1)+(1+r)^2\beta U''(C_2)}$$

Current Account Determination: Small Open (XI)

- Multiply by $\dfrac{C_2}{U'(C_2)}$:

$$\frac{dC_1}{dr} = \frac{\beta C_2 + (1+r)\beta \dfrac{U''(C_2)C_2}{U'(C_2)}(Y_1 - C_1)}{\dfrac{U''(C_1)C_2}{U'(C_2)} + (1+r)^2 \beta \dfrac{U''(C_2)C_2}{U'(C_2)}}$$

- Replace coefficient of int. subst.:

$$\frac{dC_1}{dr} = \frac{\beta C_2 - (1+r)\beta \dfrac{1}{\sigma}(Y_1 - C_1)}{\dfrac{U''(C_1)C_2}{U'(C_2)} - \dfrac{(1+r)^2 \beta}{\sigma}}$$

Current Account Determination: Small Open (XII)

- Replace $U'(C_2)$ on bottom using Euler Equation:

$$\frac{dC_1}{dr} = \frac{\beta C_2 - (1+r)\beta \frac{1}{\sigma}(Y_1 - C_1)}{\frac{\beta(1+r)U''(C_1)C_2 C_1}{U'(C_1)C_1} - \frac{(1+r)^2 \beta}{\sigma}}$$

- Replace coefficient of int. subst.:

$$\frac{dC_1}{dr} = \frac{\beta C_2 - (1+r)\beta \frac{1}{\sigma}(Y_1 - C_1)}{-\frac{\beta(1+r)C_2}{\sigma C_1} - \frac{(1+r)^2 \beta}{\sigma}}$$

Current Account Determination: Small Open (XIII)

- Multiply by $-\frac{\sigma}{\beta(1+r)}$:

$$\frac{dC_1}{dr} = \frac{Y_1 - C_1 - \sigma \frac{C_2}{1+r}}{\frac{C_2}{C_1} + 1 + r}$$

- Notice that a rise in the interest rate leads to a decline in first period consumption for borrowers, however, it is ambiguous for lenders. Why?

Current Account Determination: Infinite Horizon Perfect Foresight (I)

- Preferences:

$$\sum_{t=s}^{\infty} \beta^{t-s} U\left(C_t\right)$$

- Budget Constraint Derivation: (from Current Account Identity)

$$CA_t = B_{t+1} - B_t = Y_t + rB_t - C_t - G_t - I_t$$

- Rearranging:

$$(1+r)B_t = C_t + G_t + I_t - Y_t + B_{t+1}$$

Current Account Determination: Infinite Horizon Perf. Foresight (II)

- Budget Constraint Derivation (continued):

$$B_{t+1} = \frac{C_t + G_t + I_t - Y_t}{1+r} + \frac{B_{t+2}}{1+r}$$

- Thus:

$$(1+r)B_t = C_t + G_t + I_t - Y_t + \frac{C_t + G_t + I_t - Y_t}{1+r} + \frac{B_{t+2}}{1+r}$$

- And:

$$(1+r)B_s = \sum_{t=s}^{\infty} \frac{\left[C_t + G_t + I_t - Y_t\right]}{\left(1+r\right)^{t-s}} + \lim_{t\to\infty} \frac{B_{t+1}}{\left(1+r\right)^{t-s}}$$

Current Account Determination: Infinite Horizon Perf. Foresight (III)

- Additional Capital Market Requirement: No-Ponzi Condition Constraint:

$$\lim_{t\to\infty}\frac{B_t}{(1+r)^{t-s}}\geq 0$$

- Initial Condition on Wealth (doesn’t have to be zero):

$$B_s = 0$$

- Combining Initial Wealth and Dynamic Budget:

$$\sum_{t=s}^{\infty}\frac{[Y_t - G_t]}{(1+r)^{t-s}} = \sum_{t=s}^{\infty}\frac{[C_t + I_t]}{(1+r)^{t-s}} + \lim_{t\to\infty}\frac{B_{t+1}}{(1+r)^{t-s}}$$

Current Account Determination: Infinite Horizon Perf. Foresight (IV)

- We can now write the problem as:

$$\max_{\overrightarrow{C_t}} \sum_{t=s}^{\infty} \beta^{t-s} U(C_t)$$

- Subject to:

$$\sum_{t=s}^{\infty} \frac{[Y_t - G_t]}{(1+r)^{t-s}} = \sum_{t=s}^{\infty} \frac{[C_t + I_t]}{(1+r)^{t-s}} + \lim_{t\to\infty} \frac{B_{t+1}}{(1+r)^{t-s}}$$

- And:

$$\lim_{t\to\infty} \frac{B_t}{(1+r)^{t-s}} \geq 0$$

Current Account Determination: Infinite Horizon Perf. Foresight (V)

- Necessary and Sufficient Conditions for the solution to this problem are:

  - Euler Equations:

$$\frac{\partial U(C_t)}{\partial C_t} = \beta(1+r)\frac{\partial U(C_{t+1})}{\partial C_{t+1}}$$

  - Transversality Condition:

$$\lim_{t\to\infty}\frac{B_t}{(1+r)^{t-s}} = 0$$

Two Solution Techniques (I)

- Lagrangian:
  - Part I

$$\max_{\vec{C}_t,\lambda} L = \sum_{t=s}^{\infty} \beta^{t-s} U\left(C_t\right) - \lambda \left[\sum_{t=s}^{\infty} \frac{\left[Y_t - G_t\right]}{\left(1+r\right)^{t-s}} = \sum_{t=s}^{\infty} \frac{\left[C_t + I_t\right]}{\left(1+r\right)^{t-s}} \right]$$

  - Part II

$$\lim_{t\to\infty} \frac{B_t}{\left(1+r\right)^{t-s}} = 0$$

Two Solution Techniques (II)

- Dynamic Programming:
  - Formulation One:

$$V(B_t) = \max_{C_t, B_t} U(C_t) + \beta V(B_{t+1})$$

  - Subject to:

$$(1+r)B_t - C_t - G_t - I_t + Y_t = B_{t+1}$$

  - Alternatively:

$$V(B_t) = \max_{C_t, B_t} U(C_t) + \beta V((1+r)B_t - C_t - G_t - I_t + Y_t)$$

Two Solution Techniques (III)

- There is a third solution technique: the Hamiltonian. This is rarely used in discrete time and won't be shown here.

- After Euler Equations, to construct consumption functions:
  - Simultaneously solve infinite system of equations (i.e. through recursion)

  - Guess consumption function and check consistency with Euler Equations and Transversality condition

Current Account Determination: Infinite Horizon Stochastic (I)

- Maximize Expected Utility:

$$E_s \sum_{t=s}^{\infty} \beta^{t-s} U\left(C_t\right)$$

- Subject to Stochastic Budget Constraint:

$$E_s \sum_{t=s}^{\infty} \frac{Y_t}{\left(1+r\right)^{t-s}} = E_s \sum_{t=s}^{\infty} \frac{C_t}{\left(1+r\right)^{t-s}} \qquad \left\{Y_t\right\} \text{is a stochastic process}$$

- No Ponzi Games:

$$\lim_t \frac{B_t}{(1+r)^t} \geq 0$$

Current Account Determination: Infinite Horizon Stochastic (II)

- Solution is Characterized by Stochastic Euler Equation:

$$\frac{\partial U(C_t)}{\partial C_t} = \beta(1+r)E_t \frac{\partial U(C_{t+1})}{\partial C_{t+1}}$$

- Transversality Condition:

$$\lim_t \frac{B_t}{(1+r)^t} = 0$$

Current Account Determination: Infinite Horizon Stochastic (III)

- Functional Form Assumption (Linear-Quadratic):

$$U(C_t) = C_t - \frac{aC_t^2}{2}$$

- Further assume that:

$$\beta = \frac{1}{1 = r}$$

- The we can derive that consumption follows a martingale: $$\frac{\partial U(C_t)}{\partial C_t} = E_t \frac{\partial U(C_{t+1})}{\partial C_{t+1}} \Rightarrow C_t = E_t C_{t+1}$$

Current Account Determination: Infinite Horizon Stochastic (IV)

- Using the Dynamic Budget Constraint:

$$E_s \sum_{t=s}^{\infty} \frac{Y_t}{(1+r)^{t-s}} = E_s \sum_{t=s}^{\infty} \frac{C_s}{(1+r)^{t-s}} = C_s \left(\frac{1}{1-\frac{1}{1+r}} \right) = C_s \left(\frac{1+r}{r} \right)$$

- So we derive a consumption function:

$$C_s = \frac{r}{1+r} E_s \sum_{t=s}^{\infty} \frac{Y_t}{(1+r)^{t-s}}$$

- Notice that the consumer is not characterized by a cautionary demand for savings but rather that she demands her certainty equivalent; this is due to quadratic utility

Current Account Determination: Infinite Horizon Stochastic (V)

- If we want to go farther, we have to assume a stochastic income process. Assume a long run average income:

$$\overline{Y}$$

- Deviations from long run income follow an AR(1) process:

$$Y_{t+1} - \overline{Y} = \rho\left(Y_t - \overline{Y}\right) + \varepsilon_t$$

Current Account Determination: Infinite Horizon Stochastic (VI)

- Computing the date s expectation of future income:

$$E_s\left(Y_t - \overline{Y}\right) = \rho^{t-s}\left(Y_s - \overline{Y}\right)$$

$$E_s Y_t = \rho^{t-s} Y_s + \left(1 - \rho^{t-s}\right)\overline{Y}$$

- Now we can compute a consumption function:

$$C_s = \frac{r}{1+r}\sum_{t=s}^{\infty}\frac{\rho^{t-s} Y_s + \left(1 - \rho^{t-s}\right)\overline{Y}}{\left(1+r\right)^{t-s}}$$

Current Account Determination: Infinite Horizon Stochastic (VII)

- Rewriting the consumption function, we get:

$$C_s = \overline{Y} + \frac{r}{1+r}\sum_{t=s}^{\infty}\frac{\rho^{t-s}\left(Y_s - \overline{Y}\right)}{(1+r)^{t-s}} = \overline{Y} + \frac{r}{1+r}\frac{1}{1-\dfrac{\rho}{1+r}}\left(Y_s - \overline{Y}\right)$$

$$= \overline{Y} + \frac{r}{1+r-\rho}\left(Y_s - \overline{Y}\right)$$

- Now we can look at the impact of shocks on the current account:

$$CA_t = B_{t+1} - B_t = Y_t + rB_t - C_t = Y_t - C_t$$

Current Account Determination: Infinite Horizon Stochastic (VIII)

- We have our expressions for income and consumption:

$$Y_s - \overline{Y} = \rho\left(Y_{s-1} - \overline{Y}\right) + \varepsilon_s \Rightarrow Y_s = (1-\rho)\overline{Y} + \rho Y_s + \varepsilon_s$$

$$C_s = \overline{Y} + \frac{r}{1+r-\rho}\left(Y_s - \overline{Y}\right) = \overline{Y} + \frac{r}{1+r-\rho}\left(\rho\left(Y_{s-1} - \overline{Y}\right) + \varepsilon_s\right)$$

- Now we plug in our consumption and income equations into our current account equation:

$$CA_s = (1-\rho)\overline{Y} + \rho Y_s + \varepsilon_s - \left[\overline{Y} + \frac{r}{1+r-\rho}\left(\rho\left(Y_{s-1} - \overline{Y}\right) + \varepsilon_s\right)\right]$$

Current Account Determination: Infinite Horizon Stochastic (IX)

- Finally, we simplify our expression:

$$\Rightarrow CA_s = \frac{r\rho - \rho(1+r-\rho)}{1+r-\rho}\overline{Y} + \frac{\rho(1+r-\rho) - r\rho}{1+r-\rho}Y_s + \frac{1-\rho}{1+r-\rho}\varepsilon_s$$

$$\Rightarrow CA_s = \frac{\rho(1-\rho)}{1+r-\rho}\left(Y_s - \overline{Y}\right) + \frac{1-\rho}{1+r-\rho}\varepsilon_s$$

- Notice that the effects of permanent versus transitory shocks.

Theories of Current Account Persistence

- Does Persistent Shocks Do It?
- Precautionary Savings
- Consumer Durables
- Intertemporal Conflicts