# International Macro Lecture 5 

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## Target Zones: I

- Consider a continuous time monetary model of the exchange rate. First we specify foreign and domestic money demand:

$$
\begin{aligned}
& m_{t}-p_{t}=\phi y_{t}-\alpha i_{t} \\
& m_{t}^{*}-p_{t}^{*}=\phi y_{t}^{*}-\alpha i_{t}^{*}
\end{aligned}
$$

- Then we specify covered interest parity:

$$
i_{t}-i_{t}^{*}=\dot{s_{t}}
$$

## Target Zones: II

- Finally, we add PPP:

$$
s_{t}=p_{t}-p_{t}^{*}
$$

- We now define the exchange rate fundamental:

$$
f_{t}=m_{t}-m_{t}^{*}-\phi\left(y_{t}-y_{t}^{*}\right)
$$

- Subtracting the money demands and plugging them into covered interest parity, we get:

$$
f_{t}=s_{t}-\alpha s_{t}
$$

## Target Zones: IV

- Rewriting:

$$
s_{t}=\frac{s_{t}}{\alpha}-\frac{f_{t}}{\alpha}
$$

- We now define the exchange rate fundamental:

$$
s_{t}=\frac{1}{\alpha} \int_{t}^{\infty} e^{\frac{(t-x)}{\alpha}} f(x) d x
$$

- Why is this the solution? Differentiate it! (Leibniz' Rule):

$$
\frac{d \frac{1}{\alpha} e^{\frac{t}{\alpha}} \int_{t}^{\infty} e^{\frac{-x}{\alpha}} f(x) d x}{d t}=\frac{1}{\alpha^{2}} e^{\frac{t}{\alpha}} \int_{t}^{\infty} e^{\frac{-x}{\alpha}} f(x) d x-\frac{1}{\alpha} f_{t}=\frac{1}{\alpha} s_{t}-f_{t}
$$

## Target Zones: V

- We now turn to the stochastic case where the fundamental is determined by a Gaussian Process:

$$
\begin{gathered}
E \dot{s} t_{t}-\frac{s_{t}}{\alpha}=-\frac{f_{t}}{\alpha} \\
d f(t)=\eta d t+\sigma d z(t)
\end{gathered}
$$

- We now define the exchange rate fundamental:

$$
\begin{aligned}
& f(x)-f(t)=\int_{t}^{x} d f(r) d r= \\
& \int_{t}^{x} \eta d r+\int_{t}^{x} \sigma d z(r)=\eta(x-t)+\sigma u \sqrt{(x-t)}
\end{aligned}
$$

## Target Zones: VI

- Taking the expectation at date $t$ of $f(x)$ for the future, we get:

$$
E f(x)=E[f(x)-f(t)]+f(t)=\eta(x-t)+f(t)
$$

- No, we can plug in to our solution for $\mathrm{s}(\mathrm{t})$

$$
s_{t}=\frac{1}{\alpha} \int_{t}^{\infty} e^{\frac{(t-x)}{\alpha}}[f(t)+\eta(x-t)] d x
$$

- Rearranging, we get:

$$
s_{t}=\frac{1}{\alpha}\left[e^{\frac{t}{\alpha}}(f(t)-\eta t) \int_{t}^{\infty} e^{\frac{-x}{\alpha}} d x+\eta e^{\frac{t \infty}{\alpha}} \int_{t} x e^{\frac{-x}{\alpha}} d x\right]
$$

## Target Zones: VII

- Calculating

$$
\int_{t}^{\infty} x e^{\frac{-x}{\alpha}} d x=\alpha^{2} e^{-\frac{t}{\alpha}}\left(\frac{t}{\alpha}+1\right)
$$

- No, we can plug in to our solution for $\mathrm{s}(\mathrm{t})$

$$
s_{t}=\frac{1}{\alpha}\left[e^{\frac{t}{\alpha}}(f(t)-\eta t) e^{-\frac{t}{\alpha}}+\eta e^{\frac{t}{\alpha}} \alpha^{2} e^{-\frac{t}{\alpha}}\left(\frac{t}{\alpha}+1\right)\right]
$$

- Finally, we get:

$$
s_{t}=f(t)-\eta t+\eta t+\alpha \eta=f(t)+\alpha \eta
$$

## Target Zones: VIII

- Now, we look at the case when the government intervenes to maintain the currency for: -

$$
\underline{S}<S<S
$$

- At the edge of the currency band, the government intervenes with the money supply or interest rate to maintain the currency. The intervention is not sterilized. Thus, the fundamentals do not follow a diffusion process when the band limits are reached.
- We will now use the guess and check method of solution for the exchange rate. Assume a solution: $s(t)=G[f(t)]$


## Target Zones: IX

- Applying Ito's Lemma, we get:

$$
d s(t)=d G[f(t)]=G^{\prime}[f(t)] f(t)+\frac{\sigma^{2}}{2} G^{\prime \prime}[f(t)] d t
$$

- Substituting in for the diffusion process:

$$
d s(t)=G^{\prime}[f(t)][\eta d t+\sigma d z(t)]+\frac{\sigma^{2}}{2} G^{\prime \prime}[f(t)] d t
$$

- Taking expectations at time t , we get:

$$
E d s(t)=\eta G^{\prime}[f(t)]+\frac{\sigma^{2}}{2} G^{\prime \prime}[f(t)] d t
$$

## Target Zones: X

- Applying Ito's Lemma, we get:

$$
d s(t)=d G[f(t)]=G^{\prime}[f(t)] f(t)+\frac{\sigma^{2}}{2} G^{\prime \prime}[f(t)] d t
$$

- But, from before, we know that:

$$
d s(t)-\frac{s(t)}{\alpha}=-\frac{f(t)}{\alpha}
$$

- So that:

$$
\eta G^{\prime}[f(t)]+\frac{\sigma^{2}}{2} G^{\prime \prime}[f(t)] d t-\frac{G[f(t)]}{\alpha}=-\frac{f(t)}{\alpha}
$$

## Target Zones: XI

- Rearranging, we get a $2^{\text {nd }}$ order differential equation:

$$
G^{\prime \prime}[f(t)] d t+\frac{2 \eta}{\sigma^{2}} G^{\prime}[f(t)]-\frac{2 G[f(t)]}{\alpha \sigma^{2}}=-\frac{2 f(t)}{\alpha \sigma^{2}}
$$

- Now, we will look briefly at solutions to differential equations of the form:

$$
y^{\prime \prime}+a_{1} y^{\prime}+a_{2} y=b t
$$

- First, we look at solutions to the homogenous part:

$$
y^{\prime \prime}+a_{1} y^{\prime}+a_{2} y=0
$$

## Target Zones: XII

- Trying: $y=A e^{\lambda t}$
- We obtain $A e^{\lambda t}\left(\lambda^{2}+a_{1} \lambda+a_{2}\right)=0$
- Using the quadratic formula, we find two solutions:
- Now,

$$
\lambda_{1}=\frac{-a_{1}+\sqrt{a_{1}^{2}-4 a_{2}}}{2}, \lambda_{2}=\frac{-a_{1}-\sqrt{a_{1}^{2}-4 a_{2}}}{2}
$$

$$
\begin{aligned}
& y_{p}=\beta_{0}+\beta_{1} t \Rightarrow y_{p}{ }^{\prime}=\beta_{1} \Rightarrow y_{p}{ }^{\prime \prime}=0 \\
& \Rightarrow y_{p}{ }^{\prime \prime}+a_{1} y_{p}{ }^{\prime}+a_{2}=a_{1} \beta_{1}+a_{2}\left(\beta_{0}+\beta_{1} t\right)=b t
\end{aligned}
$$

## Target Zones: XIII

- Solving for the constant terms using the method of undetermined coefficients we get:

$$
\begin{aligned}
& \Rightarrow y_{p}{ }^{\prime}+a_{1} y_{p}{ }^{\prime}+a_{2}=a_{1} \beta_{1}+a_{2}\left(\beta_{0}+\beta_{1} t\right)=b t \\
& \Rightarrow \beta_{1}=\frac{b}{a_{2}} \Rightarrow \beta_{0}=-\frac{a_{1} b}{a_{2}^{2}}
\end{aligned}
$$

- Thus the general solution to the differential equation is given by:

$$
y(t)=A e^{\lambda_{1} t}+B e^{\lambda_{2} t}-\frac{a_{1} b}{a_{2}^{2}}+\frac{b}{a_{2}} t
$$

## Target Zones: XIV

- Going back to our original problem, we get:

$$
G[f(t)]=\eta \alpha+f(t)+A e^{\lambda_{1} f(t)}+B e^{\lambda_{2} f(t)}
$$

- Thus the general solution to the differential equation is given by:

$$
\begin{aligned}
& \lambda_{1}=-\frac{\eta}{\sigma^{2}}+\sqrt{\frac{\eta^{2}}{\sigma^{4}}+\frac{2}{\alpha \sigma^{2}}}>0 \\
& \lambda_{2}=-\frac{\eta}{\sigma^{2}}-\sqrt{\frac{\eta^{2}}{\sigma^{4}}+\frac{2}{\alpha \sigma^{2}}}<0
\end{aligned}
$$

## Target Zones: XV

- Finally, we have to solve for the constants A and B. Here we can use the fact that there is an upper bound to $f$ and a lower bound to $f$. At both bands, $\mathrm{ds}=0$. We can use this to derive two equations for our two unknowns:

$$
\begin{aligned}
& G^{\prime}[\bar{f}]=1+\lambda_{1} A e^{\lambda_{1} \bar{f}}+\lambda_{2} B e^{\lambda_{2} \bar{f}} \\
& G^{\prime}[\underline{f}]=1+\lambda_{1} A e^{\lambda_{1} \underline{f}}+\lambda_{2} B e^{\lambda_{2} \underline{f}}
\end{aligned}
$$

## Target Zones: XVI

- Solving for $A$ and $B$ is just solving a system of 2 linear equations and two unknowns. The solution is given by:

$$
\begin{aligned}
& A=\frac{e^{\lambda_{2} \bar{f}}-e^{\lambda_{2} \underline{f}}}{\lambda_{1}\left[e^{\left(\lambda_{1} \bar{f}+\lambda_{2} \underline{f}\right)}-e^{\left(\lambda_{1} \underline{f}+\lambda_{2} \bar{f}\right)}\right]}<0 \\
& B=\frac{e^{\lambda_{1} \underline{f}}-e^{\lambda_{1} \bar{f}}}{\lambda_{2}\left[e^{\left(\lambda_{1} \bar{f}+\lambda_{2} \underline{f}\right)}-e^{\left(\lambda_{1} \underline{f}+\lambda_{2} \bar{f}\right)}\right.}>0
\end{aligned}
$$

## Target Zones: XVII

- Now, we make two simplifying assumptions. First, we assume no drift in the stochastic process:

$$
\eta=0
$$

- Secondly, we assume that the boundaries on the fundamentals are equal and opposite:

$$
\bar{f}=\underline{f}
$$

- In this case, $A=-B$ and we can rewrite:

$$
G[f(t)]=f(t)-B\left[e^{\lambda f(t)}-e^{-\lambda f(t)}\right]
$$

- where $\left.\lambda=\lambda_{1}=-\lambda_{2}=\sqrt{\frac{2}{\alpha \sigma^{2}}}, \mathrm{~B}=\frac{\mathrm{e}^{\lambda \bar{f}}-\mathrm{e}^{-\lambda \bar{f}}}{\lambda\left[\mathrm{e}^{2 \lambda \bar{f}}-\mathrm{e}^{-2 \lambda \bar{f}}\right.}\right]$


## Target Zones: XVIII

- Comparing the free-floating solution with the target zone solution, we notice an S-shape to the target-zone:

$$
\begin{aligned}
& \text { Floating: } s_{t}=f(t) \\
& \text { Band: } s_{t}=f(t)-B\left[e^{\lambda f(t)}-e^{-\lambda f(t)}\right]
\end{aligned}
$$

- $s(t)$ for the target zone lies below that for the free-floating for $\mathrm{f}>0$ and below it for $\mathrm{f}<0$. In other words, the relation of the exchange rate to the fundamental has an S-shape. The expectation of hitting the edge of the band slows the growth of the exchange rate (downwards for high values of $s$ and upwards for low values of s).


## Engel-West (2005): I

- Tries to explain why exchange rates follow near-Martingales or Martingales
- Explanation: If the exchange rate is a function of a process which is integrated of order one (i.e. money supply process) and the discount factor is close to one, then the exchange rate will be close to a martingale.


## Engel-West (2005): II

- General Structure: suppose the equation of the exchange rate is given by:

$$
s_{t}=(1-b) \sum_{j=0}^{\infty} b^{j} E_{t}\left(a_{1}^{\prime} x_{t+j}\right)+b \sum_{j=0}^{\infty} b^{j} E_{t}\left(a_{2}^{\prime} x_{t+j}\right), 0<\mathrm{b}<1
$$

- Moreover, suppose that:

$$
a_{1}^{\prime} x_{t+j} \rightarrow I(1), a_{2}=0 \text { or } a_{2}^{\prime} x_{t+j} \rightarrow I(1)
$$

- Then $s_{t}$ will be nearly a unit root (which will be explained later).


## Engel-West (2005): III

- Simple intuitive model:

$$
s_{t}=(1-b) m_{t}+b \rho_{t}+b E_{t} S_{t+1}
$$

- Then, adding the no-ponzi constraint

$$
s_{t}=(1-b) \sum_{j=0}^{\infty} b^{j} E_{t}\left(m_{t+j}\right)+b \sum_{j=0}^{\infty} b^{j} E_{t}\left(\rho_{t+j}\right), 0<\mathrm{b}<1
$$

- Moreover, suppose that:

$$
\Delta m_{t}=\phi \Delta m_{t-1}+\varepsilon_{m t}, \Delta \rho_{t}=\gamma \Delta \rho_{t-1}+\varepsilon_{\rho t}
$$

## Engel-West (2005): IV

- First note that this process for $m$ is non-stationary:

$$
\begin{aligned}
& \Delta m_{t}=\phi \Delta m_{t-1}+\varepsilon_{m t} \Rightarrow \\
& m_{t+1}=\phi \Delta m_{t-1}+m_{t}+\varepsilon_{m t}=(1+\phi) m_{t}-\phi m_{t-1}+\varepsilon_{m t}
\end{aligned}
$$

- Assume stationary and compute variance of the process - it doesn't make sense:

$$
\begin{aligned}
& V\left(m_{t+1}\right)=V\left[(1+\phi) m_{t}-\phi m_{t-1}+\varepsilon_{m t}\right] \Rightarrow \\
& \sigma_{m}^{2}=(1+\phi)^{2}+\phi^{2}+\sigma_{\varepsilon}^{2} \Rightarrow \sigma_{m}^{2}=\frac{\sigma_{\varepsilon}^{2}}{-2\left(\phi^{2}+\phi\right)}
\end{aligned}
$$

## Engel-West (2005): V

- Going back to our primary equation, we replace the stochastic processes:

$$
\begin{gathered}
s_{t+1}-s_{t}=(1-b) \sum_{j=0}^{\infty} b^{j} E_{t+1}\left(m_{t+1+j}\right)+b \sum_{j=0}^{\infty} b^{j} E_{t+1}\left(\rho_{t+1+j}\right)-(1-b) \sum_{j=0}^{\infty} b^{j} E_{t}\left(m_{t+j}\right)-b \sum_{j=0}^{\infty} b^{j} E_{t}\left(\rho_{t+j}\right) \\
=(1-b) \sum_{j=0}^{\infty} b^{j}\left[E_{t+1} m_{t+1+j}-E_{t} m_{t+j}\right]+b \sum_{j=0}^{\infty} b^{j}\left[E_{t+1} \rho_{t+1+j}-E_{t} \rho_{t+j}\right] \\
=(1-b) \sum_{j=0}^{\infty} b^{j} \phi^{j}\left(\phi \Delta m_{t-1}+\varepsilon_{m t}\right)+b \sum_{j=0}^{\infty} b^{j} \gamma^{j}\left(\gamma \Delta \rho_{t-1}+\varepsilon_{\rho t}\right) \\
=\frac{\phi(1-b)}{1-b \phi} \Delta m_{t-1}+\frac{1}{1-b \phi} \varepsilon_{m t}+\frac{b \gamma}{1-b \gamma} \Delta \rho_{t-1}+\frac{b}{(1-b)(1-b \gamma)} \varepsilon_{\rho t}
\end{gathered}
$$

## Engels-West (VI)

- Starting with the previous slide's equation:

$$
\Delta s_{t}=\frac{\phi(1-b)}{1-b \phi} \Delta m_{t-1}+\frac{1}{1-b \phi} \varepsilon_{m t}+\frac{b \gamma}{1-b \gamma} \Delta \rho_{t}+\frac{b}{(1-b)(1-b \gamma)} \varepsilon_{\rho t}
$$

- First of all, as b approaches 1 ,

$$
\begin{gathered}
V\left(\Delta s_{t}\right) \approx \frac{b}{(1-b)(1-b \gamma)} \varepsilon_{\rho t} \rightarrow \infty \\
\frac{\phi(1-b)}{1-b \phi} \Delta m_{t-1}+\frac{1}{1-b \phi} \varepsilon_{m t}
\end{gathered}
$$

## Engels-West (VII)

- However, when we assume that rho equals zero, we get:

$$
\Delta s_{t}=\frac{\phi(1-b)}{1-b \phi} \Delta m_{t-1}+\frac{1}{1-b \phi} \boldsymbol{\varepsilon}_{m t}
$$

- Here, as b approaches 1, we have a finite variance but the exchange rate follows a random walk:

$$
\lim _{b \rightarrow 1} \Delta s_{t}=\frac{1}{1-b \phi} \varepsilon_{m t}
$$

TABLE 1
Population Autocorrelations and Cross Correlations of $\Delta s_{t}$

|  | $\begin{gathered} b \\ (1) \end{gathered}$ | $\begin{gathered} \varphi_{1} \\ (2) \end{gathered}$ | $\begin{gathered} \varphi \\ (3) \end{gathered}$ | Correlation of $\Delta s_{t}$ with: |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\Delta s_{t-1}$ <br> (4) | $\Delta s_{t-2}$ <br> (5) | $\Delta s_{t-3}$ <br> (6) | $\Delta x_{t-1}$ <br> (7) | $\begin{gathered} \Delta x_{t-2} \\ (8) \end{gathered}$ | $\begin{gathered} \Delta x_{t-3} \\ (9) \end{gathered}$ |
| 1. | . 50 | 1.0 | . 3 | . 15 | . 05 | . 01 | . 16 | . 05 | . 01 |
| 2. |  |  | . 5 | . 27 | . 14 | . 07 | . 28 | . 14 | . 07 |
| 3. |  |  | . 8 | . 52 | . 42 | . 34 | . 56 | . 44 | . 36 |
| 4. | . 90 | 1.0 | . 3 | . 03 | . 01 | . 00 | . 03 | . 01 | . 00 |
| 5. |  |  | . 5 | . 05 | . 03 | . 01 | . 06 | . 03 | . 01 |
| 6. |  |  | . 8 | . 09 | . 07 | . 06 | . 13 | . 11 | . 09 |
| 7. | . 95 | 1.0 | . 3 | . 02 | . 01 | . 00 | . 02 | . 01 | . 00 |
| 8. |  |  | . 5 | . 03 | . 01 | . 01 | . 03 | . 01 | . 01 |
| 9. |  |  | . 8 | . 04 | . 04 | . 03 | . 07 | . 05 | . 04 |
| 10. | . 90 | . 90 | . 5 | . 04 | -. 01 | -. 03 | . 02 | $-.03$ | -. 05 |
| 11. | . 90 | . 95 | . 5 | . 05 | . 01 | -. 01 | . 04 | $-.00$ | -. 02 |
| 12. | . 95 | . 95 | . 5 | . 02 | -. 00 | -. 01 | . 01 | -. 02 | -. 03 |
| 13. | . 95 | . 99 | . 5 | . 02 | . 01 | . 00 | . 03 | . 01 | -. 00 |

Note. - The model is $s_{t}=(1-b) \sum_{j=0}^{\infty} b^{j} E_{t} x_{t+j}$ or $s_{t}=b \sum_{j=0}^{\infty} b^{j} E_{t} x_{t+j}$. The scalar variable $x_{t}$ follows an $\operatorname{AR}(2)$ process with autoregressive roots $\varphi_{1}$ and $\varphi$. When $\varphi_{1}=1.0, \Delta x_{t} \sim \operatorname{AR}(1)$ with parameter $\varphi$. The correlations in cols. $4-9$ were computed analytically. If $\varphi_{1}=1.0$, as in rows $1-9$, then in the limit, as $b \rightarrow 1$, each of these correlations approaches zero.
sive root very near one. For a given autoregressive root less than one, the autocorrelations will not converge to zero as $b$ approaches one. But they will be very small for $b$ very near one.

Table 1 gives an indication of just how small "small" is. The table gives correlations of $\Delta s_{t}$ with time $t-1$ information when $x_{t}$ follows a scalar univariate $\operatorname{AR}(2)$. (One can think of $a_{1}=0$ and $a_{2}=1$ or $a_{1}=1$ and $a_{2}=0$. One can consider these two possibilities interchangeably since, for given $b<1$, the autocorrelations of $\Delta s_{t}$ are not affected by whether or not a factor of $1-b$ multiplies the present value of fundamentals.) Rows 1-9 assume that $x_{t} \sim \mathrm{I}(1)$-specifically, $\Delta x_{t} \sim \operatorname{AR}(1)$ with parameter $\varphi$. We see that for $b=0.5$ the autocorrelations in columns $4-6$ and the cross correlations in columns 7-9 are appreciable. Specifically, suppose that one uses the conventional standard error of $1 / \sqrt{T}$. Then when $\varphi=0.5$, a sample size larger than 55 will likely suffice to reject the null that the first autocorrelation of $\Delta s_{t}$ is zero (since row 2, col. 5 , gives $\operatorname{corr}\left(\Delta s_{t}, \Delta s_{t-1}\right)=0.269$ and $0.269 /[1 / \sqrt{55}] \approx 2.0$ ). (In this argument, we abstract from sampling error in estimation of the autocorrelation.) But for $b=0.9$, the autocorrelations are dramatically smaller. For $b=0.9$ and $\varphi=0.5$, a sample size larger than 1,600 will be required, since $0.051 /(1 / \sqrt{1,600}) \approx 2.0$. Finally, in connection with the previous paragraph's reference to autoregressive roots less than one, we see in rows $10-13$ in the table that if the unit root in $x_{t}$ is replaced by an autoregressive root of 0.9 or higher, the autocorrelations and cross correlations of $\Delta s_{t}$ are not much changed.

TABLE 3
Bivariate Granger Causality Tests, Different Measures of $\Delta f_{i}$, Full Sample: 1974:1-2001:3

|  | Canada | France | Germany | Italy | Japan | United <br> Kingdom |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A. Rejections at $1 \%(* * *), 5 \%(* *)$, and $10 \%(*)$ Levels of $\mathrm{H}_{0}: \Delta s_{t}$ Fails to Cause $\Delta f_{t}$ |  |  |  |  |  |
| 1. $\Delta\left(m-m^{*}\right)$ |  | * |  | ** | ** |  |
| 2. $\Delta\left(p-p^{*}\right)$ |  |  | *** | *** | *** |  |
| 3. $i-i^{*}$ |  | ** |  |  | ** |  |
| 4. $\Delta\left(i-i^{*}\right)$ |  | ** |  |  | *** |  |
| 5. $\Delta\left(m-m^{*}\right)-\Delta\left(y-y^{*}\right)$ |  | * |  | * |  |  |
| 6. $\Delta\left(y-y^{*}\right)$ |  |  |  |  |  |  |

B. Rejections at $1 \%(* * *), 5 \%(* *)$, and $10 \%(*)$ Levels of $\mathrm{H}_{0}: \Delta f_{t}$ Fails to Cause $\Delta s_{t}$

1. $\Delta\left(m-m^{*}\right)$
2. $\Delta\left(p-p^{*}\right)$
3. $i-i^{*}$

* $* *$

4. $\Delta\left(i-i^{*}\right)$
5. $\Delta\left(m-m^{*}\right)-\Delta\left(y-y^{*}\right)$
6. $\Delta\left(y-y^{*}\right)$

Note.-See the notes to earlier tables for variable definitions. Statistics are computed from fourth-order bivariate VARs in $\left(\Delta s_{t}, \Delta f_{t}\right)^{\prime}$. Because four observations were lost to initial conditions, the sample generally is 1975:2-2001:3, with exceptions as indicated in the note to table 2.
determined as a present value that depends in part on observable fundamentals.

Table 3 summarizes the results of our Granger causality tests on the full sample. We include a constant and four lags of each variable in all causality tests reported in this and all other tables. For all tests of no causality we use likelihood ratio statistics using the degrees of freedom correction suggested in Sims (1980).

We see in panel A that at the 5 percent level of significance, the null that $\Delta s_{t}$ fails to Granger-cause $\Delta\left(m_{t}-m_{t}^{*}\right), \Delta\left(p_{t}-p_{t}^{*}\right), i_{t}-i_{t}^{*}, \Delta\left(i_{t}-i_{t}^{*}\right)$, $\Delta\left(y_{t}-y_{t}^{*}\right)$, and $\Delta\left[m_{t}-y_{t}-\left(m_{t}^{*}-y_{t}^{*}\right)\right]$ can be rejected in nine cases at the 5 percent level and three more cases at the 10 percent level. There are no rejections for Canada and the United Kingdom but rejections in 12 of the 24 tests for the other four countries. The strongest rejections pertain to prices, where the null is rejected in three cases at the 1 percent level. ${ }^{4}$

[^0]TABLE 6
Var Causality Tests, Full Sample: 1974:1-2001:3: Rejections at 1\% (***), $5 \%(* *)$, AND $10 \%(*)$
$\left.\begin{array}{lccccc}\hline \hline \text { Variables in VAR } & \text { Canada } & \text { France } & \text { Germany } & \text { Italy } & \text { Japan }\end{array} \begin{array}{c}\text { United } \\ \text { Kingdom }\end{array}\right]$
could not help predict fundamentals. The exchange rate was found to be useful in forecasting real output in only two cases.

In summary, while the evidence is far from overwhelming, there does appear to be a link from exchange rates to fundamentals, going in the directions that exchange rates help forecast fundamentals.

## C. Correlation between $\Delta \mathrm{s}$ and the Present Value of Fundamentals

The previous subsection established a statistically significant link between exchange rates and certain fundamentals. We now examine such links to ask whether the signs of the regression coefficients are in some sense right. The statistic we propose is broadly similar to one developed in Campbell and Shiller (1987). The modification of the CampbellShiller statistic is necessary for two reasons. First, in contrast to Campbell and Shiller, our variables are not well approximated as cointegrated. Second, we allow for unobservable forcing variables, again in contrast to Campbell and Shiller.

Write the present-value relationship for exchange rates as

$$
\begin{equation*}
s_{t}=\sum_{j=0}^{\infty} b^{j} E_{t} f_{t+j}+\sum_{j=0}^{\infty} b^{j} E_{t} z_{t+j} \equiv F_{t}+U_{t} . \tag{15}
\end{equation*}
$$

TABLE 7
Correlation between $\Delta s_{t}$ and $\Delta F_{t}$

| Variables and Information Set | France | Germany | Italy | Japan |
| :---: | :---: | :---: | :---: | :---: |
|  | A. Discount Factor $b=0.5$ |  |  |  |
| 1. $\Delta\left(m-m^{*}\right)$ : |  |  |  |  |
| $F_{1 t}$ | $\begin{gathered} -.02 \\ (-.24, .16) \end{gathered}$ |  | $\begin{gathered} -.13 \\ (-.32, .06) \end{gathered}$ | $\begin{gathered} .24 \\ (.08, .37) \end{gathered}$ |
| $F_{2 t}$ | $\begin{gathered} .10 \\ (-.14, .31) \end{gathered}$ |  | $\begin{gathered} -.05 \\ (-.26, .17) \end{gathered}$ | $\begin{gathered} .23 \\ (.05, .38) \end{gathered}$ |
| $F_{2 t}-F_{1 t}$ | $\begin{gathered} .13 \\ (.05, .36) \end{gathered}$ |  | $\begin{gathered} .08 \\ (.02, .29) \end{gathered}$ | $\begin{gathered} -.01 \\ (-.08, .16) \end{gathered}$ |
| 2. $\Delta_{F_{1 t}}\left(p-p^{*}\right)$ : |  | $\begin{gathered} -.03 \\ (-.20, .14) \end{gathered}$ | $\begin{gathered} .19 \\ (-.03, .34) \end{gathered}$ | $\begin{gathered} -.21 \\ (-.35,-.06) \end{gathered}$ |
| $F_{2 t}$ |  | $\begin{gathered} .10 \\ (-.09, .28) \end{gathered}$ | $\begin{gathered} .27 \\ (.04, .44) \end{gathered}$ | $\begin{gathered} -.13 \\ (-.29, .06) \end{gathered}$ |
| $F_{2 t}-F_{1 t}$ |  | $\begin{gathered} .13 \\ (.07, .28) \end{gathered}$ | $\begin{gathered} .08 \\ (.02, .24) \end{gathered}$ | $\begin{gathered} .09 \\ (.03, .25) \end{gathered}$ |
| 3. $\Delta\left(i-i^{*}\right)$ : |  |  |  |  |
| $F_{1 t}$ | $\begin{gathered} -.21 \\ (-.38,-.03) \end{gathered}$ |  |  | $\begin{gathered} -.05 \\ (-.25, .13) \end{gathered}$ |
| $F_{2 t}$ | $\begin{gathered} -.07 \\ (-.27, .13) \end{gathered}$ |  |  | $\begin{gathered} .13 \\ (-.09, .34) \end{gathered}$ |
| $F_{2 t}-F_{1 t}$ | $\begin{gathered} .14 \\ (.07, .34) \end{gathered}$ |  |  | $\begin{gathered} .18 \\ (.11, .42) \end{gathered}$ |
| 4. $\Delta\left(m-m^{*}\right)-\Delta\left(y-y^{*}\right)$ : | $\begin{gathered} -.01 \\ (-.23, .17) \end{gathered}$ |  | $\begin{gathered} -.10 \\ (-.28, .07) \end{gathered}$ |  |
| $F_{2 t}$ | $\begin{gathered} .10 \\ (-.15, .31) \end{gathered}$ |  | $\begin{gathered} -.05 \\ (-.25, .16) \end{gathered}$ |  |
| $F_{2 t}-F_{1 t}$ | $\begin{gathered} .11 \\ (.03, .33) \end{gathered}$ |  | $\begin{gathered} .05 \\ (-.01, .31) \end{gathered}$ |  |
|  |  | B. Discount | ctor $b=0.9$ |  |
| 1. $\Delta\left(m-m^{*}\right)$ : |  |  |  |  |
| $F_{1 t}$ | $\begin{gathered} -.05 \\ (-.24, .12) \end{gathered}$ |  | $\begin{gathered} -.13 \\ (-.31, .05) \end{gathered}$ | $\begin{gathered} .19 \\ (.04, .33) \end{gathered}$ |
| $F_{2 t}$ | $\begin{gathered} .25 \\ (-.15, .55) \end{gathered}$ |  | $\begin{gathered} -.03 \\ (-.33, .34) \end{gathered}$ | $\begin{gathered} -.05 \\ (-.32, .24) \end{gathered}$ |
| $F_{2 t}-F_{1 t}$ | $\begin{gathered} .30 \\ (.05, .89) \end{gathered}$ |  | $\begin{gathered} .10 \\ (-.07, .69) \end{gathered}$ | $\begin{gathered} -.24 \\ (-.41, .30) \end{gathered}$ |
| 2. $\begin{aligned} & \Delta\left(p-p^{*}\right): \\ & F_{1 t}\end{aligned}$ |  | $\begin{gathered} -.01 \\ (-.18, .16) \end{gathered}$ | $\begin{gathered} .17 \\ (-.03, .34) \end{gathered}$ | $\begin{gathered} -.17 \\ (-.31,-.02) \end{gathered}$ |
| $F_{2 t}$ |  | $\begin{gathered} .49 \\ (.19, .68) \end{gathered}$ | $\begin{gathered} .51 \\ (.18, .71) \end{gathered}$ | $\begin{gathered} .31 \\ (.00, .53) \end{gathered}$ |
| $F_{2 t}-F_{1 t}$ |  | $\begin{gathered} .50 \\ (.32, .81) \end{gathered}$ | $\begin{gathered} .34 \\ (.16, .71) \end{gathered}$ | $\begin{gathered} .47 \\ (.28, .84) \end{gathered}$ |
| 3. $\Delta\left(i-i^{*}\right): ~$ |  |  |  |  |
| $F_{1 t}$ | $\begin{gathered} -.21 \\ (-.39,-.03) \end{gathered}$ |  |  | $\begin{gathered} -.06 \\ (-.27, .12) \end{gathered}$ |
| $F_{2 t}$ | $\begin{gathered} .15 \\ (-.19, .45) \end{gathered}$ |  |  | $\begin{gathered} .54 \\ (.19, .75) \end{gathered}$ |
| $F_{2 t}-F_{1 t}$ | $\begin{gathered} .37 \\ (.15, .86) \end{gathered}$ |  |  | $\begin{gathered} .60 \\ (.41, .95) \end{gathered}$ |

TABLE 7
(Continued)

| Variables and Information Set | France | Germany | Italy | Japan |
| :---: | :---: | :---: | :---: | :---: |
| 4. $\Delta\left(m-m^{*}\right)-\Delta\left(y-y^{*}\right)$ : |  |  |  |  |
| $F_{1 t}$ | $\begin{gathered} -.04 \\ (-.23, .14) \end{gathered}$ |  | $\begin{gathered} -.10 \\ (-.28, .06) \end{gathered}$ |  |
| $F_{2 t}$ | $\begin{gathered} .23 \\ (-.17, .53) \end{gathered}$ |  | $\begin{gathered} -.04 \\ (-.34, .31) \end{gathered}$ |  |
| $F_{2 t}-F_{1 t}$ | $\begin{gathered} .27 \\ (.02, .78) \end{gathered}$ |  | $\begin{gathered} .06 \\ (-.11, .67) \end{gathered}$ |  |

Note. $-F_{1 t}$ and $F_{2 t}$ are the expected discounted values of fundamentals, computed using lagged fundamentals alone $\left(F_{1}\right)$ or lagged fundamentals and lagged exchange rates $\left(F_{2}\right)$. The point estimates are the correlation between the change in the estimates of the expected present discounted values and the change in the actual exchange rate. They may be interpreted as correlations between fitted and actual values. The numbers in parentheses are 90 percent confidence intervals, computed from a nonparametric bootstrap.
is positive in six of the 10 cases for $b=0.5$ (though significantly different from zero at the 90 percent level in only one case [Japan, $\Delta\left(m-m^{*}\right)$ ]); it is positive in seven of the 10 cases for $b=0.9$ and significant in four of these (all three inflation series and $\Delta\left(i-i^{*}\right)$ in Japan). The sharpest result is that the correlation is higher for $\Delta F_{2 t}$ than for $\Delta F_{1 t}$ : the difference between the two is positive and significant in eight cases for $b=0.5$ and positive in nine cases and significant in seven for $b=0.9 .{ }^{9}$ The median correlations can be summarized as follows:

| Information |  |  |
| :--- | :---: | ---: |
| $\quad$ Set | $b=0.5$ | $b=0.9$ |
| $F_{1 t}$ | -.04 | -.05 |
| $F_{2 t}$ | .10 | .24 |

It is clear that using lags of $\Delta s_{t}$ to estimate the present value of fundamentals results in an estimate that is more closely tied to $\Delta s_{t}$ itself than when the present value of fundamentals is based on univariate estimates. But even when we limit ourselves to data in which there is Granger causality from $\Delta s_{t}$ to $\Delta f_{t}$, the largest single correlation in the table is 0.51 (Germany, for $\Delta\left(p_{t}-p_{t}^{*}\right)$, when $\left.b=0.9\right)$. A correlation less than one may be due to omitted forcing variables, $U_{t}$. In addition, we base our present values on the expected present discounted value of fundamental variables one at a time, instead of trying to find the appropriate linear combination (except when we use $m-y$ as a fundamental). So we should not be surprised that the correlations are still substantially below one.

The long literature on random walks in exchange rates causes us to

[^1]
## Evans/Lyons on Engel/West I

- Take a one factor exchange rate formula:

$$
s_{t}=(1-b) \sum_{t=0}^{\infty} b^{i} E_{t} f_{t+1} \Rightarrow s_{t+1}=(1-b) \sum_{t=0}^{\infty} b^{i+1} E_{t} f_{t+2}
$$

- We can rewrite the exchange rate recursively:

$$
s_{t}=E_{t} f_{t}+\frac{b}{1-b} E_{t}\left(s_{t+1}-s_{t}\right)
$$

## Evans/Lyons on Engel/West II

- Rewriting (and re-expressing the expectation under rational expectations), we get:

$$
\Delta s_{t+1}=\frac{1-b}{b}\left(s_{t}-E_{t} f_{t}\right)+\varepsilon_{t+1}
$$

- Where $\varepsilon_{t+1}=(1-b) \sum_{i=0}^{\infty} b^{i}\left(E_{t+1}-E_{t}\right) f_{t+i+1}$
- We will now compute the R-squared of the above regression. But how?


## Evans/Lyons on Engel/West III

- We now assume a fundamentals process which is integrated of order 1 :

$$
\Delta f_{t}=\phi \Delta f_{t-1}+\mu_{t+1}
$$

- You can show that the following holds:

$$
s_{t}-f_{t}=\phi\left(s_{t-1}-f_{t-1}\right)+\frac{b \phi}{1-b \phi} \mu_{t}
$$

- Where

$$
\varepsilon_{t+1}=\frac{1}{1-b \phi} \mu_{t+1}
$$

## Evans/Lyons on Engel/West IV

- To solve for the R-squared of $\Delta s_{t+1}=\frac{1-b}{b}\left(s_{t}-E_{t} f_{t}\right)+\varepsilon_{t+1}$

We need to compute $1-\frac{V\left(\varepsilon_{t+1}\right)}{V\left(\Delta s_{t+1}\right)}=1-\frac{V\left(\varepsilon_{t+1}\right)}{V\left(\frac{1-b}{b}\left(s_{t}-E_{t} f_{t}\right)\right)+V\left(\varepsilon_{t+1}\right)}$

- We start with $V\left(\varepsilon_{t+1}\right)=\left(\frac{1}{1-b \phi}\right)^{2} \sigma_{\mu}^{2}$
- For the other part, we have to calculated the variance of the exchange rate minus the fundamental, using stationarity.


## Evans/Lyons on Engel/West V

- From the orthogonality of regressors and residual, we get:

$$
V\left(s_{t}-f_{t}\right)=\phi^{2} V\left(s_{t-1}-f_{t-1}\right)+V\left(\varepsilon_{t+1}\right)
$$

- From stationarity, we then get:

$$
V\left(s_{t}-f_{t}\right)=\phi^{2} V\left(s_{t}-f_{t}\right)+V\left(\varepsilon_{t+1}\right)
$$

- Which leads us to the variance of s-f being a multiple of the variance of the residual:

$$
V\left(s_{t}-f_{t}\right)=\frac{V\left(\varepsilon_{t+1}\right)}{1-\phi^{2}}=\left(\frac{1}{1-b \phi}\right)^{2} \frac{\sigma_{\mu}^{2}}{1-\phi^{2}}
$$

## Evans/Lyons on Engel/West VI

- Now, we can calculate the variance of the change in the exchange rate:

$$
V\left(\Delta s_{t}\right)=\left(\frac{1-b}{b}\right)^{2}\left(\frac{1}{1-b \phi}\right)^{2} \frac{\sigma_{\mu}^{2}}{1-\phi^{2}}+\left(\frac{1}{1-b \phi}\right)^{2} \sigma_{\mu}^{2}
$$

- Finally, we can calculate the R-squared:

$$
R^{2}=1-\frac{\left(\frac{1}{1-b \phi}\right)^{2} \frac{\sigma_{\mu}^{2}}{1-\phi^{2}}}{\left(\frac{1-b}{b}\right)^{2}\left(\frac{1}{1-b \phi}\right)^{2} \frac{\sigma_{\mu}^{2}}{1-\phi^{2}}+\left(\frac{1}{1-b \phi}\right)^{2} \sigma_{\mu}^{2}}
$$

## Evans/Lyons on Engel/West VII

- Simplifying the R-squared of the regression:

$$
R^{2}=\frac{(1-b)^{2} \phi^{2}}{(1-b)^{2} \phi^{2}+1-\phi^{2}}
$$

- Note that as bis goes to 1 , the R-squared goes to zero.
- Another name for this is exchange rate disconnect (though in this case, fundamentals are in some sense not disconnected from exchange rates, they just can't be used to predict exchange rates).


## Order Flow (Evans \& Lyons): I

- Take a standard asset-price approach to the exchange rate:

$$
s_{t}=(1-b) \sum_{i=0}^{\infty} b^{i} E_{t} f_{t+i}
$$

- Where fundamentals follow an AR1 in first differences plus the innovation to order flow:

$$
\Delta f_{t}=f_{t}-f_{t-1}=\phi \Delta f_{t-1}+\mu_{t}+v_{t}
$$

- And order flow is determined by:

$$
x_{t}=\lambda x_{t}+v_{t}
$$

## Order Flow (Evans \& Lyons): II

- How can such a model be rationalized (microfounded)? Take the order flow model:

$$
x_{t}=\lambda x_{t}+v_{t}
$$

- Where the innovations to order flow are mean zero shocks with variance $\sigma_{v}^{2}$ and market-makers do not observe the contemparaneous order flow but instead observe:

$$
x_{t}^{i}=x_{t}+\varepsilon_{t}^{i}, \varepsilon_{t}^{i} \sim \operatorname{iid} N\left(0, \sigma_{\varepsilon}^{2}\right)
$$

## Order Flow (Evans \& Lyons): III

- What is the surprise in order flow?

$$
x_{t+1}^{i}-E_{t}^{i} x_{t+1}^{i}=x_{t+1}-E_{t}^{i} x_{t+1}+\varepsilon_{t+1}^{i}
$$

- There are two signals for this difference which the market maker has available to herself. The first is a date $t+1$ partial revelation of contemporaneous order flow. The other is the lagged actual order from the previous period:

$$
\begin{aligned}
& E_{t}^{i} x_{t+1}^{i}=\frac{\sigma_{v}^{2}}{\sigma_{v}^{2}+\sigma_{\varepsilon}^{2}} E_{t}^{i} x_{t}^{i}+\frac{\sigma_{\varepsilon}^{2}}{\sigma_{v}^{2}+\sigma_{\varepsilon}^{2}} \lambda x_{t-1}= \\
& \frac{\sigma_{v}^{2}}{\sigma_{v}^{2}+\sigma_{\varepsilon}^{2}} x_{t}+\frac{\sigma_{\varepsilon}^{2}}{\sigma_{v}^{2}+\sigma_{\varepsilon}^{2}}\left(x_{t}-v_{t}\right)=\lambda x_{t}-\frac{\sigma_{\varepsilon}^{2}}{\sigma_{v}^{2}+\sigma_{\varepsilon}^{2}} v_{t}
\end{aligned}
$$

## Order Flow (Evans \& Lyons): IV

- Pluggin this equation in to the surprise in order flow, we get:

$$
\begin{aligned}
& x_{t+1}^{i}-E_{t}^{i} x_{t+1}^{i}=x_{t+1}-E_{t}^{i} x_{t+1}+\varepsilon_{t+1}^{i}= \\
& v_{t+1}+\lambda \psi v_{t}+\varepsilon_{t+1}^{i}, \psi=\frac{\sigma_{\varepsilon}^{2}}{\sigma_{\varepsilon}^{2}+\sigma_{v}^{2}}
\end{aligned}
$$

- In other words, the surprise impact of order flow depends upon the prior realization of order flow.


## Order Flow (Evans \& Lyons): V

- Returning to the exchange rate model:

$$
\begin{aligned}
& s_{t}=(1-b) \sum_{i=0}^{\infty} b^{i} E_{t} f_{t+i} \\
& E_{t} s_{t+1}-E_{t} s_{t}=(1-b) \sum_{i=1}^{\infty}\left(b^{i-1}-b^{i}\right) E_{t} f_{t+i}-(1-b) E_{t} f_{t} \\
& =(1-b) \sum_{i=1}^{\infty} b^{i-1}(1-b) E_{t} f_{t+i}-(1-b) E_{t} f_{t} \\
& \Rightarrow \frac{b}{1-b}\left[E_{t} s_{t+1}-E s_{t}\right]=(1-b) \sum_{i=1}^{\infty} b^{i} E_{t} f_{t+i}-b E_{t} f_{t}
\end{aligned}
$$

## Order Flow (Evans \& Lyons): VI

- Continuing with our computations:

$$
\begin{aligned}
& \frac{b}{1-b}\left[E_{t} s_{t+1}-E s_{t}\right]=(1-b) \sum_{i=1}^{\infty} b^{i} E_{t} f_{t+i}-b E_{t} f_{t} \\
& \Rightarrow \frac{b}{1-b}\left[E_{t} s_{t+1}-E s_{t}\right]=(1-b) \sum_{i=0}^{\infty} b^{i} E_{t} f_{t+i}-E_{t} f_{t}
\end{aligned}
$$

- Now:

$$
\begin{aligned}
& \Rightarrow E_{t} s_{t+1}-E s_{t}=\frac{1-b}{b}\left(s_{t}-E_{t} f_{t}\right) \\
& \Rightarrow \Delta s_{t+1}=\frac{1-b}{b}\left(s_{t}-E_{t} f_{t}\right)+\varepsilon_{t+1}
\end{aligned}
$$

- Where:

$$
\varepsilon_{t+1}=(1-b) \sum_{i=0}^{\infty} b^{i}\left(E_{t+1}-E_{t}\right) f_{t+i+1}
$$

## Order Flow (Evans \& Lyons): VII

- Continuing with our computations:

$$
\begin{aligned}
& \frac{b}{1-b}\left[E_{t} s_{t+1}-E s_{t}\right]=(1-b) \sum_{i=1}^{\infty} b^{i} E_{t} f_{t+i}-b E_{t} f_{t} \\
& \Rightarrow \frac{b}{1-b}\left[E_{t} s_{t+1}-E s_{t}\right]=(1-b) \sum_{i=0}^{\infty} b^{i} E_{t} f_{t+i}-E_{t} f_{t}
\end{aligned}
$$

- Now:

$$
\begin{aligned}
& \Rightarrow E_{t} s_{t+1}-E s_{t}=\frac{1-b}{b}\left(s_{t}-E_{t} f_{t}\right) \\
& \Rightarrow \Delta s_{t+1}=\frac{1-b}{b}\left(s_{t}-E_{t} f_{t}\right)+\varepsilon_{t+1}
\end{aligned}
$$

- Where:

$$
\varepsilon_{t+1}=(1-b) \sum_{i=0}^{\infty} b^{i}\left(E_{t+1}-E_{t}\right) f_{t+i+1}
$$

## Order Flow (Evans \& Lyons): VIII

- We will now derive an equation for the change in the exchange rate which depends upon lagged order flow; this equation will be estimatable:

$$
s_{t}=\frac{1}{1-b \phi} f_{t}-\frac{b \phi}{1-b \phi} f_{t-1}-\frac{\delta}{1-b \phi} v_{t}
$$

- Now:

$$
\begin{aligned}
& E_{t}^{m} f_{t}=E_{t}\left[(1+\phi) f_{t-1}-\phi f_{t-2}+u_{t}+\delta v_{t}\right]=(1+\phi) f_{t-1}-\phi f_{t-2}+u_{t} \\
& \Rightarrow f_{t}-E_{t}^{m} f_{t}=\delta v_{t}
\end{aligned}
$$

- Also: $\varepsilon_{t+1}=s_{t+1}-E_{t}^{m} s_{t+1}$


## Order Flow (Evans \& Lyons): IX

- Substituting in, we get:

$$
\varepsilon_{t+1}=\frac{1}{1-b \phi}\left(f_{t+1}-E_{t}^{m} f_{t+1}\right)-\frac{b \phi}{1-b \phi}\left(f_{t}-E_{t}^{m} f_{t}\right)-\frac{\delta}{1-b \phi} v_{t+1}
$$

- Notice that:
$f_{t+1}-E_{t} f_{t+1}=\frac{1}{1-b \phi}\left((1+\phi) f_{t}-\phi f_{t-1}+u_{t+1}+\delta_{t+1}-E_{t}^{m}\left[(1+\phi) f_{t}-\phi f_{t-1}+u_{t+1}+\delta_{t+1}\right)\right)$
- So:

$$
\begin{aligned}
& =\frac{1}{1-b \phi}\left((1+\phi)\left(f_{t}-E_{t}^{m} f_{t}\right)+u_{t+1}+\delta v_{t+1}\right)-\frac{b \phi}{1-b \phi}\left(f_{t}-E_{t}^{m} f_{t}\right)-\frac{\delta}{1-b} \\
& =\frac{1}{1-b \phi} u_{t+1}+\frac{[1+\phi(1-b)] \delta}{1-b \phi} v_{t}
\end{aligned}
$$

## Order Flow (Evans \& Lyons): X

- We now now that the future change in the exchange rate depends upon the current order flow:

$$
\begin{aligned}
& \Delta s_{t+1}=\frac{1-b}{b}\left(s_{t}-E_{t} f_{t}\right)+\varepsilon_{t+1} \\
& \Rightarrow \Delta s_{t+1}=\frac{1-b}{b}\left(s_{t}-E_{t} f_{t}\right)+\frac{1}{1-b \phi} u_{t+1}+\frac{[1+\phi(1-b)] \delta}{1-b \phi} v_{t}
\end{aligned}
$$

- Now, as b goes to 1 , we get lack of predictability of all macro variables but order flow still matters:

$$
\lim _{b \rightarrow 1} \Delta s_{t+1}=\frac{1}{1-\phi} u_{t+1}+\frac{\delta}{1-\phi} v_{t}
$$

## Order Flow (Evans \& Lyons): XI

- So, we can estimate:

$$
\Delta s_{t+1}=\alpha+\beta x_{t}+e_{t}
$$

Table 1—Forecast Comparisons

|  | Horizon $h$ (trading days) |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Model | 1 | 5 | 10 | 15 | 20 |
| UIP |  |  |  |  |  |
| MSE | 1.001 | 1.006 | 1.012 | 1.016 | 1.021 |
| $(p$ value) | $(1.000)$ | $(1.000)$ | $(1.000)$ | $(1.000)$ | $(1.000)$ |
| $\beta$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| ( $p$ value) | $(0.058)$ | $(0.597)$ | $(0.542)$ | $(0.488)$ | $(0.414)$ |
|  |  |  |  |  |  |
| Fama |  |  |  |  |  |
| MSE Ratio | 1.005 | 1.011 | 1.022 | 1.035 | 1.054 |
| ( $p$ value) | $(1.000)$ | $(1.000)$ | $(1.000)$ | $(1.000)$ | $(1.000)$ |
| $\beta$ | 0.000 | 0.003 | 0.002 | 0.003 | 0.010 |
| ( $p$ value) | $(0.533)$ | $(0.332)$ | $(0.457)$ | $(0.452)$ | $(0.359)$ |
|  |  |  |  |  |  |
| Micro I |  |  |  |  |  |
| MSE Ratio | 1.026 | 1.015 | 1.001 | 0.946 | 0.896 |
| ( $p$ value) | $(1.000)$ | $(1.000)$ | $(1.000)$ | $(0.357)$ | $(0.106)$ |
| $\beta$ | 0.002 | 0.024 | 0.092 | 0.133 | 0.129 |
| $(p$ value) | $(0.398)$ | $(0.118)$ | $(0.000)$ | $(0.000)$ | $(0.000)$ |
|  |  |  |  |  |  |
| Micro II |  |  |  |  |  |
| MSE Ratio | 0.961 | 0.876 | 0.848 | 0.810 | 0.806 |
| $(p$ value) | $(0.124)$ | $(0.024)$ | $(0.091)$ | $(0.045)$ | $(0.055)$ |
| $\beta$ | 0.027 | 0.057 | 0.102 | 0.122 | 0.157 |
| ( $p$ value) | $(0.005)$ | $(0.018)$ | $(0.005)$ | $(0.007)$ | $(0.002)$ |

Notes: MSE ratio is the ratio of mean squared forecast errors for the non-RW model to the RW model. The $p$ value from a one-sided test for the RW null is reported in parenthesis under the MSE ratios. These $p$ values are computed as in Mark (1995) with the Andrews AR(1) rule for the truncation lag. The $p$ values below the estimates of $\beta$ are for the null $\beta=0$ and are computed from the asymptotic distribution of the ordinary least-squares estimates using NeweyWest estimator with $h-1$ lags.

These models are compared across five different forecasting horizons $h: 1,5,10,15$, and 20 trading days, using 3 June 1996 as the start of the forecasting period. ${ }^{7}$ Note that 20 trading days corresponds to four trading weeks (i.e., roughly one calendar month). In the UIP and Fama models we use euro deposit rates with maturities that match the forecast horizon. In the micro-based models order flows are derived from transactions over the $h$ trading days starting on day $t-h$.

Table 1 shows that the forecasting performance of the macro models is uniformly poor,

[^2]in keeping with results from Meese and Rogoff (1983) and the voluminous literature that followed their work (for a recent update, see YinWong Cheung et al. [2005]). In contrast, the forecasting performance of the micro models is significantly better, particularly as the forecasting horizon is extended. According to the projection statistic, the forecasting ability of the Micro I model is significantly better than the RW model at the 1-percent level at horizons of 10 days or longer. The results from the Micro II model are, if anything, even stronger. The projection statistics indicate that disaggregated order flow has statistically significant forecasting power for spot rate changes at all horizons. This finding is robust to our forecasting method. We find similar results when forecasts are based on rolling estimates of the Micro II model using a fixed number of observations.

The estimates of $\beta$ also provides us with a more economically meaningful measure of the forecasting performance. By definition, the $h$-period change in the spot rate comprises a forecastable and unforecastable component: $\Delta s_{t+h} \equiv \widehat{\Delta s_{t+h \mid t}}+\widehat{\varepsilon_{t+h \mid h}}$. Multiplying both sides of this identity by $\Delta s_{t+h}$, and taking expectations gives us a variance decomposition for spot rate changes:

$$
\begin{aligned}
\operatorname{Var}\left(\Delta s_{t+h}\right)=\operatorname{Cov}\left(\Delta s_{t+h},\right. & \widehat{\left.\Delta s_{t+h \mid t}\right)} \\
& +\operatorname{Cov}\left(\overline{\varepsilon_{t+h \mid h}} \Delta s_{t+h}\right)
\end{aligned}
$$

Since the projection coefficient $\beta$ is simply the ratio of the covariance between $\Delta s_{t+h}$ and $\widehat{\Delta s_{t+h \mid t}}$ to the variance of $\Delta s_{t+h}$, the values for $\beta$ reported in Table 1 estimate the contribution of the model forecasts to the variance of spot rate changes over the forecasting period. As the table shows, forecasts based on either micro-based model account for a greater fraction of the variance in spot rates as the forecasting horizon rises. Forecasts from the Micro II model account for almost 16 percent of the sample variance in monthly spot rate changes. By this metric, the forecasting power of disaggregated order flows is truly significant from an economic perspective.


Figure 5. U.S. News Effects as a Function of Release Time
Notes: We estimate the contemporaneous exchange-rate news response model, $R_{t}=\beta_{k} S_{k t}+\varepsilon_{t}$, where $R_{t}$ is the 5-minute return from time $t$ to time $t+1$ and $S_{k t}$ is the standardized news corresponding to announcement $k(k=1, \ldots, 17)$ made at time $t$. We estimate the regression using only those observations ( $R_{t}, S_{k t}$ ) such that an announcement was made at time $t$. On the vertical axis we display the $R^{2}$ values, and on the horizontal axis we display macroeconomic news announcements in the chronological order documented in Table 2. The "news numbers" are as follows:

| GDP | Real Activity | Investment | Forward-Looking |
| :--- | :--- | :--- | :--- |
| 1. GDP advance | 4. Payroll employment | 10. Durable goods orders | 14. Consumer confidence |
| 2. GDP preliminary | 5. Retail sales | 11. Construction spending | 15. NAPM index |
| 3. GDP final | 6. Industrial production | 12. Factory orders | 16. Housing starts |
|  | 7. Capacity utilization | 13. Business inventories | 17. Index of leading indicators |
|  | 8. Personal income |  |  |

helps with the interpretation of our earlierreported empirical results in Table 2, which indicate that only seven of the 40 announcements significantly impacted all the currency
specifications. The reason is that many of the announcements are to some extent redundant, and the market then only reacts to those released earlier. Hence, for example, U.S. durable goods

Table 4-Return and Volatility News Response Coefficients and Announcement Dummy Coefficients

| Announcement | Pound/\$ | Yen/\$ | DM/\$ | CHF/\$ | Euro/\$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Contemporaneous Return Response |  |  |  |  |  |
| Nonfarm payroll employment |  |  |  |  |  |
| $\beta_{k 0}$ | 0.091* | 0.071* | 0.159* | 0.115* | 0.079* |
| $\theta_{k 0}$ | 0.018 | 0.008 | 0.029* | -0.002 | 0.020 |
| Durable goods orders |  |  |  |  |  |
| $\beta_{k 0}$ | 0.052* | 0.028* | 0.082* | 0.083* | 0.039* |
| $\theta_{k 0}$ | -0.023* | -0.004 | -0.023* | -0.017 | -0.010 |
| Trade balance |  |  |  |  |  |
| $\beta_{k 0}$ | 0.086* | 0.121* | 0.144* | 0.131* | 0.085* |
| $\theta_{k 0}$ | 0.013 | 0.029 | 0.013 | 0.004 | 0.012 |
| Initial claims |  |  |  |  |  |
| $\beta_{k 0}$ | -0.009* | -0.009* | -0.017* | -0.023* | -0.017* |
| $\theta_{k 0}$ | 0.001 | -0.010* | -0.005 | -0.002 | -0.006 |
| Contemporaneous Volatility Response |  |  |  |  |  |
| Nonfarm payroll employment |  |  |  |  |  |
| $\beta_{k 0}$ | 0.0173* | 0.0216* | 0.0215* | 0.0169* | 0.015* |
| $\theta_{k 0}$ | 0.0566* | 0.0436* | 0.0873* | 0.0837* | 0.0597* |
| Durable goods orders |  |  |  |  |  |
| $\beta_{k 0}$ | 0.014* | 0.0098* | 0.023* | 0.0148* | 0.0144* |
| $\theta_{k 0}$ | 0.0042 | 0.0002 | 0.0048 | 0.0046 | 0.0043 |
| Trade balance |  |  |  |  |  |
| $\beta_{k 0}$ | 0.0226* | 0.0255* | 0.0214* | 0.0149* | 0.0153* |
| $\theta_{\text {k }}$ | 0.0001 | 0.0174* | 0.0162* | 0.0198* | 0.0141* |
| Initial claims |  |  |  |  |  |
| $\beta_{k 0}$ | 0.0005 | -0.0005 | 0.0039* | 0.0062* | 0.002 |
| $\theta_{k 0}$ | 0.0035* | 0.0048* | 0.0062* | 0.0038* | 0.0032* |

Notes: We add to equation (1) $J$ lags of announcement period dummies on each of $K$ fundamentals, $R_{t}=\beta_{0}+\Sigma_{i=1}^{I}$ $\beta_{i} R_{t-i}+\sum_{k=1}^{K} \sum_{j=0}^{J} \beta_{k j} S_{k, t-j}+\sum_{k=1}^{K} \sum_{j=0}^{J} \theta_{k j} D_{k, t-j}+\varepsilon_{t}$, and we report estimates of the contemporaneous return response to news and to announcement periods, $\beta_{k 0}$ and $\theta_{k 0}$, respectively. We also add to equation (2) $J^{\prime}$ lags of announcement period dummies on each of $K$ fundamentals,

$$
\begin{aligned}
\left|\hat{\varepsilon}_{t}\right|= & c
\end{aligned}+\psi \frac{\hat{\sigma}_{d(t)}}{\sqrt{288}}+\sum_{k=1}^{K} \sum_{j^{\prime}=0}^{J^{\prime}} \beta_{k j^{\prime}}\left|S_{k, t-j^{\prime}}\right|+\sum_{k=1}^{K} \sum_{j^{\prime}=0}^{J^{\prime}} \theta_{k j^{\prime}} D_{k, t-j^{\prime}}+\left(\sum_{q=1}^{Q}\left(\delta_{q} \cos \left(\frac{q 2 \pi t}{288}\right)+\phi_{q} \sin \left(\frac{q 2 \pi t}{288}\right)\right),\right.
$$

and report estimates of the contemporaneous return response to news and to announcement periods, $\beta_{k 0}$ and $\theta_{k 0}$, respectively. Asterisks denote statistical significance at the 5-percent level.

## F. News Effects V: Announcement Effects are Asymmetric-Responses Vary with the Sign of the News

We have seen that news about macroeconomic fundamentals significantly affect high-frequency exchange rates. Thus far we have allowed only for constant news effects, but it is natural to go farther and ask whether the news effects vary with the sign of the surprise. To address this issue we generalize equation (3) by allowing the impact response coefficient $\beta_{k}$ to be a linear function of the news surprise $S_{k k}$, allowing for a different constant and slope on each side of the origin,

$$
\beta_{k}= \begin{cases}\beta_{0 k}+\beta_{1 k} S_{k t} & \text { if } S_{t} \leq 0  \tag{4}\\ \beta_{2 k}+\beta_{3 k} S_{k t} & \text { if } S_{t}>0\end{cases}
$$

Inserting (4) into (3) yields the impact response specification,

$$
R_{t}= \begin{cases}\beta_{0 k} S_{k t}+\beta_{1 k} S_{k t}^{2}+\varepsilon_{t} & \text { if } S_{t} \leq 0  \tag{5}\\ \beta_{2 k} S_{k t}+\beta_{3 k} S_{k t}^{2}+\varepsilon_{t} & \text { if } S_{t}>0\end{cases}
$$

Following Robert F. Engle and Victor K. Ng (1993), we call the union of $\beta_{0 k} S_{k t}+\beta_{1 k} S_{k t}^{2}$ to the left of the origin and $\beta_{2 k} S_{k t}+\beta_{3 k} S_{k t}^{2}$ to the


Figure 6. U.S. News Impact Curves
Notes: In the top row we show the news impact curves averaged across all macroeconomic fundamentals, $k=1, \ldots, 41$. In the remaining rows we show the news impact curves for payroll employment, trade balance, durable good orders, and initial claims. See text for details.
right of the origin the "news impact curve." ${ }^{22}$ In the top row of Figure 6 we show the news impact curves averaged across all macroeconomic fundamentals, $k=1, \ldots, 41$. It is clear that, on average,

[^3]the effect of macroeconomic news often varies with its sign. In particular, negative surprises often have greater impact than positive surprises. ${ }^{23}$

It is interesting to see whether the sign effect prevails when we look separately at the most
${ }^{23}$ To the best of our knowledge, such sign effects have not previously been documented for the foreign exchange market. Evidence of asymmetric conditional-mean news effects exists in other contexts, however. For example, Jennifer Conrad et al. (2001) find asymmetric effects of earnings news on stock returns, while recent concurrent work by Hautsch and Hess (2001) details an asymmetric response to employment news in the T-bond futures market.

## PPP: I

- Law of One Price (LOP):

$$
p_{i}=e p_{i}
$$

- Absolute PPP (Using CPI - doesn't hold... why?):

$$
\sum_{i} p_{i}=e \sum_{i} p_{i}^{*}
$$

- Relative PPP:

$$
\frac{\sum_{i} p_{i t}}{\sum_{i} p_{i t-1}}=\frac{\varepsilon_{t}}{\varepsilon_{t-1}} \frac{\sum_{i} p_{i t}^{*}}{\sum_{i}^{*} p_{i t-1}^{*}}
$$

## PPP: II

| Country | Big Mac (\$) | City | Gold (US\$) |  |
| :--- | :--- | :--- | :--- | :---: |
| Switzerland | 5.20 | Hong Kong | 379.35 |  |
| Denmark | 4.92 |  | London |  |
| Japan | 4.65 | 379.25 |  |  |
| Belgium | 3.84 | Paris | 378.81 |  |
| Germany | 3.48 |  | Frankfurt |  |
| US | 2.32 | 379.87 |  |  |
| Canada | 1.99 | Zurich | 379.1 |  |
| Russia | 1.62 | New York | 379.1 |  |
| China | 1.05 |  |  |  |

## PPP: III

- Why such a large difference between hamburgers and gold?
- Non-tradable component
- Tariffs
- Imperfect Competition, Product Differentiation and 'Pricing to Market'
- Isard (1977) and Richardson (1978) looked at 4-digit and 7-digit commodity prices and found large, persistent deviations from LOP. Interpretations: tariffs and non-tariff barriers (including discriminating import networks)
- Transportation costs or international trade barriers? Egnel and Rogers (1995): 14 categories of disaggregated price indices for 23 cities in the US and Canada. Estimate gravity equation on price differentials (regress relative prices on distance between cities). The border effect on price volatility is between 2500 and 23,000 miles depending upon specification.


## PPP: IV

- Given failure of LOP, not surprising that PPP fails as well. Could this be related to Meese-Rogoff?
- Fluctuations in exchange rates plus prices that are sticky in domestic currency lead to failures of PPP (though Lothian and Taylor argue that one cannot reject the rate of convergence to PPP being larger before 1973),
- This is essentially what the Dornbusch model has except the Dornbusch model predicts somewhat quick convergence (for reasonable parameter values).
- Frankel $(1986,1990)$ argued that for slow adjustment of prices, time series were not long enough to reject PPP in the long run. He estimated a rate of decay for real exchange rates of $14 \%$ per year (half-life of 4.6 years).


## PPP: V

- Balassa-Samuelson Effect (one theory of why PP shouldnt hold) - Let P and $\mathrm{P}^{*}$ be price indices:

$$
P=1^{\gamma} p^{1-\gamma}, P^{*}=1^{\gamma}\left(p^{*}\right)^{1-\gamma}
$$

- Thus

$$
\frac{P}{P^{*}}=\left(\frac{p}{p^{*}}\right)^{1-\gamma}
$$

- Now assume CRS production with tech. change. Zero profit implies::

$$
A_{T} f\left(k_{T}\right)=r k_{T}+w
$$

## PPP: VI

- Log differentiating, using the small open economy assumption:

$$
\frac{\frac{d A_{T}}{d t}}{A_{T}}+\frac{f^{\prime}\left(k_{T}\right) \frac{d k_{T}}{d t}}{f\left(k_{T}\right)}=\frac{r \frac{d k_{T}}{d t}+\frac{d w}{d t}}{A_{T} f\left(k_{T}\right)}
$$

- Rearranging and usings FOC:

$$
\frac{\frac{d A_{T}}{d t}}{A_{T}}+\frac{\frac{r}{A_{T}} k_{T}}{f\left(k_{T}\right)} \frac{\frac{d k_{T}}{d t}}{k_{T}}=\frac{r k_{T}}{A_{T} f\left(k_{T}\right)} \frac{\frac{d k_{T}}{d t}}{k_{T}}+\frac{w}{A_{T} f\left(k_{T}\right)} \frac{\frac{d w}{d t}}{w}
$$

## PPP: VII

- Thus:
$\frac{\frac{d A_{T}}{d t}}{A_{T}}+\frac{r k_{T}}{A_{T} f\left(k_{T}\right)} \frac{\frac{d k_{T}}{d t}}{k_{T}}=\frac{r k_{T}}{A_{T} f\left(k_{T}\right)} \frac{\frac{d k_{T}}{d t}}{k_{T}}+\frac{w}{A_{T} f\left(k_{T}\right)} \frac{\frac{d w}{d t}}{w}$
- Subtracting off the middle:

$$
\hat{A}_{T}=\frac{\frac{d A_{T}}{d t}}{A_{T}}=\frac{w}{A_{T} f\left(k_{T}\right)} \frac{\frac{d w}{d t}}{w}=\mu_{L T} \hat{w}
$$

## PPP: VIII

- Similarly for the non-tradable sector, we get:

$$
\begin{aligned}
& \hat{p}+\hat{A}_{N}=\mu_{L N} \hat{w}=\frac{\mu_{L N}}{\mu_{L T}} \hat{A}_{T} \\
& \Rightarrow \hat{p}=\frac{\mu_{L N}}{\mu_{L T}} \hat{A}_{T}-\hat{A}_{N}
\end{aligned}
$$

- Going back to our equation for the aggregate price indexes, we get that if the labor share is greater in the non-tradables sector, then price indexes rise faster in countries with faster productivity growth in tradables relative to non-tradables (developing countries):

$$
\hat{P}-\hat{P}^{*}=(1-\gamma)\left(\hat{p}-\hat{p}^{*}\right)=(1-\gamma)\left[\frac{\mu_{L N}}{\mu_{L T}}\left(\hat{A}_{T}-\hat{A}_{T}^{*}\right)-\left(\hat{A}_{N}-\hat{A}_{N}^{*}\right)\right]
$$

- Basically, a rise in productivity of tradables raises demand for both non-tradables and tradables leading to a rise in the price of non-tradables relative to tradables. Similar conclusions yield from assuming imperfect capital mobility (leading to higher wages in countries with higher capital/labor ratios) - Bhagwati.


## PPP:IX

- Evidence in favor of Balassa-Samuelson
- Higher price level of more developped countries
- Faster growth in CPI (more non-tradables) than in WPI
- Other theories of long-run determinants of the real exchange rate:
- Sustained current account deficits lead to real exchange rate depreciation (less demand for non-tradables). Obstfeld and Rogoff (1995) show that the correlation between trade-weighted real exchange rate changes and net foreign asset positions is quite large and significant.
- Froot and Rogoff (1991) show that government spending is positively correlated with the real exchange rate. They interpret their evidence as due to higher propensities of public consumption to consume non-tradables.


[^0]:    ${ }^{4}$ The overall level of predictability, though not the pattern, is consistent with the point estimates in Stock and Watson (2003). Using inflation and output from the G7 countries (rather than for six countries relative to the United States) and a 1985-99 sample, Stock and Watson examine the ability of the exchange rate (and many other financial variables) to forecast out of sample. They find that the exchange rate lowers the mean squared prediction error for inflation in one country (Canada) and for GDP in four countries (Canada, Germany, Italy, and Japan). Thus the overall rate of success (five out of 14 data series) is comparable to ours, though the pattern (more success with real than nominal)

[^1]:    ${ }^{9}$ Here it is advisable to recall that we examine only series that display Granger causality. So the statistical significance of the difference is unsurprising. On the other hand, the sign of the difference (positive) was not foretold by our Granger causality tests.

[^2]:    ${ }^{7}$ The number of out-of-sample forecasts used to compute the MSE and projection statistics for $h=\{1,5,10,15$, $20\}$ are $797,793,788,783$, and 778 , respectively. Since there are at least 38 nonoverlapping observations $(778 / 20>$ 38) in the forecasting period, our results should be largely immune to the well-known small-sample problems that plague inference in long-horizon forecast comparisons conducted over standard data spans.

[^3]:    ${ }^{22}$ Despite the superficial resemblance in terms of documenting asymmetric responses to news, our work is very different from that of Engle and Ng (1993) and many subsequent related studies. In particular, the Engle-Ng news impact curve tracks the variance of equity returns conditional upon the sign and size of past returns (with no allowance for a time-varying conditional mean return), whereas our news impact curve tracks the mean of foreign exchange returns conditional upon the sign and size of macroeconomic news.

