International Macro
Lecture 8

March 15, 2006

Speculative Attacks – Krugman: I

- Krugman model shows that countries with a fixed exchange rate pursuing expansionary monetary policy will have an attack on reserves, forcing them to float, far before reserves run out.

- Start with money demand:

$$\frac{M_t}{P_t} = Ae^{-\eta i_t}$$

- And uncovered interest parity:

$$i_t = i_t^* + \frac{\dot{E}_t}{E_t}$$

Speculative Attacks – Krugman: II

- The central bank's balance sheet just says that money demand is equal to money supply. In other words, the amount of money in the economy is equal to domestic credit plus the value, in domestic currency of foreign reserves:

$$M_t = C_t + Ef_t$$

- Additionally, we assume that the government is expands domestic credit over time at a constant rate:

$$\frac{\dot{C}_t}{C_t} = \gamma$$

- Then we can show that foreign reserves decline over time.

Speculative Attacks – Krugman: III

- Deriving the time path of foreign reserves:

$$0 = \frac{\dot{M}_t}{C_t} = \frac{\dot{C}_t + \dot{f}_t}{C_t}$$

- We can solve for the rate of change in foreign reserves by solving (where omega(f) is the ratio of foreign reserves in high-powered money):

$$-\gamma = -\frac{\dot{C}_t}{C_t} = \frac{\dot{f}_t}{C_t}\frac{f_t}{f_t} = \frac{\dot{f}_t}{f_t}\frac{f_t}{C_t} \quad \Rightarrow \frac{\dot{f}_t}{f_t} = -\gamma\frac{f_t}{C_t} = -\gamma\frac{\frac{f_t}{M_t}}{\frac{C_t}{M_t}} = -\gamma\frac{1-\omega_f}{\omega_f}$$

Speculative Attacks – Krugman: IV

- So, eventually, the monetary authority will run out of foreign reserves. However, before that, it will experience an attack on the currency.
- First, we assume PPP. Therefore, the domestic price level is fixed with respect to the exchange rate. Normalizing the foreign price level to 1, we get:

$$P_t = E_t$$

- Now, we define the shadow exchange rate that would exist if the central bank sold all reserves and let the currency float. However, when reserves are gone money demand is equal to domestic credit:

$$M_t = C_t$$

- Taking logs of money demand and substituting in, we get:

$$\log C_t - \log \overline{E}_t = \log A - \eta i_t$$

Speculative Attacks – Krugman: V

- Also, we know that after the currency is allowed to float, given perfect price flexibility, the rate of depreciation of the exchange rate will be equal to the rate of domestic credit growth:

$$\frac{\dot{C}_t}{C_t} = \gamma = \frac{\dot{E}_t}{E_t}$$

- Therefore, from uncovered interest parity:

$$i_t = i^* + \gamma$$

- Thus, our shadow exchange rate equation is:

$$\log C_t + \eta\left(i^* + \gamma\right) - \ln A = \log \overline{E}_t$$

-

Speculative Attacks – Krugman: VI

- Now, whenever the shadow exchange rate is depreciated relative to the fixed exchange rate, selling short domestic currency until the central bank floats brings an infinite rate of return. Therefore, this can not happen. Since the government will not float the currency until it gives up all its reserves, after the date when the shadow exchange rate equals the fixed exchange rate, the government can not hold any reserves. Also, before such time, speculators earn a negative return from selling the domestic currency.

- Therefore, there will be a speculative attack on the currency where by (1.) there will be a discrete drop in reserves with all remaining reserves dissappearing, (2.) the money supply will drop discretely, (3.) the exchange rate will float adjusting continuously starting from the date when the shadow exchange rate equals the fixed exchange rate.

Speculative Attacks: Obstfeld I

- In the Krugman model, (1.) government wasn’t a strategic actor and (2.) individual investors were not strategic actors.

- However, when government is a strategic actor and there are many speculators who are credit constrained, then, (1.) there is a possibility that even after the first date when an attack is viable, it does not happen because investors do not coordinate for it to happen even though if all investors coordinated, it would be profitable – i.e. multiple equilibria and (2.) whether or not multiple equilibria exist depends upon the state of the fundamentals (or reserves) of the economy.

Speculative Attacks: Obstfeld II

- Suppose there are a continuum of speculators with payoff equal to:

$$-t$$

- if there is an unsuccesful attack and:

$$\bar{e}-\theta-t$$

- if the attack is succesful. The government gets a payoff of zero if it does not defend the attack otherwise gets a payoff of:

$$v-\alpha\theta$$

Speculative Attacks: Obstfeld III

- Suppose there are a large finite number of speculators. If a speculator attacks, it must be a transactions fee whether or not the attack is succesful. Thus a specualtor gets a payoff equal to:

$$-t$$

- if there is an unsuccesful attack. However, if the attack is succesful, the investors get a benefit greater than t:

$$\bar{e} - t > 0$$

- The government gets a payoff of zero if it does not defend the attack otherwise gets a payoff of:

$$\theta - n$$

- where theta is state of fundamentals and N is the number of potential specualtors and n is the number of actual ones.

Speculative Attacks: Obstfeld IV

- Note that it will only be worthwhile for a speculator to attack if the attack is succesful. Therefore, we can identify three regimes:

- (1.) $\theta > N$

  In this case, even if everyone attacks, the government will not give up the peg. Therefore, noone will ever attack.

- (2.) $\theta < 1$

  In this case, even if one person attacks, the government will give up the peg. Therefore, everyone will attack.

Speculative Attacks: Obstfeld V

- (3.) $N > \theta > 1$

  In this case, if everyone attacks, then it is rational for any speculator to attack. However, if noone attacks, then it is rational for any speculator to not attack. Therefore, there are multiple equilibria. There are, in fact, two pure strategy equilibra, one where everyone attacks and one where noone attacks. In addition, there is a symmetric equilibrium in mixed strategies where everyone attacks with the same probability and in equlibrium, the attack is succesful with a certain probability (generically not equal to the probability with which speculators mix). Notice that this equilibrium goes away with a continuum of speculators since then the outcome of the attack can not be stochastic and thus speculators can not be indifferent.

Speculative Attacks – Morris & Shin: I

- We now generalize the Obstfeld model both by allowing the costs of the government defending to vary with the state of fundamentals as well as the percentage of people attacking (α).

- Speculators have utility:

$$e^{*} - f(\theta) - t > 0$$

- The government has utility:

$$v - c(\alpha, \theta)$$

Speculative Attacks – Morris & Shin: II

- We make the following functional form and parameter assumptions:
  - (1.) Theta is bounded and transactions costs of speculation are positive

$$\theta \in [0,1], \mathrm{t} > 0$$

  - (2.)The costs of defense are increasing in the percentage of attackers and decreasing in fundamentals:

$$\frac{\partial C}{\partial \alpha} > 0, \frac{\partial C}{\partial \theta} < 0,$$

  - (3.) In the worst state of fundamentals, the cost of defending the currency exceeds the value even if no speculators attack:

$$C(0,0) > v$$

  - (4.) If all speculators attack, the costs outweigh the values even in the best state:

$$C(1,1) > v$$

  - (5.) In the best state of fundamentals, the floating exchange rate is sufficiently close to the pegged level that it is not worth the transactions cost to speculate:

$$t > e^* - f(1)$$

Speculative Attacks – Morris & Shin: III

- Then, as with the Obstfeld model, there is a tripartite division of fundamentals:
  - (1.) If fundamentals are sufficiently bad, then there will be no attack:

$$\exists \underline{\theta} \text{ s.t. } \mathrm{C}\left(0, \underline{\theta}\right) = v \Rightarrow \forall \theta \leq \underline{\theta}, \text{ there is always an attack}$$

  - (2.)The costs of defense are increasing in the percentage of attackers and decreasing in fundamentals:

$$\exists \overline{\theta} \text{ s.t. } \mathrm{f}\left(\overline{\theta}\right) = e^{*} - t, \forall \theta \geq \overline{\theta}, \text{ there is no attack}$$

  - (3.) For inbetween states of the economy, there are always multiple equilibria:

Speculative Attacks – Morris & Shin: IV

- Now we add that the state is observed with noise where every speculator observes the truth plus uniformly distributed error:

$$x \approx U[\theta - \varepsilon, \theta + \varepsilon]$$

- The government, which decides whether or not to defend, moves after attack decisions are made and thus observes both the state as well as the percentage who attack the currency.

Speculative Attacks – Morris & Shin: V

- We will now show that equilibria are unique with incomplete information. We solve by backward induction.

- Below $\underline{\theta}$, the government will not defend even if no speculator attacks. Denote by $a(\theta)$ the maximum percentage of attacking speculators for which the government will still be willing to defend as a function of the state. Note that $a(\theta)=\alpha \text{ s.t. } c(\alpha,\theta)=v$ so that a is strictly increasing for theta greater than $\underline{\theta}$.

- Also, define the set of combinations of percentage of attacking speculator as function of the state and an equilibrium strategy: $s(\theta,\pi)$

Speculative Attacks – Morris & Shin: VI

- The relation, s, can potentially be a correspondence.

- We can write s as: $s(\theta,\pi)=\frac{1}{2\varepsilon}\int_{\theta-\varepsilon}^{\theta+\varepsilon}\pi(x)dx$

- Where $\pi(x)$ is the percentage of speculators who attack given signal x.

- Now we can define the set of combinations of percentage of attackers and fundamentals such that the government will not defend:

$$A(\pi)=\{\theta|s(\theta,\pi)\geq a(\theta)\}$$

Speculative Attacks – Morris & Shin: VII

- We want to show that s intersect a only once. In other words, there exists a value of theta, θ^*, such that the government abandons the peg if and only if: $\theta \leq \theta^*$

- We first show three lemmas and then prove the main theorem:
  - Utility is increasing in the aggressiveness of bidding
  - If individuals follow cutoff simple cutoff strategies, utility is decreasing in the cutoff
  - There is a unique cutoff such that, in any equilibrium, all investors attack if they get at or above the cutoff and do not attack otherwise

- Then we conclude with a proof of the main result, that the equilibrium is unique. Also, we prove that even in the limit as the variance of noise goes to zero, equilibria remain unique.

Speculative Attacks – Morris & Shin: VIII

- Lemma 1: $\pi(x) \geq \pi'(x) \Rightarrow U(x,\pi) \geq U(x,\pi')$
- Proof:
  - If the share of investors attacking at any given signal level is higher, then the share of investors attacking given any signal and any underlying state of the world is higher which means that the set under which the government abandons the peg is larger:

$$\pi(x) \geq \pi'(x) \Rightarrow s(\theta,\pi) \geq s(\theta,\pi') \Rightarrow A(\pi) \geq A(\pi')$$

  - Then the utility of a speculator is given by:

$$u(x,\pi) = \frac{1}{2\varepsilon}\left[\int_{A(\pi)\cap[x-\varepsilon,x+\varepsilon]} \left(e^{*} - f(\theta)\right) d\theta\right] - t \geq$$

  - $$\frac{1}{2\varepsilon}\left[\int_{A(\pi')\cap[x-\varepsilon,x+\varepsilon]} \left(e^{*} - f(\theta)\right) d\theta\right] - t = u(x,\pi')$$

Speculative Attacks – Morris & Shin: VIII

- Lemma 2: $U(k, I_k)$ is continuous and strictly decreasing in k where:

$$U(k, I_k) = \begin{cases} 1 & \mathrm{x} < \mathrm{k} \\ 0 & \mathrm{x} \geq \mathrm{k} \end{cases}$$

- Proof:
  - Given that all agents follow the above equilibrium strategy, then we can solve for s:

$$s(\theta, I_k) = \begin{cases} 1 & \theta \leq \mathrm{k} - \varepsilon \\ \frac{1}{2} - \frac{1}{2\varepsilon}(\theta - k) & \mathrm{k} - \varepsilon \leq \theta \leq \mathrm{k} + \varepsilon \\ 0 & \theta \geq \mathrm{k} + \varepsilon \end{cases}$$

  –

Speculative Attacks – Morris & Shin: IX

- Denote by $\psi(k) \ni \mathrm{s}(\mathrm{k}+\psi(k)) = a(k+\psi(k))$
- Then the government abandons the peg on the interval:

$$[0, k+\psi(k)]$$

- In which case, the payoff function for an attacking investor is given by:

$$u(k, I_k) = \frac{1}{2\varepsilon} \int_{k-\varepsilon}^{k+\psi(k)} \left(e^* - f(\theta)\right) d\theta - t$$

  - Using Lebniz' rule, since we know that $e^* - f(k)$ is strictly decreasing in theta, we need only to show that phi(k) is weakly decreasing in k.
  - But $\psi(k) = \varepsilon \text{ if } \mathrm{k} \le \underline{\theta} - \varepsilon, \text{-} \varepsilon < \psi(\mathrm{k}) < \varepsilon \text{ if } \mathrm{k} > \underline{\theta} - \varepsilon$
  -

Speculative Attacks – Morris & Shin: X

- Thus, in equilibrium:

$$\mathrm{s}\left(\mathrm{k}+\psi\left(k\right)\right)=a\left(k+\psi\left(k\right)\right)\Rightarrow\frac{1}{2}-\frac{\psi\left(k\right)}{2\varepsilon}$$

- Totally differentiating, we get:

$$a'\left(k+\psi\left(k\right)\right)\left(1+\psi'\left(k\right)\right)=\frac{-\psi'\left(k\right)}{2\varepsilon}$$

- Continuing to solve, we get:

$$a'\left(\theta\right)=-\frac{\left[1+2\varepsilon a'\left(\theta\right)\right]\psi'\left(k\right)}{2\varepsilon}$$

$$\psi'\left(k\right)=-\frac{2\varepsilon a'\left(\theta\right)}{\left[1+2\varepsilon a'\left(\theta\right)\right]}<0$$

–

Speculative Attacks – Morris & Shin: XI

- Thus we have shown the the utility of the cutoff strategy is decreasing in the cutoff.

- Moreover, since the utility is given by an integral, the utility function is differentiable and thus continuous. This concludes our proof of lemma 2.

- Lemma 3: There is a unique cutoff value of the signal such that all investors attack if they receive a value lower than the cutoff and do not attack otherwise.

  –

Speculative Attacks – Morris & Shin: XII

- Proof of lemma 3:

- First note that for small enough k, the utility of the cutoff strategy is positive and for large enough k, it is negative:

$$u(k, I_k) > 0, \mathrm{k} \in [0, \underline{\theta}], u(k, I_k) < 0, \mathrm{k} > \overline{\theta}$$

- By continuity of the utility function and the fact that it is strictly decreasing in k, we know that (1.) there exists a level of k such that the utility from following the cutoff strategy is equal to zero – i.e. equal to the utility of not attacking and (2.) that such a level of k is unique.

- Now we make the following definitions:

$$x^* = x \ni u(x, I_x) = 0, \underline{\mathrm{x}} = \inf\{x \mid \pi(x) < 1\}, \overline{\mathrm{x}} = \inf\{x \mid \pi(x) > 0\},$$

–

Speculative Attacks – Morris & Shin: XIII

- We will show that:

$$x^{*} = \underline{x} = \overline{x}$$

- First note that:

$$\underline{x} \leq \overline{x}$$

- This is due to:

$$\overline{x} \geq \sup\{x \mid 0 < \pi(x) < 1\} \geq \inf\{x \mid 0 < \pi(x) < 1\} \geq \underline{x}$$

- Now it remains to show the reverse:

$$\underline{\mathrm{x}} \geq x^{*} \geq \overline{\mathrm{x}}$$

Speculative Attacks – Morris & Shin: XIV

- When pi(x)<1, then there are at least some investors who weakly prefer not to attack. Taking the limit, we get:

$$u(x,\pi(x)) \leq 0 \Rightarrow \lim_{x \to \underline{x}} u(x,\pi(x)) \leq 0 \Rightarrow u(\underline{x},\pi(x)) \leq 0$$

- But: $I_{\underline{x}}(x) > \pi(x) \forall x$

- So, from lemma 1, we get:

$$\mathrm{u}(\underline{\mathrm{x}},\mathrm{I}_{\underline{\mathrm{x}}}) \leq \mathrm{u}(\underline{\mathrm{x}},\mathrm{I}_{\underline{\mathrm{x}}}) \geq 0 \Rightarrow \underline{x} \geq x^*$$

Speculative Attacks – Morris & Shin: XV

- A symmetric argument gets us:

$$x^* \geq \bar{x}$$

- Thus, we have lemma 3. The unique strategy followed by investors is the cutoff strategy at x*:

$$I_{x^*}(x)$$

- We have figured out the unique strategy by investors and by the government. It remains to show that the equilibrium is unique.

Speculative Attacks – Morris & Shin: XVI

- The equilibrium strategy for investors is given by:

$$s\left(\theta, I_{x^*}\right) = \begin{cases} 1 & \theta \leq \mathrm{x}^* - \varepsilon \\ \dfrac{1}{2} - \dfrac{1}{2\varepsilon}\left(\theta - x^*\right) & \mathrm{x}^* - \varepsilon \leq \theta \leq \mathrm{x}^* + \varepsilon \\ 0 & \theta \geq \mathrm{x}^* + \varepsilon \end{cases}$$

- We know that:

$$\underline{\theta} + \varepsilon > x^* > \underline{\theta} - \varepsilon$$

- And s is strictly increasing over this range. Moreover, below this range, s is below a and above it, a is below s. This means that a and s cross precisely once.

Speculative Attacks – Morris & Shin: XVII

- Two additional notes:
  - There is a proof that uniqueness of equilibria remains even as epsilon (and thus the variance of the noise) goes to zero. There is a mistake in the proof by Morris and Shin in their original AER paper. The correct proof is a note in a subsequent AER edition.
  - Hellwig, Mukherji, and Tsyvinski have a paper which says that the Morris and Shin results on uniqueness are dependent upon whether or not the central bank has a discrete or continuous policy (like setting interest rates versus just defending) and whether or not the signals are public or private. This paper has a revise and resubmit right now at AER.

Speculative Attacks: Generation III

- Example: Aghion, Banerjee and Bachetta (JET)
  - Can explain why currency crises covary with recessions
  - Debt denominated in foreign currency
  - If speculators believe there will be an attack, then the value of debt will go up, making firms closer to insolvency.
  - Thus interest rates will rise and capital will flow out.
  - In order to restore low interest rates, central bank will let the currency float but this raises the value of external debt, causing inability of firms to borrow abroad and incompleted projects (recession).
  - However, if speculators did not believe there would be an attack, then none of this will happen – multiple equilibria (East Asian Currency Crisis).