Economics 326: Profit Maximization and Production

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1. Profit Maximization

What is profit maximization?

Firms decide how many inputs to purchase in order to produce:

- K for Kapital
- L for Labor
• Give capital and labor decisions, firms produce output:

\[ Y = F(K, L) \]

• This function is called the production function.

• Revenue is quantity of output (number of goods produced and sold) times price:

\[ R = pY = pF(K, L) \]

• What are the costs that the producer faces?

  – Labor costs: wage bill which is just the number of worker hours times the wage

    \[ wL \]

  – Capital costs: rental rate on capital times the amount of capital used

    \[ rK \]
• So total costs are:

\[ C = wL + rK \]

• Profits are revenues minus costs

\[ \Pi(K, L) = R - C = pF(K, L) - (wL + rK) \]
\[ = pF(K, L) - wL - rK \]

• Similar to Utility Maximization
  
  – Maximize benefits minus costs rather than Benefits subject to cost constraints
  
  – Why this differences? Benefits are in dollars and costs are in dollars

• What are the endogenous variables? What are the exogenous parameters?

• So firms choose inputs to maximize profits.
• What are the analogues of Marshallian Demand? Input Demand.

2 Production

• The production function is like the utility function of the supply side of the economy.

• The simples interesting choices are for two inputs (capital and labor) but that means 3 dimensional graphs (output, capital labor) as with utility functions.

• So, we graph the level sets of production function - the analogue of indifference curves. They are called isoquants. An isoquant (iso = same, quant = quantity) is a combination of labor and capital inputs that gives the same production level.
Different from indifference curves, isoquants have cardinal not just ordinal meaning.

- Indifference curves: the indifference curve \( \bar{U} (X, Y) = 5 \) is the set of all commodities \( X \) and \( Y \) such that utility is 5. Here utility being 5 has no meaning. Just the order of utility has meaning.

- Isoquants: \( \bar{F} (K, L) = 5 \) is the set of all input pairs \((K, L)\) such that output is 5.

Besides Isoquants, the returns to scale of production are a very important property. The returns to scale will be very important for the theory of monopoly.

\[
F (\lambda K, \lambda L) < \lambda F (K, L) : \text{ Decreasing Returns}
\]
\[
F (\lambda K, \lambda L) = \lambda F (K, L) : \text{ Constant Returns}
\]
\[
F (\lambda K, \lambda L) > \lambda F (K, L) : \text{ Increasing Returns}
\]

How do we interpret these three possible returns to scale?
– Decreasing Returns: As you produce more, you become less productive.

* Example: suppose one unit of labor and capital produce 5 units of output. moreover lets say that if you double the inputs (2 units of labor and capital), you get 8 units of output.

\[
F(1, 1) = 5 \\
F(2, 2) = 8 < 10 = 2 \times F(1, 1)
\]

* So then 1 unit of capital costs \( r \) and one unit of labor costs \( w \), the costs of producing 5 units is \( r + w \). However if I double the costs by doubling the inputs (2\( r \) + 2\( w \) costs), I less than double the output.

* Firms which have decreasing returns to scale tend to be small.

– Constant Returns: As you produce more, your productivity stays the same
* Example: suppose one unit of labor and capital produce 5 units of output. moreover lets say that if you double the inputs (2 units of labor and capital), you get 10 units of output.

\[
F(1, 1) = 5 \\
F(2, 2) = 10 = 10 = 2 \times F(1, 1)
\]

* So then 1 unit of capital costs \(r\) and one unit of labor costs \(w\), the costs of producing 5 units is \(r + w\). However if I double the costs by doubling the inputs \((2r + 2w\) costs), I exactly double the output.

* Firms which have constant returns to scale can be of any size.

  – Increasing Returns: As you produce more, your productivity increases

* Example: suppose it one unit of labor and capital produce 5 units of output. moreover lets
say that if you double the inputs (2 units of labor and capital), you get 8 units of output.

\[ F(1, 1) = 5 \]
\[ F(2, 2) = 14 > 10 = 2 \times F(1, 1) \]

So then 1 unit of capital costs \( r\) and one unit of labor costs \( w\), the costs of producing 5 units is \( r + w\). However if I double the costs by doubling the inputs (\(2r + 2w\) costs), I have more than double the output!

* Firms which have decreasing returns to scale tend to be large. They tend to be monopolists or oligopolists (more on this later in the course).