Microeconometrics: Switching Regressions

Ethan Kaplan

1 Basic Switching Regressions

- Used for estimating model of multiple regime with endogenous regime (i.e. union vs. non-union sector, household production vs. market production (potentially), services vs. industry)

- Three Equations with observables X, W_A, W_B, C, Q

  – Two Regime Equations

$$\begin{aligned} W_{Ai} &= X\beta_{Ai} + \gamma Z_{Ai} + e_{Ai} \\ W_{Bi} &= X\beta_{Bi} + \mu Z_{Bi} + e_{Bi} \end{aligned}$$

  – One Switching Equation

$$I_i = \alpha Q_i - \delta_i$$

$$\begin{aligned} I_i^* &= 1 \text{ if } I_i \leq 0 \Longleftrightarrow \alpha Q_i \geq \delta_i \\ I_i^* &= 0 \text{ if } I_i \geq 0 \Longleftrightarrow \alpha Q_i < \delta_i \end{aligned}$$

- Note that $Ee_{Ai} = 0$ but $E\left(e_{Ai} | I_i^* = 1\right) \neq 0$ so that separate estimation of OLS will induce bias (i.e. endogenous regime).

- Estimate by maximum likelihood (can also be done with Heckman Selection Correction equation by equation): jointly normal

$$
\begin{aligned}
& L\left(\alpha, \beta_{Ai}, \beta_{Bi}, \gamma, \mu\right) \\
= \; & \Pi_i \left[\int_{-\infty}^{\alpha Q_i} f\left(W_{Ai} - X\beta_{Ai} - \gamma Z_{Ai}\right) de_{Ai} \right]^{I_i^*} \\
& \bullet \left[\int_{\alpha Q_i}^{\infty} g\left(W_{Bi} - X\beta_{Bi} - \mu Z_{Bi}\right) de_{Bi} \right]^{1-I_i^*}
\end{aligned}
$$

- The variance-covariance matrix is given by:

$$
\begin{pmatrix}
\sigma_A^2 & \sigma_{AB} & \sigma_{AI} \\
\sigma_{AB} & \sigma_B^2 & \sigma_{BI} \\
\sigma_{AI} & \sigma_{BI} & \sigma_I^2
\end{pmatrix}
$$

which introduces the extra parameters:

$$\sigma_A^2, \sigma_B^2, \sigma_I^2, \sigma_{AB}, \sigma_{AI}, \sigma_{BI}$$

which can be estimated or assumed. Often σ_I^2 is normalized to 1. Interpretations of the variance elements of the variance-covariance matrix?

- Often the switching equation is just the difference between the two regime equations plus noise (i.e. on mean the differential wage plus noise for taste). This is usually the case when the switching equation reflects a choice (i.e. where the wage of the worker is higher net of idiosyncratic tastes for working in a given industry):

  $$I_i = X\beta_{Bi} + \mu Z_{Bi} + e_{Bi} - (X\beta_{Ai} + \gamma Z_{Ai} + e_{Ai}) - \delta_i$$

2 Switching Regressions with Unobservable States

- What happens if I_i^* is not observed? For example, $I_i = 1$ is a high marginal returns to education labor market and $I_i = 0$ is a low marginal returns labor market. Then instead of maximizing

$$
\begin{aligned}
& L\left(\alpha, \beta_{Ai}, \beta_{Bi}, \gamma, \mu\right) \\
= \; & \Pi_i \left[\int_{-\infty}^{\alpha Q_i} f\left(W_{Ai} - X\beta_{Ai} - \gamma Z_{Ai}\right) de_{Ai} \right]^{I_i^*} \\
& \bullet \left[\int_{\alpha Q_i}^{\infty} g\left(W_{Bi} - X\beta_{Bi} - \mu Z_{Bi}\right) de_{Bi} \right]^{1 - I_i^*}
\end{aligned}
$$

you can maximize the likelihood density for expected e (i.e. you can replace the state with the probability of a state)

$$\sum_i P\left(\alpha Q_i \geq \delta_i\right)\left[\int_{-\infty}^{\alpha Q_i} f\left(W_{Ai} - X\beta_{Ai} - \gamma Z_{Ai}\right) de_{Ai}\right]$$
$$+\sum_i P\left(\alpha Q_i < \delta_i\right)\left[\int_{\alpha Q_i}^{\infty} g\left(W_{Bi} - X\beta_{Bi} - \mu Z_{Bi}\right) de_{Bi}\right]$$

- This, in turn, equals

$$\sum_i \left[1 - \Phi\left(\alpha Q_i\right)\right]\left[\int_{-\infty}^{\alpha Q_i} f\left(W_{Ai} - X\beta_{Ai} - \gamma Z_{Ai}\right) de_{Ai}\right]$$
$$+\sum_i \Phi\left(\alpha Q_i\right)\left[\int_{\alpha Q_i}^{\infty} g\left(W_{Bi} - X\beta_{Bi} - \mu Z_{Bi}\right) de_{Bi}\right]$$