The Labor Boundaries of the Firm: Employment and Independent Contracting*

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Abstract

When contracts are incomplete and control over pre-existing assets are already assigned through onwership, the employment relation gives employers rights to expropriate worker investment. In contrast, independent contractors retain rights to walk away from their relationships without relenquishing their investments. Therefore, independent contractors have better incentives to invest in non-contractible aspects of production. However, employment-at-will gives better incentives to firms in that it safeguards them against worker holdup.

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1. Introduction

Economics lacks a theory of employment. There is an extensive literature on forms of compensation which explain what types of employment contracts exist. However, few papers address why some workers are hired within the firm (i.e. labeled employees) while others are hired externally as independent contractors. An early paper by Herbert Simon (1951) argues that the benefits to a firm of hiring using direct employment are that the firm gains the rights to control what labor does. However, in Simon's framework, it is not clear why these control rights matter. At any point in time, as long as the worker has the freedom to quit (granted by anti-slavery laws), the worker can always contest the employer's authority and bargain jointly over tasks and compensation. It is not clear why the outcome of this bargain should be different depending on whether the worker is labeled an employee as or an independent contractor. Our paper borrows from the property rights theory of the firm, developed in Grossman and Hart (1986) and Hart and Moore (1990) to argue that employment grants control rights over ex-ante non-contractible investments made by workers, and that this assignment of rights affects the efficiency of production and, therefore, the choice of the worker's legal status.

Advances have been made in understanding the ownership of capital, but there has been comparatively little work on the employment of labor. Early work in this literature viewed the firm as a "nexus of contracts." (Jensen and Meckling (1976)). Alchian and Demsetz (1972) argued that "the firm has no power of fiat, no authority, no disciplinary action any different in the slightest degree from ordinary market contracting between any two people." Others have gone beyond contractual relations to better understand "power" or "authority" that appears to mediate exchange within a firm. Oliver Hart, John Moore, and Sanford Grossman (hence referred to as GHM) have been responsible for developing the "property rights" approach to the theory of the firm. Both Grossman and Hart (1986) and Hart and Moore (1990) emphasize the ownership of "non-human assets" (physical capital, patents, etc.) as a way to exercise power when all future exchanges cannot be contracted upon. By owning assets, the owner can threaten to exclude employees who specialize themselves to these assets through relationship specific investments. As a result, ownership of assets is a way for the owner to obtain greater returns on relation-specific investments, since it allows her to threaten others with exclusion in the future. This property rights view of the firm is built upon Oliver Williamson's (1985) transactions costs theory of the firm.

In this approach a worker can be considered an "in-house" employee of company A if the assets she is specialized to are owned by that company. If these assets are owned by yet another company B, then she is "outsourced" in the GHM sense. This general approach of defining the firm as a collection of workers who are significantly specialized to a set of assets (and hence to each other) is also used by Rajan and Zingales (1998, 2001), although they allow for mechanisms other than ownership (namely access or opportunity to specialize to the asset) to exercise power as well. Overall, this represents what we call the "asset boundaries" of the firm. Ownership of (or access to) pre-existing assets can be used to exercise power *ex post* and hence can be used to structure incentives.

The property rights theory of the firm is a theory of the capital asset boundaries of the firm (i.e. which firms own which assets). Firms own capital because it gives them control over the capital. However, the property rights theory is not able to provide a satisfactory explanation of the labor boundaries of the firm. The crucial difference between labor and capital is that the law grants rights to control assets based upon ownership. However, the law does not grant control rights over labor as anti-slavery laws grant labor the right to quit. In particular, labor always has the rights to quit both as an employee and an independent contractor.

In this paper, we hope to show that though the law does not grant owners the right to control labor based upon employment status, *legal* definitions are nevertheless relevant in the decision to hire or contract out. While the GHM framework helps us address the issue of "who owns the asset," that sort of analysis is unlikely to be fruitful in explaining contracting out of labor services. Here we are specifically thinking of software consultants, management, IT contractors, accountants, writers, performing artists, etc. One defining characteristic of these cases is the lack of any obvious transference of existing assets. Rather, the decision facing companies here is whether to structure the production relation through the legal label of employment or through the legal label of independent contracting, without any change in asset ownership.

Some economists have focused on the compensation mechanism as a defining characteristic of employment (see Holmstrom and Milgrom(1994), Ellman(2004)). In this view, employment is defined as having low-powered wage contracts, whose terms are left vague. This stands in contrast to contracting out, where detailed, high-powered contracts are used. However, piece rates are sometimes used for workers hired under employment contracts and fixed wages are often used to compensate independent contractors.

Labeling a worker as an "employee" gives the owner[1] of a firm a set of rights which differ from the situation where the worker is labelled an "independent contractor." This is particularly true in the realm of intellectual property rights.

Hiring employees directly gives the owner greater property rights over investments made by workers – what we call greater rights to expropriate. (Section 2 below explores expropriability of various types of knowledge in greater depth.) When workers' investments are embodied in innovations which can be patented or copyrighted, default rules of assignment used by courts favor the owner when the worker is hired as an employee. However, the opposite is true when the worker is

[1] The worker is referred to as an "employee" and the owner as an "employer" only when employment at will is the organizational strucutre.

hired as an independent contractor. The main evidence for our theory of employment comes from employment law in the United States. However most countries similarly grant differential rights to knowledge created on the job to Independent Contractors and Employees[2]. Even when the ownership over the innovations is not assigned to the employer, he typically enjoys a "shop right" – a royalty free license to use the invention. Such default rules are clearly important when there are no separate contracts assigning property rights over future innovations. When employees' inventions are more nebulous – and not patentable or copyrightable – employers may yet be able to secure ownership (or shop rights) by claiming that these inventions are "trade secrets." Their ability to do so, however, is diminished when the workers are not directly employed. In sum, the accumulated set of statutes and precedents make the employment contract attractive to employers if they are attempting to secure property rights over employee investments. Economically, this ex-post benefit for the employer can also be an ex-ante liability. The ability of the employer to more easily expropriate the worker's investment means that the worker is less motivated to produce new knowledge.

In section 3, we present our model and develop the implications of the differential property rights accruing to the owner when using employment as opposed to contracting out. Following much of the literature, we assume that the organizational form is chosen solely based on efficiency considerations. We characterize the equilibria under employment and independent contracting. Greater rights of expropriation under employment makes the owner's threat to replace a worker more credible. We find that when workers' incentives are unimportant, they will be employed in-house. In contrast, when the owner's incentives are unimportant, the work will be performed by independent contractors. In section 5, we relax the assumption that ex ante transfers neutralize all distributional concerns. As

[2] *See a previous working paper version, (Dube and Kaplan (2005)) which documents in detail legal evidence in favor of our theory.*

a consequence, organizational form picked by the owner now does not maximize total surplus, and only maximizes his profits. It is shown that employment can be chosen purely on such distributional grounds, even when the two forms are equivalent under value maximization, providing an explanation for the prevalence of employment as an organizational form. We conclude with a discussion of possible directions for future work.

2. Knowledge and Expropriation

The property rights or incomplete contracts literature makes a distinction between assets and investments: assets can be transferred between people, but investments cannot. Therefore, for GHM and authors following this tradition, the issue of property rights has been limited to pre-existing, non-human assets, which are alienable. In contrast, investments that come about as a consequence of people's effort – especially investments in knowledge – are always considered to be inalienable. Workers own their investments simply due to the assumed nature of such investments; consequently, workers are able to withdraw them from production upon the dissolution of a relationship. The question that has been addressed is how to use property rights over alienable assets (or access to these assets) to influence inalienable investments.

In contrast, we allow investments to be partially divorceable from the worker and hence potentially contested. The firm owner's procurement of the divorceable investments created by employees is what we call "expropriation." Even when the investment is in knowledge, it might be expropriable. In general, we think of investments as expropriable to greater or lesser degrees based upon the technological and legal characteristics of production.

For knowledge to be expropriable, it first has to be divorceable. Some investments in knowledge such as workers' improved dexterity working with a particular machine, learning a programming language, or better understanding accounting rules, are not divorceable from the worker and hence not expropriable by the firm. These are typically considered to be human capital investments.

However, other types of knowledge investments are indeed divorceable. Here, rights over the investments are crucial. Sometimes, expropriation might be a contractual matter. Assignment contracts signed by parties might determine ownership of patents or copyrights over future assets. However, in some situations, the nature of the investment makes it difficult to write detailed contingent contracts assigning future ownership. Sometimes, the innovation comes as a by-product of the primary work relationship.

In their 2001 paper, Rajan and Zingales, too, discuss concerns about owners' ability to secure future property rights in the face of worker hold-up. And they also consider a situation where such property rights cannot be perfectly secured through *ex ante* contracts. However, there are two important distinctions. First, their concern is the employee's ability to expropriate the owner's initial (and critical) asset. By having "access" to the asset, employees may be able to replicate it and start their own competing venture. In this, Rajan and Zingales take the traditional view of investments as being inalienable. In contrast, our environment is one where future property rights over workers' (partially divorceable) investments are in question. Secondly, and more crucially, the margin they look at is not legal forms, although they do touch on the issue.

3. Organizational Choice with Full *Ex Ante* Transfers

3.1. The Model Setup

The owner of a primary firm is assumed to possess a critical asset which is indispensable for production. There are three dates during which strategic actions are taken: date 0, date 1, and date 2. At date 0, the firm's owner hires N workers, chooses the organizational form, and engages in any side payments that are necessary and feasible. By organizational form, we mean that she may either label each worker as a direct employee or as an independent contractor. As in most of the literature on the theory of the firm, in this section we assume that *ex ante* transfer payments can be made between the various parties. Moreover, the parties are assumed to share a common discount rate, which is taken to be zero for simplicity. We assume that for the workers' date 1 investments to be productive, the owner has to provide a basic amount of training and instructions, which is costly. When N workers are hired to invest, this preparatory cost of investment is denoted as $C(N)$, where $C' > 0$, and $C'' > 0$.

At date 1, both the workers and the firm owner make non-contractible, relation specific investments which will be usable for the production of a good X at date 2. X, too, is not contractible at date 1. Each worker who makes an investment e_j incurs a cost e_j. The owner is assumed to make an investment, denoted by i, at cost i which simultaneously affects the productivity of all the workers.

At date 2, a worker can produce a value of $X(i, e_j)$. At the beginning of date 2, workers and firms can write contracts on $X(e_j, i)$. Firms and workers then produce, and at the end of the period, the firm receives its revenue from X and meets its contractual obligations. However, what actual production occurs depends on the cooperation of various players at date 2.

We will first consider the case of employment. If an employee leaves the firm, αe_j of his investment can be expropriated, where $0 < \alpha < 1$. An α less than unity reflects the difficulty of an employer to expropriate or retain the full value of a worker's investment once that worker leaves the firm. Given this degree of expropriability, the owner of the firm can hire a replacement worker at date 2 at the reservation level of wage, $\underline{w}$, and this worker produces an overall value of $X(i, \alpha e_j)$, provided $X(i, \alpha e_j) \geq \underline{w}$. Of course, if the original employee cooperates, he and the owner together produce $X(i, e_j)$.

Next we consider an independent contracting relationship. The only difference in this situation is that the firm owner no longer retains any rights to use the worker's specific investment upon dissolution of the relationship. Without the ability to expropriate any portion of the independent contractor's investments, the owner (together with a replacement worker at cost $\underline{w}$) can only produce an overall value of $X(i, 0)$ without the independent contractor's cooperation in the second period.

Finally, we make some technical assumptions:

(1.) Decreasing marginal benefits of both firm and worker investment

:

$$
\begin{aligned}
(a.)\frac{\partial X(e_j, i)}{\partial i} &> 0, \frac{\partial^2 X(e_j, i)}{\partial i^2} < 0 \\
(b.)\frac{\partial X(e_j, i)}{\partial e_j} &> 0, \frac{\partial^2 X(e_j, i)}{\partial e_j^2} < 0
\end{aligned}
$$

(2.) Both investments are necessary for production of $X(\cdot)$:

$$X(0, i) = X(e_j, 0) = 0$$

(3.) X is bounded above, such that $\forall e, i : X(e, i) < \overline{X}$

Assumption (1) is standard. Assumption (2) means that hiring an additional workers with no specific investments is never worthwhile. Assumption (3) is not important substantively, but is used in the proofs to help rule out the possibility that N is unboundedly large.

The timeline is as follow:

Date 0 : The firm chooses employment level (N) and organizational form - employment (EMP) or Independent Contracting(IC). Side payments are made. If

N workers are employed, the firm incurs a preparatory cost of investment $C(N)$

Date 1 : Workers also choose level of e_j and the owner chooses level of i.

Date 2 : Contracts over Y and $X(e_j, i)$ are negotiated. Production ensues, and payments are made.

Production is assumed to be additively separable across workers and workers are assumed to be symmetric. The separability of production means that the date 2 bargaining games are separable as well. This has two simplifying implications. First, we can separately derive payoffs associated with each firm-worker pair. Second, whichever organizational form maximizes firm profits with respect to one worker also maximizes firm profits with respect to all the workers.

The aggregate net production function,then, is:

$$\Pi(\mathbf{e}, i, N) = \sum_{j=1}^{N} [X(e_j, i) + Y] - i - \sum_j e_j - C(N)$$

We can solve for the first best levels of investment by maximizing this function. The optimality conditions (imposing symmetry of the worker investments) are as follows:

$$\begin{aligned}
\text{Worker Investment} &: \frac{\partial X(e_j, i^*(N^*))}{\partial e_j} = 1 \\
\text{Owner Investment} &: N^* \frac{\partial X(e^*(N^*), i)}{\partial i} = 1 \\
\text{Employment} &: N^* = \arg \max_{N \in \{0, \ldots, \overline{N}\}} \Pi(\mathbf{e}^*(N), i^*(N), N)
\end{aligned}$$

These first order conditions show what the investment levels of the two parties would be if they were able to write contracts on investment (or output) at date 1. The convexity of $C(N)$ and the bound on X ensures that N is bounded as well. Since $X(e, i) < \overline{X}$, we can derive $\overline{N}$ such that $\frac{C(\overline{N})}{\overline{N}} \doteq \overline{X}$, where $\doteq$ signifies equality up to an integer constraint. It can never pay to produce more than $\overline{N}$ since the firm will be making negative profits *regardless of* $i, e, \underline{w}$. Below, we use the result that $N^* \in \{0, ..., \overline{N}\}$.

3.2. Returns from Different Organizational Forms

Having looked at what parties would do if they could contract on output (X and Y) or inputs (e_j and i) at date 1, we now look at what happens when they are not able to write contracts on output until date 2 and cannot write contracts on the level of investments at all. In this case, the second-best solution is to affect incentives through the *ex ante* allocation of property rights over the investments. We consider two organizational forms for allocating such property rights: employment (EMP), and independent contracting (IC). The owner can either directly hire the worker as an employee, in which case she obtains the future rights to the worker's knowledge, or hire the worker as an independent contractor, in which case the worker retains full rights to the knowledge he creates.

3.2.1. Independent Contracting

We first characterize the date 2 bargaining game that determines the parties' payoffs under independent contracting. If at date 2 the independent contractor leaves the relationship, there will be a loss to the firm in terms of its ability to produce X. Since the owner cannot expropriate any of the independent contractor's investments, a replacement worker can only produce a value of $X(0, i) = 0$. The owner, then, has an outside option equal to 0. Since the investment is assumed to be completely relation specific, the worker's outside option is just his outside wage $\underline{w}$. Therefore, the surplus from worker j staying in the relationship is:

$$\max\{X(e_j, i) - \underline{w}, 0\}$$

The worker's returns are then given by his outside option plus his share of the surplus. As is the convention in Nash bargaining models, we assume the share to be half. The worker's date 2 payoff is then:

$$\frac{X(e_j, i) + \underline{w}}{2}$$

The worker's expected returns (at date 1) are his ex post payments net of his effort cost:

$$\omega(e_j|i) = \frac{X(e_j, i) + \underline{w}}{2} - e_j$$

At date 2, the owner receives half the surplus. For all N workers, the owner's expected returns at date 1 are the following:

$$\pi(i|\mathbf{e}) = \frac{\sum_j X(e_j, i) - N\underline{w}}{2} - i$$

Finally, we can solve for the first order conditions that determine the equilibrium investment levels for all parties (e^*_{IC}, i^*_{IC}), imposing symmetric investment levels for workers. [3]

$$\begin{aligned} \text{Worker} &: \quad \frac{\partial X\left(e^*_{IC}, i^*_{IC}\right)}{\partial e} = 2 \\ \text{Firm} &: \quad \sum_j \frac{\partial X\left(e^*_{IC}, i^*_{IC}\right)}{\partial i} = 2 \end{aligned}$$

Note that with *ex ante* transferability at date 0, the firm extracts the difference

[3] For a non zero level of investment to take place, the "investment participation constraints" of both parties have to be satisfied:

IPC (worker) : $X(e^*_{IC}, i^*_{IC}) + \underline{w} > 2e^*_{IC}$

IPC (owner) : $X(e^*_{IC}, i^*_{IC}) - \underline{w} > \frac{2i^*_{IC}}{N}$

For simplicity, we assume that these IPC's are not binding.

between the date 2 return and the worker's outside opportunity from the worker. In other words, under independent contracting, the firm demands from a worker the following date 0 transfer:

$$t_j^* = \frac{1}{2}\left(Xe^*, i^* - \underline{w}\right) - e^*$$

This payment is based on future expected investments, and does not play a role in the determination of incentives and organizational forms. Finally, the owner hiring N workers receives the following date 0 expected profit:

$$\Pi_{IC}(N) = N\left(X(e^*_{IC}(N), i^*_{IC}(N)) - e^*_{IC}(N)\right) - i^*_{IC}(N) - N\underline{w} - C(N)$$

The optimum N is chosen as:

$$N^*_{IC} = \arg \max_{N\in\{0,\ldots,\overline{N}\}} \Pi_{IC}(N)$$

3.2.2. Employment

The case of EMP is somewhat more complicated. We proceed in two steps. First we calculate the date 2 payoffs for both parties. Next we characterize the parties' reaction functions for investment.

The key difference between employment and independent contracting is that if an employee leaves at date 2, the firm now keeps αe of that employee's investment. This means that the surplus, or the added benefit of cooperation accruing to the firm/worker pair, is no longer the full value of production on $X(e, i)$. Rather, the surplus now is the difference between full production and production with αe of the worker's investment with a replacement, provided it is greater than zero:

$$\max\{X(e_j,i)-\underline{w},\quad X(e_j,i)-X\left(\alpha e_j,i\right),\ 0\}$$

As before, the worker's outside option at date 2 is $\underline{w}$. With equal sharing of the surplus, an employee's date 2 payoff is:

$$\begin{array}{lccc} \dfrac{X(e_j,i)}{2}-\dfrac{X\left(\alpha e_j,i\right)}{2}+\underline{w} & \text{if} & e_j & > & \widetilde{e}(\underline{w},i):X(\alpha\widetilde{e},i)\geq\underline{w} \\ \dfrac{X(e_j,i)+\underline{w}}{2} & \text{if} & e_j & \leq & \widetilde{e}(\underline{w},i) \end{array}$$

If the worker's investment is smaller than $\widetilde{e}$, it will not be worth replacing her if she leaves. If her investment is greater than $\widetilde{e}$, it will be worth producing even with a replacement worker with productivity αe_j.

The owner in turn receives the following date 2 return:

$$\sum_m \frac{1}{2}\left(X(e_m,i)-\underline{w}\right)+\sum_k\left(\frac{1}{2}\left(X(e_k,i)+X\left(\alpha e_k,i\right)\right)-\underline{w}\right)$$
$$\text{where } \forall e_m,\ X(\alpha e_m,i)<\underline{w}<X(e_m,i),\ \text{and } \forall e_k,\ X(\alpha e_k,i)\geq\underline{w}$$

For a given profile of worker investment, $\mathbf{e}$, the owner will earn more at date 2 when he can credibly threaten replacement (all e_k).

The date 1 expected returns (net of investment costs) for the worker and the owner are as follows.

$$\begin{aligned} \omega(e_j|i) &= \frac{X(e_j,i)-\underline{w}}{2}+\underline{w}-e_j && \text{if } e_j\leq\widetilde{e}(\underline{w},i) \\ \omega(e_j|i) &= \frac{X(e_j,i)}{2}-\frac{X\left(\alpha e_j,i\right)}{2}+\underline{w}-e_j && \text{if } e_j\geq\widetilde{e}(\underline{w},i) \end{aligned}$$

$$\pi(i|\mathbf{e}) = \sum_m \frac{1}{2}\left(X(e_m, i) - \underline{w}\right) + \sum_k \left(\frac{1}{2}\left(X(e_k, i) + X\left(\alpha e_k, i\right)\right) - \underline{w}\right) - i$$

$$\text{where } \forall e_m,\ X(\alpha e_m, i) < \underline{w} < X(e_m, i),\ \text{ and } \forall e_k,\ X(\alpha e_k, i) \geq \underline{w}$$

The Worker's Reaction Function under Employment First, we characterize a worker's reaction function. If the worker invests more than $\widetilde{e}(\underline{w}, i)$, it is worthwhile for the owner to replace her if she leaves. However, if she invests less, she will not be replaced. The worker's marginal return from investment is greater when she is not replaced. This generates a kink in the worker's date 2 payoff at $e_j = \widetilde{e}(\underline{w}, i)$. In other words, if she is investing $e^+ > \widetilde{e}(\underline{w}, i)$, her marginal return from investment is $\left(\frac{1}{2}\left(\frac{\partial X(e_j,i^*)}{\partial e_j} - \alpha\frac{\partial X(\alpha e_j,i^*)}{\partial e_j}\right) - 1\right)$. If she leaves, it will be profitable for the owner to replace her, and so she will face a lower return on her investment. However, if she is investing $e^- < \widetilde{e}(\underline{w}, i)$, the marginal return is $\left(\frac{1}{2}\frac{\partial X(e_j,i^*)}{\partial e_j} - 1\right)$ as replacement is no longer credible; her returns in this regime are the same as in the independent contracting case. Conditional on the owner's investment i, the worker's best response investment level will vary in the three possible regimes. In order to characterize these regimes, we will need some new definitions.

$$
\begin{aligned}
(1) \quad e^*_{NR}(i) &= \arg\max_e \left(\frac{X(e_j, i)}{2} - e_j\right) \\
(2) \quad e^*_R(i) &= \arg\max_e \left(\frac{X(e_j, i)}{2} - \frac{X\left(\alpha e_j, i\right)}{2} - e_j\right) \\
(3) \quad e^*_C(i) &= \widetilde{e}(\underline{w}, i)
\end{aligned}
$$

Note that $e^*_{NR}(i)$ is the worker's best response level of investment if there were no threat of replacement; $e^*_R(i)$ is the best response level when there is replacement; and $e^*_C(i)$ is the "corner" case, when the owner is indifferent between replacing and not. Furthermore, $e^*_{NR}(i) > e^*_R(i)$ as long as $\alpha > 0$. With these definitions, we can characterize the various cases.[4]

Regime 1: *Interior Solution with Replacement.* Formally, .$\widetilde{e}(\underline{w}, i) < e^*_R(i)$

In this situation, the replacement maxima $e^*_R(i)$ is at a sufficiently high level (greater than the trigger point $\widetilde{e}(i)$). The first order conditions for the workers' investment are as follows:

$$\frac{\partial X(e^*_R, i)}{\partial e} - \alpha \frac{\partial X\left(\alpha e^*_R, i\right)}{\partial e} = 2$$

Regime 2: *Interior Solution without Replacement.* Formally, $\widetilde{e}(\underline{w}, i) > e^*_{NR}(i)$.

In this situation, the investment needed to trigger replacement is very high - greater than even the non-replacement maxima $e^*_R(i)$ (and by definition greater than the replacement optimum $e^*_R(i)$). The employee will invest $e^*_{NR}(i)$ without fear of replacement, and the first order conditions for the workers' investment are as follows:

$$\frac{\partial X(e^*_{NR}, i)}{\partial e} = 2$$

Regime 3: *Corner solution.* Formally, $e^*_R(i) < \widetilde{e}(\underline{w}, i) < e^*_{NR}(i)$.

In this situation, the worker's investments are not determined by marginal conditions. Rather the worker invests $e^*_C(i)$ - up to the level where she would not be replaced if she left. It can only occur when the trigger point $\widetilde{e}(\underline{w}, i)$ is at an intermediate level, such that at $\widetilde{e}(\underline{w}, i)$ the left derivative (return without replacement) is positive, but the right derivative (return with replacement) is negative.

[4] As in the IC case, we have assumed that parties will invest positive amounts - that their Investment Participation Constraint is not binding. Generally, this will be true if produciton is sufficiently valuable, but we suppress this consideration to keep the analysis tractable.

The Owner's Reaction Function under Employment For the owner, the marginal return from investment is greater in the replacement case. Generally, if the owner invests "enough," the value of production is higher, and replacement of a departing worker is credible. Again, let us first define some values of the owner's investment conditional on a symmetric level of worker investment, e.

$$
\begin{array}{llcl}
(1) & i^*_{NR}(e) & = & \arg\max_i \left(\frac{NX(e,i)}{2} - i \right) \\
(2) & i^*_R(e) & = & \arg\max_i \left(\frac{NX(e,i)}{2} + \frac{NX(\alpha e, i)}{2} - i \right) \\
(3) & \widetilde{i}(\underline{w}, e) & : & X\left(\alpha e, \widetilde{i}\right) = \underline{w}
\end{array}
$$

Note that it $i^*_R(e) > i^*_{NR}(e)$, which is different from the worker case.

Regime 1: Interior Solution with Replacement. Formally, either (1) $.\widetilde{i}(e) \leq i^*_{NR}(e)$, or (2) $i^*_{NR}(e) < \widetilde{i}(e) < i^*_R(e)$ and $\pi(i^*_R) > \pi(i^*_{NR})$

If the trigger point $\widetilde{i}(e)$ is smaller than the local optimum without replacement i^*_{NR}, it follows that the global optimum is with replacement. If the trigger point is greater than i^*_{NR}, then replacement requires that the local optimum i^*_R is more profitable than the local optimum i^*_{NR}. This condition $\pi(i^*_R|e) > \pi(i^*_{NR}|e)$ can be written as:

$$\frac{1}{2}\left(X(e,i^*_R) + X(\alpha e, i^*_R) - \ X(e, i^*_{NR})\right) - (i^*_R - i^*_{NR}) > \frac{\underline{w}}{2}.$$

It follows that the ex-post replacement condition $X(\alpha e, i^*_R) > \underline{w}$ is necessary but not sufficient for the owner to invest i^*_R.

The first order conditions for the owner's investment in this regime are as follows:

$$N\frac{\partial X(e,\ i_R^*)}{\partial i} + N\frac{\partial X\left(\alpha e,\ i_R^*\right)}{\partial i} = 2$$

Regime 2: Interior Solution without Replacement. Formally, either (1) $i_{NR}^*(e) < \widetilde{i}(e) < i_R^*(e)$ and $\pi(i_R^*) < \pi(i_{NR}^*)$, or (2) $\widetilde{i}(e) \geq i_R^*(e)$

If the trigger point is greater than the local optimum with replacement, replacement will never be chosen. If the trigger point is smaller than $i_R^*(e)$, non replacement might yet be preferred if that local optimum is more profitable. In this regime, the first order conditions are as follows:

$$N\frac{\partial X(e, i_{NR}^*)}{\partial i} = 2$$

Unlike the case of the worker, here there is no corner solution at the trigger point $\widetilde{i}(e)$. The critical difference is that the left derivative (without replacement) at the $\widetilde{i}(e)$ is always smaller than the right derivative (with replacement). Another way to make the same point is by noting that $i_R^*(e) > i_{NR}^*(e)$. As a result, the owner will invest at either $i_R^*(e)$ or $i_{NR}^*(e)$.

From the preceding, we can see that for a given N, EMP generically has 2 possible equilibrium configurations - one where replacement is credible in equilibrium, and one where it is not.[5]

$$\{e^*_{EMP}(N), i^*_{EMP}(N)\} = \begin{cases} (e^*_R(N), i^*_R(N)) \\ (e^*_{NR}(N), i^*_{NR}) \end{cases}$$

Finally, employment is determined maximizing the date 0 expected profit.

$$\max_{N\in\{0,\ldots,\overline{N}\}} \Pi_{EMP}(N) = N\left(X(e^*_{EMP}(N), i^*_{EMP}(N)) - e^*_{EMP}(N)\right) - i^*_{EMP}(N) - N\underline{w} - C(N)$$

3.2.3. Comparison of Investment under IC and EMP

Remark 1. *Imposing symmetric investments by workers, we have the following first order conditions for the two legal forms for a given level of employment, N:*

IC	F	$N\frac{\partial X\left(e^*_{IC}, i^*_{IC}\right)}{\partial i} = 2$
	W	$\frac{\partial X\left(e^*_{IC}, i^*_{IC}\right)}{\partial e} = 2$
EMP (No Replacement):	F	$N\frac{\partial X(e^*_{EMP}, i^*_{EMP})}{\partial i} = 2$
	W	$\frac{\partial X(e^*_{EMP}, i^*_{EMP})}{\partial e} = 2$
EMP (Replacement):	F	$N\frac{\partial X(e^*_{EMP}, i^*_{EMP})}{\partial i} + N\frac{\partial X\left(\alpha e^*_{EMP}, i^*_{EMP}\right)}{\partial i} = 2$
	W	$\frac{\partial X(e^*_{EMP}, i^*_{EMP})}{\partial e} - \alpha\frac{\partial X\left(\alpha e^*_{EMP}, i^*_{EMP}\right)}{\partial e} = 2$

First let us consider the replacement equilibrium. The owner's FOC has an extra term, $\frac{\partial X(\alpha e^*, i^*)}{\partial i}$, entering positively; and the worker's FOC now has the same

[5] It can be easily shown that generically, the corner level of investment e^*_C cannot be part of an equilibrium. Assuming the contrary, if the worker invested e^*_C, at date 2 it must be the case that $X(\alpha e^*_C, i^*) = \underline{w}$, and the owner is indifferent regarding replacement, i.e., $i^* = \widetilde{i}(e^*_C)$. However, as we showed above, either $\pi(i^*_R)$ or $\pi(i^*_{NR})$ always exceeds $\pi(\widetilde{i})$, so this cannot be an equilibrium. We say generically because it is possible that the owner *happens to be* indifferent between replacing and not replacing because $i^*_{NR}(e^*_C) < \widetilde{i}(e) < i^*_R(e^*_C)$ and $\pi(i^*_R) = \pi(i^*_{NR})$. However, this happens only for a paramter set of measure zero; if we perturbed $\underline{w}$ for instance, this regime would collapse.

extra term multiplied by alpha, $-\alpha\frac{\partial X(\alpha e^*,i^*)}{\partial e_j}$, but entering negatively. This term represents the added amount produced by the firm under employment when the worker does not cooperate and when replacement is credible. Relative to the independent contracting case above, the inclusion of this term represents a net transfer from the worker to the firm. If replacement is credible, *ceteris paribus*, employment gives better incentives to the owner at the expense of the worker, as it allows some production of X to take place even without the original worker. Unsurprisingly, when replacement is not credible, IC and EMP have exactly the same incentives, even if $\alpha > 0$.

3.3. Characterizing Equilibrium Outcomes

Thus far, we have characterized what incentives are like for each party under the two organizational forms by comparing first order conditions for investment. However, the choice of organizational form will be based on actual equilibrium outcomes involving e,i, and N. Our main results for this section of the paper are that when worker incentives are not important, the firm owner will incentivize herself by choosing employment as the organizational form. Similarly, when the owner's incentives are not important, workers will be incentivized through the choice of independent contracting. Of course, organizational forms will affect incentives only when replacement is credible. As a first step, we establish when the threat of replacement exists in an equilibrium sense. The proof is contained in the appendix.

Proposition 1. *When outside wage $\underline{w}$ is low as compared to the value of production and it is worth hiring at least one worker, the relevant equilibrium configuration for EMP will be one with replacement. $\exists\underline{\underline{\mu}}$ such that $\forall\underline{w} < \underline{\underline{\mu}}$, $(e^*_{EMP}, i^*_{EMP}) = (e^*_R, i^*_R)$*

This result justifies looking at the replacement configuration for EMP for the purposes of our analysis; in other words, below we always assume $(e^*_{EMP}, i^*_{EMP}) = (e^*_R, i^*_R)$ because $\underline{w} < \underline{\underline{\mu}}$. This is after all the interesting case: if production with a replacement worker is not profitable, then the property rights are economically irrelevant.

Next, we define the concept of "unimportance of incentives" and add some technical assumptions. Intuitively, we would like to define incentives as being unimportant when the equilibrium investment made by a party is not very sensitive to the share of output which she receives in compensation for her investment. In such cases, a shift in the organizational form is unlikely to cause much of a change in investment patterns. One way that a party's incentives can be unimportant is from his investment being unimportant. In a limiting case, if a party's investment does not add anything to production, then changing the share of productive output that he receives will clearly not change his investment behavior. Although investments being unimportant for a party does indeed imply that the party's incentives are unimportant, the converse need not be true in general. It could be the case, for example, that investment is discrete so that it only makes sense to invest fully or not at all. In such a case, even relatively large changes in the share of output received by the investing party will lead to little or no change in actual investment. Hence, unimportance of investments is a sufficient though not necessary condition for unimportance of incentives.

In our model, incentives being "unimportant" is similar to the discrete investment case described above. Our definition has two components. First, we assume that when incentives for a party are unimportant, the net marginal returns of investment are almost everywhere negative in the sense that the marginal costs of investment outweighs the benefits. Again, this does not mean that marginal benefits are less than zero but rather less than marginal effort costs. Second, we stipulate that it is worthwhile to invest in the sense that the value of production when incentives are unimportant is greater than the cost of investing if the worker (or the firm) invests the minimum amount that makes such production possible. Notationally, we denote by $\bar{i}$ the minimum amount of investment by the firm which allows for positive production when firm incentives are unimportant and we denote by $\bar{e}$ the minimum amount of investment by the worker which allows for positive production when worker incentives are unimportant.

Definition 1. ***(UNIM1)*** *The firm's incentives are said to be "unimportant" when:*

(1.) $\overline{N}\frac{\partial X(e_j,i)}{\partial i} < \overline{N}\gamma < 1 \; \forall i \neq \bar{i}$, *and (2.)* $X(e_j,\bar{i}) - \underline{w} > 2\bar{i} \; \forall e_j > 0$

Definition 2. ***(UNIM2)*** *The worker's incentives are said to be "unimportant" when:*

(1.) $\frac{\partial X(e_j,i)}{\partial e_j} < \gamma < 1 \; \forall e \neq \bar{e}$, *and (2.)* $X(\bar{e},i) - \alpha X(\alpha\bar{e},i) > 2\bar{e} \; \forall i > 0$

For a given production technology, we define the production technology when firms' incentives are unimportant, I, to be a function such that (1.) $\frac{\partial I(e_j,i)}{\partial e_j} = \frac{\partial X(e_j,i)}{\partial e_j} \; \forall e_j$ and (2.) **UNIM1** holds. Similarly, we define the production technology associated with X when workers' incentives are unimportant, E, to be a production function such that (1.) $\frac{\partial E(e,i)}{\partial i} = \frac{\partial X(e,i)}{\partial i} \; \forall i$ and (2.) **UNIM2** holds. We can now parameterize production technologies by saying that as firm incentives (worker incentives) become unimportant, IC (EMP) will tend to be chosen. Our parameterization is a simple convex combination of the production function and the production function when the firm's (workers') incentives are unimportant : $tI(e_j,i) + (1-t)X(e_j,i) \quad (tE(e_j,i) + (1-t)X(e_j,i))$. As t increases, the incentives of the party in question become less important. We now make a technical assumption :

Condition 1. *We assume that for all production functions, X, there exists an upper bound $\overline{X}_i$ on the marginal product of the firm's investment and an upper bound on the marginal product of the worker's investment $\overline{X}_e$.*

This assumption makes the proofs easier and more transparent. To clarify, the condition above does not even impose an universal upper bound on the marginal product of investments over the function space of all production functions, since the bound is not uniform. We impose one last technical condition :

Condition 2. *We assume unique interior equilibria*

The above condition is standard in the literature. Alternatively, we could prove our comparative statics results allowing multiple equilibria and establishing the results for the equilibrium set. However, we feel that making the interior equilibrium assumption is simpler and, again, makes our argument more transparent to the reader. We now state two of our main results, whose proofs are presented in the Appendix:

Proposition 2. *As employer incentives become less important, IC will strictly dominate EMP provided production occurs.* $\forall X \epsilon \widetilde{X}$ $\exists \bar{t}$ *such that* $\forall t \geq \bar{t}$ $\Pi_{IC}(N_{IC}^*) = N_{IC}^* X^t \left(i_{IC}^*, e_{IC}^*\right) - i_{IC}^* - N_{IC}^* e_{IC}^* - C(N_{IC}^*) > \Pi_{EMP}(N_{EMP}^*) = N_{EMP}^* X^t \left(i_{EMP}^*, e_{EMP}^*\right) - i_{EMP}^* - N_{EMP}^* e_{EMP}^* - C(N_{EMP}^*)'$

All else equal, independent contracting provides the best incentives for workers, and employment provides the best incentives for the owner. Therefore, when firm incentives are not important, the firm wishes to tie its hands from expropriating worker knowledge precisely because it is relatively important to the firm's production. We obtain a similar result indicating a preference for employment when worker incentives are unimportant:

Proposition 3. *As employee incentives become less important, EMP will dominate IC provided production occurs.* $\forall X \epsilon \widetilde{X}\ \exists \bar{t}$ *such that* $\forall t \geq \bar{t}\ \Pi_{EMP}(N^*_{EMP}) = N^*_{EMP}X^t\left(i^*_{EMP}, e^*_{EMP}\right) - i^*_{EMP} - N^*_{EMP}e^*_{EMP} - C(N^*_{EMP}) > \Pi_{IC}(N^*_{IC}) = N^*_{IC}X^t\left(i^*_{IC}, e^*_{IC}\right) - i^*_{IC} - N^*_{IC}e^*_{IC} - C(N^*_{IC})$

4. Organizational Choice without *Ex Ante* Transfers

Thus far, our explanation of the labor boundary of the firm has been completely based on efficiency grounds. In other words, the work arrangement is chosen to provide incentives for investment in a fashion which maximizes the joint surplus. However, when we relax the assumption that firms can extract all *ex post* rents through *ex ante* transfers, it is no longer true that the choice made by a firm between IC and EMP is the one which maximizes the joint surplus. In particular, by giving no rights of expropriation to the firm, IC gives more *ex post* rents to the worker (conditional on investment levels), which can affect organizational choice.

The rent-sharing and efficiency wage literatures discuss various reasons why workers may earn *ex ante* rents, and we are not going to review the entire literature here. But we point out some key arguments why *ex ante* transfers might not neutralize *ex post* distributional consequences. Perhaps the most obvious rationale is liquidity constraints. In general, differential access to credit markets will imply that, *ceteris paribus*, workers' relative valuation of a dollar today versus a dollar tomorrow will be greater than that of a wealthier firm owner. As a result, what workers would be willing or able to pay the owner at date 0 to be hired as an independent contractor would fall short of the owner's expected future loss in rent. Although in this model we assume risk neutrality for all players, adding differential risk aversion (or risk aversion and differential wealth) would also lead to a divergence between the workers' and the owner's valuation of future rents.

In this section, we look at the case where no *ex-ante* transfers are possible.

When full transfers were possible, a worker's expected returns at date 0 was simply $\underline{w}$, across organizational forms. His future expected returns in excess of $\underline{w}$ were commonly known, and hence collected by the firm at date 0 as a condition of employment. Consequently, under the "full transfers" assumption, the date 0 returns to the owner were as follows for a given N:

$$\begin{aligned}
\Pi_{IC}^{FT}(N) &= N\left(X(e_{IC}^{*},(N)i_{IC}^{*}(N)) - e_{IC}^{*}(N)\right) - i_{IC}^{*}(N) - N\underline{w} - C(N) \\
\Pi_{EMP}^{FT}(N) &= N\left(X(e_{EMP}^{*}(N),i_{EMP}^{*}(N)) - e_{EMP}^{*}(N)\right) - i_{EMP}^{*}(N) - N\underline{w} - C(N) \\
&\quad -N\underline{w} - C(N)
\end{aligned}$$

Now we look at the payoffs to the firm under different organizational forms without the possibility of *ex ante* transfers. The firm still has to pay each worker at least his outside wage at date 2. However, the firm now shares X with the worker according to the property rights implicit in employment and contracting out. This leads to the following date 0 returns for the firm owner under the "no transfers" assumption for a given N:

$$\begin{aligned}
\Pi_{IC}^{NT}(N) &= N\left(\frac{X(e_{IC}^{*}(N),i_{IC}^{*}(N)) - \underline{w}}{2}\right) - i_{IC}^{*}(N) - C(N) \\
\Pi_{EMP}^{NT}(N) &= N\left(\frac{X(e_{EMP}^{*}(N),i_{EMP}^{*}(N)) + X(\alpha e_{EMP}^{*}(N),i_{EMP}^{*}(N))}{2}\right) \\
&\quad -i_{EMP}^{*}(N) - N\underline{w} - C(N)
\end{aligned}$$

In the no transfers case, the owner receives only a share of the output, and he does not compensate the worker for her investment e. These two factors change the net benefit calculation for the two legal forms. From the derivation above, it is clear that for the same level of investments and employment, the owner's expected profits are larger under EMP. To make the point in an equilibrium context, we look at the case when both the owner's and the worker's incentives are sufficiently "unimportant" in the sense defined in the previous section.

Proposition 4. *When both parties' incentives are unimportant: (1) with full transfers, profits and employment with IC and EMP are identical; but (2) under no transfers, EMP is more profitable, and the optimum employment under EMP is greater. When UNIM1 and UNIM2 hold,* $\Pi_{EMP}^{FT}(N_{FT,EMP}^{*}) = \Pi_{IC}^{FT}(N_{FT,IC}^{*}) = \Pi^{FT}(N_{FT}^{*})$*, but* $\Pi_{EMP}^{NT}(N_{NT,EMP}^{*}) > \Pi_{IC}^{NT}(N_{NT,IC}^{*})$*,and* $N_{NT,EMP}^{*} > N_{NT,IC}^{*}$

When both parties' incentives are unimportant, efficiency reasons do not discriminate between EMP and IC. However, an owner can capture a greater share of the rent under EMP, and this provides an additional consideration beyond the incentive reasons discussed in the previous section.

5. Possible Extensions and Conclusion

The labor boundaries of the firm is an understudied area of economics. In light of the growth in contracting out in the recent past, it is an area that deserves more attention. In this chapter, we argue that at least some part of contracting out might have a simple explanation – that the law treats employees differently than it treats contractors or employees of contractors. The employment relationship gives the employer certain rights to expropriate worker investment, and these rights of expropriation determine the costs and benefits which define when the employment relationship is profitable for a firm.

Overall, we associate independent contracting with worker incentives and employment with firm incentives. When the incentives of the firm owner are unimportant, she will hire workers as independent contractors. In such cases, only worker incentives matter, and by giving workers full rights to keep their own investments, independent contracting better incentivizes workers to make investments in noncontractables. Similarly, when worker incentives are unimportant, the owner will choose to employ . Employment assigns significant expropriation rights to the owner, and hence provides her with better incentives. Moreover, without ex ante transfers, there is yet another reason to choose employment: by making worker replacement more credible, it offers a greater share of the surplus to the owner.

Here we assumed that the knowledge created by a worker cannot be used outside of the production relationship. But a practical concern, highlighted by Rajan and Zingales, is the possibility that a worker might leave the firm and compete with his old employer. In this context, the legal rights over intellectual property not only affects the owner's right to use it to continue production, but also the worker's right to use it elsewhere. The choice of legal forms in this context would be guided by both concerns of minimizing future competition and of providing sufficient incentives to workers.

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6. APPENDIX: Proofs of Propositions

Proposition 1. ***When outside wage*** $\underline{w}$ ***is low as compared to the value of production and it is worth hiring at least one worker, the relevant equilibrium configuration for EMP will be one with replacement.*** $\exists \underline{\underline{w}}$ ***such that*** $\forall \underline{w} < \underline{\underline{\mu}}, (e^*_{EMP}, i^*_{EMP}) = (e^*_R, i^*_R)$

Proof. Define μ (N)=min$\{X(\alpha e^*_R(N), i^*_R(N)), X(\alpha e^*_{NR}(N), i^*_{NR}(N)\}$, where:

$$\begin{aligned}
(1.)\ i^*_{NR}(N) &= \arg\max_i \left(\frac{NX(e^*_{NR}, i)}{2} - i \right) \\
(2.)\ e^*_{NR}(N) &= \arg\max_e \left(\frac{X(e, i^*_R)}{2} - e \right) \\
(3.)\quad i^*_R(N) &= \arg\max_i \left(\frac{NX(e^*_R, i)}{2} + \frac{NX\ (\alpha e^*_R, i)}{2} - i \right) \\
(4.)\quad e^*_R(N) &= \arg\max_e \left(\frac{X(e, i^*_R)}{2} - \frac{X\ (\alpha e, i^*_R)}{2} - e \right)
\end{aligned}$$

We know that $N^*(\underline{w})$ is bounded above by $\overline{N}$ derived in the body of the chapter, and by assumption, $N^*(\underline{w}) \geq 1$ Define $\underline{\underline{\mu}} = \min_{N \in \{1\ ...\ \overline{N}\}} \mu(N)$. .

We know that $\underline{\underline{\mu}} > 0$, since $X(e, i) > 0$ iff $e > 0, i > 0$, and this must be true if there is some production. Take a $\underline{w}$ such that $0 < \underline{w} < \underline{\underline{\mu}}$.

First we show that ex-post replacement is indeed credible at the (e^*_R, i^*_R, N^*_R) equilibrium defined by (3.), (4.) and the optimumlity condition for N. First, note that $X(\alpha e^*_R(N), i^*_R(N)) > \underline{w}$ for all $N \in \{1\ ...\ \overline{N}\}$ since $\underline{w} < \underline{\underline{\mu}}$. It is then also true that $X(\alpha e^*_R(N^*_R), i^*_R(N^*_R)) > \underline{w}$ for the optimuml $N^*_R(\underline{w})$. Therefore, at (e^*_R, i^*_R, N^*_R), production with a replacement worker is ex-post profitable, and this is indeed an equilibrium.

To prove that $(e^*_{NR}, i^*_{NR}, N^*_{NR})$ is not an equilibrium, we first assume the contrary. We assume that there exists a $(e^*_{NR}, i^*_{NR}, N^*_{NR})$ at outside wage $\underline{w}$, defined by (1.), (2.), and the FOC for employment, and where replacement is not credible in equilibrium at date 2. Note that by construction, however, $X(\alpha e^*_{NR}(N), i^*_{NR}(N)) > \underline{w}$ for all $N \in (0, \overline{N})$, and hence $X(\alpha e^*_{NR}(N^*_{NR}), i^*_{NR}(N^*_{NR})) > \underline{w}$ for optimuml $N^*_{NR}(\underline{w})$. implying that at $(e^*_{NR}, i^*_{NR}, N^*_{NR}(\underline{w}))$, replacing a worker is indeed credible. Therefore, $(e^*_{NR}, i^*_{NR}, N^*_{NR}(w))$ can be ruled out as an actual equilibrium. ■

Proposition 2. ***As employer incentives become less important, IC will dominate EMP.*** $\forall X \epsilon \widetilde{X}$ $\exists \overline{t}$ ***such that*** $\forall t \geq \overline{t}$ $\Pi_{IC}(N^*_{IC}) = N^*_{IC}X^t\left(i^*_{IC}, e^*_{IC}\right) - i^*_{IC} - N^*_{IC}e^*_{IC} - C(N^*_{IC}) > \Pi_{EMP}(N^*_{EMP}) = N^*_{EMP}X^t\left(i^*_{EMP}, e^*_{EMP}\right) - i^*_{EMP} - N^*_{EMP}e^*_{EMP} - C(N^*_{EMP})$

Proof. First, we prove that $\Pi_{IC}(N) > \Pi_{EMP}(N)$ at any given level of $N \in \{1 \ldots \overline{N}\}$, where $\overline{N}$ is the upper bound on N as defined previously. Then we prove that $\Pi_{IC}(N^*_{IC}) > \Pi_{EMP}(N^*_{EMP})$

(I.) Subclaim : IC(N)$\succ$EMP(N) for a given N if t=1 :

Note that for a given N, $\Pi_{IC}(N) > \Pi_{EMP}(N)$ iff $NX_{IC}\left(i^*_{IC}, e^*_{IC}\right) - i^*_{IC} - Ne^*_{IC} > NX_{EMP}\left(i^*_{EMP}, e^*_{EMP}\right) - i^*_{EMP} - Ne^*_{EMP}$

First, we take $NX^1_{IC}\left(i^*_{IC}, e^*_{IC}\right) - i^*_{IC} - Ne^*_{IC} = NI\left(i^*_{IC}, e^*_{IC}\right) - i^*_{IC} - Ne^*_{IC}$. It is true that $\forall e > 0, \frac{N}{2}\left[I\left(e, \overline{i}\right) - \underline{w}\right] - \overline{i} = \frac{N}{2}I\left(e, \overline{i}\right) - \overline{i} \geq \frac{1}{2}[I\left(e, \overline{i}\right) - \underline{w}] - \overline{i} > 0$; therefore, with IC, the owner finds it worthwhile to invest $\overline{i}$. Furthermore, $\overline{i}$ is the owner's optimum level since $\arg\max_i \left\{\max_{i<\overline{i}} \frac{1}{2}I\left(e, i\right) - i, \max_{i>\overline{i}} \frac{1}{2}I\left(e, i\right) - i\right\}$ which equals $\arg\max_i \left\{0, = \frac{1}{2}I\left(e, \overline{i}\right) - \overline{i}\right\} = \overline{i}$ as long as $e^*_{IC} > 0$. Similarly, $e^*_{IC} = e\left(\overline{i}\right)$ as long as $e^*_{IC} > 0$. From the first order conditions for e in IC, we have $I_e\left(e, \overline{i}\right) = 2$ and for EMP, we have $I_e\left(e, \overline{i}\right) - \alpha I_e\left(\alpha e, \overline{i}\right) = 2 \Rightarrow e^*_{IC} > e^*_{EMP} > 0$

if $i^*_{EMP} > 0$. Furthermore, in the efficient solution the first order condition is $I_e\left(e,\bar{i}\right) = 1$, implying $e^*_{EFF} > e^*_{IC} > e^*_{EMP}$. The concavity of $L(e,i)$ in e implies that for all $e \leq e^*_{IC}$ and $i = \bar{i}$ we have $I\left(e^*_{IC},\bar{i}\right) - e^*_{IC} > I\left(e^*_{EMP},\bar{i}\right) - e^*_{EMP} \Longrightarrow$ $N\left(I\left(e^*_{IC},\bar{i}\right) - e^*_{IC}\right) - \bar{i} > N\left(I\left(e^*_{EMP},\bar{i}\right) - e^*_{EMP}\right) - \bar{i} \Longrightarrow$

$\Pi_{IC}(N) > \Pi_{EMP}(N) \Longrightarrow$ IC(N)$\succ$EMP(N) when t=1.

(II.) Subclaim : $\exists \bar{\mathbf{t}}_{IC} < 1$ **such that** $\forall t > \bar{\mathbf{t}}_{IC}$, $\left(e_{ic}(\bar{i}),\bar{i}\right)$ **is the unique IC equilibrium for** $\forall N \in \{1 \ldots \overline{N}\}$.

Denote $N\overline{X_i} = \max_i \frac{N}{2}\frac{\partial X\left(e_{IC}(\bar{i}),i\right)}{\partial i}$. Then denote by $\bar{t}_{IC}$ the t such that $(1-t)\overline{NX_i} + t\gamma = 1 \Rightarrow \bar{t}_{IC} = \frac{\overline{NX_i}-1}{\overline{NX_i}-\gamma}$, and $\frac{N\overline{X_i}-1}{N\overline{X_i}-\gamma} < \bar{t}_{IC} < 1$ (since $\gamma < 1$). Then for all $t > \bar{t}_{IC}$, $tN\frac{\partial I(i)}{\partial i} + (1-t)\frac{\partial X(e,i)}{\partial i} < t\gamma + (1-t)N\overline{X_i} < 1 \Rightarrow$

$\arg\max_i \left\{ \max_{i<\bar{i}} \frac{NX^t\left(\bar{i},e_{IC}(\bar{i})\right)}{2} - i, \max_{i\geq\bar{i}} \frac{NX^t(i,e_{IC}(i))}{2} - i \right\} =$ $\arg\max_i \left\{0, \frac{NX\left(\bar{i}\right)}{2} - \bar{i}\right\} = \bar{i} \Rightarrow \left(\bar{i}, e_{IC}(\bar{i})\right)$ is an equilibrium and the only interior equilibrium by the interior uniqueness assumption for all t greater than $\bar{t}_{IC}$.

(III.) Subclaim : $\forall t > \bar{\mathbf{t}}_{IC}$ **and** $\forall N \in \{1 \ldots \overline{N}\}$, **IC(N)$\succ$EMP(N) for a given** N.

Now, we argue that i^*_{EMP}, e^*_{EMP} are invariant to t for EMP for a given N. Clearly, $i(\bar{e})$ is the same for all t by construction. And given our assumption about $I_e(e,i) = X^t_e(e,i)$ for all i and t , $e(i)$ is the same for all t. Thus, by **subclaim (I.),** IC(N)$\succ$EMP(N), or $\Pi_{IC}(N) > \Pi_{EMP}(N)$

(IV.) Subclaim : IC$(N^*_{IC}) \succ$**EMP**$(\mathbf{N}^*_{EMP})$**.** $\forall t > \bar{\mathbf{t}}_{IC}, \Pi_{IC}(N^*_{IC}) > \Pi_{EMP}(N^*_{EMP})$

We know that $N^*_{IC,EMP}$ are bounded such that $N^*_{IC,EMP} \in \{1 \ldots \overline{N}\}$, and by **subclaim (III.),** $\Pi_{IC}(N) > \Pi_{EMP}(N)$ for all $t > \bar{\mathbf{t}}_{IC}, N \in \{1 \ldots \overline{N}\}$. But this implies that for all $t > \bar{\mathbf{t}}_{IC}$, $\max_N(\Pi_{IC}(N)) > \max_N(\Pi_{EMP}(N))$. ■

Proposition 3. ***As employee incentives become less important, EMP***

will dominate IC. $\forall X \epsilon \widetilde{X}$ $\exists \bar{t}$ ***such that*** $\forall t \geq \bar{t}$ $\Pi_{EMP}(N^*_{EMP}) = N^*_{EMP} X^t\left(i^*_{EMP}, e^*_{EMP}\right) - i^*_{EMP} - N^*_{EMP} e^*_{EMP} - C(N^*_{EMP}) > \Pi_{IC}(N^*_{IC}) = N^*_{IC} X^t\left(i^*_{IC}, e^*_{IC}\right) - i^*_{IC} - N^*_{IC} e^*_{IC} - C(N^*_{IC})$

Proof. First, we prove that $\Pi_{EMP}(N) > \Pi_{IC}(N)$ at any given level of $N \in \{1 \,...\, \overline{N}\}$, where $\overline{N}$ is the upper bound on N as defined previously. Then we prove that $\Pi_{EMP}(N^*_{EMP}) > \Pi_{IC}(N^*_{IC})$

(I.) Subclaim : EMP(N)$\succ$IC(N) for a given N if t=1 :

Note that for a given N, $\Pi_{EMP}(N) > \Pi_{IC}(N)$ iff $NX_{EMP}\left(i^*_{EMP}, e^*_{EMP}\right) - i^*_{EMP} - Ne^*_{EMP} > NX_{IC}\left(i^*_{IC}, e^*_{IC}\right) - i^*_{IC} - Ne^*_{IC}$

We take $NX^1_{EMP}\left(i^*_{EMP}, e^*_{EMP}\right) - i^*_{EMP} - Ne^*_{EMP} = NE\left(i^*_{EMP}, e^*_{EMP}\right) - i^*_{EMP} - Ne^*_{EMP}$. It is true that $\forall i > 0$, $\frac{1}{2}\left[E\left(\overline{e}, i\right) - E\left(\alpha\overline{e}, i\right)\right] - \overline{e} = \frac{1}{2}E\left(\overline{e}, i\right) - \overline{e} > 0$, so it is worthwhile for the worker to invest $\overline{e}$. Moreover, $\overline{e}$ is the worker's optimuml level of investment since:

$$\arg\max_e \left\{ \begin{array}{c} \max_{e<\overline{e}} \frac{1}{2}\left[E\left(\overline{e}, i\right) - E(\alpha\overline{e}, i)\right] - \overline{e}, \\ \max_{e>\overline{e}} \frac{1}{2}\left[E\left(\overline{e}, i\right) - E\left(\alpha\overline{e}, i\right)\right] - \overline{e} \end{array} \right\} =$$

$\arg\max_e \left\{0, \frac{1}{2}\left[E\left(\overline{e}, i\right) - E\left(\alpha\overline{e}, i\right)\right] - \overline{e}\right\} = \overline{e}$ as long as $i^*_{IC} > 0$; similarly, $i^*_{EMP} = i\left(\overline{e}\right) > 0$ as long as $e^*_{EMP} > 0$. From the first order conditions for i in EMP, we have $NE_i\left(\overline{e}, i\right) + NE_i\left(\alpha\overline{e}, i\right) = 2$ and for IC, we have $NE_i\left(\overline{e}, i\right) = 2 \Rightarrow i^*_{IC} > i^*_{EMP} > 0$. Furthermore, in the efficient solution the first order condition is $E_i\left(\overline{e}, i\right) = 1$, implying $i^*_{EFF} > i^*_{IC} > i^*_{EMP}$. The concavity of $E(e, i)$ in i implies that for all $i \leq i^*_{EMP}$ and $e = \overline{e}$ we have $NE\left(\overline{e}, i^*_{EMP}\right) - i^*_{EMP} > NE\left(\overline{e}, i^*_{IC}\right) - i^*_{IC} \Longrightarrow NE\left(\overline{e}, i^*_{EMP}\right) - i^*_{EMP} - N(\underline{w} + \overline{e}) - C(N) > NE\left(\overline{e}, i^*_{IC}\right) - i^*_{IC} - N(\underline{w} + \overline{e}) - C(N) \Longrightarrow$ EMP(N) $\succ$IC(N)

(II.) Subclaim : $\exists \overline{\mathbf{t}}_{EMP} < 1$ **such that** $\forall t > \overline{\mathbf{t}}$ and $\forall N \in \{1 \,...\overline{N}\}$, $(\overline{e}, i_{EMP}(\overline{e}, N))$

is the unique EMP equilibrium

Denote $\overline{X}_e = \max_e \frac{1}{2}\left(\frac{\partial X(i_{EMP}(\overline{e}),e)}{\partial e} - \alpha \frac{\partial X(i_{EMP}(\overline{e}),\alpha e)}{\partial e}\right)$. Then denote by $\overline{t}_{EMP}$ the t such that $(1-t)\overline{X_e} + t\gamma = 1 \Rightarrow \overline{t}_{EMP} = \frac{\overline{X_e}-1}{\overline{X}_e-\gamma} < 1$ (since $\gamma < 1$). Then for all $t > \overline{t}_{EMP}$, $t\frac{\partial E(e)}{\partial e} + (1-t)\frac{\partial X(e,i)}{\partial e} < t\gamma + (1-t)\overline{X}_e < 1 \Rightarrow$
$\arg\max_e \left\{ \begin{array}{c} \max_{e<\overline{e}} \frac{X^t(e,i_{EMP}(\overline{e}))+X^t(\alpha e,i_{EMP}(\overline{e}))}{2} - e, \\ \max_{e>\overline{e}} \frac{X^t(e,i_{EMP}(\overline{e}))+X^t(\alpha e,i_{EMP}(\overline{e}))}{2} - e \end{array} \right\} =$
$\arg\max_e \left\{0, \frac{X^t(\overline{e},i_{EMP}(\overline{e}))+X^t(\alpha\overline{e},i_{EMP}(\overline{e}))}{2} - \overline{e} - i_{EMP}(\overline{e})\right\} = \overline{e} \Rightarrow (\overline{e}, i_{EMP}(\overline{e},N))$ is an equilibrium and the only interior equilibrium by the interior uniqueness assumption for all t greater than $\overline{t}_{EMP}$.

(III.) Subclaim : $\forall t > \overline{\mathbf{t}}_{EMP}$, **EMP(N)$\succ$IC(N) for all** $.N \in \{1 ...\overline{N}\}$

Now, we argue that e^*_{IC}, i^*_{IC} are invariant to t for EMP. Clearly, $e(\overline{i})$ is the same for all $t > \overline{\mathbf{t}}_{EMP}$ by construction. And given our assumption about $E_i(e,i) = X^t_i(e,i)$ for all e and $t > \overline{\mathbf{t}}_{EMP}$, $i(e,N)$ is the same for all t. Thus, by **subclaim (I.),** when $t > \overline{t}$, EMP(N)$\succ$IC(N)

(IV.) Subclaim $.\forall t > \overline{\mathbf{t}}_{EMP}$, **EMP($N^*_{EMP}$)$\succ$IC($N^*_{EMP}$). That is,** $\Pi_{EMP}(N^*_{EMP}) > \Pi_{IC}(N^*_{IC})$.

We know that $N^*_{IC,EMP}$ are bounded such that $N^*_{IC,EMP} \in \{1 ...\overline{N}\}$, and by **subclaim (III.),** $\Pi_{EMP}(N) > \Pi_{IC}(N)$ for all $t > \overline{\mathbf{t}}_{EMP}, N \in \{1 ...\overline{N}\}$. But this implies that for all $t > \overline{\mathbf{t}}_{EMP}$, $\max_N(\Pi_{EMP}(N)) > \max_N(\Pi_{IC}(N))$. ■

Proposition 4. ***When both parties' incentives are unimportant: (1) with full transfers, profits and employment with IC and EMP are identical; but (2) under no transfers, EMP is more profitable, and the optimuml employment under EMP is greater. When UNIM1 and UNIM2 hold,*** $\Pi^{FT}_{EMP}(N^*_{EMP}) = \Pi^{FT}_{IC}(N^*_{IC})$, ***but*** $\Pi^{NT}_{EMP}(N^*_{EMP}) > \Pi^{NT}_{IC}(N^*_{IC})$,***and*** $N^*_{NT,EMP} > N^*_{NT,IC}$

Proof. When UNIM1 and UNIM2 hold, as shown before, $e^*_{IC} = e^*_{EMP} = \overline{e}$, and $i^*_{IC} = i^*_{EMP} = \overline{i}$. Under full transfer, $\Pi^{FT}_{EMP}(N) = NX\left(\overline{i}, \overline{e}\right) - \overline{i} - N(\underline{w} + \overline{e}) - C(N) = \Pi^{FT}_{IC}(N)$, and, therefore, $N^*_{FT,EMP} = N^*_{FT,IC} = N^*_{FT}$; consequently $\Pi^{FT}_{EMP}(N^*_{FT,EMP}) = \Pi^{FT}_{IC}(N^*_{FT,IC})$
In the no transfer case, $\Pi^{NT}_{EMP}(N) = N\left(\frac{X(\overline{e},i)+X(\alpha\overline{e},\overline{i})}{2}\right) - \overline{i} - N\underline{w} - C(N)$, and $\Pi^{NT}_{EMP}(N) = N\left(\frac{X(\overline{e},i)-w}{2}\right) - \overline{i} - C(N)$. But the assumption of replacement equilibrium implies that $X(\alpha\overline{e}, \overline{i}) > \underline{w}$, and hence $\Pi^{NT}_{EMP}(N) > \Pi^{NT}_{IC}(N)$. It follows that $\max_N \Pi^{NT}_{EMP}(N) > \max_N \Pi^{NT}_{IC}(N)$. Moreover, by convexity of $C(N)$, $N^*_{NT,EMP} \geq N^*_{NT,IC}$. ■