Outline of Lectures on Monetary Policy

- Why do we have money?
- Real Versus Nominal Effects of Money
- Money and Interest Rates
  - Short Term
  - Term Structure
- Money and Inflation (Prices):
  - Supply and Demand (Cagan)
  - Optimal Seignorage
  - Friedman Rule
- Monetary Policy and Time Inconsistency
- Optimal Monetary Policy
- Monetary Policy Rules
- Transparency

Theories of Money

- Three values of Money
  - (1.) Store of Value (OLG Models of Money)
  - (2.) Unit of Account (Bounded Rationality)
  - (3.) Transactions Value

- Different Modelling Approaches to a Value for Money
  - Transactions times reductions (3.)
  - Cash in Advance (3.)
  - Overlapping Generations (1.)
  - Money in Utility Function (1.), (2.) & (3.)

Real Versus Nominal Effects of Money

- Money in General Equilibrium: general equilibrium determines relative prices not the price level.
- Can add in money to pin down the price level but this implicitly assumes velocity is one.
  - $MV=PY$
- No good theories of the determination of velocity (which is unstable)

Money and Interest Rates I

- Real Money Supply = Real Money Demand (note money demand depends upon nominal not real interest rate… why?):

$$\frac{M}{P} = L(i, Y); \frac{\partial \mathrm{L}}{\partial \mathrm{i}} < 0, \frac{\partial \mathrm{L}}{\partial \mathrm{Y}} > 0$$

- Fisher Equation:

$$1 + i_t = (1 + r_t)\frac{P_{t+1}}{p_t} = (1 + r_t)(1 + \pi_t) = 1 + r_t + \pi_t + r_t \pi_t$$

- When r and π are small, then:

$$i \approx r + \pi$$

Money and Interest Rates II

- Start with full price flexilibity: no Keynesian effects on output. Then:

$$\frac{M}{L(\bar{r} + \pi^e, \bar{Y})} = P$$

- Without effects on inflation expectations and thus nominal interest rates, increases in growth rates of money would increase growth rates of prices proportionally.

- With effects on nominal interest rates, there should be a one time increase in nominal interest rates, leading to a one time decline in money (liquidity) demand, meaning that prices have to experience a discrete jump up.

- Fixed Prices: Money Supply Increase leads to nominal interest rate decline

Money and Interest Rates III

- So: if prices are fixed in the short run, then we should see:
  - A one-time increase in the money stock leading to a short-run decline in interest rates followed by a rise in nominal interest rates to pre-shock levels
  - An increase in money growth leading to a short-run decline in interest rates followed by an increase in nominal interest rates beyond the pre-shock nominal interest rate level
- Question: Which interest rates are important for lowering unemployment? What should the FED look at?

Money and Interest Rates IV

- What do we actually see?
  - Cook and Hahn (1989) ran the following regression of interest rate changes at time horizon i on changes in the Federal Fund Rate:
  - We should see increases in the short run and decreases in the long run.

$$\Delta I_t^i = \alpha_i + \beta_i \Delta FF_t + \varepsilon_{it}$$

  - We actually see a positive effect for interest rates even as far out as 20 years (thought the effect declines over the term structure):
    - 3 months: 55 basis points for every 100 basis points in the FF
    - 1 year: 50 basis points for every 100 basis points in the FF
    - 5 years: 21 basis points for every 100 basis points in the FF
    - 20 years: 10 basis points for every 100 basis points in the FF
- So: how do we interpret these results… what do they suggest?

Money and Interest Rates V

- The term structure of interest rates is set of interest rates of different maturities.
- The Expectations Hypothesis of the term structure states that:

$$i_t^k = \sum_{j=0}^{k-1} E_t i_{j+t}^1$$

- What are the assumptions behind this hypothesis?

Money and Inflation I

- Money Demand: Assume a specific functional form and then assume processes for money supply over time and look at what happens to the price level (constant elasticity of inflation):

$$\frac{M_t^D}{P_t} = \left(\frac{EP_{t+1}}{P_t} \right)^{-\eta}$$

- Taking logs, we get (small letters denote natural logs of big letters):

$$m_t^D - p_t = -\eta \left(E_t p_{t+1} - p_t \right)$$

Money and Inflation II

- Rearranging terms, we get:

$$p_t = \frac{m_t^D}{1+\eta} + \frac{\eta E_t p_{t+1}}{1+\eta}$$

- Iterating forward, we get:

$$p_t = \frac{m_t^D}{1+\eta} + \frac{\eta E_t \left[\frac{m_{t+1}^D}{1+\eta} + \frac{\eta E_{t+1} p_{t+2}}{1+\eta} \right]}{1+\eta}$$

Money and Inflation III

- Remebering the law of iterated expectations:

$$E_t E_{t+1} P_{t+2} = E_t P_{t+2}$$

- And continuing the iteration, we derive:

$$p_t = \frac{1}{1+\eta} \sum_{j=t}^{T} \left[\frac{\eta^{j-t} E_t m_t^D}{(1+\eta)^{j-t}} \right] + \left(\frac{\eta}{1+\eta} \right)^{T+1-t} E_t p_{T+1}$$

- Finally, equating money supply with money demand in every period, we get:

$$p_t = \frac{1}{1+\eta} \sum_{j=t}^{\infty} \left[\frac{\eta^{j-t} E_t m_j}{(1+\eta)^{j-t}} \right] + \lim_{j \to \infty} \left(\frac{\eta}{1+\eta} \right)^{j+1-t} E_t p_{t+j}$$

Money and Inflation IV

- An infinite number of solutions exist to the previous equation. Not true with a finite horizon problem.
- $p_t = \frac{1}{1+\eta}\sum_{j=t}^{\infty}\left[\frac{\eta^{j-t}E_t m_t^D}{(1+\eta)^{j-t}}\right]$ is a solution to the difference equation $p_t = \frac{m_t^D}{1+\eta} + \frac{\eta E_t p_{t+1}}{1+\eta}$.
- However, $\tilde{p}_t = p_t + b_t$ where $b_t = \frac{1+\eta}{\eta} b_{t-1} + v_t$ and where $E_t v_{t+1} = 0$. These solutions are called bubble solutions. They are different from the baseline solution above by a permanent constant term.

Money and Inflation V

- In the case of bubbles:

$$\lim_{T\to\infty}\left(\frac{\eta}{1+\eta}\right)^{T} E_t p_{t+T} = \lim_{T\to\infty}\left(\frac{\eta}{1+\eta}\right)^{T} E_t b_{t+T} = b_t$$

- We now consider examples of stochastic (or non-stochastic) processes for money supply

Money and Inflation VI: Examples

- (A.) Constant Money: Non-Stochastic:

$$m_t = m \,\forall m$$

$$p_t = \frac{1}{1+\eta}\sum_{j=t}^{\infty}\left[\frac{\eta^{j-t}E_t m_j}{(1+\eta)^{j-t}}\right] = \sum_{j=t}^{\infty}\left[\frac{m}{(1+\eta)^{j-t}}\right]$$

  – Aside:

$$P = \sum_{t=0}^{T} r^t m = m\left[1 + r + r^2 + \ldots + r^T\right]$$

$$\Rightarrow (1-r)P = \sum_{t=0}^{T} r^t m = m\left[1 - r^{T+1}\right]$$

$$\Rightarrow P = m\frac{\left[1 - r^{T+1}\right]}{1-r}$$

Money and Inflation VII: Examples

- Taking the limit as T goes to infinity:

$$\Rightarrow \lim_{T\to\infty} P = \lim_{T\to\infty} \frac{m\left[1-r^{T+1}\right]}{1-r} = \frac{m}{1-r}$$

- So: $\sum_{t=0}^{\infty} r^t m = \frac{m}{1-r}$

- Thus

$$p_t = \frac{1}{1+\eta}\sum_{j=t}^{\infty}\left[\frac{\eta^{j-t}m}{(1+\eta)^{j-t}}\right] = \frac{1}{1+\eta}\frac{m}{1-\frac{\eta}{1+\eta}} = \frac{1+\eta}{1+\eta}m = m$$

Money and Inflation VIII: Examples

- Case 2: Constant growth rate.

- Suppose money grows at rate g. This means that:

$$\frac{M_{t+1}}{M_t} = 1 + g$$

- Or alternatively that (for small g): $m_{t+1} - m_t \approx g$

- Then $p_t = \frac{1}{1+\eta} \sum_{j=t}^{\infty} \left[\frac{\eta^{j-t} (m_t + gt)}{(1+\eta)^{j-t}} \right] = \frac{1}{1+\eta} \frac{m + gt}{1 - \frac{\eta}{1+\eta}} = \frac{1+\eta}{1+\eta} (m + gt) = m + gt$

Money and Inflation IX: Examples

- Case 3: AR(1) process:

$$m_t = \rho m_{t-1} + \varepsilon_t$$

- Then

$$p_t = \frac{1}{1+\eta}\sum_{j=t}^{\infty}\left[\frac{\eta E_t\left(m_{j-t}\right)}{\left(1+\eta\right)^{j-t}}\right] = \frac{1}{1+\eta}\sum_{j=t}^{\infty}\left[\frac{\eta^{j-t}\rho^{j-t}m_t}{\left(1+\eta\right)^{j-t}}\right] =$$

$$\frac{1}{1+\eta}\frac{m_t}{1-\frac{\eta\rho}{1+\eta}} = \frac{1}{1+\eta\left(1-\rho\right)}m_t$$

Money and Inflation X: Examples

- Case 4: At time 0, there is an announcement that at time T>0, the money supply will be increased from m_1 to m_2.

- Then, before time 0, the price level will be constant at m_1. After time T, the price level will be constant at m_2.

- In between, 0 and T, however, the price level will be given by: $\frac{1}{1+\eta}\sum_{j=t}^{T-1}\left(\frac{\eta}{1+\eta}\right)^{j-t} m_1 + \frac{1}{1+\eta}\sum_{j=T}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{j-t} m_2$

Money and Inflation XI: Examples

- We can now rewrite this equation as:

$$\frac{1}{1+\eta}\sum_{t=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{t} m_1 + \frac{1}{1+\eta}\sum_{t=T}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{t}\left(m_2 - m_1\right)$$

- But we can rewrite this as:

$$m_1 + \left(\frac{\eta}{1+\eta}\right)^{T-t}\left(m_2 - m_1\right)$$

Seignorage I

- Seignorage is the real value of the money a government gets from printing money. Note that in developping countries, governments use the money supply but in developped countries, they mostly set interest rates.

- So, we can formally define seignorage as the difference in the money supply between the previous and current time periods divided by the current price level: $\frac{M_t - M_{t-1}}{P_t}$

Seignorage II

- Another way to write seignorage:

$$\frac{M_t - M_{t-1}}{P_t} * \frac{M_t}{M_t} = \frac{M_t - M_{t-1}}{M_{t-1}} \frac{M_{t-1}}{M_t} \frac{M_t}{P_t}$$

- This is interpretable as the growth rate of money (g) divided by one plus the growth rate of money multiplied by the value of the real money stock.

- Government can use monetary policy to affect unemployment as well as for seignorage.

Seignorage III

- Lets suppose the government wanted to maximize seignorage revenue. Would it set the inflation rate to infinity?
  - Intuitive answer: No. Always a gain in seignorage from increasing the money supply but if real money balances decrease because inflationary expectations rise, lowering the demand for money and causing a massive price inflation, then the marginal seignorage from an increase in the inflation rate can be negative!

Seignorage IV

- Another way to see seignorage:

$$\frac{M_t - M_{t-1}}{P_t} = \left(\frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}}\right) + \left(\frac{M_{t-1}}{P_{t-1}} - \frac{M_{t-1}}{P_t}\right)$$

- The first expression is the change in real balances and the second is the inflation tax.

- Assume a steady state rate of inflation. Then, Government solves following problem:

$$\underset{g}{Max}\, g \frac{M}{P}(g)$$

Seignorage V

- To solve the government problem in general, we have to figure out how the growth rate of money affects real balances (price level). So, we have to assume a money demand function. We choose Cagan's constant elasticity of inflation demand function:

$$\frac{M_t}{P_t} = \left(\frac{P_{t+1}}{P_t}\right)^{-\eta}$$

- From before, we know that the inflation rate for a constant elasticity of inflation money demand function with a constant rate of money growth, g, is just the same growth rate, g.

Seignorage VI

- We now replace money and price level variables with the growth rate of money in our expression for seignorage:

$$\frac{M_t - M_{t-1}}{M_t}\frac{M_t}{P_t} = \left[1 - \frac{1}{1+g}\right]\left(\frac{P_t}{P_{t-1}}\right)^{-\eta} = \frac{g}{1+g}(1+g)^{-\eta}$$

- Reformulating, we get:

$$\frac{g}{1+g}(1+g)^{-\eta} = g(1+g)^{-\eta-1}$$

Seignorage VII

- Taking the derivative of seignorage, we get:

$$\frac{dS}{dg} = (1+g)^{-\eta-1} - (1+\eta)(1+g)^{-\eta-2} = 0$$

- In which case, we get:

$$g = \frac{1}{\eta}$$

- Remember to check SOCs.
- Interpretation: the more the elastic the response of money demand to inflation, the higher the adjustment of the price level in response to a given level of money and thus the less the gain in real money from from additional inflation.

Seignorage VIII

- Given the Laffer curve for inflation, that marginal increases in inflation after a certain point can yield negative marginal seignorage, why do we ever see hyperinflation?
  - Hyperinflation defined by Cagan as 50% or more per month (almost 13,000% per year)
  - Highest ever: Hungary 19,800% per month!
  - Bolivia: April, 84' – July, 85': 23,000% rise.

- Answer: Adaptive expectations and surprise! Government fixes inflation expectations… then surprises… fixes again… then surprises… etc…

Time Inconsistency I

- Kydland and Prescott (1977) argue that time inconsistency leads to inflationary bias:
  - Central bankers promise low inflation. Then, after prices have been set, they surprise everyone by increasing the money supply, leading to an increase in output. (Dynamic inconsistency of central bank)
  - Problem: producers foresee this when setting prices and so set prices high

Time Inconsistency II

- Assume the Lucas Supply Curve:

$$y_t = \bar{y} - (w_t - p_t) - z_t$$

- Wage contracts are set in advance according to inflationary expectations in the following period:

$$w_t = E_{t-1} p_t$$

- The price at time t-1 is given at time t-1 when wages are set. Inflation is equal to the change in the price level: $\pi_t = p_t - p_{t-1}$

Time Inconsistency III

- Central banks set inflation directly by setting the price level. They do so after observing wages. They try to minimize both inflation and deviations from the natural level of output:

$$L = \left(y_t - \tilde{y}\right)^2 + \gamma \pi_t^2$$

- Plugging in the Lucas supply curve, we get:

$$L = \left(\overline{y} - \left(w_t - p_t\right) - z_t - \tilde{y}\right)^2 + \gamma \pi_t^2$$

- Replacing the wage term with expected inflation, and simplifying, we get:

$$L = \left(E_{t-1} p_t - p_t - z_t - k\right)^2 + \gamma \pi_t^2$$

Time Inconsistency IV

- Where we define $k = \tilde{y} - \bar{y} > 0$. In other words, the central bank targets a level of output higher than the natural level.

- Then, we add and subtract p_{t-1} :

$$L = \left(p_t - p_{t-1} + p_{t-1} - E_{t-1}p_t - z_t - k\right)^2 + \gamma\pi_t^2$$

- Finally, we replace price level terms with terms for inflation and expected inflation:

$$L = \left(\pi_t - E_{t-1}\pi_t - z_t - k\right)^2 + \gamma\pi_t^2$$

Time Inconsistency V

- First, suppose that central banks can fully commit to a level of inflation. Then, this must be observed ahead of time, in which case, the federal reserve simultaneously determines inflation and correct inflationary expectations. Thus, the maximization problem reduces to:

$$L = \left(-z_t - k\right)^2 + \gamma \pi_t^2$$

- The obvious welfare maximizing choice here is to set $\pi_t^* = 0$.

Time Inconsistency VI

- Lack of commitment: central banks set inflation but firms and wage setters anticipate correctly the inflation rate that the central bank will choose to implement.

$$\frac{dL}{d\pi} = 2\left(\pi_t - E_{t-1}\pi_t - z_t - k\right) + 2\lambda\pi_t = 0$$

- Solving for the inflation level:

$$\pi_t^* = \frac{E_{t-1}\pi_t + z_t + k}{1+\gamma}$$

- However, from rational expectations, we know that people on average believe correctly.

$$E_{t-1}\pi_t^* = \frac{E_{t-1}\pi_t + E_{t-1}z_t + k}{1+\gamma}$$

Time Inconsistency VII

- Solving for $E_{t-1}\pi_t$, we get $E_{t-1}\pi_t^* = \frac{E_{t-1}z_t + k}{\gamma} = \frac{k}{\gamma}$

- Solving for the equilibrium inflation level, we get:

$$\pi_t = \frac{\frac{k}{\lambda} + z_t + k}{1+\gamma} = \frac{k}{\gamma} + \frac{z_t}{1+\gamma}$$

- Notice that since the the shock, z, is mean zero, the average inflation rate is equal to the date t-1 expectation of inflation, namely: $\frac{k}{\gamma}$

Time Inconsistency VIII

- Notice that we have an inefficiently high level of inflation.

$\frac{k}{\lambda}$

- Expected inflation is equal to $$E_{t-1}\pi_t^* = \frac{E_{t-1}z_t + k}{\gamma} = \frac{k}{\gamma}$$

- Actual inflation is equal to $$\pi_t = \frac{\frac{k}{\gamma} + z_t + k}{1+\gamma} = \frac{k}{\gamma} + \frac{z_t}{1+\gamma}$$

- If the central bank credibly commit in advance to a policy rule: $\pi_t^* = \frac{z_t}{1+\gamma}$ then output would on average be the same and inflation would be lower... Time inconsistency leads to inflation bias!

Policy Rules I

- One solution: Pass a law mandating inflation to be zero.

- Then expected inflation is equal to actual inflation is equal to zero. So the loss function then ends up being:

$$E_{t-1}L = E_{t-1}\left(-z_t - k\right)^2 = \sigma_z^2 + k^2$$

- Expected utility from policy could be higher by allowing for commitment to a rule that allows the bank to fight shocks, z, but not on raise output above the natural rate, k.

Delegation I

- One solution: delegate authority to a conservative central banker... i.e. To a banker who has a strong dislike for inflation.

- Let the banker dislike inflation with parameter λ greater than γ. Then $\pi_t^* = \frac{z_t}{1+\lambda} + \frac{k}{\lambda}$ and $E_{t-1}\pi_t^* = \frac{k}{\lambda}$

- Notice that $\lim_{\lambda \to \infty} \pi_t^* = E_{t-1}\pi_t = 0$ so that we get our constitutionally set zero inflation rule again. This shows us that sufficiently conservative central bankers may not be sensitive enough to inflation and may be dominated by policy rules.

Practical Solutions

- Legally set inflation rate : New Zealand
- Give incentives to central bankers : New Zealand
- Use Policy Rules not Discretion: Canada, New Zealand, Sweden

Critiques of Time Inconsistency Model I

- Evidence: Alesina (1988) says that central bank independence is evidence of delegation. Finds independent central banks have lower inflation rates.
  - But not clear that voters wont vote for central bankers so not clear that independence should help reduce inflation
  - Do central banks with low inflation become independent or does independence cause low inflation?
  - Even if independence lowers inflation, is it due to time inconsistency reduction?

Critiques of Time Inconsistency Model II

- Countries like Canada and New Zealand which have made credible commitments to inflation reduction have experienced inflation reduction but usually before such policies have been implemented.
- Time inconsistency models dont account for time variation in inflation. Very high inflation in developed countries only lasted for less than a decade in the late 70s and early 80s.
- Sargent (1999) and Delong (1997) argue that the 70s were different because policy makers thought they could permanently change the unemp. rate by increasing monetary growth rates. However, once it was learned that this was not possible, central banks didnt need institutional reforms.

Critiques of Time Inconsistency Model III

- Fuhrer (1997) fails to find evidence of forward as opposed to backward looking expectations setting in US inflation.

- Debelle (1996) finds that institutional variables associated with reductions in time inconsistency seem not be correlated with inflation.

Policy Rules II

- Problems of discretion, Kydland and Prescott (1977)

- Long and Variable Lags: Friedman (1968)

- Creation of common knowledge: Svensson (2001)

Policy Rules III

- Arguments against rules: creates inertia, Amato and Shin (2004)

- Arguments against rules: not flexible enough.

Policy Rules IV

- If rules are followed, what should be set?
  - Interest Rate Rules
    - Taylor Rules: i.e. set nominal interest rates as a linear function of inflation and the output gap

$$i_t - \pi_t^* = a + b\pi_t + c\left(\ln Y_t - \ln \overline{Y_t}\right) + de_t$$

    - Taylor claims that the US followed such a rule with b=c=.5 since the move to targeting interest rates.
  - Money Supply Rules: Historically used... Still used in developing countries
  - Inflation Rules: Becoming more common

Inflation Targeting

- Use inflation as a specific target variable.
- Central Bank Transparency
- Accountability
- Debate: Effective or not (Sweden has it, US doesnt but US inflation is very low anyway).
- Debate: Is it just conservative policy in disguise (i.e. People who dont care about unemployment)?

Optimal Monetary Policy

- A framework for policy analysis: used in central banks.

- Define objectives: i.e.

$$L = E\left(\left(y - \overline{y}\right)^2\right) + \gamma E\pi^2$$

- And a model of the economy: i.e.

$$y_t = -\beta r_{t-1} + \rho y_{t-1} + \varepsilon_t,\ \beta \geq 0, 0 \geq \rho \geq 1$$

$$\pi_t = \pi_{t-1} + \alpha y_{t-1} + \delta_t,\ \alpha \geq 0$$

- And policy instruments: the central bank sets r_t after observing ε_t and δ_t . What is the optimal rule?

Friedman Rule

- Max $U\left(C_t, \frac{M_t}{P_t}\right)$

- Assume a satiation point for real balances for each level of consumption.

- Then, set $\frac{\partial U\left(C_t, \frac{M_t}{P_t}\right)}{\partial \frac{M_t}{P_t}} = 0$

- But from intertemporal maximizations, we get

Friedman Rule II

- But from intertemporal maximizations, we get

$$\frac{\dfrac{\partial U\left(C_t, \dfrac{M_t}{P_t}\right)}{\partial \dfrac{M_t}{P_t}}}{\dfrac{\partial U\left(C_t, \dfrac{M_t}{P_t}\right)}{\partial C_t}} = \frac{i_t}{1+i_t} = 0 \Leftrightarrow i_t = 0$$

- So, the optimal nominal interest rate is zero. This implies that $i_t \approx r_t + \pi_t = 0 \Rightarrow \pi_t = -r_t$