Mundell-Fleming I: Setup

- In ISLM, we had:

$$E = C\left(Y - T\right) + I(i - \pi^{e}) + G - T$$

- To this, we now add net exports, which is a function of the exchange rate:

$$E = C\left(Y - T\right) + I(i - \pi^{e}) + G - T + NX\left(\frac{\varepsilon P^{*}}{P}\right)$$

- Where NX is assumed (Marshall Lerner condition) increasing in the real exchange rate:

$$\frac{\varepsilon P^{*}}{P}$$

Mundell-Fleming II: Setup

- Note that P is the domestic price level, $P*$ the foreign price level, and ε the nominal exchange rate (i.e. the price of foreign currency expressed in domestic currency).

- Now we look at the nominal side of the economy:

$$\frac{M}{P}^{D} = L(i, Y)$$

- We also assume static expectations and perfect capital mobility which in this context means that: $i = i^{*}$

Mundell-Fleming III: Floating Exchange Rate

- Static expectations means that future expectations equal the current state.

- So, with a floating exchange rate, we end up with two equations:

$$Y = E(Y, i^* - \pi^e, G, T, \frac{\varepsilon P^*}{P})$$

$$\frac{M}{P} = L\left(i^*, Y\right)$$

- Notice that the LM* curve is vertical and the IS* (both graphed in exchange rate versus output space) curve is upward sloping.

Mundell-Fleming IV: Floating Exchange Rate

- Monetary expansion with a flexible exchange rate:
  - Money supply increases
  - Temporary fall in domestic interest rate
  - Capital outflow
  - Fall in demand for domestic currency
  - Depreciation
  - Output expansion

- Fiscal expansion with a flexible exchange rate:
  - Government expenditures increase
  - Temporary rise in domestic interest rate
  - Capital inflow
  - Increase in demand for domestic currency
  - Appreciation
  - Greater domestic demand but lower net exports offset each other

Mundell Fleming V: Floating Exchange Rate

- Why is the fiscal expansion exactly offset by appreciation and decline in net exports?
  - Suppose not… then for a given M and P, Y would be higher

  - Then in order for money demand to equal money supply, the domestic interest rate would have to be higher which goes against the assumption of perfect capital mobility.

Mundell Fleming VI: Fixed Exchange Rates

- Two changes need to be made for the model of fixed exchange rates:
- (1.) $\varepsilon = \bar{\varepsilon}$
- (2.) M is now endogenous in order to make sure that the exchange remains fixed (it is bought and sold by the central bank to retain the exchange rate target).

Mundell-Fleming VII: Fixed Exchange Rate

- So, with a fixed exchange rate, we end up with three equations:

$$Y = E(Y, i^* - \pi^e, G, T, \frac{\varepsilon P^*}{P})$$

$$\frac{M}{P} = L\left(i^*, Y\right)$$

$$\varepsilon = \bar{\varepsilon}$$

- Now the LM* curve is replaced by a horizontal exchange rate curve and the IS* (both graphed in exchange rate versus output space) curve is upward sloping.

Mundell-Fleming VIII: Fixed Exchange Rate

- Monetary expansion with a fixed exchange rate:
  - Money supply increases
  - Temporary fall in domestic interest rate
  - Capital outflow
  - Fall in demand for domestic currency
  - Incipient Depreciation
  - Central bank buys back money to restore the exchange rate

- Fiscal expansion with a flexible exchange rate:
  - Government expenditures increase
  - Temporary rise in domestic interest rate
  - Capital inflow
  - Increase in demand for domestic currency
  - Appreciation
  - Monetary authority prints money, restoring exchange rate
  - Two sources of demand increase: fiscal and monetary

Dornbusch I: Setup

- Return on a domestic bond: $1+i_t$

- Exchange rate = price of foreign currency: i.e. if US is the home country and Sweden the foreign country, then the exchange rate would be that it cost 1/7 of a dollar divided by one dollar or just 1/7.

- Return on a foreign bond: $\left[1+i_t^*\right]\frac{\varepsilon_{t+1}}{\varepsilon_t}$

- Taking logs: $\log\left[1+i_t^*\right]\frac{\varepsilon_{t+1}}{\varepsilon_t} \approx i_t^* + e_{t+1} - e_t$

Dornbusch II: Setup

- So, we get uncovered interest parity:

$$i_t = i_t^* + e_{t+1} - e_t$$

- Under what assumptions will this hold?
- Covered interest parity: replace the future spot rate with the current future rate. Essentially, this MUST hold. If it doesn't, why not?
- Second equation: money demand

$$m_t - p_t = -\eta i_{t+1} + \phi y_t$$

Dornbusch III: Setup

- Definition of real exchange rate:

$$q = e + p^* - p$$

- In theory, the real exchange rate should have a price of 1 for tradable goods. Why?
  - Empirically, we don't see this.

- Demand is determined by the real exchange rate relative to the full employment REX:

$$y_t^d = \overline{y} + \delta\left[e_t + p^* - p_t - \overline{q}\right] = \overline{y} + \delta\left[q_t - \overline{q}\right]$$

Dornbusch IV: Setup

- Interpretation of real exchange rate: the real price of foreign to domestic goods (i.e. the price of foreign TV expressed in domestic currency to price of domestic TV).
- Real exchange rate of 1 is called PPP (Purchasing Power Parity).
- Dornbusch model allows for deviations from PPP.

Dornbusch V: Setup

- Motivation for assumption that demand for a country's output is a decreasing function of the real exchange rate
  - (1.) Monopoly power by home firm in own markets (so price adjustment doesn't lead to infinite or zero demand)
  - (2.) Home-produced tradables goods are more important to the home country
  - (3.) Domestic demand switches from foreign tradables to domestic non-tradables.

Dornbusch VI: Setup

- Dornbusch model is fully dynamic; shows period by period price adjustment.

- Assumption: price adjustment happens according to an expectations-adjusted philips curve:

$$p_{t+1} - p_t = \psi\left[y_t^d - \overline{y}\right] + \left(\tilde{p}_{t+1} - \tilde{p}_t\right)$$

- Where $\tilde{p}_t$ is the price level that would occur if the price level cleared:

$$\tilde{p}_t = e_t + p_t * - \overline{q}_t$$

Dornbusch VII: Setup

- First differencing the definition of $\tilde{p}_t$, we get:

$$\tilde{p}_{t+1} - \tilde{p}_t = \left(e_{t+1} + p_{t+1}* - \bar{q}_{t+1}\right) - \left(e_t + p_t* - \bar{q}_t\right)$$

- Plugging this into the price adjustment philips curve, we get:

$$p_{t+1} - p_t = \psi\left[y_t^d - \bar{y}\right] + e_{t+1} - e_t$$

- Now that we have specified the model, we will review the main equations: notice that this model can much better look at the time path of exchange rate dynamics in comparison with the Mundell-Fleming model which can only be used to analyze changes from one long-term equilibrium to another.

Dornbusch VIII: Setup

- Thus, we have four unknowns (e_t, p_t, y_t, i_t) and the following equations:
  - Uncovered Interest Parity: $i_t = i_t^* + e_{t+1} - e_t$
  - Money Demand: $m_t - p_t = -\eta i_{t+1} + \phi y_t$
  - Domestic Tradables Demand: $y_t^d = \overline{y} + \delta\left[q_t - \overline{q}\right]$
  - Price Adjustment: $p_{t+1} - p_t = \psi\left[y_t^d - \overline{y}\right] + e_{t+1} - e_t$

Dornbusch IX: Graphical Solution

- First, from the definition of the real exchange rate, we get

$$q_{t+1} - q_t = e_{t+1} + p_{t+1}* - p_{t+1} - [e_t + p_t* - p_t]$$

$$\Rightarrow q_{t+1} - q_t = e_{t+1} - p_{t+1} - [e_t - p_t]$$

- Then, combining the money demand equation with the price adjustment equation, we get:

$$q_{t+1} - q_t = -\psi\left[y_t^d - \overline{y}\right] = -\psi\delta\left(q_t - \overline{q}\right)$$

- This is one of two equations whose dynamics we will need… the other is the nominal exchange rate.

Dornbusch X: Graphical Solution

- First, we normalize the parameters to zero:

$$p^* = \bar{y} = i^* = 0$$

- Then, using uncovered interest parity, we get:

$$e_{t+1} - e_t = i_t$$

- From money demand, we can solve for the interest rate:

$$i_t = \frac{p_t + \phi y_t - m_t}{\eta}$$

Dornbusch XI: Graphical Solution

- Now we need to get rid of all endogenous variables besides nominal and real exchange rates (i.e. price and output). We can replace output using the demand equation (and normalizing):

$$y_t = \bar{y} + \delta\left(q_t - \bar{q}\right) = \delta\left(q_t - \bar{q}\right)$$

- Similarly, we can get price from the definition of the real exchange rate (normalized):

$$q_t = p_t^* + e_t - p_t = e_t - p_t$$

Dornbusch XII: Graphical Solution

- Replacing the expressions for price and output into the interest rate equation, we get:

$$i_t = \frac{e_t - q_t + \phi\delta\left(q_t - \bar{q}\right) - m_t}{\eta} = \frac{e_t - \left[1 - \phi\delta\right]q_t - \left(\phi\delta\bar{q} + m_t\right)}{\eta}$$

- Replacing the interest rate expression from uncovered interest parity, we get:

$$e_{t+1} - e_t = \frac{e_t - \left[1 - \phi\delta\right]q_t - \left(\phi\delta\bar{q} + m_t\right)}{\eta}$$

Dornbusch XIII: Graphical Solution

- So, we have two equations of motion, each of which are functions of two dynamic variables:

- The equation of motion for the real exchange rate:

$$q_{t+1} - q_t = -\psi\delta\left(q_t - \bar{q}\right)$$

- And the equation of motion for the nominal exchange rate:

$$e_{t+1} - e_t = \frac{e_t - [1 - \phi\delta]q_t - \left(\phi\delta\bar{q} + m_t\right)}{\eta}$$

Dornbusch XIV: Graphical Solution

- First, we start with solving for the long-run equilibrium:

$$q_{t+1} - q_t = 0 \Rightarrow q_t = \overline{q}$$

$$e_{t+1} - e_t = 0 \Rightarrow e_t = [1 - \phi\delta]q_t + (\phi\delta\overline{q} + m_t)$$

- But since in equilibrium, we have $q_t = \overline{q}$, we get:

$$\overline{e} = m + \overline{q}$$

- So the q schedule is a vertical line at $q_t = \overline{q}$ and the e schedule is a straight line with slope $1 - \phi\delta$ and intercept $\phi\delta q + m$

Dornbusch XV: Graphical Solution

- Now we draw the phase diagram and look at stability properties of the equilibrium.

$$q > \bar{q} \Rightarrow \Delta q < 0 \qquad\qquad q < \bar{q} \Rightarrow \Delta q > 0$$

and

$$e_t > [1 - \phi\delta]q_t + \left(\phi\delta\bar{q} + m_t\right) \Rightarrow \Delta e > 0$$

$$e_t < [1 - \phi\delta]q_t + \left(\phi\delta\bar{q} + m_t\right) \Rightarrow \Delta e < 0$$

- These patterns imply something called saddle-path stability: convergence to the long-run equilibrium is only along a unique saddle path. Elsewhere in the space, we get divergence. Moreovere, this is no real economic reason to expect the economy to be on the saddle path.

Dornbusch XVI: Graphical Solution

- Now consider a one-time permanent increase in the money supply… a monetary shock.

- In the long run, since $q = e + \underline{p}^* - p = e - p$ and also $e = m + q$ then we have , $p = m$. As a result, in the long run, an increase in the money supply from m to $\hat{m}$ will lead to an increase in the price level to $\hat{m}$ and given that the long-run real exchange rate must remain constant, an increase in the long-run exchange rate by the amount $\hat{m} - m$

Dornbusch XVII: Graphical Solution

- What about in the short run? In the short run, since prices are sticky, so $p_0 = m$ which means that the real exchange rate, nominal exchange rate combination are on the line given by:

$$q_0 = e_0 - m$$

- The initial new nominal and real exchange rate combination are given by the intersection of the above line and the new saddle path (which intersects the new long run equilibrium and has a slope of less than the exchange rate curve which itself has a slope less than 1).

Dornbusch XVIII: Graphical Solution

- Thus, as long as the exchange rate curve is positively sloped ($1-\phi\delta > 0$), there will be overshooting of the nominal exchange rate.

- What is the intuition for this? An increase in the money stock leads to an increase in real money balances because prices are fixed. If the $1-\phi\delta$ exchange jumped to its new equilibrium with p fixed, that would cause a real depreciation of the currency and thus output would increase by $\phi\delta$. If the above condition is satisfied, then real money supply will be greater than real money demand and the domestic interest rate will have to fall.

Dornbusch XIX: Graphical Solution

- If the domestic interest rate falls, then there should be an accompanying expected appreciation of the nominal exchange rate… in other words, the exchange has to depreciate more than implied by the expansion of the money supply and then it must appreciate slowly afterwards as prices increase.

Dornbusch Model XX: Analytical Solution

- We start with the real-side difference equation which is just an equation with real endogenous variables:

$$q_{t+1} - q_t = -\psi\delta\left(q_t - \overline{q}\right)$$

$$\Rightarrow q_{t+1} - \overline{q} = [1 - \psi\delta]\left(q_t - \overline{q}\right)$$

$$\Rightarrow q_{t+2} - \overline{q} = [1 - \psi\delta]\left(q_{t+1} - \overline{q}\right) = [1 - \psi\delta]^2\left(q_t - \overline{q}\right)$$

$$\Rightarrow q_{t+N} - \overline{q} = [1 - \psi\delta]^{N-t}\left(q_t - \overline{q}\right)$$

Dornbusch Model XXI: Analytical Solution

- Now taking the equation for the nominal side of the economy, which does depend upon the real side, we get:

$$e_{t+1} - e_t = \frac{e_t - [1-\phi\delta]q_t - \left(\phi\delta\bar{q} + m_t\right)}{\eta}$$

$$\Rightarrow \frac{1+\eta}{\eta} e_t = e_{t+1} + \frac{[1-\phi\delta]q_t + \left(\phi\delta\bar{q} + m_t\right)}{\eta}$$

$$\Rightarrow e_t = \frac{\eta}{1+\eta} e_{t+1} + \frac{[1-\phi\delta]q_t + \left(\phi\delta\bar{q} + m_t\right)}{1+\eta}$$

$$\Rightarrow e_t - \bar{q} = \frac{\eta}{1+\eta}\left(e_{t+1} - \bar{q}\right) + \frac{[1-\phi\delta]\left(q_t - \bar{q}\right) + m_t}{1+\eta}$$

Dornbusch Model XXII: Analytical Solution

- Iterating forward, we get:

$$\Rightarrow e_t - \bar{q} = \lim_{j\to\infty}\left(\frac{\eta}{1+\eta}\right)^{j-t} e_j + \frac{[1-\phi\delta]\sum_{j=t}^{\infty}\left[\frac{\eta}{1+\eta}\right]^{j-t}\left(q_j - \bar{q}\right) + \sum_{j=t}^{\infty}\left[\frac{\eta}{1+\eta}\right]^{j-t} m_j}{1+\eta}$$

- Imposing the no bubbles condition (which rules out paths except the saddle path):

$$\lim_{j\to\infty}\left(\frac{\eta}{1+\eta}\right)^{j-t} e_j = 0$$

Dornbusch Model XXIII: Analytical Solution

- Rewriting:

$$\Rightarrow e_t - \bar{q} = \frac{[1-\phi\delta]\sum_{j=t}^{\infty}\left[\frac{\eta}{1+\eta}\right]^{j-t}\left(q_j - \bar{q}\right) + \sum_{j=t}^{\infty}\left[\frac{\eta}{1+\eta}\right]^{j-t} m_j}{1+\eta}$$

- Now, assuming a constant money supply, we first note that:

$$\sum_{j=t}^{\infty}\left[\frac{\eta}{1+\eta}\right]^{j-t} = \frac{1}{1-\frac{\eta}{1+\eta}} = 1+\eta$$

Dornbusch Model XXIV: Analytical Solution

- Continuing to solve:

$$\Rightarrow e_t = \bar{q} + m + \frac{[1-\phi\delta]\sum_{j=t}^{\infty}\left[\frac{\eta}{1+\eta}\right]^{j-t}\left(q_j - \bar{q}\right)}{1+\eta}$$

- Replacing for the real exchange rate:

$$\Rightarrow e_t = \bar{q} + m + \frac{[1-\phi\delta]\left(q_t - \bar{q}\right)\sum_{j=t}^{\infty}\left[\frac{\eta}{1+\eta}\right]^{j-t}[1-\psi\delta]^{j-t}}{1+\eta}$$

$$\sum_{j=t}^{\infty}\left[\frac{\eta}{1+\eta}\right]^{j-t}[1-\psi\delta]^{j-t} = \frac{1}{1-\left(1-\psi\delta\right)\frac{\eta}{1-\eta}} = \frac{1+\eta}{1+\eta\psi\delta}$$

Dornbusch Model XXV: Analytical Solution

- Finally, we arrive at the equation for the saddle path:

$$\Rightarrow e_t = \bar{q} + m + \frac{[1 - \phi\delta](q_t - \bar{q})}{1 + \psi\delta\eta}$$

- Now we look at shocks:

- Suppose that at date zero, the money supply unexpectedly increases from m to m'

Dornbusch Model XXVI: Analytical Solution

- From before, we know that prices are stuck in the short run so that:

$$q_0 = e_0 - p_0 = e_0 - m$$

- We also have derived the equation for the saddle path:

$$\Rightarrow e_0 = \bar{q} + m' + \frac{[1-\phi\delta](q_0 - \bar{q})}{1+\psi\delta\eta}$$

- We have two equations and two unknowns so that we can solve for the initial q and e.

Dornbusch Model XXVII: Analytical Solution

- Plugging the initial condition equation into the saddle path equation, we get:

$$\Rightarrow q_0 + m = \bar{q} + m' + \frac{[1 - \phi\delta](q_0 - \bar{q})}{1 + \psi\delta\eta}$$

- Solving for the initial real exchange rate, we get:

$$q_0 = \bar{q} + \frac{1 + \psi\delta\eta}{\phi\delta + \psi\delta\eta}(m' - m)$$

- We can also solve for the initial nominal exchange rate:

$$e_0 = \bar{q} + m + \frac{1 + \psi\delta\eta}{\phi\delta + \psi\delta\eta}(m' - m)$$

Dornbusch Model XXVIII: Analytical Solution

- Since, by assumption, $1 > \phi\delta$, we find that
  $e_0 > \bar{q} + m' = \bar{e}$

- In other words we get overshooting of the nominal exchange rate!

- Note that we can continue to solve for other period values by iterating forward on the real exchange rate equation and then plugging the real exchange rate into the equation for the nominal exchange rate.