# Insider/Outsider Theory

- Developped by Lindbeck-Snower
- Firms maximize profits, taking as given number of insiders, expected wage of insiders, wage differential between insiders and outsiders and choosing level of outsider employment and wage levels given productivity level.

$$\Psi = \sum_{i} p_i \left[ A_i F \left[ \overline{L}_I + L_{Oi} \right] - w_{Ii} \overline{L}_I - (w_{Ii} - r) L_{Oi} \right] + \lambda \left[ \sum_{i} p_i U(w_{Ii}) - U_0 \right]$$

# Insider/Outsider Theory II

- What is  $W_{Oi}$ ?
- FOCs

$$\frac{\partial \Psi}{\partial L_{Oi}} = 0 \Longrightarrow A_i F' \left( \overline{L_I} + L_{Oi} \right) = w_{Ii} - C$$

$$\frac{\partial \Psi}{\partial w_{Ii}} = -p_i \left(\overline{L_I} + L_{Oi}\right) + \lambda p_i U'(w_{Ii}) = 0 \Longrightarrow U'(w_{Ii}) = \frac{\overline{L_I} + L_{Oi}}{\lambda}$$

# Insider/Outsider III

 If we assume that outsider labor supply is completely inelastic and sufficiently large, we have equilibrium unemployment as a consequence.

# Hysteresis

- Sometimes shocks so large that some insiders are laid off.
- Then, new group of insiders rebargains wage contracts, choosing higher wages over employment since there are less insiders now.
- This means that a negative shock to productivity leads to low employment this period and low employment the period after: positive serial correlation in unemployment levels or hysteresis.

### Search Unemployment I: Broad Overview

- People have differentiated between different types of unemployment: search (or frictional unemployment) and structural unemployment (i.e. due to aggregate demand failures).
- Chicago-View:
  - Frictional unemployment exists but government can not influence its level
  - Structural employment fluctuations occur but there is no structural unemployment
- MIT/Harvard/Princeton/Berkeley View:
  - Frictional unemployment can be inefficiently high due to market failures in lack of centralized market for employment (lack of Walrasian Auctioneer)
  - Structural unemployment exists and can be inefficiently high due to aggregate demand failures

#### Search Unemployment II: Model Setup I

- Workers choose whether to be in the labor market (if they are unemployed, this means they search for a job; if they are employed, it means they remain at their job)
- Firms choose whether or not to post a position (if they already employ someone, this means they retain the position; if they do not already employ someone, they post a vacancy)
- Matches (firm/worker pairs) find eachother with rate M(U,V) depending upon the unemployment rate and vacancy rate.
- An equilibrium, (a level of employment: E and a set of decisions by firms and workers to post vacancies and search for work) is a steady state level of employment such that no firms have incentives to change their vacancy posting decision and no workers have the incentive to change their decisions to look for work.

# Search Unemployment III: Solving the Model I

- We will describe how to solve for the level of employment, E, and its properties (how it changes with parameters).
- First, compute the value of posting a vacancy, posting a filled position, being employed, and being unemployed.
- Then, equate the difference in values between a vacancy and filled position with an unemployed and employed position and solve for the wage as a function of employment.
- Having calculated the wage, we can solve for the value of posting a vacancy. Equilibrium will occur when the value of posting a vacancy is zero. This value of posting a vacancy function will be a function of the employment level and will define equilibrium employment and unemployment.

# Search Unemployment IV: Model Setup II

Firms choose  $V_v$  or 0 if vacant They choose  $V_F$  or 0 if filled

Workers choose  $V_E$  or 0 if employed Workers choose  $V_U$  or 0 if unemployed

### Search Unemployment V: Model Setup III

- Solve for Steady State Level of Emp./Unemp.  $bE = M(U, V) = KU^{\beta}V^{\gamma}$
- M(U,V) is called the matching function
- Rate at Which Workers Find Jobs

$$a = \frac{M(U,V)}{U}$$

• Rate at Which Jobs Find Workers

$$\alpha = \frac{M(U,V)}{V}$$

# Search Unemployment II: Constructing Value Functions

• Value of Being Employed:

$$\mathbf{r}\mathbf{V}_{\mathrm{E}} = w - b(V_{E} - V_{U})$$

- Value of Being Unemployed:  $\mathbf{rV}_{\mathrm{U}} = a \big( V_E V_U \big)$
- Value of A Filled Position:  $rV_F = A - w - C - b(V_F - V_V)$
- Value of A Vacant Position:

$$\mathrm{rV}_{\mathrm{v}} = -C + \alpha \left( V_F - V_V \right)$$

### Search Unemployment: Solving the Model II

• Split the surplus

$$\mathbf{V}_{\mathrm{F}} - \boldsymbol{V}_{V} = \boldsymbol{V}_{E} - \boldsymbol{V}_{U}$$

Subtract Equations from Previous Slide

$$rV_{F} - rV_{V} = A - w - C - b(V_{F} - V_{V}) + C - \alpha(V_{F} - V_{V})$$

$$\Longrightarrow V_F - V_V = \frac{A - w}{\alpha + b + r}$$

• Similarly

$$V_E - V_U = \frac{w}{a+b+r}$$

#### Search Unemployment VIII: Solving the Model III

• So  $\frac{A-w}{\alpha+b+r} = \frac{w}{a+b+r}$ 

$$w = \frac{(a+b+r)A}{\alpha+a+2b+2r}$$

But



# Search Unemployment IX: Computing Equilibrium Emp.

 Inflow into Unemployment Equals Outflow From Unemployment

$$a\left[\overline{L} - E\right] = bE$$

 Inflow Into Vacancies Equals Outflow from Vacancies

$$bE = M(U, V) \Rightarrow \alpha = \frac{M(U, V)}{V}$$
$$bE = KU^{\beta}V^{\gamma} \Rightarrow V = \left(\frac{bE}{KU^{\beta}}\right)^{\frac{1}{\gamma}} = \left(\frac{bE}{K(\overline{L} - E)^{\beta}}\right)^{\frac{1}{\gamma}}$$
$$\alpha = \frac{bE}{\left[\frac{bE}{K(\overline{L} - E)}\right]^{\frac{1}{\gamma}}}$$

# Search Unemployment X: Computing Equilibrium Emp. II

• Model Closure: Firms are indifferent between posting a vacancy and not (like labor demand):

$$rV_{V} = -C + \frac{\alpha(E)}{\alpha(E) + \alpha(E) + 2b + 2r}A = 0$$

• Equilibrium Employment is whatever E solves the above equation; unemployment is then  $\overline{L} - E$ .

#### Search Unemployment XI: Uniqueness of Equilibria

• Want to show that  $\frac{dV_V}{dE} < 0$ 

$$\frac{da(E)}{dE} = \frac{d\frac{bE}{\overline{L}-E}}{dE} = \frac{b}{\overline{L}-E} + \frac{bE}{\left(\overline{L}-E\right)^2} > 0$$

• Remember that  $\gamma \leq 1$ 

$$\frac{d\alpha(E)}{dE} = \frac{d(bE)^{\frac{\gamma-1}{\gamma}} \left[K^{\frac{1}{\gamma}} \left(\overline{L} - E\right)^{\frac{\beta-\gamma}{\gamma}}\right]}{dE} = \left(1 - \frac{1}{\gamma}\right) b(bE)^{-\frac{1}{\gamma}} \left[K \left(\overline{L} - E\right)^{\beta}\right]^{\frac{1}{\gamma}} - K^{\beta} \left(\overline{L} - E\right)^{\frac{\beta-\gamma}{\gamma}} < 0$$

# Search Unemployment XII: Uniqueness of Equilibria II

• Differentiating, we get:

 $\frac{dV_{V}}{dE} = \frac{\frac{d\alpha(E)}{dE}}{\alpha(E) + a(E) + 2r + 2b} A - \frac{\alpha(E)}{\left[\alpha(E) + a(E) + 2r + 2b\right]^{2}} A \left[\frac{d\alpha(E)}{dE} + \frac{da(E)}{dE}\right]$ 

$$\frac{dV_{V}}{dE} = \left[1 - \frac{\alpha(E)}{D}\right] A \frac{d\alpha(E)}{dE} - \frac{\alpha(E)}{D^{2}} A < 0$$

- Where D=  $\alpha(E) + a(E) + 2r + 2b$
- From this we know that equilibria are unique if they exist

#### Search Unemplomyent: Existence of Equilibria

- Note that E ranges from zero to  $\overline{L}$
- We now take the limits of the value of a vacancy as employment varies from zero to full employment:

$$\lim_{E \to 0} \alpha(E) = \infty, \lim_{E \to 0} \alpha(E) = 0 \Longrightarrow \lim_{E \to 0} rV_V = A - C$$
$$\lim_{E \to L} \alpha(E) = 0, \lim_{E \to 0} \alpha(E) = \infty \Longrightarrow \lim_{E \to 0} rV_V = -C$$

 Given continuity of the value of posting a vacancy in the range of zero to full employment, we have existence of an equilibrium!

# Search Unemployment: Comparative Statics I

 How do we look at Cyclical Changes in this model?

• How do we look at long-term changes in this model (i.e. the effect of growth)?

## Search Unemployment: Comparative Statics II

- A moves for cyclical fluctuations; the ratio of A and C for growth.
- Impact of Growth: None... an increase in A and C leaving the ratio unchanged has no impact on labor demand:

$$rV_{V} = \frac{\alpha(E)}{\alpha(E) + \alpha(E) + 2b + 2r} = \frac{C}{A}$$

# Search Unemployment: Comparative Statics III

• Equilibrium Vacancies are given by:

$$M(U,V) = KU^{\beta}V^{\gamma} = bE \Longrightarrow V = \left(\frac{bE}{KU^{\beta}}\right)^{\frac{1}{\gamma}} = \left(\frac{b(\overline{L}-U)}{KU^{\beta}}\right)^{\frac{1}{\gamma}}$$

- Negative Relationship between vacancies and unemployment: Beveridge Curve
- Short term changes lead to smaller employment effects and larger wage adjustment: think about infinitessimally short reduction in A: comes out of wages entirely because there are no changes in value of vacancies and value of unemployment which means no change in M(U,V) or bE.
- Procyclical Wage: How do we figure this out?

#### Search Unemployment: Comparative Statics IV

• Wage expression:

$$\Rightarrow w = \frac{(a+b+r)A}{\alpha + a + 2b + 2r}$$

• Total differentiate:

$$\Rightarrow dw = \frac{(\alpha + a + 2b + 2r)A\frac{\partial a(E)}{\partial E}dE - (a + b + r)A\frac{\partial a(E)}{\partial E}dE - (\alpha + b + r)A\frac{\partial a(E)}{\partial E}dE}{(\alpha + a + 2b + 2r)^2}dE$$

• Solve:  $\Rightarrow \frac{dw}{dE} = \frac{(\alpha + b + r)A\left[\frac{\partial a(E)}{\partial E} - \frac{\partial a(E)}{\partial E}\right]}{(\alpha + a + 2b + 2r)^2} > 0$ 

# Search Unemployment: Recent Empirical Results

• US manufacturing: 3% of workers leave jobs in a typical month and 10% of jobs disappear in a typical year.

 Volatility in unemployment mostly derived from increased job destruction not decreased job creation.

# Other Models of Unemployment

• Why assume supply equals demand?

Weitzman: Continuum of Equilibria

 Out of Equilibrium Dynamics Models: Assume price adjustment mechanism (i.e. price adjusts to reduce net excess demand but this feeds back into demand)

# Phillips Curve I:

- Originally studied by Phillips, A. W. (1958), "The Relationship between Unemployment and the Rate of Change of Money Wages in the United Kingdom, 1861-1957", *Economica* 25, pp. 283-299.
- ISLM interpretation: Monetary Expansion:

$$\frac{dY^*}{dM} > 0$$

# Phillips Curve II:



# Phillips Curve III:

 New Classical View: Lucas Supply Curve; monetary confusion... when firms notice high nominal demand, they mistake it for firm-specific demand and expand output. However, monetary expansions that are expected have no effect on employment... just on prices.

# Lucas Supply Curve I: Solution Concept I

- (1.) Satisfies individual supply equals individual demand (i.e. for each good)
- (2.) Satisfies aggregate demand (output equals real money supply)
- Equilibrium is a set of prices, *p<sub>i</sub>* and quantities, *q<sub>i</sub>*, such that (1.) and (2.) are satisfied.

# Lucas Supply Curve II: Solution Concept II

- Two versions of the model:
  - Complete Information
  - Incomplete Information
- Complete Information: Take money supply as given (commonly known) and price level... Money has no effects.
- Incomplete Information:
  - Prices used to figure out state of demand. Agents use true relative variances of aggregate (nominal) versus individual (real) shocks to figure out level of demand.
  - First paper to use prices as revealing information.

#### Lucas Supply Curve III: Solution Concept III

- Three types of shocks:
  - Shocks to relative demand
  - Expected shocks to money supply (Monetary policy)
  - Unexpected shocks to money supply

# Lucas Supply Curve III: Complete Information I

• **Production:**  $Q_i = L_i$ 

• Consumption: 
$$U_i = C_i - \frac{L_i^{\gamma}}{\gamma}, \gamma \ge 1$$

 Budget Constraint: Value of individual consumption equals value of individual production

$$P_i Q_i = P C_i$$

 Plugging Production into Budget Constraint and Budget Constraint into Consumption:

$$U_i = \frac{P_i L_i}{P} - \frac{L_i^{\gamma}}{\gamma}$$

### Lucas Supply Curve IV: Complete Information II

• FOC:

$$\frac{dU_i}{dL_i} = \frac{P_i}{P} - L_i^{\gamma - 1} = 0 \Longrightarrow L_i^* = \left(\frac{P_i}{P}\right)^{\frac{1}{\gamma - 1}}$$

• Translate into Logs:

$$\ell_i^* = \frac{1}{\lambda - 1} (p_i - p)$$

• Add individual demand (can we derive this from utility functions?):  $q_i = y + z_i - \eta(p_i - p); z_i \sim N(0, \sigma_z^2)$ 

# Lucas Supply Curve V: Complete Information III

• Means of distributions:

$$\int_{D} z_i f(z_i) = 0$$

$$\int_{D} p_i f(p_i) = p$$

$$\int_{D} q_i f(q_i) = y$$

# Lucas Supply Curve VI: Complete Information IV

Add aggregate demand (real money equals output)

$$y = m - p$$

• From 
$$Q_i = L_i$$
, we get:  
 $q_i = \ell_i \Rightarrow \frac{1}{\gamma - 1} (p_i - p) = y + z_i - \eta (p_i - p)$ 

• Solving for  $P_i$ , we get:

$$p_i = \frac{\gamma - 1}{1 + \eta \gamma - \eta} (y + z_i) + p$$

## Lucas Supply Curve VII: Complete Information V

• Taking the average of both sides of:

$$p_i = \frac{\gamma - 1}{1 + \eta \gamma - \eta} (y + z_i) + p$$

• We get:

$$p = \frac{\gamma - 1}{1 + \eta \gamma - \eta} y + p \Longrightarrow y = 0$$

• This implies that:

$$p = m$$

# Lucas Supply Curve VIII: Complete Information Interpretation

• An increase in the money supply is reflected one for one in higher prices.

- There is no effect of an increase in money supply on output: money is neutral.
- y=0: output is uncorrelated with inflation

# Lucas Supply Curve IX: Incomplete Information I

- Same as complete information model except that there are two sources of information which is not observed: shocks to money supply, m, and shocks to productivity, z;
- Why do we need two shocks to be unobserved?
- The distribution of each of the shocks is known:

$$\mathbf{z}_{i} \sim N(\mathbf{0}, \sigma_{z}^{2})$$
  $\mathbf{m}_{i} \sim N(\mathbf{0}, \sigma_{m}^{2})$ 

• We will know derive a correlation between the aggregate price level, m, and aggregate output, y

## Lucas Supply Curve X: Incomplete Information II

• Producers maximize:

$$EU_{i} = E\left[\frac{P_{i}L_{i}}{P} - \frac{L_{i}^{\gamma}}{\gamma}|p_{i}\right]$$

• Approximate actual solution with certainequivalence:

$$\ell_i = \frac{1}{\gamma - 1} E[p_i - p|p_i]$$

• Denote relative price by:

$$r_i = p_i - p$$

#### Lucas Supply Curve XI: Incomplete Information III

Assume rational expections: the expectations of the relative price given the observed market-specific price is the true distribution of the relative price given the observed market-specific price.

# Lucas Supply Curve XII: Incomplete Information IV

Need to calculate

 $E[r_i|p_i] = E[p_i - p|p + (p_i - p)]$ 

 For now, assume that p<sub>i</sub> and r<sub>i</sub> are normally distributed and independent of eachother. This will be verified later once we have calculated the equilibrium p<sub>i</sub> and r<sub>i</sub> (this is called the guess and check method).

# Lucas Supply Curve XIII: Incomplete Information V

 Linear Conditional Expectations given two signals: individual price and average price level:

$$E[r_i|p_i] = \alpha p_i - \beta p$$

The coefficients are just the precisions of the estimates in the normal distribution model (remember variances of p and r are endogenous):

$$\alpha = \beta = \frac{\overline{\sigma_p^2}}{\frac{1}{\sigma_r^2} + \frac{1}{\sigma_p^2}}$$

## Lucas Supply Curve XIV: Incomplete Information VI

• Multiplying top and bottom by  $\sigma_r^2 \sigma_p^2$  gives us

$$\alpha = \beta = \frac{\sigma_r^2}{\sigma_r^2 + \sigma_p^2}$$

 Thus our expression for the conditional expectation is:

$$E[r_i|p_i] = \frac{\sigma_r^2}{\sigma_p^2 + \sigma_r^2} (p_i - Ep)$$

• Plugging this into labor supply, we get:

$$\ell_i = \frac{1}{\gamma - 1} \frac{\sigma_r^2}{\sigma_p^2 + \sigma_r^2} (p_i - Ep)$$

# Lucas Supply Curve XV: Incomplete Information VII

 Averaging labor input across workers gives us aggregate output

$$y = \frac{1}{\gamma - 1} \frac{\sigma_r^2}{\sigma_p^2 + \sigma_r^2} (p - Ep)$$

• Defining a constant b to be equal to  $\frac{1}{\gamma-1}\frac{\sigma_r^2}{\sigma_p^2+\sigma_r^2}$ , we obtain the Lucas Supply Curve (how do we know it is a supply curve and not a demand curve?)

$$y = b(p - Ep)$$

# Lucas Supply Curve XVI: Incomplete Information VIII

• Combine Lucas supply curve with aggregate demand curve:

$$y = m - p$$
$$y = b(p - Ep)$$

• Solve for p and y  $\begin{pmatrix} 1 & 1 \\ -b & 1 \end{pmatrix} \begin{pmatrix} p \\ y \end{pmatrix} = \begin{pmatrix} m \\ -bEp \end{pmatrix} \Rightarrow$  $\begin{pmatrix} p \\ y \end{pmatrix} = \frac{1}{1+b} \begin{pmatrix} 1 & -1 \\ b & 1 \end{pmatrix} \begin{pmatrix} m \\ -bEp \end{pmatrix}$ 

#### Lucas Supply Curve XVII: Incomplete Information IX

• Thus:



• What is still endogenous?

#### Lucas Supply Curve XVIII: Incomplete Information X

• Solving for Ep:

$$Ep = \frac{1}{1+b}Em + \frac{b}{1+b}EEp$$

• From the law of iterated expectations, we get:

$$EEp = Ep$$

• Thus:

$$\frac{1}{1+b}Ep = \frac{1}{1+b}Em \Longrightarrow Ep = Em$$

## Lucas Supply Curve XIX: Incomplete Information XI

• Rewriting the solutions to y and p, we get:

$$p = \frac{1}{1+b}m + \frac{b}{1+b}Em = Em + \frac{1}{1+b}(m - Em)$$
$$y = \frac{b}{1+b}(m - Em)$$

- Are we finished yet?
- How do we interpret these equations? How are they different from the complete information model? What is the role of monetary policy? Surprises in money supply? How do we interpret m? Em? m-Em?

Lucas Supply Curve XX: Incomplete Information XII

• Remember that 
$$b = \frac{1}{\gamma - 1} \frac{\sigma_r}{\sigma_p^2 + \sigma_r^2}$$

 The variances are endogenous but we are now in a position to solve for them. First, taking variances of the equation for p, we get:

$$\sigma_p^2 = \frac{1}{\left(1+b\right)^2} \sigma_m^2$$

• To solve for the variance of r, we substitute the lucas supply curve into the individual demand equation:  $q_i = b(p - Ep) + z_i - \eta r_i$ 

# Lucas Supply Curve XXI: Incomplete Information XIII

• We do the same for individual labor supply:

$$\ell_i = br_i + b(p - Ep)$$

Equation individual labor supply and individual labor demand, we get:

$$br_i + b(p - Ep) = b(p - Ep) + z_i - \eta r_i$$

• Thus we can solve for  $\mathcal{V}_i$  as a function of exogenous variables:

$$r_i = \frac{Z_i}{b + \eta}$$

## Lucas Supply Curve XXII: Incomplete Information XIV

• Thus the variance of  $r_i$  is:

$$\sigma_r^2 = \frac{\sigma_z^2}{\left(b+\eta\right)^2}$$

 Finally, plugging our expressions for variances of endogenous variables into the expression for b, we get:

$$b = \frac{1}{\gamma - 1} \left[ \frac{\sigma_z^2}{\sigma_z^2 + \frac{(\eta + b)^2}{(1 + b)^2} \sigma_m^2} \right]$$

#### Lucas Supply Curve XXIII: Incomplete Information XV

 In general we can not solve for b parametrically... only implicitly. However, if η = 1, then we can solve explicitly for b:

$$b = \frac{1}{\gamma - 1} \frac{\sigma_z^2}{\sigma_z^2 + \sigma_m^2}$$

• In this case, we can express our solutions for p and y:  $y = \frac{\sigma_z^2}{\gamma \sigma_z^2 + (\gamma - 1)\sigma_w^2} (m - Em)$ 

$$p = Em + \left(1 - \frac{\sigma_z^2}{(\gamma - 1)\sigma_m^2 + \sigma_z^2}\right) (m - Em)$$