A Spatial Cliff-Ord-type Model with Heteroskedastic Innovations: Small and Large Sample Results[1]

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Abstract

The purpose of this paper is two fold. First, we specify a linear Cliff and Ord-type spatial model. The model allows for spatial lags in the dependent variable, the exogenous variables, and disturbances. The innovations in the disturbance process are assumed to be heteroskedastic with an unknown form. We formulate a multi-step GMM/IV type estimation procedure for the parameters of the model. We then establish the limiting distribution of our suggested estimators, and give consistent estimators for their asymptotic variance covariance matrices, utilizing results given in Kelejian and Prucha (2007). Second, we conduct a Monte Carlo study to show that the derived large sample distribution provides a good approximation to the actual small sample distribution of our estimators.

1 Introduction

Spatial econometric models, which are variants of the ones suggested by Cliff and Ord (1973, 1981), have been widely used in regional science, geography, and economics.[1] Maximum likelihood is one method of estimating the parameters of these models. However, until Lee (2004), there was no formal theory establishing large-sample results for maximum-likelihood estimators of the parameters of these models. In addition, as Kelejian and Prucha (1999) indicate, under certain conditions the implementation of maximum-likelihood procedures involves computational difficulties. Kelejian and Prucha (1999) suggested an alternative method of estimating these models to overcome these hurdles. Their procedure was based on a generalized method-of-moments (GMM) estimator of the autoregressive parameter in the disturbance process. Although they demonstrated the consistency of their GMM estimator, they did not determine its large-sample distribution and so tests relating to that autoregressive parameter could not be carried out in their framework. Kelejian and Prucha (1999) gave their results under the usual assumption that the innovations of the disturbance process were homoskedastic. This homoskedasticity assumption restricts the scope of applications of their procedure because cross-sectional spatial units often differ in size and other characteristics which causes one to suspect that the innovations to the disturbance process are heteroskedastic.

In a later paper, Kelejian and Prucha (2007) extended their earlier results in a variety of directions. In particular, they considered a Cliff-Ord-type spatial-autoregressive process with heteroskedastic innovations and suggested a GMM estimator, say $\hat{\rho}$, for the autoregressive parameter of that process. That GMM estimator was assumed to be based on estimated residuals that were formulated in terms of a regression parameter estimator, say $\hat{\delta}$. Under reasonably general conditions, they gave the large-sample distribution of their GMM estimator, $\hat{\rho}$. They also gave the joint large-sample distribution of their GMM estimator $\hat{\rho}$ and the regression parameter estimator $\hat{\delta}$.

Given their generality the results in Kelejian and Prucha (2007) cover a wide array of settings. For example, their assumptions underlying their GMM estimator $\hat{\rho}$ were general enough to include cases in which the residuals that were used in their GMM estimation could have come from a linear, or a non-linear spatial regression model. Similarly, their results concerning the large-sample joint distribution of their GMM estimator $\hat{\rho}$ and the regression parameter estimator $\hat{\delta}$ were general enough to include a variety of cases. For example, their regression parameter estimator could be a 2SLS estimator, a feasible generalized 2SLS estimator, etc. Similarly, their GMM estimator $\hat{\rho}$ could be based on estimated residuals obtained from a 2SLS estimator, a feasible generalized-spatial 2SLS estimator (GS2SLS), etc. Their results

[1] Classic references to spatial models are Cliff and Ord (1973, 1981), Anselin (1988), and Cressie (1993). Some recent applications are Anselin and Gallo (2006), Gallo and Dall'erba (2006), Baltagi, Egger, and Pfaffermayr (2006), Cohn and Morrison (2005, 2003), Keller and Shiue (2006), Cohen and Morrison Paul (2004), Hanushek et al. (2003), Topa (2001), Sacrerdote (2001), Betrand, Luttmer and Mullainathan (2000), Bell and Bockstael (2000), Audretsch and Feldmann (1996), Besley and Case (1995), Shroder (1995), Holtz-Eakin (1994), and Case, Hines, and Rosen (1993).

concerning the joint large-sample distribution of $\hat{\rho}$ and $\hat{\delta}$ were general enough to cover a variety of combinations of $\hat{\rho}$ and $\hat{\delta}$. For instance, among other things, $\hat{\rho}$ could be based on 2SLS residuals, and $\hat{\delta}$ could be the GS2SLS estimator.

Given their aim of providing a general estimation theory, Kelejian and Prucha (2007) do not provide specific expressions for the large-sample distribution for specific estimators. Because of this practitioners may find it challenging and/or tedious to specialize the general distributional results for a particular estimator-model combination.

The purpose of this paper is three fold. First, we specify a typical linear spatial model that might be considered in practice and demonstrate that our suggested estimators of its parameters satisfy the general assumptions in Kelejian and Prucha (2007). This model allows for spatial lags in the dependent variable, the exogenous variables, and disturbances, and allows for heteroskedasticity of unknown form in the innovations. Second, we specialize the general distributional results in Kelejian and Prucha (2007) for our estimators of the parameters in this model. These results make estimation of and inference about the parameters of this spatial model, and special cases of it, straight forward. Third, we give Monte Carlo results which describe the small-sample properties of our estimators, the estimators of their variances, as well as corresponding Wald-type tests.

Our Monte Carlo results suggest that our estimators behave quite nicely in small samples. They also suggest that the maximum-likelihood estimator of the autoregressive parameter in the disturbance process can be substantially biased in certain circumstances.

2 A Spatial Cliff-Ord-Type Model

2.1 Specifications

In this section we specify a linear spatial model that allows for spatial lags in the dependent variable, the exogenous variables and disturbances. Consistent with the terminology introduced by Anselin (1988), and used elsewhere in the literature, e.g. in Kelejian and Prucha (2007), we refer to this model as a spatial ARAR(1,1) model, i.e., SARAR(1,1). The specification does not assume homoskedastic innovations, but instead allows for heteroskedasticity of unknown form. Apart from allowing for heteroskedasticity the assumptions are similar to those made in the existing literature. Since those assumptions have been discussed in detail before,[2] our discussion of them will be brief.

Consider the following spatial model relating to n cross sectional units:

$$\begin{aligned} y_n &= X_n\beta + \lambda W_n y_n + u_n \\ &= Z_n\delta + u_n, \end{aligned} \tag{1}$$

and

$$u_n = \rho M_n u_n + \varepsilon_n, \tag{2}$$

[2] Among other studies, see Kelejian and Prucha (1998,2004,2005,2007) for a more extensive discussion of these assumptions.

where $Z_n = [X_n, W_n y_n]$, $\delta = [\beta', \lambda]'$, y_n is the $n \times 1$ vector of observations of the dependent variable, X_n is the $n \times k$ matrix of observations on non-stochastic (exogenous) regressors, W_n and M_n are $n \times n$ non-stochastic weights matrices, u_n is the $n \times 1$ vector of regression disturbances, ε_n is an $n \times 1$ vector of innovations, λ and ρ are scalar parameters, and β is a $k \times 1$ vector of parameters. The subscript n denotes dependence on the sample size and so (1) and (2) allow for triangular arrays. Consequently, this specification allows some or all of the exogenous variables to be spatial lags of exogenous variables. Thus the model is fairly general in that it allows for spatial spill-overs in the endogenous variables, exogenous variables and disturbances.

Our discussions will also utilize the following spatial Cochrane-Orcutt transformation of (1) and (2):

$$y_{n*}(\rho) = Z_{n*}(\rho)\delta + \varepsilon_n, \tag{3}$$

where $y_{n*}(\rho) = y_n - \rho M_n y_n$ and $Z_{n*}(\rho) = Z_n - \rho M_n Z_n$. The transformed model is readily obtained by pre-multiplying (1) by $I_n - \rho M_n$.

The spatial weights matrices and the autoregressive parameters are assumed to satisfy the following assumption.

Assumption 1 *(a) All diagonal elements of W_n and M_n are zero. (b) $\lambda \in (-1, 1)$, $\rho \in (-1, 1)$. (c) The matrices $I_n - \overline{\lambda} W_n$ and $I_n - \overline{\rho} M_n$ are nonsingular for all $\overline{\lambda} \in (-1, 1)$, and $\overline{\rho} \in (-1, 1)$.*

Assumption 2 *The innovations $\{\varepsilon_{i,n} : 1 \leq i \leq n,\ n \geq 1\}$ satisfy $E\varepsilon_{i,n} = 0$, $E(\varepsilon_{i,n}^2) = \sigma_{i,n}^2$ with $0 < \underline{a}^\sigma \leq \sigma_{i,n}^2 \leq \overline{a}^\sigma < \infty$, and $\sup_{1 \leq i \leq n, n \geq 1} E\left|\varepsilon_{i,n}\right|^{4+\eta} < \infty$ for some $\eta > 0$. Furthermore, for each $n \geq 1$ the random variables $\varepsilon_{1,n}, \ldots, \varepsilon_{n,n}$ are totally independent.*

Assumption 3 *The row and column sums of the matrices W_n and M_n are bounded uniformly in absolute value by, respectively, one and some finite constant, and the row and column sums of the matrices $(I_n - \lambda W_n)^{-1}$ and $(I_n - \rho M_n)^{-1}$ are bounded uniformly in absolute value by some finite constant.*

It is evident from (1) and (2) that, under typical specifications, $W_n y_n$ will be correlated with the disturbances u_n, which motivates the use of the instrumental variable procedure. The selection of instruments as an approximation to ideal instruments is discussed in Kelejian and Prucha (1998, 2005, 2007), and a review of that discussion is given below. At this point let H_n be an $n \times p$ matrix of non-stochastic instruments where $p \geq k + 1$, and note that in practice H_n would depend upon X_n. Our assumptions concerning X_n and H_n are given below.

Assumption 4 *: The regressor matrices X_n have full column rank (for n large enough). Furthermore, the elements of the matrices X_n are uniformly bounded in absolute value.*

Assumption 5 *: The instrument matrices H_n have full column rank $p \geq k+1$ (for all n large enough). Furthermore, the elements of the matrices H_n are uniformly bounded in absolute value. Additionally H_n is assumed to, at least, contain the linearly independent columns of $(X_n, M_n X_n)$.*

Assumption 6 *: The instruments H_n satisfy furthermore:*

(a) $Q_{HH} = \lim_{n\to\infty} n^{-1} H_n' H_n$ *is finite, and nonsingular.*

(b) $Q_{HZ} = \text{plim}_{n\to\infty} n^{-1} H_n' Z_n$ *and* $Q_{HMZ} = \text{plim}_{n\to\infty} n^{-1} H_n' M_n Z_n$ *are finite and have full column rank. Furthermore,* $Q_{HZ*}(\rho) = Q_{HZ} - \rho Q_{HMZ}$ *has full column rank.*

(c) $Q_{H\mathbf{\Sigma}H} = \lim_{n\to\infty} n^{-1} H_n' \mathbf{\Sigma}_n H_n$ *is finite and nonsingular, where* $\mathbf{\Sigma}_n = diag_{i=1}^n \sigma_{i,n}^2)$.

In treating X_n and H_n as non-stochastic our analysis should be viewed as conditional on X_n and H_n.

2.2 A Brief Discussion of the Assumptions

Among other things, Assumption 1 implies that the model is complete in that the dependent vector y_n can be solved for in terms of X_n and the innovation ε_n. Specifically,

$$\begin{aligned} y_n &= (I_n - \lambda W_n)^{-1}[X_n\beta + u_n] \\ u_n &= (I_n - \rho M_n)^{-1}\varepsilon_n. \end{aligned} \tag{4}$$

For a detailed discussion of the specification of the parameter space for the autoregressive parameters and normalizations of the spatial weights matrices see Kelejian and Prucha (2007).

Assumption 2 allows the innovations to be heteroskedastic with uniformly bounded variances .

Given (4), Assumption 2 implies that $E(y_n) = (I_n - \lambda W_n)^{-1} X_n \beta$. Since under Assumptions 1 and 3 the roots of W_n are all less than one in absolute value,

$$E(y_n) = [I_n + \lambda W_n + \lambda^2 W_n^2 + ...] X_n \beta. \tag{5}$$

We suggest a multi-step estimation procedure below. In the first step instruments are needed for Z_n, and in a later step instruments are needed for $M_n Z_n$. The ideal instruments are

$$\begin{aligned} E(Z_n) &= [X_n, W_n E(y_n)], \\ E(M_n Z_n) &= [M_n X_n, M_n W_n E(y_n)]. \end{aligned} \tag{6}$$

In light of (5), all of the columns of $E(Z_n)$ and $E(M_n Z_n)$ are linear in

$$X_n, W_n X_n, W_n^2 X_n, ..., M_n X_n, M_n W_n X_n, M_n W_n^2 X_n, ... \tag{7}$$

Let H_n be a subset of the columns in (7), say

$$H_n = (X_n, W_n X_n, ..., W_n^q X_n, M_n X_n, M_n W_n X_n, ..., M_n W_n^q X_n) \tag{8}$$

where, typically, $q \leq 2$. Then the evident approximation to the ideal instruments for Z_n and $M_n Z_n$ is $P_n Z_n$ and $P_n M_n Z_n$ where P_n is the projection matrix: $P_n = H_n(H_n' H_n)^{-1} H_n$. In passing note that, via Assumption 5, H_n is assumed to contain at least the linearly independent columns of X_n and $M_n X_n$, and therefore

$$\begin{aligned} P_n Z_n &= (X_n, P_n W_n y_n), \\ P_n M_n Z_n &= (M_n X_n, P_n M_n W_n y_n). \end{aligned} \tag{9}$$

Assumption 3 is a technical assumption which is used in the large-sample derivation of the regression parameter estimator. Among other things, this assumption limits the extent of spatial autocorrelation.

Assumption 4 rules out multicollinearity problems, as well as unbounded exogenous variables. Among other things, Assumption 5 implies that there are at least as many instruments as there are regression parameters. Assumption 6 rules out redundant instruments and specifies conditions which ensure the identifiability of the regression parameter estimators.

3 Estimators

In this section we specify GMM and instrumental variable (IV) estimators for the model parameters ρ and δ. The suggested estimation procedure consists of two steps. Each step consists of substeps involving the estimation of ρ and δ by GMM and IV methods. In step 1, estimates are computed from the original model (1). Those estimates are used in step 2 to compute estimates from the transformed model (3), with ρ replaced by an estimator.

3.1 Moment Conditions

Following Kelejian and Prucha (2007) our estimators for ρ will be GMM estimators corresponding to the following population moment conditions:

$$\begin{aligned} n^{-1} E \overline{\varepsilon}_n' \overline{\varepsilon}_n &= n^{-1} tr \left\{ M_n \left[diag_{i=1}^n (E\varepsilon_{i,n}^2) \right] M_n' \right\}, \\ n^{-1} E \overline{\varepsilon}_n' \varepsilon_n &= 0, \end{aligned} \tag{10}$$

with $\overline{\varepsilon}_n = M_n \varepsilon_n$. Let $A_{1,n} = M_n' M_n - diag_{i=1}^n (m_{.i,n}' m_{.i,n})$ and $A_{2,n} = M_n$. It is readily seen that these moment conditions can also be written as.

$$\begin{aligned} n^{-1} E \varepsilon_n' A_{1,n} \varepsilon_n &= n^{-1} E[u_n - \rho \overline{u}_n]' A_{1,n} [u_n - \rho \overline{u}_n] = 0, \\ n^{-1} E \varepsilon_n' A_{2,n} \varepsilon_n &= n^{-1} E[u_n - \rho \overline{u}_n]' A_{2,n} [u_n - \rho \overline{u}_n] = 0, \end{aligned} \tag{11}$$

with $\overline{u}_n = M_n u_n$.

3.2 GMM/IV Estimators, Original Model

Step 1a: 2SLS Estimator
In the first step, δ is estimated by 2SLS applied to model (1) using the instrument matrix H_n in Assumption 5. Let $\widetilde{\delta}_n$ denote the 2SLS estimator, then

$$\widetilde{\delta}_n = (\widetilde{Z}_n' Z_n)^{-1} \widetilde{Z}_n' y_n, \tag{12}$$

where $\widetilde{Z}_n = P_H Z_n = (X_n, \widetilde{W_n y_n})$, $\widetilde{W_n y_n} = P_H W_n y_n$, and where $P_H = H_n(H_n' H_n)^{-1} H_n'$. An instrument matrix such as H_n was suggested originally in Kelejian and Prucha (1998).

Step 1b: Initial GMM Estimator of ρ Based on 2SLS Residuals
In light of (1) and (12), the 2SLS residuals are $\widetilde{u}_n = y_n - Z_n \widetilde{\delta}_n$. Let $\widetilde{\overline{u}}_n = M_n \widetilde{u}_n$, and $\widetilde{\overline{\overline{u}}}_n = M_n^2 \widetilde{u}_n$. Consider the following sample moments corresponding to (11) based on estimated residuals:

$$\begin{aligned} m(\rho, \widetilde{\delta}_n) &= n^{-1} \begin{bmatrix} (\widetilde{u}_n - \rho\widetilde{\overline{u}}_n)' A_1 (\widetilde{u}_n - \rho\widetilde{\overline{u}}_n) \\ (\widetilde{u}_n - \rho\widetilde{\overline{u}}_n)' A_2 (\widetilde{u}_n - \rho\widetilde{\overline{u}}_n) \end{bmatrix} \\ &= g_n(\widetilde{\delta}_n) - G_n(\widetilde{\delta}_n) \begin{bmatrix} \rho \\ \rho^2 \end{bmatrix}, \end{aligned} \tag{13}$$

where the elements of the 2×1 vector g_n and the 2×2 matrix G_n are defined in Appendix B.1. Equation (13) shows that the elements of $g_n(\widetilde{\delta}_n)$ and $G_n(\widetilde{\delta}_n)$ are observable functions of $\widetilde{u}_n$, $\widetilde{\overline{u}}_n$, and $\widetilde{\overline{\overline{u}}}_n$. Our initial GMM estimator for ρ is defined as

$$\check{\rho}_n = \underset{\rho \in [-a^\rho, a^\rho]}{argmin} \left[m(\rho, \widetilde{\delta}_n)' m(\rho, \widetilde{\delta}_n) \right] \tag{14}$$

where $a^\rho \geq 1$. In light of the second expression in (13) the estimator can be viewed as an unweighted nonlinear least squares estimator. Given further assumptions listed below, it is consistent, but not efficient because of this lack of weighting.

Step 1c: Efficient GMM Estimator of ρ Based on 2SLS Residuals
As might be anticipated from the discussion above, our efficient GMM estimator of ρ is a weighted nonlinear least squares estimator. Specifically, this estimator is $\widetilde{\rho}_n$ where

$$\widetilde{\rho}_n = \underset{\rho \in [-a^\rho, a^\rho]}{argmin} \left[m(\rho, \widetilde{\delta}_n)' \widetilde{\Psi}_n^{-1} m(\rho, \widetilde{\delta}_n) \right] \tag{15}$$

and where the weighting matrix is $\widetilde{\Psi}_n^{-1}$. $\widetilde{\Psi}_n = \widetilde{\Psi}_n(\check{\rho}_n)$, defined in Appendix B.2, is an estimator of the variance-covariance matrix of the limiting distribution of the normalized sample moments $n^{1/2} m(\rho, \widetilde{\delta}_n)$.

3.3 GMM/IV Estimators, Transformed Model

Step 2a: GS2SLS Estimator

Consider the spatial Cochrane-Orcutt transformed model in (3). Analogous to Kelejian and Prucha (1998) we now define a generalized spatial two-stage least squares (GS2SLS) estimator of δ as the 2SLS estimator of the transformed model in (3) after replacing the parameter ρ by $\widetilde{\rho}_n$ computed in Step 1c. Specifically, the GS2SLS estimator is defined as

$$\widehat{\delta}_n(\widetilde{\rho}_n) = [\widehat{Z}_{n*}(\widetilde{\rho}_n)' Z_{n*}(\widetilde{\rho}_n)]^{-1} \widehat{Z}_{n*}(\widetilde{\rho}_n)' y_{n*}(\widetilde{\rho}_n) \tag{16}$$

where $y_{n*}(\widetilde{\rho}_n) = y_n - \widetilde{\rho}_n M_n y_n$, $Z_{n*}(\widetilde{\rho}_n) = Z_n - \widetilde{\rho}_n M_n Z_n$, $\widehat{Z}_{n*}(\widetilde{\rho}_n) = P_H Z_{n*}(\widetilde{\rho}_n)$, and where $P_H = H_n(H_n' H_n)^{-1} H_n'$.

Step 2b: Efficient GMM Estimator of ρ Based on GS2SLS Residuals

The GS2SLS residuals are given by $\widehat{u}_n = y_n - Z_n \widehat{\delta}_n(\widetilde{\rho}_n)$. Let $\widehat{\overline{u}}_n = M_n \widehat{u}_n$, and $\widehat{\overline{\overline{u}}}_n = M_n^2 \widehat{u}_n$. Now consider the the sample moments $m(\rho, \widehat{\delta}_n)$ obtained by replacing the 2SLS residuals in (13) by the GS2SLS residuals $\widehat{u}_n$, $\widehat{\overline{u}}_n$, and $\widehat{\overline{\overline{u}}}_n$. The efficient GMM estimator for ρ based on GS2SLS residuals is now given by

$$\widehat{\rho}_n = \underset{\rho \in [-a^\rho, a^\rho]}{argmin} \left[m(\rho, \widehat{\delta}_n)' \widehat{\Psi}_n^{-1} m(\rho, \widehat{\delta}_n) \right], \tag{17}$$

where the weighting matrix is $\widehat{\Psi}_n^{-1}$ $\widehat{\Psi}_n$, defined in Appendix B.3, is an estimator of the variance-covariance matrix of the limiting distribution of the normalized sample moments $n^{1/2} m(\rho, \widehat{\delta}_n)$.[3]

4 Large Sample Distribution

In this section we give results on the joint limiting distribution of the initial 2SLS estimator, $\widetilde{\delta}_n$, and the efficient GMM estimator of ρ based on 2SLS residuals, namely $\widetilde{\rho}_n$. These estimators relate to the untransformed model. We also give the joint limiting distribution of the GS2SLS estimator $\widehat{\delta}_n$, and the efficient GMM estimator of ρ which is based on GS2SLS residuals, namely $\widehat{\rho}_n$. These estimators correspond to the transformed model. Proofs are given in Appendix C.

4.1 GMM/IV Estimators, Original Model

In Appendix C we prove the following theorem concerning the join limiting distribution of $\widetilde{\rho}_n$ and $\widetilde{\delta}_n$.

[3] $n^{1/2} m(\rho, \widehat{\delta}_n)$ and $n^{1/2} m(\rho, \widetilde{\delta}_n)$ have different limiting distributions.

Theorem 1 *Suppose Assumptions 1-6 above and Assumptions A.1 and A.2 in the appendix hold. Then,* $\widetilde{\rho}_n$ *is efficient among the class of GMM estimators based on 2SLS residuals, and*

$$\begin{bmatrix} n^{1/2}(\widetilde{\delta}_n - \delta) \\ n^{1/2}(\widetilde{\rho}_n - \rho) \end{bmatrix} \xrightarrow{D} N(0, \text{plim}_{n\to\infty}\widetilde{\Omega}_n) \tag{18}$$

where $\text{plim}_{n\to\infty}\widetilde{\Omega}_n$ *is a positive definite matrix. For applied purposes, an expression is needed for* $\widetilde{\Omega}_n$. *This expression is given in the Appendix B.2.*

The result in (18) indicates that both $\widetilde{\delta}_n$ and $\widetilde{\rho}_n$ are consistent. It also suggests that small-sample inferences concerning either ρ, δ, or both can be based on the small-sample approximation

$$\begin{bmatrix} (\widetilde{\delta}_n - \delta) \\ (\widetilde{\rho}_n - \rho) \end{bmatrix} \dot{\sim} N\left(\begin{bmatrix} \delta \\ \rho \end{bmatrix}, n^{-1}\widetilde{\Omega}_n \right).$$

4.2 GMM/IV Estimators, Transformed Model

In Appendix C we prove the following theorem concerning the joint limiting distribution of $\widehat{\rho}_n$ and $\widehat{\delta}_n$.

Theorem 2 *Suppose Assumptions 1-6 above and Assumptions A.1 and A.3 in the appendix hold. Then,* $\widehat{\rho}_n$ *is efficient among the class of GMM estimators based on GS2SLS residuals, and*

$$\begin{bmatrix} n^{1/2}(\widehat{\delta}_n - \delta) \\ n^{1/2}(\widehat{\rho}_n - \rho) \end{bmatrix} \xrightarrow{D} N(0, \text{plim}_{n\to\infty}\widehat{\Omega}_n)$$

where $\text{plim}_{n\to\infty}\widehat{\Omega}_n$ *is a positive definite matrix. For applied purposes, an expression is needed for* $\widehat{\Omega}_n$. *This expression is given in the appendix B.3.*

Clearly, Theorem 2 implies that both $\widehat{\delta}_n$ and $\widehat{\rho}_n$ are consistent. It also suggests that small-sample inferences can be based on the approximation

$$\begin{bmatrix} (\widehat{\delta}_n - \delta) \\ (\widehat{\rho}_n - \rho) \end{bmatrix} \dot{\sim} N\left(\begin{bmatrix} \delta \\ \rho \end{bmatrix}, n^{-1}\widehat{\Omega}_n \right).$$

5 Monte Carlo Experiments

In this section we give Monte Carlo results which suggest that our estimators and corresponding test statistics behave well in finite samples. Our Monte Carlo model is a special case of the one specified in (1) and (2). Our experimental design is somewhat similar to those used in the literature by Kelejian and Prucha (1999, 2007) and by Anselin and Florax (1995).

5.1 The Model

The model underlying our Monte Carlo experiments is a special case of the model specified in (1) and (2) with two exogenous regressors, i.e., $X_n = [x_{n,1}, x_{n,2}]$ and $\beta = (\beta_1, \beta_2)'$, and with $M_n = W_n$.

We consider two cases for the innovation vector ε_n. In one of these cases the elements of the innovation vector are i.i.d. $N(0, c^2)$, and so their standard deviation is c. In our second case the elements of the innovation vector are heteroskedastic. In this case we take the i-th element of the innovation vector ε_n as

$$\varepsilon_{n,i} = \sigma_{n,i}\zeta_{n,i}, \qquad (19)$$
$$\sigma_{n,i} = c\frac{d_{n,i}}{\Sigma_{j=1}^{n} d_{n,j}/n},$$

where $\zeta_{n,i}$ is, for each of our considered sample sizes, i.i.d. $N(0,1)$, and $d_{n,i}$ is the number of neighbors the i-th unit has, which will be defined by the sample size, and weights matrices described below. At this point note that the average of the standard deviations of the elements of ε_n is c, and thus the average standard deviation is identical to that in the homoskedastic case. Also note that these standard deviations are related to the number of neighbors each unit has. One example in which units might have different numbers of neighbors is the case in which the units differ in size. If neighbors are defined as units falling within a certain distance, then each unit in a group of smaller units could have many neighbors, while each unit in a group of larger units could have fewer neighbors. This scenario could relate to the northeastern portion of the US, as compared to western states in the US.

The parameters of the model which we will estimate are $\delta = (\beta_1, \beta_2, \lambda)'$ and ρ. The specifications we use to generate 2000 repetitions for each Monte Carlo experiment are described below.

The two $n \times 1$ regressors $x_{n,1}$ and $x_{n,2}$ are normalized versions of income per-capita and the proportion of housing units which are rental in 1980, in 760 counties in US mid-western states. These data were taken from Kelejian and Robinson (1993). We normalized the 760 observations on these variables by subtracting from each observation the corresponding sample average, and then dividing that result by the sample standard deviation. The first n values of these normalized variables were used in our Monte Carlo experiments of sample size n. For sample sizes larger than 760 the observations were repeated. Finally, the same set of observations on these variables were used in all Monte Carlo repetitions.

We considered five experimental values for λ and for ρ, namely: $-.8$, $-.3$, 0, .3. .8. In all of our experiments we took $\beta_1 = \beta_2 = 1$. We consider two values for the (average) standard deviation c, namely: .5 and 1.

For each approximate[4] sample size we consider three weights matrices. The first is a variation of those that were considered in Kelejian and Prucha (1999, 2004). This variation is considered because it allows a convenient formulation of the innovation heteroskedasticity, which is a major focus of

[4]Our discussion below will clarify this notion of “approximate" sample size.

this paper. Specifically, this variation is an $n \times n$ matrix whose first $n/3$ rows, except for the first row, have zeroes everywhere except for the elements in positions $(i, i+1)$ and $(i, i-1)$. [5] In the first row, the non-zero elements are in position $(1, 2)$ and $(1, n)$ so that it relates to a circular world. The non-zero elements in the first $n/3$ rows are all $1/2$, e.g., these rows are row normalized because each row has 2 neighbors. The next $n/3$ rows, say $j = n/3+1, ..., 2n/3$, have zeroes everywhere except in positions $(j, j \pm r)$, where $r = 1, 2, ..., 5$. The non-zero elements in these rows are all $1/10$. The last $n/3$ rows are defined in a similar manner to the first $n/3$ rows. Specifically, the non-zero elements in rows $j = 2n/3 + 1, ..., n-1$ are in positions $(j, j+1)$, and $(j, j-1)$; in the last row the non-zero element are in positions $(n, 1)$ and $(n, n-1)$. The non-zero elements in these rows are all $1/2$. This matrix was considered for sample sizes $n = 500$ and $n = 1000$. We refer to this matrix below as the "circular world matrix".

The second and third sets of weights matrices are matrices which correspond to a "space" in which units located in the northeast portion of that space are smaller, closer to each other, and have more neighbors than the units corresponding to other quadrants of that space. Again, one might think of the states located in the northeastern portion of the US, as compared to western states.

To define these matrices, think of the matrix in terms of a square grid with both the x and y coordinates only taking on the values 1, 1.5, 2, 2.5, ...$\bar{m}$. Let the units in the northeast quadrant of this matrix be at the indicated discrete coordinates: $m \leq x \leq \bar{m}$ and $m \leq y \leq \bar{m}$. Let the remaining units be located only at integer values of the coordinates: $x = 1, 2, ..., m-1$ and $y = 1, 2, ..., m-1$. In this set-up it should be clear that the number of units located in the northeast quadrant is inversely related to m.

For this matrix we define a distance measure between any two units, i_1 and i_2, which have coordinates respectively, (x_1, y_1) and (x_2, y_2) as the Euclidean distance between them, namely

$$d(i_1, i_2) = \left[(x_1 - x_2)^2 + (y_1 - y_2)^2\right]^{1/2}.$$

Given this distance measure we define the (i, j)-th element of our row normalized weights matrix W as

$$w_{ij} = w_{ij}^* / \sum_{j=1}^{n} w_{ij}^*,$$
$$w_{ij}^* = \begin{cases} 1 & \text{if} \quad 0 < d(i_1, i_2) \leq 1 \\ 0 & \text{else} \end{cases}.$$

For our experiments with a sample size of approximately 500, we considered two cases of this matrix namely $(m = 5, \bar{m} = 15)$ and $(m = 14, \bar{m} = 20)$. These values of m and $\bar{m}$ imply sample sizes of respectively, $n = 486$ and $n = 485$. These values of m and $\bar{m}$ were selected because they correspond to different proportions of units in the northeast quadrant, where each unit has more neighbors than units located in the other quadrants. As indicated,

[5] When $n/3$ is not an integer, we take the smallest integer greater than $n/3$.

the number of neighbors each unit has is important because it is a determinant of the standard deviation of the innovation, see (19). In our first small-sample case, namely $(m = 5, \bar{m} = 15)$, approximately 25% of the units are located in the northeast quadrant; in our second case, $(m = 14, \bar{m} = 20)$, approximately 75% of the units are located in the northeast quadrant.

For our experiments with a sample size of approximately 1000, the two variations of this matrix we considered are $(m = 7, \bar{m} = 21)$ and $(m = 20, \bar{m} = 28)$. The implied sample sizes are, respectively, $n = 974$ and $n = 945$. In these two cases, the proportion of units located in the northeast quadrant is, respectively approximately 25% and 76%. Below we refer to all of these matrices as north-east modified-rook matrices. For future reference we summarize the characteristics of these four "modified rook" matrices in Table 1 below. We also illustrate a north-east modified-rook matrix, with the units indicated by the stars, in Figure 1 for the case in which $m = 2$ and $\overline{m} = 5$

Table 1: North-East Modified-Rook Matrices

Matrix R1	Matrix R2	Matrix R3	Matrix R4
$(m = 5, \bar{m} = 15)$	$(m = 7, \bar{m} = 21)$	$(m = 14, \bar{m} = 20)$	$(m = 20, \bar{m} = 28)$
$n = 486$	$n = 974$	$n = 485$	$n = 945$
$\%NE : 25\%$	$\%NE : 25\%$	$\%NE : 75\%$	$\%NE : 76\%$

Figure 1: Example of a North-East Modified-Rook Matrix: $m = 2$ and $\overline{m} = 5$

5.0	*		*		*	*	*	*	*
4.5					*	*	*	*	*
4.0	*		*		*	*	*	*	*
3.5					*	*	*	*	*
3.0	*		*		*	*	*	*	*
2.5									
2.0	*		*		*		*		*
1.5									
1.0	*		*		*		*		*
	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0

5.2 Monte Carlo Results

Our Monte Carlo results are given in Tables 2-14 below. These tables contain results for the generalized spatial 2SLS estimator $\widehat{\delta}(\hat{\rho})$ defined in (16) which is based on the instrument matrix $H = [X, WX, W^2X]$, where $\tilde{\rho}$ is replaced by $\hat{\rho}$ which is the efficient GM estimator given in (17). Only results for the estimators of λ and ρ, which are denoted in the tables as λ_{GS} and ρ_{GS}, are reported. For purposes of comparison, we also report the quasi-maximum likelihood estimators of these two parameters, denoted in the tables as λ_{ML} and ρ_{ML}.

The results in Tables 2-7 are from the heteroskedastic case with the larger average standard deviation in which $c = 1.0$, see (19). The results in the first four of these tables are based on north-east modified-rook matrices 1-4 as described in Table 1. Tables 6 and 7 report results based on our circular world matrices, with sample sizes 500 and 1000 respectively. Tables 8-13 have the same format as Tables 2-7 except they correspond to our smaller standard deviation case $c = .5$. Finally, Table 14 reports results for the homoskedastic case in which the elements of the innovation vector are $i.i.d.$ as $N(0, c^2)$, $c = 1$ with the north-east modified rook matrix III R1.III The experiments underlying Table 14 correspond to those underlying Table 2, the difference being that the former reports on the heteroskedastic case while the latter reports on the homoskedastic case. We have also performed Monte Carlo experiments under homoskedasticity corresponding to all cases considered in

Tables 3-13, but for space reasons we have not included corresponding tables below. A full set of our results is available on our web site.

The results in Tables 2-13 are consistent with our large-sample theory, namely that λ_{GS} and ρ_{GS} are consistent estimators and, in the presence of heteroskedastic innovations, the quasi-maximum-likelihood estimators λ_{ML} and ρ_{ML} are in general not consistent. For example, notice that in all of the tables the biases of λ_{GS} and ρ_{GS} are so small that the root mean square error is approximately equal to the standard deviation. Also note that in all of the tables the rejection rates corresponding to λ_{GS} and to ρ_{GS} are quite close to the theoretical .05 level. Indeed, in each of the Tables 2-13 the average of these rejection rates over all of the experiments considered relating to both λ_{GS} and ρ_{GS} are "very" close to the theoretical .05 level. For future reference we note that these averages do not mask outliers; the largest of these outliers, namely .1200, relates λ_{GS} in Table 4 and corresponds to the experiment $\rho = .8$ and $\lambda = .3$. In the tables there are no rejection rate outliers that relate to the estimator ρ_{GS}.

In contrast, the results for λ_{ML} and ρ_{ML} in the heteroskedastic cases show that the biases are typically large and, consequently, the rejection rates, especially for ρ_{ML}, deviate from the theoretical .05 level in many of the considered experiments. Indeed, in Tables 3-7, 9, and 11-13 the rejection rates corresponding to ρ_{ML} exceed .9 in some experiments. In most of these experiments, the III true III value of $\rho = -.8$. The rejection rates relating to λ_{ML} are more moderate but still have outlier values ranging from $.85 - 1.0$ in Tables 5-7, and 12-13. In most of these cases at least one of the III true III values for ρ and λ are negative; actually the only exception to this is in Table 7 when the III true III values are $\rho = .8$ and $\lambda = 0$. Interestingly, extreme rejection rates, say over .8, for ρ_{ML} and λ_{ML} do not always occur for the same set of III true III parameter values – see, e.g., Tables 3-5. Of course, when they do occur simultaneously at least one of the III true III values for ρ or λ are negative, but typically not both.

Intuitive explanations of the table results thus far discussed as they relate to the III true III parameters are not straight forward. As one example, the reduced form for y_n from the model (1) and (2) is

$$y_n = (I_n - \lambda W_n)^{-1} X_n \beta + (I_n - \lambda W_n)^{-1} (I_n - \rho W_n)^{-1} \varepsilon_n \qquad (20)$$

If λ is large in absolute value, say close 1.0, the variances of the elements of error vector in (20), namely $(I_n - \lambda W_n)^{-1}(I_n - \rho W_n)^{-1}\varepsilon_n$, will, ceteris paribus, tend to be large since $\lambda = 1.0$ is a singular point of the inverse matrix. These larger variances will obviously have a negative effect on estimation precision. On the other hand, increased variation of the vector y_n will, ceteris paribus, increase the variation in $W_n y_n$, which is a right hand side variable, and this should increase estimation precision. The net effect on estimation precision of a large value of λ will obviously be the result of these two effects and it is not clear which of these two effects would dominate in a particular case. Similar concerns relate to the III true III value of ρ since, on the negative side, it also enters the error term in (20) in the same fashion as λ; on the positive side ρ can be viewed as a regression parameter in (2) and the larger the value of ρ the more $W_n u_n$ varies and so the more precision is increased!

Of course, intuitive interpretations of our results are made still more complex by the interactive effects of ρ and λ as is evident in (20).

Returning to the tables, note from Tables 2-13 that, on average, the root mean square errors relating to ρ_{GS} and to λ_{GS} decrease as the sample size increases in every "comparable" case considered. As an example of "comparable" cases, Tables 2 and 3 both relate to a north-east modified-rook matrix in which the north east quadrant contains 25% of the units; also the results in both of these tables are based on $c = 1.0$. The main difference in the design underlying Tables 2 and 3 is the sample size, namely $n = 486$ for Table 2 and $n = 974$ for Table 3. Other comparable tables are 4 and 5, 6 and 7, etc.

The root mean square errors for ρ_{ML} and λ_{ML} typically decrease in relevant comparisons as the sample size increases. A glance at the tables suggests that the reason for this is that the standard deviations, not the biases, decreases with the sample size. However, in all cases considered, the average root mean square errors for ρ_{GS} and λ_{GS} are less than those for ρ_{ML} and λ_{ML}. For example, if in each table the ratio of the average of the root mean square error of ρ_{GS} to ρ_{ML} is taken, and then these ratios are averaged over the 12 tables the result is .63; this average of the root mean square errors of λ_{GS} and λ_{ML} is .70. Thus, over our experiments the increase in efficiency of ρ_{GS} relative to ρ_{ML} seems to be be larger than the increase in λ_{GS} relative to λ_{ML}.

The "comparable" cases above focused attention on the effects of the sample size in the modified rook matrix cases by holding constant the relative size of the north-east quadrant of those matrices. We now focus attention on comparisons relating to the relative size of the north-east quadrant of those modified rook matrices by holding constant the sample size. For instance, consider the results in Tables 2 and 4. In these tables the sample sizes are, respectively 486 and 485; the proportion of units located in the north-east quadrant are, respectively, 25% and 75%.

The root mean square errors for ρ_{GS} and λ_{GS} are lower in Table 4 than they are in Table 2, as are the averages of these root mean square errors. The same result holds for the root mean square errors of ρ_{GS} and λ_{GS} in Tables 3 and 5, Tables 8 and 10, and Tables 9 and 11. Thus, the larger the size of the north-east quadrant, the more precise the estimation is.

Given the complexity of our model and our estimators, there does not seem to be a simple explanation of these results. On an intuitive level, one suspects that the particular values of the instrument matrix and the variances are at least part of the explanation. For example, if $\rho = 0$ the only term in the large sample distribution of $\tilde{\delta}_n$ that would involve the variances would be $\lim_{n\to\infty} n^{-1} H_n' \mathbf{\Sigma}_n H_n$, which clearly involves the products of the variances and the elements of H_n. On another, and simpler issue, we note that the root mean square errors are lower in Tables 8-11 than they are, respectively in 2-5. The reason for this is that $c = .5$ in Tables 8-11, while $c = 1.0$ in Tables 2-5.

Table 14 contains results for the homoskedastic case in which the weights matrix is north-east modified-rook matrix R1 and $c = 1.0$. Under homoskedasticity the quasi-maximum-likelihood estimator is the maximum-likelihood estimator, so it is consistent and efficient. Of course, in this case both ρ_{GS} and λ_{GS} are also consistent. Table 14 shows that these results hold in our simulations. For instance, the biases are small for all four of these estimators, and the rejection rates are reasonably close to the theoretical .05 level. Although the root mean square errors are relatively small for both ρ_{GS} and λ_{GS} they are typically larger than those of ρ_{ML} and λ_{ML}. On average the root mean square errors of ρ_{GS} and λ_{GS} are, respectively 5% and 8% larger than those of ρ_{ML} and λ_{ML}.

Table 2: Heteroskedasticity with c=1, Modified Rook Matrix R1 (n=486)

		ρ_{GS}				ρ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.7870	0.1242	0.0425	0.1248	-0.6464	0.0634	0.3000	0.1661
-.8	-.3	-0.7899	0.1167	0.0420	0.1172	-0.5918	0.0659	0.7100	0.2184
-.8	0	-0.7902	0.1117	0.0420	0.1121	-0.5847	0.0676	0.7625	0.2256
-.8	.3	-0.7923	0.1081	0.0430	0.1083	-0.5863	0.0675	0.7715	0.2241
-.8	.8	-0.7922	0.1025	0.0470	0.1028	-0.6036	0.0658	0.7200	0.2071
-.3	-.8	-0.2974	0.1285	0.0480	0.1285	-0.3047	0.0949	0.0190	0.0950
-.3	-.3	-0.2964	0.1344	0.0485	0.1345	-0.2473	0.0907	0.0410	0.1049
-.3	0	-0.2960	0.1317	0.0500	0.1317	-0.2281	0.0913	0.0675	0.1162
-.3	.3	-0.2949	0.1279	0.0490	0.1280	-0.2185	0.0936	0.0915	0.1241
-.3	.8	-0.2944	0.1198	0.0535	0.1199	-0.2269	0.0916	0.0910	0.1172
0	-.8	0.0013	0.1190	0.0510	0.1190	-0.0685	0.1023	0.0810	0.1232
0	-.3	-0.0007	0.1262	0.0505	0.1262	-0.0276	0.0961	0.0335	0.1000
0	0	-0.0011	0.1267	0.0480	0.1267	-0.0072	0.0971	0.0265	0.0973
0	.3	-0.0011	0.1254	0.0465	0.1254	0.0073	0.0977	0.0310	0.0980
0	.8	0.0013	0.1177	0.0510	0.1177	0.0074	0.0986	0.0380	0.0989
.3	-.8	0.3019	0.1012	0.0510	0.1012	0.2013	0.1012	0.1805	0.1414
.3	-.3	0.2967	0.1092	0.0560	0.1092	0.2182	0.0974	0.1190	0.1272
.3	0	0.2958	0.1116	0.0500	0.1117	0.2356	0.0972	0.0855	0.1166
.3	.3	0.2947	0.1122	0.0500	0.1123	0.2529	0.0978	0.0665	0.1085
.3	.8	0.2971	0.1076	0.0475	0.1077	0.2642	0.0980	0.0670	0.1044
.8	-.8	0.7992	0.0511	0.0535	0.0512	0.7364	0.0608	0.2960	0.0880
.8	-.3	0.7937	0.0572	0.0560	0.0575	0.7240	0.0659	0.2785	0.1005
.8	0	0.7885	0.0615	0.0695	0.0625	0.7254	0.0691	0.2210	0.1017
.8	.3	0.7822	0.0685	0.0695	0.0708	0.7330	0.0733	0.1700	0.0993
.8	.8	0.7814	0.0692	0.0575	0.0717	0.7525	0.0819	0.1500	0.0947
average		0.0000	0.1068	0.0509	0.1072	0.0207	0.0851	0.2167	0.1279
		λ_{GS}				λ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.8093	0.0627	0.0525	0.0634	-0.7650	0.0384	0.0145	0.0520
-.8	-.3	-0.3050	0.0612	0.0495	0.0614	-0.3409	0.0494	0.0525	0.0642
-.8	0	-0.0031	0.0505	0.0515	0.0506	-0.0510	0.0454	0.1070	0.0683
-.8	.3	0.2982	0.0373	0.0505	0.0373	0.2570	0.0357	0.1265	0.0559
-.8	.8	0.7996	0.0120	0.0495	0.0120	0.7899	0.0112	0.0710	0.0151
-.3	-.8	-0.8014	0.0433	0.0380	0.0434	-0.7307	0.0329	0.1220	0.0767
-.3	-.3	-0.3028	0.0574	0.0485	0.0575	-0.2957	0.0476	0.0075	0.0478
-.3	0	-0.0019	0.0520	0.0490	0.0520	-0.0208	0.0455	0.0230	0.0500
-.3	.3	0.2994	0.0410	0.0480	0.0410	0.2722	0.0376	0.0450	0.0468
-.3	.8	0.7997	0.0140	0.0480	0.0140	0.7905	0.0130	0.0480	0.0161
0	-.8	-0.8005	0.0367	0.0380	0.0367	-0.7279	0.0300	0.1825	0.0781
0	-.3	-0.2996	0.0552	0.0455	0.0552	-0.2734	0.0473	0.0190	0.0542
0	0	0.0004	0.0543	0.0475	0.0543	-0.0020	0.0481	0.0110	0.0482
0	.3	0.3002	0.0455	0.0460	0.0455	0.2828	0.0419	0.0260	0.0453
0	.8	0.8001	0.0167	0.0505	0.0167	0.7910	0.0154	0.0405	0.0178
.3	-.8	-0.7991	0.0327	0.0440	0.0327	-0.7307	0.0286	0.2295	0.0750
.3	-.3	-0.2972	0.0578	0.0455	0.0578	-0.2523	0.0497	0.0550	0.0689
.3	0	0.0028	0.0587	0.0465	0.0588	0.0187	0.0530	0.0240	0.0562
.3	.3	0.3027	0.0530	0.0510	0.0531	0.2959	0.0487	0.0175	0.0489
.3	.8	0.8013	0.0216	0.0485	0.0216	0.7913	0.0204	0.0390	0.0222
.8	-.8	-0.7980	0.0288	0.0520	0.0288	-0.7421	0.0243	0.2160	0.0627
.8	-.3	-0.2877	0.0659	0.0735	0.0671	-0.2189	0.0570	0.1175	0.0992
.8	0	0.0186	0.0791	0.1000	0.0813	0.0657	0.0678	0.0820	0.0944
.8	.3	0.3260	0.0858	0.1120	0.0897	0.3394	0.0740	0.0670	0.0838
.8	.8	0.8158	0.0474	0.0975	0.0500	0.7962	0.0535	0.0920	0.0536
average		0.0024	0.0468	0.0553	0.0473	0.0136	0.0407	0.0734	0.0561

Table 3: Heteroskedasticity with c=1, Modified Rook Matrix R2 (n=974)

		ρ_{GS}				ρ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.7963	0.0945	0.0495	0.0946	-0.6485	0.0471	0.6840	0.1587
-.8	-.3	-0.7977	0.0853	0.0400	0.0853	-0.5884	0.0494	0.9760	0.2173
-.8	0	-0.7979	0.0819	0.0390	0.0820	-0.5796	0.0501	0.9855	0.2260
-.8	.3	-0.7980	0.0800	0.0415	0.0800	-0.5806	0.0504	0.9860	0.2251
-.8	.8	-0.7985	0.0762	0.0445	0.0763	-0.6005	0.0492	0.9770	0.2055
-.3	-.8	-0.3003	0.0983	0.0530	0.0983	-0.3132	0.0678	0.0245	0.0691
-.3	-.3	-0.3006	0.0991	0.0525	0.0991	-0.2463	0.0663	0.0600	0.0854
-.3	0	-0.3001	0.0968	0.0480	0.0968	-0.2238	0.0671	0.1105	0.1015
-.3	.3	-0.3003	0.0935	0.0480	0.0935	-0.2126	0.0679	0.1575	0.1107
-.3	.8	-0.3003	0.0872	0.0470	0.0872	-0.2215	0.0670	0.1530	0.1032
0	-.8	-0.0006	0.0914	0.0515	0.0914	-0.0790	0.0735	0.1630	0.1079
0	-.3	-0.0016	0.0952	0.0540	0.0952	-0.0282	0.0705	0.0425	0.0760
0	0	-0.0021	0.0938	0.0510	0.0938	-0.0050	0.0711	0.0315	0.0713
0	.3	-0.0018	0.0914	0.0510	0.0914	0.0128	0.0714	0.0360	0.0726
0	.8	-0.0009	0.0838	0.0530	0.0838	0.0151	0.0697	0.0495	0.0714
.3	-.8	0.2981	0.0746	0.0540	0.0747	0.1914	0.0711	0.3455	0.1298
.3	-.3	0.2959	0.0814	0.0515	0.0815	0.2163	0.0702	0.2030	0.1093
.3	0	0.2951	0.0856	0.0530	0.0858	0.2371	0.0709	0.1275	0.0948
.3	.3	0.2938	0.0858	0.0530	0.0860	0.2563	0.0715	0.0765	0.0838
.3	.8	0.2967	0.0777	0.0505	0.0778	0.2732	0.0704	0.0715	0.0753
.8	-.8	0.7994	0.0356	0.0540	0.0356	0.7321	0.0428	0.5045	0.0803
.8	-.3	0.7970	0.0393	0.0600	0.0394	0.7240	0.0471	0.4875	0.0894
.8	0	0.7938	0.0435	0.0645	0.0439	0.7274	0.0507	0.3800	0.0886
.8	.3	0.7896	0.0496	0.0660	0.0507	0.7346	0.0547	0.2775	0.0853
.8	.8	0.7844	0.0588	0.0660	0.0608	0.7544	0.0625	0.1640	0.0774
average		-0.0021	0.0792	0.0518	0.0794	0.0219	0.0620	0.3230	0.1126
		λ_{GS}				λ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.8025	0.0538	0.0460	0.0538	-0.7595	0.0303	0.0635	0.0506
-.8	-.3	-0.2994	0.0519	0.0505	0.0519	-0.3418	0.0388	0.1100	0.0570
-.8	0	0.0009	0.0438	0.0475	0.0438	-0.0537	0.0367	0.2250	0.0651
-.8	.3	0.3005	0.0326	0.0485	0.0326	0.2530	0.0304	0.2855	0.0560
-.8	.8	0.8002	0.0111	0.0495	0.0111	0.7874	0.0106	0.1745	0.0164
-.3	-.8	-0.7992	0.0400	0.0490	0.0400	-0.7162	0.0283	0.5305	0.0885
-.3	-.3	-0.2984	0.0482	0.0450	0.0483	-0.2923	0.0383	0.0175	0.0391
-.3	0	0.0016	0.0434	0.0440	0.0434	-0.0202	0.0379	0.0455	0.0429
-.3	.3	0.3008	0.0354	0.0465	0.0354	0.2703	0.0324	0.0955	0.0440
-.3	.8	0.8007	0.0135	0.0485	0.0136	0.7886	0.0127	0.1140	0.0170
0	-.8	-0.7989	0.0352	0.0500	0.0352	-0.7103	0.0267	0.6985	0.0936
0	-.3	-0.2974	0.0489	0.0515	0.0490	-0.2675	0.0395	0.0590	0.0512
0	0	0.0023	0.0471	0.0505	0.0472	0.0007	0.0406	0.0225	0.0406
0	.3	0.3021	0.0391	0.0470	0.0391	0.2826	0.0361	0.0510	0.0400
0	.8	0.8011	0.0159	0.0500	0.0159	0.7894	0.0152	0.0910	0.0185
.3	-.8	-0.7972	0.0317	0.0560	0.0318	-0.7115	0.0260	0.7645	0.0922
.3	-.3	-0.2956	0.0494	0.0560	0.0496	-0.2422	0.0420	0.1520	0.0714
.3	0	0.0039	0.0517	0.0575	0.0518	0.0235	0.0442	0.0575	0.0500
.3	.3	0.3035	0.0451	0.0550	0.0452	0.2979	0.0411	0.0330	0.0412
.3	.8	0.8020	0.0206	0.0480	0.0207	0.7906	0.0197	0.0650	0.0218
.8	-.8	-0.7980	0.0277	0.0590	0.0278	-0.7279	0.0205	0.7180	0.0750
.8	-.3	-0.2909	0.0536	0.0730	0.0543	-0.2094	0.0455	0.3025	0.1013
.8	0	0.0158	0.0645	0.0885	0.0664	0.0697	0.0545	0.1665	0.0885
.8	.3	0.3191	0.0688	0.1010	0.0714	0.3409	0.0584	0.0990	0.0713
.8	.8	0.8146	0.0430	0.1030	0.0454	0.7999	0.0442	0.1010	0.0442
average		0.0037	0.0406	0.0568	0.0410	0.0177	0.0340	0.2017	0.0551

Table 4: Heteroskedasticity with c=1, Modified Rook Matrix R3 (n=485)

		ρ_{GS}				ρ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.7813	0.0853	0.0630	0.0873	-0.6909	0.0531	0.2560	0.1213
-.8	-.3	-0.7885	0.0668	0.0500	0.0678	-0.6307	0.0521	0.8755	0.1771
-.8	0	-0.7918	0.0592	0.0485	0.0597	-0.6264	0.0524	0.9240	0.1814
-.8	.3	-0.7936	0.0556	0.0485	0.0559	-0.6328	0.0517	0.9240	0.1750
-.8	.8	-0.7935	0.0517	0.0510	0.0521	-0.6552	0.0468	0.8845	0.1522
-.3	-.8	-0.2858	0.1008	0.0560	0.1018	-0.3133	0.0788	0.0375	0.0799
-.3	-.3	-0.2907	0.1052	0.0590	0.1056	-0.2420	0.0696	0.0815	0.0906
-.3	0	-0.2919	0.1026	0.0590	0.1029	-0.2163	0.0703	0.1300	0.1093
-.3	.3	-0.2912	0.0994	0.0580	0.0998	-0.2029	0.0730	0.1860	0.1215
-.3	.8	-0.2899	0.0921	0.0610	0.0926	-0.2138	0.0730	0.1810	0.1130
0	-.8	0.0103	0.1026	0.0570	0.1031	-0.0789	0.0905	0.1490	0.1200
0	-.3	0.0070	0.1073	0.0595	0.1075	-0.0350	0.0812	0.0610	0.0884
0	0	0.0059	0.1087	0.0620	0.1089	-0.0084	0.0815	0.0490	0.0820
0	.3	0.0056	0.1070	0.0610	0.1072	0.0108	0.0824	0.0600	0.0831
0	.8	0.0070	0.1005	0.0580	0.1007	0.0135	0.0829	0.0755	0.0840
.3	-.8	0.3068	0.0960	0.0620	0.0962	0.1839	0.0969	0.3425	0.1512
.3	-.3	0.3025	0.0989	0.0600	0.0989	0.1976	0.0871	0.2550	0.1344
.3	0	0.3025	0.1014	0.0595	0.1014	0.2182	0.0855	0.1625	0.1183
.3	.3	0.3016	0.1032	0.0595	0.1032	0.2398	0.0858	0.1085	0.1048
.3	.8	0.3027	0.0993	0.0580	0.0993	0.2563	0.0898	0.1165	0.0999
.8	-.8	0.7994	0.0541	0.0530	0.0541	0.7207	0.0610	0.5010	0.1001
.8	-.3	0.7949	0.0558	0.0595	0.0560	0.7061	0.0661	0.5225	0.1149
.8	0	0.7887	0.0610	0.0700	0.0621	0.7063	0.0672	0.4675	0.1153
.8	.3	0.7828	0.0687	0.0700	0.0708	0.7130	0.0705	0.3630	0.1120
.8	.8	0.7794	0.0693	0.0720	0.0723	0.7330	0.0836	0.2655	0.1071
average		0.0040	0.0861	0.0590	0.0867	0.0061	0.0733	0.3192	0.1175
		λ_{GS}				λ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.8131	0.0593	0.0795	0.0607	-0.7748	0.0365	0.0140	0.0443
-.8	-.3	-0.3065	0.0710	0.0640	0.0713	-0.3591	0.0517	0.1480	0.0785
-.8	0	-0.0024	0.0605	0.0535	0.0606	-0.0756	0.0510	0.2875	0.0912
-.8	.3	0.2994	0.0459	0.0535	0.0459	0.2350	0.0426	0.3420	0.0777
-.8	.8	0.8001	0.0153	0.0560	0.0153	0.7847	0.0139	0.1975	0.0207
-.3	-.8	-0.8019	0.0309	0.0455	0.0310	-0.7364	0.0291	0.2970	0.0699
-.3	-.3	-0.3024	0.0537	0.0510	0.0538	-0.2926	0.0416	0.0105	0.0422
-.3	0	-0.0004	0.0538	0.0485	0.0538	-0.0282	0.0438	0.0490	0.0521
-.3	.3	0.3006	0.0452	0.0530	0.0452	0.2583	0.0405	0.1250	0.0581
-.3	.8	0.8005	0.0168	0.0535	0.0168	0.7847	0.0159	0.1565	0.0221
0	-.8	-0.8003	0.0255	0.0430	0.0255	-0.7375	0.0255	0.4520	0.0675
0	-.3	-0.2997	0.0477	0.0525	0.0477	-0.2655	0.0392	0.0395	0.0522
0	0	0.0001	0.0507	0.0535	0.0507	-0.0007	0.0426	0.0200	0.0426
0	.3	0.3008	0.0454	0.0540	0.0454	0.2759	0.0399	0.0555	0.0466
0	.8	0.8009	0.0191	0.0535	0.0192	0.7853	0.0181	0.1275	0.0233
.3	-.8	-0.7999	0.0228	0.0495	0.0228	-0.7424	0.0230	0.5150	0.0620
.3	-.3	-0.2970	0.0476	0.0540	0.0477	-0.2431	0.0411	0.1240	0.0701
.3	0	0.0041	0.0535	0.0555	0.0536	0.0257	0.0454	0.0375	0.0521
.3	.3	0.3036	0.0498	0.0565	0.0499	0.2950	0.0449	0.0345	0.0451
.3	.8	0.8013	0.0235	0.0545	0.0235	0.7868	0.0230	0.0945	0.0265
.8	-.8	-0.7993	0.0228	0.0535	0.0229	-0.7542	0.0201	0.4745	0.0500
.8	-.3	-0.2925	0.0542	0.0730	0.0547	-0.2156	0.0484	0.2940	0.0973
.8	0	0.0153	0.0711	0.0910	0.0728	0.0717	0.0590	0.1730	0.0928
.8	.3	0.3237	0.0816	0.1190	0.0850	0.3436	0.0660	0.1030	0.0791
.8	.8	0.8180	0.0476	0.1100	0.0509	0.7982	0.0547	0.1515	0.0547
average		0.0021	0.0446	0.0612	0.0451	0.0088	0.0383	0.1729	0.0568

Table 5: Heteroskedasticity with c=1, Modified Rook Matrix R4 (n=945)

		ρ_{GS}				ρ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.7895	0.0639	0.0525	0.0647	-0.6909	0.0393	0.5945	0.1160
-.8	-.3	-0.7982	0.0495	0.0400	0.0495	-0.6243	0.0390	0.9990	0.1799
-.8	0	-0.7994	0.0442	0.0415	0.0442	-0.6187	0.0397	1.0000	0.1856
-.8	.3	-0.7993	0.0400	0.0425	0.0400	-0.6250	0.0380	1.0000	0.1791
-.8	.8	-0.7996	0.0376	0.0470	0.0376	-0.6488	0.0343	0.9975	0.1551
-.3	-.8	-0.2930	0.0778	0.0470	0.0781	-0.3106	0.0593	0.0330	0.0602
-.3	-.3	-0.2958	0.0770	0.0490	0.0771	-0.2370	0.0518	0.1320	0.0816
-.3	0	-0.2972	0.0765	0.0520	0.0765	-0.2065	0.0522	0.2940	0.1071
-.3	.3	-0.2969	0.0743	0.0475	0.0744	-0.1899	0.0530	0.4330	0.1222
-.3	.8	-0.2967	0.0672	0.0505	0.0673	-0.2001	0.0541	0.4100	0.1136
0	-.8	0.0068	0.0769	0.0520	0.0772	-0.0779	0.0675	0.2420	0.1031
0	-.3	0.0050	0.0824	0.0505	0.0825	-0.0318	0.0592	0.0625	0.0672
0	0	0.0026	0.0800	0.0545	0.0801	-0.0006	0.0581	0.0380	0.0581
0	.3	0.0033	0.0802	0.0555	0.0803	0.0240	0.0593	0.0545	0.0640
0	.8	0.0040	0.0747	0.0520	0.0748	0.0302	0.0621	0.0795	0.0690
.3	-.8	0.3064	0.0708	0.0485	0.0711	0.1836	0.0690	0.5340	0.1353
.3	-.3	0.3033	0.0758	0.0510	0.0759	0.1974	0.0638	0.4130	0.1208
.3	0	0.3020	0.0788	0.0485	0.0789	0.2220	0.0620	0.2635	0.0996
.3	.3	0.3011	0.0789	0.0505	0.0789	0.2484	0.0620	0.1350	0.0807
.3	.8	0.3034	0.0736	0.0525	0.0737	0.2721	0.0656	0.0990	0.0713
.8	-.8	0.8019	0.0389	0.0460	0.0389	0.7198	0.0462	0.7115	0.0925
.8	-.3	0.7999	0.0402	0.0435	0.0402	0.7036	0.0493	0.7560	0.1083
.8	0	0.7979	0.0423	0.0485	0.0423	0.7053	0.0500	0.6895	0.1071
.8	.3	0.7951	0.0469	0.0535	0.0472	0.7147	0.0509	0.5570	0.0994
.8	.8	0.7892	0.0548	0.0545	0.0559	0.7377	0.0612	0.2700	0.0873
average		0.0023	0.0641	0.0492	0.0643	0.0119	0.0539	0.4319	0.1066
		λ_{GS}				λ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.8104	0.0452	0.0740	0.0464	-0.7735	0.0279	0.0330	0.0385
-.8	-.3	-0.3040	0.0556	0.0535	0.0557	-0.3621	0.0387	0.2695	0.0732
-.8	0	-0.0017	0.0488	0.0525	0.0488	-0.0825	0.0390	0.5250	0.0912
-.8	.3	0.2997	0.0372	0.0515	0.0372	0.2270	0.0332	0.6190	0.0802
-.8	.8	0.8004	0.0130	0.0515	0.0130	0.7812	0.0120	0.4055	0.0223
-.3	-.8	-0.8020	0.0221	0.0550	0.0222	-0.7334	0.0218	0.7245	0.0701
-.3	-.3	-0.3016	0.0405	0.0505	0.0405	-0.2898	0.0300	0.0155	0.0317
-.3	0	-0.0016	0.0412	0.0465	0.0412	-0.0297	0.0319	0.0770	0.0436
-.3	.3	0.2994	0.0352	0.0495	0.0352	0.2517	0.0296	0.2605	0.0567
-.3	.8	0.8003	0.0141	0.0505	0.0141	0.7802	0.0137	0.3240	0.0241
0	-.8	-0.8010	0.0184	0.0535	0.0185	-0.7350	0.0185	0.8600	0.0675
0	-.3	-0.3009	0.0359	0.0505	0.0359	-0.2609	0.0288	0.0815	0.0485
0	0	-0.0004	0.0394	0.0495	0.0394	-0.0013	0.0314	0.0155	0.0314
0	.3	0.2996	0.0357	0.0440	0.0357	0.2705	0.0303	0.0850	0.0423
0	.8	0.8004	0.0161	0.0490	0.0161	0.7808	0.0157	0.2290	0.0247
.3	-.8	-0.8005	0.0172	0.0535	0.0172	-0.7406	0.0165	0.8870	0.0617
.3	-.3	-0.3006	0.0360	0.0590	0.0361	-0.2365	0.0298	0.3075	0.0701
.3	0	0.0001	0.0394	0.0495	0.0394	0.0280	0.0331	0.0515	0.0434
.3	.3	0.3005	0.0391	0.0480	0.0391	0.2930	0.0335	0.0205	0.0342
.3	.8	0.8008	0.0195	0.0470	0.0196	0.7828	0.0198	0.1350	0.0262
.8	-.8	-0.8009	0.0162	0.0555	0.0162	-0.7549	0.0150	0.8085	0.0476
.8	-.3	-0.2988	0.0389	0.0545	0.0389	-0.2119	0.0354	0.5825	0.0950
.8	0	0.0051	0.0497	0.0625	0.0499	0.0746	0.0432	0.3110	0.0862
.8	.3	0.3101	0.0556	0.0760	0.0565	0.3446	0.0462	0.1120	0.0642
.8	.8	0.8093	0.0386	0.0790	0.0397	0.7956	0.0418	0.0945	0.0421
average		0.0001	0.0339	0.0546	0.0341	0.0079	0.0287	0.3134	0.0527

Table 6: Heteroskedasticity with c=1, Circular Matrix (n=500)

		ρ_{GS}				ρ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.7898	0.0836	0.0660	0.0843	-0.5705	0.0344	1.0000	0.2321
-.8	-.3	-0.7921	0.0797	0.0460	0.0801	-0.4478	0.0342	1.0000	0.3539
-.8	0	-0.7935	0.0730	0.0400	0.0733	-0.4203	0.0374	1.0000	0.3815
-.8	.3	-0.7944	0.0702	0.0400	0.0705	-0.4099	0.0400	1.0000	0.3921
-.8	.8	-0.7922	0.0660	0.0440	0.0665	-0.4290	0.0410	1.0000	0.3732
-.3	-.8	-0.2910	0.1103	0.0605	0.1106	-0.2724	0.0524	0.0295	0.0592
-.3	-.3	-0.2932	0.1157	0.0575	0.1159	-0.1713	0.0443	0.4670	0.1361
-.3	0	-0.2939	0.1159	0.0555	0.1161	-0.1368	0.0457	0.7575	0.1695
-.3	.3	-0.2941	0.1146	0.0550	0.1148	-0.1074	0.0485	0.8930	0.1986
-.3	.8	-0.2933	0.1109	0.0530	0.1111	-0.0863	0.0537	0.9315	0.2203
0	-.8	0.0085	0.1100	0.0610	0.1104	-0.1177	0.0702	0.3415	0.1370
0	-.3	0.0038	0.1121	0.0610	0.1122	-0.0442	0.0566	0.0525	0.0718
0	0	0.0018	0.1137	0.0590	0.1137	-0.0089	0.0561	0.0225	0.0568
0	.3	-0.0007	0.1157	0.0580	0.1157	0.0258	0.0582	0.0440	0.0637
0	.8	0.0006	0.1124	0.0580	0.1124	0.0615	0.0635	0.1380	0.0884
.3	-.8	0.3047	0.0959	0.0615	0.0961	0.0827	0.0835	0.8580	0.2328
.3	-.3	0.2999	0.0997	0.0610	0.0997	0.1195	0.0695	0.7800	0.1934
.3	0	0.2960	0.1011	0.0560	0.1012	0.1506	0.0674	0.6225	0.1639
.3	.3	0.2963	0.1029	0.0535	0.1029	0.1869	0.0674	0.4010	0.1316
.3	.8	0.2952	0.1029	0.0585	0.1030	0.2398	0.0744	0.1410	0.0957
.8	-.8	0.7986	0.0452	0.0540	0.0453	0.6343	0.0657	0.9735	0.1783
.8	-.3	0.7960	0.0477	0.0510	0.0479	0.6058	0.0681	0.9875	0.2058
.8	0	0.7921	0.0513	0.0525	0.0519	0.6086	0.0652	0.9815	0.2022
.8	.3	0.7863	0.0560	0.0615	0.0577	0.6254	0.0632	0.9475	0.1857
.8	.8	0.7786	0.0616	0.0620	0.0652	0.6823	0.0716	0.5430	0.1378
average		0.0012	0.0907	0.0554	0.0911	0.0320	0.0573	0.6365	0.1865
		λ_{GS}				λ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.8058	0.0440	0.0685	0.0444	-0.7195	0.0225	0.6520	0.0836
-.8	-.3	-0.3014	0.0750	0.0590	0.0750	-0.3527	0.0308	0.1090	0.0610
-.8	0	0.0003	0.0685	0.0550	0.0685	-0.1229	0.0355	0.7945	0.1279
-.8	.3	0.3016	0.0538	0.0560	0.0538	0.1544	0.0367	0.9660	0.1501
-.8	.8	0.8008	0.0177	0.0510	0.0177	0.7455	0.0153	0.9600	0.0566
-.3	-.8	-0.8001	0.0159	0.0470	0.0159	-0.6842	0.0172	0.9995	0.1170
-.3	-.3	-0.2998	0.0408	0.0425	0.0408	-0.2608	0.0246	0.0285	0.0463
-.3	0	0.0009	0.0455	0.0475	0.0455	-0.0367	0.0291	0.0435	0.0468
-.3	.3	0.3011	0.0415	0.0480	0.0415	0.2152	0.0304	0.6065	0.0901
-.3	.8	0.8008	0.0169	0.0510	0.0169	0.7509	0.0158	0.8880	0.0516
0	-.8	-0.8000	0.0141	0.0505	0.0141	-0.6882	0.0164	1.0000	0.1130
0	-.3	-0.2981	0.0331	0.0485	0.0332	-0.2310	0.0249	0.2555	0.0734
0	0	0.0027	0.0390	0.0460	0.0391	-0.0023	0.0291	0.0040	0.0292
0	.3	0.3020	0.0371	0.0470	0.0372	0.2441	0.0300	0.2230	0.0635
0	.8	0.8012	0.0168	0.0485	0.0168	0.7563	0.0170	0.7330	0.0470
.3	-.8	-0.7998	0.0156	0.0510	0.0156	-0.6947	0.0158	1.0000	0.1065
.3	-.3	-0.2966	0.0336	0.0540	0.0338	-0.2002	0.0265	0.7210	0.1032
.3	0	0.0039	0.0392	0.0535	0.0394	0.0364	0.0303	0.0615	0.0474
.3	.3	0.3043	0.0387	0.0490	0.0389	0.2764	0.0326	0.0235	0.0402
.3	.8	0.8016	0.0192	0.0485	0.0193	0.7621	0.0207	0.4850	0.0432
.8	-.8	-0.7992	0.0201	0.0540	0.0201	-0.7046	0.0178	1.0000	0.0970
.8	-.3	-0.2927	0.0553	0.0560	0.0557	-0.1173	0.0393	0.9895	0.1869
.8	0	0.0153	0.0698	0.0820	0.0714	0.1475	0.0440	0.8185	0.1539
.8	.3	0.3238	0.0755	0.1065	0.0791	0.3815	0.0459	0.3185	0.0936
.8	.8	0.8165	0.0389	0.1120	0.0422	0.7866	0.0431	0.0910	0.0451
average		0.0033	0.0386	0.0573	0.0390	0.0177	0.0277	0.5509	0.0830

Table 7: Heteroskedasticity with c=1, Circular Matrix (n=1000)

		ρ_{GS}				ρ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.7926	0.0599	0.0490	0.0604	-0.5718	0.0244	1.0000	0.2295
-.8	-.3	-0.7947	0.0571	0.0360	0.0573	-0.4447	0.0254	1.0000	0.3562
-.8	0	-0.7953	0.0532	0.0360	0.0534	-0.4151	0.0278	1.0000	0.3859
-.8	.3	-0.7960	0.0507	0.0375	0.0509	-0.4025	0.0304	1.0000	0.3987
-.8	.8	-0.7953	0.0474	0.0395	0.0476	-0.4226	0.0315	1.0000	0.3787
-.3	-.8	-0.2919	0.0821	0.0570	0.0825	-0.2689	0.0383	0.0405	0.0493
-.3	-.3	-0.2918	0.0843	0.0555	0.0847	-0.1656	0.0333	0.8980	0.1385
-.3	0	-0.2920	0.0863	0.0555	0.0867	-0.1289	0.0348	0.9910	0.1746
-.3	.3	-0.2936	0.0865	0.0540	0.0867	-0.0961	0.0371	0.9995	0.2072
-.3	.8	-0.2937	0.0830	0.0525	0.0833	-0.0708	0.0419	1.0000	0.2330
0	-.8	0.0088	0.0825	0.0560	0.0830	-0.1141	0.0509	0.6005	0.1250
0	-.3	0.0068	0.0843	0.0550	0.0846	-0.0385	0.0418	0.0790	0.0568
0	0	0.0062	0.0848	0.0545	0.0850	-0.0016	0.0422	0.0230	0.0423
0	.3	0.0054	0.0861	0.0530	0.0863	0.0363	0.0437	0.0950	0.0568
0	.8	0.0060	0.0837	0.0535	0.0839	0.0788	0.0493	0.3340	0.0929
.3	-.8	0.3055	0.0715	0.0545	0.0717	0.0872	0.0620	0.9845	0.2217
.3	-.3	0.3036	0.0746	0.0550	0.0747	0.1252	0.0516	0.9630	0.1822
.3	0	0.3031	0.0762	0.0575	0.0763	0.1577	0.0495	0.8630	0.1506
.3	.3	0.3022	0.0782	0.0565	0.0782	0.1968	0.0498	0.6045	0.1146
.3	.8	0.3029	0.0762	0.0555	0.0763	0.2574	0.0563	0.1645	0.0706
.8	-.8	0.8007	0.0328	0.0560	0.0328	0.6361	0.0485	1.0000	0.1709
.8	-.3	0.7996	0.0350	0.0560	0.0350	0.6049	0.0499	1.0000	0.2014
.8	0	0.7974	0.0372	0.0585	0.0373	0.6074	0.0485	1.0000	0.1986
.8	.3	0.7938	0.0422	0.0600	0.0426	0.6261	0.0470	0.9965	0.1802
.8	.8	0.7877	0.0493	0.0630	0.0508	0.6906	0.0572	0.7100	0.1235
average		0.0037	0.0674	0.0527	0.0677	0.0385	0.0429	0.7339	0.1816
		λ_{GS}				λ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.8036	0.0337	0.0760	0.0339	-0.7188	0.0165	0.9835	0.0828
-.8	-.3	-0.3024	0.0556	0.0625	0.0556	-0.3543	0.0227	0.3115	0.0588
-.8	0	-0.0014	0.0506	0.0600	0.0506	-0.1273	0.0268	0.9880	0.1301
-.8	.3	0.2994	0.0399	0.0575	0.0399	0.1482	0.0266	1.0000	0.1541
-.8	.8	0.7997	0.0131	0.0555	0.0131	0.7428	0.0120	0.9995	0.0585
-.3	-.8	-0.8011	0.0129	0.0565	0.0129	-0.6790	0.0134	1.0000	0.1217
-.3	-.3	-0.3002	0.0313	0.0550	0.0313	-0.2591	0.0188	0.1320	0.0450
-.3	0	-0.0004	0.0353	0.0580	0.0353	-0.0376	0.0224	0.1390	0.0438
-.3	.3	0.3002	0.0318	0.0555	0.0318	0.2109	0.0230	0.9280	0.0920
-.3	.8	0.8001	0.0128	0.0545	0.0128	0.7471	0.0127	0.9935	0.0544
0	-.8	-0.8002	0.0110	0.0580	0.0110	-0.6818	0.0130	1.0000	0.1189
0	-.3	-0.3000	0.0271	0.0600	0.0271	-0.2269	0.0192	0.7380	0.0756
0	0	0.0010	0.0311	0.0565	0.0312	-0.0013	0.0225	0.0080	0.0226
0	.3	0.3012	0.0304	0.0525	0.0305	0.2405	0.0239	0.5370	0.0641
0	.8	0.8007	0.0138	0.0525	0.0138	0.7519	0.0145	0.9575	0.0502
.3	-.8	-0.7997	0.0121	0.0610	0.0121	-0.6878	0.0132	1.0000	0.1130
.3	-.3	-0.2989	0.0275	0.0615	0.0275	-0.1929	0.0208	0.9850	0.1091
.3	0	0.0012	0.0323	0.0640	0.0324	0.0401	0.0249	0.1760	0.0471
.3	.3	0.3019	0.0328	0.0640	0.0329	0.2761	0.0267	0.0785	0.0359
.3	.8	0.8013	0.0163	0.0550	0.0163	0.7577	0.0183	0.7890	0.0461
.8	-.8	-0.7989	0.0150	0.0555	0.0151	-0.6961	0.0135	1.0000	0.1048
.8	-.3	-0.2938	0.0389	0.0650	0.0394	-0.1027	0.0298	1.0000	0.1996
.8	0	0.0102	0.0504	0.0795	0.0514	0.1616	0.0347	0.9915	0.1653
.8	.3	0.3151	0.0589	0.0925	0.0608	0.3909	0.0376	0.6330	0.0984
.8	.8	0.8101	0.0342	0.0975	0.0357	0.7856	0.0366	0.1440	0.0394
average		0.0017	0.0300	0.0626	0.0302	0.0195	0.0218	0.7005	0.0852

Table 8: Heteroskedasticity with c=.5, Modified Rook Matrix R1 (n=486)

		ρ_{GS}				ρ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.7928	0.1067	0.0430	0.1069	-0.6260	0.0664	0.5350	0.1862
-.8	-.3	-0.7921	0.1046	0.0455	0.1049	-0.6077	0.0653	0.6865	0.2031
-.8	0	-0.7918	0.1033	0.0475	0.1037	-0.6060	0.0652	0.7075	0.2046
-.8	.3	-0.7923	0.1028	0.0475	0.1031	-0.6063	0.0651	0.7095	0.2043
-.8	.8	-0.7920	0.1006	0.0460	0.1009	-0.6111	0.0626	0.6885	0.1990
-.3	-.8	-0.2940	0.1212	0.0520	0.1214	-0.2601	0.0914	0.0475	0.0997
-.3	-.3	-0.2944	0.1214	0.0520	0.1216	-0.2421	0.0895	0.0650	0.1067
-.3	0	-0.2936	0.1206	0.0535	0.1207	-0.2360	0.0903	0.0745	0.1107
-.3	.3	-0.2939	0.1207	0.0530	0.1209	-0.2344	0.0903	0.0800	0.1116
-.3	.8	-0.2942	0.1185	0.0515	0.1186	-0.2376	0.0902	0.0760	0.1097
0	-.8	0.0047	0.1150	0.0510	0.1151	-0.0248	0.0980	0.0515	0.1010
0	-.3	0.0016	0.1177	0.0510	0.1178	-0.0125	0.0961	0.0370	0.0969
0	0	0.0014	0.1177	0.0515	0.1177	-0.0070	0.0957	0.0375	0.0960
0	.3	0.0010	0.1164	0.0500	0.1164	-0.0033	0.0963	0.0365	0.0964
0	.8	0.0043	0.1130	0.0530	0.1131	-0.0038	0.0953	0.0410	0.0953
.3	-.8	0.3027	0.1004	0.0530	0.1005	0.2397	0.0948	0.1180	0.1124
.3	-.3	0.2998	0.1017	0.0505	0.1017	0.2420	0.0936	0.1030	0.1101
.3	0	0.2999	0.1024	0.0495	0.1024	0.2467	0.0941	0.0955	0.1082
.3	.3	0.2998	0.1021	0.0505	0.1021	0.2516	0.0937	0.0880	0.1055
.3	.8	0.3000	0.1014	0.0525	0.1014	0.2533	0.0928	0.0905	0.1039
.8	-.8	0.7991	0.0497	0.0545	0.0497	0.7494	0.0562	0.2285	0.0756
.8	-.3	0.7977	0.0522	0.0540	0.0522	0.7462	0.0590	0.2350	0.0799
.8	0	0.7957	0.0537	0.0555	0.0539	0.7453	0.0594	0.2150	0.0808
.8	.3	0.7938	0.0562	0.0535	0.0566	0.7461	0.0613	0.1995	0.0816
.8	.8	0.7934	0.0590	0.0535	0.0594	0.7497	0.0641	0.1915	0.0814
average		0.0026	0.0992	0.0510	0.0993	0.0261	0.0811	0.2175	0.1184
		λ_{GS}				λ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.8031	0.0318	0.0470	0.0319	-0.7878	0.0247	0.0110	0.0276
-.8	-.3	-0.3018	0.0305	0.0480	0.0305	-0.3125	0.0283	0.0320	0.0310
-.8	0	-0.0014	0.0250	0.0500	0.0250	-0.0150	0.0240	0.0435	0.0283
-.8	.3	0.2991	0.0187	0.0505	0.0187	0.2878	0.0180	0.0430	0.0217
-.8	.8	0.7997	0.0059	0.0480	0.0060	0.7973	0.0056	0.0310	0.0062
-.3	-.8	-0.8007	0.0214	0.0405	0.0214	-0.7786	0.0179	0.0205	0.0279
-.3	-.3	-0.3014	0.0288	0.0475	0.0288	-0.2992	0.0267	0.0110	0.0268
-.3	0	-0.0011	0.0261	0.0470	0.0261	-0.0062	0.0248	0.0200	0.0255
-.3	.3	0.2995	0.0205	0.0480	0.0205	0.2920	0.0197	0.0295	0.0212
-.3	.8	0.7997	0.0070	0.0465	0.0070	0.7974	0.0065	0.0310	0.0070
0	-.8	-0.8002	0.0184	0.0390	0.0184	-0.7786	0.0152	0.0235	0.0262
0	-.3	-0.3004	0.0281	0.0420	0.0281	-0.2924	0.0268	0.0085	0.0278
0	0	-0.0006	0.0273	0.0455	0.0273	-0.0011	0.0263	0.0125	0.0263
0	.3	0.2995	0.0228	0.0440	0.0228	0.2950	0.0222	0.0245	0.0228
0	.8	0.7998	0.0083	0.0475	0.0083	0.7976	0.0078	0.0345	0.0082
.3	-.8	-0.7998	0.0163	0.0420	0.0163	-0.7796	0.0136	0.0245	0.0245
.3	-.3	-0.2992	0.0291	0.0430	0.0291	-0.2864	0.0279	0.0190	0.0311
.3	0	0.0003	0.0298	0.0435	0.0298	0.0046	0.0298	0.0175	0.0302
.3	.3	0.3002	0.0263	0.0460	0.0263	0.2984	0.0262	0.0240	0.0263
.3	.8	0.8004	0.0107	0.0500	0.0107	0.7978	0.0101	0.0410	0.0103
.8	-.8	-0.7994	0.0143	0.0525	0.0143	-0.7830	0.0120	0.0170	0.0208
.8	-.3	-0.2967	0.0320	0.0600	0.0321	-0.2747	0.0305	0.0260	0.0396
.8	0	0.0056	0.0385	0.0655	0.0389	0.0202	0.0384	0.0275	0.0434
.8	.3	0.3075	0.0414	0.0725	0.0421	0.3118	0.0420	0.0350	0.0436
.8	.8	0.8039	0.0268	0.0745	0.0271	0.7991	0.0279	0.0750	0.0279
average		0.0004	0.0234	0.0496	0.0235	0.0041	0.0221	0.0273	0.0253

Table 9: Heteroskedasticity with c=.5, Modified Rook Matrix R2 (n=974)

		ρ_{GS}				ρ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.7976	0.0812	0.0460	0.0812	-0.6273	0.0489	0.8935	0.1795
-.8	-.3	-0.7973	0.0784	0.0435	0.0785	-0.6071	0.0486	0.9670	0.1989
-.8	0	-0.7982	0.0771	0.0425	0.0771	-0.6050	0.0486	0.9705	0.2009
-.8	.3	-0.7979	0.0770	0.0450	0.0770	-0.6055	0.0484	0.9725	0.2005
-.8	.8	-0.7977	0.0760	0.0450	0.0760	-0.6115	0.0481	0.9685	0.1945
-.3	-.8	-0.3006	0.0874	0.0495	0.0874	-0.2652	0.0649	0.0530	0.0736
-.3	-.3	-0.3002	0.0871	0.0500	0.0871	-0.2422	0.0641	0.0950	0.0863
-.3	0	-0.3006	0.0870	0.0510	0.0870	-0.2362	0.0644	0.1120	0.0906
-.3	.3	-0.3004	0.0870	0.0500	0.0870	-0.2341	0.0648	0.1200	0.0924
-.3	.8	-0.2993	0.0844	0.0480	0.0844	-0.2361	0.0631	0.1210	0.0898
0	-.8	-0.0010	0.0826	0.0495	0.0826	-0.0262	0.0683	0.0575	0.0731
0	-.3	-0.0001	0.0842	0.0520	0.0842	-0.0105	0.0674	0.0425	0.0682
0	0	0.0003	0.0841	0.0520	0.0841	-0.0038	0.0670	0.0435	0.0671
0	.3	-0.0003	0.0843	0.0520	0.0843	0.0005	0.0677	0.0450	0.0677
0	.8	-0.0014	0.0805	0.0510	0.0805	0.0007	0.0660	0.0495	0.0661
.3	-.8	0.3004	0.0703	0.0550	0.0703	0.2378	0.0649	0.1735	0.0899
.3	-.3	0.2988	0.0715	0.0500	0.0715	0.2449	0.0644	0.1475	0.0848
.3	0	0.2987	0.0729	0.0515	0.0729	0.2509	0.0645	0.1260	0.0811
.3	.3	0.2985	0.0722	0.0530	0.0722	0.2559	0.0642	0.1090	0.0778
.3	.8	0.2988	0.0711	0.0530	0.0711	0.2603	0.0623	0.1050	0.0739
.8	-.8	0.7998	0.0342	0.0535	0.0342	0.7501	0.0387	0.3575	0.0632
.8	-.3	0.7988	0.0361	0.0585	0.0361	0.7468	0.0400	0.3625	0.0666
.8	0	0.7977	0.0376	0.0570	0.0376	0.7473	0.0408	0.3345	0.0666
.8	.3	0.7965	0.0393	0.0565	0.0395	0.7494	0.0432	0.2935	0.0665
.8	.8	0.7942	0.0427	0.0545	0.0431	0.7546	0.0465	0.2460	0.0650
average		-0.0004	0.0714	0.0508	0.0715	0.0275	0.0572	0.3106	0.0994
		λ_{GS}				λ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.8005	0.0270	0.0455	0.0270	-0.7845	0.0209	0.0450	0.0260
-.8	-.3	-0.2995	0.0259	0.0515	0.0259	-0.3125	0.0227	0.0580	0.0259
-.8	0	0.0004	0.0219	0.0480	0.0219	-0.0153	0.0203	0.0870	0.0254
-.8	.3	0.3002	0.0163	0.0495	0.0163	0.2870	0.0157	0.0970	0.0204
-.8	.8	0.8000	0.0056	0.0495	0.0056	0.7968	0.0053	0.0630	0.0062
-.3	-.8	-0.7996	0.0203	0.0495	0.0203	-0.7720	0.0165	0.1370	0.0325
-.3	-.3	-0.2992	0.0243	0.0445	0.0243	-0.2977	0.0224	0.0250	0.0225
-.3	0	0.0006	0.0219	0.0440	0.0219	-0.0057	0.0208	0.0365	0.0215
-.3	.3	0.3003	0.0178	0.0460	0.0178	0.2918	0.0171	0.0535	0.0189
-.3	.8	0.8002	0.0068	0.0470	0.0068	0.7971	0.0065	0.0565	0.0071
0	-.8	-0.7995	0.0176	0.0480	0.0176	-0.7716	0.0144	0.1660	0.0319
0	-.3	-0.2991	0.0248	0.0480	0.0248	-0.2901	0.0234	0.0400	0.0254
0	0	0.0008	0.0237	0.0500	0.0237	0.0005	0.0228	0.0290	0.0228
0	.3	0.3007	0.0197	0.0460	0.0197	0.2954	0.0194	0.0415	0.0199
0	.8	0.8004	0.0080	0.0480	0.0080	0.7974	0.0077	0.0590	0.0081
.3	-.8	-0.7990	0.0160	0.0535	0.0161	-0.7727	0.0128	0.1805	0.0301
.3	-.3	-0.2986	0.0247	0.0515	0.0247	-0.2825	0.0244	0.0635	0.0300
.3	0	0.0013	0.0259	0.0490	0.0259	0.0067	0.0253	0.0405	0.0262
.3	.3	0.3010	0.0231	0.0500	0.0231	0.2996	0.0227	0.0395	0.0227
.3	.8	0.8007	0.0104	0.0480	0.0104	0.7977	0.0098	0.0615	0.0101
.8	-.8	-0.7995	0.0140	0.0595	0.0140	-0.7780	0.0101	0.1300	0.0243
.8	-.3	-0.2967	0.0265	0.0620	0.0267	-0.2714	0.0242	0.0880	0.0375
.8	0	0.0049	0.0309	0.0665	0.0313	0.0228	0.0300	0.0705	0.0377
.8	.3	0.3066	0.0333	0.0690	0.0340	0.3130	0.0329	0.0620	0.0354
.8	.8	0.8038	0.0229	0.0690	0.0232	0.8005	0.0234	0.0895	0.0234
average		0.0012	0.0204	0.0517	0.0204	0.0061	0.0189	0.0728	0.0237

Table 10: Heteroskedasticity with c=.5, Modified Rook Matrix R3 (n=485)

		ρ_{GS}				ρ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.7910	0.0609	0.0555	0.0615	-0.6728	0.0499	0.6015	0.1366
-.8	-.3	-0.7922	0.0557	0.0505	0.0562	-0.6540	0.0489	0.8570	0.1539
-.8	0	-0.7929	0.0540	0.0550	0.0544	-0.6545	0.0483	0.8755	0.1534
-.8	.3	-0.7941	0.0527	0.0545	0.0530	-0.6567	0.0473	0.8745	0.1509
-.8	.8	-0.7935	0.0517	0.0490	0.0521	-0.6636	0.0457	0.8655	0.1439
-.3	-.8	-0.2877	0.0926	0.0620	0.0934	-0.2575	0.0741	0.0830	0.0854
-.3	-.3	-0.2897	0.0935	0.0610	0.0941	-0.2365	0.0708	0.1140	0.0950
-.3	0	-0.2917	0.0932	0.0595	0.0936	-0.2296	0.0706	0.1320	0.0997
-.3	.3	-0.2911	0.0924	0.0595	0.0928	-0.2262	0.0708	0.1450	0.1023
-.3	.8	-0.2905	0.0903	0.0600	0.0908	-0.2283	0.0704	0.1460	0.1004
0	-.8	0.0109	0.0975	0.0615	0.0981	-0.0258	0.0853	0.0835	0.0892
0	-.3	0.0088	0.0989	0.0595	0.0993	-0.0138	0.0810	0.0730	0.0822
0	0	0.0065	0.0989	0.0605	0.0991	-0.0060	0.0812	0.0735	0.0814
0	.3	0.0062	0.0996	0.0595	0.0998	-0.0012	0.0814	0.0735	0.0814
0	.8	0.0076	0.0976	0.0590	0.0979	-0.0014	0.0817	0.0775	0.0817
.3	-.8	0.3083	0.0921	0.0630	0.0924	0.2259	0.0888	0.2275	0.1156
.3	-.3	0.3047	0.0932	0.0610	0.0933	0.2288	0.0860	0.1990	0.1117
.3	0	0.3040	0.0926	0.0585	0.0927	0.2337	0.0842	0.1745	0.1072
.3	.3	0.3025	0.0938	0.0585	0.0939	0.2391	0.0840	0.1605	0.1037
.3	.8	0.3043	0.0937	0.0595	0.0938	0.2425	0.0850	0.1620	0.1026
.8	-.8	0.8007	0.0536	0.0540	0.0536	0.7351	0.0574	0.4255	0.0867
.8	-.3	0.7991	0.0538	0.0570	0.0538	0.7300	0.0588	0.4355	0.0914
.8	0	0.7973	0.0550	0.0580	0.0550	0.7293	0.0601	0.4220	0.0928
.8	.3	0.7940	0.0565	0.0660	0.0568	0.7298	0.0606	0.3930	0.0927
.8	.8	0.7919	0.0595	0.0600	0.0601	0.7321	0.0654	0.3475	0.0943
average		0.0053	0.0789	0.0585	0.0793	0.0119	0.0695	0.3209	0.1054
		λ_{GS}				λ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.8037	0.0307	0.0640	0.0309	-0.7916	0.0244	0.0230	0.0258
-.8	-.3	-0.3020	0.0355	0.0555	0.0355	-0.3189	0.0315	0.0840	0.0367
-.8	0	-0.0008	0.0303	0.0520	0.0303	-0.0227	0.0287	0.1240	0.0366
-.8	.3	0.2997	0.0230	0.0525	0.0230	0.2811	0.0223	0.1410	0.0292
-.8	.8	0.8000	0.0077	0.0570	0.0077	0.7959	0.0070	0.0890	0.0081
-.3	-.8	-0.8005	0.0156	0.0455	0.0156	-0.7808	0.0146	0.0635	0.0241
-.3	-.3	-0.3006	0.0269	0.0480	0.0269	-0.2981	0.0242	0.0180	0.0242
-.3	0	-0.0001	0.0270	0.0495	0.0270	-0.0078	0.0251	0.0390	0.0262
-.3	.3	0.3001	0.0227	0.0535	0.0227	0.2884	0.0217	0.0670	0.0246
-.3	.8	0.8002	0.0084	0.0550	0.0084	0.7960	0.0079	0.0825	0.0089
0	-.8	-0.8001	0.0127	0.0445	0.0127	-0.7817	0.0120	0.0910	0.0219
0	-.3	-0.2999	0.0241	0.0510	0.0241	-0.2898	0.0226	0.0220	0.0248
0	0	0.0001	0.0255	0.0500	0.0255	-0.0004	0.0247	0.0255	0.0247
0	.3	0.3001	0.0228	0.0500	0.0228	0.2933	0.0225	0.0450	0.0234
0	.8	0.8004	0.0095	0.0575	0.0095	0.7961	0.0091	0.0750	0.0099
.3	-.8	-0.8000	0.0114	0.0495	0.0114	-0.7833	0.0107	0.1090	0.0198
.3	-.3	-0.2992	0.0237	0.0495	0.0237	-0.2834	0.0231	0.0460	0.0285
.3	0	0.0012	0.0271	0.0530	0.0272	0.0078	0.0263	0.0360	0.0275
.3	.3	0.3010	0.0252	0.0550	0.0253	0.2988	0.0248	0.0435	0.0248
.3	.8	0.8005	0.0119	0.0530	0.0119	0.7965	0.0117	0.0770	0.0122
.8	-.8	-0.7998	0.0115	0.0515	0.0115	-0.7863	0.0101	0.1165	0.0170
.8	-.3	-0.2976	0.0269	0.0565	0.0270	-0.2741	0.0256	0.0945	0.0365
.8	0	0.0044	0.0340	0.0615	0.0343	0.0220	0.0327	0.0675	0.0394
.8	.3	0.3066	0.0384	0.0740	0.0390	0.3134	0.0369	0.0565	0.0392
.8	.8	0.8047	0.0269	0.0740	0.0273	0.7997	0.0282	0.1240	0.0282
average		0.0006	0.0224	0.0545	0.0224	0.0028	0.0211	0.0704	0.0249

Table 11: Heteroskedasticity with c=.5, Modified Rook Matrix R4 (n=945)

		ρ_{GS}				ρ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.7967	0.0454	0.0440	0.0455	-0.6720	0.0372	0.9220	0.1333
-.8	-.3	-0.7981	0.0408	0.0440	0.0409	-0.6503	0.0345	0.9965	0.1536
-.8	0	-0.7987	0.0388	0.0475	0.0388	-0.6496	0.0339	0.9975	0.1542
-.8	.3	-0.7991	0.0384	0.0475	0.0385	-0.6517	0.0338	0.9970	0.1521
-.8	.8	-0.7991	0.0378	0.0465	0.0378	-0.6590	0.0330	0.9960	0.1448
-.3	-.8	-0.2954	0.0698	0.0470	0.0699	-0.2540	0.0548	0.1145	0.0715
-.3	-.3	-0.2967	0.0701	0.0525	0.0702	-0.2308	0.0517	0.2115	0.0864
-.3	0	-0.2961	0.0686	0.0520	0.0687	-0.2216	0.0513	0.2610	0.0937
-.3	.3	-0.2965	0.0673	0.0520	0.0674	-0.2174	0.0515	0.2975	0.0973
-.3	.8	-0.2965	0.0661	0.0485	0.0662	-0.2221	0.0523	0.2870	0.0939
0	-.8	0.0067	0.0739	0.0465	0.0742	-0.0233	0.0618	0.0865	0.0661
0	-.3	0.0040	0.0746	0.0510	0.0747	-0.0099	0.0595	0.0595	0.0603
0	0	0.0032	0.0737	0.0550	0.0737	-0.0013	0.0590	0.0610	0.0590
0	.3	0.0039	0.0735	0.0535	0.0736	0.0053	0.0596	0.0595	0.0598
0	.8	0.0043	0.0724	0.0485	0.0725	0.0064	0.0604	0.0650	0.0607
.3	-.8	0.3067	0.0699	0.0480	0.0702	0.2271	0.0640	0.3240	0.0970
.3	-.3	0.3055	0.0703	0.0485	0.0705	0.2301	0.0628	0.2905	0.0939
.3	0	0.3051	0.0703	0.0485	0.0705	0.2376	0.0620	0.2545	0.0880
.3	.3	0.3042	0.0706	0.0510	0.0707	0.2453	0.0619	0.2110	0.0826
.3	.8	0.3035	0.0710	0.0495	0.0711	0.2510	0.0627	0.2000	0.0796
.8	-.8	0.8020	0.0392	0.0430	0.0392	0.7342	0.0434	0.6165	0.0789
.8	-.3	0.8017	0.0393	0.0445	0.0394	0.7294	0.0436	0.6380	0.0830
.8	0	0.8008	0.0395	0.0465	0.0395	0.7295	0.0445	0.6150	0.0834
.8	.3	0.7991	0.0402	0.0430	0.0402	0.7316	0.0446	0.5640	0.0816
.8	.8	0.7987	0.0432	0.0450	0.0433	0.7376	0.0484	0.4360	0.0789
average		0.0031	0.0586	0.0481	0.0587	0.0161	0.0509	0.4225	0.0933
		λ_{GS}				λ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.8028	0.0233	0.0565	0.0235	-0.7906	0.0186	0.0360	0.0208
-.8	-.3	-0.3012	0.0279	0.0515	0.0280	-0.3199	0.0241	0.1175	0.0313
-.8	0	-0.0004	0.0245	0.0500	0.0245	-0.0250	0.0222	0.2025	0.0334
-.8	.3	0.3000	0.0186	0.0510	0.0186	0.2788	0.0177	0.2295	0.0276
-.8	.8	0.8002	0.0065	0.0515	0.0065	0.7949	0.0059	0.1405	0.0078
-.3	-.8	-0.8007	0.0110	0.0525	0.0111	-0.7793	0.0104	0.1985	0.0232
-.3	-.3	-0.3006	0.0201	0.0490	0.0201	-0.2968	0.0179	0.0195	0.0182
-.3	0	-0.0006	0.0205	0.0465	0.0205	-0.0090	0.0190	0.0440	0.0211
-.3	.3	0.2997	0.0177	0.0515	0.0177	0.2860	0.0163	0.1045	0.0215
-.3	.8	0.8001	0.0071	0.0505	0.0071	0.7947	0.0067	0.1195	0.0085
0	-.8	-0.8004	0.0093	0.0515	0.0093	-0.7805	0.0086	0.2580	0.0213
0	-.3	-0.3006	0.0180	0.0515	0.0181	-0.2883	0.0168	0.0390	0.0204
0	0	-0.0003	0.0198	0.0500	0.0198	-0.0005	0.0187	0.0230	0.0187
0	.3	0.2997	0.0176	0.0450	0.0176	0.2914	0.0168	0.0480	0.0189
0	.8	0.8001	0.0081	0.0495	0.0081	0.7949	0.0076	0.0955	0.0091
.3	-.8	-0.8003	0.0086	0.0510	0.0086	-0.7826	0.0077	0.2845	0.0190
.3	-.3	-0.3008	0.0180	0.0545	0.0180	-0.2814	0.0171	0.0740	0.0253
.3	0	-0.0005	0.0200	0.0525	0.0200	0.0082	0.0193	0.0370	0.0210
.3	.3	0.2997	0.0197	0.0485	0.0197	0.2975	0.0192	0.0275	0.0194
.3	.8	0.8002	0.0097	0.0480	0.0097	0.7952	0.0095	0.0770	0.0107
.8	-.8	-0.8004	0.0080	0.0550	0.0081	-0.7869	0.0071	0.2560	0.0150
.8	-.3	-0.3002	0.0193	0.0505	0.0193	-0.2732	0.0185	0.1690	0.0326
.8	0	0.0011	0.0242	0.0515	0.0243	0.0231	0.0233	0.0905	0.0329
.8	.3	0.3023	0.0271	0.0565	0.0272	0.3133	0.0264	0.0395	0.0296
.8	.8	0.8017	0.0201	0.0545	0.0202	0.7985	0.0214	0.0690	0.0214
average		-0.0002	0.0170	0.0512	0.0170	0.0025	0.0159	0.1120	0.0211

Table 12: Heteroskedasticity with c=.5, Circular Matrix (n=500)

		ρ_{GS}				ρ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.7927	0.0705	0.0655	0.0709	-0.5169	0.0371	1.0000	0.2856
-.8	-.3	-0.7942	0.0685	0.0510	0.0688	-0.4650	0.0361	1.0000	0.3369
-.8	0	-0.7935	0.0670	0.0430	0.0673	-0.4568	0.0366	1.0000	0.3452
-.8	.3	-0.7930	0.0664	0.0445	0.0667	-0.4553	0.0376	1.0000	0.3467
-.8	.8	-0.7917	0.0653	0.0555	0.0658	-0.4624	0.0380	1.0000	0.3397
-.3	-.8	-0.2910	0.1074	0.0620	0.1078	-0.1954	0.0523	0.2975	0.1170
-.3	-.3	-0.2923	0.1097	0.0605	0.1100	-0.1599	0.0481	0.6005	0.1481
-.3	0	-0.2945	0.1106	0.0580	0.1107	-0.1477	0.0483	0.6945	0.1598
-.3	.3	-0.2948	0.1100	0.0575	0.1101	-0.1384	0.0493	0.7490	0.1690
-.3	.8	-0.2933	0.1088	0.0570	0.1090	-0.1336	0.0499	0.7730	0.1737
0	-.8	0.0088	0.1085	0.0590	0.1089	-0.0440	0.0663	0.0850	0.0796
0	-.3	0.0048	0.1102	0.0620	0.1103	-0.0202	0.0603	0.0480	0.0636
0	0	0.0010	0.1112	0.0620	0.1112	-0.0085	0.0608	0.0450	0.0614
0	.3	0.0007	0.1110	0.0625	0.1110	0.0027	0.0616	0.0435	0.0617
0	.8	0.0012	0.1107	0.0595	0.1107	0.0116	0.0630	0.0530	0.0640
.3	-.8	0.3053	0.0932	0.0620	0.0933	0.1455	0.0767	0.6700	0.1725
.3	-.3	0.3041	0.0959	0.0625	0.0960	0.1552	0.0726	0.6290	0.1620
.3	0	0.2995	0.0965	0.0615	0.0965	0.1649	0.0719	0.5790	0.1530
.3	.3	0.2979	0.0972	0.0610	0.0972	0.1770	0.0721	0.5170	0.1425
.3	.8	0.2969	0.0974	0.0625	0.0974	0.1919	0.0735	0.4290	0.1307
.8	-.8	0.7987	0.0444	0.0565	0.0444	0.6620	0.0602	0.9445	0.1506
.8	-.3	0.7982	0.0451	0.0550	0.0451	0.6527	0.0619	0.9580	0.1598
.8	0	0.7972	0.0454	0.0545	0.0454	0.6522	0.0616	0.9550	0.1601
.8	.3	0.7953	0.0476	0.0540	0.0478	0.6571	0.0607	0.9390	0.1553
.8	.8	0.7920	0.0501	0.0505	0.0507	0.6748	0.0624	0.8020	0.1399
average		0.0028	0.0859	0.0576	0.0861	0.0377	0.0567	0.6325	0.1711
		λ_{GS}				λ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.8015	0.0231	0.0525	0.0231	-0.7685	0.0145	0.2230	0.0347
-.8	-.3	-0.2998	0.0383	0.0545	0.0383	-0.3200	0.0228	0.0585	0.0304
-.8	0	0.0003	0.0345	0.0540	0.0345	-0.0438	0.0240	0.3260	0.0499
-.8	.3	0.3007	0.0268	0.0555	0.0268	0.2522	0.0213	0.5460	0.0523
-.8	.8	0.8003	0.0088	0.0540	0.0088	0.7842	0.0075	0.5230	0.0175
-.3	-.8	-0.8000	0.0080	0.0480	0.0080	-0.7626	0.0081	0.8925	0.0383
-.3	-.3	-0.2997	0.0204	0.0445	0.0204	-0.2862	0.0157	0.0090	0.0209
-.3	0	0.0005	0.0227	0.0490	0.0227	-0.0125	0.0182	0.0200	0.0220
-.3	.3	0.3005	0.0208	0.0505	0.0208	0.2726	0.0177	0.1885	0.0326
-.3	.8	0.8003	0.0084	0.0530	0.0084	0.7860	0.0074	0.3945	0.0158
0	-.8	-0.7999	0.0071	0.0495	0.0071	-0.7650	0.0072	0.9175	0.0357
0	-.3	-0.2993	0.0167	0.0495	0.0167	-0.2765	0.0151	0.0475	0.0280
0	0	0.0011	0.0193	0.0495	0.0193	-0.0007	0.0180	0.0035	0.0180
0	.3	0.3008	0.0186	0.0470	0.0186	0.2818	0.0180	0.0590	0.0256
0	.8	0.8004	0.0084	0.0485	0.0084	0.7875	0.0082	0.2930	0.0150
.3	-.8	-0.8000	0.0079	0.0510	0.0079	-0.7678	0.0074	0.9080	0.0330
.3	-.3	-0.2990	0.0171	0.0535	0.0171	-0.2667	0.0164	0.1780	0.0371
.3	0	0.0011	0.0194	0.0480	0.0195	0.0122	0.0192	0.0255	0.0227
.3	.3	0.3015	0.0197	0.0490	0.0197	0.2925	0.0198	0.0150	0.0212
.3	.8	0.8006	0.0096	0.0455	0.0097	0.7893	0.0100	0.1830	0.0146
.8	-.8	-0.7997	0.0101	0.0540	0.0101	-0.7708	0.0090	0.8375	0.0305
.8	-.3	-0.2979	0.0269	0.0530	0.0270	-0.2380	0.0244	0.6055	0.0667
.8	0	0.0045	0.0343	0.0580	0.0346	0.0538	0.0298	0.3420	0.0615
.8	.3	0.3063	0.0365	0.0715	0.0370	0.3302	0.0318	0.1525	0.0439
.8	.8	0.8043	0.0217	0.0680	0.0222	0.7964	0.0243	0.1025	0.0246
average		0.0011	0.0194	0.0524	0.0195	0.0064	0.0166	0.3140	0.0317

Table 13: Heteroskedasticity with c=.5, Circular Matrix (n=1000)

		ρ_{GS}				ρ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.7950	0.0503	0.0440	0.0505	-0.5171	0.0274	1.0000	0.2842
-.8	-.3	-0.7960	0.0499	0.0375	0.0500	-0.4625	0.0268	1.0000	0.3386
-.8	0	-0.7961	0.0487	0.0380	0.0489	-0.4533	0.0276	1.0000	0.3478
-.8	.3	-0.7953	0.0480	0.0390	0.0482	-0.4505	0.0282	1.0000	0.3507
-.8	.8	-0.7945	0.0464	0.0430	0.0467	-0.4583	0.0278	1.0000	0.3428
-.3	-.8	-0.2908	0.0805	0.0565	0.0810	-0.1912	0.0396	0.6415	0.1158
-.3	-.3	-0.2926	0.0821	0.0545	0.0824	-0.1529	0.0369	0.9435	0.1517
-.3	0	-0.2925	0.0821	0.0555	0.0824	-0.1401	0.0370	0.9750	0.1641
-.3	.3	-0.2924	0.0815	0.0550	0.0818	-0.1301	0.0374	0.9865	0.1740
-.3	.8	-0.2915	0.0806	0.0525	0.0811	-0.1240	0.0390	0.9900	0.1803
0	-.8	0.0077	0.0824	0.0540	0.0828	-0.0394	0.0488	0.1220	0.0627
0	-.3	0.0077	0.0811	0.0540	0.0815	-0.0135	0.0454	0.0500	0.0474
0	0	0.0068	0.0825	0.0525	0.0828	-0.0011	0.0455	0.0425	0.0456
0	.3	0.0063	0.0823	0.0520	0.0825	0.0113	0.0461	0.0530	0.0475
0	.8	0.0069	0.0822	0.0495	0.0825	0.0224	0.0482	0.0795	0.0532
.3	-.8	0.3071	0.0701	0.0525	0.0704	0.1513	0.0574	0.8845	0.1594
.3	-.3	0.3052	0.0719	0.0540	0.0721	0.1628	0.0537	0.8530	0.1474
.3	0	0.3054	0.0726	0.0530	0.0728	0.1730	0.0533	0.8070	0.1377
.3	.3	0.3042	0.0728	0.0525	0.0729	0.1858	0.0531	0.7205	0.1259
.3	.8	0.3038	0.0734	0.0495	0.0735	0.2034	0.0548	0.5825	0.1111
.8	-.8	0.8008	0.0325	0.0560	0.0325	0.6654	0.0433	0.9970	0.1414
.8	-.3	0.8004	0.0327	0.0570	0.0327	0.6557	0.0456	0.9975	0.1514
.8	0	0.7995	0.0339	0.0575	0.0339	0.6546	0.0447	0.9975	0.1521
.8	.3	0.7982	0.0347	0.0575	0.0347	0.6597	0.0448	0.9955	0.1472
.8	.8	0.7964	0.0370	0.0555	0.0372	0.6805	0.0477	0.9235	0.1287
average		0.0048	0.0637	0.0513	0.0639	0.0437	0.0424	0.7457	0.1643
		λ_{GS}				λ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.8010	0.0173	0.0635	0.0173	-0.7682	0.0110	0.6105	0.0337
-.8	-.3	-0.3011	0.0281	0.0620	0.0282	-0.3216	0.0169	0.1265	0.0275
-.8	0	-0.0009	0.0255	0.0615	0.0255	-0.0469	0.0179	0.6385	0.0502
-.8	.3	0.2996	0.0200	0.0570	0.0200	0.2483	0.0158	0.8735	0.0541
-.8	.8	0.7998	0.0066	0.0550	0.0066	0.7830	0.0056	0.8560	0.0179
-.3	-.8	-0.8004	0.0064	0.0580	0.0065	-0.7602	0.0066	0.9995	0.0403
-.3	-.3	-0.3001	0.0158	0.0550	0.0158	-0.2856	0.0122	0.0350	0.0189
-.3	0	-0.0000	0.0177	0.0585	0.0177	-0.0131	0.0142	0.0535	0.0193
-.3	.3	0.3000	0.0159	0.0570	0.0159	0.2708	0.0136	0.4355	0.0322
-.3	.8	0.8001	0.0064	0.0550	0.0064	0.7846	0.0058	0.7345	0.0165
0	-.8	-0.8002	0.0056	0.0610	0.0056	-0.7626	0.0062	1.0000	0.0379
0	-.3	-0.3000	0.0136	0.0615	0.0136	-0.2749	0.0124	0.2045	0.0280
0	0	0.0004	0.0157	0.0570	0.0157	-0.0004	0.0145	0.0110	0.0145
0	.3	0.3005	0.0153	0.0560	0.0153	0.2808	0.0141	0.1860	0.0238
0	.8	0.8003	0.0069	0.0550	0.0069	0.7861	0.0068	0.5650	0.0154
.3	-.8	-0.7999	0.0061	0.0585	0.0061	-0.7651	0.0061	0.9995	0.0355
.3	-.3	-0.2997	0.0137	0.0615	0.0137	-0.2637	0.0134	0.5210	0.0387
.3	0	0.0002	0.0164	0.0680	0.0164	0.0135	0.0160	0.0775	0.0209
.3	.3	0.3006	0.0167	0.0620	0.0167	0.2921	0.0165	0.0590	0.0183
.3	.8	0.8006	0.0082	0.0580	0.0082	0.7880	0.0087	0.3945	0.0148
.8	-.8	-0.7995	0.0075	0.0560	0.0076	-0.7675	0.0067	0.9960	0.0332
.8	-.3	-0.2977	0.0194	0.0570	0.0195	-0.2314	0.0188	0.9335	0.0711
.8	0	0.0034	0.0250	0.0670	0.0252	0.0596	0.0231	0.6850	0.0640
.8	.3	0.3045	0.0281	0.0720	0.0284	0.3344	0.0259	0.2875	0.0430
.8	.8	0.8028	0.0188	0.0720	0.0190	0.7958	0.0215	0.1720	0.0219
average		0.0005	0.0151	0.0602	0.0151	0.0070	0.0132	0.4982	0.0317

Table 14: Homoskedasticity with c=1, Modified Rook Matrix R1 (n=486)

		ρ_{GS}				ρ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.7724	0.1019	0.0895	0.1056	-0.7932	0.0855	0.0815	0.0858
-.8	-.3	-0.7939	0.0704	0.0555	0.0707	-0.7970	0.0588	0.0455	0.0589
-.8	0	-0.7979	0.0642	0.0485	0.0643	-0.7966	0.0565	0.0545	0.0566
-.8	.3	-0.7994	0.0610	0.0480	0.0610	-0.7970	0.0532	0.0580	0.0533
-.8	.8	-0.8003	0.0576	0.0550	0.0576	-0.7978	0.0513	0.0590	0.0514
-.3	-.8	-0.2926	0.1124	0.0555	0.1127	-0.3038	0.1108	0.0450	0.1108
-.3	-.3	-0.2981	0.1052	0.0500	0.1052	-0.3022	0.1031	0.0475	0.1031
-.3	0	-0.3010	0.1032	0.0505	0.1032	-0.3026	0.0989	0.0475	0.0990
-.3	.3	-0.3026	0.0975	0.0510	0.0975	-0.3026	0.0956	0.0520	0.0956
-.3	.8	-0.3033	0.0901	0.0545	0.0901	-0.3019	0.0890	0.0545	0.0890
0	-.8	0.0015	0.1038	0.0490	0.1038	-0.0048	0.1023	0.0420	0.1024
0	-.3	-0.0015	0.1069	0.0510	0.1069	-0.0041	0.1048	0.0465	0.1049
0	0	-0.0038	0.1054	0.0490	0.1055	-0.0037	0.1029	0.0485	0.1030
0	.3	-0.0059	0.1036	0.0500	0.1037	-0.0041	0.1019	0.0480	0.1020
0	.8	-0.0063	0.0935	0.0535	0.0937	-0.0035	0.0924	0.0515	0.0925
.3	-.8	0.2994	0.0885	0.0495	0.0885	0.2975	0.0871	0.0455	0.0872
.3	-.3	0.2960	0.0944	0.0545	0.0945	0.2980	0.0940	0.0530	0.0940
.3	0	0.2938	0.0949	0.0550	0.0951	0.2958	0.0954	0.0465	0.0955
.3	.3	0.2924	0.0959	0.0495	0.0962	0.2950	0.0948	0.0460	0.0949
.3	.8	0.2933	0.0884	0.0485	0.0887	0.2967	0.0878	0.0470	0.0879
.8	-.8	0.7995	0.0427	0.0500	0.0427	0.7972	0.0399	0.0530	0.0400
.8	-.3	0.7956	0.0503	0.0580	0.0505	0.7960	0.0448	0.0500	0.0449
.8	0	0.7906	0.0576	0.0635	0.0583	0.7955	0.0503	0.0525	0.0505
.8	.3	0.7841	0.0657	0.0720	0.0676	0.7939	0.0564	0.0560	0.0568
.8	.8	0.7791	0.0656	0.0660	0.0688	0.7962	0.0674	0.0900	0.0675
average		-0.0021	0.0848	0.0551	0.0853	-0.0021	0.0810	0.0528	0.0811
		λ_{GS}				λ_{ML}			
rho	lambda	Median	Std. err	Rej. Rate	RMSE	Median	Std. err	Rej. Rate	RMSE
-.8	-.8	-0.8282	0.0736	0.1220	0.0788	-0.8010	0.0689	0.0835	0.0689
-.8	-.3	-0.3105	0.0640	0.0655	0.0649	-0.3053	0.0584	0.0480	0.0587
-.8	0	-0.0041	0.0510	0.0550	0.0511	-0.0041	0.0484	0.0425	0.0486
-.8	.3	0.2990	0.0373	0.0520	0.0373	0.2973	0.0358	0.0455	0.0359
-.8	.8	0.8001	0.0119	0.0515	0.0119	0.7990	0.0112	0.0425	0.0112
-.3	-.8	-0.8081	0.0542	0.0600	0.0548	-0.7975	0.0497	0.0520	0.0498
-.3	-.3	-0.3051	0.0619	0.0550	0.0621	-0.3017	0.0618	0.0500	0.0618
-.3	0	-0.0021	0.0537	0.0505	0.0537	-0.0026	0.0528	0.0465	0.0528
-.3	.3	0.2996	0.0417	0.0495	0.0417	0.2981	0.0400	0.0440	0.0400
-.3	.8	0.8000	0.0140	0.0495	0.0140	0.7990	0.0132	0.0445	0.0133
0	-.8	-0.8040	0.0479	0.0545	0.0481	-0.7971	0.0424	0.0450	0.0425
0	-.3	-0.3019	0.0638	0.0535	0.0639	-0.3011	0.0639	0.0480	0.0639
0	0	-0.0017	0.0587	0.0525	0.0587	-0.0018	0.0578	0.0495	0.0578
0	.3	0.2997	0.0464	0.0510	0.0464	0.2980	0.0459	0.0460	0.0460
0	.8	0.8002	0.0165	0.0480	0.0165	0.7984	0.0154	0.0430	0.0155
.3	-.8	-0.8019	0.0448	0.0515	0.0449	-0.7977	0.0387	0.0455	0.0388
.3	-.3	-0.2987	0.0676	0.0600	0.0676	-0.2988	0.0683	0.0475	0.0683
.3	0	0.0018	0.0660	0.0575	0.0660	0.0003	0.0666	0.0500	0.0666
.3	.3	0.3020	0.0553	0.0595	0.0554	0.2983	0.0551	0.0510	0.0551
.3	.8	0.8006	0.0211	0.0485	0.0212	0.7980	0.0202	0.0460	0.0203
.8	-.8	-0.7994	0.0403	0.0555	0.0403	-0.7965	0.0321	0.0515	0.0323
.8	-.3	-0.2894	0.0820	0.0765	0.0827	-0.2946	0.0698	0.0550	0.0700
.8	0	0.0188	0.0974	0.0940	0.0992	0.0055	0.0808	0.0575	0.0810
.8	.3	0.3276	0.0997	0.1055	0.1034	0.3039	0.0867	0.0695	0.0868
.8	.8	0.8176	0.0486	0.1045	0.0517	0.7991	0.0525	0.0915	0.0525
average		0.0005	0.0528	0.0633	0.0534	-0.0002	0.0495	0.0518	0.0495

A Appendix: Additional Assumptions

In this appendix we state the additional assumptions needed to formally establish the limiting distribution of the GMM/IV estimators.

Assumption A.1 *Let* $\Gamma_n = [\gamma_{rs,n}]_{r,s=1,2}$ *and* $\gamma_n = [\gamma_{1,n}, \gamma_{2,n}]'$ *where, dropping the subscript* n *temporarily for notational convenience,*

$$\begin{aligned}
\gamma_{11} &= 2n^{-1}E\left\{\overline{\overline{u}}'\overline{u} - Tr\left[M\left[diag_{i=1}^{n}(\overline{u}_i u_i)\right]M'\right]\right\} = 2n^{-1}Eu'M'A_1 u, \\
\gamma_{12} &= -n^{-1}E\left\{\overline{\overline{u}}'\overline{\overline{u}} + Tr\left[M\left[diag_{i=1}^{n}(\overline{u}_i^2)\right]M'\right]\right\} = -n^{-1}Eu'M'A_1 Mu, \\
\gamma_{21} &= n^{-1}E(u'\overline{\overline{u}} + \overline{u}'\overline{u}) = n^{-1}Eu'M'(A_2 + A_2')u, \\
\gamma_{22} &= -n^{-1}E\overline{u}'\overline{\overline{u}} = -n^{-1}Eu'M'A_2 Mu, \\
\gamma_1 &= n^{-1}E\left\{\overline{u}'\overline{u} - Tr\left[M\left[diag_{i=1}^{n}(u_i^2)\right]M'\right]\right\} = n^{-1}Eu'A_1 u, \\
\gamma_2 &= n^{-1}Eu'\overline{u} = n^{-1}Eu'A_2 u,
\end{aligned} \tag{A.1}$$

with $\overline{u} = Mu$, *and* $\overline{\overline{u}} = M\overline{u} = M^2 u$. *Then* Γ_n *is nonsingular for all* n *sufficiently large and* $\lim_{n\to\infty}\Gamma_n = \Gamma$ *is finite and nonsingular.*

Assumption A.2 *Let* $\Psi_n = (\psi_{rs,n})$ *where for* $r, s = 1, 2$

$$\psi_{rs,n} = (2n)^{-1}tr\left[\left(A_{r,n} + A_{r,n}'\right)\mathbf{\Sigma}_n\left(A_{s,n} + A_{s,n}'\right)\mathbf{\Sigma}_n\right] + n^{-1}a_{r,n}'\mathbf{\Sigma}_n a_{s,n} \tag{A.2}$$

with $a_{r,n} = (I_n - \rho M_n')^{-1} H_n P \alpha_{r,n}$ *where*

$$\begin{aligned}
\alpha_{r,n} &= -n^{-1}E\left[Z_n'(I_n - \rho M_n')(A_{r,n} + A_{r,n}')(I_n - \rho M_n)u_n\right], \\
P &= Q_{HH}^{-1}Q_{HZ}[Q_{HZ}'Q_{HH}^{-1}Q_{HZ}]^{-1},
\end{aligned} \tag{A.3}$$

and $\mathbf{\Sigma}_n = diag(\sigma_{i,n}^2)$, *where* $\sigma_{i,n}^2 = E\varepsilon_{i,n}^2$. *Furthermore, let*

$$\Psi_{\circ,n} = \begin{bmatrix} \Psi_{\Delta\Delta,n} & \Psi_{\Delta\rho,n} \\ \Psi_{\Delta\rho,n}' & \Psi_n \end{bmatrix} \tag{A.4}$$

with

$$\begin{aligned}
\Psi_{\Delta\Delta,n} &= n^{-1}H_n'(I_n - \rho M_n)^{-1}\Sigma_n(I_n - \rho M_n')^{-1}H_n, \\
\Psi_{\Delta\rho,n} &= n^{-1}H_n'(I_n - \rho M_n)^{-1}\Sigma_n[a_{1,n}, a_{2,n}].
\end{aligned}$$

Then Ψ_n *and* $\Psi_{\circ,n}$ *are nonsingular for all* n *sufficiently large and* $\lim_{n\to\infty}\Psi_n = \Psi$ *and* $\lim_{n\to\infty}\Psi_{\circ,n} = \Psi_\circ$ *are finite and nonsingular.*

Assumption A.3 *Let* $\Psi_n = (\psi_{rs,n})$ *where for* $r, s = 1, 2$

$$\psi_{rs,n} = (2n)^{-1}tr\left[\left(A_{r,n} + A_{r,n}'\right)\mathbf{\Sigma}_n\left(A_{s,n} + A_{s,n}'\right)\mathbf{\Sigma}_n\right] + n^{-1}a_{r,n}'\mathbf{\Sigma}_n a_{s,n} \tag{A.5}$$

with

$$\begin{aligned}
a_{r,n} &= H_n P \alpha_{r,n}, \\
\alpha_{r,n} &= -n^{-1}E\left[Z_n'(I_n - \rho M_n')(A_{r,n} + A_{r,n}')(I_n - \rho M_n)u_n\right], \\
P &= Q_{HH}^{-1}Q_{HZ*}[Q_{HZ*}'Q_{HH}^{-1}Q_{HZ*}]^{-1},
\end{aligned} \tag{A.6}$$

and $\mathbf{\Sigma}_n = diag(\sigma_{i,n}^2)$*, where* $\sigma_{i,n}^2 = E\varepsilon_{i,n}^2$*. Furthermore, let*

$$\Psi_{\circ,n} = \begin{bmatrix} \Psi_{\delta\delta,n} & \Psi_{\delta\rho,n} \\ \Psi_{\delta\rho,n}' & \Psi_n \end{bmatrix} \tag{A.7}$$

with

$$\Psi_{\delta\delta,n} = n^{-1}H_n'\Sigma_n H_n,$$
$$\Psi_{\delta\rho,n} = n^{-1}H_n'\Sigma_n\left[a_{1,n}, a_{2,n}\right].$$

Then Ψ_n *and* $\Psi_{\circ,n}$ *are nonsingular for all* n *sufficiently large and* $\lim_{n\to\infty}\Psi_n = \Psi$ *and* $\lim_{n\to\infty}\Psi_{\circ,n} = \Psi_\circ$ *are finite and nonsingular.*

B Appendix: Estimators for Ψ and Ω

For simplicity of notation we drop subscript n in the following.

B.1 Definition of G and g

Let $\widetilde{\delta}$ be some estimator for δ, let $\widetilde{u} = y - Z\widetilde{\delta}$ be the corresponding estimated residuals, and let $\widetilde{\overline{u}}_n = M_n\widetilde{u}_n$, $\widetilde{\overline{\overline{u}}}_n = M_n^2\widetilde{u}_n$. Then, $G(\widetilde{\delta}) = \left[g_{rs}(\widetilde{\delta})\right]_{r,s=1,2}$ and $g(\widetilde{\delta}) = \left[g_1(\widetilde{\delta}), g_2(\widetilde{\delta})\right]'$ are obtained from the expressions for the elements of $\Gamma = [\gamma_{rs}]_{r,s=1,2}$ and $\gamma = [\gamma_1, \gamma_2]'$ in (A.1) by suppressing the expectations operator, and replacing the disturbance vectors u, $\overline{u}$, and $\overline{\overline{u}}$ by their predictors $\widetilde{u}$, $\widetilde{\overline{u}}$, and $\widetilde{\overline{\overline{u}}}$.

B.2 Definition of $\widetilde{\Psi}$ and $\widetilde{\Omega}$

Let $\widetilde{u} = y - Z\widetilde{\delta}$ denote the 2SLS residuals, and let $\bar{\rho}$ be some estimator for ρ. Then $\widetilde{\Psi} = \left[\widetilde{\psi}_{rs}\right]_{r,s=1,2}$ with

$$\widetilde{\psi}_{rs} = (2n)^{-1}tr\left[(A_r + A_r')\,\widetilde{\mathbf{\Sigma}}\,(A_s + A_s')\,\widetilde{\mathbf{\Sigma}}\right] + n^{-1}\widetilde{a}_r'\widetilde{\mathbf{\Sigma}}\widetilde{a}_s, \tag{B.1}$$

where

$\widetilde{\mathbf{\Sigma}} = diag_{i=1,\ldots,n}(\widetilde{\varepsilon}_i^2)$ $\qquad$ $\widetilde{\varepsilon} = (I - \bar{\rho}M)\,\widetilde{u}$

$\widetilde{a}_r = (I - \bar{\rho}M')^{-1}\,H\widetilde{P}\widetilde{\alpha}_r$ $\qquad$ $\widetilde{\alpha}_r = -n^{-1}\left[Z'\,(I - \bar{\rho}M')\,(A_r + A_r')\,(I - \bar{\rho}M)\,\widetilde{u}\right]$

Changed $\widetilde{\rho}$ to $\bar{\rho}$ in definition of $\widetilde{a}_r$

$$\widetilde{P} = (n^{-1}H'H)^{-1}(n^{-1}H'Z)\left[(n^{-1}Z'H)(n^{-1}H'H)^{-1}(n^{-1}H'Z)\right]^{-1}.$$

Let $\widetilde{\Gamma} = G(\widetilde{\delta})$, where $G(.)$ is defined in Appendix B.1, and let $\widetilde{J} = \widetilde{\Gamma}[1, 2\bar{\rho}]'$, then the estimator $\widetilde{\Omega}$ is given by

$$\widetilde{\Omega} = \begin{bmatrix} \widetilde{P}' & 0 \\ 0 & (\widetilde{J}'\widetilde{\Psi}^{-1}\widetilde{J})^{-1}\widetilde{J}'\widetilde{\Psi}^{-1} \end{bmatrix} \widetilde{\Psi}_\circ \begin{bmatrix} \widetilde{P} & 0 \\ 0 & \widetilde{\Psi}^{-1}\widetilde{J}(\widetilde{J}'\widetilde{\Psi}^{-1}\widetilde{J})^{-1} \end{bmatrix}, \tag{B.2}$$

$$\widetilde{\Psi}_\circ = \begin{bmatrix} \widetilde{\Psi}_{\delta\delta} & \widetilde{\Psi}_{\delta\rho} \\ \widetilde{\Psi}_{\delta\rho}' & \widetilde{\Psi} \end{bmatrix}, \quad \widetilde{\Psi}_{\delta\delta} = n^{-1}H'\,(I - \bar{\rho}M)\,\widetilde{\mathbf{\Sigma}}\,(I - \bar{\rho}M')\,H,$$

$$\widetilde{\Psi}_{\delta\rho} = n^{-1}H'\,(I - \bar{\rho}M)\,\widetilde{\mathbf{\Sigma}}\,[\widetilde{a}_1, \widetilde{a}_2].$$

We will also write $\widetilde{\Psi}(\bar{\rho})$, $\widetilde{\Omega}(\bar{\rho})$, and $\widetilde{\Psi}_{\circ}(\bar{\rho})$ for $\widetilde{\Psi}$, $\widetilde{\Omega}$, and $\widetilde{\Psi}_{\circ}$ to explicitly denote the dependence on $\bar{\rho}$.

B.3 Definition of $\widehat{\Psi}$ and $\widehat{\Omega}$

Let $\widehat{u} = y - Z\widehat{\delta}$ denote the GS2SLS residuals, and let $\bar{\rho}$ be some estimator for ρ. Then $\widehat{\Psi} = \left[\widehat{\psi}_{rs}\right]_{r,s=1,2}$ with

$$\widehat{\psi}_{rs} = (2n)^{-1} tr \left[(A_r + A_r') \widehat{\mathbf{\Sigma}} (A_s + A_s') \widehat{\mathbf{\Sigma}} \right] + n^{-1} \widehat{a}_r' \widehat{\mathbf{\Sigma}} \widehat{a}_s,$$

where

$$\begin{array}{ll} \widehat{\mathbf{\Sigma}} = diag_{i=1,\ldots,n}(\widehat{\varepsilon}_i^2) & \widehat{\varepsilon} = (I - \bar{\rho}M)\,\widehat{u} \\ \widehat{a}_r = H\widehat{P}\widehat{\alpha}_r & \widehat{\alpha}_r = -n^{-1}\left[Z'\,(I - \bar{\rho}M')\,(A_r + A_r')\,(I - \bar{\rho}M)\,\widehat{u}\right] \end{array}$$

and

$$\widehat{P} = (n^{-1}H'H)^{-1}(n^{-1}H'Z_*(\bar{\rho})) \left[(n^{-1}Z_*'(\bar{\rho})H)(n^{-1}H'H)^{-1}(n^{-1}H'Z_*(\bar{\rho})) \right]^{-1},$$

and $Z_*(\bar{\rho}) = Z - \bar{\rho}MZ$. Let $\widehat{\Gamma} = G(\widehat{\delta})$, where $G(.)$ is defined in Appendix B.1, then the estimator $\widehat{\Omega}$ is given by

$$\widehat{\Omega} = \begin{bmatrix} \widehat{P}' & 0 \\ 0 & (\widehat{J}'\widehat{\Psi}^{-1}\widehat{J})^{-1}\widehat{J}'\widehat{\Psi}^{-1} \end{bmatrix} \widehat{\Psi}_{\circ} \begin{bmatrix} \widehat{P} & 0 \\ 0 & \widehat{\Psi}^{-1}\widehat{J}(\widehat{J}'\widehat{\Psi}^{-1}\widehat{J})^{-1} \end{bmatrix},$$

$$\widehat{\Psi}_{\circ} = \begin{bmatrix} \widehat{\Psi}_{\delta\delta} & \widehat{\Psi}_{\delta\rho} \\ \widehat{\Psi}_{\delta\rho}' & \widehat{\Psi} \end{bmatrix}, \qquad \widehat{\Psi}_{\delta\delta} = n^{-1}H'\widehat{\mathbf{\Sigma}}H, \qquad \widehat{\Psi}_{\delta\rho} = n^{-1}H'\widehat{\mathbf{\Sigma}}\left[\widehat{a}_1, \widehat{a}_2\right].$$

We will also write $\widehat{\Psi}(\bar{\rho})$, $\widehat{\Omega}(\bar{\rho})$, and $\widehat{\Psi}_{\circ}(\bar{\rho})$ for $\widehat{\Psi}$, $\widehat{\Omega}$, and $\widehat{\Psi}_{\circ}$ to explicitly denote the dependence on $\bar{\rho}$.

C Appendix: Proofs

Proof of Theorem 1: Consider the 2SLS residuals $\widetilde{u}_n = y_n - Z_n\widetilde{\delta}_n$. Then clearly $\widetilde{u}_n - u_n = D_n\Delta_n$ with $D_n = -Z_n$ and $\Delta_n = \widetilde{\delta}_n - \delta$. Next observe that under our Assumptions 1-3 and 4-6, Assumptions 1-3 and 8-10 in Kelejian and Prucha (2007) clearly hold. Since β does not vary with n it now follows directly from Lemma 3 in Kelejian and Prucha (2007) that the fourth moments of the elements of $D_n = -Z_n$ are uniformly bounded, that Assumption 6 in Kelejian and Prucha (2007) holds, and:
(a) $n^{1/2}(\widetilde{\delta}_n - \delta) = n^{-1/2}T_n'\varepsilon_n + o_p(1)$ with $T_n = F_nP$ and where

$$\begin{aligned} P &= Q_{HH}^{-1}Q_{HZ}[Q_{HZ}'Q_{HH}^{-1}Q_{HZ}]^{-1}, \\ F_n &= (I_n - \rho M_n')^{-1} H_n. \end{aligned}$$

(b) $n^{-1/2}T_n'\varepsilon_n = O_p(1)$.
(c) $P = O_p(1)$ and $\widetilde{P}_n - P = o_p(1)$ for

$$\widetilde{P}_n = (n^{-1}H_n'H_n)^{-1}(n^{-1}H_n'Z_n)\times [(n^{-1}Z_n'H_n)(n^{-1}H_n'H_n)^{-1}(n^{-1}H_n'Z_n)]^{-1}.$$

From this we see that also Assumptions 4 and 7 in Kelejian and Prucha (2007) are satisfied.

By Assumption A.1 we have Γ_n is nonsingular for all n sufficiently large and $\lim_{n\to\infty}\Gamma_n = \Gamma$ is finite and nonsingular. Consequently the $\lambda_{\min}(\Gamma_n'\Gamma_n) \geq const > 0$ for n sufficiently large and thus also Assumption 5(a) in Kelejian and Prucha (2007) holds. Furthermore observe that for $\widetilde{\Upsilon}_n = \Upsilon_n = I_2$ also Assumption 5(b),(c) in Kelejian and Prucha (2007) are trivially satisfied.

By Assumption A.2 Ψ_n and $\Psi_{\circ,n}$ are nonsingular for all n sufficiently large and $\lim_{n\to\infty}\Psi_n = \Psi$ and $\lim_{n\to\infty}\Psi_{\circ,n} = \Psi_\circ$ are finite and nonsingular, and thus the smallest [largest] eigenvalues of Ψ_n, Ψ_n^{-1}, $\Psi_{\circ,n}$ and $\Psi_{\circ,n}^{-1}$ are bounded away from zero [bounded from above] for sufficiently large n.

It now follows immediately from Theorems 1-3 in Kelejian and Prucha (2007) that the initial GMM estimator for ρ, $\check{\rho}_n$, is $n^{1/2}$- consistent and that $\operatorname{plim}_{n\to\infty}\widetilde{\Psi}_n(\check{\rho}_n) = \Psi$ and $\operatorname{plim}_{n\to\infty}\widetilde{\Psi}_n^{-1}(\check{\rho}_n) = \Psi^{-1}$.

The estimator $\widetilde{\rho}_n$ is a special case of the GMM estimators for ρ defined in equation (9) in Kelejian and Prucha with $\widetilde{\Upsilon}_n = \widetilde{\Psi}_n^{-1}(\check{\rho}_n)$ and $\Upsilon_n = \Psi_n^{-1}$. Recalling that the smallest [largest] eigenvalues of Ψ_n^{-1} bounded away from zero [bounded from above] for sufficiently large n we see from Theorem 3 in Kelejian and Prucha (2007) that also in this case Assumption 5(b),(c) in that paper are satisfied. All other assumptions maintained by Theorems 1-3 in Kelejian and Prucha (2007) have already been verified, which establishes $n^{1/2}$-consistency of $\widetilde{\rho}_n$ and its asymptotic efficiency.

The joint limiting distribution of $n^{1/2}(\widetilde{\delta}_n - \delta)$ and $n^{1/2}(\widetilde{\rho}_n - \rho)$ given by the theorem now follows immediately from Theorem 4 in Kelejian and Prucha (2007). ■

Proof of Theorem 2:

Consider the GS2SLS residuals $\widehat{u}_n = y_n - Z_n\widehat{\delta}_n$. Then clearly $\widehat{u}_n - u_n = D_n\Delta_n$ with $D_n = -Z_n$ and $\Delta_n = \widehat{\delta}_n - \delta$. As in the proof of Theorem 1, observe that under our Assumptions 1-3 and 4-6, Assumptions 1-3 and 8-10 in Kelejian and Prucha (2007) clearly hold. Also recall that in the proof of Theorem 1 we have established that the fourth moments of the elements of $D_n = -Z_n$ are uniformly bounded, and that Assumption 6 in Kelejian and Prucha (2007) holds. It now follows from Lemma 4 in Kelejian and Prucha (2007) that[6]
(a) $n^{1/2}(\widehat{\delta}_n(\widetilde{\rho}_n) - \delta) = n^{-1/2}T_n'\varepsilon_n + o_p(1)$ with $T_n = F_n\ P$ and where

$$P = Q_{HH}^{-1}Q_{HZ^*}(\rho)[Q_{HZ^*}'(\rho)Q_{HH}^{-1}Q_{HZ^*}(\rho)]^{-1},$$
$$F_n = H_n.$$

[6]The argument, and hence the Theorem, also holds if $\widetilde{\rho}_n$ is replaced by any other $n^{1/2}$-consistent estimator for ρ.

(b) $n^{-1/2}T_n'\varepsilon_n = O_p(1)$.
(c) $P = O_p(1)$ and $\widetilde{P}_n - P = o_p(1)$ for

$$\widetilde{P}_n = (n^{-1}H_n'H_n)^{-1}(n^{-1}H_n'Z_n^*(\widetilde{\rho}_n))\times$$
$$[(n^{-1}Z_n^*(\widetilde{\rho}_n)'H_n)(n^{-1}H_n'H_n)^{-1}(n^{-1}H_n'Z_n(\widetilde{\rho}_n))]^{-1}.$$

From this we see that Assumptions 4 and 7 in Kelejian and Prucha (2007) are also satisfied.

By Assumption A.1 we have Γ_n is nonsingular for all n sufficiently large and $\lim_{n\to\infty}\Gamma_n = \Gamma$ is finite and nonsingular. Consequently the $\lambda_{\min}(\Gamma_n'\Gamma_n) \geq$ *const* > 0 for n sufficiently large and thus Assumption 5(a) in Kelejian and Prucha (2007) also holds.

By Assumption A.3 Ψ_n and $\Psi_{\circ,n}$ are nonsingular for all n sufficiently large and $\lim_{n\to\infty}\Psi_n = \Psi$ and $\lim_{n\to\infty}\Psi_{\circ,n} = \Psi_\circ$ are finite and nonsingular, and thus the smallest [largest] eigenvalues of Ψ_n, Ψ_n^{-1}, $\Psi_{\circ,n}$ and $\Psi_{\circ,n}^{-1}$ are bounded away from zero [bounded from above] for sufficiently large n.

The estimator $\widehat{\rho}_n$ is a special case of the GMM estimators for ρ defined in equation (9) in Kelejian and Prucha with $\widetilde{\Upsilon}_n = \widehat{\Psi}_n^{-1}(\check{\rho}_n)$ and $\Upsilon_n = \Psi_n^{-1}$. As remarked above, the smallest [largest] eigenvalues of Ψ_n^{-1} bounded away from zero [bounded from above] for sufficiently large n, and thus we see from Theorem 3 in Kelejian and Prucha (2007) that in this case Assumption 5(b),(c) in that paper are also satisfied. All other assumptions maintained by Theorems 1-3 in Kelejian and Prucha (2007) have already been verified, which establishes $n^{1/2}$-consistency of $\widehat{\rho}_n$ and its asymptotic efficiency.

The joint limiting distribution of $n^{1/2}(\widehat{\delta}_n - \delta)$ and $n^{1/2}(\widehat{\rho}_n - \rho)$ given by the theorem now follows immediately from Theorem 4 in Kelejian and Prucha (2007). ■

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