

Table 1

Sample Size N=128									
Model: $y_t = \phi y_{t-1} + \epsilon_t - \theta \epsilon_{t-1}, \epsilon_t = u_t \sqrt{h_t}, h_t = \gamma_0 + \gamma_1 \epsilon_{t-1}^2$									
Estimator	ϕ	γ_1	Empirical Quantiles			Mean	MAE	Var	Rel Eff
			.05	.5	.95				
ϕ_{OLS}	-0.9	0.5	-0.955	-0.901	-0.799	-0.89156	0.03770	0.00258	1.00000
$\tilde{\phi}(\hat{a})$			-0.946	-0.893	-0.790	-0.88376	0.03880	0.00272	1.05356
ϕ_{OLS}	-0.6	0.5	-0.757	-0.600	-0.407	-0.59317	0.08402	0.01140	1.00000
$\tilde{\phi}(\hat{a})$			-0.744	-0.590	-0.395	-0.58490	0.08519	0.01142	1.00172
ϕ_{OLS}	-0.3	0.5	-0.565	-0.308	0.036	-0.29356	0.14023	0.03307	1.00000
$\tilde{\phi}(\hat{a})$			-0.555	-0.297	0.056	-0.28097	0.14158	0.03473	1.05026
ϕ_{OLS}	-0.0	0.5	-0.468	-0.042	0.445	-0.03444	0.23502	0.08461	1.00000
$\tilde{\phi}(\hat{a})$			-0.475	0.012	0.602	0.03480	0.25536	0.10495	1.24032
ϕ_{OLS}	0.3	0.5	-0.807	0.020	0.805	0.03337	0.44887	0.22780	1.00000
$\tilde{\phi}(\hat{a})$			-0.861	0.095	0.823	0.06568	0.47611	0.28708	1.26022
ϕ_{OLS}	0.6	0.5	-0.495	0.463	0.899	0.37276	0.34112	0.17357	1.00000
$\tilde{\phi}(\hat{a})$			-0.646	0.517	0.929	0.37507	0.37173	0.22390	1.28994
ϕ_{OLS}	0.9	0.5	0.758	0.905	0.966	0.88964	0.04730	0.00515	1.00000
$\tilde{\phi}(\hat{a})$			0.755	0.895	0.957	0.87900	0.04868	0.00848	1.64731

Simulations are based on 1000 replications. For all simulations $\theta = .4$ and $\gamma_0 = .1$. The columns .05, .5 and .95 are the 5, 50 and 95 percent quantiles of the empirical distribution of the 1000 parameter estimates centered by $(\hat{\beta} - \beta_0)$. The columns Mean, MAE and Var are the mean, mean absolute error and variance of $\hat{\theta}$. The column Rel Eff is $\text{Var}(\phi_{IV}) / \text{Var}(\phi_{OLS})$.

Table 2

Sample Size N=1024									
Model: $y_t = \phi y_{t-1} + \epsilon_t - \theta \epsilon_{t-1}, \epsilon_t = u_t \sqrt{h_t}, h_t = \gamma_0 + \gamma_1 \epsilon_{t-1}^2$									
Estimator	ϕ	γ_1	Empirical Quantiles			Mean	MAE	Var	Rel Eff
			.05	.5	.95				
ϕ_{OLS}	-0.9	0.5	-0.923	-0.899	-0.867	-0.89778	0.01362	0.00029	1.00000
$\tilde{\phi}(\hat{a})$			-0.921	-0.899	-0.869	-0.89722	0.01311	0.00027	0.90732
ϕ_{OLS}	-0.6	0.5	-0.667	-0.599	-0.532	-0.59932	0.03314	0.00180	1.00000
$\tilde{\phi}(\hat{a})$			-0.664	-0.598	-0.533	-0.59801	0.03215	0.00167	0.92366
ϕ_{OLS}	-0.3	0.5	-0.411	-0.299	-0.180	-0.29801	0.05513	0.00504	1.00000
$\tilde{\phi}(\hat{a})$			-0.410	-0.299	-0.179	-0.29678	0.05476	0.00492	0.97570
ϕ_{OLS}	-0.0	0.5	-0.183	0.003	0.187	0.00247	0.08668	0.01296	1.00000
$\tilde{\phi}(\hat{a})$			-0.179	0.000	0.187	0.00134	0.08643	0.01390	1.07293
ϕ_{OLS}	0.3	0.5	-0.443	0.226	0.754	0.19449	0.28975	0.12765	1.00000
$\tilde{\phi}(\hat{a})$			-0.471	0.263	0.861	0.23880	0.31438	0.15716	1.23116
ϕ_{OLS}	0.6	0.5	0.254	0.591	0.752	0.56296	0.11514	0.02767	1.00000
$\tilde{\phi}(\hat{a})$			0.332	0.594	0.783	0.58142	0.11118	0.02588	0.93553
ϕ_{OLS}	0.9	0.5	0.863	0.900	0.927	0.89783	0.01611	0.00044	1.00000
$\tilde{\phi}(\hat{a})$			0.863	0.900	0.927	0.89716	0.01585	0.00040	0.91736

Table 3

Sample Size N=128									
Model: $y_t = \phi y_{t-1} + \epsilon_t - \theta \epsilon_{t-1}, \epsilon_t = u_t \sqrt{h_t}, h_t = \gamma_0 + \gamma_1 \epsilon_{t-1}^2$									
Estimator	ϕ	γ_1	Empirical Quantiles			Mean	MAE	Var	Rel Eff
			.05	.5	.95				
θ_{OLS}	-0.9	0.5	0.206	0.420	0.612	0.41076	1.31076	0.01568	1.00000
$\tilde{\theta}(\hat{a})$			0.168	0.389	0.588	0.38325	1.28325	0.01776	1.13274
θ_{OLS}	-0.6	0.5	0.162	0.429	0.608	0.41149	1.01149	0.01890	1.00000
$\tilde{\theta}(\hat{a})$			0.149	0.415	0.596	0.40033	1.00033	0.01921	1.01608
θ_{OLS}	-0.3	0.5	0.070	0.424	0.664	0.40273	0.70329	0.03258	1.00000
$\tilde{\theta}(\hat{a})$			0.111	0.412	0.685	0.40833	0.71080	0.03552	1.09029
θ_{OLS}	-0.0	0.5	-0.095	0.396	0.798	0.36121	0.39530	0.08695	1.00000
$\tilde{\theta}(\hat{a})$			-0.115	0.455	0.944	0.43205	0.47440	0.10065	1.15753
θ_{OLS}	0.3	0.5	-0.796	0.076	0.914	0.13512	0.42971	0.25224	1.00000
$\tilde{\theta}(\hat{a})$			-0.888	0.257	0.947	0.16763	0.47977	0.32497	1.28835
θ_{OLS}	0.6	0.5	-0.617	0.192	0.782	0.16428	0.47862	0.17372	1.00000
$\tilde{\theta}(\hat{a})$			-0.731	0.228	0.821	0.15652	0.49950	0.22096	1.27192
θ_{OLS}	0.9	0.5	0.116	0.404	0.627	0.39147	0.50853	0.02441	1.00000
$\tilde{\theta}(\hat{a})$			0.111	0.381	0.594	0.37073	0.52927	0.02796	1.14545

Table 4

Sample Size N=1024									
Model: $y_t = \phi y_{t-1} + \epsilon_t - \theta \epsilon_{t-1}, \epsilon_t = u_t \sqrt{h_t}, h_t = \gamma_0 + \gamma_1 \epsilon_{t-1}^2$									
Estimator	ϕ	γ_1	Empirical Quantiles			Mean	MAE	Var	Rel Eff
			.05	.5	.95				
θ_{OLS}	-0.9	0.5	0.314	0.403	0.478	0.40025	1.30025	0.00276	1.00000
$\tilde{\theta}(\hat{a})$			0.309	0.398	0.472	0.39546	1.29546	0.00266	0.96414
θ_{OLS}	-0.6	0.5	0.307	0.406	0.486	0.40128	1.00128	0.00354	1.00000
$\tilde{\theta}(\hat{a})$			0.308	0.404	0.481	0.40027	1.00027	0.00340	0.96041
θ_{OLS}	-0.3	0.5	0.295	0.408	0.506	0.40337	0.70337	0.00428	1.00000
$\tilde{\theta}(\hat{a})$			0.296	0.406	0.504	0.40288	0.70288	0.00433	1.01298
θ_{OLS}	-0.0	0.5	0.229	0.410	0.547	0.40313	0.40319	0.01007	1.00000
$\tilde{\theta}(\hat{a})$			0.230	0.408	0.550	0.40259	0.40340	0.01098	1.09060
θ_{OLS}	0.3	0.5	-0.358	0.353	0.806	0.29456	0.28015	0.12588	1.00000
$\tilde{\theta}(\hat{a})$			-0.399	0.394	0.931	0.34175	0.31433	0.15653	1.24343
θ_{OLS}	0.6	0.5	0.000	0.382	0.612	0.36104	0.24552	0.03222	1.00000
$\tilde{\theta}(\hat{a})$			0.097	0.386	0.650	0.37908	0.24110	0.03434	1.06582
θ_{OLS}	0.9	0.5	0.299	0.401	0.491	0.39782	0.50218	0.00388	1.00000
$\tilde{\theta}(\hat{a})$			0.300	0.401	0.487	0.39535	0.50465	0.00554	1.42871

Table 5

Sample Size N=128									
Model: $y_t = \phi y_{t-1} + \epsilon_t - \theta \epsilon_{t-1}, \epsilon_t = u_t \sqrt{h_t}, h_t = \gamma_0 + \gamma_1 \epsilon_{t-1}^2$									
Estimator	ϕ	γ_1	Empirical Quantiles			Mean	MAE	Var	Rel Eff
			.05	.5	.95				
ϕ_{OLS}	-0.9	0.5	-0.955	-0.901	-0.799	-0.89156	0.03770	0.00258	1.00000
$\tilde{\phi}(\hat{a}(5))$			-0.950	-0.893	-0.795	-0.88493	0.03807	0.00254	0.98611
$\tilde{\phi}(\hat{a}(10))$			-0.956	-0.897	-0.795	-0.88809	0.03812	0.00258	0.99808
$\tilde{\phi}(\hat{a}(\sqrt{n}))$			-0.957	-0.897	-0.797	-0.88881	0.03862	0.00262	1.01336
$\tilde{\phi}(\hat{a}(n^{2/5}))$			-0.950	-0.894	-0.796	-0.88573	0.03781	0.00251	0.97072
ϕ_{OLS}	-0.6	0.5	-0.757	-0.600	-0.407	-0.59317	0.08402	0.01140	1.00000
$\tilde{\phi}(\hat{a}(5))$			-0.757	-0.598	-0.397	-0.59105	0.08729	0.01217	1.06787
$\tilde{\phi}(\hat{a}(10))$			-0.766	-0.600	-0.386	-0.59241	0.09133	0.01383	1.21323
$\tilde{\phi}(\hat{a}(\sqrt{n}))$			-0.767	-0.601	-0.387	-0.59259	0.09206	0.01389	1.21860
$\tilde{\phi}(\hat{a}(n^{2/5}))$			-0.761	-0.597	-0.393	-0.59130	0.08837	0.01310	1.14946
ϕ_{OLS}	-0.3	0.5	-0.565	-0.308	0.036	-0.29356	0.14023	0.03307	1.00000
$\tilde{\phi}(\hat{a}(5))$			-0.576	-0.306	0.047	-0.28672	0.14963	0.03743	1.13189
$\tilde{\phi}(\hat{a}(10))$			-0.584	-0.311	0.068	-0.29041	0.15538	0.03961	1.19781
$\tilde{\phi}(\hat{a}(\sqrt{n}))$			-0.581	-0.309	0.069	-0.29086	0.15736	0.04033	1.21962
$\tilde{\phi}(\hat{a}(n^{2/5}))$			-0.583	-0.305	0.042	-0.28847	0.15090	0.03755	1.13543
ϕ_{OLS}	-0.0	0.5	-0.468	-0.042	0.445	-0.03444	0.23502	0.08461	1.00000
$\tilde{\phi}(\hat{a}(5))$			-0.507	0.023	0.627	0.03491	0.26556	0.11351	1.34154
$\tilde{\phi}(\hat{a}(10))$			-0.511	0.020	0.634	0.02537	0.26963	0.11692	1.38186
$\tilde{\phi}(\hat{a}(\sqrt{n}))$			-0.519	0.008	0.645	0.02330	0.27087	0.11824	1.39743
$\tilde{\phi}(\hat{a}(n^{2/5}))$			-0.499	0.022	0.630	0.03151	0.26577	0.11299	1.33540
ϕ_{OLS}	0.3	0.5	-0.807	0.020	0.805	0.03337	0.44887	0.22780	1.00000
$\tilde{\phi}(\hat{a}(5))$			-0.850	0.106	0.860	0.07318	0.47295	0.28543	1.25297
$\tilde{\phi}(\hat{a}(10))$			-0.874	0.106	0.880	0.06449	0.48434	0.30121	1.32225
$\tilde{\phi}(\hat{a}(\sqrt{n}))$			-0.885	0.098	0.881	0.06644	0.49138	0.30908	1.35682
$\tilde{\phi}(\hat{a}(n^{2/5}))$			-0.871	0.098	0.876	0.07370	0.48030	0.29423	1.29160
ϕ_{OLS}	0.6	0.5	-0.495	0.463	0.899	0.37276	0.34112	0.17357	1.00000
$\tilde{\phi}(\hat{a}(5))$			-0.650	0.521	0.932	0.39153	0.36045	0.21753	1.25326
$\tilde{\phi}(\hat{a}(10))$			-0.660	0.525	0.935	0.38953	0.36243	0.22062	1.27107
$\tilde{\phi}(\hat{a}(\sqrt{n}))$			-0.631	0.522	0.951	0.39077	0.36288	0.22069	1.27147
$\tilde{\phi}(\hat{a}(n^{2/5}))$			-0.589	0.512	0.936	0.39196	0.35895	0.21481	1.23760
ϕ_{OLS}	0.9	0.5	0.758	0.905	0.966	0.88964	0.04730	0.00515	1.00000
$\tilde{\phi}(\hat{a}(5))$			0.763	0.897	0.960	0.88349	0.04697	0.00460	0.89297
$\tilde{\phi}(\hat{a}(10))$			0.745	0.899	0.963	0.88455	0.04682	0.00522	1.01486
$\tilde{\phi}(\hat{a}(\sqrt{n}))$			0.751	0.901	0.965	0.88579	0.04688	0.00519	1.00907
$\tilde{\phi}(\hat{a}(n^{2/5}))$			0.753	0.898	0.959	0.88355	0.04694	0.00456	0.88544

Table 6

Sample Size N=1024									
Model: $y_t = \phi y_{t-1} + \epsilon_t - \theta \epsilon_{t-1}, \epsilon_t = u_t \sqrt{h_t}, h_t = \gamma_0 + \gamma_1 \epsilon_{t-1}^2$									
Estimator	ϕ	γ_1	Empirical Quantiles			Mean	MAE	Var	Rel Eff
			.05	.5	.95				
ϕ_{OLS}	-0.9	0.5	-0.923	-0.899	-0.867	-0.89778	0.01362	0.00029	1.00000
$\tilde{\phi}(\hat{a}(5))$			-0.921	-0.899	-0.868	-0.89731	0.01303	0.00026	0.89462
$\tilde{\phi}(\hat{a}(10))$			-0.921	-0.900	-0.869	-0.89784	0.01283	0.00026	0.87525
$\tilde{\phi}(\hat{a}(\sqrt{n}))$			-0.923	-0.900	-0.869	-0.89855	0.01316	0.00028	0.93718
$\tilde{\phi}(\hat{a}(n^{2/5}))$			-0.922	-0.900	-0.869	-0.89827	0.01290	0.00026	0.89411
ϕ_{OLS}	-0.6	0.5	-0.667	-0.599	-0.532	-0.59932	0.03314	0.00180	1.00000
$\tilde{\phi}(\hat{a}(5))$			-0.666	-0.599	-0.533	-0.59939	0.03199	0.00161	0.89495
$\tilde{\phi}(\hat{a}(10))$			-0.664	-0.600	-0.534	-0.59946	0.03202	0.00163	0.90252
$\tilde{\phi}(\hat{a}(\sqrt{n}))$			-0.665	-0.600	-0.534	-0.59960	0.03256	0.00169	0.93683
$\tilde{\phi}(\hat{a}(n^{2/5}))$			-0.662	-0.601	-0.533	-0.59977	0.03203	0.00166	0.91796
ϕ_{OLS}	-0.3	0.5	-0.411	-0.299	-0.180	-0.29801	0.05513	0.00504	1.00000
$\tilde{\phi}(\hat{a}(5))$			-0.414	-0.300	-0.184	-0.29826	0.05489	0.00493	0.97761
$\tilde{\phi}(\hat{a}(10))$			-0.414	-0.302	-0.185	-0.29839	0.05417	0.00482	0.95500
$\tilde{\phi}(\hat{a}(\sqrt{n}))$			-0.417	-0.301	-0.186	-0.29928	0.05534	0.00494	0.97891
$\tilde{\phi}(\hat{a}(n^{2/5}))$			-0.411	-0.299	-0.187	-0.29761	0.05455	0.00488	0.96805
ϕ_{OLS}	-0.0	0.5	-0.183	0.003	0.187	0.00247	0.08668	0.01296	1.00000
$\tilde{\phi}(\hat{a}(5))$			-0.164	0.001	0.182	0.00259	0.08408	0.01262	0.97421
$\tilde{\phi}(\hat{a}(10))$			-0.167	0.001	0.181	0.00153	0.08382	0.01280	0.98762
$\tilde{\phi}(\hat{a}(\sqrt{n}))$			-0.180	-0.000	0.180	0.00099	0.08644	0.01305	1.00752
$\tilde{\phi}(\hat{a}(n^{2/5}))$			-0.176	-0.000	0.176	-0.00048	0.08448	0.01367	1.05504
ϕ_{OLS}	0.3	0.5	-0.443	0.226	0.754	0.19449	0.28975	0.12765	1.00000
$\tilde{\phi}(\hat{a}(5))$			-0.472	0.262	0.823	0.23169	0.30489	0.14780	1.15781
$\tilde{\phi}(\hat{a}(10))$			-0.470	0.263	0.813	0.22714	0.30534	0.14724	1.15341
$\tilde{\phi}(\hat{a}(\sqrt{n}))$			-0.475	0.275	0.807	0.22659	0.30403	0.14775	1.15744
$\tilde{\phi}(\hat{a}(n^{2/5}))$			-0.476	0.270	0.797	0.22892	0.30289	0.14735	1.15432
ϕ_{OLS}	0.6	0.5	0.254	0.591	0.752	0.56296	0.11514	0.02767	1.00000
$\tilde{\phi}(\hat{a}(5))$			0.330	0.592	0.776	0.58073	0.10771	0.02356	0.85146
$\tilde{\phi}(\hat{a}(10))$			0.340	0.598	0.776	0.58329	0.10682	0.02307	0.83399
$\tilde{\phi}(\hat{a}(\sqrt{n}))$			0.342	0.600	0.793	0.58453	0.10530	0.02224	0.80396
$\tilde{\phi}(\hat{a}(n^{2/5}))$			0.342	0.598	0.783	0.58221	0.10664	0.02353	0.85050
ϕ_{OLS}	0.9	0.5	0.863	0.900	0.927	0.89783	0.01611	0.00044	1.00000
$\tilde{\phi}(\hat{a}(5))$			0.864	0.900	0.927	0.89747	0.01581	0.00040	0.90842
$\tilde{\phi}(\hat{a}(10))$			0.864	0.900	0.927	0.89727	0.01575	0.00039	0.89651
$\tilde{\phi}(\hat{a}(\sqrt{n}))$			0.864	0.900	0.928	0.89819	0.01590	0.00040	0.91352
$\tilde{\phi}(\hat{a}(n^{2/5}))$			0.864	0.900	0.927	0.89788	0.01555	0.00039	0.88298

Table 7

Sample Size N=128									
Model: $y_t = \phi y_{t-1} + \epsilon_t - \theta \epsilon_{t-1}, \epsilon_t = u_t \sqrt{h_t}, h_t = \gamma_0 + \gamma_1 \epsilon_{t-1}^2$									
Estimator	ϕ	γ_1	Empirical Quantiles			Mean	MAE	Var	Rel Eff
			.05	.5	.95				
θ_{OLS}	-0.9	0.5	0.206	0.420	0.612	0.41076	1.31076	0.01568	1.00000
$\tilde{\theta}(\hat{a}(5))$			0.175	0.386	0.573	0.37861	1.27861	0.01593	1.01593
$\tilde{\theta}(\hat{a}(10))$			0.163	0.384	0.581	0.37748	1.27748	0.01677	1.06957
$\tilde{\theta}(\hat{a}(\sqrt{n}))$			0.157	0.383	0.583	0.37722	1.27722	0.01697	1.08244
$\tilde{\theta}(\hat{a}(n^{2/5}))$			0.167	0.385	0.574	0.37764	1.27764	0.01616	1.03061
θ_{OLS}	-0.6	0.5	0.162	0.429	0.608	0.41149	1.01149	0.01890	1.00000
$\tilde{\theta}(\hat{a}(5))$			0.149	0.412	0.587	0.39467	0.99467	0.01952	1.03255
$\tilde{\theta}(\hat{a}(10))$			0.149	0.413	0.601	0.39469	0.99469	0.02112	1.11753
$\tilde{\theta}(\hat{a}(\sqrt{n}))$			0.141	0.411	0.608	0.39473	0.99473	0.02137	1.13060
$\tilde{\theta}(\hat{a}(n^{2/5}))$			0.143	0.414	0.586	0.39352	0.99352	0.02001	1.05877
θ_{OLS}	-0.3	0.5	0.070	0.424	0.664	0.40273	0.70329	0.03258	1.00000
$\tilde{\theta}(\hat{a}(5))$			0.095	0.411	0.675	0.40155	0.70260	0.03441	1.05616
$\tilde{\theta}(\hat{a}(10))$			0.071	0.414	0.671	0.39839	0.69936	0.03561	1.09296
$\tilde{\theta}(\hat{a}(\sqrt{n}))$			0.074	0.410	0.674	0.39859	0.69957	0.03621	1.11144
$\tilde{\theta}(\hat{a}(n^{2/5}))$			0.084	0.415	0.675	0.40086	0.70180	0.03440	1.05581
θ_{OLS}	-0.0	0.5	-0.095	0.396	0.798	0.36121	0.39530	0.08695	1.00000
$\tilde{\theta}(\hat{a}(5))$			-0.171	0.451	0.914	0.42548	0.47008	0.09929	1.14188
$\tilde{\theta}(\hat{a}(10))$			-0.149	0.456	0.894	0.41882	0.46682	0.10223	1.17573
$\tilde{\theta}(\hat{a}(\sqrt{n}))$			-0.153	0.454	0.891	0.41593	0.46290	0.10179	1.17071
$\tilde{\theta}(\hat{a}(n^{2/5}))$			-0.173	0.450	0.911	0.42252	0.46768	0.09888	1.13720
θ_{OLS}	0.3	0.5	-0.796	0.076	0.914	0.13512	0.42971	0.25224	1.00000
$\tilde{\theta}(\hat{a}(5))$			-0.868	0.241	0.947	0.17698	0.47296	0.31613	1.25330
$\tilde{\theta}(\hat{a}(10))$			-0.884	0.255	0.952	0.16685	0.48248	0.32830	1.30154
$\tilde{\theta}(\hat{a}(\sqrt{n}))$			-0.897	0.244	0.960	0.16911	0.48888	0.33448	1.32605
$\tilde{\theta}(\hat{a}(n^{2/5}))$			-0.877	0.231	0.955	0.17610	0.47858	0.32312	1.28100
θ_{OLS}	0.6	0.5	-0.617	0.192	0.782	0.16428	0.47862	0.17372	1.00000
$\tilde{\theta}(\hat{a}(5))$			-0.742	0.236	0.844	0.17426	0.48674	0.21360	1.22957
$\tilde{\theta}(\hat{a}(10))$			-0.707	0.240	0.835	0.16947	0.49082	0.21558	1.24095
$\tilde{\theta}(\hat{a}(\sqrt{n}))$			-0.688	0.238	0.845	0.17104	0.49123	0.21694	1.24878
$\tilde{\theta}(\hat{a}(n^{2/5}))$			-0.695	0.224	0.844	0.17427	0.48566	0.21134	1.21657
θ_{OLS}	0.9	0.5	0.116	0.404	0.627	0.39147	0.50853	0.02441	1.00000
$\tilde{\theta}(\hat{a}(5))$			0.120	0.380	0.605	0.36972	0.53028	0.02401	0.98333
$\tilde{\theta}(\hat{a}(10))$			0.097	0.379	0.602	0.36524	0.53476	0.02554	1.04625
$\tilde{\theta}(\hat{a}(\sqrt{n}))$			0.099	0.378	0.599	0.36794	0.53206	0.02521	1.03277
$\tilde{\theta}(\hat{a}(n^{2/5}))$			0.116	0.378	0.602	0.36707	0.53293	0.02406	0.98551

Table 8

Sample Size N=1024									
Model: $y_t = \phi y_{t-1} + \epsilon_t - \theta \epsilon_{t-1}, \epsilon_t = u_t \sqrt{h_t}, h_t = \gamma_0 + \gamma_1 \epsilon_{t-1}^2$									
Estimator	ϕ	γ_1	Empirical Quantiles			Mean	MAE	Var	Rel Eff
			.05	.5	.95				
θ_{OLS}	-0.9	0.5	0.314	0.403	0.478	0.40025	1.30025	0.00276	1.00000
$\tilde{\theta}(\hat{a}(5))$			0.311	0.399	0.468	0.39548	1.29548	0.00230	0.83251
$\tilde{\theta}(\hat{a}(10))$			0.310	0.399	0.471	0.39545	1.29545	0.00241	0.87393
$\tilde{\theta}(\hat{a}(\sqrt{n}))$			0.310	0.397	0.472	0.39537	1.29537	0.00247	0.89604
$\tilde{\theta}(\hat{a}(n^{2/5}))$			0.313	0.397	0.471	0.39532	1.29532	0.00244	0.88618
θ_{OLS}	-0.6	0.5	0.307	0.406	0.486	0.40128	1.00128	0.00354	1.00000
$\tilde{\theta}(\hat{a}(5))$			0.307	0.403	0.482	0.39940	0.99940	0.00311	0.87951
$\tilde{\theta}(\hat{a}(10))$			0.309	0.402	0.481	0.39942	0.99942	0.00295	0.83481
$\tilde{\theta}(\hat{a}(\sqrt{n}))$			0.306	0.401	0.481	0.39885	0.99885	0.00310	0.87526
$\tilde{\theta}(\hat{a}(n^{2/5}))$			0.310	0.403	0.483	0.39921	0.99921	0.00302	0.85474
θ_{OLS}	-0.3	0.5	0.295	0.408	0.506	0.40337	0.70337	0.00428	1.00000
$\tilde{\theta}(\hat{a}(5))$			0.292	0.404	0.506	0.40162	0.70162	0.00432	1.01026
$\tilde{\theta}(\hat{a}(10))$			0.293	0.405	0.502	0.40134	0.70134	0.00424	0.99030
$\tilde{\theta}(\hat{a}(\sqrt{n}))$			0.292	0.407	0.504	0.40127	0.70127	0.00435	1.01766
$\tilde{\theta}(\hat{a}(n^{2/5}))$			0.291	0.406	0.508	0.40219	0.70219	0.00440	1.02892
θ_{OLS}	-0.0	0.5	0.229	0.410	0.547	0.40313	0.40319	0.01007	1.00000
$\tilde{\theta}(\hat{a}(5))$			0.244	0.407	0.548	0.40271	0.40293	0.00923	0.91666
$\tilde{\theta}(\hat{a}(10))$			0.236	0.406	0.549	0.40174	0.40221	0.00953	0.94657
$\tilde{\theta}(\hat{a}(\sqrt{n}))$			0.239	0.404	0.542	0.40113	0.40141	0.00968	0.96206
$\tilde{\theta}(\hat{a}(n^{2/5}))$			0.239	0.405	0.545	0.40039	0.40162	0.01009	1.00236
θ_{OLS}	0.3	0.5	-0.358	0.353	0.806	0.29456	0.28015	0.12588	1.00000
$\tilde{\theta}(\hat{a}(5))$			-0.386	0.389	0.876	0.33326	0.30211	0.14403	1.14418
$\tilde{\theta}(\hat{a}(10))$			-0.397	0.389	0.875	0.32826	0.30228	0.14317	1.13736
$\tilde{\theta}(\hat{a}(\sqrt{n}))$			-0.406	0.392	0.870	0.32675	0.30159	0.14356	1.14042
$\tilde{\theta}(\hat{a}(n^{2/5}))$			-0.408	0.390	0.864	0.32942	0.30085	0.14375	1.14194
θ_{OLS}	0.6	0.5	0.000	0.382	0.612	0.36104	0.24552	0.03222	1.00000
$\tilde{\theta}(\hat{a}(5))$			0.093	0.380	0.632	0.37508	0.23988	0.03104	0.96340
$\tilde{\theta}(\hat{a}(10))$			0.103	0.385	0.639	0.37809	0.23671	0.03050	0.94662
$\tilde{\theta}(\hat{a}(\sqrt{n}))$			0.106	0.388	0.657	0.37981	0.23678	0.02995	0.92948
$\tilde{\theta}(\hat{a}(n^{2/5}))$			0.097	0.384	0.641	0.37741	0.23687	0.03064	0.95106
θ_{OLS}	0.9	0.5	0.299	0.401	0.491	0.39782	0.50218	0.00388	1.00000
$\tilde{\theta}(\hat{a}(5))$			0.300	0.398	0.488	0.39491	0.50509	0.00345	0.88820
$\tilde{\theta}(\hat{a}(10))$			0.301	0.395	0.484	0.39297	0.50703	0.00390	1.00505
$\tilde{\theta}(\hat{a}(\sqrt{n}))$			0.296	0.395	0.482	0.39373	0.50627	0.00359	0.92596
$\tilde{\theta}(\hat{a}(n^{2/5}))$			0.297	0.396	0.487	0.39331	0.50669	0.00396	1.01969