

Table 1: ARCH(1)

Relative Efficiency of OLS					
Sample Size:256					
γ_1	ϕ	0	.5	.7	.9
0.0		1.002	1.005	1.009	1.042
		(0.002)	(0.002)	(0.003)	(0.006)
0.5		0.988	0.955	0.917	0.944
		(0.003)	(0.004)	(0.005)	(0.008)
0.9		0.958	0.912	2.640	0.729
		(0.004)	(0.005)	(1.761)	(0.019)

Table 2: ARCH(1)

Relative Efficiency of OLS					
Sample Size:512					
γ_1	ϕ	0	.5	.7	.9
0.0		1.000	1.003	1.009	1.024
		(0.001)	(0.002)	(0.002)	(0.004)
0.5		0.997	0.941	0.901	0.886
		(0.002)	(0.004)	(0.006)	(0.008)
0.9		0.961	0.916	0.818	0.588
		(0.005)	(0.007)	(0.020)	(0.019)

Table 3: ARCH(1)

Relative Efficiency of OLS					
Sample Size:1024					
γ_1	ϕ	0	.5	.7	.9
0.0		1.000	0.998	1.005	1.012
		(0.001)	(0.001)	(0.001)	(0.002)
0.5		1.002	0.940	0.861	0.840
		(0.003)	(0.004)	(0.008)	(0.016)
0.9		0.965	0.991	0.814	0.523
		(0.005)	(0.093)	(0.019)	(0.045)

Relative Efficiency is defined as S_{IV}^2/S_{OLS}^2 where S^2 is the estimated variance of the estimator $\hat{\phi}$. Numbers in parenthesis are asymptotic standard deviations of the variance ratio. Results are based on 10,000 replications.

Table 4: ARCH(1)

Sample Size N=32										
Model: $y_t = \phi y_{t-1} + \varepsilon_t$ $\varepsilon_t = u_t h_t^{1/2}$ $h_t = 0.1 + \gamma_1 \varepsilon_{t-1}^2$										
Estimator	ϕ	γ_1	Empirical Quantiles			Mean	MAE	Var	ASDE	Size
			.05	.5	.95					
$\hat{\phi}_{OLS}$	0.0	0.0	-0.277	0.003	0.286	0.00145	0.13847	0.02934	0.16326	0.24550
$\hat{\phi}_{FD}$			-0.277	0.000	0.285	0.00104	0.13827	0.02926	0.20789	0.31180
$\tilde{\phi}_{OLS}$	0.0	0.5	-0.363	0.003	0.365	0.00268	0.17902	0.04992	0.20455	0.24560
$\tilde{\phi}_{FD}$			-0.355	0.003	0.360	0.00260	0.17475	0.04774	0.22183	0.28040
$\hat{\phi}_{OLS}$	0.0	0.9	-0.432	-0.004	0.440	-0.00054	0.21191	0.07051	0.23157	0.23310
$\tilde{\phi}_{FD}$			-0.415	-0.001	0.418	0.00032	0.20355	0.06600	0.23048	0.25090
$\hat{\phi}_{OLS}$	0.5	0.0	0.198	0.487	0.698	0.47333	0.12004	0.02308	0.14348	0.26860
$\hat{\phi}_{FD}$			0.178	0.470	0.688	0.45730	0.12363	0.02361	0.18201	0.32020
$\hat{\phi}_{OLS}$	0.5	0.5	0.106	0.480	0.738	0.45906	0.15407	0.03786	0.17238	0.24760
$\tilde{\phi}_{FD}$			0.094	0.461	0.717	0.43892	0.15485	0.03675	0.19272	0.27860
$\hat{\phi}_{OLS}$	0.5	0.9	0.039	0.470	0.764	0.44382	0.17753	0.05141	0.19429	0.24380
$\tilde{\phi}_{FD}$			0.029	0.445	0.726	0.38482	0.21166	12.14425	0.20099	0.26040
$\hat{\phi}_{OLS}$	0.7	0.0	0.414	0.680	0.846	0.66137	0.10549	0.01791	0.12168	0.25530
$\hat{\phi}_{FD}$			0.386	0.657	0.828	0.63829	0.11316	0.01905	0.15310	0.30300
$\hat{\phi}_{OLS}$	0.7	0.5	0.334	0.674	0.863	0.64723	0.12905	0.02742	0.14094	0.24920
$\tilde{\phi}_{FD}$			0.308	0.647	0.836	0.62030	0.13727	0.05083	0.16101	0.27610
$\hat{\phi}_{OLS}$	0.7	0.9	0.285	0.665	0.878	0.63532	0.14813	0.03687	0.15633	0.23310
$\tilde{\phi}_{FD}$			0.256	0.635	0.844	0.60396	0.15774	0.10272	0.16725	0.25470
$\hat{\phi}_{OLS}$	0.9	0.0	0.654	0.877	0.972	0.85409	0.07781	0.01054	0.08298	0.25310
$\hat{\phi}_{FD}$			0.600	0.844	0.935	0.81433	0.10001	0.09315	0.10287	0.30430
$\hat{\phi}_{OLS}$	0.9	0.5	0.629	0.875	0.974	0.84744	0.08508	0.01308	0.08903	0.25300
$\tilde{\phi}_{FD}$			0.576	0.841	0.936	0.81022	0.10305	0.02118	0.10679	0.29620
$\hat{\phi}_{OLS}$	0.9	0.9	0.557	0.872	0.978	0.83519	0.10038	0.01981	0.09683	0.23700
$\hat{\phi}_{FD}$			0.512	0.838	0.935	0.79325	0.12347	0.12058	0.11137	0.29590

Simulations are based on 10,000 replications. The columns .05, .5 and .95 are the 5, 50 and 95 percent quantiles of the empirical distribution of the 10,000 parameter estimates. The columns Mean, MAE and Var are the mean, mean absolute error and variance of the parameter estimates. The column ASDE is the average estimated standard error based on White's covariance matrix for OLS and $\hat{\Xi}$ for IV. Size is the relative frequency at which a t-test for $H_0 : \phi = \phi_0$ rejects the null when it is true with nominal size equal to .05.

Table 5: ARCH(1)

Sample Size N=256										
Model: $y_t = \phi y_{t-1} + \varepsilon_t$ $\varepsilon_t = u_t h_t^{1/2}$ $h_t = 0.1 + \gamma_1 \varepsilon_{t-1}^2$										
Estimator	ϕ	γ_1	Empirical Quantiles			Mean	MAE	Var	ASDE	Size
			.05	.5	.95					
$\hat{\phi}_{OLS}$	0.0	0.0	-0.102	0.001	0.102	0.00067	0.04946	0.00382	0.06182	0.08960
$\hat{\phi}_{FD}$			-0.102	0.001	0.103	0.00059	0.04954	0.00383	0.07615	0.11110
$\tilde{\phi}_{OLS}$	0.0	0.5	-0.166	0.000	0.164	-0.00032	0.07925	0.01043	0.09583	0.09370
$\tilde{\phi}_{FD}$			-0.163	-0.000	0.164	-0.00023	0.07871	0.01030	0.09195	0.09250
$\hat{\phi}_{OLS}$	0.0	0.9	-0.264	0.003	0.273	0.00202	0.12515	0.02699	0.14419	0.09400
$\tilde{\phi}_{FD}$			-0.258	0.003	0.266	0.00195	0.12229	0.02586	0.11797	0.08150
$\hat{\phi}_{OLS}$	0.5	0.0	0.404	0.500	0.583	0.49730	0.04300	0.00289	0.05356	0.09420
$\hat{\phi}_{FD}$			0.402	0.498	0.580	0.49537	0.04317	0.00291	0.06601	0.11490
$\hat{\phi}_{OLS}$	0.5	0.5	0.355	0.494	0.617	0.49145	0.06384	0.00654	0.07829	0.09770
$\tilde{\phi}_{FD}$			0.353	0.491	0.611	0.48795	0.06281	0.00625	0.07738	0.09560
$\hat{\phi}_{OLS}$	0.5	0.9	0.251	0.487	0.679	0.47856	0.10158	0.01810	0.11766	0.09130
$\tilde{\phi}_{FD}$			0.250	0.480	0.657	0.46966	0.09802	0.01651	0.09793	0.07770
$\hat{\phi}_{OLS}$	0.7	0.0	0.617	0.696	0.764	0.69420	0.03610	0.00206	0.04442	0.10000
$\hat{\phi}_{FD}$			0.613	0.694	0.762	0.69146	0.03650	0.00208	0.05468	0.12270
$\hat{\phi}_{OLS}$	0.7	0.5	0.578	0.694	0.788	0.68932	0.05033	0.00413	0.06122	0.09370
$\tilde{\phi}_{FD}$			0.578	0.690	0.779	0.68527	0.04877	0.00379	0.06207	0.10150
$\hat{\phi}_{OLS}$	0.7	0.9	0.485	0.684	0.818	0.67185	0.07998	0.01181	0.09110	0.08750
$\hat{\phi}_{FD}$			0.489	0.678	0.797	0.66657	0.07637	0.03118	0.07722	0.08010
$\hat{\phi}_{OLS}$	0.9	0.0	0.842	0.897	0.934	0.89336	0.02266	0.00083	0.02759	0.09640
$\hat{\phi}_{FD}$			0.837	0.893	0.931	0.88962	0.02370	0.00087	0.03386	0.11410
$\hat{\phi}_{OLS}$	0.9	0.5	0.828	0.896	0.938	0.89098	0.02707	0.00122	0.03257	0.09330
$\tilde{\phi}_{FD}$			0.826	0.892	0.933	0.88739	0.02700	0.00115	0.03591	0.10760
$\hat{\phi}_{OLS}$	0.9	0.9	0.777	0.891	0.948	0.88011	0.04149	0.00359	0.04710	0.09180
$\hat{\phi}_{FD}$			0.793	0.889	0.934	0.87856	0.03571	0.00262	0.04181	0.09740

Simulations are based on 10,000 replications. The columns .05, .5 and .95 are the 5, 50 and 95 percent quantiles of the empirical distribution of the 10,000 parameter estimates. The columns Mean, MAE and Var are the mean, mean absolute error and variance of the parameter estimates. The column ASDE is the average estimated standard error based on White's covariance matrix for OLS and $\hat{\Xi}$ for IV. Size is the relative frequency at which a t-test for $H_0 : \phi = \phi_0$ rejects the null when it is true with nominal size equal to .05.

Table 6: ARCH(1)

Sample Size N=1024										
Model: $y_t = \phi y_{t-1} + \varepsilon_t$ $\varepsilon_t = u_t h_t^{1/2}$ $h_t = 0.1 + \gamma_1 \varepsilon_{t-1}^2$										
Estimator	ϕ	γ_1	Empirical Quantiles			Mean	MAE	Var	ASDE	Size
			.05	.5	.95					
$\hat{\phi}_{OLS}$	0.0	0.0	-0.051	-0.000	0.050	-0.00021	0.02474	0.00096	0.03116	0.04870
$\hat{\phi}_{FD}$			-0.051	-0.000	0.050	-0.00020	0.02475	0.00096	0.03822	0.05860
$\tilde{\phi}_{OLS}$	0.0	0.5	-0.090	0.001	0.091	0.00096	0.04401	0.00328	0.05394	0.05020
$\tilde{\phi}_{FD}$			-0.089	0.001	0.091	0.00095	0.04396	0.00329	0.04940	0.04680
$\hat{\phi}_{OLS}$	0.0	0.9	-0.201	0.003	0.205	0.00199	0.09448	0.01667	0.11111	0.04510
$\tilde{\phi}_{FD}$			-0.200	0.003	0.201	0.00194	0.09262	0.01608	0.08362	0.03690
$\hat{\phi}_{OLS}$	0.5	0.0	0.454	0.499	0.542	0.49899	0.02137	0.00073	0.02698	0.04440
$\hat{\phi}_{FD}$			0.453	0.499	0.542	0.49852	0.02136	0.00072	0.03311	0.05880
$\hat{\phi}_{OLS}$	0.5	0.5	0.421	0.496	0.570	0.49603	0.03633	0.00214	0.04407	0.04770
$\tilde{\phi}_{FD}$			0.422	0.495	0.567	0.49507	0.03535	0.00202	0.04120	0.04530
$\hat{\phi}_{OLS}$	0.5	0.9	0.308	0.491	0.642	0.48368	0.07898	0.01194	0.08986	0.04390
$\tilde{\phi}_{FD}$			0.310	0.486	0.621	0.47648	0.07506	0.01183	0.06735	0.03470
$\hat{\phi}_{OLS}$	0.7	0.0	0.660	0.699	0.734	0.69819	0.01792	0.00050	0.02229	0.04860
$\hat{\phi}_{FD}$			0.659	0.699	0.733	0.69748	0.01800	0.00050	0.02735	0.06090
$\hat{\phi}_{OLS}$	0.7	0.5	0.639	0.697	0.751	0.69609	0.02721	0.00122	0.03373	0.05070
$\tilde{\phi}_{FD}$			0.641	0.696	0.747	0.69524	0.02560	0.00105	0.03235	0.05230
$\hat{\phi}_{OLS}$	0.7	0.9	0.551	0.690	0.799	0.68291	0.05892	0.00711	0.06844	0.04700
$\tilde{\phi}_{FD}$			0.564	0.687	0.773	0.67880	0.05173	0.00579	0.04994	0.03950
$\hat{\phi}_{OLS}$	0.9	0.0	0.874	0.899	0.920	0.89841	0.01101	0.00019	0.01366	0.05240
$\hat{\phi}_{FD}$			0.873	0.899	0.919	0.89753	0.01113	0.00019	0.01674	0.06160
$\hat{\phi}_{OLS}$	0.9	0.5	0.866	0.898	0.924	0.89711	0.01400	0.00032	0.01716	0.04910
$\tilde{\phi}_{FD}$			0.868	0.898	0.921	0.89636	0.01309	0.00027	0.01803	0.05540
$\hat{\phi}_{OLS}$	0.9	0.9	0.823	0.894	0.938	0.88821	0.02829	0.00194	0.03288	0.04280
$\hat{\phi}_{FD}$			0.848	0.895	0.924	0.89090	0.01941	0.00102	0.02353	0.05220

Simulations are based on 10,000 replications. The columns .05, .5 and .95 are the 5, 50 and 95 percent quantiles of the empirical distribution of the 10,000 parameter estimates. The columns Mean, MAE and Var are the mean, mean absolute error and variance of the parameter estimates. The column ASDE is the average estimated standard error based on White's covariance matrix for OLS and $\hat{\Xi}$ for IV. Size is the relative frequency at which a t-test for $H_0 : \phi = \phi_0$ rejects the null when it is true with nominal size equal to .05.

Table 7: IGARCH(1,1)

Sample Size N=1024										
Model: $y_t = \phi y_{t-1} + \varepsilon_t$ $\varepsilon_t = u_t h_t^{1/2}$ $h_t = 0.1 + \gamma_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$; $\beta_1 + \gamma_1 = 1$										
Estimator	ϕ	γ_1	Empirical Quantiles			Mean	MAE	Var	ASDE	Size
			.05	.5	.95					
$\hat{\phi}_{OLS}$	0.5	0.0	0.452	0.499	0.544	0.49861	0.02261	0.00080	0.02812	0.04680
$\hat{\phi}_{FD}$			0.451	0.498	0.544	0.49790	0.02262	0.00080	0.03358	0.05730
$\tilde{\phi}_{OLS}$	0.5	0.4	0.365	0.495	0.615	0.49263	0.05815	0.00602	0.07012	0.04820
$\tilde{\phi}_{FD}$			0.364	0.493	0.613	0.49132	0.05818	0.00601	0.05703	0.03560
$\hat{\phi}_{OLS}$	0.5	0.6	0.330	0.490	0.632	0.48579	0.07144	0.00930	0.08449	0.04510
$\tilde{\phi}_{FD}$			0.328	0.488	0.628	0.48336	0.07082	0.00913	0.06604	0.03730
$\hat{\phi}_{OLS}$	0.5	0.8	0.295	0.489	0.650	0.48186	0.08146	0.01239	0.09579	0.04290
$\tilde{\phi}_{FD}$			0.293	0.484	0.638	0.47700	0.08004	0.01196	0.07260	0.03550
$\hat{\phi}_{OLS}$	0.5	1.0	0.261	0.490	0.669	0.48052	0.09230	0.01649	0.10613	0.04600
$\tilde{\phi}_{FD}$			0.262	0.483	0.642	0.46972	0.08866	0.01845	0.07664	0.03490
$\hat{\phi}_{OLS}$	0.7	0.0	0.659	0.700	0.736	0.69886	0.01878	0.00055	0.02321	0.04880
$\tilde{\phi}_{FD}$			0.658	0.699	0.735	0.69784	0.01884	0.00055	0.02770	0.05880
$\hat{\phi}_{OLS}$	0.7	0.4	0.589	0.694	0.783	0.69020	0.04619	0.00377	0.05700	0.04870
$\tilde{\phi}_{FD}$			0.589	0.692	0.780	0.68885	0.04587	0.00371	0.04665	0.04010
$\hat{\phi}_{OLS}$	0.7	0.6	0.557	0.690	0.797	0.68526	0.05628	0.00593	0.06706	0.04530
$\tilde{\phi}_{FD}$			0.559	0.689	0.789	0.68264	0.05462	0.00554	0.05256	0.03400
$\hat{\phi}_{OLS}$	0.7	0.8	0.532	0.689	0.805	0.68114	0.06295	0.00759	0.07428	0.04610
$\tilde{\phi}_{FD}$			0.534	0.686	0.790	0.67630	0.06020	0.01034	0.05584	0.04080
$\hat{\phi}_{OLS}$	0.7	1.0	0.515	0.689	0.810	0.67893	0.06848	0.00921	0.07985	0.04650
$\tilde{\phi}_{FD}$			0.526	0.685	0.779	0.67338	0.06046	0.00821	0.05589	0.03630
$\hat{\phi}_{OLS}$	0.9	0.0	0.873	0.899	0.921	0.89812	0.01161	0.00021	0.01426	0.04620
$\tilde{\phi}_{FD}$			0.872	0.898	0.919	0.89677	0.01176	0.00021	0.01699	0.05360
$\hat{\phi}_{OLS}$	0.9	0.4	0.827	0.894	0.939	0.88970	0.02675	0.00138	0.03218	0.04670
$\tilde{\phi}_{FD}$			0.828	0.893	0.934	0.88791	0.02587	0.00130	0.02688	0.04440
$\hat{\phi}_{OLS}$	0.9	0.6	0.813	0.893	0.942	0.88700	0.03069	0.00193	0.03609	0.04690
$\tilde{\phi}_{FD}$			0.820	0.892	0.931	0.88544	0.02747	0.00157	0.02814	0.04160
$\hat{\phi}_{OLS}$	0.9	0.8	0.803	0.892	0.942	0.88448	0.03313	0.00243	0.03850	0.04400
$\tilde{\phi}_{FD}$			0.820	0.893	0.928	0.88518	0.02636	0.00166	0.02798	0.04880
$\hat{\phi}_{OLS}$	0.9	1.0	0.799	0.893	0.944	0.88404	0.03414	0.00282	0.03926	0.04410
$\tilde{\phi}_{FD}$			0.835	0.895	0.924	0.88811	0.02225	0.00151	0.02572	0.05000

Simulations are based on 10,000 replications. The columns .05, .5 and .95 are the 5, 50 and 95 percent quantiles of the empirical distribution of the 10,000 parameter estimates. The columns Mean, MAE and Var are the mean, mean absolute error and variance of the parameter estimates. The column ASDE is the average estimated standard error based on White's covariance matrix for OLS and $\hat{\Xi}$ for IV. Size is the relative frequency at which a t-test for $H_0 : \phi = \phi_0$ rejects the null when it is true with nominal size equal to .05.

Table 8: Stochastic Volatility

Sample Size N=1024										
Model: $y_t = \phi y_{t-1} + \varepsilon_t$ $\varepsilon_t = u_t \exp(h_t/2)$ $h_t = 0.1 + \gamma_1 h_{t-1} + v_t$										
Estimator	ϕ	γ_1	Empirical Quantiles			Mean	MAE	Var	ASDE	Size
			.05	.5	.95					
$\hat{\phi}_{OLS}$	0.7	0.8	0.655	0.699	0.740	0.69839	0.02051	0.00067	0.02563	0.04850
$\tilde{\phi}_{FD}$			0.654	0.698	0.739	0.69769	0.02052	0.00066	0.02877	0.05250
$\hat{\phi}_{OLS}$	0.7	0.9	0.646	0.698	0.747	0.69746	0.02448	0.00095	0.03026	0.04800
$\tilde{\phi}_{FD}$			0.645	0.697	0.746	0.69662	0.02445	0.00094	0.03096	0.04980
$\hat{\phi}_{OLS}$	0.7	1.0	0.605	0.695	0.775	0.69260	0.04118	0.00283	0.05066	0.05210
$\tilde{\phi}_{FD}$			0.606	0.694	0.773	0.69164	0.04069	0.00273	0.04229	0.04310
$\hat{\phi}_{OLS}$	0.9	0.8	0.871	0.899	0.921	0.89779	0.01202	0.00023	0.01497	0.05150
$\tilde{\phi}_{FD}$			0.871	0.898	0.920	0.89687	0.01209	0.00023	0.01732	0.05690
$\hat{\phi}_{OLS}$	0.9	0.9	0.866	0.898	0.924	0.89723	0.01409	0.00031	0.01733	0.04500
$\tilde{\phi}_{FD}$			0.865	0.897	0.923	0.89620	0.01401	0.00030	0.01839	0.05290
$\hat{\phi}_{OLS}$	0.9	1.0	0.838	0.896	0.937	0.89230	0.02372	0.00094	0.02919	0.04700
$\tilde{\phi}_{FD}$			0.839	0.894	0.934	0.89111	0.02317	0.00087	0.02481	0.04250

Simulations are based on 10,000 replications. The columns .05, .5 and .95 are the 5, 50 and 95 percent quantiles of the empirical distribution of the 10,000 parameter estimates. The columns Mean, MAE and Var are the mean, mean absolute error and variance of the parameter estimates. The column ASDE is the average estimated standard error based on White's covariance matrix for OLS and $\hat{\Xi}$ for IV. Size is the relative frequency at which a t-test for $H_0 : \phi = \phi_0$ rejects the null when it is true with nominal size equal to .05.

Relative Efficiency

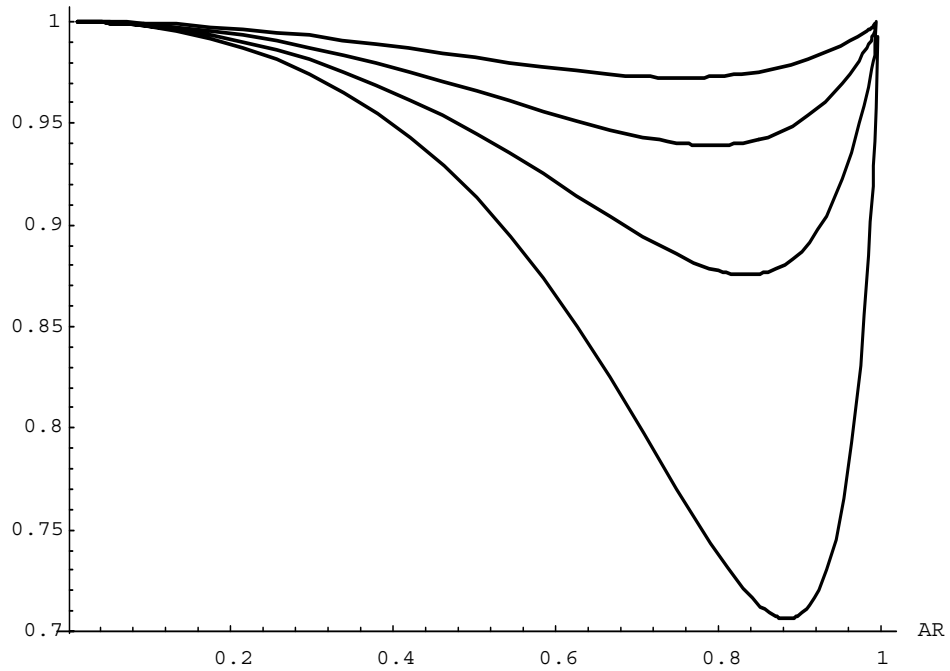


Figure 3.1: *Asymptotic efficiency of OLS relative to the IV estimator as a function of the parameter ϕ . Generating mechanisms considered are from bottom to top: $\gamma_1 = .5$, $\gamma_1 = .4$, $\gamma_1 = .3$ and $\gamma_1 = .2$.*