

Corporate-Sovereign Debt Nexus and Externalities*

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Abstract

I show that corporate debt accumulation during booms can explain increases in sovereign risk during stress periods. Using idiosyncratic shocks to large firms as instruments for aggregate corporate leverage, I show that rising corporate leverage during the period 2002-2007 causally increases sovereign spreads in six Eurozone countries during the debt crisis period of 2008-2012. To explain these findings, I build a dynamic quantitative model in which both firms and the government can default. Rising corporate debt increases sovereign default risk, as tax revenues are expected to decrease. Externalities arise because it can be privately optimal but socially suboptimal for firms to default given their limited liability. The fact that firms do not take into account the effect of their debt accumulation on aggregate sovereign spreads is an important externality, rationalizing macroprudential interventions in corporate debt markets. I propose a set of such optimal debt policies that reduce the number of defaulting firms, increase fiscal space, and boost household consumption during financial crises. Both constant and countercyclical debt tax schedules can correct overborrowing externalities. Contrary to conventional wisdom, countercyclical debt policy is less effective than constant debt policy, as the countercyclical policy induces more firm defaults.

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1 Introduction

Eurozone sovereign spreads surged during the European debt crisis of 2010 to 2012, reflecting sharp increases in the perceived probability of sovereign default. Previous research on sovereign default has identified high sovereign indebtedness as one of the precursors of sovereign debt crises. However, the average Eurozone government-debt-to-GDP ratio rose by only 19.7 percentage points from 2000 to 2011, while the non-financial corporate-debt-to-GDP ratio rose by 41.6 percentage points during the same period. [Reinhart and Rogoff \(2009\)](#) show that corporate defaults have also been a predictor of government defaults or reschedulings in many episodes.¹ Consistent with their finding, the Eurozone non-financial corporate interest rate spreads peaked before and during the sovereign debt crisis, as plotted in [Figure 1](#).

The empirical literature faces the challenge of establishing causal mechanisms. Based on the long history of credit cycles across countries, the consensus view holds that credit booms end up with various forms of financial crises. However, this view arguably has not been properly supported by causal inference. One of the main empirical difficulties is that there is likely an omitted variable linking credit booms and financial crises. For instance, optimism among investors can generate a credit boom and a subsequent correction. In this case, the cause of the subsequent correction might be sudden changes in investors' sentiment, rather than the credit boom itself. A possible remedy to this endogeneity problem is to find instruments for credit that are unrelated to other macroeconomic shocks, but it is difficult to find (excludable) instruments for macroeconomic variables.

One of the main contributions of this paper is to establish a causal relationship running from corporate debt to sovereign default risk during the European debt crisis. I run instrumental variable (IV) regressions to show that a rising corporate debt-to-GDP ratio causally increases sovereign spreads during the pre-crisis period, using a weighted sum of idiosyncratic shocks to large firms as an instrument for aggregate corporate leverage. [Gabaix and Koijen \(2020\)](#) develop this type of general identification strategy for estimating aggregate relationships using idiosyncratic shocks to variables of interest as instruments. An instrumental variable regression suggests that a one standard deviation (23%p) increase in the corporate debt-to-GDP ratio causally increases sovereign spreads by 253 basis points, using a sample of six Eurozone countries during the period 2002-2012.² This regression takes account of financial conditions in the banking sector together with country and time fixed effects.

¹[Reinhart and Rogoff \(2011\)](#) report that rapidly rising private indebtedness precedes banking crises, and that banking crises increase the likelihood of sovereign default, using a long-term historical database which dates back to the 1800s and covers 70 countries. See also [Schularick and Taylor \(2012\)](#), [Drehmann and Juselius \(2012\)](#), and many others.

²The maximum increase during this period for these countries' spreads is 883 basis points, and the average increase is 320 basis points.

Core identification assumptions in my empirical strategy are that idiosyncratic shocks to large firms are correlated with aggregate corporate debt (relevance) and that these shocks affect sovereign spreads only through corporate debt after controlling for country-specific GDP growth and time-varying factors common to all countries (exclusion). The exclusion condition states that the error term in the sovereign spread regression reflects systematic shocks to government solvency, which are purged by looking at idiosyncratic shocks. Using idiosyncratic shocks to large firms as an instrument addresses potential reverse causality (government solvency shocks spill over to corporate debt) and omitted variables bias (both corporate and government debt are affected by liquidity shocks). To the best of my knowledge, my paper is the first to present evidence for a causal linkage between corporate debt and sovereign default risk, combining detailed firm-level balance sheet data with a new identification technique.³ Also, local projections regression suggests that rising corporate leverage has persistent effects on government solvency, raising 3-year-ahead sovereign spreads.

Next, to explain the link between corporate debt and sovereign default risk, I build a model in which both firms and the government can default on debt, based on the models of [Jermann and Quadrini \(2012\)](#) and [Arellano \(2008\)](#). Rising corporate debt increases sovereign default risk, as government tax revenues are expected to decrease, given that firms pay fewer taxes when they default and that rising firms' borrowing costs dampen economic activity and reduce overall tax revenues including household income taxes. When the government may not be able to raise enough taxes to finance public expenditure, it may choose to accumulate debt or repudiate debt to finance this expenditure. This means that the government has more incentive to default on its debt, and thus sovereign risk increases. [Arellano et al. \(2019\)](#) also build a model that connects firms and the government via the tax revenue channel. However, my model differs in that firms choose leverage endogenously and can default on debt, while firm leverage in their model is exogenously given by the working capital constraint. Endogenous firm leverage is a key ingredient in my model that enables assessment of overborrowing externalities as described below.

To provide empirical evidence on the tax revenue channel, I run a country-level tax revenue regression, using the weighted sum of idiosyncratic shocks to large firms as an instrument for corporate leverage, and find that a one standard deviation increase in the ratio of corporate debt to GDP (23%p) leads on average to a 7.4%p decrease in one-year-ahead tax revenue growth. Moreover, a difference-in-difference regression using firm-level

³Several papers estimate the causal link running from other variables (excluding corporate debt) to sovereign spreads. For example, [Acharya et al. \(2014\)](#) run regressions of daily European sovereign credit default swap (CDS) rates on bank CDS rates before, during, and after the governments' announcements on bank bailouts. [Wang \(2020\)](#) finds that surprise increases in corporate CDS rates lead to increases in sovereign CDS rates in emerging markets, using a high-frequency event-study analysis. [Bernoth and Herwartz \(2019\)](#) estimate a causal linkage between exchange rates and sovereign spreads in emerging market economies during 2004-2016, using structural vector autoregressive (SVAR) models.

data suggests that highly-leveraged firms pay fewer taxes compared to less-leveraged firms during and after the 2008 global financial crisis, while controlling for interest payments.⁴

I use my model to perform a normative analysis in which firms do not internalize the effects of their borrowing on the welfare of households, and the government needs to intervene in corporate debt markets to correct this negative externality. When firms default, the economy bears economic costs of bankruptcies. However, firms do not internalize these negative spillovers, since their liability is limited in the event of default. In this environment, firms tend to over-borrow and increase corporate default risk, which is likely to reduce household consumption and raise the risk of government default. Externalities arise because it is privately optimal but socially suboptimal for firms to default given their limited liability. Most papers on optimal macroprudential policies have been silent on externalities due to limited liability. The limited liability externality is qualitatively different from previously identified externalities such as pecuniary and aggregate demand externalities. The limited liability externality hinges on the corporate law, which is operative when a firm is a separate legal entity apart from its owners. This means that this type of externality can arise even if firms are not atomistic internalizing their effects on aggregate prices, unlike pecuniary and aggregate demand externalities.

I calibrate the model to six Eurozone countries during the period 2000-2012. This model reproduces dynamics in the data during the period 2007-2017, successfully matching the duration and magnitude of each of the corporate credit boom-bust cycle and fluctuations in sovereign spreads and output. Using the model, I uncover a set of time-consistent optimal policies (a constant debt tax rate and subsidies to firms during the crisis) that correct the limited liability externality. The welfare gain from these policies is substantial, amounting to around a 12.8% increase in permanent consumption. The reason is that optimal policies mitigate corporate debt cycles and associated firm defaults, leading to more household consumption and tax revenues. As a result, government interest rates fall, and the government can finance more subsidies to firms, which again reduces firm defaults. This positive feedback loop of optimal policies gives rise to the large welfare gain by alleviating the causal linkage between corporate debt and government spreads. In reality, this type of state-contingent optimal policy might not be available to policymakers. Thus, I explore alternative simple debt policies. I find that both constant and countercyclical debt tax schedules can increase welfare by correcting overborrowing externalities. However, contrary to conventional wisdom, the model shows that countercyclical debt policy is less effective than constant debt policy, resulting in welfare gains of 0.1% and 2.1%, respectively. Intuitively, the government raises the debt tax rate during corporate credit booms after observing the current level of

⁴The ratio of tax payments to value added of highly-leveraged firms drops more by around 0.28%p than that of less-leveraged firms, while the average tax payment ratio of the regression sample is 4.15%.

aggregate corporate debt, and this backward-looking stance of the simple countercyclical debt policy leads to more firm defaults during the crisis. Furthermore, in the presence of firm default risk, it is optimal for the government to cut the debt tax rate to reduce firm default risk when corporate leverage is high. This result is in sharp contrast to the existing models based on borrowing constraints (Bianchi, 2011; Bianchi and Mendoza, 2018), as these models typically recommend raising of the debt tax rate by the government when the borrowing constraint is more likely to bind due to high leverage.

Related Literature

I contribute to three strands of research. First, I add to the empirical literature that has identified key covariates that are highly correlated with sovereign spreads (Longstaff et al., 2011; Aguiar et al., 2016; Hilscher and Nosbusch, 2010; Bevilaqua et al., 2020). This literature typically does not attempt to estimate a causal relationship between fundamentals and spreads. Also, such regressions usually do not include corporate debt as an explanatory variable, though corporate debt is known to predict financial crises well (e.g., Reinhart and Rogoff (2011)). An exception that considers corporate debt includes Du and Schreger (2017). They run spread regressions for emerging economies, using foreign currency corporate debt borrowed from foreign lenders as an explanatory variable. In contrast, I show that total corporate debt (denominated in all currencies and borrowed from all lenders) in advanced economies (the Eurozone) helps to explain increases in sovereign spreads after controlling for currency risk. Moreover, other variables typically included in existing research, such as GDP growth and government debt, fail to explain the bulk of variation in sovereign spreads empirically, as reported in Aguiar et al. (2016). Specifically, the growth rate of output and the government debt-to-GDP ratio together explain less than 20 percent of sovereign spread variation (within countries and across time) in emerging economies. In my IV spread regression for Eurozone countries, including country and year fixed effects, GDP growth and the government debt-to-GDP ratio together explain about 27 percent of sovereign spread variation, while the corporate debt-to-GDP ratio accounts for an additional 19 percent of this variation.

Second, I build a sovereign default model that sheds light on a new causal mechanism. The canonical quantitative sovereign default models aim to explain sovereign default events as resulting from exogenous aggregate shocks to income flows and adjustments in external government debt (Arellano (2008), Aguiar and Gopinath (2006) and other papers that build on Eaton and Gersovitz (1981)).⁵ I contribute to this literature by building a model in which firms finance investment with internal funds, debt, or equity, and both firms and

⁵See Mendoza and Yue (2012), Arellano et al. (2019), Bocola et al. (2019), Rojas (2020), and many others for recent sovereign default models.

the government can default. In my model, endogenous firm leverage generates time-varying firm default risk, which affects the expected path of dividends and taxes. This generates time-varying risk in government tax revenue, which is a new channel that helps to match the crisis dynamics of corporate leverage and sovereign spreads observed in the data. [Wu \(2020\)](#) builds a similar model in which exchange rate shocks and corresponding time-varying risk premia are key sources of variation in sovereign spreads, but his model abstracts from equity issuance and limited liability. [Kaas et al. \(2020\)](#) also build a model with sovereign and private default risk without modeling physical investment and limited liability. Their model explains business cycle regularities in emerging economies.

Third, I contribute to the literature on optimal macroprudential policies. The existing literature justifies the use of macroprudential policies as arising from deviations from the conditions assumed in the first welfare theorem (see [Kehoe and Levine \(1993\)](#) and [Lorenzoni \(2008\)](#)). The first welfare theorem states that a competitive equilibrium is Pareto-efficient under several conditions. One of these conditions is that there should be no externality in the economy, in the sense that each person should be able to internalize their effects on other people via market prices. However, if people are atomistic and do not internalize their effects on market prices, the competitive equilibrium does not guarantee efficient allocations. In this environment, people can over-borrow compared to the socially optimal level of debt. The government can intervene in debt markets to correct externalities and increase social welfare. [Bianchi \(2011\)](#), [Brunnermeier and Sannikov \(2015\)](#), [Benigno et al. \(2016\)](#), [Jeanne and Korinek \(2020\)](#) and others identify this type of externality (pecuniary externality) in various settings and investigate optimal debt policies.⁶ I introduce a new concept of the limited liability externality into the framework and analyze the resulting optimal policies. [Wu \(2020\)](#) also identifies a novel externality which emerges because firms do not internalize their effects on sovereign spreads, as sovereign spreads do not enter firms' budget constraint. His model assumes that owners do not walk away from firms when firms go bankrupt. On the other hand, my model assumes that owners exit the business upon bankruptcy, which creates a fundamental externality arising from limited liability. Also, unlike [Wu \(2020\)](#), I analyze a set of optimal policies to correct externalities. [Aguiar and Amador \(2016\)](#) study optimal fiscal policy with implicit sovereign and private default risk in which the government minimizes tax distortions arising from financing exogenous public expenditure. I complement their work by introducing explicit corporate default decisions into the sovereign default model and analyzing optimal debt policy to address the limited liability externality.

⁶Also, [Farhi and Werning \(2016\)](#), [Korinek and Simsek \(2016\)](#), and [Schmitt-Grohé and Uribe \(2016\)](#) identify aggregate demand externalities in the presence of nominal rigidities that macroprudential policies can correct. [Basu et al. \(2020\)](#) consider both pecuniary and aggregate demand externalities. See [Erten et al. \(Forthcoming\)](#) for discussion of various types of externalities that can be corrected by macroprudential policies.

2 Empirical Evidence: Corporate-Sovereign Linkage

2.1 Data Description

I estimate the impact of corporate leverage on sovereign spreads in the EU. Out of the 25 EU countries, only 20 have data on the ratio of corporate debt to GDP available from BIS. Out of these 20 countries, 11 have sovereign interest rate data for 10 year bonds denominated in euro available from Bloomberg during the period 1999q1–2012q4. I focus on this period to mitigate potential reverse causality running from sovereign risk to corporate debt during the post-2012 crisis. I drop Germany since the German rate is used as the reference rate in measuring the sovereign spread. I also focus on countries that adopted the euro in 1999 to eliminate currency risk as a source of yield differences between the German government bond and other countries’ government bonds. The resulting sample consists of country-quarter observations during the period 1999q1–2012q4 across nine Eurozone countries (Austria, Belgium, Finland, France, Ireland, Italy, Netherlands, Portugal, and Spain). Summary statistics are presented in Table A.1 for this sample.

The sovereign spread is measured as the difference between a country’s 10-year government bond rate and the 10-year German government bond rate. Both bond rates are denominated in euro. Quarterly GDP growth is the log difference of seasonally-and-calendar-adjusted real GDP (Eurostat). Debt-to-GDP ratios (based on core debt at market value) for government and non-financial corporates come from BIS total credit statistics.⁷ These debt measures include credit from all sources in all currencies. VIX is the implied volatility of the S&P 500 (CBOE) based on option prices. The literature has identified GDP growth, the ratio of government debt to GDP, and a global common factor as being important to explaining sovereign spreads (see Longstaff et al. (2011), Aguiar et al. (2016), Bai et al. (2019) and others). I choose VIX as the global factor to capture changes in global investors’ risk aversion. Additionally, I use the difference between US 3-month treasury rates and the 3-month LIBOR (TED spread) and the difference between US 10-year and 3-month treasury rates (term spread) as additional controls for global credit conditions. Both spreads and the VIX are available from FRED (Economic Data by the Federal Reserve Bank of St. Louis). I also control for country-level banking sector leverage, measured as the ratio of selected financial assets to total equity, available from OECD.⁸ Banking sector leverage is only available at an annual frequency and thus will be used only for annual regressions. Tax revenue is measured as total receipts from taxes and social contributions and comes from Eurostat. Real tax revenue is calculated as nominal revenue divided by the GDP deflator (obtained from Eurostat).

⁷BIS (2019), Credit to the non-financial sector, <https://www.bis.org/statistics/totcredit.htm>.

⁸OECD (2020), Banking sector leverage (indicator). doi: 10.1787/027a0800-en.

For IV regressions, I use the weighted sum of idiosyncratic productivity shocks to top 50 large firms as an instrument for aggregate corporate debt. Thus, I must restrict my IV regression sample to countries for which I can estimate total factor productivity in firm-level data (ORBIS-AMADEUS). Summary statistics for the IV regression sample are shown in Table A.4. As I also use two years of lagged variables in the IV regressions, my final IV sample is limited to annual observations from six Eurozone countries (Italy, Spain, Portugal, Belgium, Finland, and France) during the period 2002-2012. The ORBIS-AMADEUS database is compiled by Bureau van Dijk Electronic Publishing (BvD), and AMADEUS is the European subset of ORBIS. This dataset has detailed annual firm-level information, including balance sheets, income statements, and profit and loss accounts. The main advantage of this data is that it contains both publicly and privately held companies, which distinguishes ORBIS from Compustat and Worldscope, which only contain large listed firms. I refer to [Kalemli-Ozcan et al. \(2019\)](#) for the construction of data.⁹ [Kalemli-Ozcan et al. \(2019\)](#) show that the ORBIS-AMADEUS database is nationally representative in the sense that aggregated firm-level data covers a considerable part of total gross output and employment (as compiled by official Bureaus of Statistics). Furthermore, I compare debt-to-GDP ratios from ORBIS-AMADEUS and BIS in Figure 2. I calculate debt-to-value-added ratios from ORBIS-AMADEUS as the sum of debt over the sum of value added (operating revenue net of material costs) across firms in a given country. I measure debt as the sum of loans and long-term debt, which represents financial debt excluding other accounts payable. I find that the time series in debt-to-GDP ratios from ORBIS-AMADEUS and BIS track each other well for the six countries on average.

I use data from the ORBIS-AMADEUS database on non-financial sector firms only, because financial intermediaries such as banks and insurance companies likely have different decision rules regarding leverage from the non-financial sector. I also exclude the mining and oil-related sectors, since measurement of idiosyncratic shocks is problematic in these sectors, given that their revenues depend strongly on aggregate commodity price shocks. For firm-level regressions, I use a full sample covering the period 2004-2012 in the six Eurozone countries used for the aggregate IV regressions (Italy, Portugal, Spain, Belgium, Finland, and France). I exclude the recession period of 2000 to 2003. I include firm observations missing in some years to account for behaviors of defaulting firms. The leverage measure is the ratio of financial debt to value added (b/y), where value added is operating revenue minus materials cost. Nominal variables are deflated by a 2-digit sector gross output deflator (EU KLEMS). To measure firm-level total factor productivity, I implement the [Wooldridge \(2009\)](#) extension of the [Levinsohn and Petrin \(2003\)](#) methodology.¹⁰ I closely follow [Gopinath et](#)

⁹See [Data Appendix](#) for data cleaning.

¹⁰As I do not observe firm-level prices, this measure of TFP might capture both productivity and demand shocks. However, as long as these measured shocks are exogenous with respect to sovereign risk, I can use

al. (2017) for the TFP estimation. Capital is the book value of tangible fixed assets, and labor is measured as the wage bill, reflecting the quantity and quality of labor.¹¹ Capital is deflated by a 2-digit sector investment deflator, and labor and materials costs are deflated by the 2-digit sector gross output deflator (EU KLEMS). All firm-level regression variables measured with ratios are winsorized at the 3rd and 97th percentiles.

2.2 Sovereign Spread OLS Regression

To begin, I estimate the following OLS regression using quarterly observations:

$$\begin{aligned} \text{Gov't Spread}_{c,t} = & \beta_y \text{GDP Growth}_{c,t-1} + \beta_{gov} \text{Gov't Debt}/\text{GDP}_{c,t-1} \\ & + \beta_{corp} \text{Corp Debt}/\text{GDP}_{c,t-1} + \delta_c + \gamma_t + \epsilon_{c,t} \end{aligned} \quad (1)$$

where δ_c and γ_t are country and quarter fixed effects, respectively.

Table 1 presents OLS regression results using the sample of six Eurozone countries that will be used for IV regressions. In column (1), government spreads are negatively related to GDP growth and positively related to government debt. These results suggest that the government is more likely to default in bad times when it has a large amount of debt. In column (2), corporate debt is positively related to spreads, suggesting that rising corporate leverage raises the perceived probability of sovereign default. The within- R^2 increases significantly (from 0.366 to 0.445) after including corporate debt, which suggests that corporate leverage is an important variable to explain within-country variation in sovereign spreads. In column (3), adding the log of VIX does not change the other coefficients much, and VIX is not a significant predictor for sovereign spreads. Adding the TED spread or term spread does not affect the coefficients on corporate debt in columns (4) and (5). The insignificant coefficients on these variables suggest that global credit conditions as measured by these variables do not have significant impacts on these Eurozone countries. Adding quarter fixed effects in columns (6) and (7) yields similar coefficients for corporate leverage. These fixed effects generate a substantial increase in within- R^2 . This is consistent with Longstaff et al. (2011), who find that global common factors explain a large amount of variation in sovereign spreads. In Table A.2, the same OLS regressions are presented using the larger sample of nine Eurozone countries during the period 1999q1–2012q4, and results are similar: the coefficients on corporate debt are positive and significant at the 1% level in all specifications. In Table A.3, I run similar regressions for individual Eurozone countries including a linear

this measure to construct instruments for my identification strategy.

¹¹Since I use year-on-year variation of productivity rather than its level, investment goods purchases are largely measured at current prices.

time trend, using all available observations for each country. Basic results do not change. Corporate debt is positively and significantly correlated with sovereign spreads in six of the nine sample countries (Spain, Portugal, Ireland, Belgium, Finland, and Austria).

In Figure 3, I plot regression coefficients estimated by Jordà (2005)-style local projections, as follows:

$$\begin{aligned} \text{Gov't Spread}_{c,t+h} = & \beta_{y,h} \text{GDP Growth}_{c,t} + \beta_{gov,h} \text{Gov't Debt}/\text{GDP}_{c,t} \\ & + \beta_{corp,h} \text{Corp Debt}/\text{GDP}_{c,t} + \delta_c + \gamma_t + \epsilon_{c,t+h} \end{aligned} \quad (2)$$

for each horizon $h = 0, 1, 2, \dots$, where δ_c and γ_t are country and time fixed effects.

I find that a one standard deviation increase in the corporate debt-to-GDP ratio has persistent positive impacts on sovereign spreads (from 30 to 50 basis points) up to 12 quarters ahead. An increase in GDP growth reduces sovereign spreads up to 6 quarters ahead, while an increase in the ratio of sovereign debt-to-GDP raises spreads up to 6 quarters ahead.

2.3 Sovereign Spread IV Regression

OLS estimates of β_{corp} in equation (1) might be biased due to correlation between corporate debt and unobservables that lead to variation in sovereign risk. Consider the following OLS regression:

$$Y_t = \alpha + \beta X_{t-1} + \xi_t \quad (3)$$

where X is corporate debt, β is the true coefficient on X , and Y is the sovereign spread, where both X and Y are purged of their correlations with GDP growth, government debt, and common time-varying factors such as VIX.

Suppose market liquidity W is omitted from the regression. Thus, $\xi_t = \gamma W_{t-1} + \epsilon_t$ where ϵ_t is white noise.

Then, the OLS estimator is given by

$$\hat{\beta}^{OLS} = \frac{\sum_{t=1}^T (X_{t-1} - \bar{X}) Y_t}{\sum_{t=1}^T (X_{t-1} - \bar{X})^2} = \beta + \gamma \frac{\sum_{t=1}^T (X_{t-1} - \bar{X}) W_{t-1}}{\sum_{t=1}^T (X_{t-1} - \bar{X})^2} + \frac{\sum_{t=1}^T (X_{t-1} - \bar{X}) \epsilon_t}{\sum_{t=1}^T (X_{t-1} - \bar{X})^2} \quad (4)$$

where $\bar{X} = 1/T \sum_{t=1}^T X_{t-1}$. Notice that $\sum_{t=1}^T (X_{t-1} - \bar{X}) W_{t-1}$ might be positive, since tighter liquidity will reduce equilibrium corporate debt. The coefficient γ might be negative, as sovereign risk decreases with more liquidity. This implies that the OLS estimator might be biased downward, $\mathbb{E}[\hat{\beta}^{OLS}] < \beta$.

To estimate a causal relationship running from corporate debt to sovereign spreads, I use

a weighted sum of idiosyncratic productivity shocks to each country’s top 50 largest firms (sorted by sales in the previous year) as an instrument for that country’s aggregate corporate leverage. The application of this type of identification strategy to the spread regressions is motivated by the fact that large firms drove the aggregate corporate leverage cycle during this period. Figure 4 shows that the top 50 large non-financial firms in each Eurozone country on average increased their debt (both relative to value-added and in levels) prior to 2012, while small firms’ debt remained relatively stable. To my knowledge, this finding is novel to the literature.¹² To construct idiosyncratic shocks, I estimate the following firm-level productivity growth ($g_{i,t}$) decomposition for each country in the spirit of Gabaix and Koijen (2020):

$$g_{i,t} = \beta_s \eta_t + u_{i,t} \quad (5)$$

where $g_{i,t} = (z_{i,t} - z_{i,t-1}) / (0.5 \times (z_{i,t} + z_{i,t-1}))$, $z_{i,t}$ is firm-level productivity, η_t is an aggregate shock, and $u_{i,t}$ is an idiosyncratic shock to firm i at time t .

I assume that (i) the responsiveness (β_s) of firm-level productivity growth ($g_{i,t}$) with regard to an aggregate shock (η_t) is identical within a 4-digit sector s , and that (ii) aggregate and idiosyncratic shocks are separable in the growth decomposition. Under this assumption, regressing firm-level productivity growth on sector \times year fixed effects gives residuals ($\hat{u}_{i,t}$) that are consistent estimators for idiosyncratic shocks ($u_{i,t}$).¹³¹⁴ I use total factor productivity (TFP) estimated by the Wooldridge (2009) method as a productivity measure. These productivity measures are obtained from the ORBIS-AMADEUS database for firms in the six sample Eurozone countries, following Gopinath et al. (2017).

I construct the granular residual Γ_t as the weighted sum of idiosyncratic shocks to the top 50 firms for each country c , as follows:

¹²Adrian and Shin (2010) and Kalemli-Ozcan et al. (2012) find that large banks drive the procyclicality of aggregate bank leverage. Alfaro et al. (2019) find that highly levered large firms in emerging markets are more vulnerable to exchange rate shocks compared to small firms.

¹³Yeh (2019) uses U.S. Census Bureau sources that include the universe of employer firms and trade transactions at the firm-destination-year level. He controls for differential responses of firms across destinations to estimate firm-level idiosyncratic shocks. Gabaix and Koijen (2020) suggest joint estimation of firm-specific responsiveness (β_i), which is a function of firm characteristics, and residuals. I find that this procedure is not feasible in my firm-level dataset, as it requires fully-balanced firm-year observations. This procedure would drop a considerable amount of observations during the period 2000–2012, which would complicate the precise estimation of residuals. The assumption that aggregate and idiosyncratic shocks in growth are separable is consistent with a standard firm dynamics model in which aggregate and idiosyncratic shocks in levels are multiplicative in the production function.

¹⁴Regression (5) is estimated using a sample containing only the top 50 largest firms in each 4-digit sector for a given country. All residuals (growth rate) are winsorized at 20 and -20 percent following Gabaix (2011)’s calculations of “granular residuals”.

$$\Gamma_{c,t} = \sum_{i=1}^{50} \frac{\text{Sales}_{i,c,t-1}}{\text{GDP}_{c,t-1}} \hat{u}_{i,c,t} \quad (6)$$

where $\text{Sales}_{i,c,t-1}$ is sales of firm i in country c at time $t - 1$.¹⁵

The assumptions needed for identification are as follows: First, idiosyncratic firm-level productivity shocks are correlated with firm-level leverage. Second, large firms make up a substantial share of aggregate activity, so that the law of large numbers does not hold and $\Gamma_{c,t}$ will be relevant for aggregate leverage. Third, idiosyncratic shocks to large firms affect sovereign spreads only through their effect on corporate debt after controlling for alternative channels and thus are uncorrelated with unobservable shocks affecting sovereign spreads. This will imply exogeneity of the instrument. I expect idiosyncratic firm-level productivity shocks to firms will be negatively correlated with their leverage following the previous finance literature (Rajan and Zingales (1995), Harris and Raviv (1991), Titman and Wessels (1988) and others). This literature has established that profitability is negatively associated with leverage, since lower cash on hand implies more need to finance operating costs externally. As idiosyncratic productivity shocks are positively correlated with profitability, firms' optimal financing decisions in response to negative shocks can explain the negative within-firm correlation between leverage and idiosyncratic productivity shocks. Regarding the exclusion restriction, it is difficult to see how idiosyncratic shocks to large firms would affect sovereign spreads directly without working through firm variables. These idiosyncratic shocks can affect sovereign risk because they reduce aggregate output growth or because they increase aggregate corporate debt. As I control for aggregate output growth, while instrumenting for aggregate corporate debt, it is less likely that idiosyncratic shocks to large firms are correlated with other systematic unobservable shocks to government solvency. One concern is that negative shocks to firms can increase their probability of default on bank loans, and thus increase the probability of the government bailing out these banks. However, this does not necessarily mean that the instrument is invalid, as this banking channel works through rising corporate debt and associated corporate default risk, and the IV estimate can be interpreted as the impact of rising corporate debt on sovereign risk via this banking channel together with the tax revenue channel. Nevertheless, to tease out the tax revenue channel, I also control for bank leverage in some IV regressions.

In the IV regression, I use lagged granular residuals ($\Gamma_{c,t-1}$ and $\Gamma_{c,t-2}$) as excluded instruments for the lagged corporate debt to GDP ratio ($\text{Corp Debt}/\text{GDP}_{c,t-1}$). I use lagged values as instruments to better capture the relationship between firm-level productivity and firm borrowing, as lagged shocks are likely to be important determinants of firms' current borrowing decisions. I also include country fixed effects, year fixed effects, aggregate GDP growth,

¹⁵See [Data Appendix](#) for the discussion of the alternative measure of idiosyncratic shocks.

and government debt as controls in the regression. I run the following annual regression:

$$\begin{aligned} \text{Gov't Spread}_{c,t} = & \beta_y \text{GDP Growth}_{c,t-1} + \beta_{gov} \text{Govt Debt/GDP}_{c,t-1} \\ & + \beta_{corp} \text{Corp Debt/GDP}_{c,t-1} + \beta_{bank} \text{Bank Leverage}_{c,t-1} + \delta_c + \gamma_t + \epsilon_{c,t} \end{aligned} \quad (7)$$

where I instrument for corporate debt using the granular residuals $\Gamma_{c,t-1}$ and $\Gamma_{c,t-2}$. I add banking sector leverage as an additional control in subsequent regressions.

Table 2 presents IV regression results. In column (1), I reproduce the OLS regression results from column (7) in Table 1 using annual data, in which corporate debt is positively correlated with sovereign spreads. In column (2), I present IV regression results, which establish that corporate leverage causally increases sovereign spreads. These effects are both statistically and economically significant. A one standard deviation increase in the corporate debt-to-GDP ratio (23%p) increases sovereign spreads by about 345 basis points, which is significantly different from zero at the 5% significance level. Notice that the IV estimate is larger than the OLS estimate, consistent with the idea that the IV regression identifies variation in corporate leverage that is orthogonal to unobservable shocks to sovereign spreads that bias OLS estimates of β_{corp} downward. In column (3), I add bank leverage to the OLS regression in column (1). The results show that bank leverage is positively correlated with sovereign spreads, which is consistent with existing findings that the need for government to bail out financially distressed banks increased government default risk during the Eurozone debt crisis (see Acharya et al. (2014), Farhi and Tirole (2017), and many others). Column (4) presents results instrumenting corporate debt, while controlling for bank leverage. The estimates imply that a one standard deviation increase in the corporate debt-to-GDP ratio (23%p) increases sovereign spreads by about 253 basis points, and this effect is significant at the 5% level. The magnitude of the coefficient on corporate debt decreases after controlling for bank leverage. This suggests that the IV estimate on corporate debt not controlling for bank leverage includes the impacts of the corporate balance sheet channel working through bank balance sheets.

The first stage regression results reported in panel B of Table 2 suggest that the instruments are negatively correlated with corporate debt. Figure A.4 confirms that the instrument and corporate debt are negatively correlated, and that this correlation is not driven by outliers. The first-stage effective F statistic (calculated following Olea and Pflueger (2013)) reported in column (4) is 5.15. The rule-of-thumb threshold value is 10, above which instruments are considered as being highly correlated with the instrumented variables. While the coefficients on instruments are significant at the 5% level, the F statistic suggests that the instruments might be weak, and thus I conduct weak IV robust inference, as recommended by

Andrews et al. (2019) in the presence of weak instruments. I find that the p-value associated with these author’s proposed *CLR* statistic is 0.0628. This test rejects the null hypothesis that the coefficient on corporate debt is zero at the 10% significance level. The Hansen (1982) test of overidentifying restrictions also does not reject the null that instruments are excludable.

There might be concerns that estimates of equation (5) do not identify true idiosyncratic shocks $u_{i,t}$ if firms’ responsiveness (β_s) to aggregate shocks (η_t) is firm-specific. To address this concern, I relax one of the identification assumptions by allowing the response coefficient (β_s) in equation (5) to vary over firm size as well. Specifically, I regress firm-level productivity growth on sector×size×year fixed effects, where size dummies represent each quintile (q) of firm size. A caveat is that these estimates of idiosyncratic shocks are more likely to be imprecise, as the number of observations used to estimate the coefficient $\beta_{s,q}$ for each sector and size group becomes smaller. Estimated residuals from this regression are used to construct alternative granular residuals. Another concern is that weights for idiosyncratic shocks might drive results rather than idiosyncratic shocks themselves. Thus, I also construct alternative granular residuals as a simple average of idiosyncratic shocks to top 50 firms in each country. The other granular residuals are constructed using idiosyncratic shocks to top 100 firms in each country to test robustness of the instrument. Table A.5 presents IV regression results in which corporate debt is instrumented using the alternative measures. Results are robust. In general, the coefficients on corporate debt are still positive and are statistically and economically significant with bank leverage as a control. The instruments are significantly negatively correlated with corporate debt in the first-stage regressions. Figure A.3 plots estimated alternative residuals $\hat{u}_{i,t}$ (with sector×size×year fixed effects) for each top 50 firm i in each country in a given year t . There is no clustering of these residuals indicating a trend, which suggests that the residuals are independent of aggregate shocks, as implied by the estimation procedure itself. Table A.6 presents similar IV regression results replacing the ratio of corporate debt to GDP with the ratio of corporate debt to corporate value added. Results are robust to alternative corporate leverage. Even after including the ratio of household debt to GDP, the causal relationship between corporate debt and government spreads does not disappear as shown in Table A.7.

Next, I calculate the contributions of each explanatory variable to variation in sovereign spreads, using the IV regression results in column (4) of Table 2. First, I obtain predicted values of corporate debt-to-GDP ratios from the first-stage regression. Second, I purge sovereign spreads, GDP growth, predicted corporate debt-to-GDP ratios, sovereign debt-to-GDP ratios, and bank leverage of country- and year-fixed effects. To calculate the contribution of corporate debt, I multiply purged corporate debt by its coefficient $\hat{\beta}_{corp}$ and divide the variance of this multiplied term by the variance of purged sovereign spreads. I calculate the

contributions of sovereign debt, growth, and bank leverage in the same fashion. Corporate debt accounts for about 19% of variation in sovereign spreads, while government debt, bank leverage, and GDP growth explain around 20%, 6%, and 7% of this variation, respectively.

2.4 Tax Revenue IV Regression

The model outlined in the next section proposes that corporate risk is linked to sovereign risk through total tax revenues. Rising corporate default risk implies higher borrowing costs and lower aggregate output. This reduces the general tax base, including not only corporate taxes but also labor income taxes. To test this mechanism directly, I run the following annual regression using the same sample of six Eurozone countries:

$$\begin{aligned} \text{Tax Revenue}_{c,t} = & \beta_y \text{Growth}_{c,t-1} + \beta_{gov} \text{Govt Debt/GDP}_{c,t-1} \\ & + \beta_{corp} \text{Corp Debt/GDP}_{c,t-1} + \beta_{bank} \text{Bank Leverage}_{c,t-1} + \delta_c + \gamma_t + \epsilon_{c,t} \end{aligned} \quad (8)$$

where the dependent variable is now real tax revenue growth in country c at time t . Table 3 presents both OLS and IV results. Column (1) presents OLS regression and finds that corporate debt-to-GDP ratios are negatively correlated with one-year ahead tax revenue growth. These results suggest that expected government tax revenue becomes lower with rising corporate debt. My model below will provide a mechanism to explain this empirical finding, namely that rising corporate debt implies higher default rates on corporate taxes. In column (2), the dependent variable is one-year ahead tax revenue growth, and the corporate debt-to-GDP ratio is instrumented with the granular residuals as in Table 2. I find that a one standard deviation increase in corporate leverage (23%p) leads to a 7.4%p decrease in one-year ahead tax revenue growth, which is economically significant given that average tax revenue growth was only around 1% during the sample period. These effects are also statistically significant at the 1% level. The coefficients on the excluded instruments in the first-stage regressions are all significant, and the *CLR* statistics suggest that the IV results are robust to the presence of weak instruments. The Hansen (1982) test of overidentifying restrictions again does not reject the null that the granular residual instruments are excludable. In columns (3) OLS and (4) IV, I additionally control for contemporaneous GDP growth to tease out the tax revenue channel via pure firm default risk by removing the negative effects of corporate debt overhang on output. I find that a one standard deviation increase in corporate leverage (23%p) causes tax revenue growth to decrease by around 6.7%p, and this coefficient is significant at the 5% level. This suggests that corporate default risk captures expectations about future developments beyond current fundamentals. Moreover, Table A.8 adds future tax revenue growth in years t and $t + 1$ to sovereign spread regressions in

Table 2. After including tax revenue growth, the impacts of corporate debt on sovereign spreads are muted, suggesting that future tax revenue is an important channel to explain the relationship between corporate debt and sovereign spreads.

2.5 Firm-level Tax Revenue Regression

I run the following difference-in-difference regressions using annual firm-level observations for the same countries as in the previous IV regression:

$$\begin{aligned} \text{Tax Payment}_{i,t} = & \beta_1 \text{HighLev}_i \times \text{Crisis}_t + \beta_2 \log(z_{i,t-1}) + \beta_3 \log(k_{i,t-1}) \\ & + \beta_4 \log(1 + b_{i,t-1}) + \beta_5 \text{Interest Payment}_{i,t} + \delta_i + \gamma_{c,s,t} + \epsilon_{i,t} \end{aligned} \quad (9)$$

where $\text{Tax Payment}_{i,t}$ is the ratio of firm i 's tax payments to its value-added in year t . I include firm fixed effects δ_i and country \times sector \times year fixed effects $\gamma_{c,s,t}$. In different specifications, I replace country \times sector \times year fixed effects with country- and year-fixed effects or country \times year fixed effects. I use the four-digit sector NACE code to construct sector dummies. I use a sample covering 2004-2012 to exclude the recession in the early 2000s, which is likely to include other large exogenous shocks to firms and complicate the quasi-experiment using the 2008 crisis shock. Figure 5 shows that there is a parallel trend in tax payment rates between high-leverage and low-leverage firms before 2008, which validates the use of difference-in-difference regressions. I control for key firm-level variables (productivity z , capital k , and debt b) that are state variables in typical firm dynamics models. The Crisis_t dummy equals 1 in or after 2008 and 0 otherwise. The HighLev_i dummy equals 1 if the firm's average leverage before 2008 is higher than the aggregate median before 2008, and 0 otherwise. Leverage is measured as the ratio of financial debt to value added. I include the ratio of interest payment to value added to control for the effect of interest tax shields. See Table A.9 for firm summary statistics on this regression sample. I expect the coefficient on the interaction term $\text{HighLev}_i \times \text{Crisis}_t$ to be negative, meaning that highly-leveraged firms will decrease their tax payments more than less-leveraged firms in response to negative shocks from the 2008 global financial crisis. Columns (1)-(3) of Table 4 present results without interest payments as a control, while columns (4)-(6) take account of interest payments. I include country- and year-fixed effects in columns (1) and (4), country \times year fixed effects in columns (2) and (5), and country \times sector \times year fixed effects in columns (3) and (6). In all specifications, the coefficients on $\text{HighLev}_i \times \text{Crisis}_t$ are negative and statistically significant at the 1 percent level. Following the 2008 crisis, highly-leveraged firms decrease tax payment rates by around 0.38%p more than less-leveraged firms in the specification reported in column (3), reflecting increasing interest tax shields with higher leverage. This response reduces to 0.28%p when I control for interest payments. This effect is economically

significant, given that the average tax payment rate (relative to value-added) across firms was 4.15% in the regression sample during the period 2004-2012. Also, increases in the debt level are associated with lower tax payment rates, suggesting that tax revenues are expected to be low in general with higher corporate debt.

Table A.10 further controls for other state variables interacted with time dummies and shows that results for leverage are robust.¹⁶ Similarly, Kalemli-Ozcan et al. (2020) find that highly-leveraged firms tend to invest less in response to the 2008 global financial shocks, using ORBIS-AMADEUS data matched to banks' balance sheets (available from BANKSCOPE and ECB confidential data). Importantly, they show that corporate leverage is an important variable to account for sluggish post-crisis investment even after controlling for the bank lending channel using information on firm-bank relationships. This finding suggests that corporate indebtedness might also explain the behavior of firms' tax payments even after controlling for banks' balance sheets.

3 Model, Mechanism, and Identifying Externalities

3.1 Competitive Equilibrium

3.1.1 Firms

A continuum of firms with unit measure operate with a production function $F(z_t, k_t^i) = z_t(k_t^i)^\alpha$, where z_t is aggregate productivity, and k_t^i and b_t^i are firm-specific capital and one-period non-state-contingent debt, respectively. i refers to firms, and t is time. For notational simplicity, I omit superscripts i for firm variables unless stated otherwise. As discussed below, it is not necessary to track firm-specific capital k^i and debt b^i to infer the dynamics of aggregate variables, because all firms will choose the same levels of capital and debt. A firm's budget constraint is given by

$$e_t + b_t + i_t \leq (1 - \tau_t^y)F(z_t, k_t) + q(z_t, k_{t+1}, b_{t+1})b_{t+1} \quad (10)$$

where e_t are dividend payments, and investment is $i_t = k_{t+1} - (1 - \delta)k_t$ with a depreciation rate δ . $q(\cdot)$ is the schedule of firm debt price, which is determined by risk-neutral investors factoring in firm default risk. Firms take this schedule as given. τ_t^y is a corporate income tax rate, which firms also take as given.

Key aggregate state variables are productivity, capital, corporate and government debt,

¹⁶Using an alternative leverage measure such as a debt-to-tangible-fixed-assets ratio does not change the results.

and share purchases. I collect these state variables as $X = (z, k, b, B, s)$, which are sufficient statistics to determine the evolution of all relevant variables in this model. I assume that the supply of shares s is fixed at 1 for simplicity, which firms and the government internalize. Pension funds purchase these shares on behalf of households. Let x' denote the one-period ahead realization of the variable x . Firms' objective is to maximize the discounted flow benefits resulting from dividend payments e , as shown in the following recursive form:

$$V^f(k, b; X) = \max_{e, k', b'} \phi(e) + \mathbb{E} \left[m(X, X') \max_{d'_i} \langle V^f(k', b'; X'), \nu_i^{d'} \rangle \right] \quad (11)$$

subject to

$$e \leq (1 - \tau^y)F(z, k) + (1 - \delta)k - k' - b + q(z, k', b')b' \quad (12)$$

$$\phi(e) = e - \kappa(e - \bar{e})^2 \quad (13)$$

$$X' = \Gamma(X) \quad (14)$$

Firms take the stochastic discount factor $m(\cdot)$ as given. $\nu_i^{d'}$ is an idiosyncratic firm default shock (*i.i.d.* across time and firms) capturing the benefit from defaulting next period, generated from the cumulative distribution function, $\Omega(\nu_{i,t+1}^d)$. This default shock represents the limited liability of the owners of firms, where owners get the value of $\nu_i^{d'}$ when they decide to default on their firms' debt and stop paying dividends to households and taxes to the government. Equation (13) captures frictions in equity adjustment as in [Jermann and Quadrini \(2012\)](#), where \bar{e} is a steady-state level of dividend payments. Aggregate productivity z_{t+1} is drawn from the conditional cumulative distribution function, $\Pi(z_{t+1}|z_t)$. Stochastic variables belong to compact sets such that $\nu_{i,t+1}^d \in [\nu_{min}^d, \nu_{max}^d]$ and $z_{t+1} \in [z_{min}, z_{max}]$. Firms and the government choose their actions simultaneously, based on the forecast rule for aggregate state variables given by equation (14). Firms choose next period capital k_{t+1} and debt b_{t+1} , taking into account their effects on the firm debt price $q(z_t, k_{t+1}, b_{t+1})$ and their default cutoff $\bar{\nu}^d(k_{t+1}, b_{t+1}; X_{t+1})$ for each state z_{t+1} . The cutoff with regard to the firm default decision at time t is determined by

$$\bar{\nu}^d(k_t, b_t; X_t) \in \{ \nu_t^d | V^f(k_t, b_t; X_t) = \nu_t^d \} \quad (15)$$

and the corresponding firm default decision rule is given by

$$d(k_t, b_t, \nu_{i,t}^d; X_t) = \begin{cases} 1 \text{ (default), if } \nu_{i,t}^d \geq \bar{\nu}^d(k_t, b_t; X_t) \\ 0 \text{ (repay), otherwise} \end{cases} \quad (16)$$

When a firm defaults, creditors take over the firm as new owners and continue business as usual, issuing new debt and making investment. I assume that the internal funds that would have been used for dividend payments e_t , income taxes $\tau_t^y F(z_t, k_t)$, and debt repayment b_t had the firm not defaulted are used to pay bankruptcy costs, implying the following budget constraint for defaulting firms:

$$\underbrace{e_t + \tau_t^y F(z_t, k_t) + b_t}_{\text{bankruptcy costs}} + i_t \leq F(z_t, k_t) + q(z_t, k_{t+1}, b_{t+1})b_{t+1} \quad (17)$$

This budget constraint can be rewritten to be identical to the budget constraint (10) of non-defaulting firms, implying that the new owners of defaulting firms choose the same b_{t+1} and k_{t+1} as non-defaulting firms.¹⁷ For this reason, firm-specific capital k^i and debt b^i need not be tracked to infer the dynamics of aggregate variables, and I can omit superscript i . Bankruptcy costs capture not only direct administrative costs but also broadly defined macroeconomic costs entailed by bankruptcies.¹⁸

I assume that there is no passthrough of sovereign default risk into the corporate sector, and thus firm values $V^f(k, b; X)$ are independent of government debt B . This assumption implies that firm default decisions and debt prices do not depend on government debt choices, such that $d_t = d(z_t, k_t, b_t, \nu_{i,t}^d)$.

I use the multiplier λ_t for the firm budget constraint (12), which is interpreted as the shadow value of marginal funds. Henceforth, I assume that first-order conditions are necessary and sufficient and that all allocations are interior, implicitly imposing regularity assumptions such as concavity, monotonicity, and Inada conditions on $F(\cdot)$. Also, debt price functions are assumed to be differentiable. The optimality conditions for dividends, capital, and debt are as follows:

¹⁷Gomes et al. (2016) adopt a similar setting for computational tractability.

¹⁸See Hotchkiss et al. (2008) for the overview of corporate bankruptcy costs. Benmelech et al. (2018) and Bernstein et al. (2019) empirically show that there exist spillover costs of bankruptcies affecting local economies.

$$e_t \quad :: \quad \lambda_t = \phi_e(e_t) \quad (18)$$

$$k_{t+1} \quad :: \quad \left[1 - \frac{\partial q(z_t, k_{t+1}, b_{t+1})}{\partial k_{t+1}} b_{t+1}\right] \lambda_t = \int_{z_{min}}^{z_{max}} \int_{\nu_{min}^d}^{\bar{\nu}^d(X_{t+1})} [m(X_t, X_{t+1})((1 - \tau_{t+1}^y) F_k(z_{t+1}, k_{t+1}) + 1 - \delta) \lambda_{t+1}] d\Omega(\nu_{i,t+1}^d) d\Pi(z_{t+1}|z_t) \quad (19)$$

$$b_{t+1} \quad :: \quad \left[\frac{\partial [q(z_t, k_{t+1}, b_{t+1}) b_{t+1}]}{\partial b_{t+1}}\right] \lambda_t = \int_{z_{min}}^{z_{max}} \int_{\nu_{min}^d}^{\bar{\nu}^d(X_{t+1})} [m(X_t, X_{t+1}) \lambda_{t+1}] d\Omega(\nu_{i,t+1}^d) d\Pi(z_{t+1}|z_t) \quad (20)$$

Equation (18) shows that the shadow value of marginal funds must at an optimum equal the marginal equity adjustment cost, $\phi_e(e_t) = 1 - 2\kappa(e - \bar{e})$. The left-hand side of equation (19) represents the marginal cost of investment, adjusting for the effect of additional capital on the firm debt price and the shadow value of current funds λ_t . The right-hand side of the same equation captures the expected discounted marginal product of capital conditional on no firm default, adjusting for the shadow value of future funds λ_{t+1} . Firms choose capital so that the marginal cost equals the marginal benefit. Equation (20) shows that firms choose debt so that the additional funds that they can raise today (marginal benefit) equals the expected discounted debt payments conditional on no firm default (marginal cost), adjusting for the shadow values of funds today and tomorrow.

Risk-neutral lenders' profit from making a loan to the firm is given by

$$\pi_t = -q(z_t, k_{t+1}, b_{t+1}) b_{t+1} + \frac{1 - \int_{z_{min}}^{z_{max}} \mu(z_{t+1}, k_{t+1}, b_{t+1}) d\Pi(z_{t+1}|z_t)}{1 + r} b_{t+1} \quad (21)$$

where $\mu(z_{t+1}, k_{t+1}, b_{t+1}) = \int_{\bar{\nu}^d(z_{t+1}, k_{t+1}, b_{t+1})}^{\nu_{max}^d} d\Omega(\nu_{i,t+1}^d)$ is the default probability of the firm in the next period for each state z_{t+1} , and r is a risk-free rate. The zero-profit condition $\pi_t = 0$ pins down the firm debt price $q(\cdot)$.

3.1.2 Households

There is a continuum with unit measure of identical households. Household utility is given by

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t) \quad (22)$$

and the budget constraint is

$$C_t \leq s_t[(1 - \mu(z_t, k_t, b_t))\phi(e_t) + p_t] - s_{t+1}p_t + G_t \quad (23)$$

Households hold shares of firms s_t which pay dividends net of equity adjustment costs $\phi(e_t)$ and can be sold at a price p_t . On behalf of households, public pension funds purchase corporate shares s_{t+1} at a price p_t . The pension funds' problem will be defined after the government problem.¹⁹ $\mu(z_t, k_t, b_t)$ is the fraction of firms defaulting on their debt. Shares issued by defaulting firms pay zero dividends. Notice that this fraction of defaulting firms equals the firm default probability, determined by firms' optimality conditions, as the law of large numbers holds with a continuum of firms. G_t is lump-sum transfers (or taxes) to households from the government.²⁰

3.1.3 The Government

The government's objective is to maximize households' utility after taking into account the costs of government default. The government chooses transfer payments G and government debt B' and chooses whether to default D' as follows:

$$V^g(B; X) = \max_{G, B'} u(C) + \beta \mathbb{E} \max_{D'} \langle V^g(B'; X'), V^g(0; X') - \xi' \rangle \quad (24)$$

subject to the dividend equation (12), the household budget constraint (23), and

$$G + B \leq (1 - \mu(z, k, b))\tau^y F(z, k) + Q(z, k', b', B')B' \quad (25)$$

$$X' = \Gamma(X) \quad (26)$$

$$p = p(X, X') \quad (27)$$

where ξ is an *i.i.d.* government default cost shock, which is microfounded in standard

¹⁹Households could instead purchase shares directly, and the main results would not change. The purpose of having pension funds is to show the case in which the first welfare theorem holds without firm and government default risk (and without tax distortions), when the social planner's objective is to maximize households' utility after adjusting for government default costs. Pension funds take into account government default costs on top of households' utility as the social planner does, which is the reason why pension funds are required to compare market allocations with social planner's allocations.

²⁰The government could finance a fixed path of final good spending instead of financing transfers. However, this assumption would generate distortions – due to changes in tax rates needed to finance the exogenous government expenditure – that the Ramsey planner would need to address. The objective of this paper is to study optimal policy to address externalities, rather than distortions that arise in a public finance problem without lump-sum taxes, and hence I abstract from this traditional public finance problem.

sovereign default models as reputation costs or punishment by creditors. In equation (25), the government collects taxes $(1 - \mu(z, k, b))\tau^y F(z, k)$ from non-defaulting firms and issues one-period non-state-contingent debt B' at a price Q . The government forecasts aggregate state variables using equation (26). The share price equation (27) is determined by the pension funds' problem described below.

When the government defaults, its debt obligation becomes zero, $B' = 0$, and the government continues with a value of $V^g(B' = 0; X')$ but pays default costs ξ' . The threshold government default cost shock $\bar{\xi}$ with regard to government default D at time t is

$$\bar{\xi}(B_t; X_t) \in \{\xi_t | V^g(B_t; X_t) = V^g(0; X_t) - \xi_t\} \quad (28)$$

and the corresponding government default decision rule is given by

$$D(B_t, \xi_t; X_t) = \begin{cases} 1 \text{ (default), if } \xi_t \leq \bar{\xi}(B_t; X_t) \\ 0 \text{ (repay), otherwise} \end{cases} \quad (29)$$

The government debt is priced by risk-neutral competitive lenders as below:

$$Q(z_t, k_{t+1}, b_{t+1}, B_{t+1}) = \frac{1 - \int_{z_{t+1}} \int_{\xi_{t+1}} D(B_{t+1}, \xi_{t+1}; X_{t+1}) d\Pi(z_{t+1}|z_t) d\Xi(\xi_{t+1})}{1 + r} \quad (30)$$

where Π and Ξ are cumulative distributive functions for productivity and government default costs, respectively.

The associated government spread is

$$SPRD(z_t, k_{t+1}, b_{t+1}, B_{t+1}) = \frac{1}{Q(z_t, k_{t+1}, b_{t+1}, B_{t+1})} - (1 + r) \quad (31)$$

The government takes into account its effects on the default cutoff $\bar{\xi}(B_t; X_t)$. The optimality condition for government debt B_{t+1} is given by

$$B_{t+1} :: \frac{\partial [Q(z_t, k_{t+1}, b_{t+1}, B_{t+1}) B_{t+1}]}{\partial B_{t+1}} u'(C_t) = \beta \left[\int_{z_{min}}^{z_{max}} \int_{\bar{\xi}(X_{t+1})}^{\xi_{max}} u'(C_{t+1}) \right] d\Pi(z_{t+1}|z_t) d\Xi(\xi_{t+1}) \quad (32)$$

The left-hand side of equation (32) shows the marginal benefit of financing additional household consumption with the marginal funds raised by government borrowing, accounting for the impact of government borrowing on its bond price. The right-hand side represents the

expected discounted marginal cost of paying debt and the resulting decreases in households' future consumption, conditional on no government default. These benefits and costs are measured in terms of households' marginal utility $u'(C)$.

3.1.4 Pension Funds

Pension funds purchase shares s_{t+1} on behalf of households to maximize the households' utility as in the government problem after firms choose their capital k_{t+1} and debt b_{t+1} , and the government chooses its debt B_{t+1} and expenditure G_t . This pension fund problem is given by

$$V^g(B; X) = \max_{s'} u(C) + \beta \mathbb{E} \max_{D'} \langle V^g(B'; X'), V^g(0; X') - \xi \rangle \quad (33)$$

subject to the dividend equation (12), equity adjustment costs (13), and the following household budget constraint

$$C = s([1 - \mu(X)]\phi(e) + p) - s'p + G \quad (34)$$

$$(35)$$

where k', b', B', G are taken as given by pension funds. The pension funds' first-order condition with regard to s' gives the following share price equation:

$$p(X, X') = \int_{z_{min}}^{z_{max}} \int_{\xi_{min}}^{\xi_{max}} m(X, X') [[1 - \mu(z', k', b')] \phi(e') + p(X', X'')] d\Pi(z'|z) d\Xi(\xi') \quad (36)$$

where $m(X, X') = \beta \frac{u'(C')}{u'(C)}$ is a stochastic discount factor. It can be shown that this stochastic discount factor is identical to the one in the firm value function (11), in which firms maximize the shareholders' net present value of $\phi(e) + p(X, X')$.

3.1.5 Equilibrium

The equity market clears in equilibrium as follows:

$$s_t = 1 \quad (37)$$

where each firm issues one unit of shares to households for every period t , and in turn

total share supply is 1 when there is a continuum of firms with unit measure. I define a recursive competitive equilibrium of the model as follows.

Definition 1 *A recursive competitive equilibrium consists of firms' equity value $\{V^f(X)\}$ and policies $\{e(X), k'(X), b'(X), d'(X)\}$, the government's value $V^g(X)$ and its policies $\{B'(X), D'(X), G(X)\}$, households' policy $\{C(X)\}$, the pension funds' policy $\{s'(X)\}$ and the associated share price equation $\{p(X, X')\}$, debt prices $\{q(z, k', b'), Q(z, k', b', B')\}$ and the fraction of defaulting firms $\{\mu(X)\}$, and a stochastic discount factor $\{m(X, X')\}$, given the corporate income tax rate $\{\tau^y\}$ and laws of motion for aggregate productivity z , firm default shocks ν_i^d , and the government default cost shocks ξ , such that (a) firms' policies and equity value solve (11); (b) the government solves (24); (c) households solve (22); (d) pension funds solve (33); (e) the equity market clears, $s(X) = 1$; (f) the stochastic discount factor for firms is given by the households' marginal rate of substitution, $m(X, X') = \beta \frac{u'(C')}{u'(C)}$; (g) the law of large numbers holds, $\mu(X) = \int_{\bar{\nu}^d(X)}^{\nu^{d, max}} d\Omega(\nu_i^d)$; and (h) the actual law of motion for the aggregate variables X is consistent with the forecasting rule (14) and stochastic processes for z , ν_i^d , and ξ .*

3.2 Corporate-Sovereign Debt Nexus

Government tax revenue in the next period is given by

$$TR_{t+1} = [1 - \mu(z_{t+1}, k_{t+1}, b_{t+1})][\tau_{t+1}^y F(z_{t+1}, k_{t+1})] \quad (38)$$

Rising corporate debt implies higher firm default risk μ_{t+1} and lower expected tax revenues $\mathbb{E}_t[TR_{t+1}]$. The following proposition states that expected future tax revenue falls when corporate debt increases today.

Proposition 1 *The distribution of tax revenue TR_{t+1} in an economy with low firm debt b_{t+1} first-order stochastically dominates the one in an economy with high firm debt b_{t+1} . This implies that the realization of tax revenue TR_{t+1} with higher firm debt b_{t+1} is lower over the entire distribution of subsequent shocks to productivity z_{t+1} .*

Proof: See [Appendix A](#) ■

When tax revenue decreases, the government might not be able to finance transfers that are essential to households. Transfers to households are essential, especially when the marginal utility of consumption for households is high in bad times. The government is thus more likely to prioritize transfers to households by defaulting on debt during bad times. Hence, government default serves as an insurance device to households. Moreover,

households are more likely to have a high marginal utility of consumption when more firms default, since defaulting firms lead to bankruptcy costs in the economy. The following proposition formalizes this risk spillover from the corporate sector to the government.

Proposition 2 *Suppose surviving firms pay non-negative net dividends $\phi(e_t) \geq 0$. If firm debt b_{t+1} increases, firm default risk $\mu(z_{t+1}, k_{t+1}, b_{t+1})$ (weakly) rises, and the government debt price $Q(z_t, k_{t+1}, b_{t+1}, B_{t+1})$ (weakly) decreases (the government spread increases).*

Proof: See [Appendix B](#) ■

The causal linkage that rising corporate debt implies higher sovereign risk is reinforced by overborrowing externalities. I now identify externalities by comparing the market allocations of the competitive equilibrium with the constrained-efficient allocations.

3.3 Constrained Efficiency

3.3.1 Constrained Social Planner

I consider a constrained social planner who can choose $\{k_{t+1}, b_{t+1}, e_t, B_{t+1}, D_{t+1}\}$ directly, but who is constrained to allow firms to choose whether to default or not, and faces the equity adjustment cost (13) as well as risk-neutral debt price schedules. The social planner's problem is as follows:

$$V^g(k, b, B; z) = \max_{k', b', e, B'} u(C) + \beta \mathbb{E} \max_{D'} \langle V^g(k', b', B'; z'), V^g(k', b', 0; z') - \xi' \rangle \quad (39)$$

subject to

$$C = [1 - \mu(z, k, b)](F(z, k) + (1 - \delta)k - k' - b + q(z, k', b')b' - \kappa(e - \bar{e})^2) + Q(z, k', b', B')B' - B \quad (40)$$

where the fraction of defaulting firms, $\mu(z, k, b) = \int_{\bar{\nu}^d(X_t)}^{\nu_{max}^d} d\Omega(\nu_{i,t}^d)$, and the firm bond price $q(z, k', b')$ are determined by (15), (16) and the associated pricing equation (21); the government bond price $Q(z, k', b', B')$ is determined by government default decisions according to equation (28) and (29), and the associated pricing equation (30); the resource constraint (40) combines budget constraints of firms (12), households (23), and the government (25); and the equity market clearing condition is given by $s = s' = 1$.

I focus on the allocations for key state variables $\{k_{t+1}, b_{t+1}, B_{t+1}\}$, which together with aggregate productivity z_{t+1} summarize all allocations in the recursive problem. For convenience, define firms' cash on hand before taxes as $e^{SP} = F(z, k) + (1 - \delta)k - k' - b + q(z, k', b')b'$. The optimality conditions from the social planner's problem are as follows:

$$\begin{aligned}
k_{t+1} &:: \underbrace{\left(1 - \frac{\partial q(z_t, k_{t+1}, b_{t+1})}{\partial k_{t+1}} b_{t+1}\right)}_{\text{private marginal cost of capital}} \underbrace{\left(1 - \mu(z_t, k_t, b_t)\right)}_{\text{externality}} - \underbrace{\frac{\partial Q(z_t, k_{t+1}, b_{t+1}, B_{t+1})}{\partial k_{t+1}} B_{t+1}}_{\text{externality}} \\
&= \int_{z_{\min}}^{z_{\max}} \int_{\xi_{\min}}^{\xi_{\max}} m(X_t, X_{t+1}) \left[\underbrace{\left(1 - \mu(z_{t+1}, k_{t+1}, b_{t+1})\right) (F_k(z_{t+1}, k_{t+1}) + 1 - \delta)}_{\text{private marginal product of capital without income taxes}} \right. \\
&\quad \left. - \underbrace{\frac{\partial \mu(z_{t+1}, k_{t+1}, b_{t+1})}{\partial k_{t+1}} e_{t+1}^{SP}}_{\text{externality}} \right] d\Pi(z_{t+1}|z_t) d\Xi(\xi_{t+1}) \tag{41}
\end{aligned}$$

$$\begin{aligned}
b_{t+1} &:: \underbrace{\frac{\partial [q(z_t, k_{t+1}, b_{t+1}) b_{t+1}]}{\partial b_{t+1}}}_{\text{private marginal benefit of borrowing}} \times \underbrace{\left(1 - \mu(z_t, k_t, b_t)\right)}_{\text{externality}} + \underbrace{\frac{\partial Q(z_t, k_{t+1}, b_{t+1}, B_{t+1})}{\partial b_{t+1}} B_{t+1}}_{\text{externality}} = \\
&\int_{z_{\min}}^{z_{\max}} \int_{\xi_{\min}}^{\xi_{\max}} m(X_t, X_{t+1}) \left[\underbrace{1 - \mu(z_{t+1}, k_{t+1}, b_{t+1})}_{\text{private marginal cost of borrowing}} + \underbrace{\frac{\partial \mu(z_{t+1}, k_{t+1}, b_{t+1})}{\partial b_{t+1}} e_{t+1}^{SP}}_{\text{externality}} \right] d\Pi(z_{t+1}|z_t) d\Xi(\xi_{t+1}) \tag{42}
\end{aligned}$$

$$B_{t+1} :: \frac{\partial [Q(z_t, k_{t+1}, b_{t+1}, B_{t+1}) B_{t+1}]}{\partial B_{t+1}} u'(C_t) = \beta \int_{z_{\min}}^{z_{\max}} \int_{\bar{\xi}(X_{t+1})}^{\xi_{\max}} u'(C_{t+1}) d\Pi(z_{t+1}|z_t) d\Xi(\xi_{t+1}) \tag{43}$$

where the social planner optimally sets $e = \bar{e}$ to minimize equity adjustment costs. Comparing the social planner's optimality conditions (41) and (42) with their market counterparts (19) and (20), the externality terms in (41) and (42) show that firms do not internalize the effects of their capital and debt choices on the future equilibrium firm default rate μ_{t+1} and the associated total bankruptcy costs resulting from more firm defaults given their limited liability. Also, firms do not internalize their effects on bankruptcy costs of each firm given the current firm default rate μ_t . The limited liability externality is distinct from externalities identified in the literature on optimal macroprudential policy in the sense that the externality terms due to firm default risk do not disappear even when firms internalize their effects on the firm debt price. In this setting, firm default serves as a put option for firms, and in turn firms are likely to take more risk than is socially optimal. In contrast, a pecuniary externality works because firms do not internalize their effects on asset prices that determine the tightness of their borrowing constraints. Also, in my model, firms do not internalize their effects on the government bond price Q , which appear as externality terms in conditions (41)

and (42). In these terms, $\frac{\partial Q_{t+1}}{\partial k_{t+1}}$ and $\frac{\partial Q_{t+1}}{\partial b_{t+1}}$ are not zero. The reason is that Q does not enter the firms' budget constraint. Without limited liability, externality terms regarding firm default rates μ_t and μ_{t+1} would disappear. Without limited liability and government default risk, all externality terms would disappear. The government's optimality condition (43) is identical to condition (32) in the competitive equilibrium. The fraction of defaulting firms in the next period $1 - \mu_{t+1}$ is a part of the private marginal product of capital or marginal cost of borrowing, since firms take into account their default probabilities, which equal $1 - \mu_{t+1}$ due to the law of large numbers.

3.4 Optimal Policy

3.4.1 Regulated Competitive Equilibrium

Next, I consider a decentralized economy in which debt taxes τ^b , transfers to firms T , and investment credits τ^k can be implemented. The Ramsey planner (or the government) chooses its policies $(\tau_t^b, T_t, \tau_t^k)$ first, and then firms choose their actions taking policies as given. The firms' budget constraint is given by

$$e_t = (1 - \tau_t^y)F(z_t, k_t) + (1 - \delta)k_t - (1 - \tau_t^k)k_{t+1} - b_t + (q(z_t, k_{t+1}, b_{t+1}) - \tau_t^b)b_{t+1} + T_t \quad (44)$$

where the path of the corporate income tax rate τ_t^y is given.²¹

Firms' optimality conditions for k_{t+1} and b_{t+1} are given by

$$k_{t+1} \quad :: \quad \left[1 - \tau_t^k - \frac{\partial q(z_t, k_{t+1}, b_{t+1})}{\partial k_{t+1}} b_{t+1}\right] \lambda_t = \int_{z_{min}}^{z_{max}} \int_{\nu_{min}^d}^{\bar{\nu}^d(X_{t+1})} [m(X_t, X_{t+1})((1 - \tau_{t+1}^y)F_k(z_{t+1}, k_{t+1}) + (1 - \delta)) \lambda_{t+1}] d\Omega(\nu_{i,t+1}^d) d\Pi(z_{t+1}|z_t) \quad (45)$$

$$b_{t+1} \quad :: \quad \left(\frac{\partial [q(z_t, k_{t+1}, b_{t+1}) b_{t+1}]}{\partial b_{t+1}} - \tau_t^b\right) \lambda_t = \int_{z_{min}}^{z_{max}} \int_{\nu_{min}^d}^{\bar{\nu}^d(X_{t+1})} [m(X_t, X_{t+1}) \lambda_{t+1}] d\Omega(\nu_{i,t+1}^d) d\Pi(z_{t+1}|z_t) \quad (46)$$

The government budget constraint is

²¹The corporate income tax rate τ_t^y could also be implemented optimally by a planner. Rather, I assume that τ_t^y follows a fixed path to consider the case when transfers T_t are flexibly adjusted by the planner, which belongs to a more realistic set of policies.

$$B_t + G_t = (1 - \mu(z_t, k_t, b_t))[\tau_t^y F(z_t, k_t) - \tau_t^k k_{t+1} + \tau_t^b b_{t+1} - T_t] + Q(z_t, k_{t+1}, b_{t+1}, B_{t+1})B_{t+1} \quad (47)$$

where firms cannot receive investment credits $\tau_t^k k_{t+1}$ or transfers T_t if they default on income taxes $\tau_t^y F(z_t, k_t)$ and debt taxes $\tau_t^b b_{t+1}$.

By combining (44), (47), and the household budget constraint (23), I get the associated resource constraint as follows:

$$C_t = (1 - \mu(z_t, k_t, b_t)) [F(z_t, k_t) + (1 - \delta)k_t - k_{t+1} - b_t + q(z_t, k_{t+1}, b_{t+1})b_{t+1} - \kappa(e_t - \bar{e})^2] + Q(z_t, k_{t+1}, b_{t+1}, B_{t+1})B_{t+1} - B_t \quad (48)$$

This equation is identical to the resource constraint that the social planner faces. Following definitions and a proposition characterize the Ramsey planner's policy.

Definition 2 *A regulated competitive equilibrium is a sequence of prices $\{q_t, Q_t, p_t\}$, allocations $\{e_t, k_{t+1}, b_{t+1}, d_{t+1}^i, C_t, s_{t+1}\}$, and government policies $\{\tau_t^y, T_t, \tau_t^k, \tau_t^b, B_{t+1}, D_{t+1}, G_t\}$ such that (i) the modified firms' budget constraint (44) and firm optimality conditions (45) and (46) are satisfied; (ii) the modified government budget constraint (47) is satisfied; and (iii) all remaining conditions defined in the competitive equilibrium (Definition 1) are satisfied.*

Definition 3 *The Ramsey problem is to solve the problem (24) over the regulated competitive equilibrium by setting optimal paths for policy instruments $\{T_t, \tau_t^k, \tau_t^b, B_{t+1}, D_{t+1}, G_t\}$ given the path of the corporate income tax rate $\{\tau_t^y\}$.*

Proposition 3 *The Ramsey planner implements policy instruments $\{T_t, \tau_t^k, \tau_t^b, B_{t+1}, D_{t+1}, G_t\}$ such that the outcome of the Ramsey problem solves the problem of the constrained social planner (39). The implemented set of policies is time-consistent.*

Proof: See [Appendix C](#) ■

Notice that the Ramsey planner and the social planner face common constraints, including the firm and government debt pricing equations and equity adjustment costs. The only difference is that the social planner can directly allocate firm capital $\{k_t\}$, debt $\{b_t\}$, and dividends $\{e_t\}$, while the Ramsey planner indirectly chooses the path of these three variables mainly by adjusting three policy instruments $\{\tau_t^k, \tau_t^b, T_t\}$. Thus, it can be shown that the Ramsey planner solves the social planner problem, which leads to Pareto-optimal

allocations under the common constraints. Optimal policies $\{\tau_t^k, \tau_t^b, T_t\}$ are obtained by plugging the social planner’s allocations into the optimality conditions ((45) and (46)) and the firms’ budget constraint (44). These are time-consistent policies because the policy functions are solutions to the recursive equilibrium, which implies that optimal paths for policies are invariant in every period. These policies are also called Markov stationary policy, as the planner does not have the incentive to deviate from policy rules determined by a Markov perfect equilibrium.²²

3.5 Discussion of Assumptions

1. I have assumed that there is no passthrough of the sovereign debt price $Q(\cdot)$ into the private capital market to highlight the key mechanism causally linking corporate debt to sovereign spreads. This assumption is also consistent with my empirical analysis, in which I exclude post-crisis observations and use instrumental variables to overcome reverse causality running from sovereign spreads to corporate debt. This assumption can be relaxed by allowing firms to lose access to financial markets when the government defaults, as in [Mendoza and Yue \(2012\)](#). This setting will generate an endogenous firm debt price that decreases in government debt, generating endogenous costs of government default. These endogenous government default costs will be another externality that firms do not internalize.
2. I have assumed that ν_i^{df} is an *i.i.d.* firm default shock for computational tractability. As in [D’Erasmus and Boedo \(2012\)](#), this assumption can be relaxed so that firms compare the continuation value of repayment against the value of liquidating capital and defaulting. However, relaxing this assumption is not likely to change the results, as liquidation also serves as a put option for firms and encourages overborrowing.
3. I have assumed that ξ represents the costs of repudiating government debt. This assumption can be microfounded and is commonly used in the literature to capture sovereign default costs associated with reputation, roll-over risk, and other factors (see, for example, [Arellano et al. \(2020\)](#)).
4. I have abstracted from labor income taxes and capital adjustment costs. In the next section, I add labor income tax revenues and firms’ capital adjustment costs to the quantitative model in a simple way.

²²See [Bianchi and Mendoza \(2018\)](#) for more discussions about the properties of both time-consistent macroprudential policies and time-inconsistent policies under commitment.

4 Quantitative Model and Policy Analysis

4.1 Additional Ingredients

To better explain the data, I add additional ingredients to the model including capital adjustment costs, labor income, and debt recovery rates. Capital adjustment costs paid by firms are given by

$$\Psi(k_t, k_{t+1}) = \psi k_t \left(\frac{k_{t+1} - (1 - \delta)k_t}{k_t} \right)^2 \quad (49)$$

Firms pay the fraction θ of output $F(z, k)$ to households as wages, which are not defaultable. The government receives labor income taxes $\tau^l \theta F(z, k)$ from households. Investors get fractions R^f and R^g of defaulted debt from firms and the government, respectively. Also, for computational tractability, I assume that the firms' stochastic discount factor is given by $m(X, X') = \beta$. This type of assumption on stochastic discount factors is common in the small open economy literature in that the discount factor is not fully determined by domestic factors. The productivity process is as follows:

$$\log(z_{t+1}) = \mu_z + \rho_z \log(z_t) + \sigma_z u_{t+1} \quad (50)$$

where u_{t+1} follows a standard normal distribution, and $\mu_z = \frac{-\sigma_z^2}{2(1+\rho_z)}$ so that the level of z equals one on average.

Following [Arellano \(2008\)](#), I assume that investors price government debt as follows:

$$Q(z_t, k_{t+1}, b_{t+1}, B_{t+1}) = \int_{z_{t+1}} \int_{\xi_{t+1}} M(z_{t+1}, z_t) [1 - (1 - R^g)D(B_{t+1}, \xi_{t+1}; X_{t+1})] d\Pi(z_{t+1}|z_t) d\Xi(\xi_{t+1}) \quad (51)$$

where $M(z_{t+1}, z_t) = \frac{1}{1+r} - \gamma v_{t+1}$ is a stochastic discount factor, and v_{t+1} is an innovation to productivity, $\log(z_{t+1}) - \log(z_t)$. γ is non-negative and represents the sensitivity of the investors' sentiment. This assumption implies that debt repayments in relatively bad states are more valuable to investors than those in good states.

4.2 Numerical Solution

I solve the model using value function iteration. Key state variables $X = (z, k, b, B)$ are discretized so that firms and the government choose their actions out of a finite set of options. The potential challenge facing this solution method is that the existence of equilibrium is not guaranteed when two types of players move simultaneously and can only use pure strategies.

To overcome this problem and improve the convergence of the algorithm, I adopt a discrete choice model with taste shocks such that firms and the government can assign probabilities to their actions. To be specific, I add taste shocks ϵ' associated with the choice of capital k' , firm debt b' , and firm default d' to firms' value functions, and taste shocks ϵ' associated with the choice of government debt B' to the government's value function. Firms and the government choose the probability of taking certain actions out of a choice set before taste shocks realize. Realized taste shocks determine actual actions of firms and the government. This type of algorithm has recently been formalized by [Dvorkin et al. \(forthcoming\)](#). See [Appendix D](#) for more details.

4.3 Calibration

The model is solved for the competitive equilibrium and calibrated to six European countries during the period 2000-2012. The six countries are those included in the main IV spread regression. [Table 5](#) presents the parameter values assigned for the quantitative model. First, I set four structural parameters following the standard literature. The capital income share α and labor income share θ are set to 0.35 and 0.5, respectively. Households' preferences are given by the following CRRA utility function:

$$u(C_t) = \frac{C_t^{1-\nu}}{1-\nu} \quad (52)$$

where I assume a risk aversion parameter ν of 2. The capital depreciation rate δ is set to 0.06 at an annual frequency. Following [Arellano et al. \(2019\)](#), corporate and government discount factors β and β_g are set to 0.98 and 0.90, respectively, as governments are usually considered less patient than the private sector. As firms are more patient than the government, the overborrowing problem for firms is less severe, and thus welfare gains from correcting overborrowing externalities are lower with this parameterization. The annual risk-free rate is set to 1.9 percent, which is the average German nominal interest rate minus the average inflation rate of the six sample countries during 2000-2012. The corporate income tax rate is set to 32.7 percent, using the average combined income tax rates of the six sample countries during 2000-2012 (OECD corporate tax statistics database).²³ The government debt recovery rate R^g is set to 63 percent, based on the average global government debt haircut ratio ([Cruces and Trebesch, 2013](#)), and the corporate debt recovery rate R^f is set to 70 percent, based on the average debt recovery rate for the six sample countries (World Bank Doing Business database).²⁴ I set the equity issuance cost parameter κ to 0.426 following

²³OECD (2020), Corporate Tax Statistics Database, <https://www.oecd.org/tax/beps/corporate-tax-statistics-database.htm>

²⁴World Bank (2020), Doing Business 2020, <https://www.doingbusiness.org/>

Jermann and Quadrini (2012). The volatility of taste shocks is set to 0.001, a sufficiently small number such that solutions to the model with taste shocks are close to those without taste shocks. This parameter value is commonly used in the literature (see Dvorkin et al. (forthcoming)).

As shown in Panel B of Table 5, I set the remaining parameters by matching moments observed in data to those obtained in the model simulation. I set the persistence of the productivity process ρ_z by targeting the autocorrelation of log GDP, where the autocorrelation is obtained by regressing log real GDP on its own lag (after pooling country-year observations of the sample six countries and controlling for country fixed effects). Productivity volatility σ_z is set to match the standard deviation of the same data on log GDP. I set the capital adjustment cost parameter ϕ by targeting the standard deviation of log gross fixed assets (divided by the GDP deflator) of the six sample countries (after pooling country-year observations and controlling for country fixed effects). The log GDP series and log gross fixed assets are initially detrended using the Hamilton (2018) filter. The mean parameter of the corporate default shock ν^d is set to target the average corporate spread of Euro area non-financial corporate bonds during the period 2000-2012, using spread data (Euro area Bund NFC) obtained from Gilchrist and Mojon (2017). Similarly, the mean and standard deviation of the government default cost ξ are set by targeting the mean and standard deviation of government spreads for the six sample countries during the period 2000-2012. To better match the standard deviation of government spreads, I adjust the sensitivity of investors' sentiment γ .²⁵

Table 6 presents moments calculated from model simulations and data. To obtain model moments, I simulate the model economy for 10,000 periods and take averages of moments from 1,000 simulations after dropping the first 500 periods and excluding periods of government default for each simulation. All targeted moments from the model and data are close to each other. The table also shows non-targeted moments from the simulations and data, which are reasonably close to each other. For example, the annual corporate default rate is 4.4% in the model and 4.1% in the data.²⁶ Also, the government default rate is 1.9% in the model and 1.5% in data.²⁷ I reproduce the government spread regression using simulated data, treating variables in the same way as in the actual data. I report the 5th and 95th

²⁵ Aguiar et al. (2016) find that increases in the standard deviation of government spreads are associated with increases in the average government spread in standard sovereign default models, and it is difficult to match both the mean and standard deviation without modeling a time-varying risk premium. As shown by Chatterjee and Eyigungor (2012), adding long-duration bonds and asymmetric output costs caused by government default can solve this puzzle in an endowment economy.

²⁶ The annual corporate default rate in the data is measured as the historical global corporate default rate (speculative-grade) obtained from Moody's (2020).

²⁷ The government default rate in the data is the average default rate of the six sample Eurozone countries during 1900–2014, calculated using Table 6.4 of Reinhart and Rogoff (2009).

percentiles of the model-simulated estimates out of 1,000 simulations in brackets, together with their median reported above the brackets. The model-implied regression coefficients qualitatively match the main IV regression coefficients from column 4 of Table 2, confirming that this model is a good representation of the data generating process.

I quantify the magnitude of overborrowing externalities using the optimality condition for firm borrowing b' presented in equation (42). I calculate each term in equation (53) (shown below), using the model simulations in the competitive equilibrium as before, and I obtain the values for each term as the average across simulations. I normalize these values so that the private marginal benefit and cost of borrowing are 100, respectively. The social marginal benefit is the sum of the (i) private marginal benefit, (ii) the externality associated with increasing bankruptcy costs regarding debt issuance of each firm given the current firm default rate μ_t (externality A), and (iii) the externality associated with the falling government debt price (externality B). I find that the social marginal benefit of borrowing is about 5.1% lower than the private marginal benefit, and that the externality term A accounts for about 84% of this difference ($=4.3/5.1$), while the externality term B only explains 16%. At the same time, the social marginal cost of borrowing is 2.8% higher than the private marginal cost, where the externality term associated with the rising future firm default rate μ_{t+1} and the resulting increase in bankruptcy costs (externality C) solely explains this difference. These results show that externality terms (A and C) associated with firm default rates drive most of the overborrowing externalities quantitatively, while the externality term B – directly associated with the government bond price – does not play a large role. Without the limited liability associated with the firm default shock ν_i^d , the externality terms A and C would disappear. Wu (2020) identifies the externality term B without allowing for limited liability of firms.

$$\begin{aligned}
& \overbrace{\frac{\partial[q(z_t, k_{t+1}, b_{t+1})b_{t+1}]}{\partial b_{t+1}} - \mu(z_t, k_t, b_t) \frac{\partial[q(z_t, k_{t+1}, b_{t+1})b_{t+1}]}{\partial b_{t+1}} + \frac{\partial Q(z_t, k_{t+1}, b_{t+1}, B_{t+1})}{\partial b_{t+1}} B_{t+1}}^{\text{social marginal benefit of borrowing} = 94.9} \\
& \underbrace{\frac{\partial[q(z_t, k_{t+1}, b_{t+1})b_{t+1}]}{\partial b_{t+1}}}_{\text{private marginal benefit} = 100} \quad \underbrace{- \mu(z_t, k_t, b_t) \frac{\partial[q(z_t, k_{t+1}, b_{t+1})b_{t+1}]}{\partial b_{t+1}}}_{\text{externality A} = -4.3} \quad \underbrace{+ \frac{\partial Q(z_t, k_{t+1}, b_{t+1}, B_{t+1})}{\partial b_{t+1}} B_{t+1}}_{\text{externality B} = -0.8} \\
& = \mathbb{E} \left\langle \underbrace{\left[\beta(1 - \mu(z_{t+1}, k_{t+1}, b_{t+1})) \right]}_{\text{private marginal cost} = 100} + \underbrace{\beta \frac{\partial \mu(z_{t+1}, k_{t+1}, b_{t+1})}{\partial b_{t+1}} e_{t+1}^{SP}}_{\text{externality C} = 2.8} \right\rangle \tag{53}
\end{aligned}$$

4.4 Decision Rules and Government Spread Functions

Figure 6 shows the equilibrium decision rules plotted against corporate debt. Firms choose a higher level of debt b_{t+1} when they have higher initial debt b_t , as firms need to borrow money

to pay off existing debt. Firms choose to reduce the stock of capital k_{t+1} as they accumulate more debt, because the corporate debt price decreases due to rising default risk. Firms choose to pay constant dividends net of equity adjustment costs $\phi(e_t)$ regardless of their debt level, which reflects firms' dividend-smoothing motive. Firms' debt price q_t decreases with rising new debt issuance b_{t+1} due to an increase in firm default risk. The fraction of defaulting firms $\mu(z_t, k_t, b_t)$ increases with more debt b_t . The government chooses to issue more debt B_{t+1} when corporate debt b_t increases. The reason is that tax revenues decrease as more firms default, and the government is induced to finance transfers to households externally to smooth households' consumption.

Figure 7 presents government default probabilities and the associated government spread function over the levels of next-period corporate and government debt. Panel A shows that the government is more likely to default when next-period government or corporate debt is higher. The mechanism is as follows: With higher government debt, the government's value of defaulting increases, as it can finance a larger amount of transfers to households to increase households' consumption by repudiating its debt. Higher corporate debt increases the fraction of defaulting firms, which in turn reduces government tax revenue, while households receive fewer dividends from firms. This leads to lower consumption and gives the government more incentive to repudiate its debt to increase transfers to households. A higher government default probability is associated with higher government spreads, as investors need to be compensated for higher government default risk. Thus, as shown in Panel B, government spreads increase in both government and corporate debt. Interestingly, the government spread increases at a faster rate in government debt when corporate debt is higher. This is because households' consumption is lower when a larger fraction of firms stop paying taxes and dividends, so that the marginal gain from repudiating debt is higher for the government. At the same time, the government is willing to pay high interest rates to finance transfers to households when households are suffering from low consumption, and the government tends to increase its debt. These forces make the government spread more sensitive to rising government debt when the corporate debt level is higher.

4.5 Optimal Policy, Simple Debt Policies, and Welfare Gains

To explore model dynamics during a sovereign debt crisis, I perform an event study using the model. Using the solution to the competitive equilibrium, I simulate the economy for a million periods and drop the first 500 periods. I identify a period t as a sovereign debt crisis if government spreads at time t increase by more than two standard deviations of government spreads across all periods. Next, I obtain the paths of variables within each event window ranging from $t-5$ to $t+5$ and calculate the deviation of each variable from its average within each event window. I plot the average of the demeaned variables across event windows.

Figure 8 presents simulation results with no policy and their data counterparts. I mark time t with the shaded area which corresponds to a year of 2012 when the average sovereign spread in the six sample countries peaked. In the model without any policies (blue dashed lines), output decreases by about 3% during the crisis, which matches the magnitude of the recession in the data.²⁸ In the model, investment and consumption drop by around 12% and 8%, respectively. The corporate debt-to-output ratio increases before the crisis. As a result, corporate spreads increase by more than 50 basis points, and the fraction of defaulting firms rises by more than 2%p. As more firms default, bankruptcy costs increase, and firms are deleveraged sharply, reducing consumption drastically. Furthermore, corporate income tax revenue decreases, and government spreads increase by around 150 basis points. At the same time, the government issues more debt to finance transfers to households (government expenditure), but the amount of transfers decreases in equilibrium due to higher government interest rates. This adds to the reduction in household consumption during the crisis. The sovereign debt crisis is associated with negative shocks to productivity. These model-simulated time series qualitatively matches the data counterparts, although there are some differences in terms of the timing of shocks and the magnitude of responses. In the data, consumption does not decrease as much as implied by the model, possibly because the European governments and the European Central Bank intervene, as evidenced by rising government debt and sustained government expenditure during and after the crisis. Firm default rate peaks during the 2008-2009 global financial crisis in the data, in response to a large negative shock to total factor productivity during the same period. In contrast, the model generates a negative productivity shock in 2012 and the corresponding increase in the firm default rate.

Figure 9 plots these simulation results for economies with various policies in which the path of productivity and the timing of the crises are identical to the previous economy without policy. To study optimal policy in the presence of corporate-sovereign linkages, solutions to a set of time-consistent optimal policies are obtained as described in section 3.4, using the same parameters of the quantitative model used for the competitive equilibrium. The bottom panels of Figure 9 present the set of implemented optimal policies as red solid lines. It is optimal for the government to impose a low constant tax rate on debt, which is around 0.13% of new firm debt issuance. The government subsidizes firms during the crisis by reducing their lump-sum taxes relative to output by about 0.3%p. The government also subsidizes firms for their investment during the crises. Intuitively, the government uses the constant

²⁸Output is measured net of bankruptcy and equity adjustment costs, $F(z_t, k_t) - DW_t - \kappa(e_t - \bar{e})^2$. The resource constraint can be written as follows: $C_t + I_t + DW_t + \kappa(e_t - \bar{e})^2 = F(z_t, k_t) - (1 - \mu_t)b_t - \mu_t R^f b_t + q_t b_{t+1} - B_t + Q_t B_{t+1}$, where $I_t = k_{t+1} - (1 - \delta)k_t + \Psi(k, k')$ is gross investment, $DW_t = \mu_t(\phi(e_t) + \tau^y(1 - \theta)F(z_t, k_t) + (1 - R^f)b_t)$ is deadweight costs due to corporate defaults, and $\kappa(e_t - \bar{e})^2$ is the equity adjustment cost.

debt tax rate to incentivize firms to reduce debt, discouraging over-borrowing externalities due to the limited liability of firms. At the same time, it implements lump-sum taxes and investment credits to reduce firm default risk in bad times and to increase tax revenue buffers in good times in preparation for future negative shocks. With this set of optimal policies, output decreases only about 2% during the crisis compared to a 3% decrease in the economy without policy. Moreover, investment and consumption do not collapse as much as in the economy without policy. The reason is that, as the government subsidizes firms by reducing lump-sum taxes and increasing investment credits, firms' financing needs decrease, and firm default risk decreases. This translates into smoothed paths of corporate debt and income tax revenue. As the government has enough tax revenue to finance transfers to households, it has weaker incentive to default. Hence, government spreads do not respond much during the crisis, and the government has more than enough room to provide transfers to households and implement its set of optimal policies. Again, these policies reduce firm defaults. This positive feedback loop reinforces the effectiveness of optimal policy in smoothing out the path of consumption and increasing welfare.

I calculate welfare gains from implementing this set of optimal policies. To be specific, I compute the permanent increase in consumption ω_0 that households would require as compensation when they move to the economy without policy from the one with optimal policy, using the following equation:

$$\sum_{t=1}^{100} \beta_g^{t-1} u(C_t^{NP}(1 + \omega_0)) = \sum_{t=1}^{100} \beta_g^{t-1} u(C_t^{OP}) \quad (54)$$

where household consumption is C^{NP} and C^{OP} in the economy without and with optimal policy, respectively. I simulate the economy 100,000 times for 200 periods, dropping the first 100 periods. I compute the average welfare gain ω_0 from these simulations. I find that the average welfare gain from implementing optimal policy is substantial (a 12.8% increase in permanent consumption), because optimal policy alleviates the risk spillover from corporate debt to government spreads with a positive feedback loop as described above. Importantly, these results are not driven by direct bankruptcy costs, which are only about 2.2% of output on average in the simulations, but rather are driven by corporate credit cycles and the amplification mechanism through the corporate-sovereign linkage.²⁹

Although optimal policy improves welfare dramatically, this type of policy is arguably not realistic, in the sense that the government may not be able to assess the economic conditions so precisely as to be able to implement state-contingent optimal policies, or it

²⁹The ratio of total direct bankruptcy costs to output are calculated as the average of $\mu_t(\phi(e_t) + \tau^y(1 - \theta)F(z_t, k_t) + (1 - R^f)b_t)/F(z_t, k_t)$ across simulations. Average bankruptcy costs associated with the tax revenue channel, $\tau^y(1 - \theta)F(z_t, k_t)/F(z_t, k_t)$, are about 0.7%, while average bankruptcy costs for dividend cuts ($\phi(e_t)/F(z_t, k_t)$) and direct debt repudiation ($(1 - R^f)b_t)/F(z_t, k_t)$) are about 0.3% and 1.2%, respectively.

is politically not viable. For this reason, I consider two types of simple debt policies: (i) a constant debt tax and (ii) a countercyclical debt tax rate. When imposing a constant debt tax rate $\bar{\tau}^b$, the government does not change the debt tax rate along the credit cycles. In contrast, with the countercyclical debt tax, the government raises the debt tax rate τ_t^b when the current corporate debt-to-output ratio b_t/Y_t exceeds a target leverage ratio \bar{b}/\bar{Y} , as shown in equation (55) below:

$$\tau_t^b = \max[\bar{\tau}^b + \beta_\tau(b_t/Y_t - \bar{b}/\bar{Y}), 0] \quad (55)$$

where the debt tax rate τ_t^b is always non-negative and β_τ is positive. I assume that the government cannot collect debt taxes from defaulting firms.³⁰

Figure 10 Panel A plots average welfare gains from imposing different constant debt tax rates $\bar{\tau}^b$ ranging from 0% to 20%, while the slope β_τ is set to zero. These welfare gains are obtained in the same way as when considering optimal policy, and each average welfare gain is calculated by repeating simulation exercises across different debt tax rates. Panel A shows that, as the constant debt tax rate increases, the welfare gain increases and reaches a maximum of 2.1% with a debt tax rate of 6%. After reaching the peak, the welfare gain decreases monotonically in the debt tax rate and eventually becomes negative. Intuitively, raising the debt tax rate corrects overborrowing externalities, but at the same time increases firm default risk. Given that the constant debt tax rate $\bar{\tau}^b$ of 6% approximately gives the maximum welfare gain, I calculate welfare gains for different debt tax rule slopes β_τ ranging from -5 to 1, while I set $\bar{\tau}^b = 6\%$. I set the target leverage \bar{b}/\bar{Y} to 0.91, which is the average corporate leverage ratio across simulations in the regulated equilibrium with a constant debt tax rate of 6%. Surprisingly, the welfare rarely increases or even decreases when the slope of debt tax rule is positive. As the government implements a more countercyclical debt tax rate, the welfare gain decreases. The reasons are that firms face higher default risk, as they have to pay higher debt taxes and run out of cash during credit booms, and that the government collects too many debt taxes than are optimal. Moreover, the problem is that (i) it raises the debt tax rate based on the current corporate debt b_t , which is out-of-date information compared to planned debt issuance b_{t+1} ,³¹ and (ii) this type of policy is not sophisticated enough to replicate the state contingency of optimal policies, with which the government subsidizes firms during the crisis.

³⁰The government could regulate the individual firms' planned corporate debt issuance b_{t+1} using microprudential measures, but in practice the government can regulate firms based only on current aggregate debt b_t when it comes to macroprudential policies.

³¹I find that raising a debt tax rate based on the planned firm debt issuance b_{t+1} gives larger welfare gains than raising a debt tax rate based on the current firm debt b_t , but smaller welfare gains than imposing a constant debt tax rate. Levying debt taxes based on b_{t+1} is close to microprudential policy in the sense that planned debt choice b_{t+1} can be regulated immediately and that regulated firms internalize the effect of their individual debt choice on a debt tax rate.

Figure 9 again presents dynamics during the crisis implied by simple debt policies. The evolutions of variables in the economy with the countercyclical debt tax rule ($\bar{\tau}^b = 6\%$, $\beta_\tau = 0.25$) are presented as black solid lines with yellow circles. Output decreases more compared to the economy with optimal policy, due to the larger bankruptcy costs induced by more firm defaults during the crisis. Despite the higher firm default risk and the resulting higher government spreads, the countercyclical debt tax increases welfare by about 0.1% relative to the economy without policy, because this debt tax rule tames the corporate credit cycles and smoothes out the paths of investment and consumption. The economy with a constant debt tax rate ($\bar{\tau}^b = 6\%$, $\beta_\tau = 0.0$) is plotted as green dotted lines. The constant debt tax rate similarly smoothes out the paths of corporate credit, investment, and consumption. Compared to the countercyclical debt tax, the constant debt tax induces fewer defaulting firms, resulting in a larger welfare gain of 2.1%. To sum up, the welfare gain from the optimal policy is the largest (12.8%), which is followed by the constant debt tax (2.1%) and the countercyclical debt tax (0.1%). Table 7 shows that the welfare rank between optimal policy, the constant debt tax, and the countercyclical debt tax is robust to alternative parameterizations.

To better understand the nature of the cyclical debt tax rule, consider the following formula of equation (46) when lump-sum taxes and investment credits are not available.

$$\tau_t^b = \underbrace{\frac{\partial[q(\cdot)b_{t+1}]}{\partial b_{t+1}}}_{\text{marginal funds that can be borrowed}} - \beta \underbrace{\int_{z_{min}}^{z_{max}} \int_{\nu_{min}^d}^{\bar{\nu}^d(X_{t+1})} \frac{\lambda_{t+1}}{\lambda_t} d\Omega(\nu_{i,t+1}^d) d\Pi(z_{t+1}|z_t)}_{\text{expected appreciation of marginal funds}} \quad (56)$$

When corporate leverage and corporate default risk increases, marginal funds that can be borrowed decrease, and the value of marginal funds is likely to increase, implying that the planner should cut the debt tax rate to reduce firm default risk. Notice that the debt tax rate still should be positive to correct the overborrowing externality under reasonable parameterizations. Confirming this intuition, Figure 10 Panel B shows that the welfare gain increases as the planner cuts the debt tax rate more aggressively during corporate credit booms. The welfare gain reaches a maximum of about 6.7% when the planner cuts the debt tax rate by 2.75%p in response to an 1%p increase in the corporate debt-to-output ratio gap $b_t/Y_t - \bar{b}/\bar{Y}$ (while the constant term of debt tax rate $\bar{\tau}^b$ is set to 6%). In contrast, models with a borrowing constraint (based on a pecuniary externality) typically show that the planner should raise the debt tax rate when corporate leverage increases, and the borrowing constraint is more likely to bind. The reason is that these models abstract from firm defaults and associated bankruptcy costs, and the planner faces smaller costs when raising the debt tax rate, compared to my model with firm bankruptcy costs.

5 Conclusion

In this paper, I find that corporate debt causally affects sovereign default risk and show that this corporate-sovereign debt nexus is an important amplification mechanism driven by externalities that call for macroprudential policies. I run instrumental variable regressions to estimate a causal relationship running from aggregate corporate leverage to sovereign spreads. I use the weighted sum of idiosyncratic shocks to top 50 large firms in each Eurozone country as an instrument for aggregate corporate leverage to rule out potential reverse causality and omitted variable bias. The regressions suggest that rising corporate leverage causes sovereign spreads to rise, which confirms the existence of the corporate-sovereign nexus. To understand the mechanism, I build a model in which both firms and the government can default. When corporate debt increases, tax revenues are expected to be lower, as firms stop paying taxes and dividends when they default, which raises sovereign default risk. This tax revenue channel is supported by empirical evidence. Country-level tax revenue regressions show that increases in corporate debt-to-GDP ratios reduce future tax revenue growth. Difference-in-difference regressions using firm-level data suggest that highly-leveraged firms reduce tax payments more compared to less-leveraged firms in response to the 2008 global financial crisis. Moreover, I analyze an externality that arises from firms' limited liability, which is distinct from the pecuniary and aggregate demand externalities. I find that there exist time-consistent optimal policies that correct the limited liability externality. A quantitative model calibrated to six Eurozone countries shows that such policies consists of a low constant debt tax rate together with transfers and investment credits to firms during the crisis. Implementing these policies alleviates the corporate-sovereign linkage, so that the number of defaulting firms decreases, and the government has enough fiscal space to provide transfers to households suffering from low consumption. Furthermore, practical policies such as either constant or countercyclical debt tax schedules can correct overborrowing externalities. However, the countercyclical debt policy induces more firm defaults during the crisis, and thus it is less effective than the constant debt policy. This suggests that policymakers should be cautious about implementing countercyclical debt policies such as countercyclical capital buffers and should even consider relaxing regulations when corporate default risk is high.

Taxes on total corporate debt have not been given much attention by policymakers or researchers, as previous research typically discusses currency and maturity mismatch in debt (in either the corporate or banking sector) and associated bank regulations. In this regard, it would be important to regulate total corporate leverage when firms' liability is limited. Moreover, it would be useful to study the consequences of limited liability and the implied optimal mix of macroprudential policies and capital controls in future research.

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Figures

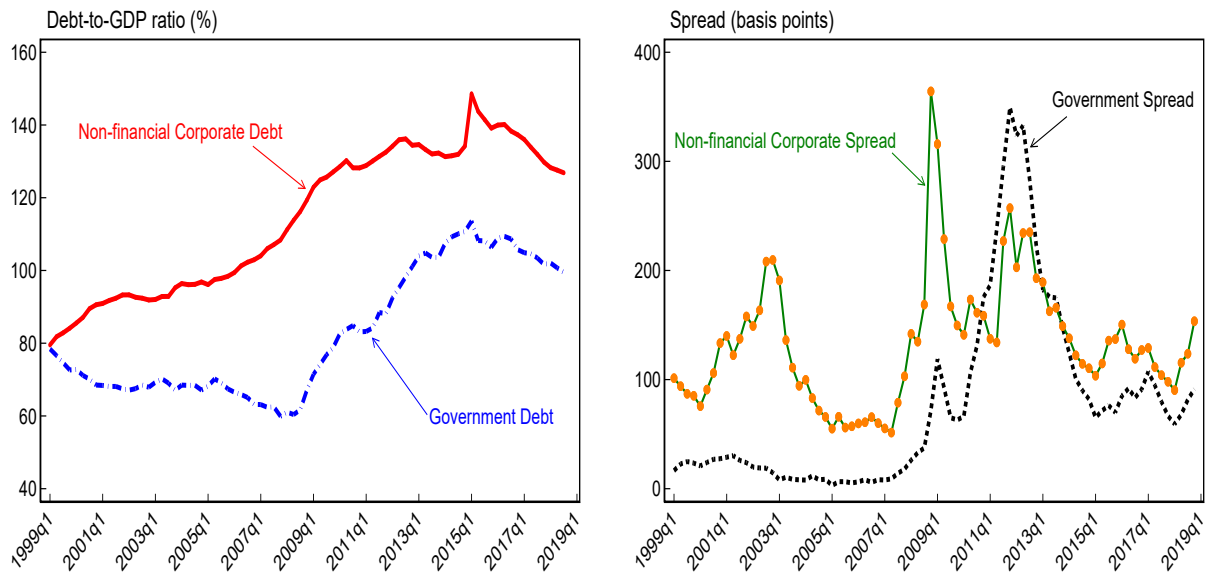


Figure 1. Eurozone Leverage and Interest Rate Spreads

Source: BIS, Bloomberg, [Gilchrist and Mojon \(2017\)](#)

Notes: The figure shows the averages of debt-to-GDP ratios and sovereign spreads of 9 Eurozone countries (Austria, Belgium, Finland, France, Ireland, Italy, Netherlands, Portugal, and Spain). Debt is core debt (BIS), which consists of the following financial instruments as defined in the System of National Accounts (SNA): debt securities, loans, and currency and deposits. Core debt excludes special drawing rights, insurance, pensions, standardized guarantee schemes, and other accounts payable. Sovereign spreads (Bloomberg) are measured as the difference between the 10-year government bond yield of each country and that of Germany. Both government bond yields are denominated in euro and are daily averages by quarters. The corporate spread of Euro area non-financial corporate bonds (Euro area Bund NFC) is obtained from [Gilchrist and Mojon \(2017\)](#).

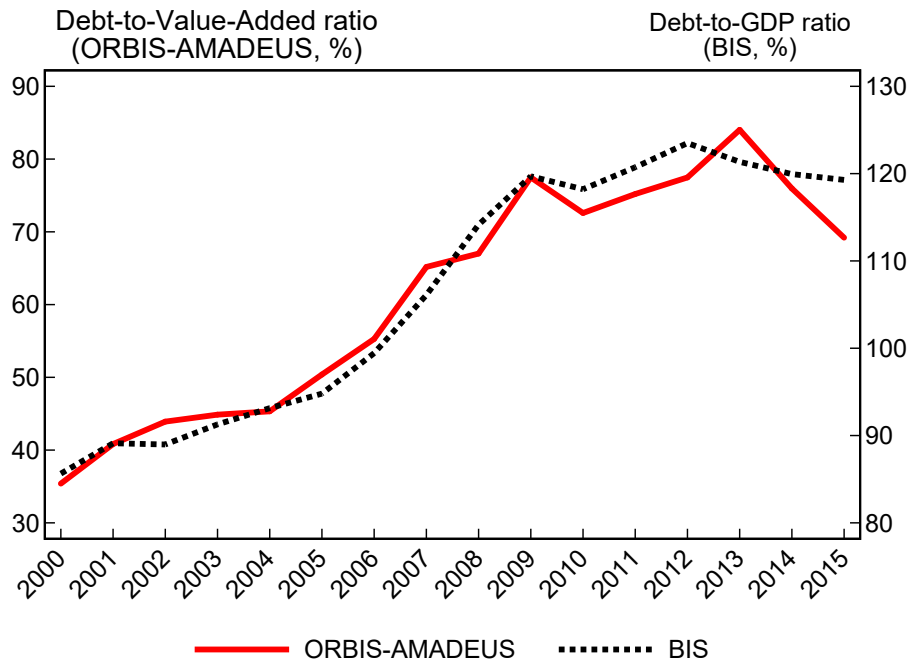
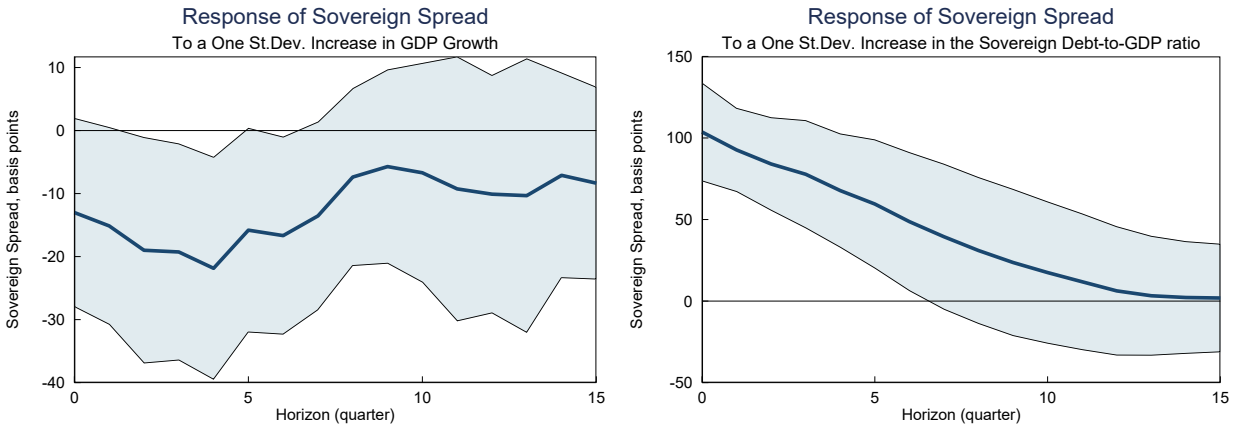


Figure 2. Corporate Debt: ORBIS vs. BIS Comparison

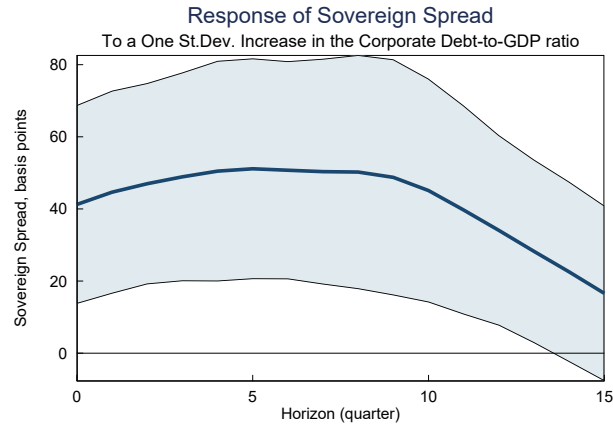
Source: Author's calculation based on ORBIS-AMADEUS and BIS

Notes: The figure shows the averages of debt-to-value-added ratios (ORBIS-AMADEUS) and debt-to-GDP ratios (BIS) for six Eurozone countries (Italy, Spain, Portugal, Belgium, Finland, and France). For the ORBIS-AMADEUS data, debt is financial debt measured as the sum of loans and long-term debt, which excludes other accounts payable. Leverage is calculated as the average leverage of six Eurozone countries, where each country's leverage is calculated as the sum of individual firms' debt over the sum of value added, where value added is operating revenue minus materials cost. For BIS statistics, debt is core debt (BIS), which consists of the following financial instruments as defined in the System of National Accounts (SNA): debt securities, loans, and currency and deposits. Core debt excludes special drawing rights, insurance, pensions, standardized guarantee schemes, and other accounts payable.



(a) GDP Growth

(b) Sovereign Debt



(c) Corporate Debt

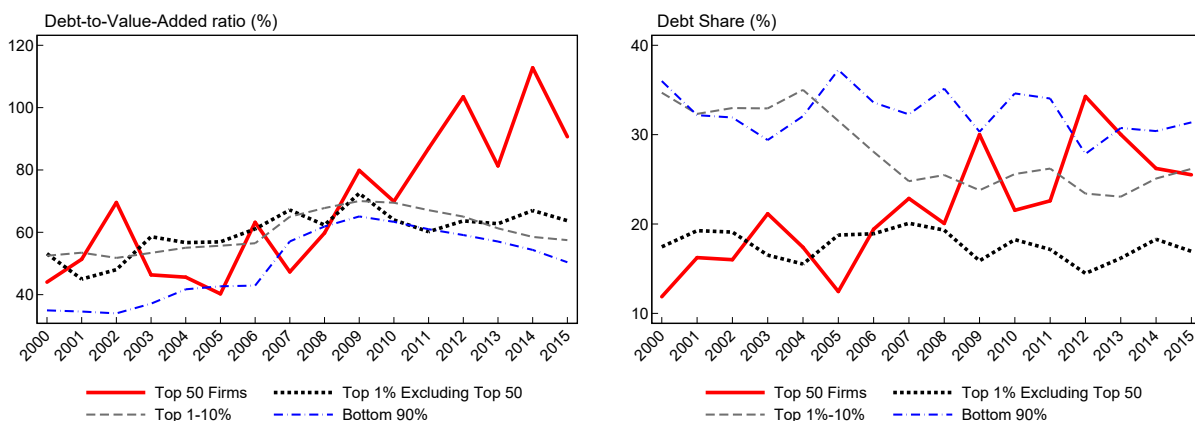
Figure 3. Sovereign Spread Regression Coefficients with Different Horizons

Source: Author's calculation

Notes: I run the following Jordà (2005)-style local projections regression:

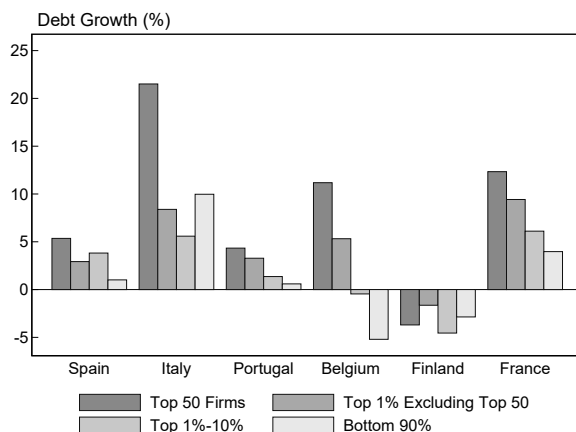
$$\text{Spread}_{c,t+h} = \beta_{y,h} \text{Growth}_{c,t} + \beta_{gov,h} \text{Govt Debt/GDP}_{c,t} + \beta_{corp,h} \text{Corp Debt/GDP}_{c,t} + \delta_c + \gamma_t + \epsilon_{c,t+h}$$

for each horizon $h = 0, 1, 2, \dots$, where δ_c and γ_t are country and time fixed effects. The figures plot regression coefficients for (a) GDP growth ($\beta_{y,h}$), (b) sovereign debt ($\beta_{gov,h}$), and (c) corporate debt ($\beta_{corp,h}$) that are multiplied by a one standard shock to GDP growth, sovereign debt, and corporate debt, respectively. The shaded area presents 95 percent confidence intervals, calculated using Driscoll and Kraay (1998) standard errors (robust to heteroskedasticity and clustering on date and kernel-robust to common correlated disturbances) with a lag length of 4 quarters. The regression sample consists of observations for nine Eurozone countries (Austria, Belgium, Finland, France, Ireland, Italy, Netherlands, Portugal, and Spain) during the period 1999q1–2012q4.



(a) Leverage

(b) Debt Share



(c) Debt Growth

Figure 4. Corporate Debt in Average Eurozone Countries by Firm Size Class

Source: Author’s calculation based on ORBIS-AMADEUS

Notes: Debt is financial debt measured as the sum of loans and long-term debt, which excludes other accounts payable. Leverage reported in panel (a) is calculated as the average over six Eurozone countries (Finland, France, Italy, Spain, Belgium, and Portugal), where each country’s leverage is calculated as the weighted average of individual firms’ debt-to-value-added ratio within each size class, where value added is operating revenue minus materials cost, and weights are sales of each firm in a given year. In panel (b), I measure the debt share by each size class in a given country as the sum of financial debt within each size class over the sum of financial debt of all firms in a given country. I plot the average debt share of six Eurozone countries for each size class. To obtain debt growth rates in panel (c), I deflate debt using 2-digit sector gross output prices (EU KLEMS) and calculate growth rates — measured as $(debt_t - debt_{t-1}) / (0.5 \times (debt_t + debt_{t-1}))$ — for each firm. Debt growth rates of each size class are weighted averages within each size class during the period 2000–2015, where weights are sales of each firm in a given year $t - 1$. These time-varying weights are consistent with the weights used later in constructing the granular residual, which reflect the time-varying contribution of large firms. Size classes are determined by percentiles of sales.

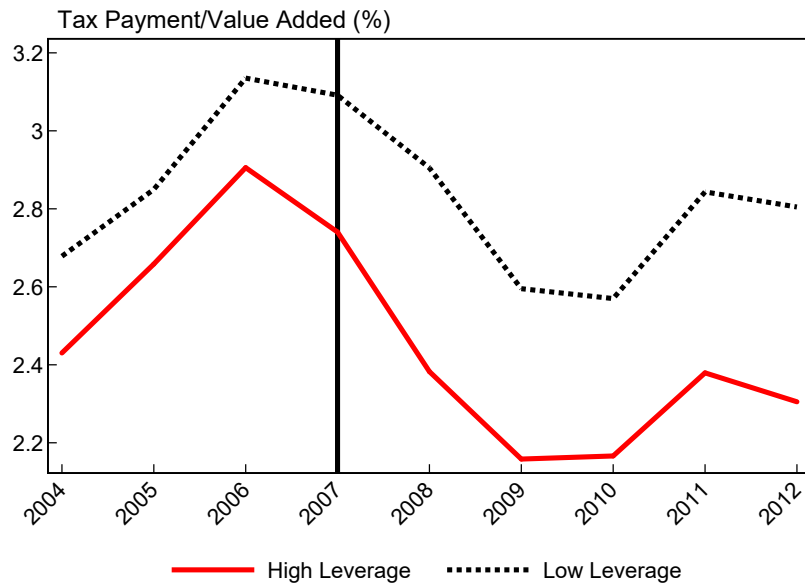


Figure 5. Firm Tax Payment by Leverage Group

Source: Author's calculation based on ORBIS-AMADEUS

Notes: I plot averages of firm tax payment across firms within each leverage group in a given year using a sample of six Eurozone countries (Italy, Portugal, Spain, Belgium, Finland, and France). Firm tax payment for each firm i is the ratio of tax payment to value added (operating revenue – materials cost). Leverage is the ratio of financial debt to value added in which financial debt b is the sum of loans and long-term debt. A firm belongs to the high leverage group if average firm leverage before 2008 of a given firm is higher than the aggregate median before 2008. I use a full sample during the period 2000–2007 in six Eurozone countries to calculate average firm leverage of each firm and aggregate median leverage.

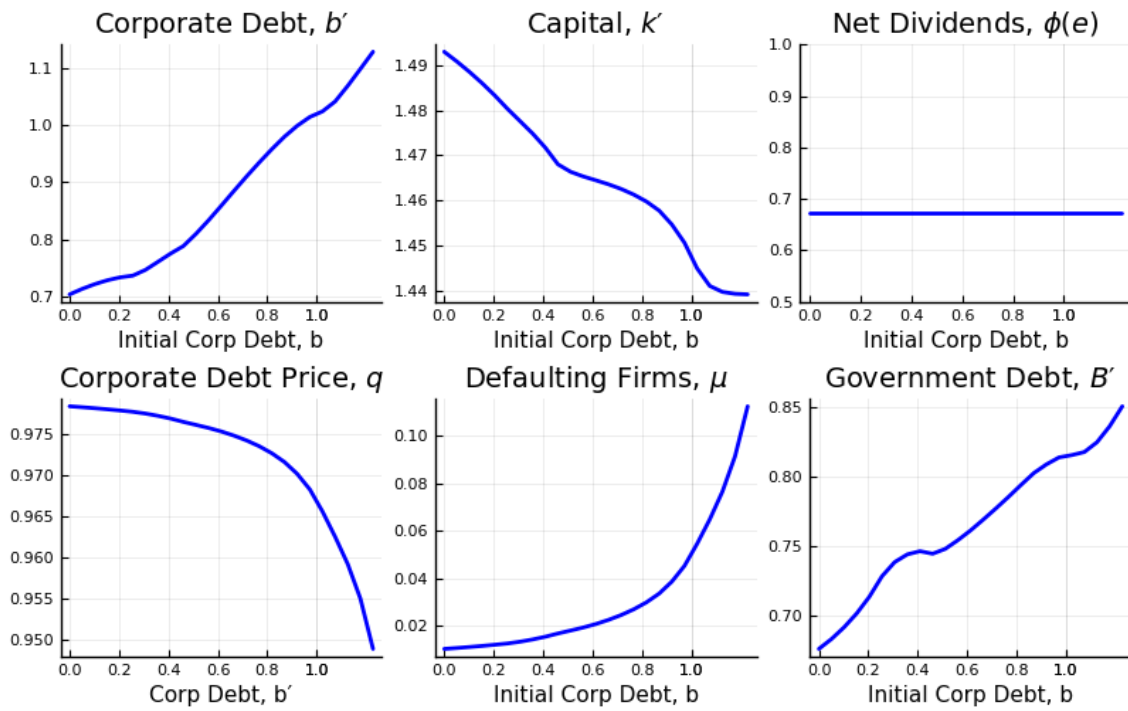
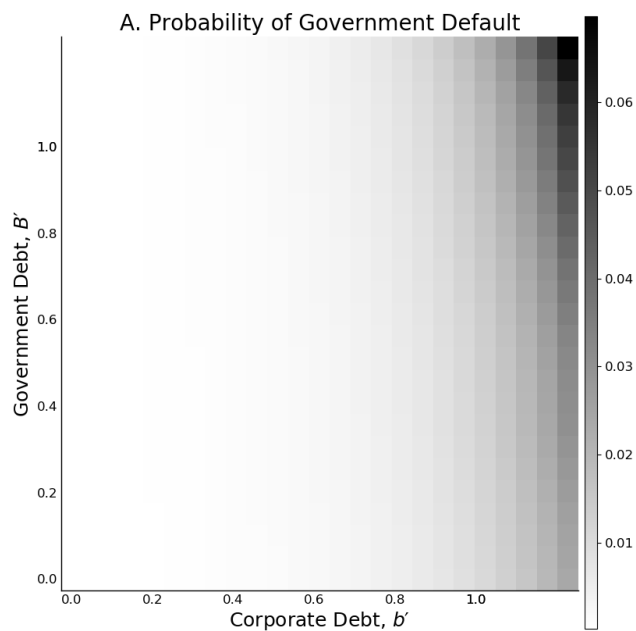


Figure 6. Decision Rules

Notes: The figure shows decision rules in the competitive equilibrium of the quantitative model. Productivity z and capital k are set to the minimum level so that substantial firm default risk exists, and government debt B is set to the median level. Decision rules are averaged over the distribution of taste shocks.



B. Government Spread (basis points)

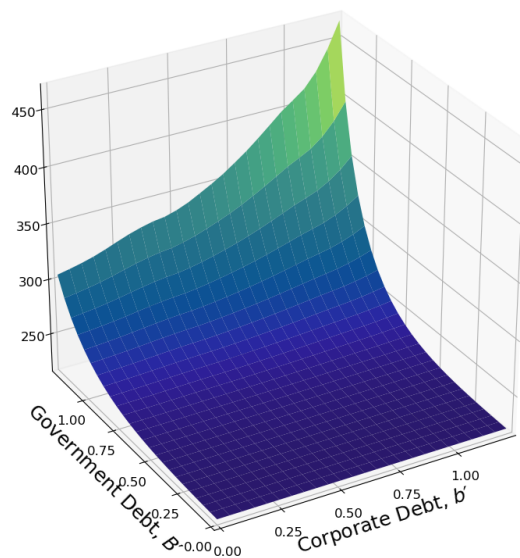


Figure 7. Government Default Probability and Spread Function

Notes: The figure shows government default probabilities and the spread function in the competitive equilibrium of the quantitative model. Productivity z and capital k are set to the minimum level so that substantial firm default risk exists.

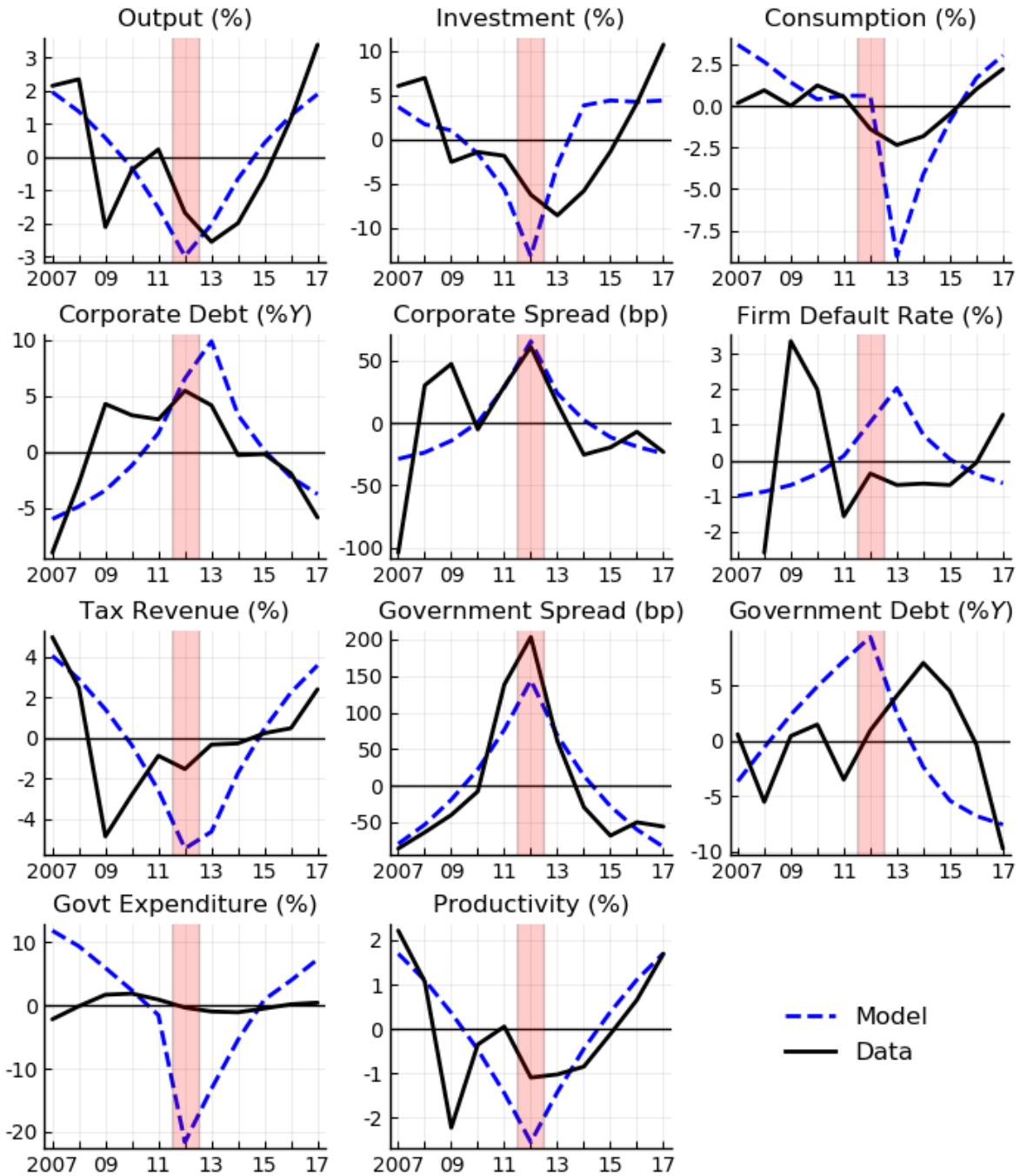


Figure 8. Event Study: Sovereign Debt Crisis

Notes: For model-simulated time series, I simulate an economy (competitive equilibrium) for a million periods and drop the first 500 periods. I identify a period t as a sovereign debt crisis if government spreads at time t increase by more than two standard deviations of government spreads across all periods. Next, I obtain the paths of variables within each event window ranging from $t - 5$ to $t + 5$, and I calculate the deviation of each variable from its average within each event window. Time t is marked with the shaded area which corresponds to the year of 2012. I plot the average of the demeaned variables across event windows. See the notes of Figure 9 for more details. Data counterparts are linearly detrended series during the period of 2007-2017. See Data Appendix for more details.

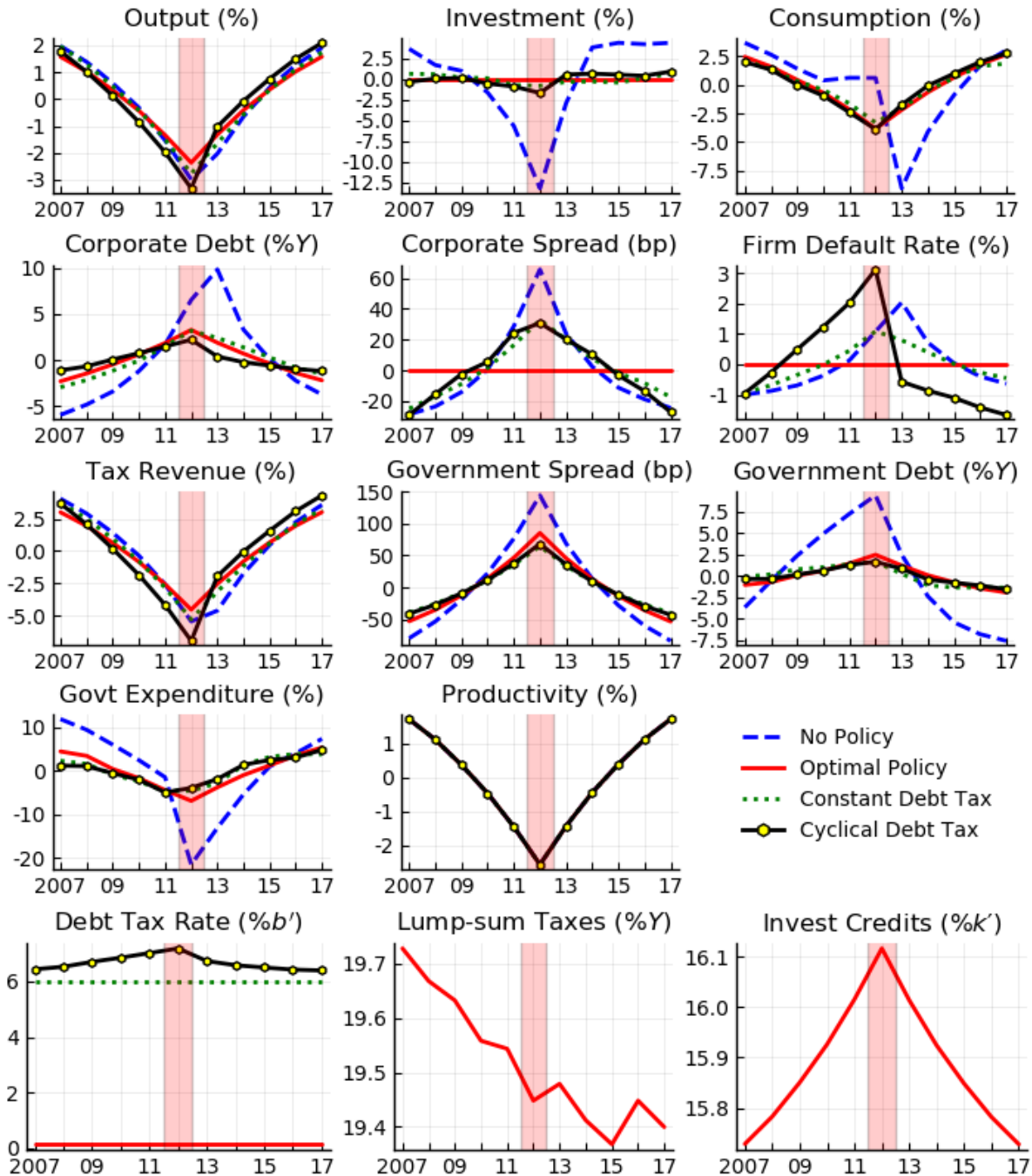


Figure 9. Event Study: Policy Analysis

Notes: I simulate each economy with different policies, using the same path of productivity and the timing of the crisis events as shown in Figure 8. Crisis time t is marked with the shaded area which corresponds to the year of 2012. I plot the average of the demeaned variables across event windows. Output, investment, consumption, corporate income tax revenue, government expenditure, productivity are demeaned and normalized by their steady state values and are expressed in percentage deviations. Corporate debt and government debt are initially normalized by output and are demeaned later. Corporate spreads and government spreads are demeaned and are in basis points. The fraction of defaulting firms is demeaned and is in percentage. The debt tax rate τ_t^b , the ratio of lump-sum taxes to output $-T_t/Y_t$, and investment credits τ_t^k are plotted as their original levels in percentage.

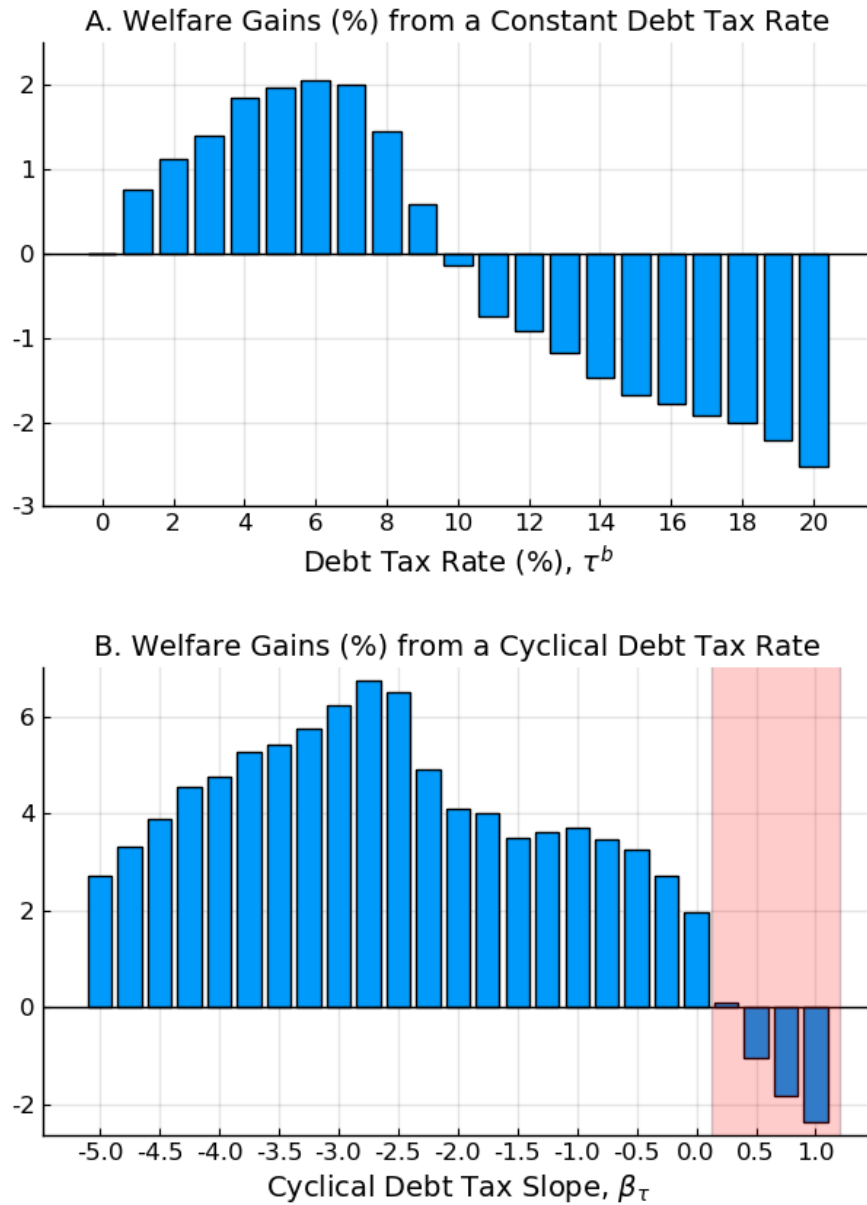


Figure 10. Welfare Gains From Simple Debt Tax Policies

Notes: A permanent increase in consumption by implementing different debt policies is calculated. For each policy, the average welfare gains from 100,000 simulations of 200 periods after dropping first 100 periods for each simulation are presented. The debt tax rate is given by $\tau_t^b = \max[\bar{\tau}^b + \beta_\tau(b_t/Y_t - \bar{b}/\bar{Y}), 0]$. The target corporate leverage ratio \bar{b}/\bar{Y} is set to 0.91. In Panel A, β_τ is set to zero, and only the constant debt tax rate $\bar{\tau}^b$ changes. In Panel B, $\bar{\tau}^b$ is set to 6%, and the debt tax slope β_τ changes.

Tables

Table 1. OLS Sovereign Spread Regression with six Eurozone Countries

Dependent Variable:	Sovereign Spread _{c,t}						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
GDP Growth _{c,t-1}	-0.45** (0.19)	-0.28* (0.16)	-0.35** (0.16)	-0.29 (0.20)	-0.30* (0.16)	-0.48** (0.23)	-0.43** (0.19)
Sovereign Debt/GDP _{c,t-1}	0.08*** (0.02)	0.07*** (0.01)	0.07*** (0.01)	0.07*** (0.01)	0.08*** (0.01)	0.04** (0.02)	0.06*** (0.01)
Corporate Debt/GDP _{c,t-1}		0.04*** (0.01)	0.04*** (0.01)	0.04*** (0.01)	0.04*** (0.01)		0.05*** (0.01)
log(VIX) _{t-1}			-0.33 (0.21)				
TED Spread _{t-1}				-0.08 (0.30)			
Term Spread _{t-1}					-0.11 (0.09)		
Within R^2	0.3659	0.4448	0.4474	0.4429	0.4488	0.4774	0.5180
$N_{Observations}$	248	248	248	248	248	248	248
Country Fixed Effects	yes	yes	yes	yes	yes	yes	yes
Quarter Fixed Effects	no	no	no	no	no	yes	yes

Notes: Driscoll and Kraay (1998) standard errors (robust to heteroskedasticity and clustering on date and kernel-robust to common correlated disturbances) with a lag length of 4 quarters are in parentheses. ***, **, * denote significance at the 1%, 5%, and 10% levels, respectively. The regression sample is quarterly and covers the same countries as in annual IV regressions (Italy, Spain, Portugal, Belgium, Finland, and France) over the period 2002q1–2012q4.

Table 2. IV Spread Regression

	(1)	(2)	(3)	(4)
	Baseline		Bank Control	
	OLS	IV	OLS	IV
A. Second-Stage Regression				
Dependent Variable:	Sovereign Spread _{c,t}			
Corporate Debt/GDP _{c,t-1}	0.03** (0.01)	0.15** (0.06)	0.04*** (0.02)	0.11** (0.05)
Sovereign Debt/GDP _{c,t-1}	0.03 (0.02)	0.10** (0.05)	0.04* (0.02)	0.08** (0.03)
GDP Growth _{c,t-1}	-0.23** (0.12)	-0.08 (0.13)	-0.25** (0.11)	-0.19** (0.09)
Bank Leverage _{c,t-1}			0.07*** (0.01)	0.09*** (0.02)
B. First-Stage Regression				
Dependent Variable:	Corporate Debt/GDP _{c,t-1}			
Granular Residual _{c,t-1}	n/a	-5.10** (2.37)	n/a	-5.86** (2.32)
Granular Residual _{c,t-2}	n/a	-3.88* (2.18)	n/a	-4.67** (2.00)
Adjusted R^2	0.6547	0.4800	0.6833	0.6275
$N_{Observation}$	60	60	60	60
Country Fixed Effect	yes	yes	yes	yes
Time Fixed Effect	yes	yes	yes	yes
Hansen J (p-value)	n/a	0.5528	n/a	0.3880
First-stage F^{eff}	n/a	3.43	n/a	5.15
CLR (p-value, $H_0 : \beta_{corp} = 0$)	n/a	0.0250	n/a	0.0628

Notes: Annual sovereign spreads are regressed on lagged explanatory variables over the period 2002–2012. In instrumental variable (IV) regressions, excluded instruments for corporate debt are 1 and 2 years lagged granular residuals, which are based on idiosyncratic total factor productivity shocks to large firms estimated by the method of Wooldridge (2009). The granular residual is a weighted sum of idiosyncratic shocks to top 50 firms in each country c , using lagged Domar weights ($sales_{i,c,t-1}/GDP_{c,t-1}$) for a given firm i . Idiosyncratic shocks are residuals from the regression of firm-level productivity on 4-digit sector×year fixed effects. In column 2, lagged GDP growth, the lagged government debt to GDP ratio, country fixed effects, and year fixed effects are included in both the first and second stage regressions. Lagged bank leverage is added to the first and second stage regressions in column 4. The Hansen (1982) J statistic tests the null that instruments are excludable. The first-stage effective F statistic of Olea and Pflueger (2013) tests the null that the excluded instruments are not relevant. The p-value for CLR statistic (Andrews et al. (2019)) is reported to test the null hypothesis that the coefficient on corporate debt in the second stage regression is zero. This test is robust to weak instruments. Robust standard errors (calculated using the 2-step GMM method and robust to arbitrary clusters for IV regressions) are in parentheses. ***, **, * denote significance at the 1%, 5%, and 10% levels, respectively.

Table 3. Tax Revenue Regression

	(1)	(2)	(3)	(4)
	OLS	IV	OLS	IV
A. Second-Stage Regression				
Dependent Variable:	Tax Revenue Growth _{c,t}		Tax Revenue Growth _{c,t+1}	
Corporate Debt/GDP _{c,t-1}	-0.22*** (0.07)	-0.32*** (0.12)	-0.20*** (0.07)	-0.29** (0.12)
Sovereign Debt/GDP _{c,t-1}	-0.08 (0.05)	-0.14* (0.08)	-0.08 (0.05)	-0.12 (0.08)
GDP Growth _{c,t}			0.43* (0.26)	0.34 (0.30)
GDP Growth _{c,t-1}	0.39* (0.21)	0.26 (0.22)	0.28 (0.19)	0.18 (0.21)
Bank Leverage _{c,t-1}	-0.05 (0.06)	-0.08 (0.07)	0.05 (0.06)	-0.07 (0.07)
B. First-Stage Regression				
Dependent Variable:	Corporate Debt/GDP _{c,t-1}			
Granular Residual _{c,t-1}	n/a	-5.86** (2.32)	n/a	-5.30** (2.32)
Granular Residual _{c,t-2}	n/a	-4.67** (2.00)	n/a	-4.49** (2.05)
Adjusted R^2	0.6247	0.5928	0.6356	0.6106
$N_{Observation}$	60	60	60	60
Country Fixed Effect	yes	yes	yes	yes
Time Fixed Effect	yes	yes	yes	yes
Hansen J (p-value)	n/a	0.2373	n/a	0.1511
First-stage F^{eff}	n/a	5.15	n/a	4.25
CLR (p-value, $H_0 : \beta_{corp} = 0$)	n/a	0.0879	n/a	0.1936

Notes: Annual real tax revenue growth is regressed on lagged explanatory variables over the period 2000–2012. In instrumental variable (IV) regressions, excluded instruments for corporate debt are 1 and 2 years lagged granular residuals, which are based on idiosyncratic total factor productivity shocks to large firms estimated by the method of [Wooldridge \(2009\)](#). The granular residual is a weighted sum of idiosyncratic shocks to top 50 firms in each country c , using lagged Domar weights ($sales_{i,c,t-1}/GDP_{c,t-1}$) for a given firm i . Idiosyncratic shocks are residuals from the regression of firm-level productivity on 4-digit sector \times year fixed effects. In column 2, lagged GDP growth, the lagged government debt to GDP ratio, country fixed effects, and year fixed effects are included in both the first and second stage regressions. Lagged bank leverage is added to the first and second stage regressions in column 4. The [Hansen \(1982\)](#) J statistic tests the null that instruments are excludable. The first-stage effective F statistic of [Olea and Pflueger \(2013\)](#) tests the null that the excluded instruments are not relevant. The p-value for CLR statistic ([Andrews et al. \(2019\)](#)) is reported to test the null hypothesis that the coefficient on corporate debt in the second stage regression is zero. This test is robust to weak instruments. Robust standard errors (calculated using the 2-step GMM method and robust to arbitrary clusters for IV regressions) are in parentheses. ***, **, * denote significance at the 1%, 5%, and 10% levels, respectively.

Table 4. Firm Tax Payment Regression

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent Variable:	Tax Payment $_{i,t}$					
HighLev $_i \times$ Crisis $_t$	-0.39*** (0.07)	-0.39*** (0.07)	-0.38*** (0.07)	-0.30*** (0.07)	-0.30*** (0.07)	-0.28*** (0.06)
$\log(z_{i,t-1})$	1.56*** (0.13)	1.56*** (0.13)	1.47*** (0.14)	1.53*** (0.13)	1.53*** (0.13)	1.45*** (0.14)
$\log(k_{i,t-1})$	-0.16*** (0.03)	-0.16*** (0.03)	-0.17*** (0.04)	-0.13*** (0.03)	-0.13*** (0.03)	-0.14*** (0.04)
$\log(b_{i,t-1})$	-0.02** (0.01)	-0.02** (0.01)	-0.02** (0.01)	-0.00 (0.01)	-0.00 (0.01)	-0.01 (0.01)
Interest Payment $_{i,t}$				-0.18*** (0.02)	-0.18*** (0.02)	-0.19*** (0.02)
$N_{\text{Observation}}$	35,187	35,187	34,463	35,187	35,187	34,463
R^2	0.65	0.65	0.65	0.65	0.65	0.65
Firm Fixed Effects (FE)	yes	yes	yes	yes	yes	yes
Country and Year FE	yes	n/a	n/a	yes	n/a	n/a
Country \times Year FE	no	yes	n/a	no	yes	n/a
Country \times Sector \times Year FE	no	no	yes	no	no	yes

Notes: Standard errors are in parentheses and clustered at the 4-digit sector level. ***, **, * denote significance at the 1%, 5%, and 10% levels, respectively. z is total factor productivity estimated by the Wooldridge (2009) method. k is tangible fixed assets. b is the sum of loans and long-term debt. Leverage is measured as a debt-to-value-added ratio (b/y), where value added y is measured as operating revenue minus materials cost. Interest payments are measured as the ratio of interest paid to value added. The Crisis $_t$ dummy equals 1 in or after 2008 and 0 otherwise. The HighLev $_i$ dummy equals 1 if average leverage before 2008 of a given firm is higher than the aggregate median before 2008 and 0 otherwise. The regression sample covers the period 2004–2012 in six Eurozone countries (Italy, Portugal, Spain, Belgium, Finland, and France).

Table 5. Parameterization

Panel A. Parameters set independently			
interpretation	symbol	value	source
capital income share	α	0.35	standard literature
labor income share	θ	0.50	
risk aversion	ν	2	
capital depreciation rate	δ	0.06	
corporate discount factor	β	0.98	Arellano et al. (2019)
government discount factor	β_g	0.90	
risk-free rate	r	0.019	average German real interest rate
corporate income tax rate	τ^y	0.327	average corporate income tax rate
government debt recovery rate	R^g	0.63	Cruces and Trebesch (2013)
corporate debt recovery rate	R^f	0.70	World Bank Doing Business Database
equity issuance cost	κ	0.426	Jermann and Quadrini (2012)
volatility of taste shocks	σ_ϵ	0.001	Dvorkin et al. (forthcoming)
Panel B. Parameters set by simulation			
interpretation	symbol	value	target (6 European countries)
productivity persistence	ρ_z	0.85	autocorrelation of log GDP
productivity volatility	σ_z	0.015	std. dev. of log GDP
capital adjustment cost	ψ	28	std. dev. of log fixed assets
corporate default value	μ_{ν^d}	0.011	average corporate spread
average government default cost	μ_ξ	0.35	average government spread
government default cost volatility	σ_ξ	0.1	std. dev. of government spread
sensitivity of investor sentiment	γ	1.7	std. dev. of government spread

Table 6. Moments from Model and Data

Panel A. Targeted Moments	Model	Data
autocorrelation of log GDP	0.454	0.470
std. dev. of log GDP	0.025	0.026
std. dev. of log fixed assets	0.024	0.024
average corporate spread	136 bp	139 bp
average government spread	79 bp	74 bp
std. dev. of government spread	108 bp	107 bp
Panel B. Non-targeted Moments	Model	Data
corporate default rate (%)	4.4	4.1
government default rate (%)	1.9	1.5
std. dev. of corporate spread	79 bp	60 bp
average corporate debt to GDP ratio	0.86	1.03
average government debt to GDP ratio	1.02	0.79
<u>Government Spread Regression Coefficients:</u>		
corporate debt/GDP	0.017 [0.015 0.018]	0.11 (0.05)
government debt/GDP	0.021 [0.019 0.023]	0.08 (0.03)
GDP Growth	-0.121 [-0.124 -0.117]	-0.19 (0.09)

Notes: For the model-simulated regression, 5th and 95th percentiles of estimates out of 1,000 simulations are in brackets together with their median above the brackets. For the empirical regression, robust standard errors (calculated using the 2-step GMM method for IV regressions) are in parentheses together with estimates above the parentheses.

Table 7. Model Sensitivity

	Baseline	Identical Discount Factor $\beta = \beta_g = 0.98$	Small Defaultable Tax $\zeta = 0.5$	No Investor's Sentiment $\gamma = 0$	High Equity Issuance Cost $\kappa = 1$
Panel A. Statistics:					
Corporate Debt/Output (%)	86	86	86	86	89
Firm Default Rate (%)	4.4	4.4	4.4	4.4	5.5
Tax Revenue (% Y_{ss})	29.1	29.1	29.1	29.1	28.7
Government Spread (bp)	78	3	74	75	76
Correlation(b/Y , Govt Spread)	0.258	0.200	0.277	0.328	0.282
Panel B. Welfare Gains (%):					
Optimal Policy	12.8	18.9	12.3	12.8	12.9
Constant Debt Tax	2.1	2.1	2.2	2.0	0.5
Countercyclical Debt Tax	0.09	0.03	0.7	-0.1	-1.8

Notes: Panel A presents average statistics of the competitive equilibrium from 100,000 simulations of 200 periods after dropping first 100 periods for each simulation. Tax revenue is in percentage relative to the steady state output. Panel B reports the averages of a permanent increase in consumption across simulations by implementing different debt policies. The debt tax rate is given by $\tau_t^b = \max[\bar{\tau}^b + \beta_\tau(b_t/Y_t - \bar{b}/\bar{Y}), 0]$. The target corporate leverage ratio \bar{b}/\bar{Y} is set to 0.91. The constant debt tax rate $\bar{\tau}^b$ is 6% (β_τ is set to zero), For the countercyclical debt tax rate, $\bar{\tau}^b$ is set to 6%, and the debt tax slope β_τ is set to 0.25. When only the half of the corporate income tax is defaultable, firms cannot default on the fraction $\zeta = 0.5$ of their income tax, and the government receives non-defaultable corporate income tax from defaulting firms.

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Figure Appendix

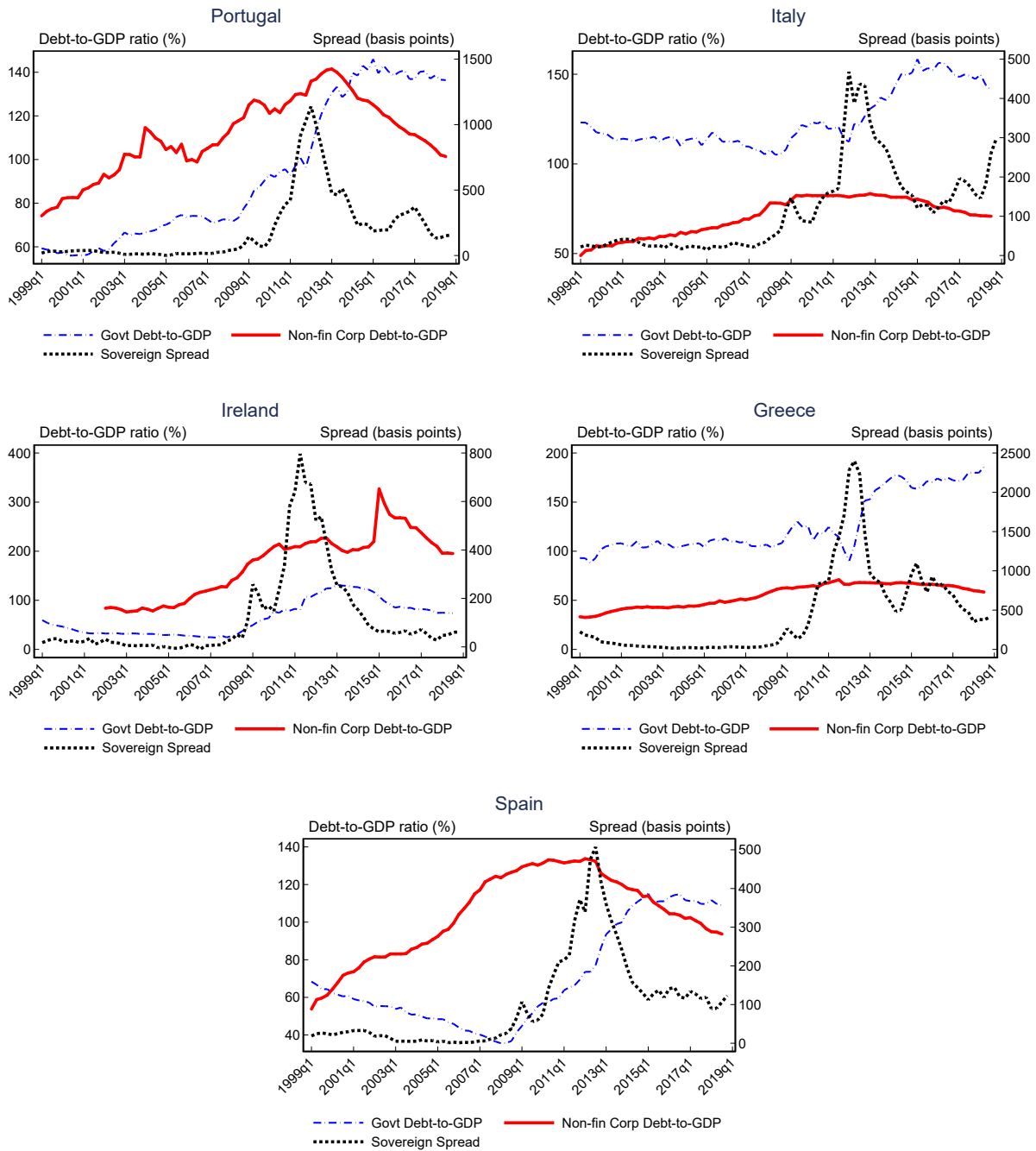


Figure A.1. Leverage and Sovereign Spread in Peripheral Countries

Source: BIS, Bloomberg

Notes: See Figure A.1 for notes.

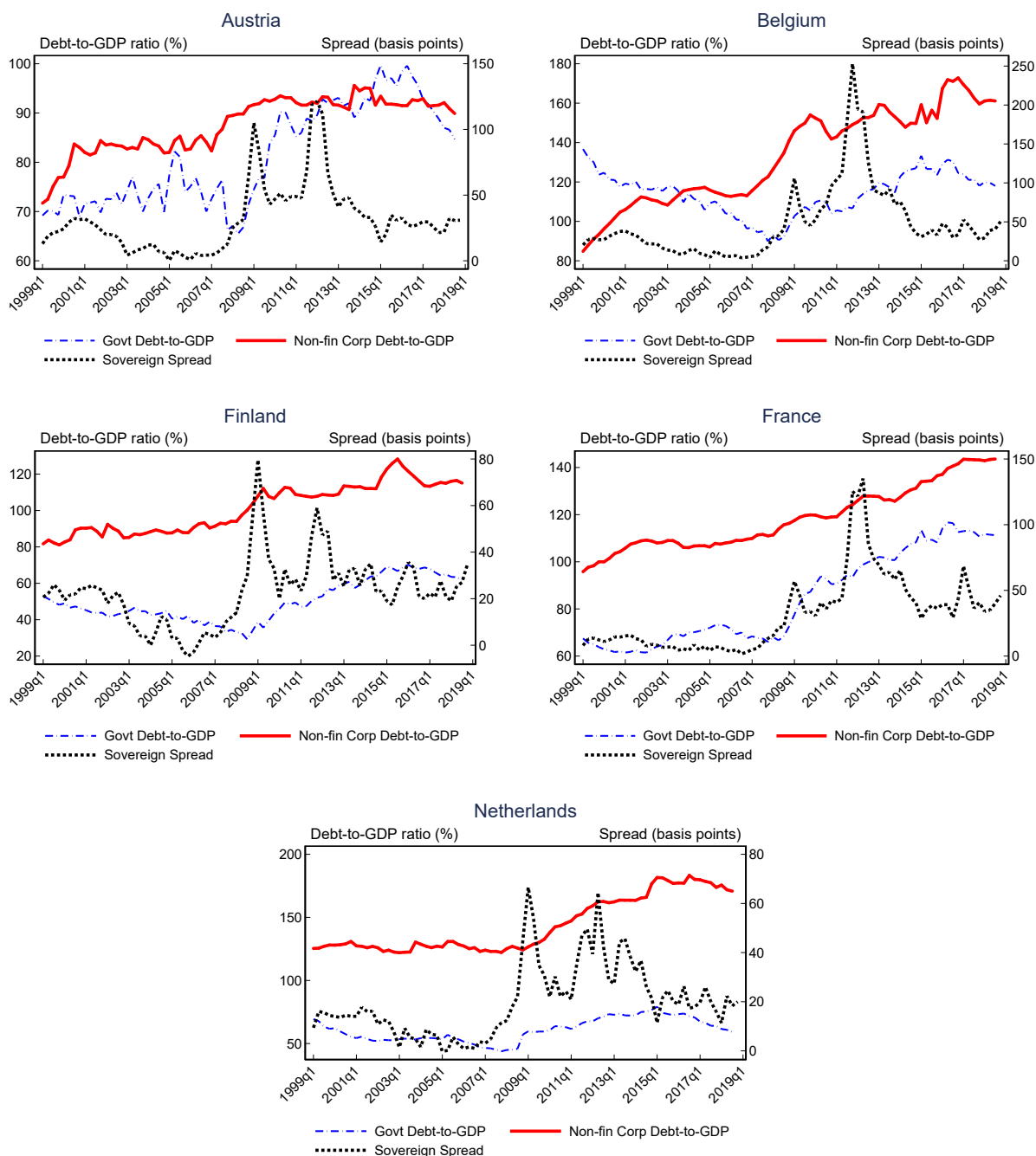


Figure A.2. Leverage and Sovereign Spread in Non-Peripheral Countries

Source: BIS, Bloomberg

Notes: The figure shows debt-to-GDP ratios and sovereign spreads for each country. Debt is core debt (BIS) which consists of the following financial instruments as defined in the System of National Accounts (SNA): debt securities, loans, and currency and deposits. Core debt excludes special drawing rights, insurance, pension, standardized guarantee schemes, and other accounts payable. Sovereign spreads (Bloomberg) are measured as the difference between the 10-year government bond yield of each country and that of Germany. Both government bond yields are denominated in euro and daily averages of the period.

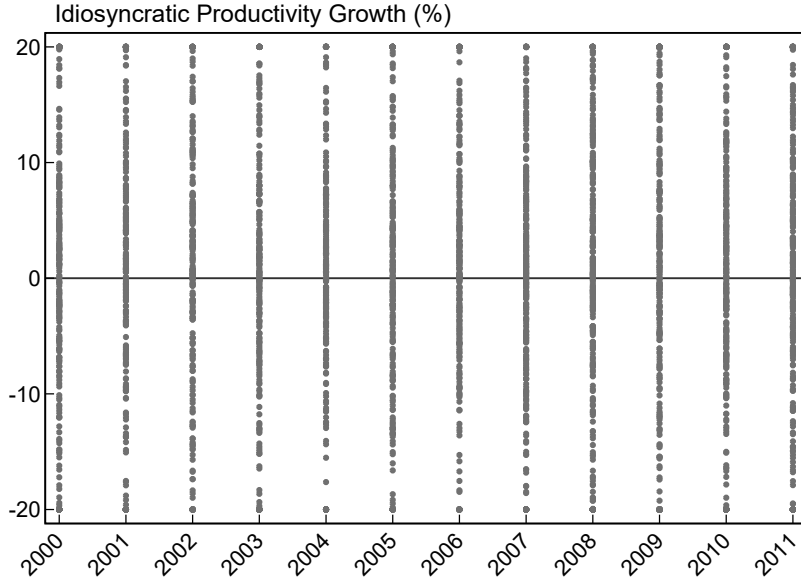


Figure A.3. Idiosyncratic Shocks to Top 50 Firms in six Eurozone Countries

Source: Author's calculation based on ORBIS-AMADEUS

Notes: To construct idiosyncratic shocks, I estimate the following firm-level productivity growth ($g_{i,t}$) decomposition:

$$g_{i,t} = \beta_{s,q}\eta_t + u_{i,t}$$

where η_t is an aggregate shock, and $u_{i,t}$ is an idiosyncratic shock to firm i at time t . I plot estimated residuals $\hat{u}_{i,t}$ for top 50 large firms in each Eurozone country (Italy, Spain, Portugal, Belgium, Finland, and France) together. I assume that (i) the responsiveness ($\beta_{s,q}$) of firm-level productivity growth ($g_{i,t}$) with regard to an aggregate shock (η_t) is identical within a 4-digit sector s and a firm size quintile q , and that (ii) aggregate and idiosyncratic shocks are separable in the growth decomposition. Under this assumption, regressing firm-level productivity growth on sector \times size \times year fixed effects gives residuals ($\hat{u}_{i,t}$) that are consistent estimators for idiosyncratic shocks ($u_{i,t}$). These residuals are calculated using top 50 largest firms in each 4-digit sector and winsorized at 20 and -20 percent following [Gabaix \(2011\)](#).

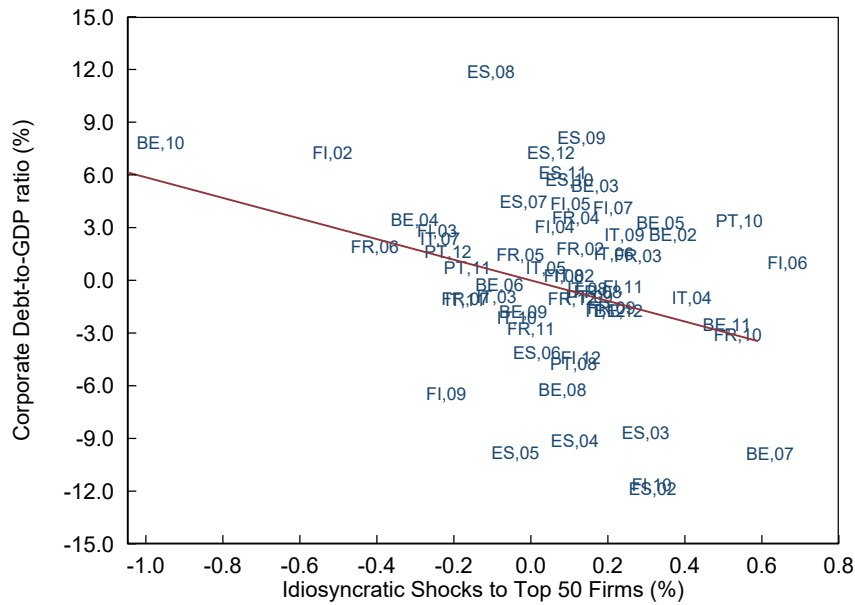


Figure A.4. First-stage Regression Plot

Source: Author's calculation based on BIS and ORBIS-AMADEUS

Notes: The figure plots the weighed idiosyncratic shocks to top 50 firms in each country against corporate debt-to-GDP ratios for six Eurozone countries using the first-stage regression in Table 2, column (4). These variables are purged of GDP growth, government debt-to-GDP ratio, lagged bank leverage, lagged granular residual, and country- and year- fixed effects. Labels denote the two-digit letter country code combined with two-digit years in the 2000s. IT: Italy, PT: Portugal, ES: Spain, BE: Belgium, FI: Finland, FR: France.

Table Appendix

Table A.1. Summary Statistics: Spread Regression Sample (1999q1-2012q4)

	Mean	Std. Dev.	Min	Max	$N_{Observations}$
Panel A. 9 Eurozone Countries					
Sovereign Spread (%p)	0.68	1.47	-0.10	11.39	489
GDP Growth (%)	0.35	0.99	-7.09	4.90	489
Sovereign Debt/GDP (%)	74	27	24	137	489
Corporate Debt/GDP (%)	107	30	52	227	489
Panel B. Italy, Spain, Portugal, Ireland					
Sovereign Spread (%p)	1.21	2.10	-0.10	11.39	210
GDP Growth (%)	0.27	1.12	-3.86	4.90	210
Sovereign Debt/GDP (%)	75	31	24	131	210
Corporate Debt/GDP (%)	103	40	52	227	210
Panel C. Belgium, Finland, France, Austria, Netherlands					
Sovereign Spread (%p)	0.29	0.34	-0.05	2.53	279
GDP Growth (%)	0.41	0.87	-7.09	3.30	279
Sovereign Debt/GDP (%)	72	24	29	137	279
Corporate Debt/GDP (%)	109	21	73	163	279

Notes: This table summarizes statistics for observations in nine Eurozone countries (Italy, Spain, Portugal, Belgium, Finland, France, Ireland, Austria, and Netherlands) during the period 1999q1-2012q4. Quarter-on-quarter GDP growth is measured as a log difference.

Table A.2. OLS Sovereign Spread Regression with nine Eurozone Countries

Dependent Variable:	Sovereign Spread _{c,t}						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
GDP Growth _{c,t-1}	-0.28*** (0.09)	-0.15* (0.08)	-0.13 (0.09)	-0.14 (0.09)	-0.15* (0.08)	-0.17* (0.09)	-0.13 (0.09)
Sovereign Debt/GDP _{c,t-1}	0.06*** (0.01)	0.05*** (0.01)	0.05*** (0.01)	0.05*** (0.01)	0.05*** (0.01)	0.05*** (0.01)	0.03*** (0.01)
Corporate Debt/GDP _{c,t-1}		0.02*** (0.00)	0.02*** (0.00)	0.02*** (0.00)	0.02*** (0.00)		0.02*** (0.01)
log(VIX) _{t-1}			0.12 (0.15)				
TED Spread _{t-1}				0.05 (0.12)			
Term Spread _{t-1}					-0.02 (0.04)		
Within R ²	0.4150	0.4853	0.4850	0.4844	0.4844	0.5191	0.5435
N _{Observations}	489	489	489	489	489	489	489
Country Fixed Effect	yes	yes	yes	yes	yes	yes	yes
Quarter Fixed Effect	no	no	no	no	no	yes	yes

Notes: Driscoll and Kraay (1998) standard errors (robust to heteroskedasticity and clustering on date and kernel-robust to common correlated disturbances) with a lag length of 4 quarters are in parentheses. ***, **, * denote significance at the 1%, 5%, and 10% levels, respectively. The regression sample consists of observations for nine Eurozone countries (Austria, Belgium, Finland, France, Ireland, Italy, Netherlands, Portugal, and Spain) during the period 1999q1–2012q4.

Table A.3. Country-by-country Spread Regression

Dependent Variable:	Sovereign Spread _{c,t}								
	IT (1)	ES (2)	PT (3)	IE (4)	BE (5)	FI (6)	FR (7)	AT (8)	NL (9)
GDP Growth _{t-1}	-0.30 (0.23)	-1.16*** (0.43)	-0.78 (0.48)	-0.13*** (0.05)	-0.13** (0.06)	-0.04*** (0.01)	-0.16*** (0.05)	-0.11** (0.05)	-0.08*** (0.02)
Sovereign Debt/GDP _{t-1}	-0.00 (0.02)	0.05*** (0.01)	0.02 (0.03)	0.03** (0.01)	-0.00 (0.01)	-0.00 (0.00)	0.01** (0.00)	0.00 (0.00)	0.01*** (0.00)
Corporate Debt/GDP _{t-1}	0.03 (0.02)	0.04** (0.01)	0.07*** (0.02)	0.02* (0.01)	0.02*** (0.01)	0.01*** (0.00)	-0.01 (0.01)	0.03** (0.01)	-0.01*** (0.00)
N _{Observations}	79	94	80	67	80	80	80	79	80
Trend	yes	yes	yes	yes	yes	yes	yes	yes	yes

Notes: Newey-West standard errors with a lag length of 4 quarters are in parentheses. ***, **, * denote significance at the 1%, 5%, and 10% levels, respectively. Regressions use all available data for each country. IT: Italy, ES: Spain, PT: Portugal, IE: Ireland, BE: Belgium, FI: Finland, FR: France, AT: Austria, NL: Netherland.

Table A.4. Summary Statistics: IV Regression

	Mean	Std. Dev.	Min	Max	$N_{\text{Observation}}$
Sovereign Spread (%p)	0.89	1.68	0.00	9.05	60
GDP Growth (%)	0.86	2.59	-8.63	5.06	60
Sovereign Debt/GDP (%)	80	29	32	125	60
Corporate Debt/GDP (%)	107	23	59	152	60
Bank Leverage (%)	16	7	5	46	60
Granular Residuals (Γ , %)	0.03	0.32	-0.86	0.93	60
Tax Revenue Growth (%)	1.04	3.89	-10.99	6.56	60

Notes: This table summarizes statistics for observations in Italy, Spain, Portugal, Belgium, Finland, and France during the period 2002–2012. Observations during the period 2000–2005 from Portugal are dropped due to insufficient firm-level observations used to calculate the granular residuals (weighted sum of idiosyncratic shocks to top 50 large firms) in each country. As the IV regression uses lagged variables up to two years back as explanatory variables using the sample ranging from 2000 to 2012, the final observations used in regressions run from 2002 to 2012 for Italy, Spain, Belgium, Finland, and France (55 observations) and from 2008 to 2012 for Portugal (5 observations).

Table A.5. Spread Regression with Alternative Instruments

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Baseline				Bank Control			
	OLS	IV (Size)	IV (Weight)	IV (T100)	OLS	IV (Size)	IV (Weight)	IV (T100)
A. Second-Stage Regression								
Dependent Variable:	Sovereign Spread _{c,t}							
Corporate Debt/GDP _{c,t-1}	0.03** (0.01)	0.11** (0.05)	0.10 (0.06)	0.14** (0.07)	0.04*** (0.02)	0.07* (0.03)	0.10* (0.06)	0.11** (0.05)
Sovereign Debt/GDP _{c,t-1}	0.03 (0.02)	0.08** (0.04)	0.07 (0.05)	0.10** (0.05)	0.04* (0.02)	0.05* (0.03)	0.08* (0.05)	0.08** (0.04)
GDP Growth _{c,t-1}	-0.23** (0.12)	-0.12 (0.10)	-0.15 (0.12)	-0.09 (0.13)	-0.25** (0.11)	-0.22** (0.10)	-0.20* (0.10)	-0.18** (0.09)
Bank Leverage _{c,t-1}					0.07*** (0.01)	0.08*** (0.02)	0.09*** (0.02)	0.09*** (0.02)
B. First-Stage Regression								
Dependent Variable:	Corporate Debt/GDP _{c,t-1}							
Granular Residual _{c,t-1}	n/a	-5.50** (2.69)	-0.73 (0.47)	-4.84** (2.29)	n/a	-6.47** (2.62)	-0.67 (0.49)	-5.43** (2.28)
Granular Residual _{c,t-2}	n/a	-4.25* (2.45)	-0.96** (0.41)	-3.31 (2.09)	n/a	-5.30** (2.17)	-0.95** (0.41)	-3.81* (1.95)
Adj. R-squared	0.6547	0.5765	0.5930	0.4887	0.6833	0.6737	0.6308	0.6232
Number of Observations	60	60	60	60	60	60	60	60
Country Fixed Effects	yes	yes	yes	yes	yes	yes	yes	yes
Time Fixed Effects	yes	yes	yes	yes	yes	yes	yes	yes
First-stage F	n/a	2.89	3.26	3.18	n/a	4.69	2.96	4.33
Hansen J (p-value)	n/a	0.5674	0.5165	0.5922	n/a	0.3460	0.3846	0.4638
CLR (p-value, $H_0 : \beta_{corp} = 0$)	n/a	0.0919	0.1640	0.0337	n/a	0.2289	0.1435	0.0706

Notes: Annual sovereign spreads are regressed on lagged explanatory variables over the period 2002–2012. In instrumental variable (IV) regressions, excluded instruments for corporate debt are 1 and 2 years lagged granular residuals, which are based on idiosyncratic total factor productivity shocks to large firms estimated by the method of [Wooldridge \(2009\)](#). The granular residual for IV (Size) is a weighted sum of idiosyncratic shocks to top 50 firms in each country c , using lagged Domar weights ($\text{sales}_{i,c,t-1}/\text{GDP}_{c,t-1}$) for a given firm i . Idiosyncratic shocks are residuals from the regression of firm-level productivity on sector \times size \times year fixed effects, in which sector dummies represent 4-digit sectors, and size dummies are quintiles determined by sales of each firm. The granular residual for IV (Weight) is a simple average of idiosyncratic shocks to top 50 firms in each country c , using residuals from the regression of firm-level productivity on sector \times year fixed effects. The granular residual for IV (T100) is a weighted sum of idiosyncratic shocks to top 100 firms in each country c , using lagged Domar weights and residuals from the regression of firm-level productivity on sector \times year fixed effects. In IV regressions, lagged GDP growth, the lagged government debt to GDP ratio, country fixed effects, and year fixed effects are included in both the first and second stage regressions. Lagged bank leverage is added to the first and second stage regressions in columns 5–8. The [Hansen \(1982\)](#) J statistic tests the null that instruments are excludable. The first-stage effective F statistic of [Olea and Pflueger \(2013\)](#) tests the null that the excluded instruments are not relevant. The p-value for CLR statistic ([Andrews et al. \(2019\)](#)) is reported to test the null hypothesis that the coefficient on corporate debt in the second stage regression is zero. This test is robust to weak instruments. Robust standard errors (calculated using the 2-step GMM method and robust to arbitrary clusters for IV regressions) are in parentheses. ***, **, * denote significance at the 1%, 5%, and 10% levels, respectively.

Table A.6. Spread Regression with Alternative Corporate Leverage

	(1)	(2)	(3)	(4)
	Baseline IV		Alternative IV	
A. Second-Stage Regression				
Dependent Variable:	Sovereign Spread _{c,t}			
Corporate Debt/Corporate Value Added _{c,t-1}	0.07** (0.03)	0.06** (0.03)	0.06** (0.03)	0.04* (0.02)
Sovereign Debt/GDP _{c,t-1}	0.10** (0.04)	0.08** (0.04)	0.08** (0.04)	0.06* (0.03)
GDP Growth _{c,t-1}	-0.11 (0.11)	-0.19* (0.10)	-0.14 (0.10)	-0.22** (0.11)
Bank Leverage _{c,t-1}		0.07*** (0.02)		0.07*** (0.01)
B. First-Stage Regression				
Dependent Variable:	Corporate Debt/Corp Value Added _{c,t-1}			
Granular Residual _{c,t-1}	-9.65** (4.21)	-10.37** (4.34)	-10.50** (4.55)	-11.44** (4.62)
Granular Residual _{c,t-2}	-7.78** (3.83)	-8.53** (3.79)	-8.32* (4.18)	-9.34** (3.98)
Adjusted R^2	0.4594	0.5590	0.5512	0.6296
$N_{\text{Observation}}$	60	60	60	60
Country Fixed Effect	yes	yes	yes	yes
Time Fixed Effect	yes	yes	yes	yes
Hansen J (p-value)	0.5137	0.4003	0.5528	0.3813
First-stage F^{eff}	4.10	4.68	3.58	4.39
CLR (p-value, $H_0 : \beta_{corp} = 0$)	0.0230	0.0618	0.0831	0.2154

Notes: Columns (1) and (2) present instrumental variable regression results of Table 2, using alternative corporate leverage (the ratio of non-financial corporations total debt (BIS) to non-financial corporations value added (Eurostat)). Columns (3) and (4) present similar regression results of Table A.5 IV (Size), using alternative corporate leverage. Robust standard errors (calculated using the 2-step GMM method and robust to arbitrary clusters) are in parentheses. ***, **, * denote significance at the 1%, 5%, and 10% levels, respectively.

Table A.7. Spread Regression with Household Debt

	(1)	(2)	(3)	(4)
	Baseline IV		Alternative IV	
A. Second-Stage Regression				
Dependent Variable:	Sovereign Spread _{c,t}			
Corporate Debt/GDP _{c,t-1}	0.12*** (0.05)	0.08*** (0.03)	0.12** (0.06)	0.07** (0.03)
Sovereign Debt/GDP _{c,t-1}	0.07** (0.04)	0.05** (0.02)	0.07* (0.04)	0.04* (0.02)
GDP Growth _{c,t-1}	-0.24** (0.09)	-0.31*** (0.11)	-0.22** (0.09)	-0.28** (0.11)
Bank Leverage _{c,t-1}		0.08*** (0.02)		0.08*** (0.02)
Household Debt/GDP _{c,t-1}	-0.16*** (0.05)	-0.12*** (0.04)	-0.16** (0.06)	-0.10** (0.04)
B. First-Stage Regression				
Dependent Variable:	Corporate Debt/GDP _{c,t-1}			
Granular Residual _{c,t-1}	-5.30** (2.32)	-6.14*** (2.05)	-4.75* (2.77)	-5.74** (2.49)
Granular Residual _{c,t-2}	-5.34** (2.31)	-6.24*** (1.59)	-4.63* (2.64)	-5.71*** (2.01)
Adjusted R^2	0.6556	0.7155	0.6637	0.7174
$N_{Observation}$	60	60	60	60
Country Fixed Effect	yes	yes	yes	yes
Time Fixed Effect	yes	yes	yes	yes
Hansen J (p-value)	0.3195	0.1743	0.3642	0.1491
First-stage F^{eff}	4.61	9.99	2.43	5.21
CLR (p-value, $H_0 : \beta_{corp} = 0$)	0.0309	0.1117	0.1152	0.4602

Notes: Columns (1) and (2) present instrumental variable regression results of Table 2, adding the ratio of household total debt to GDP (BIS). Columns (3) and (4) present similar regression results of Table A.5 IV (Size), adding the same household leverage. Robust standard errors (calculated using the 2-step GMM method and robust to arbitrary clusters) are in parentheses. ***, **, * denote significance at the 1%, 5%, and 10% levels, respectively.

Table A.8. Spread Regression: Importance of the Tax Revenue Channel

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent Variable:	Sovereign Spread $_{c,t}$					
Corporate Debt/GDP $_{c,t-1}$	0.03** (0.01)	0.02 (0.02)	-0.01 (0.02)	0.04*** (0.02)	0.03* (0.02)	-0.00 (0.02)
Government Debt/GDP $_{c,t-1}$	0.03 (0.02)	0.03 (0.02)	0.00 (0.02)	0.04* (0.02)	0.04* (0.02)	-0.00 (0.02)
GDP Growth $_{c,t-1}$	-0.23** (0.12)	-0.21* (0.11)	-0.03 (0.05)	-0.25** (0.11)	-0.23** (0.11)	-0.03 (0.05)
Tax Revenue Growth $_{c,t}$		-0.06 (0.06)	-0.02 (0.03)		-0.05 (0.06)	-0.02 (0.03)
Tax Revenue Growth $_{c,t+1}$			-0.10 (0.07)			-0.09 (0.07)
Bank Leverage $_{c,t-1}$			0.07***	0.07*** (0.01)	0.05** (0.01)	 (0.03)
Adjusted R^2	0.6547	0.6542	0.5797	0.6833	0.6806	0.5834
$N_{Observations}$	60	60	54	60	60	54
Country Fixed Effect	yes	yes	yes	yes	yes	yes
Time Fixed Effect	yes	yes	yes	yes	yes	yes

Notes: Columns (1) and (4) present OLS regression results of Table 2. Columns (2), (3), (5), and (6) present similar regression results, adding real tax revenue growth in years t and $t+1$. Robust standard errors are in parentheses. ***, **, * denote significance at the 1%, 5%, and 10% levels, respectively. Using Driscoll and Kraay (1998) standard errors gives similar results.

Table A.9. Summary Statistics: Firm-level Regression Sample

	Mean	Std. Dev.	Min	Max	N_{obs}
Tax Payment / Value Added (%)	4.15	3.79	0.00	12.76	35,187
Log(TFP)	5.24	1.01	0.80	10.45	35,187
Log(Tangible Fixed Assets)	13.20	2.01	2.20	20.98	35,187
Log(Debt)	10.39	6.20	0.00	22.40	35,187
High Leverage Dummy	0.47	0.50	0.00	1.00	35,187
Interest Payment / Value Added (%)	2.48	3.91	0.00	19.38	35,187

Notes: Total factor productivity (TFP) is estimated by the Wooldridge (2009) method. Debt is the sum of loans and long-term debt. The HighLev $_i$ dummy equals 1 if average leverage before 2008 of a given firm is higher than the aggregate median before 2008 and 0 otherwise. Firm leverage is measured as the ratio of debt to value added. Value added is measured as operating revenue minus materials cost. The sample covers the period 2004–2012 in six Eurozone countries (Italy, Portugal, Spain, Belgium, Finland, and France).

Table A.10. Robustness Check: Firm Tax Payment Regression

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent Variable:	Tax Payment $_{i,t}$					
HighLev $_i \times$ Crisis $_t$	-0.54*** (0.07)	-0.54*** (0.07)	-0.51*** (0.07)	-0.41*** (0.07)	-0.41*** (0.07)	-0.37*** (0.07)
$\log(z_{i,t-1})$	1.58*** (0.13)	1.58*** (0.13)	1.61*** (0.16)	1.56*** (0.13)	1.56*** (0.13)	1.55*** (0.15)
$\log(k_{i,t-1})$	-0.15*** (0.04)	-0.15*** (0.04)	-0.13** (0.05)	-0.12** (0.04)	-0.12** (0.04)	-0.09** (0.05)
$\log(b_{i,t-1})$	-0.04*** (0.01)	-0.04*** (0.01)	-0.04*** (0.01)	-0.02** (0.01)	-0.02** (0.01)	-0.02** (0.01)
$\log(z_{i,t-1}) \times$ Crisis $_t$	-0.07 (0.05)	-0.07 (0.05)	-0.28** (0.11)	-0.07 (0.05)	-0.07 (0.05)	-0.21* (0.11)
$\log(k_{i,t-1}) \times$ Crisis $_t$	-0.01 (0.03)	-0.01 (0.03)	-0.05 (0.04)	-0.01 (0.03)	-0.01 (0.03)	-0.06* (0.03)
$\log(b_{i,t-1}) \times$ Crisis $_t$	0.03*** (0.01)	0.03*** (0.01)	0.03*** (0.01)	0.03** (0.01)	0.03** (0.01)	0.03** (0.01)
Interest Payment $_{i,t}$				-0.18*** (0.02)	-0.18*** (0.02)	-0.18*** (0.02)
$N_{Observation}$	35,187	35,187	34,463	35,187	35,187	34,463
R^2	0.65	0.65	0.65	0.65	0.65	0.65
Firm Fixed Effect (FE)	yes	yes	yes	yes	yes	yes
Country and Year FE	yes	n/a	n/a	yes	n/a	n/a
Country \times Year FE	no	yes	n/a	no	yes	n/a
Country \times Sector \times Year FE	no	no	yes	no	no	yes

Notes: Standard errors are in parentheses and clustered at the 4-digit sector level. ***, **, * denote significance at the 1%, 5%, and 10% levels, respectively. z is total factor productivity estimated by the Wooldridge (2009) method. k is tangible fixed assets. b is the sum of loans and long-term debt. Leverage is measured as a debt-to-value-added ratio (b/y), where value added y is measured as operating revenue minus materials cost. Interest payments are measured as the ratio of interest paid to value added. The Crisis $_t$ dummy equals 1 in or after 2008 and 0 otherwise. The HighLev $_i$ dummy equals 1 if average leverage before 2008 of a given firm is higher than the aggregate median before 2008 and 0 otherwise. The sample covers the period 2004-2012 in six Eurozone countries (Italy, Portugal, Spain, Belgium, Finland, and France).

Data Appendix

1. Firm-level Data Cleaning

ORBIS-AMADEUS data is constructed following [Kalemli-Ozcan et al. \(2019\)](#) unless stated otherwise.

- I limit the sample to the period of 1999 to 2015 and exclude the financial sector (NACE code 64–66) and the mining and oil-related sector (NACE code 05–09). I only use unconsolidated balance sheets, as many firms do not report consolidated balance sheets.
- I drop firm-year observations if any of the following variables are missing or negative in a given year for each firm: total assets, operating revenue (turnover), sales, number of employees, costs of employees, material costs, and financial debt. Also, I drop entire firm observations if any of the following variables are negative for each firm: total assets, sales, tangible fixed assets, and number of employees. If the number of employees exceeds 2 millions for any year observations in a given firm, I drop entire observations of this firm.
- To further mitigate measurement errors arising from reporting mistakes and associated outliers, I perform the following procedure. First, I calculate growth rates of a variable x at time t as $(x_{it} - x_{it-1}) / (0.5 \times (x_{it} + x_{it-1}))$ for each firm i and year t . Next, if the growth rate at time t is greater than 150% and the growth rate at time $t + 1$ is smaller than -150%, the value x_t is replaced with a simple average $(x_{t-1} + x_{t+1}) / 2$. I repeat this procedure three times for the following variables: total assets, operating revenue (turnover), sales, number of employees, costs of employees, material costs, and financial debt. I also winsorize these variables at the bottom 1st percentile. Moreover, I manually check whether variables have obvious mistakes and correct data accordingly. For example, if a firm's sales are \$100 for three years, increase to \$10,000 and go back to \$100 in subsequent years, I replace the value of \$10,000 with \$100.
- To obtain estimates for production function parameters used for total factor productivity (TFP) calculation, I follow [Gopinath et al. \(2017\)](#) that impose stricter criteria in data cleaning and drop many observations, as quality of data needs to be much higher for TFP estimation. Final firm-level TFP is constructed by plugging cleaned data described above into the production function estimated using cleaned data as in [Gopinath et al. \(2017\)](#).

2. Alternative Measure of Idiosyncratic Shocks

Beside estimating equation (5) in a similar vein with [Gabaix and Koijen \(2020\)](#), an alternative way of estimating firm-level idiosyncratic shocks is as follows. An econometrician regresses firm-level total factor productivity $z_{i,t}$ for each firm i and year t on its lagged value using the following equation:

$$\log(z_{i,t}) = \mu + \rho \log(z_{i,t-1}) + u_{i,t} \quad (57)$$

Residuals $\hat{u}_{i,t}$ estimated from the above equation are innovations to productivity and could be called idiosyncratic shocks. However, the problem of this procedure is that the OLS estimator of ρ is biased due to the nature of autoregressive process, and thus the estimates $\hat{u}_{i,t}$ are also biased. Even if residuals are measured precisely, the granular residuals constructed using the above procedure are less likely to be relevant instruments for corporate leverage. The reason is that firms are more likely to make financing decisions, mainly based on productivity changes instead of unexpected innovations to productivity. Notice that firms' decision rules in my model depend on the level of their productivity z rather than innovations.

3. Event Study

To obtain detrended series for event study in Figure 8, I regress the following variables of each six Eurozone countries (Italy, Spain, Portugal, Belgium, Finland, and France) on a common linear trend and get residuals, using the sample of the period 2007-2017. In the figure, I plot the cross-country average of each series over time.

- Output: log of annual gross domestic product (chain linked volumes, Eurostat)
- Investment: log of annual gross fixed capital formation (chain linked volumes, Eurostat)
- Consumption: log of the sum of the annual final consumption expenditure of households and general government (chain linked volumes, Eurostat), which is consistent with the model assumption that household utility depends on the sum of dividends, labor income, and government expenditure
- Government expenditure: log of the sum of the annual final consumption expenditure of general government (chain linked volumes, Eurostat)
- Tax revenue: log of total receipts from taxes and social contributions (Eurostat) where real tax revenue is calculated as nominal revenue divided by the GDP deflator (obtained from Eurostat)
- Productivity: log of total factor productivity of the total economy obtained from European Commission AMECO Database
- Corporate debt: annual average of quarterly total credit to non-financial corporations to GDP ratios (BIS total credit statistics)
- Government debt: annual average of quarterly total credit to the government sector at market value to GDP ratios (BIS total credit statistics)
- Corporate spread: annual average of monthly corporate spreads of Euro area non-financial corporate bonds (Euro area Bund NFC), using spread data obtained from [Gilchrist and Mojon \(2017\)](#)
- Government spread: annual average of the difference between a country's 10-year government bond rate and the 10-year German government bond rate where both bond rates are denominated in euro (daily, Bloomberg)
- Firm default rate: annual average of monthly speculative-grade non-financial corporate default rates of Europe obtained from "Moody's 1Q 2020 Asia-Pacific Default Report - Non-financial High-Yield Corporate Default Rates Data" (only the aggregate European default rate from 2008 to 2020 is publicly available in this report)

Appendix A. Proof of Proposition 1

Consider the time after firms choose k_{t+1} together with b_{t+1} given B_{t+1} and before productivity z_{t+1} realizes. I show that the cumulative probability of the government revenue TR_{t+1} being equal the level of TR with low firm debt b_{t+1}^l is smaller than the one with high firm debt b_{t+1}^h , which means $H(TR|b_{t+1}^l) \leq H(TR|b_{t+1}^h)$ for every TR . By the proposition 6.D.1 of Mas-Colell et al.(1995),³² this statement is true if and only if the distribution $H(TR|b_{t+1}^h)$ first-order stochastically dominates $H(TR|b_{t+1}^l)$, which means that the following equation holds:

$$\int U(TR)dH(TR|b_{t+1}^l) \geq \int U(TR)dH(TR|b_{t+1}^h) \quad (58)$$

for every nondecreasing function $U : \mathbb{R} \rightarrow \mathbb{R}$.

The government revenue decreases in firm debt b_{t+1} as follows:

$$\frac{\partial[TR_{t+1}]}{\partial b_{t+1}} = -\frac{\partial\mu(X_{t+1})}{\partial b_{t+1}}F(z_{t+1}, k_{t+1}) \leq 0 \quad (59)$$

for every z_{t+1} , where $\frac{\partial\mu(X_t)}{\partial b_t} \geq 0$ by the proposition 2.

This implies that $TR(z_{t+1}, b_{t+1}^l) = TR(z_{t+1}, b_{t+1}^h) + \epsilon(z_{t+1}, b_{t+1}^h)$ for every z_{t+1} and any $\epsilon(z_{t+1}, b_{t+1}^h) \geq 0$.

Thus, I have $H(TR|b_{t+1}^l) \leq H(TR|b_{t+1}^h)$ for every TR :

$$\begin{aligned} H(TR|b_{t+1}^l) &= P[TR(z_{t+1}, b_{t+1}^l) \leq TR] = P[TR(z_{t+1}, b_{t+1}^h) + \epsilon(z_{t+1}, b_{t+1}^h) \leq TR] \\ &= P[TR(z_{t+1}, b_{t+1}^h) \leq TR - \epsilon(z_{t+1}, b_{t+1}^h)] = H(TR - \epsilon|b_{t+1}^h) \leq H(TR|b_{t+1}^h) \end{aligned} \quad (60)$$

Q.E.D.

Appendix B. Proof of Proposition 2

I show $\frac{Q(z_t, k_{t+1}, b_{t+1}, B_{t+1})}{b_{t+1}} \leq 0$. From the government's optimization problem (24), the government default probability is given by

³²Mas-Colell, Andreu, Michael D. Whinston, and Jerry R. Green, *Microeconomic Theory*, Oxford University Press, 1995

$$Pr[D(X_{t+1}) = 1] = \int_{\xi_{t+1}} D(X_{t+1}, \xi_{t+1}) d\Xi(\xi_{t+1}) = \tag{61}$$

$$Pr[\xi_{t+1} < V^g(0; X_{t+1}) - V^g(B_{t+1}; X_{t+1}) = \bar{\xi}_{t+1}] = \Xi[\bar{\xi}_{t+1}]$$

where Ξ is the cumulative distribution function of *i.i.d.* government default costs.

If the threshold government default cost shock $\bar{\xi}$ increases in firm debt b such that $\frac{\partial \bar{\xi}_{t+1}}{\partial b_{t+1}} \geq 0$, then the government default probability increases in firm debt $\frac{\partial Pr[D(X_{t+1})=1]}{\partial b_{t+1}} \geq 0$, and in turn the government debt price Q decreases in firm debt according to the debt pricing equation:

$$Q(z_t, k_{t+1}, b_{t+1}, B_{t+1}) = \frac{1 - \int_{z_{t+1}} \int_{\xi_{t+1}} D(X_{t+1}, \xi_{t+1}) d\Pi(z_{t+1}|z_t) d\Xi(\xi_{t+1})}{1 + r} \tag{62}$$

$$= \frac{1 - \int_{z_{t+1}} Pr[D(X_{t+1}) = 1] d\Pi(z_{t+1}|z_t)}{1 + r}$$

To show $\frac{\partial \bar{\xi}}{\partial b} \geq 0$, first, I show that the government's value $V^g(B_{t+1}; X_{t+1})$ is decreasing in firm debt b_{t+1} . The envelope condition implies that

$$\frac{\partial V^g(B_t; X_t)}{\partial b_t} = - \left[\frac{\partial \mu(X_t)}{\partial b_t} \phi(e_t) + (1 - \mu(X_t)) \lambda_t \right] u'(C_t) \leq 0 \tag{63}$$

where I use $\frac{\partial \mu(X_t)}{\partial b_t} \geq 0$, $\phi(e_t) \geq 0$, $\lambda_t \geq 0$, and $u'(C_t) > 0$. $\frac{\partial \mu(X_t)}{\partial b_t} \geq 0$ follows from the relationship that the firms' value function is decreasing in firm debt as follows:

$$\frac{\partial V^f(k_t, b_t; X_t)}{\partial b_t} = -\phi_e(e_t) = -\lambda_t \leq 0 \tag{64}$$

where I use the firms' optimality condition (18) and $\lambda_t \geq 0$. Then, combined with (15), I have $\frac{\partial v^d(X_t)}{\partial b_t} \leq 0$. This means $\frac{\partial d(X_t, \nu_{i,t}^d)}{\partial b_t} \geq 0$ and $\frac{\partial \mu(X_t)}{\partial b_t} \geq 0$.

Next, I need to show the following:

$$\frac{\partial \bar{\xi}_{t+1}}{\partial b_{t+1}} = \frac{\partial V^g(0; X_{t+1})}{\partial b_{t+1}} - \frac{\partial V^g(B_{t+1}; X_{t+1})}{\partial b_{t+1}} \geq 0 \tag{65}$$

It can be shown that $u'(C_{t+1}|B_{t+1} \geq 0) \geq u'(C_{t+1}|B_{t+1} = 0)$ and that firm variables $\left[\frac{\partial \mu(X_t)}{\partial b_t} \phi(e_t) + (1 - \mu(X_t)) \lambda_t \right]$ in the equation (63) do not change with B_t , when firm values $V^f(k, b; X)$ are independent of government debt B . Thus, the above inequality holds.

Q.E.D.

Appendix C. Proof of Proposition 3

The complete problem of the constrained social planner is as follows:

$$V^g(z, k, b, B) = \max_{e, k', b', B', D', C} u(C) + \beta \mathbb{E} \max_{D'} \langle V^g(z', k', b', B'), V^g(z', k', b', 0) - \xi' \rangle$$

subject to

$$\begin{aligned} C &= [1 - \mu(z, k, b)](F(z, k) + (1 - \delta)k - k' - b + q(z, k', b')b' - \kappa(e - \bar{e})^2) \\ &+ Q(z, k', b', B')B' - B \end{aligned} \quad (\text{SP1})$$

$$q(z, k', b') = \frac{1 - \int_{z'} \mu(z', k', b') d\Pi(z'|z)}{1 + r} \quad (\text{SP2})$$

$$\mu(z', k', b') = \int_{\bar{\nu}^d(z', k', b')}^{\nu_{max}^d} d\Omega(\nu_i^d) \quad (\text{SP3})$$

$$Q(z, k', b', B') = \frac{1 - \int_{z'} \int_{\xi'} D(z', k', b', B', \xi') d\Pi(z'|z) d\Xi(\xi')}{1 + r} \quad (\text{SP4})$$

Let's define a complete Ramsey problem. The Ramsey problem with debt taxes (τ^b), transfers to firms (T), and investment credits (τ^k) solves the following:

$$V^g(z, k, b, B) = \max_{\tau^b, T, \tau^k, B', D', G, C} u(C) + \beta \mathbb{E} \max_{D'} \langle V^g(z', k', b', B'), V^g(z', k', b', 0) - \xi' \rangle$$

subject to

$$e = (1 - \tau^y)F(z, k) \tag{RP1}$$

$$+ (1 - \delta)k - (1 - \tau^k)k' - b + (q(z, k', b') - \tau^b)b' + T$$

$$C = s([1 - \mu(z, k, b)]\phi(e) + p) - s'p + G \tag{RP2}$$

$$G + B = (1 - \mu(z, k, b))[\tau^y F(z, k) - \tau^k k' + \tau^b b' - T] \tag{RP3}$$

$$+ Q(z, k', b', B')B'$$

$$s = 1 \tag{RP4}$$

$$\lambda = \phi_e(e) \tag{RP5}$$

$$[1 - \tau_t^k - \frac{\partial q(z_t, k_{t+1}, b_{t+1})}{\partial k_{t+1}} b_{t+1}] \lambda_t = \int_{z_{min}}^{z_{max}} \int_{\nu_{min}^d}^{\bar{\nu}^d(X_{t+1})} \tag{RP6}$$

$$[m(X_t, X_{t+1})((1 - \tau_{t+1}^y)F_k(z_{t+1}, k_{t+1}) + (1 - \delta))\lambda_{t+1}] d\Omega(\nu_{i,t+1}^d) d\Pi(z_{t+1}|z_t)$$

$$\left(\frac{\partial [q(z_t, k_{t+1}, b_{t+1})b_{t+1}]}{\partial b_{t+1}} - \tau_t^b \right) \lambda_t = \tag{RP7}$$

$$\int_{z_{min}}^{z_{max}} \int_{\nu_{min}^d}^{\bar{\nu}^d(X_{t+1})} [m(X_t, X_{t+1})\lambda_{t+1}] d\Omega(\nu_{i,t+1}^d) d\Pi(z_{t+1}|z_t)$$

$$q(z, k', b') = \frac{1 - \int_{z'} \mu(z', k', b') d\Pi(z'|z)}{1 + r} \tag{RP8}$$

$$\mu(z', k', b') = \int_{\bar{\nu}^d(z', k', b')}^{\nu_{max}^d} d\Omega(\nu_i^d) \tag{RP9}$$

$$Q(z, k', b', B') = \frac{1 - \int_{z'} \int_{\xi'} D(z', k', b', B', \xi') d\Pi(z'|z) d\Xi(\xi')}{1 + r} \tag{RP10}$$

$$p(X, X') = \tag{RP11}$$

$$\int_{z_{min}}^{z_{max}} \int_{\xi_{min}}^{\xi_{max}} m(X, X') [[1 - \mu(z', k', b')] \phi(e') + p(X', X'')] d\Pi(z'|z) d\Xi(\xi')$$

$$m(X, X') = \beta \frac{u'(C')}{u'(C)} \tag{RP12}$$

First, I prove that the allocations in the Ramsey problem satisfy the equations in the constrained social planner problem. Notice that the objective functions of these two problems are identical. Combining equations (RP1), (RP2), (RP3), and (RP4) leads to the social planner's resource constraint (SP1) with the definition of net dividends $\phi(e) = e - \kappa(e - \bar{e})^2$. Variables $\{\lambda, \tau^k, \tau^b, m\}$ can be set so that allocations satisfying the implementability constraints (RP5), (RP6), and (RP7) given the stochastic discount factor (RP12) are identical to allocations in the social planner problem. Implementability constraints (RP8), (RP9), and (RP10) are common to the problems of Ramsey and social planners. The remaining constraint (RP11) is satisfied by choosing p so that allocations in two problems are identical.

Next, I show that the allocations in the constrained social planner problem satisfy the equations in the Ramsey problem. The constrained-efficient allocations $\{e, k, b, B, D\}$ are set by the social planner, and C is determined by the resource constraint (SP1) and other implementability constraints. Variables $\{\lambda, \tau^k, \tau^b, m\}$ can be chosen to satisfy constraints (RP5), (RP6), and (RP7) given (RP12) after plugging constrained-efficient allocations into these constraints. Given $\{\tau^y, \tau^k, \tau^b\}$ and constrained-efficient allocations, (RP1) is satisfied by choosing a proper T . Given the stock market clearing condition (RP4) and the constrained efficient allocations including C , the government expenditure G can be chosen to satisfy (RP2). As constraints (SP1), (RP1), (RP2), and (RP4) are satisfied, combining these constraints gives the government budget constraint (RP3). The equation (RP11) is slack as p can be set freely given the constrained-efficient allocations. All remaining implementability constraints (RP8), (RP9), and (RP10) in the Ramsey problem are identical to those of the social planner problem.

The implemented set of policies $\{\tau^b, T, \tau^k, B', D', G\}$ are functions of key state variables $X = \{z, k, b, B\}$, as they are solutions to the recursive equilibrium. This means that equilibrium policy functions are time-invariant, and thus policies are time-consistent.

Q.E.D.

Appendix D. Computational Details

To obtain the equilibria of the model, I adopt a discrete choice model with taste shocks following [Dvorkin et al. \(forthcoming\)](#). For the quantitative model, I add and modify the relevant ingredients of this model as described in section 4.1.

D1. Competitive Equilibrium

Consider the firm's maximization problem (11). I add taste shocks to the firm's value functions as follows:

$$V^f(z, k, b) = \mathbb{E}_\epsilon \max_{k', b'} [J^f(z, k, b, k', b') + \epsilon_{(k', b')}] \quad (66)$$

$$\text{where } J^f(z, k, b, k', b') = \phi(e) + \beta \int W^f(z', k', b') d\Pi(z'|z)$$

$$e = (1 - \tau^y)(1 - \theta)F(z, k) + (1 - \delta)k - k' - b + q(z, k', b')b' - \Psi(k, k')$$

$$\phi(e) = e - \kappa(e - \bar{e})^2$$

$$W^f(z', k', b') = \mathbb{E}_{\epsilon'} \max_{d'_i} \left\langle V^f(z', k', b') + \epsilon'_{i, \text{repay}}, \mu_{\nu^d} + \epsilon'_{i, \text{default}} \right\rangle$$

where taste shocks ϵ are *i.i.d* and distributed Gumbel (Extreme Value Type 1) with the variance of σ_ϵ . I use $\sigma_\epsilon = 0.001$.³³ Each taste shock is associated with the discrete choice of control variables. Average firms' defaulting value is given by μ_{ν^d} which corresponds to the mean parameter of firms' *i.i.d.* enforcement shocks ν_i^d . These enforcement shocks are treated as *i.i.d.* taste shocks $\epsilon_{i, \text{default}} - \epsilon_{i, \text{repay}}$ in actual computation for notational convenience. The number of grid points for the combination of firm capital and debt choices (k', b') is 275. Then, firms draw a vector of random variable $\epsilon_{(k', b')}$ from the Gumbel distribution that assigns different values to each choice of (k', b') , and it will choose (k'^*, b'^*) which maximizes the *ex post* value. As is analogous to [Chatterjee and Eyigungor \(2012\)](#) who add small shocks to output, taste shocks perturb value functions to improve the convergence property of the value function iteration method. To put it differently, taste shocks allow firms and the government to implement mixed strategies. With mixed strategies, it is easier for the algorithm to find a solution than pure strategies, since a mixed Nash Equilibrium always exists with a finite set of actions according to the Nash Theorem.

It can be shown that *ex ante* choice probabilities that firms will choose k'^* and b'^* con-

³³As σ_ϵ approaches zero, solutions become close to the original problem without taste shocks, but the algorithm becomes unstable.

ditional on state variables (z, k, b) are

$$P(k'^*, b'^* | z, k, b) = \frac{\exp[J^f(z, k, b, k'^*, b'^*)/\sigma_\epsilon]}{\sum_{k', b'} \exp[J^f(z, k, b, k', b')/\sigma_\epsilon]} \quad (67)$$

The firm value function is given by

$$V^f(z, k, b) = \sigma_\epsilon \log \left\langle \sum_{k', b'} \exp[J^f(z, k, b, k', b')/\sigma_\epsilon] \right\rangle \quad (68)$$

Other value functions and choice probabilities can be obtained in a similar fashion. Firms' default probability is given by

$$P(d = 1 | z, k, b) = \frac{\exp(\mu_{\nu^d}/\sigma_\epsilon)}{\exp[V^f(z, k, b)/\sigma_\epsilon] + \exp[\mu_{\nu^d}/\sigma_\epsilon]} \quad (69)$$

and the associated corporate bond price is

$$q(z, k', b') = \frac{1 - (1 - R^f) \int P(d' = 1 | z', k', b') d\Pi(z' | z)}{1 + r} \quad (70)$$

Another firm value function is given by

$$W^f(z, k, b) = \sigma_\epsilon \log \left\langle \exp[V^f(z, k, b)/\sigma_\epsilon] + \exp[\mu_{\nu^d}/\sigma_\epsilon] \right\rangle \quad (71)$$

Combined with relevant constraints (12), (13), (23), (25), and (26), the government problem (24) can be written compactly as

$$V^g(z, k, b, B) = \mathbb{E}_\epsilon \max_{B'} [J^g(z, k, b, B, B') + \epsilon_{B'}] \quad (72)$$

where $J^g(z, k, b, B, B') =$

$$\begin{aligned} & \sum_{k', b'} P(k', b' | z, k, b) [u(C) + \beta_g \int \mathbb{E}_{\xi'} \max_{D'} \langle V^g(z', k', b', B'), V^g(z', k', b', 0) - \xi' \rangle d\Pi(z' | z)] \\ & C = (1 - \mu(z, k, b)) [(1 - \theta)F(z, k) + (1 - \delta)k - k' - b + q(z, k', b')b' \\ & \quad - \kappa(e - \bar{e})^2 - \Psi(k, k')] + Q(z, k', b', B')B' - B + \theta F(z, k) \end{aligned}$$

Ex ante choice probabilities that the government will choose B'^* conditional on state variables (z, k, b, B) are

$$P(B'^* | z, k, b, B) = \frac{\exp[J^g(z, k, b, B, B'^*)/\sigma_\epsilon]}{\sum_{B'} \exp[J^g(z, k, b, B, B')/\sigma_\epsilon]} \quad (73)$$

The government value function is given by

$$V^g(z, k, b, B) = \sigma_\epsilon \log \left\langle \sum_{B'} \exp[J^g(z, k, b, B, B')/\sigma_\epsilon] \right\rangle \quad (74)$$

The cumulative distribution function (CDF) Ξ of government default costs ξ is a normal CDF with mean μ_ξ and standard deviation σ_ξ . Then, the government default probability is given by

$$\begin{aligned} P(D = 1|z, k, b, B) &= P(V^g(z, k, b, B) \leq V^g(z, k, b, 0) - \xi) \\ &= \Xi[V^g(z, k, b, 0) - V^g(z, k, b, B)] \end{aligned} \quad (75)$$

The government bond price can be expressed using the choice probability of the government default as below:

$$Q(z, k', b', B') = \int M(z', z) [1 - (1 - R^g)P(D = 1|z', k', b', B')] d\Pi(z'|z) \quad (76)$$

I get value functions, choice probabilities, and bond price functions by implementing the standard value function iteration method on discrete grid points.³⁴

1. Guess the firm value functions V^f and the firm bond price function q over the bounded grid points on z, k and b . I use the Tauchen method to discretize z with 5 grid points, and (11, 25) grid points are used for (k', b') .
2. Given these guesses, update associated firm value functions using (66), (68) and (71), choice probabilities for firm capital k' and debt b' (67) and the probability of firm default d (68), and the firm bond price function (70).
3. Continue until distances between values in the previous iteration and those in the current iteration (using the maximum distance evaluated at each grid point) goes below 10^{-7} for V^f and q .
4. Guess the government value function V^g and the government bond price function Q over the bounded grid points on z, k, b and B . I use 25 grid points on B' .
5. Given the firm's choice probabilities (67) obtained in the firms' problem and new guesses, update associated government value functions using (72) and (74), choice

³⁴I solve the model using Julia version 1.4.0 with the QuantEcon package version 0.5.0 and 16 threads parallelization on the University of Maryland Economics cluster. Exponential terms are calculated using the Arbnumeric package that ensures accuracy and thread-safe parallelization without high memory usage.

probabilities for government borrowing B' (73) and the probability of government default (75), and the government bond price function (76).

6. Continue until distances between values in the previous iteration and those in the current iteration (using the maximum distance evaluated at each grid point) goes below 10^{-7} for V^g and Q .

D2. Constrained Efficient Equilibrium

The constrained social planner problem (39) can be written as

$$V^g(z, k, b, B) = \mathbb{E}_\epsilon \max_{k', b', B'} J^g(z, k, b, B, k', b', B') + \epsilon_{B'} \quad (77)$$

where $J^g(z, k, b, B, k', b', B') =$

$$u(C) + \beta_g \mathbb{E}_{z'|z, \xi'} \max_{D'} \langle V^g(z', k', b', B'), V^g(z', k', b', 0) - \xi' \rangle$$

$$C = (1 - \mu(z, k, b)) [(1 - \theta)F(z, k) + (1 - \delta)k - k' - b + q(z, k', b')b' - \kappa(e - \bar{e})^2 - \Psi(k, k')] + Q(z, k', b', B')B' - B + \theta F(z, k)$$

subject to

corporate default decision from (66), corporate and government bond prices (70) and (76)

It can be shown that *ex ante* choice probabilities that the planner will choose B'^* conditional on state variables (z, k, b) and choices of k' and b' are

$$P(B'^*|z, k, b, k', b') = \frac{\exp[J^g(z, k, b, k', b', B'^*)/\sigma_\epsilon]}{\sum_{B'} \exp[J^g(z, k, b, k', b', B')/\sigma_\epsilon]} \quad (78)$$

The government value function is given by

$$V^g(z, k, b, B) = \sigma_\epsilon \log \left\langle \sum_{B'} \exp[J^g(z, k, b, k'^*, b'^*, B')/\sigma_\epsilon] \right\rangle \quad (79)$$

where k'^* and b'^* are maximizers of the expected value function $\mathbb{E}_\epsilon[J^g(z, k, b, B, k', b', B') + \epsilon_{B'}]$. Solutions to this problem are obtained as in the competitive equilibrium.