The Effects of Monetary Policy on Consumption and Inequality

Donggyu Lee

University of Maryland

This version : 29th September 2020

Abstract

Recent work with heterogeneous agent New Keynesian (HANK) models has shown that monetary policy shocks affect consumption mostly through a general equilibrium change in the real wage and have distinct distributional consequences. I show the fragility of these results, demonstrating how they depend on (1) the severity of wage rigidities and labor market frictions, (2) the cyclicality of profits, and (3) the progressivity of fiscal policy. I develop a HANK model that generates movements of the real wage and profits that are consistent with the data: in response to an expansionary monetary policy shock, profits increase and the real wage responds little. Indirect effects still dominate the direct effects of policy on consumption, as in Kaplan, Moll and Violante (2018). However, the effect on consumption through real wage changes is limited. Instead, a fall in unemployment risk contributes to a large increase in poor households' consumption, while an increase in profits leads to a significant response of wealthy households' consumption. Since both poor and wealthy households increase their consumption substantially, the distributional consequences of monetary policy shocks are much weaker than in previous work. What emerges instead is a hollowing out of the distribution, as middle-income households benefit the least. In contrast, in the flexible wage specification, the top quintile of the wealth distribution unambiguously loses, while the lower quintiles gain. Moreover, the distributional consequences depend crucially on fiscal policy. An expansionary monetary policy shock reduces inequality only when the government increases lump-sum transfers in response to a fall in the real value of government debt. When the fiscal authority adjusts government purchases or tax rates, welfare gains are larger for wealthier households.

1 Introduction

The distributional consequences of monetary policy have received little attention in the past decades. Traditionally, central banks have focused on the aggregate implications of their actions, such as the effects of monetary policy on the nation's output, inflation rate, unemployment rate, and so on. Theoretical models used to examine these aggregate implications of monetary policy have mostly utilized a representative agent framework, which abstracts from any meaningful differences across agents. As a result, these models could not evaluate heterogeneity in the effects of monetary policy across agents. Recently, however, the interactions between the agent's heterogeneity and monetary policy started to gain attention in the literature as rising inequality has become a center of the economic debates in the U.S. since the Great Recession. In this paper, I examine the implications of monetary policy for the distribution of wealth and income, by linking the transmission mechanisms and the distributional consequences of monetary policy.

For an accurate theoretical evaluation of the transmission mechanisms and the distributional consequences of monetary policy, a model should be able to produce plausible responses of these variables to monetary policy shocks since households are heterogeneous in terms of their income and portfolio compositions. According to the Survey of Consumer Finance (SCF hereinafter), relatively poor households in terms of net worth rely almost exclusively on labor income. By contrast, asset and profit income account for a much larger share of wealthy households' income. Regarding portfolio composition, the SCF reveals that wealthy households' portfolios consist mostly of equity, which is a claim to profits in the economy. This observation on households' income and portfolio composition shows that how wages and profits respond to monetary policy shocks is of great importance in determining the transmission mechanisms and the distributional consequences of monetary policy.

Christiano et al. (2005) and Christiano et al. (2016) show that the real wage responds little to monetary policy shocks. The unemployment rate, however, responds substantially in their theoretical and empirical analysis. Meanwhile, profits increase significantly after an expansionary monetary policy shock in the VAR analysis of Christiano et al. (2005). Similarly, Coibion et al. (2017) find that in response to a contractionary monetary policy shock, i.e., an increase in the federal funds rate, business income falls substantially, while real wages and salaries remain relatively unchanged. An important goal of the model that I develop here is to generate such responses of wages and profits to monetary policy shocks.

In many New Keynesian models, one of the reasons profits fall after a decrease in the policy rate is that the real marginal cost of production (or equivalently the mark-up) responds too flexibly to changes in aggregate demand. This is mainly due to the fact that factor prices are flexible, while goods prices are rigid. When the aggregate demand increases because of a lower real rate, firms want to hire more labor and utilize more capital to produce more goods. Such a higher demand for production factors increases the marginal cost of production, which leads to a smaller mark-up.

As a result, even though firms could produce and sell more goods, profits can fall. Thus, a key to generating a pro-cyclical response of profits to demand shocks is to dampen the response of the real marginal cost. The first ingredient for a dampened response of marginal cost to demand shocks is wage rigidity.[1] In particular, I introduce nominal wage rigidity, based on the literature that has found that such rigidity is prevalent from various types of data.[2] Assuming that the nominal wage is rigid, however, has distributional consequences that work against poor households which rely almost solely on labor income. As Gornemann et al. (2016) emphasize, poor households can benefit significantly from expansionary monetary policy even though wages are rigid, if the unemployment risk decreases substantially because of higher aggregate demand. Thus, I also introduce search and matching labor market frictions into a heterogeneous agent New Keynesian (HANK hereinafter) model. Households supply labor via labor agencies. Firms rent labor services from labor agencies. While the wage paid to households is rigid, the rental rate of labor services is assumed to be flexible. This set-up of the model enables firms to increase labor input without a significant increase in the labor rental rate by creating a substantial increase in labor agencies' mark-up after an expansionary monetary policy shock.[3] In response to an increase in the mark-up, labor agencies post more vacancies. Thus, the aggregate labor supply increases, and the unemployment rate falls. This mechanism dampens the response of marginal cost to demand shocks.

Another factor that contributes to a significant increase in marginal cost (or a fall in the mark-up) after an expansionary monetary policy shock is firms' reliance on new production in meeting changes in demand. In reality, firms adjust not only production but also their inventory stock in response to an evolution of demand. If firms rely in part on inventory stock rather than new production, the increase in factor demand is limited, and so is the response of marginal cost to monetary policy shocks. Existing empirical evidence has shown that firms manage their inventory stock in response to monetary policy shocks.[4] When firms face unexpectedly high demand, they reduce their inventory stock and, consequently, the sales-to-stock ratio rises. To reflect this dynamic response of firms, I allow firms to accumulate inventory stock as a second ingredient of the model. Dynamic adjustments of inventory stock enable firms to smooth out the marginal cost of production.

The first contribution of this paper to the recently burgeoning HANK literature is to develop a model

[1] Broer et al. (2018) also rely on wage rigidity to have pro-cyclical responses of profits to monetary policy shocks. They also discuss the importance of profit responses in determining the aggregate consequences of monetary policy shocks, and resulting distributional consequences in a simple HANK model.

[2] See, for instance, Akerlof et al. (1996), Kahn (1997), Elsby (2009), Barattieri et al. (2010), Kurmann and McEntarfer (2019) among others.

[3] Introducing wage rigidity without an extensive margin adjustment does not necessarily lead to a substantially dampened response of the marginal cost, especially when income effect regarding labor supply is absent. In such a case, the aggregate labor supply also does not increase much when the wage responds little to shocks. When there is another factor of production, e.g., capital, firms then rely more intensively on the other factor. Thus, capital utilization increases inefficiently high, which results in a substantial increase in the marginal cost.

[4] Jung and Yun (2005) show that a contractionary monetary policy shock leads to an increase in firms' finished goods inventory stock and a decrease in the ratio of sales to stocks available for sales.

that generates plausible responses of the real wage and profits to monetary policy shocks. Because of the two features explained so far, the response of firms' marginal cost to monetary policy shocks is dampened. As a result, profits increase in response to an expansionary monetary policy shock, unlike in many standard New Keynesian models. In contrast, the response of the real wage is minimal due to the assumption of nominal wage rigidity. Instead, unemployment risk falls substantially as more labor agencies post vacancies. These responses are more in line with existing empirical evidence than the responses in standard New Keynesian models, which imply counter-factual real wage increases and declining profits after an expansionary monetary policy shock.

The second contribution is to complement the existing discussion on the transmission mechanisms of monetary policy shocks in a heterogeneous agent environment. To this end, I calibrate the model to match asset distributions and portfolio compositions in the data. Importantly, in the model, substantial masses of households hold either no assets or only illiquid assets (wealthy hand-to-mouth households), as in the data. Because these subsets of households do not respond to changes in the real interest rate, the direct effect accounts for only a small fraction of the total aggregate consumption response to monetary policy shocks. By contrast, indirect effects, including the effects of changes in the real wage, profits, the job-finding rate, and fiscal instruments, explain most of the aggregate consumption response as in Kaplan et al. (2018). However, the decomposition of the indirect effects is different from existing models. Since the real wage increases only a little following an expansionary monetary policy shock, its effects on the aggregate consumption are muted. Instead, the increased job-finding rate increases the consumption of households that are relatively more vulnerable to unemployment risk.[5] Moreover, an increase in profits leads wealthy households to increase their consumption significantly because they are mostly major claimants to profits in the model. Under my calibration, the effects of the job-finding rate and profits on aggregate consumption are larger than those of real wage changes.

The third contribution of my analysis is to provide a new perspective on the distributional consequences of monetary policy shocks. Existing work argues that, expansionary monetary policy shock decreases inequality among households. However, I show that such an outcome is not guaranteed. Depending on model features, an expansionary monetary policy shock can be more beneficial for wealthy households. An unexpected increase in the inflation rate causes losses to net nominal asset holders and redistributes wealth to net borrowers. Since wealthy households tend to hold a large amount of nominal assets and poor households are usually net borrowers, this *Fisher channel* contributes to a fall in wealth and consumption inequality after an expansionary monetary policy shock. However, wealthy households' portfolio mostly consists of illiquid assets. Thus, if the return on illiquid assets increases after an expansionary monetary policy shock, total asset income can be higher than before the shock despite the presence of the Fisher channel. In the model, the return on illiquid assets is proportional

[5] A rise in the job-finding rate leads both employed and unemployed households to increase their consumption. First, a larger fraction of unemployed households become employed and increase their consumption as their income rises significantly. Second, even employed households increase their consumption by reducing their precautionary savings.

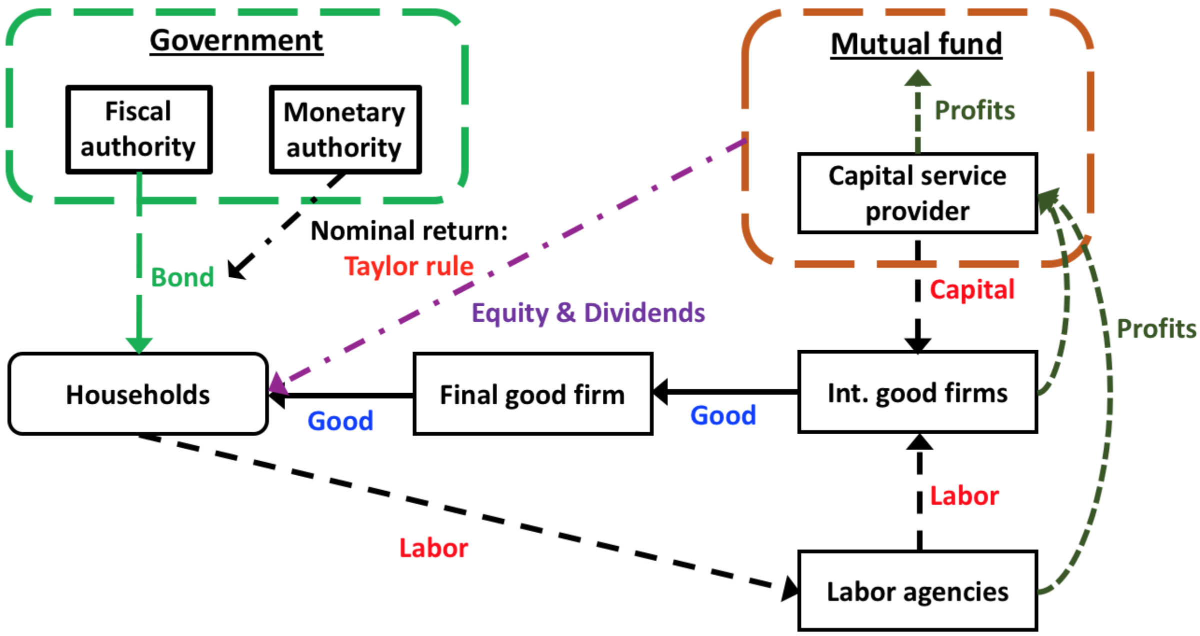
to profits. Hence, an increase in profits after an expansionary monetary policy shock translates into a higher return on illiquid assets. If the positive response of profits to expansionary monetary policy is large enough to offset the fall in the real interest rate and is much larger in magnitude than the changes in the real wage, an expansionary monetary policy shock can lead to a rise in inequality. This is because wealthy households hold the majority of claims to profits while other households rely almost exclusively on labor income. In the baseline model, an expansionary monetary policy shock leads to more pronounced consumption responses from wealthy households than from other households. In the long-run, it also leads to a higher welfare gain, measured as consumption equivalence, for the households at the top of wealth distribution.[6]

Finally, I show that the distributional consequences of monetary policy depend importantly on the fiscal policy response. Monetary policy shocks result in changes in the real value of government debt. In response to these changes, the fiscal authority needs to adjust its fiscal instruments, including government purchases, lump-sum transfers, and tax rates, to meet the solvency condition or inter-temporal budget constraint. When the government increases purchase or reduces tax rates, an expansionary monetary policy shock has weak distributional consequences. An increase in government purchases implies a one-to-one increase in aggregate demand. Such an increase in demand leads to a larger increase in profits, which benefits wealthy households disproportionately. A proportional decrease in all tax rates is also regressive. In the model, the only case in which an expansionary monetary policy shock obviously reduces inequality is when the fiscal authority increases lump-sum transfer. The result that the distributional consequences of monetary policy heavily depend on the nature of fiscal responses casts doubt on the argument that an expansionary monetary policy shock reduces inequality without further conditioning on the fiscal response.

The rest of paper is organized as follows. Section 2 develops the model. Section 3 discusses the calibration of the model. Section 4 analyzes the transmission mechanisms and the distributional consequences of monetary policy shocks in the baseline model. Section 5 compares the baseline model with a more standard model with a flexible wage, focusing on the response of aggregate variables and their implications for the effects of monetary policy shocks on inequality. Section 6 examines the implications of alternative fiscal responses. Section 7 concludes.

[6] An expansionary monetary policy shock benefits households at the left-end of wealth distribution by increasing the job-finding rate and reducing the real value of their debt. In contrast, the benefit for households at the middle of wealth distribution is limited. This is because they mostly rely on labor income, which does not respond much, and are, on average, net holders of government bond, whose real return falls in response to a fall in the policy rate. Thus, inequality among these households tends to shrink after an expansionary monetary policy shock. However, households at the right-end of wealth distribution also enjoy a substantial amount of gain because of an increase in profits or, equivalently, the return on illiquid asset. Thus overall inequality can increase after an expansionary monetary policy shock.

Figure 1: Model overview



Notes: The figure shows how economic agents in the model interact with each other. The arrows represent the movements of goods, production factors, assets, and profits. The payments associated with factors, goods, and asset transactions are not shown.

2 Model

In this section, I develop a New Keynesian model with heterogeneous agents augmented with two assets, labor market frictions combined with wage rigidity, and inventory holding. The model consists of households, firms, mutual fund, and government authorities. Households provide labor, consume the final good, and invest in two types of assets. Firms produce goods by utilizing labor and capital services. A mutual fund owns all the firms in the model. It collects all profits, and distributes a part of them to households as dividends in proportion to their equity holding. The remaining portion is given as compensation to a certain fraction of households that serve as entrepreneurs. Fiscal authority collects taxes, decides the amount of government purchase and lump-sum transfers, and pays the unemployment benefit. It also issues a one-period nominal government bond. The monetary authority sets the nominal rate of return on a government bond. Figure 1 shows an overview of the model. In the following subsections, I describe each agent's problem and their optimal choices in more detail.

2.1 Households

There is a unit mass of households, which are characterized by their idiosyncratic productivity, employment status, asset holding, and adjustment costs. Each period they earn income and choose the amount of consumption and savings. When they are employed, they also decides the amount of their labor supply. Finally, a certain fraction of households die randomly at the beginning of each period.[7] The Bellman equation for the household is given as follows.

$$
\begin{aligned}
V(a_t, b_t | s_t, e_t, \chi_t, X_t) = \max_{\{c_t, n_t, a_{t+1}, b_{t+1}\}} & \; u(c_t, n_t) - \mathbb{1}_{\{a_{t+1} \neq a_t\}} \chi_t \\
& + \beta(1-\zeta)\mathbb{E}\Big[V(a_{t+1}, b_{t+1} | s_{t+1}, e_{t+1}, \chi_{t+1}, X_{t+1})\Big]
\end{aligned} \tag{1}
$$

s.t.

$$
c_t + q_t a_{t+1} + b_{t+1} = (1 - \tau_t^y) y_t + (q_t + \tilde{r}_t^a) a_t + \frac{\tilde{R}_t}{\pi_t} b_t + T_t \tag{2}
$$

$$
a_{t+1} \geq 0 \; , \quad b_{t+1} \geq \underline{\mathrm{b}} \tag{3}
$$

where a_t, b_t, c_t, and n_t are equity and bond holding, consumption, and the amount of labor supply respectively. $u(c_t, n_t)$ is an instantaneous utility from consuming c_t amount of the final good and supplying n_t amount of labor. q_t and $\tilde{r}_t^a$ are the price and the dividend rate of the equity respectively. $\tilde{R}_t$ is the gross nominal rate of return that is applied to a one-period government bond issued in period $t-1$. π_t is the gross inflation rate from period $t-1$ to period t. τ_t^y and T_t are the income tax rate and the lump-sum transfer from the government respectively. ζ is the probability of death. X_t denotes the aggregate state of the model economy after the employment status transition and before the production of goods. Specifically, X_t consists of the aggregate capital stock, aggregate shocks, the size of the aggregate labor force after the labor market transitions, and households' distribution over their idiosyncratic states.

The household's income y_t depends on the level of idiosyncratic productivity s_t and its employment status e_t. The former evolves according to a first-order Markov process. Also, the household's employment status varies over time. Specifically, the household can be employed, unemployed or work as an entrepreneur. While employed ($e_t = 1$), the household earns a wage in proportion to its idiosyncratic productivity and the amount of labor supply. If unemployed ($n_t = 2$), the household receives an unemployment benefit from the fiscal authority based on the steady state wage that it could earn if it were employed. Finally, if the household becomes an entrepreneur ($n_t = 3$), its income is equal to a fraction of the profits. Formally, y_t is given as follows.

$$
y_t = \begin{cases} w_t s_t n_t & \text{for employed} \quad (e_t = 1) \\ \bar{w} \upsilon \min\{s_t, \bar{s}\} & \text{for unemployed} \;\; (e_t = 2) \\ \nu \Pi_t & \text{for entrepreneur} \; (e_t = 3) \end{cases} \tag{4}
$$

where υ is the replacement ratio and ν is the share of the profits that is given to the entrepreneurs.

[7] Upon death, all of the dead households' assets are collected and distributed to the surviving households as a return in proportion to their asset holding. To denote the rate of return that reflects such a hypothetical annuity arrangement, I use tilde in the notation.

w_t is the market wage and $\bar{w}$ is its steady state value. $\bar{s}$ and Π_t denote the average productivity and profits respectively.

Households' employment status transitions are governed by exogenous probabilities and endogenous labor market outcomes. First, the transitions in and out of the entrepreneurial state occur at the end of a period with given probabilities. Specifically, households that are entrepreneurs at the end of the period lose their job and become unemployed with the probability $\tilde{P}_e$. At the same time, P_e fraction of the households that are not entrepreneurs is newly hired as entrepreneurs. Then, at the beginning of the new period, employed households lose their job with an exogenous probability λ. Once households become unemployed, they look for a new job by searching in the labor market.[8] The mass of job seekers and the number of vacancies jointly determine the job-finding rate, which I denote with f. Unemployed households become employed with probability f. Once the employment status in a given period is determined, the household provides labor or works as an entrepreneur and earns income according to its employment status.

Regarding savings, the household chooses not only the total amount but also the composition of its portfolio. The model has two types of assets, government bonds and equity. Equity is assumed to be illiquid, i.e., adjusting the amount of equity holding incurs an adjustment cost. Specifically, the household suffers a loss of utility of χ_t when $a_{t+1} \neq a_t$. As in Bayer et al. (2018), the utility adjustment cost χ_t is assumed to follow a logistic distribution, which is identically and independently distributed across households.[9]

$$\chi_t \ \sim \ F(\chi_t) = \frac{1}{1 + \exp\{-\frac{\chi_t - \mu_\chi}{\sigma_\chi}\}} \tag{5}$$

where $F(\cdot)$ denotes the CDF of the logistic distribution function. Based on the realized value for χ_t, households decide whether to adjust their equity holding or not. If they choose not to adjust, they receive the dividends and adjust bond holding.[10] In contrast, if households choose to adjust, they can buy and sell equity at price q_t, which varies each period based on the aggregate status of the economy, but experience a loss of utility of χ_t.

On the liability side, households can borrow up to an exogenous amount $\underline{b}$ in the form of the liquid asset. To prevent arbitrage and excessive borrowing in the model, I assume that households must pay a premium $\underline{R}$ if they borrow.[11] Consequently, the borrowing rate is higher than the saving rate and

[8] I assume that the search intensity is exogenous, i.e., households do not decide whether to participate in the labor market or not. Instead, all the unemployed households become job seekers.

[9] Bayer et al. (2018) show that an introduction of stochastic adjustment costs in the form of utility costs preserves the concavity of the household's value functions in a similar environment. Moreover, the assumption of the logistic distribution provides a closed-form solution for the expected adjustment cost, which facilitates the numerical computation further. Finally, Kaplan and Violante (2014) find that the choice of pecuniary or utility adjustment costs does not make any significant differences in the results for the portfolio composition problem.

[10] The dividends that households receive consists of a fraction profits given by the mutual fund and annuity payments regarding equity holding.

[11] Following Kaplan et al. (2018), I assume that the borrowing premium is a wasteful intermediation cost. That is, the

they are given as follows.

$$\tilde{R}_{t+1} = \begin{cases} R_{t+1}/(1-\zeta) & \text{if } b_{t+1} \geq 0 \\ (R_{t+1}+\underline{\text{R}})/(1-\zeta) & \text{if } b_{t+1} < 0 \end{cases} \tag{6}$$

For the household's utility, I use the following form of Greenwood-Hercowitz-Huffman (GHH) preference.

$$u(c_t, n_t) = \frac{\left[c_t - \psi s_t \frac{n_t^{1+\xi}}{1+\xi}\right]^{1-\sigma}}{1-\sigma} \tag{7}$$

where ξ and σ are the inverse of the Frisch elasticity of labor supply and the coefficient of relative risk aversion respectively. By normalizing the disutility of labor supply with the level of idiosyncratic productivity, I guarantee that all employed households supply the same amount of labor based on the market wage. ψ is another normalization factor that ensures a steady state labor supply of one.

Before deriving the household's optimality conditions under the environment explained so far, I first express the household's next period expected value as follows, depending on its current period employment status. For brevity of notation, I drop households' idiosyncratic productivity and the utility cost of adjustment from these expressions.

$$\begin{aligned}
&\mathbb{E}\Big[V(a_{t+1}, b_{t+1}|e_{t+1}, X_{t+1})|e_t = 1\Big] = \mathbb{E}\Big[P_e V(a_{t+1}, b_{t+1}|3, X_{t+1}) + (1-P_e) \\
&\times \big[\{1-\lambda+\lambda f(\tilde{X}_{t+1})\}V(a_{t+1}, b_{t+1}|1, X_{t+1}) + (\lambda\{1-f(\tilde{X}_{t+1})\})V(a_{t+1}, b_{t+1}|2, X_{t+1})\big]\Big] && (8) \\
&\mathbb{E}\Big[V(a_{t+1}, b_{t+1}|e_{t+1}, X_{t+1})|e_t = 2\Big] = \mathbb{E}\Big[P_e V(a_{t+1}, b_{t+1}|3, X_{t+1}) \\
&+ (1-P_e) \times \big[f(\tilde{X}_{t+1})V(a_{t+1}, b_{t+1}|1, X_{t+1}) + \{1-f(\tilde{X}_{t+1})\}V(a_{t+1}, b_{t+1}|2, X_{t+1})\big]\Big] && (9) \\
&\mathbb{E}\Big[V(a_{t+1}, b_{t+1}|e_{t+1}, X_{t+1})|e_t = 3\Big] = \mathbb{E}\Big[(1-\tilde{P}_e)V(a_{t+1}, b_{t+1}|3, X_{t+1}) \\
&+ \tilde{P}_e \times \big[f(\tilde{X}_{t+1})V(a_{t+1}, b_{t+1}|1, X_{t+1}) + \{1-f(\tilde{X}_{t+1})\}V(a_{t+1}, b_{t+1}|2, X_{t+1})\big]\Big] && (10)
\end{aligned}$$

where $\tilde{X}$ denotes the aggregate state before labor market transitions. Specifically, it consists of the aggregate capital stock, aggregate shocks, the size of labor force and households' distribution over idiosyncratic states before labor market transitions. The equation (8) shows the next period's expected value of the households that are currently employed. Equations (9) and (10) shows the expected values of currently unemployed households and households that currently work as entrepreneurs respectively. The above expressions take into account both exogenous transition probabilities and the endogenous evolution of the job finding rate.

Now, I describe the first order conditions to the household's problem. First, the employed households

intermediation cost dose not appear in the government's budget constraint or firms' cash flow.

decide their labor supply according to the following intra-temporal optimality condition.

$$\psi n_t^{\xi} = w_t \tag{11}$$

If households are unemployed or work as entrepreneurs, $n_t = 0$ since their income does not depend on n_t. Now, let me turn into the optimality conditions regarding asset holding. Let V_a and V_b denote the partial derivatives of the value function with respect to equity and bond holding respectively. In addition, let u_c denote the derivative of the instantaneous utility function with respect to consumption. Then, the household's optimality conditions regarding asset holding can be expressed as follows. For simplicity of notation, I drop all exogenous states in the following.

$$q_t u_c(c_t, n_t) \geq \beta(1-\zeta)\mathbb{E}\Big[V_a(a_{t+1}, b_{t+1})\Big] \quad \text{with equality if } a_{t+1} > 0 \tag{12}$$

$$u_c(c_t, n_t) \geq \beta(1-\zeta)\mathbb{E}\Big[V_b(a_{t+1}, b_{t+1})\Big] \quad \text{with equality if } b_{t+1} > 0 \tag{13}$$

Note that equation (12) may not hold if $a_{t+1} = a_t$. The Envelope conditions are given as follows.

$$V_a(a_t, b_t) = \begin{cases} (q_t + \tilde{r}_t^a) u_c(c_t^A, n_t) & \text{if } a_{t+1} \neq a_t \\ \tilde{r}_t^a u_c(c_t^N, n_t) + \beta(1-\zeta)\mathbb{E}\Big[V_a(a_t, b_{t+1})\Big] & \text{if } a_{t+1} = a_t \end{cases} \tag{14}$$

$$V_b(a_t, b_t) = \begin{cases} \frac{\tilde{R}_t}{\pi_t} u_c(c_t^A, n_t) & \text{if } a_{t+1} \neq a_t \\ \frac{\tilde{R}_t}{\pi_t} u_c(c_t^N, n_t) & \text{if } a_{t+1} = a_t \end{cases} \tag{15}$$

where c_t^A and c_t^N are the optimal consumption when the household chooses to adjust and not to adjust its equity holding respectively. As shown in the above Envelope conditions, the household's optimality conditions depend on its expectation on the adjustment of the equity holding in the next period. The household adjusts its equity holding in period t if the following condition is satisfied.

$$V^A(a_t, b_t) - \chi_t \geq V^N(a_t, b_t) \tag{16}$$

where V^A and V^N denote the value of households when they adjust or do not adjust their equity holding respectively. Since the utility transaction cost is drawn from the logistic distribution, I can describe the probability of adjustment $P^*(a_t, b_t)$ as follows.

$$\begin{aligned} P^*(a_t, b_t) &= P\Big[\chi_t \leq V^A(a_t, b_t) - V^N(a_t, b_t)\Big] \\ &= F\Big[V^A(a_t, b_t) - V^N(a_t, b_t)\Big] \end{aligned} \tag{17}$$

Given the probability of adjustment, the household's Euler equation with respect to each asset holding can be described as follows.

$$q_t u_c(c_t, n_t) \geq \beta \mathbb{E}\Big[P^*(a_{t+1}, b_{t+1})\big\{q_{t+1} + r^a_{t+1}\big\}u_c(c^A_{t+1}, n_{t+1}) + \big\{1 - P^*(a_{t+1}, b_{t+1})\big\}r^a_{t+1}u_c(c^N_{t+1}, n_{t+1})$$
$$+ \big\{1 - P^*(a_{t+1}, b_{t+1})\big\}\mathbb{E}\big[V_a(a_{t+1}, b_{t+2})\big]\Big] \quad \text{with equality if } a_{t+1} > 0 \text{ and } a_{t+1} \neq a_t \tag{18}$$
$$u_c(c_t, n_t) \geq \beta \mathbb{E}\Big[P^*(a_{t+1}, b_{t+1})\frac{R_{t+1}}{\pi_{t+1}}u_c(c^A_{t+1}, n_{t+1}) + \{1 - P^*(a_{t+1}, b_{t+1})\}\frac{R_{t+1}}{\pi_{t+1}}u_c(c^N_{t+1}, n_{t+1})\Big]$$
$$\text{with equality if } b_{t+1} > 0 \tag{19}$$

2.2 Final good firm

The final good firm is a standard CES aggregator of the intermediate goods. It produces a final good using the intermediate goods as inputs in a perfectly competitive environment, taking goods' prices as given. Note that the final good firm does not distinguish inventory stock and newly-produced goods. That is, both newly-produced intermediate goods and intermediate goods produced in previous periods can be used as inputs for the final good. The final good firm's problem is described as follows.

$$\max_{S_t, \{S_{jt}\}_{j \in (0,1)}} P_t S_t - \int_0^1 P_{jt} S_{jt} dj \quad \text{s.t. } S_t = \left\{ \int_0^1 \left(\frac{a_{jt}}{a_t}\right)^{\frac{\varphi}{\eta}} S_{jt}^{\frac{\eta-1}{\eta}} dj \right\}^{\frac{\eta}{\eta-1}} \tag{20}$$

where S_t and S_{jt} denote the final good sales and intermediate firm j's sales respectively. a_t and a_{jt} are the aggregate stock and the firm j's stock available for sale respectively.[12] φ denotes the intermediate good sale's elasticity with respect to the available stock for sale. The above aggregator implies that firm j's sales are higher than other firms' sales if firm j has a larger amount of stock for sale than other firms. This feature provides an additional incentive for firms to accumulate inventory stock. In Bils and Khan (2000) and Jung and Yun (2005), such an incentive is necessary to make firms hold a meaningful amount of inventory stock at the steady state. η is the elasticity of substitution across differentiated intermediate goods. The first order conditions imply the following demand for intermediate goods and the aggregate price index.

$$S_{jt} = \left(\frac{a_{jt}}{a_t}\right)^{\varphi} \left(\frac{P_{jt}}{P_t}\right)^{-\eta} S_t \tag{22}$$

$$P_t = \left\{ \int_0^1 P_{jt}^{1-\eta} dj \right\}^{\frac{1}{1-\eta}} \tag{23}$$

[12] a_t is given by

$$a_t = \int_0^1 a_{j,t} dj \tag{21}$$

2.3 Intermediate good firms

In the model, a unit mass of intermediate good firms indexed by $j \in (0,1)$ produces differentiated goods, using labor and capital rental services. Each firm faces a downward-sloping demand curve and has monopoly power to set a price for its product. Firms' price setting is subject to a nominal rigidity in the form of Rotemberg (1982) adjustment costs. In addition, a working capital constraint applies to the firms as in Christiano et al. (2005). Specifically, firms have to pay the labor rental and capital rate in advance by borrowing the amount at interest rate R_{t+1}. Let $J_I(i_{jt-1}, P_{jt-1}|X_t)$ denote the value of the firm j with the previous period price P_{jt-1} and inventory holding i_{jt-1} under the aggregate state X_t. Importantly, firms are allowed to hold inventory stock. Thus, intermediate good firms do not sell all the goods they produce in the current period. They decide not only how much to produce but also how much to sell. The intermediate good firm's problem can be described as follows.

$$
\begin{aligned}
J_I(i_{jt-1}, P_{jt-1}|X_t) = \max_{\{P_{jt}, L_{jt}, K_{jt}\}} & \left(\frac{P_{jt}}{P_t}\right) S_{jt} - R_{t+1}(r_t^l L_{jt} + r_t^k K_{jt}) - \Xi - \frac{\eta}{2\kappa}\left(\log\frac{P_{jt}}{P_{jt-1}} - \log\bar{\pi}\right)^2 S_t \\
& - \frac{\phi_x}{2}\left(\log x_{jt} - \log\bar{x}\right)^2 S_t + \mathbb{E}\big[\Lambda_{t,t+1} J_I(i_{jt}, P_{jt}|X_{t+1})\big] \qquad (24)
\end{aligned}
$$

s.t.

$$S_{jt} = \left(\frac{a_{jt}}{a_t}\right)^{\varphi}\left(\frac{P_{jt}}{P_t}\right)^{-\eta} S_t \qquad (25)$$

$$a_{jt} = Y_{jt} + (1-\delta_i) i_{jt-1} \qquad (26)$$

$$Y_{jt} = Z_t L_{jt}^{1-\theta} K_{jt}^{\theta} \qquad (27)$$

$$i_{jt} = a_{jt} - S_{jt} \qquad (28)$$

$$x_{jt} = S_{jt}/a_{jt} \qquad (29)$$

$$\log(Z_{t+1}) = \rho_Z \log(Z_t) + \epsilon_{Z,t} \ \ , \ \epsilon_{Z,t} \sim \text{ i.i.d } \ N(0, \sigma_Z^2) \qquad (30)$$

where r_t^l and r_t^k are the labor and capital rental rates respectively. L_{jt} and K_{jt} denote the amount of labor and capital rental services used by firm j. R_{t+1} is the nominal interest rate that is applied to the working capital. Ξ is a fixed cost of operation.[13] κ determines the degree of nominal rigidity in the model jointly with the elasticity of substitution η. $\bar{\pi}$ is the steady state inflation rate.[14] If the individual firm's price variation is not equal to the steady state inflation rate, it has to pay the adjustment costs in units of the final good. $\Lambda_{t,t+1}$ denotes the stochastic discount factor that each firm

[13] In principle, an intermediate good firm should choose not to operate if the revenue that it can achieve in a given period is less than the fixed cost. I do not allow such a choice. However, under the parameterizations examined in this paper, the firm's profit is always positive and they optimally continue to produce.

[14] The current expression for the adjustment cost implies that the firm can index its price to the steady state inflation rate without paying any costs.

uses to discount future cash flows. θ is the capital share of the production. Total factor productivity Z_t follows an AR(1) process as shown in equation (30). ρ_Z and σ_Z denote the autocorrelation coefficient and the standard deviation of Z respectively.

Equation (25) describes the demand for firm j's good, which reflects a benefit of holding inventory stock, i.e., boosting sales. Equation (26) shows that the firm j's current period stock of goods for sale is the sum of the newly-produced good and the un-depreciated inventory stock. δ_i denotes the rate of depreciation of inventory stock. The inventory stock at the end of the current period is the remaining stock after sales as shown in equation (28). Equation (29) is the definition of the sales-to-stock ratio. I assume that, if the firm j's sales-to-stock ratio deviates from its steady state target, the firm has to pay adjustment costs, following Jung and Yun (2005). This feature prevents overly volatile inventory stock adjustments.

The intermediate good firm's problem can be divided into two parts. First, the intermediate good firm minimizes total costs by choosing the optimal amount of labor L_{jt} and capital services K_{jt} that can produce a given amount of the intermediate good Y_{jt}. Then, for the given marginal cost, the firm sets the price for its good and the amount of inventory holding that maximizes expected discounted cash-flows. The former determines the amount of sales and the latter determines the amount of production. Note that the firm's sales may or may not equal production Y_{jt} if it accumulates inventory stock. The firm holds unsold goods as inventory stock. The optimality conditions and the symmetric equilibrium assumption lead to the following expressions for marginal cost and New Keynesian Phillips curve.[15]

$$
\begin{aligned}
MC_t &= R_{t+1}\frac{(r_t^k)^\theta (r_t^l)^{1-\theta}}{Z_t} \times \left(\frac{1}{\theta}\right)^\theta \times \left(\frac{1}{1-\theta}\right)^{1-\theta} \\
&= \varphi x_t + \phi_x(1-\varphi)\big(\log x_t - \log \bar{x}\big)x_t + (1-\varphi x_t)(1-\delta_i)\mathbb{E}\Big[\Lambda_{t,t+1}MC_{t+1}\Big] \qquad (31)
\end{aligned}
$$

$$
\begin{aligned}
\log \pi_t - \log \bar{\pi} &= \mathbb{E}\left[\Lambda_{t,t+1}\frac{S_{t+1}}{S_t}\Big\{\log \pi_{t+1} - \log \bar{\pi}\Big\}\right] \\
&\quad + \kappa\Big\{(1-\delta_i)\mathbb{E}\Big[\Lambda_{t,t+1}MC_{t+1}\Big] - \frac{\eta-1}{\eta} + \phi_x\big(\log x_t - \log \bar{x}\big)\Big\} \qquad (32)
\end{aligned}
$$

As shown in the above, the current period inflation rate depends on the current sales-to-stock ratio adjustment cost, the expected marginal costs as well as the expected future inflation rate.

2.4 Labor agencies

I model labor agencies as in Gornemann et al. (2016). Agencies porst vacancies to hire labor from households. While a labor agency is matched to a household, it rents labor services to intermediate good

[15] For a symmetric equilibrium, I assume that the initial inventory stock $i_{j,-1}$, and the initial price level $P_{j,-1}$ are equal across firms.

firms at a competitive rental rate r^l_t.[16] A match between a household and a labor agency is terminated if the household dies; if the household survives but becomes an entrepreneur with probability P_e; and randomly with an exogenous probability λ. Given the description of the problem so far, the labor agency's value function can be expressed as follows.

$$J_L(s_t|\tilde{X}_t) = (r^l_t - w_t)s_t n_t + \mathbb{E}\Big[\Lambda_{t,t+1}(1-\zeta)(1-\lambda)(1-P_e)J_L(s_{t+1}|\tilde{X}_{t+1})\Big] \tag{33}$$

where w_t is the real wage that the labor agency pays to the household per labor efficiency unit and $s_t n_t$ is the household's supply of efficiency units. In principle, labor agencies and households should negotiate the wage that applies to their match.[17] However, for simplicity, I follow Gornemann et al. (2016) and assume that wage determination takes the following form.[18]

$$\frac{w_t}{\bar{w}} = \left(\frac{r^l_t}{\bar{r}^l}\right)^{1-\rho_w}\left\{\frac{w_{t-1}}{\bar{w}} \times \left(\frac{\bar{\pi}}{\pi_t}\right)^d\right\}^{\rho_w} \tag{34}$$

where variables with bars denote the steady state values. Equation (34) implies nominal wage rigidity and can be interpreted as follows. First, $1-\rho_w$ fraction of the real wage is adjusted in proportion to changes in the labor rental price, which reflects the marginal productivity of labor. A nominal wage rigidity is applied to the remaining ρ_w fraction. Among the remaining fraction of the nominal wage, d fraction is indexed to the steady state inflation rate. While the rest is indexed to the current period inflation rate. Due to nominal wage rigidity and insufficient indexation, an increase in the inflation rate leads to a fall in the real wage. Thus, in response to a demand shock, the real wage exhibits rigidity.[19] This wage formula resembles Calvo-type nominal wage rigidity.

For the matching function, I follow den Hann et al. (2000) and assume the following form.

$$M(\tilde{X}_t, V_t) = \frac{\Big(U(\tilde{X}_t) + \lambda N(\tilde{X}_t)\Big)V_t}{\left\{\Big(U(\tilde{X}_t) + \lambda N(\tilde{X}_t)\Big)^\alpha + V_t^\alpha\right\}^{\frac{1}{\alpha}}} \quad , \quad \alpha > 0 \tag{35}$$

where V_t is the number of vacancies while $U(\cdot)$ and $N(\cdot)$ are the mass of unemployed and employed households.[20] The above matching function guarantees that the job finding rate and the vacancy filling

[16] Labor agencies do not have a monopoly power in supplying labor service. However, it does not necessarily imply that they cannot enjoy a mark-up. Because of the fixed cost of posting vacancy, a certain amount of mark-up should be guaranteed so that labor agencies post a vacancy, thereby supplying labor services.

[17] Since each household's outside option depends not only on their idiosyncratic productivity but also on their asset holding and the level of adjustment cost, the equilibrium wage can differ at each point in the idiosyncratic state space.

[18] Note that, for the wage implied by the equation (34) to be an equilibrium wage, both labor agencies and households should not want to terminate the match between them. Under the parameterizations and the simulations examined in this paper, wages implied by the wage equation always remain in the bargaining set. Thus, to maintain a match is always beneficial for both labor agencies and households.

[19] By contrast, in response to supply shocks such as total factor productivity (TFP), the wage varies substantially because higher TFP leads to an increase in the labor rental price and a fall in the inflation rate at the same time.

[20] I assume that the mass of the households is one in the model. However, since there is a mass of the households that

rate are always between zero and one. Finally, the labor agencies are subject to the following free entry condition for vacancies.

$$\iota = \frac{M(\tilde{X}_t, V_t)}{V_t} \int J_L(s_t|\tilde{X}_t) d\mu_t \tag{36}$$

where μ_t is the distribution of households over their idiosyncratic states. The above condition implies that labor agencies keep posting vacancies until the vacancy posting cost ι (LHS) becomes equal to the expected value of matching (RHS).

2.5 Mutual fund

All firms in the model are owned by a mutual fund, which hires a fraction of households as entrepreneurs for the management of these firms. Also, the fund holds the capital stock, rents it to the firms at a competitive rate, and issues equity to finance investment. The dividend on equity is a fraction of the profits. The other fraction is given to entrepreneurs as compensation for their work. The collection of the profits, capital rental and investment are done by a capital service provider in the mutual fund. The provider determines the utilization rate of capital based on the market rental rate and converts the final good into new capital stock. Let $J_K(K_t|X_t)$ denote the value of the capital service provider with the capital stock K_t and under the aggregate state X_t. Then,

$$J_K(K_t|X_t) = \max_{\{v_t, K_{t+1}\}} (1-\tau_t^a)(1-\nu)\Pi_t - K_{t+1} + K_t - \Phi(K_{t+1}, K_t) + \mathbb{E}_t\Big[\Lambda_{t,t+1} J_K(K_{t+1}|X_{t+1})\Big] \tag{37}$$

where τ_t^a and v_t are the profit tax rate and the capital utilization rate respectively. Φ is the capital adjustment cost, whose functional form is given as follows.

$$\Phi(K_{t+1}, K_t) = \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - 1\right)^2 K_t \tag{38}$$

where the coefficient $\phi > 0$ represents the extent of adjustment costs. Π_t is before-tax profits, given as follows.

$$\Pi_t = \{r_t^k v_t - \delta(v_t) + \Psi_t\} K_t \tag{39}$$

$$\Psi_t = \left[S_t\left\{1 - \frac{\eta}{2\kappa}\left(\log\frac{\pi_t}{\bar{\pi}}\right)^2 - \frac{\phi_x}{2}\left(\log\frac{x_t}{\bar{x}}\right)^2\right\} - MC_t Y_t - \Xi + (r_t^l - w_t)\bar{s}L_t - \iota V_t\right]/K_t \tag{40}$$

where Ψ_t is the sum of the profits from intermediate good firms and labor agencies per unit of capital stock. I assume that the capital service provider takes Ψ_t as given. $\delta(\cdot)$ and L_t are the variable depreciation rate and the aggregate hours of work . For the depreciation rate, I follow Greenwood et

work as an entrepreneur, U is not equal to $1 - N$.

al. (1988).

$$\delta(v_t) = \delta_0 v_t^{\delta_1} \quad , \quad \delta_1 > 1 \tag{41}$$

where δ_0 is the depreciation rate under full utilization and δ_1 governs the degree of acceleration of depreciation with utilization. An optimal choice for the utilization rate satisfies the following optimality condition.

$$r_t^k \geq \delta'(v_t) \quad \text{with equality if } \ 0 < v_t < 1 \tag{42}$$

Finally, the inter-temporal optimality condition of the capital service provider is given as follows.

$$1 = \mathbb{E}\left[\Lambda_{t,t+1}\left\{\frac{(1-\tau_{t+1}^a)(1-\nu)\Pi_{t+1} + 1 - \Phi_2(K_{t+2}, K_{t+1})}{1 + \Phi_1(K_{t+1}, K_t)}\right\}\right] \tag{43}$$

where Φ_1 and Φ_2 are the partial derivatives of the adjustment cost Φ with respect to the first and second argument respectively.

Once the capital service provider collects all the profits, it passes them to the mutual fund. Then, the fund chooses the amount of equity issuance and the dividend rate. Importantly, I assume that the fund operates in a perfectly competitive environment. Thus, the fund cannot have retained earnings. Then, the following equation should hold.[21]

$$(1-\tau_t^a)(1-\nu)\Pi_t + K_t - K_{t+1} - \Phi(K_{t+1}, K_t) + q_t(K_{t+1} - K_t) = r_t^a K_t \tag{44}$$

Since the fund faces perfect competition, the price of the equity, which is equivalent to capital holding, should be equal to marginal cost of capital accumulation. That is, we have the following expression for the equity price.

$$q_t = 1 + \Phi_1(K_{t+1}, K_t) \tag{45}$$

By plugging (45) into the equation (44) and exploiting the homogeneity of degree zero of $\Phi(\cdot)$, we have the following expression for the dividend rate.

$$r_t^a = (1-\tau_t^a)(1-\nu)\Pi_t - \Phi_1(K_{t+1}, K_t) - \Phi_2(K_{t+1}, K_t) \tag{46}$$

Equation (45) and (46) always hold regardless of the stochastic discount factor. In this paper, I assume

[21] Equation (44) reflects the market clearing condition for equity. That is, $A_t = K_t$ for all t, where A_t is the aggregate equity demand.

the following form of the stochastic discount factor.

$$\Lambda_{t,t+1} = \frac{q_t}{q_{t+1} + r^a_{t+1}} \tag{47}$$

The above stochastic discount factor guarantees that the inter-temporal optimality condition regarding capital accumulation always holds. Thus, in the current setting, households collectively decide the next period aggregate capital stock, and the capital service provider simply meets the demand for new capital stock.[22]

2.6 Government

Fiscal authority

The fiscal authority collects taxes and issues bonds to finance government purchases, lump-sum transfers and unemployment benefits. Regarding bond issuance, following Bayer et al. (2018), I assume that the government abides by the following rule to stabilize its debt level around its steady state.[23]

$$\frac{B_{t+1}}{\bar{B}} = \left(\frac{R_t/\pi_t \times B_t}{\bar{R}/\bar{\pi} \times \bar{B}}\right)^{\rho_B} \quad , \ 0 \leq \rho_B < 1 \tag{48}$$

where variables with bars denote steady state values. The parameter ρ_B governs how rapidly the fiscal authority stabilizes its debt. For instance, if $\rho_B = 0$, the fiscal authority reverts the level of government debt back to the steady state level immediately by adjusting available fiscal instruments, i.e. government purchases, lump-sum transfers, or tax rates. If ρ_B is close to one, the government debt level can deviate from its steady state level for a substantial amount of time.

Since the economic agents in the model form rational expectations, the government must meet its solvency condition or inter-temporal budget constraint.

$$B_t = \sum_{l \geq t}^{\infty} \left\{ \prod_{i=t}^{l} \left(\frac{\pi_i}{R_i}\right) \right\} \left\{ \mathbb{T}_l - (G_l + T_l + D_l) \right\} \tag{49}$$

where $\mathbb{T}$, G, T, and D are the tax revenue, government purchases, lump-sum transfers (or taxes) and the unemployment benefits respectively. Equation (49) implies that, each period, the debt level should be equal to the present discounted value of all future government surpluses. When the real value of government debt changes, at least one of the four instruments on the right-hand side must adjust to meet the solvency condition. In the baseline model, I assume that the government adjusts its goods

[22] I experimented with different stochastic factors, such as the household's discount factor, the inverse of the rate of return on both assets, and the marginal rate of substitution of the aggregate consumption, to examine the robustness of the results. None of the results presented in this paper were significantly affected by choice of the stochastic discount factor.

[23] The households in the model are not Ricardian in the sense that they value the liquidity provided by the bond, and thus, not indifferent between tax-financing and debt-financing. Therefore such a rule is very important for the existence of a stable equilibrium. For detailed discussion, see Bayer et al. (2018).

purchases to meet the following budget constraint on a period-by-period basis.

$$\frac{R_t}{\pi_t}B_t - B_{t+1} = \left[\tau_t^y\left\{\int \mathbb{1}_{\{n_t=1\}}s_t d\mu_t + \int \mathbb{1}_{\{n_t=2\}}\min\{\upsilon s_t, \bar{s}\}d\mu_t + \nu\Pi_t\right\} + \tau_t^a(1-\nu)\Pi_t\right] - (G_t + T_t + D_t) \tag{50}$$

Monetary authority

The monetary authority is standard. It sets the nominal rate of the return on the government bond, following a Taylor-type rule with interest rate smoothing as follows.

$$\frac{R_{t+1}}{\bar{R}} = \left(\frac{R_t}{\bar{R}}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi}\left\{\exp\left(u_t - \bar{u}\right)\right\}^{\phi_u}\right]^{1-\rho_R}\exp(\epsilon_{R,t}) \quad , \ \epsilon_{R,t} \ \sim \ \text{i.i.d } N(0,\sigma_R^2) \tag{51}$$

where $0 < \rho_R < 1$ is the interest rate smoothing parameter, $\phi_\pi > 1$ and $\phi_u > 0$ are the responsiveness of the nominal rate to inflation and the unemployment rate gap, respectively, and $\epsilon_{R,t}$ denotes a monetary policy shock.

2.7 Market clearing conditions

In the model, there are five markets. The market clearing condition for the final good market is given as follows.[24]

$$S_t = \int\left[c(a_t,b_t) + \underline{\mathrm{R}} \times \mathbb{1}_{\{b_t<0\}}b_t\right]d\mu_t + \int_0^1\left[\left\{\frac{\eta}{2\kappa}\left(\log\frac{\pi_{j,t}}{\bar{\pi}}\right)^2 + \frac{\phi_x}{2}\left(\log\frac{x_{j,t}}{\bar{x}}\right)^2\right\}S_t + (R_{t+1}-1)(r_t^l L_{j,t} + r_t^k K_{j,t})\right]dj + \Xi + I_t + \Phi(K_{t+1},K_t) + G_t \tag{52}$$

where $I_t = K_{t+1} - \{1-\delta(v_t)\}K_t$ is aggregate investment. Equation (52) reflects the assumption that the household borrowing premium and the interest on working capital are not a part of the government revenue but represent a waste of resources. As shown in the above, the final good is used for consumption, investment, government purchases and other miscellaneous costs in the model.

Market clearing for capital rental services, the capital stock utilized in the current period must equal the capital services demanded by the intermediate good firms.

$$v_t K_t = \int_0^1 K_{j,t}dj \tag{53}$$

Likewise, the labor supplied by households (via labor agencies) must equal the labor services de-

[24] For the ease of notation, I drop idiosyncratic states in household's optimal policies.

manded by the intermediate good firms.

$$\int \mathbb{1}_{\{e_t=1\}} s_t n_t d\mu_t = \int_0^1 L_{j,t} dj \tag{54}$$

The market clearing condition for the equity market is given as follows.[25]

$$K_{t+1} = (1-\zeta)\int a_{t+1}(a_t, b_t) d\mu_t \tag{55}$$

That is, the sum of the surviving households' equity holding should be equal to the next period aggregate capital stock.

Finally, the market clearing condition for the bond market is given as follows.

$$B_{t+1} = \bar{B}\left(\frac{R_t/\pi_t \times B_t}{\bar{R}/\bar{\pi} \times \bar{B}}\right)^{\rho_B} = \int b_{t+1}(a_t, b_t) d\mu_t \tag{56}$$

where the government's bond issuance rule is taken into account.

2.8 A recursive equilibrium

Economic agents in the model, such as households and firms, need to forecast future variables that are relevant for their optimal decision. Especially, households and firms need to know the evolution of the households' distribution across their idiosyncratic states, since it determines the aggregate capital stock, bond holding, and the size of labor force. Let $\mu_t(\tilde{a}, \tilde{b}, \tilde{s}, \tilde{e}, \tilde{\chi})$ denote the mass of households with $(\tilde{a}, \tilde{b}, \tilde{s}, \tilde{e}, \tilde{\chi})$ in period t, where $\tilde{a}$ is the equity holding, $\tilde{b}$ is the bond holding, $\tilde{s}$ is idiosyncratic productivity, $\tilde{e}$ is the working status in period t, and $\tilde{\chi}$ is the equity adjustment cost respectively. Then, the law of motion for the distribution is described as follows.[26]

$$\begin{aligned} \mu_{t+1}(\tilde{a}, \tilde{b}, \tilde{s}, \tilde{e}, \tilde{\chi}) = \sum_{s\in\mathbb{S}} P_s(s_{t+1} = \bar{s}|s_t = s) \times \sum_{e\in\{1,2,3\}} P_e(e_{t+1} = \bar{e}|e_t = e) \times P_\chi(\chi_{t+1} = \tilde{\chi}) \\ \times \int \mathbb{1}_{\{a_{t+1}(a_t,b_t|s_t,e_t)=\bar{a}\}} \times \int \mathbb{1}_{\{b_{t+1}(a_t,b_t|s_t,e_t)=\bar{b}\}} d\mu_t(a_t, b_t, s_t, e_t) \end{aligned} \tag{57}$$

where P_e is the probability of working status transition, which is determined by the stochastic process for s_t, the exogenous transition probability in and out of the entrepreneurial state, the exogenous separation rate, and the endogenous job-finding rate.

[25] To introduce a stochastic death and a hypothetical annuity arrangement into the model, I assume that the capital service provider accumulates the capital stock that is equal to the equity holding of surviving households. Similarly, the mutual fund only receive the payment from households that will survive next period. The annuity arrangement keeps the remaining payments and use them for the annuity payment next period. Since there are infinitely many households and the law of large numbers holds, the amounts of these payments are known without uncertainty.

[26] I describe the law of motion as if the adjustment costs are drawn from a discretized process though they are assumed to follow a continuous distribution.

With the market clearing conditions and the law of motion for the households' distribution, a recursive equilibrium in the model is defined as follows.

Definition (Recursive equilibrium): A recursive equilibrium is a set of variables and functions $\{a_{t+1}$, b_{t+1}, c_t, n_t, q_t, w_t, r^a_t, r^l_t, r^k_t, v_t, u_t, V_t, M_t, $V(a_t, b_t|s_t, e_t, \chi_t, X_t)$, $J_L(s_t|\tilde{X}_t)$, $J_K(K_t|X_t)$, K_{t+1}, Π_t, Ψ_t, π_t, $\{J_I(i_{j,t-1}, P_{j,t-1}|X_t)$, $Y_{j,t}$, $L_{j,t}$, $K_{j,t}$, $\pi_{j,t}$, $S_{j,t}$, $a_{j,t}$, $i_{j,t}\}$, MC_t, $f(\tilde{X}_t)$, S_t, G_t, L_t, D_t, T_t, R_{t+1}, $\Lambda_{t,t+1}\}$ and laws of motion such that:

1. Given the laws of motion, w_t, q_t, r^a_t, R_t and π_t, the value function $V(a_t, b_t|s_t, e_t, \chi_t, X_t)$ is a solution to the household's problem. $a_{t+1}(a_t, b_t|s_t, e_t, \chi_t, X_t)$, $b_{t+1}(a_t, b_t|s_t, e_t, \chi_t, X_t)$, $c(a_t, b_t|s_t, e_t, \chi_t, X_t)$, $n(a_t, b_t|s_t, e_t, \chi_t, X_t)$ are the associated optimal decision rules.
2. Given r^k_t, r^l_t, R_{t+1}, S_t, P_t, MC_t, and $\Lambda_{t,t+1}$, value function $J_I(i_{j,t-1}, P_{j,t-1}|X_t)$ solves the problem of an intermediate good producer. $K_{j,t} = K(i_{j,t-1}, P_{j,t-1}|X_t)$, $L_{j,t} = L(i_{j,t-1}, P_{j,t-1}|X_t)$, and $P_{j,t} = P(i_{j,t-1}, P_{j,t-1}|X_t)$ are the associated optimal decision rules. Sales $S_{j,t}$ and new production $Y_{j,t}$ are determined by (25) and (27) respectively. The profit of the intermediate good firm is given by $S_{j,t} - \{\frac{\eta}{2\kappa}\left(\log\frac{\pi_{j,t}}{\bar{\pi}}\right)^2 + \frac{\phi_x}{2}\left(\log\frac{x_{j,t}}{\bar{x}_j}\right)^2\}S_t - MC_tY_{j,t} - \Xi$.
3. The final good sale S_t and the price index P_t are given by (20) and (23).
4. The marginal cost of production MC_t and the gross inflation rate π_t are given by (31) and (32).
5. Given r^l_t, w_t and $\Lambda_{t,t+1}$, the value of labor agency $J_L(s_t|\tilde{X}_t)$, the number of matching M_t, vacancies V_t, the job-finding rate $f(\tilde{X}_t)$, and the unemployment rate u_t are determined by (33), (35), and (36).
6. For given r^k_t, the capital service firm decides v_t following (41).
7. The equity price q_t and return r^a_t are given by equation (45) and (46).
8. The fiscal authority decides the bond supply according to equation (48).
9. The monetary authority decides the nominal return on bond R_{t+1} according equation (51).
10. G_t (or T_t, τ_a, τ_y) adjusts to balance the government budget constraint.
11. The law of motion for μ_t is consistent with the relevant optimal decision rules.
12. All firms make the same choices (a symmetric equilibrium).
13. All markets clear.

2.9 Numerical solution

The model is solved using a perturbation method developed by Reiter (2009). The method has been further improved by Winberry (2015), Bayer and Luetticke (2018), and others. The method relies on a linearized system of the model around its steady state. The method adopted in this paper is based on the method of Bayer and Luetticke (2018).

The steady state of the model is solved using the endogenous grid method of Carrol (2006). Because of stochastic equity adjustment costs, the household value functions are globally concave, and the endogenous grid method can be used to find a solution to the household's problem. Once households' decision rules over the state space are determined, I compute a stationary distribution, following a histogram approach developed by Young (2009). That is, the distribution is expressed as a histogram and, if a mass falls between the grid points, it is allocated to the nearest grid points based on the distance to such points.

After solving for the steady state, I linearize the model around it using Bellman equations, laws of motions for the distribution and exogenous processes, and other equilibrium conditions. The most critical problem associated with applying a perturbation method to a HANK model is the size of the state space. Since the model has many idiosyncratic states, including holdings of each asset, idiosyncratic productivity, and working status, it is almost infeasible to compute the Jacobian of the linearized system without a state-space reduction even though a relatively small number of grid points are used to approximate each dimension of the state space. Thus, for a state space reduction, I adopt a methodology used by Bayer and Luetticke (2018). Specifically, I use Chebyshev polynomials to approximate the value functions and a fixed copula to reduce the dimension of the distribution. Once I reduce the dimension of the linearized system, I solve the model by using the perturbation method developed by Schmitt-Gróhe and Uribe (2004). Under the parameterizations explored in this paper, the model has a unique perturbed solution around the steady state.

3 Calibration

One of the primary goals of this paper is to examine the transmission mechanisms of monetary policy shocks. On this subject, Kaplan et al. (2018) argue that indirect effects, such as the effects of changes in the real wage, far outweigh the direct effect of changes in the real interest rate in determining the response of the aggregate consumption to monetary policy shocks. For their result, matching the empirical asset holding distribution is of great importance.[27] Thus, to have a fair comparison of the results, I seek for a calibration for which the model can match similar targets used in Kaplan et al. (2018). Specifically, I calibrate the model to match moments of asset distribution such as the share of borrowers, the mass of households with zero assets, the mass of wealthy hand-to-mouth households (households with zero liquid but a significant amount of illiquid assets), and so on. Given that the model has many features and parameters, I use standard parameter values used in the literature for the most part and limit the number of parameters that are internally calibrated. One period in the model corresponds to one quarter in the data.

[27] Kaplan et al. (2018) emphasize that the mass of households with low liquid wealth is directly related to the overall sensitivity of consumption to interest rate changes while the top end of the liquid asset distribution is critical for plausible redistributive effects of monetary policy shocks.

Table 1: Parametrization 1/3

Parameter	Value	Description	Target or reference
Households			
σ	1.5	Relative risk aversion	Standard value
β	0.9909	Household's discount factor	Bond to GDP ratio
ξ	3	Inverse Frisch elasticity	Chetty et al. (2011)
ψ	0.7164	Scale parameter for disutility of labor	Normalization
ζ	1/181	Probability of death	Average life-span
μ_χ	10.6229	Mean of χ dist	Average prob. of adjust
σ_χ	3.7543	Scale parameter for χ dist	Top 10% illiquid asset share
Labor market			
λ	0.1	Exogenous job separation rate	Standard value
ι	0.1	Vacancy posting cost	Gornemann et al. (2016)
$\bar{w}$	1.0235	SS wage	SS unemployment rate
α	1.7127	Matching elasticity	SS vacancy filling rate
ρ_w	0.75	Degree of staggered wage setting	Calvo sticky wage lit.
d	0.75	Indexation to the steady state inflation rate	Calvo sticky wage lit.

3.1 Data and parametrization

For the calibration, I intentionally use the same data, i.e., SCF 2004, and the same categorization of assets and liabilities as Kaplan et al. (2018). In the model, there are two kinds of assets, liquid and illiquid assets. In the data, I consider all deposits in financial institutions (checking, savings, call, and money market accounts), and bonds net of revolving consumer credit as liquid assets. I define illiquid assets as the sum of real estate wealth net of mortgage debt and equity in the corporate and non-corporate business sectors. Consumer durables, such as vehicles, net of non-revolving debt are considered illiquid assets as well. Households with negative illiquid assets are excluded, as short-positions in the illiquid asset are not allowed in the model.

Table 1 shows the set of parameters related to households and the labor market. For the coefficient of relative risk aversion, I use 1.5, which is within a range of values used in the literature. The household's subjective discount factor is closely related to the household's demand for liquid assets. Thus, it is internally calibrated at 0.9909 to match the bond to GDP ratio in the model.[28] For the inverse of Frisch elasticity, I use the value reported in Chetty et al. (2011) for the Frisch elasticity along the intensive margin, which is 3. The scaling parameter for the disutility of labor is set to guarantee that the steady state labor supply is equal to one. The probability of death used in the model implies an average working life-span of 45 years.

The distribution of the utility cost of adjustment mainly determines the frequency of adjustment and households' demand for the illiquid asset. The calibrated adjustment costs imply an average adjustment frequency at the steady state of 5.7% per quarter, which is slightly higher than but close to

[28]Since all aspects of the model are inter-connected, there is no clear one-to-one relation between a parameter and a moment. Though, I explain as if there exist such one-to-one relations based on the relevance between the model parameters and the moments.

Table 2: Parametrization 2/3

Parameter	Value	Description	Target or reference
Int. good			
η	3	Elasticity of substitution	Gornemann et al. (2016)
Ξ	0.1266	Fixed cost of operation	Capital income share
θ	0.27	Capital share of the production	Labor income share
κ	0.08	Slope of the Phillips curve	Standard value
$\bar{\pi}$	1.005	steady state inflation rate	Standard value
φ	0.1967	Sales elasticity w.r.t. the stock	SS sales-to-stock ratio of 0.25
ϕ_x	0.01	Sales-to-stock adj. costs	Jung and Yun (2005)
δ_i	0.01	Inventory depreciation rate	Jung and Yun (2005)
Mutual fund			
δ_0	0.02	Depreciation rate under full utilization	SS depreciation rate
δ_1	1.69	Elasticity of depreciation rate	SS utilization rate
ϕ	10	Capital adjustment cost	Standard value
ν	0.2299	Ent. share of profits	Gini Networth

the implied adjustment probability in Kaplan and Violante (2014).[29] Also, the utility cost distribution leads to an illiquid asset share of top 10% wealthy households of about 70%.

As for the labor market parameters, I set the probability of exogenous job separation rate at 10%, which is a standard value used in the literature. Following Christiano et al. (2014), I assume that the steady state unemployment rate and vacancy-filling rate are 5.5% and 70% respectively. Given a value of 0.1 for the vacancy posting cost, which is slightly smaller the value used in Gornemann et al. (2016), these lead to a steady state wage of 1.0235 and a matching elasticity with respect to the number of searchers of 1.7127. Two parameters governing wage rigidity in the model, ρ_w and d, are set to 0.75, based on the standard parameter values used in models with Calvo wage setting, for instance Erceg et al. (2000) and Schmitt-Grohe and Uribe (2006).

I set the elasticity of substitution among intermediate inputs to 3, the same value as Gornemann et al. (2016). Such a low value for the parameter plays several important roles in the model. First, it results in a substantial amount of profits at the steady state. Thus, for high enough aggregate capital stock, the dividend rate can be much higher than the rate of return on the liquid asset even after a fraction of profits is given to entrepreneur households. This facilitates matching the illiquid asset-to-GDP ratio in the data. Second, a lower elasticity of substitution leads to a higher degree of quantity response of the intermediate good firms. The elasticity of substitution represents the degree of competition in the intermediate goods market. If η is high, i.e., the competition in the market is severe, it is hard for firms to raise their prices. Hence, the decrease in mark-ups is larger, and, thus, profits fall in response to an expansionary monetary policy shock. In contrast, if the elasticity of substitution is low, the fall in the mark-up is limited, and hence, profits can increase in response to a decrease in the real interest rate.

[29] Kaplan and Violante (2014) compute a participation frequency of 4.5% given a fixed-adjustment cost of \$500.

Table 3: Parametrization 3/3

Parameter	Value	Description	Target or reference
Government			
τ^w	0.3	SS Income tax rate	Data
τ^a	0.3	SS Profit tax rate	Data
υ	0.4	Replacement ratio	Data
ρ^B	0.85	Degree of debt stabilization	Bayer et al. (2018)
T	10.7%	SS lump-sum transfer to GDP ratio	Top 10% other income share
$\underline{\mathrm{R}}$	3.5%	Borrowing premium	Share of wealthy HtM HH
$\underline{\mathrm{b}}$	1.2705	Borrowing limit	Fraction of borrowers
Central bank			
$\bar{R}$	1.0080	SS policy rate	Share of HH with zero assets
$\bar{\pi}$	1.005	Target gross inflation rate	Fed's current target
ρ^R	0.85	Degree of interest rate smoothing	Standard value
ϕ_π	1.5	Response of nominal rate to inflation gap	Standard value
ϕ_u	0.1	Response to nominal rate to unemployment gap	Standard value
TFP shock			
ρ_Z	0.9	Auto-correlation of TFP shock	Standard value

I set the slope of Phillips curve to 0.08, a standard value used in the literature. The fixed cost of operation is set to imply a steady state capital income share of about 30%.[30] The steady state inflation rate is set to 2% per annum, which is consistent with the Fed's current inflation target. The capital share of the production is set to 0.27, which leads to a steady state labor income share of around 60%. For the parameters related to inventory holding, I follow Jung and Yun (2005). The elasticity of sales with respect to the stock of goods is calibrated at 0.1967, which implies a steady state sales-to-stock ratio of 25%. For the coefficient of sales-to-stock adjustment costs and the inventory depreciation rate, I use the values in Jung and Yun (2005).

The first parameter regarding the variable depreciation rate δ_0 is set to 0.02, which implies an annual depreciation rate of 8% under full utilization. The parameter that governs the degree of the acceleration of depreciation δ_1 is set to 1.69. This choice of parameter values regarding variable depreciation leads to a steady state capital utilization rate of about 65%. The coefficient of the capital adjustment costs is set to 10, which is a standard value used in the literature. The entrepreneur share of profits is internally calibrated to match the Gini index of networth, i.e., assets minus liabilities, since entrepreneur households mainly drive the overall wealth inequality by holding a significantly larger amount of assets than other households in the model.

For calibrating the fiscal and monetary authorities, and exogenous processes, I mostly use values that are widely used in the literature. For tax rates, I assume that both income and profit tax rates are 30%.[31] I set the replacement ratio of unemployment benefits to 40% as in Shimer (2005). The degree

[30]The current calibration for the elasticity of substitution implies a steady state mark-up of 50%. Thus, even though a fraction of profits is given to some households as a compensation for working as entrepreneurs, a high mark up can lead to too high capital income share if there is no fixed cost.

[31]Before 2017, the U.S. profit tax rate was above 30%. However, it fell to 21% after the introduction of the 2017 Tax Cuts

Table 4: Productivities

Symbol	Value
s_1	0.2488
s_2	0.9148
s_3	1.0000
s_4	1.0931
s_5	5.0287
Ent	-

Table 5: Transition matrix

		tomorrow					
		s_1	s_2	s_3	s_4	s_5	Ent
today	s_1	0.9308	0.0567	0.0020	0.0000	0.0100	0.0005
	s_2	0.0098	0.8194	0.1587	0.0016	0.0100	0.0005
	s_3	0.0020	0.1599	0.6677	0.1599	0.0100	0.0005
	s_4	0.0000	0.0016	0.1603	0.8276	0.0100	0.0005
	s_5	0.0389	0.0389	0.0389	0.0389	0.8440	0.0005
	Ent	0.0385	0.0385	0.0385	0.0385	0.0385	0.8076

of persistence in the bond issuance rule is set to 0.85, following Bayer et al. (2018). The amount of lump-sum transfer is calibrated at 10.7% of GDP in the model. This implies a share of other income of top 10% wealthy households of 5% in the model.[32] The borrowing premium is set to 3.5% per quarter to enable the model to match the share of wealthy hand-to-mouth households in SCF 2004, which is 20%. The borrowing limit is also internally calibrated to match the fraction of households with debt in the data, which is 15% in the data.

The steady state nominal rate of return on the government bond is set to 1.008 to match the share of households with zero assets. Combined with the inflation target of 2% per annum in the model, the calibrated policy rate implies a real liquid asset return of 1.2% per annum. I set parameters related to the Taylor rule to standard values. The degree of interest rate smoothing is set to 0.85. The responsiveness of nominal rate to inflation gap and unemployment gap are set to 1.5 and 0.1 respectively. I set the auto-correlation of TFP shock to 0.9, which is also a standard value.[33]

Finally, the income process, which is the ultimate source of the inequality in the model, is reverse-engineered to match asset holding and wealth inequality in the data. First, I set the normal income process for s_t as a standard AR(1) process with three states using Tauchen's method for the discretization. I set the autocorrelation and standard deviation of quarterly income to 0.93 and 0.03 respectively, both of which are close to values used in the literature.[34] In addition to this standard part, two boundary states (super low-skilled and super high-skilled) are added to enable the model to generate realistic wealth inequality. I set the probability of becoming an entrepreneur P_e to 0.05%. Then, I calibrate the probability of leaving the entrepreneur state to match the liquid asset share of top 10% wealthy households. The resulting parameter value for $\tilde{P}_e$ is 19.24%.[35] These parameter values imply that the profit income share for top 10% wealthy households is 21%, which is close to the corresponding value

and Jobs Act. In case of the income tax rate, it ranges from 10% up to around 40% based on the amount of income.

[32]Other income share of top 10% wealthy households in the model is slightly lower than the corresponding value in SCF 2004, which is 7%.

[33]Since I rely on a linearization of the model to solve for a recursive equilibrium, the size of the shocks does not matter especially for the impulse response analysis.

[34]For instance, McKay et al. (2016) uses 0.966 and 0.33 for their income AR(1) income process in a version of their HANK model.

[35]Since entrepreneur households earn a large amount of income and face a very high unemployment risk, they accumulate a lot of wealth especially in the form of liquid asset.

Table 6: Model fit 1

	Data	Model
Capital to output ratio	**2.92**	**2.92**
Bond to output ratio	**0.26**	**0.26**
Gini index for net worth	**0.81**	**0.82**
Fraction with b < 0	**0.15**	**0.15**
Fraction with b = 0 and a >0	**0.20**	**0.20**
Fraction with b = 0 and a = 0	**0.10**	**0.10**
Fraction with b = 0	0.29	0.30
Fraction with a = 0	0.21	0.25

Notes: The data moments are computed from SCF 2004.

Table 7: Model fit 2

	Liquid		**Illiquid**	
Moments	Data	Model	Data	Model
Top 0.1 percent share	17	18	12	4
Top 1 percent share	47	49	33	22
Top 10 percent share	**86**	**87**	**70**	**73**
Bottom 50 percent share	-4	0	3	1
Bottom 25 percent share	-5	-2	0.1	0
Gini Coefficient	0.98	0.94	0.81	0.84

Notes: The data moments are computed from SCF 2004.

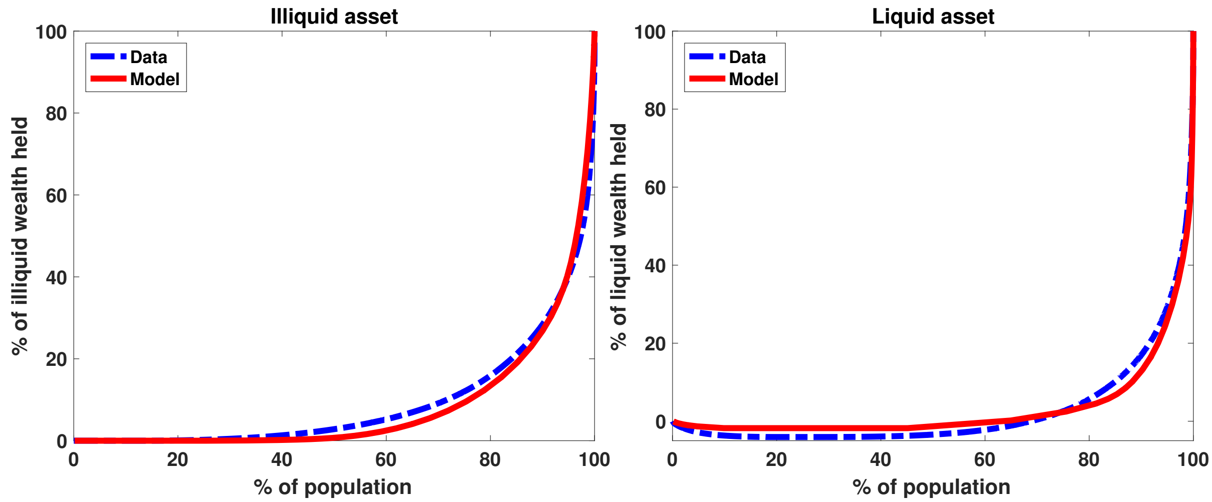
of 23% in SCF 2004. Tables 4 and 5 show the values for idiosyncratic productivity and the transition matrix of skills and the entrepreneur state.[36]

3.2 The fit of the model

Tables 6 and 7 show the model's performance in matching the targets related to aggregate asset holding and the distribution of assets. As discussed in the previous section, I use the capital-to-output ratio, bond-to-output ratio, and Gini index for net worth as targets. In addition, I target the fraction of borrowers, the share of wealthy hand-to-mouth households, and the share of households with zero assets because these moments determine the sensitivity of the aggregate consumption to changes in the real interest rate and relevant distributional effects in the model. As shown in Tables 6, the model does a fairly good job in matching these targets. Except for the Gini index for net worth, the model matches the targets up to two decimal points. Moreover, the model is reasonably good at matching non-targeted moments. In the model, the fraction of households with zero liquid assets is about 30% while it is 29% in SCF 2004. Similarly, the fraction of households with zero illiquid assets is 25% in the model, which is slightly higher than but close to the corresponding value of 21% in the data.

[36]Table 5 does not take into account employment status transition probabilities, i.e., job-finding and separation rates.

Figure 2: Lorenz curves in the data and the model



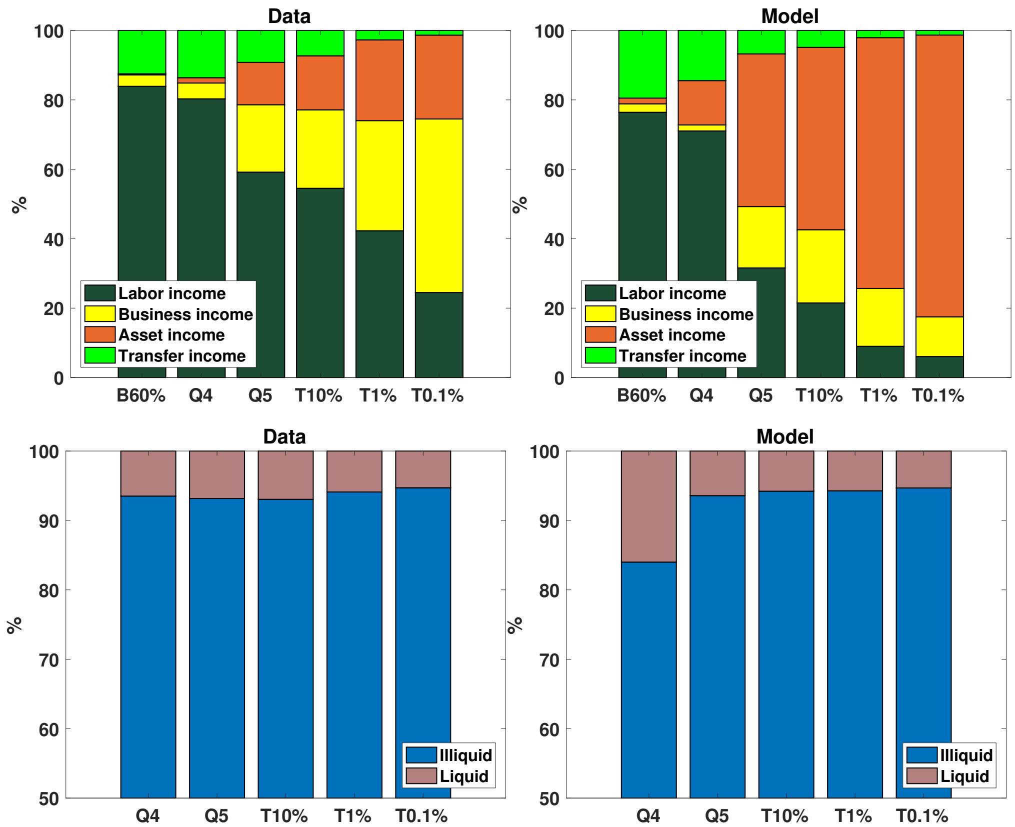
Notes: The figure shows asset holding inequality in the data and in the model using Lorenz curves. For the definition of liquid and illiquid asset in the data, see the main text.

Table 7 looks more closely into the distribution of asset holdings in the model and in the data. Among the moments presented in the table, only the top 10% shares of liquid and illiquid asset holding are targeted. Nevertheless, the model captures overall inequality in each type of asset holding very well. In case of the liquid asset, the model matches even the top 0.1% and top 1% asset shares very closely. The liquid asset shares of the top 0.1% and 1% wealthy households are 18% and 49% in the model while they are 17% and 47% in the data. By contrast, households in the model are less indebted than households in the data as shown in relatively poor households' liquid asset shares. The liquid asset share of households with the bottom 25% wealth is -5% in the data but only -2% in the model. Regarding illiquid asset holding, the model is less successful in matching the top right-end of the distribution. The illiquid asset share of top 0.1% and 1% wealthy households are 12% and 33% in the data while they are only 4% and 22% in the model. However, given that matching the right-tail of wealth distribution is known to be difficult, the model does an excellent job in generating realistic asset holding inequality.

Figure 2 shows the degree of asset holding inequality using Lorenz curves. The Lorenz curves in the model and in the data look very similar to each other, which confirms the model's ability to capture the degree of asset holding inequality observed in the data. Illiquid asset holding is slightly more concentrated in the right-tail of the distribution in the model than in the data, but the differences are not substantial. The liquid asset Lorenz curve again confirms that households at the left-tail of the distribution are less indebted in the model than in the data. However, the model captures well the concentration of liquid asset holding among households at the right-tail of the distribution.

Regarding income composition, the model captures essential characteristics of heterogeneity de-

Figure 3: Income composition in the data (SCF 2004) and in the model



Notes: The above and bottom panels show income and asset composition in the data and the model respectively. For the classification of income and asset types, see the main text.

pending on households' wealth.[37] Both in the data and the model, the main source of income of the bottom 80% of households is labor income, which accounts for around 80% of their total income. In contrast, top 10% households earn a substantial amount of income from assets and business activities. However, the combined share of business and asset income in the total income of wealthy households is larger in the model than in the data.

In case of the portfolio composition, more than 90% of wealthy households' portfolio is in illiquid assets both in the model and in the data. However, there are seemingly no noticeable differences across different wealth groups in terms of portfolio composition in the data. This is mainly because most of households in the data own houses, an important illiquid asset. If I consider only a fraction of residential houses as a productive illiquid asset that yields return, the illiquid asset share in households' portfolio tends to increase in households' wealth as in the model.

[37]In the data, labor income consists of income from wages and salaries. Business income is income from ownership of a business. Asset income includes interests, dividends, capital gains or losses. Transfer income consists of social security benefits, including unemployment benefits. In the model, labor income is income from wages. Entrepreneurs earn business income in the model. Asset income consists of the return on the bond and the equity. Transfer income consists of the lump-sum transfer from the fiscal authority and unemployment benefits.

4 The effects of monetary policy

In this section, I present the main results from the model regarding the effects of monetary policy shocks. Throughout the section, I focus on the effects of an expansionary monetary policy shock.[38] First, I show how aggregate variables such as output, consumption, investment, the inflation rate, wages and asset returns respond to a fall in the policy rate. Especially, I focus on how relevant features of the model help generate an increase in profits when an expansionary monetary policy shock hits the economy. Then, I decompose the total effect of an expansionary monetary policy shock into direct and indirect effects to examine the transmission mechanisms. Finally, I study the implications of the transmission channels on their distributional consequences.

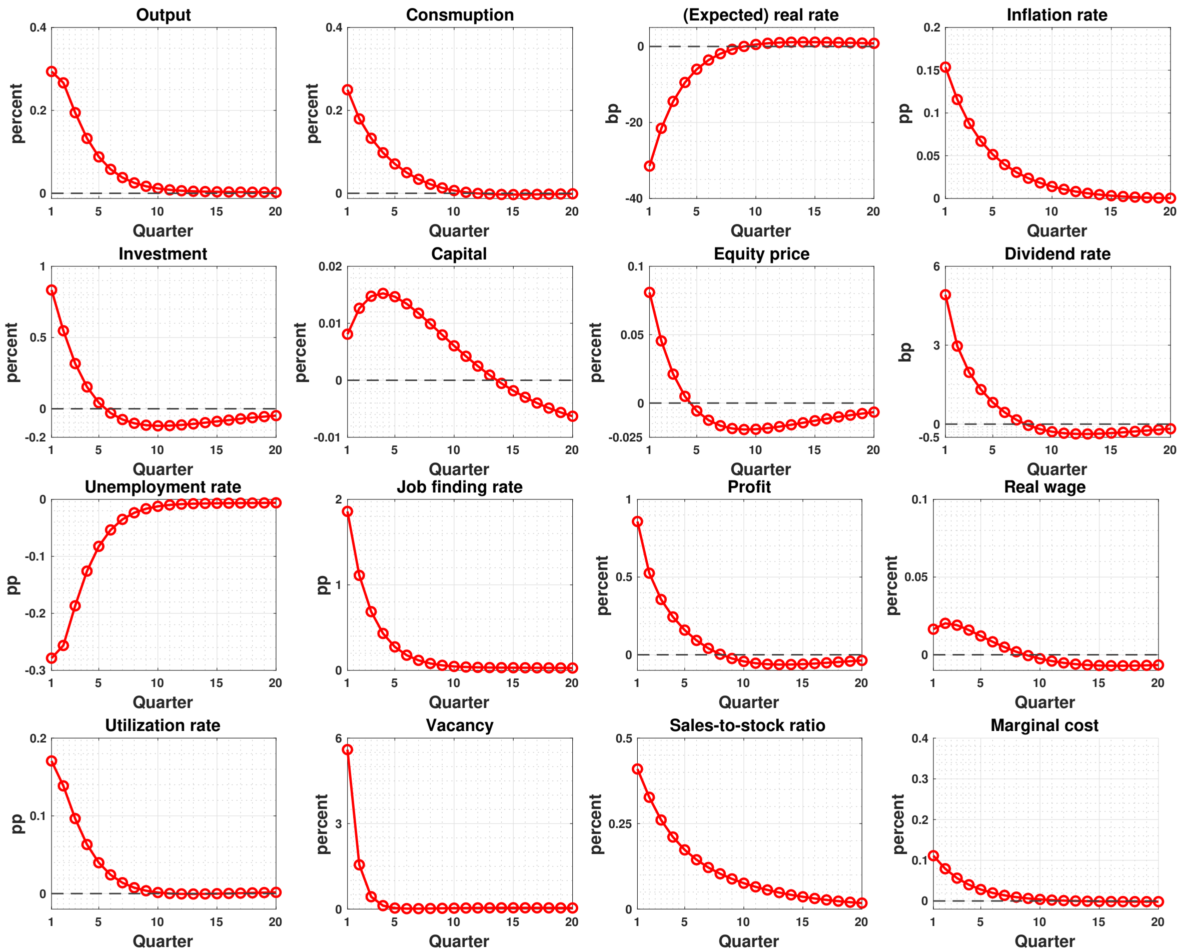
4.1 Impulse responses of aggregate variables

In this subsection, I show the impulse responses of aggregate variables to a minus 6.25 bp (minus 25 bp per annum) shock to the policy rate ($\epsilon_{R,0} = -0.000625$). After an expansionary monetary policy shock, the expected real interest rate falls and, by inter-temporal substitutions, households increase their demand for goods. In response to an increase in demand, the firms raise their prices. However, due to price adjustment costs (nominal rigidity) and the possible substitution across different goods (real rigidity), an increase in the prices is limited. Instead, firms increase their production, which leads to a rise in factor demands. Such an increase in factor demands results in higher rental rates for both labor and capital. As the market wage exhibits rigidity, a higher rental rate for labor services implies a greater mark-up for labor agencies, which makes them post more vacancies. Thus, the job-finding rate increases, and the unemployment rate falls. As more households become employed, the demand for goods further rises. Even households whose employment status has not changed also increase their demand, since the precautionary motive for savings becomes weaker due to lower unemployment risk and a higher job-finding rate. Similarly, a higher rental rate for capital leads to a higher capital utilization rate.

In principle, higher demand for goods can contribute to an increase in profits. However, if all firms want to increase production, marginal cost can rise substantially. As a consequence, mark-ups may shrink significantly, and profits may fall. In the model, firms avoid such a result by increasing production only partially, and instead, resorting to their inventory stock to meet demand. That is, firms reduce their inventory stock, and increase the sales-to-stock ratio. As a result, an increase in the marginal cost is limited. This dynamic adjustment of inventory stock, jointly with wage rigidity, contributes to an increase in profits. Hence, the dividend rate also rises. As the return on the illiquid asset becomes higher than before, households shift their portfolio towards the illiquid asset. Hence, investment and the aggregate capital stock increase.

[38] Since the solution method relies on a linearization of the system around the steady state, the results from a contractionary monetary policy shock are symmetric.

Figure 4: IRFs to an expansionary MP shock



Notes: Impulses responses to annualized 25 bp expansionary monetary policy shock. Inflation rate, unemployment rate, job finding rate, and capital utilization rate are expressed in terms of percentage point deviations from steady state. Nominal and real rate, and dividend rate are shown as basis point deviations from steady state. Inflation rate, nominal and real rate, and dividend rate are annualized. All other variables are shown as percent deviations from steady state.

Quantitatively, after a 25 bp (annualized) expansionary monetary policy shock, output increases by 0.3% on impact and gradually converges back to the initial steady state level.[39] Consumption increaseS on impact by 0.25%, which is slightly lower than that of output. As is well-known, the response of investment is much larger, increasing by more than 0.8% on impact. In order to meet the expansion in activity, the capital utilization rate increases by 0.17%. Importantly, the demand for labor services goes up, but, due to the assumed wage rigidity, the real wage does not respond much. The real wage increases only by less than 0.02% on impact. Such a response of the real wage is in line with what is reported in Christiano et al. (2005), Christiano et al. (2016), and Gornemann et al. (2016) in

[39]Like many other HANK models in the literature, the model in this paper cannot generate hump-shaped responses in output, consumption, and other aggregate variables to monetary policy shocks because it lacks mechanisms such as habit in consumption and investment adjustment costs, which help generate a hump-shaped response of these variables in a representative agent model.

terms of magnitude. Since the real wage does not respond much, a rise in labor rental rate leads to a substantial increase in the number of vacancies. On impact, vacancies rise by almost 6%.[40] As a result, the job-finding rate increases by 1.9%, and the unemployment rate falls by 0.28%.

Firms reduce their inventory stocks gradually over several periods in response to an increase in factor prices. On impact, firms reduce their inventory by 0.06% and the magnitude of the decrease peaks at 0.12% after six quarters. The response of the sales-to-stock ratio is more immediate. It rises by 0.4% on impact and converges back to the steady state level monotonically. These responses of inventory stock and the sales-to-stock ratio are consistent with the findings of Jung and Yun (2005).[41]

Because of wage rigidity and inventory stock adjustments, profits increase substantially in response to an expansionary monetary policy shock. After a minus 25 bp fall in the annualized policy rate, profits increase by 0.85% on impact. Such a response of profits is in stark contrast to the profit response in standard New Keynesian models, in which they decrease after an expansionary monetary policy shock. As I will show momentarily, such a difference in the response of profits (and the real wage) leads to different results on the transmission mechanisms as well as the distributional consequences of monetary policy shocks.

4.2 Transmission mechanisms of monetary policy in a HANK

To analyze the transmission mechanisms, I decompose the effects of an expansionary monetary policy shock on consumption into direct and indirect effects, following Kaplan et al. (2018). Note that aggregate consumption in any given period t can be written as a function of both current and future (expected) values of real interest rate r, equity price q, profits Π, real wage w, job finding rate f, and relevant fiscal instruments. That is, we have

$$C_t^* = C_t(\{r_\tau^*, q_\tau^*, \Pi_\tau^*, w_\tau^*, f_\tau^*\}_{\tau=t}^{\infty}) \tag{58}$$

in the baseline model, where I assume that the fiscal authority adjusts government purchases in response to changes in the real value of government debt. By totally differentiating (58), we have the following decomposition.[42]

[40] The vacancy posting responds in a front-loaded manner. After an initial peak, it rapidly goes back to the steady state level. This is because vacancy posting decision takes into account the expectation on all future value from a matching. Though the response of vacancy posting is transient, the unemployment and the job-finding rate exhibit respond more persistently.

[41] Like in other aggregate variables, the model cannot generate a hump-shaped response of the sales-to-stock ratio. In case of inventory stock response, the magnitude of the response is somewhat smaller in the model than in Jung and Yun (2005).

[42] I compute (59) numerically as follows. First, I compute the path of equilibrium prices and the job finding rate under an expansionary monetary policy shock using the solution that I get from the perturbation method. With that path, I solve the household's problem backwardly from T to 0 for a large T, where T is chosen to guarantee that the model economy has reverted back to the steady state in period T. A shock occurs in period 0. At each period, I compute the household's value function by taking into account the household's expectation on the future prices and labor market conditions. With these value functions, I derive the household's optimal decisions and compute the aggregate consumption by summing up individual consumption over the state space. I repeat this computation with different paths in which only one variable follows the equilibrium path and others remain at their steady state values. Then, I compute the difference between the

Figure 5: Consumption decomposition

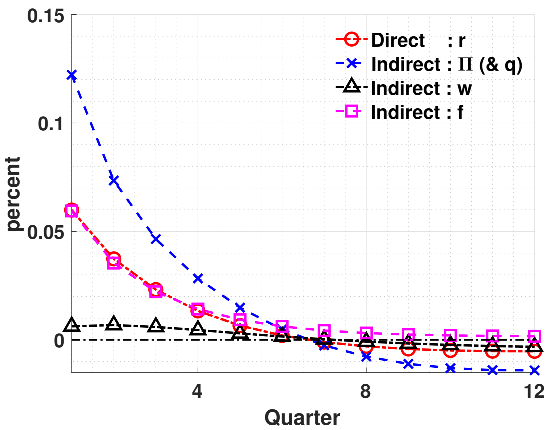


Table 8: Average response over the year

r change (pp annual)	-0.21
Elasticities $(\epsilon_X = \frac{\Delta X}{X}/\Delta r)$	
Output (Y)	-4.29
Investment (I)	-8.94
Consumption (C)	-3.19
Bond (B)	1.89
Equity (A)	-0.24
Decomposition of C (%)	
Direct effect (r)	**24**
Indirect effects	**76**
Profits (and equity price) (Π & q)	48
Wage (w)	4
Job finding rate (f)	24

Notes: The figure shows the percentage deviations of consumption from its steady state level. Elasticities are shown as semi-elasticities. The decomposition into the direct and indirect effects is approximately exhaustive. Both effects add up to 100%. The fiscal authority is assumed to adjust lump-sum transfer for given bond-issuance rule in response to monetary policy shocks. Profit is the profit given to entrepreneurs as compensations. The profits given to the equity holders are reflected in the equity return.

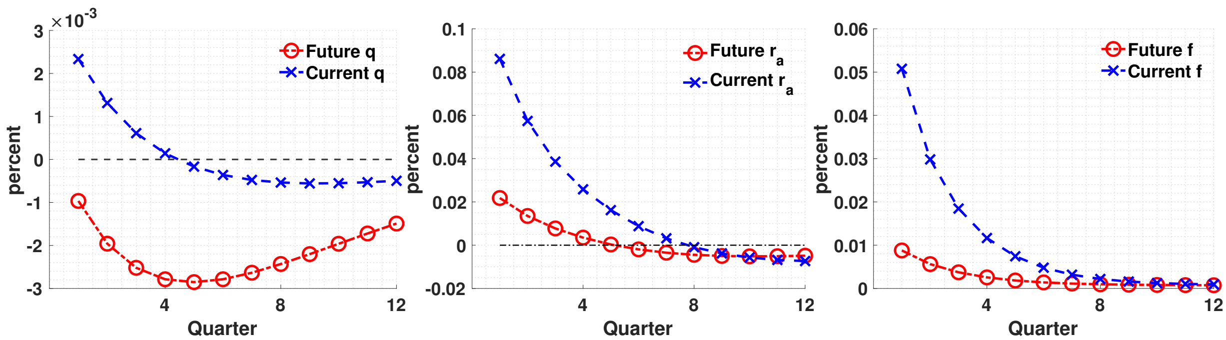
$$dC_t^* = \underbrace{\sum_{\tau=t}^{\infty} \frac{\partial C_t}{\partial r_\tau} dr_\tau}_{\text{direct effect}} + \underbrace{\sum_{\tau=t}^{\infty} \left(\frac{\partial C_t}{\partial q_\tau} dq_\tau + \frac{\partial C_t}{\partial \Pi_\tau} d\Pi_\tau + \frac{\partial C_t}{\partial w_\tau} dw_\tau + \frac{\partial C_t}{\partial f_\tau} df_\tau \right)}_{\text{indirect effects}} \tag{59}$$

Figure 5 shows the direct and indirect effects of an expansionary monetary policy shock on the aggregate consumption over the first twelve quarters. Table 8 shows the average effect over the first year. First, as in Kaplan et al. (2018), the direct effect, i.e., the effect of changes in the real interest rate, only accounts for a small proportion of the overall consumption response. Under the current calibration, the direct effect explains only 24% of the consumption response over the first year. As discussed in Kaplan et al. (2018), this is because a substantial fraction of households do not hold liquid assets at the steady state. Especially, wealthy hand-to-mouth households play a crucial role in driving this result. Since they do not hold liquid assets and face a borrowing premium, they do not respond to a small change in the real interest rate. Moreover, since they are relatively wealthy, their consumption response is important in determining the overall consumption response.

As discussed so far, the results on the transmission mechanisms are consistent with what Kaplan et al. (2018). That is, the most of the aggregate consumption response is due to the indirect effects. In the baseline model, the indirect effects account for 76% of the consumption response over the first year. However, unlike in Kaplan et al. (2018), the effect of changes in the real wage is very limited due to wage rigidity. Over the first year, the real wage changes account for only 4% of the total response.

computed consumption response and the steady state consumption level to measure the effect of each variable shown in the decomposition.

Figure 6: Effects of current and future variables



Notes: The left, middle and right panel show the effect of current and future equity price, dividend rate, and job finding rate respectively. When a current variable changes, future values of the variable are expected to take on its steady state value. When future variables are expected to take on new equilibrium values, the current value for the variable is held fixed at the steady state value. All panels shows the percentage deviations from the steady state.

Instead, increases in profits and the job-finding rate contribute substantially more to the increase in aggregate consumption than the real wage changes. Surprisingly, an increase in profits accounts for almost half of the total consumption response over the first year. This is because the magnitude of an increase in profits is much larger than that of the real wage changes. Even though profits are mostly distributed to wealthy households whose marginal propensity to consume is low, an increase in profits leads to a substantial response of the aggregate consumption because the magnitude of the profit response is sizable. An increase in the job-finding rate also leads to a large response of aggregate consumption, accounting for 24% of the aggregate consumption response over the first year.

ase.

In Figure 6, I further decompose the effect of equity price, dividend rate (profits), and job-finding rate on consumption into the effect of current and future variables. Specifically, I compute the consumption response when only the current value of a variable changes while future values are expected to remain at its steady state level. Also, I compute the consumption response when future values of a variable are expected to take on new values while the current value is held fixed at the steady state level. The effects of the equity price are shown in the left panel of the figure. In case of the equity price, positive consumption response mostly comes from the change in the current price. The higher current price of the equity has a positive income effect for the households that choose to sell their equity. In addition, higher price for a given expectation on future prices implies lower expected capital gain. Thus, households save less in the equity and consume more instead. From these two effects, consumption increases in response to an increase in the equity price. In contrast, an increase in the future expected equity price implies a higher expected capital gain, and thus, leads to a fall in consumption. Since the effects of current and future equity prices offset each other, the overall effect of equity prices on the aggregate consumption is not large.

Changes in the dividend rate induced by an increase in profits have more substantial effects than the equity price on aggregate consumption. Both changes in current and future dividend rates lead to an increase in current consumption. A higher current dividend rate contributes to higher consumption via positive income effects. What is surprising is the effect of the future dividend rate. In general, a higher return on saving leads to a fall in current consumption via inter-temporal substitution. In this model, however, higher future dividend rates also result in higher current period consumption because of the two asset structure of the model. When the expected return on equity rises, households re-optimize their portfolio towards equity. That is, households reduce bond holdings and, instead, increase savings into equity. However, the amount of withdrawal from bond holding is larger than the amount of saving into equity since the return on the equity is much higher than that of the bond. Households can achieve the desired return on their portfolio with a smaller amount of savings. Thus, households increase both consumption and equity investment at the same time when future dividend rates increase. This is the reason why dividend rate has a strong effect on aggregate consumption. Both income and substitution effects lead to an increase in current aggregate consumption.

Finally, increases in both current and future job-finding rates increase aggregate consumption. A rise in current job-finding rate allows more unemployed households to find a job. For such households, the income effect is substantial since their income increases by more than 60% on average, given the replacement ratio of 40%. Thus, these households increase their consumption substantially. On the contrary, higher future job-finding rates do not have any income effect. No households earn more in the current period from higher future job-finding rates. However, households save less or borrow more in the face of lower unemployment risk. This effect applies to all households in the model regardless of their current employment status unless they are constrained by the borrowing limit. Quantitatively, the latter effects, i.e., the effects of lower unemployment risk, are much smaller than the income effect of the current job-finding rate. Since a substantial fraction of households hold zero liquid assets and face a high borrowing premium, they can not reduce their savings and do not borrow even though unemployment risk has fallen. The increase in the future job-finding rate only leads to a 0.01% increase in aggregate consumption on impact. In contrast, a higher current job-finding rate results in about five times larger response of aggregate consumption on impact.

In sum, indirect effects are dominant in determining the aggregate consumption response, as in Kaplan et al. (2018). However, the main driver of the indirect effect is not the changes in the real wage, but rather the increase in profits and a higher job-finding rate.

4.3 Distributional consequences of monetary policy

Now, I turn to the distributional implications of monetary policy. To this end, I sort households into quintiles based on their net worth and examine the effects of an expansionary monetary policy shock

Figure 7: Decomposition across wealth groups

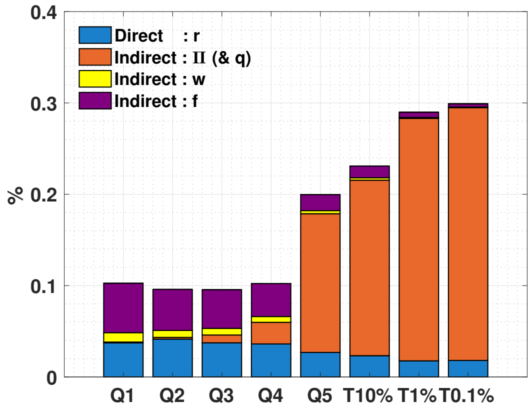


Table 9: Consumption equivalents

Q1	1.01
Q2	0.65
Q3	0.43
Q4	0.37
Q5	1.24
Top 10 %	1.64
Top 1 %	1.77
Top 0.1 %	1.87
Employed	0.61
Unemployed	2.26
Entrepreneur	11.9
Average	0.73

Notes: The figure shows the decomposition of each group's average consumption response over the first year in terms of the percentage deviations from the steady state. The table shows welfare gain of each group from an expansionary monetary policy shock in terms of the consumption equivalent. The unit for consumption equivalent is basis point.

on each wealth group.[43] In addition, I analyze the households with top 10%, 1% and 0.1% net worth for a better understanding of the effects of monetary policy on wealthy households, which is one of the focuses of this paper.

As shown in Figure 7, the average consumption response to an expansionary monetary policy shock is much larger for households with top 20% or larger wealth than for other households. Households in the lower four quintiles increase their consumption by around 0.1% over the first year. In contrast, households in the fifth quintile increase their consumption by 0.2%. The increase in consumption of top 0.1% wealthy households is almost 0.3%. The result that wealthier households increase their consumption by more is quite striking, given the heterogeneity in MPC across wealth groups. As shown in Table 10, households' MPC is decreasing in the amount of wealth.

The difference in consumption responses is due to the heterogeneity in households' portfolio and income composition as well as different magnitudes of the responses of aggregate variables to monetary policy shocks. Figure 7 shows that not only the magnitude of the response but also the decomposition are different across wealth groups. For households in the bottom 80% wealth groups, labor income is the most important component of their total income. Since the real wage does not respond much to an expansionary monetary policy shock, the benefit from changes in the real wage is limited for these households. They, instead, benefit from a higher job-finding rate and a lower real debt burden. By contrast, households in higher wealth groups do not rely much on labor income, so changes in the real wage or the job-finding rate do not affect their consumption much. Instead, most of consumption

[43] A quintile analysis is based on the classification of households in period 0. Over time, households' working status changes and, consequently, the transitions of households across different wealth groups occurs. I do not take into account such transitions in the analysis. Instead, I follow the same households over time. For instance, households in the third quintile in the third period after the shock are households in the third quintile on impact. Thus, the value of their net worth may be greater than bottom 60% or less than bottom 40% of households' net worth in the third period.

Figure 8: MPC over asset distribution

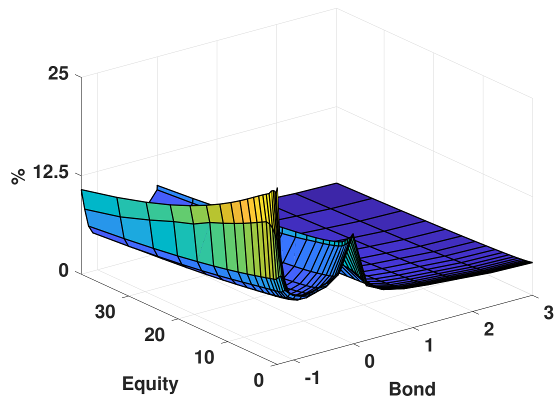


Table 10: Average MPC across wealth groups

Wealth group	**MPC**
Q1	21.0
Q2	12.8
Q3	8.3
Q4	6.4
Q5	5.7
Top 10 %	5.6
Top 1 %	2.7
Top 0.1 %	1.6
Average	10.6

Notes: Marginal propensity to consume is computed as the ratio of the difference in consumption to an unexpected lump-sum transfer of 0.05. The figure shows mean MPC over idiosyncratic productivity and working status. Average MPC for each wealth group is shown in the table. The unit for average MPC is percentage.

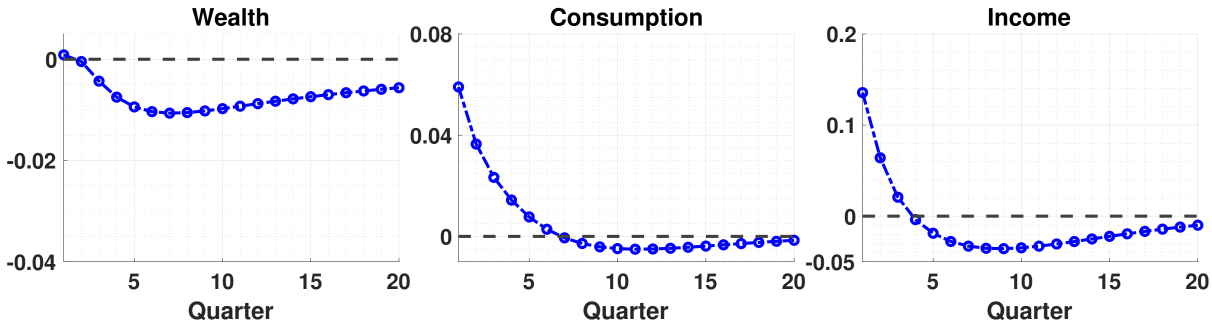
response comes from a rise in profits, because these households hold a large amount of the illiquid asset, which is a claim on profits. Since the increase in profits is much larger than changes in the real wage or the job-finding rate, consumption responses over the first year are much larger for wealthy households than for other households.

So far, I have focused on short-term responses. To evaluate long-term distributional consequences of monetary policy shocks, I compare welfare gains or losses in terms of lifetime consumption equivalents. Specifically, I compute the ratio of steady state consumption that households would require each period as compensation for forgoing a 25 bp expansionary monetary policy shock (per annum). For instance, the entry of "1.01" in row "Q1" indicates that the households in the first wealth quintile on average would feel indifferent between the steady state and experiencing a 25 bp expansionary monetary policy shock if they receive a 1.01 basis points of extra steady state consumption each period. As shown in Table 8, this welfare gain is relatively small because the size of the shock is small and a substantial amount of the benefits from an expansion is wasted as a form of government purchases. The average consumption equivalent across all households is only 0.73 basis points.

More interestingly, it is noteworthy that the magnitude of welfare gain is not monotonically decreasing in wealth. In Table 9, the welfare gain is actually largest for the wealthiest households and smallest for households in the fourth quintile. This result is in stark contrast with existing results, e.g., Gornemann et al. (2016), showing that the welfare gain is decreasing in wealth. In the first quintile, relatively more households are unemployed and in debt.[44] Thus, an increase in the job-finding rate and a fall in the real interest rate both provide benefits for these households. However, households in the third and fourth quintile are less likely to be unemployed and have a positive amount of liquid

[44]The unemployment rate among households in the first quintile is 6.5%, which is 1% larger than the average in the model economy. Also, average liquid asset holding is negative for these households.

Figure 9: Responses of Gini indices



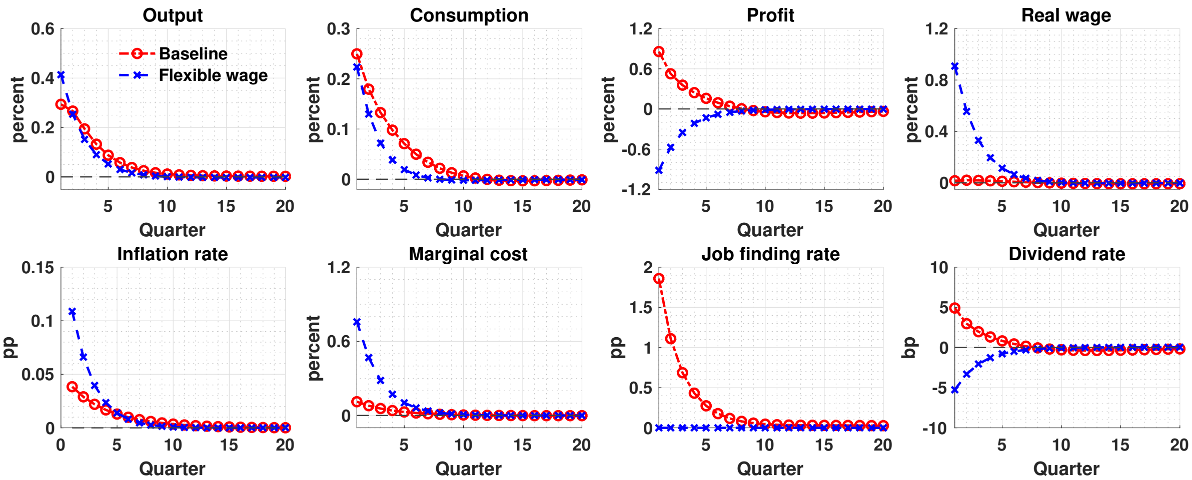
Notes: Impulse responses of Gini indices are shown as the absolute differences in indices. Wealth consists of bond and equity holdings. Income includes all types of income, including labor earning, asset income such as interest, dividends, and capital gain, unemployment benefit and government transfer.

assets on average. Thus, the benefits from an expansion is limited for these groups unless the real wage increases substantially. In contrast, wealthier households enjoy a significantly larger welfare gain, because they are major claimants to profits and the response of profits is much larger than that of the real wage and the job-finding rate. Regarding working status, employed households enjoy the smallest amount of benefits while entrepreneurs experiences the biggest gains, which 20 times larger than those of employed households. This is because the most of entrepreneurs' income comes from profits.

As another way of measuring distributional consequences, I compute the impulse responses of Gini indices for wealth, consumption, and total income. The vertical axis of the figure indicates the absolute differences of the indices. For instance, if the index rises from 65.0 to 65.2, then this will appear as 0.2 on the vetical axis. As shown in Figure 9, the impulse responses of Gini indices are consistent with the results so far. After the shock, both consumption and income Ginis increase from their steady state value, indicating higher consumption and income inequality. As before, this is mainly due to the substantial increase in profits after an expansionary monetary policy shock. Since the right-tail of wealth distribution mostly consists of entrepreneurs and households that hold a large amount of equity, a disproportionately large increase in profits leads to higher inequality in income and consumption. Meanwhile, wealth inequality decreases after an expansionary monetary policy shock. However, this does not imply that the absolute differences in wealth across households have decreased. The increase in wealth levels is actually larger for wealthier households. However, since the steady state level of wealth is much smaller for poor households, an increase in wealth is proportionally much larger for poor households. Indeed, the fall in wealth inequality is mostly driven by the responses of households in the first and second quintiles, whose absolute level of wealth is very small.

So far I have studied the distributional consequences of monetary policy shocks using various measures. From the results presented in this section, it is hard to argue that an expansionary monetary policy shock reduces inequality. In my baseline model, an expansionary monetary policy shock increases

Figure 10: IRFs to an expansionary MP shock: rigid vs flexible ($\rho_w = 0$, $\epsilon_w = 1$) wage



Notes: Impulse responses after 25 bp (annualized) expansionary monetary policy shock. Output, consumption, profit, marginal cost and wage are shown as the percent deviations from the steady state. Real interest rate and job finding rate are expressed in terms of the percentage point deviations from the steady state. Dividend rate is shown as the basis point deviations. Real interest rate and dividend rate are annualized rate.

income and consumption inequality and provides a larger welfare gain to households in the right-tail of the wealth distribution.

5 A model with flexible wages

Wage rigidity is one of the key features in this model that contributes to pro-cyclical responses of profits to monetary policy shocks. Then, a natural question that arises is how the results would change if I instead assumed a flexible wage setting as in standard New Keynesian models. In this section, I solve the model with flexible wages and compare the results with those presented in the previous section. For comparison, I maintain the structure of the previous model but set ρ_w to zero so that the real wage follows the labor service rental rate, which is close to the marginal productivity of labor, one-to-one. Moreover, I do not allow changes in inventory stock or the number of vacancies in the model.[45] Not surprisingly, profits fall and the real wage increases substantially after an expansionary monetary policy shock in the alternative model as in many standard New Keynesian models.

5.1 Impulse responses and transmission mechanisms

Figure 10 compares the impulse responses of the two models. The key differences between the two models are in the responses of profits and the real wage. As shown in the figure, the real wage increases

[45] As sales change while inventory stock remains fixed, the sales-to-stock ratio varies. To eliminate any losses associated with the changes in the sales-to-stock ratio, I additionally assume that the sales-to-stock ratio adjustment cost is zero in the alternative model.

Figure 11: Consumption decomposition

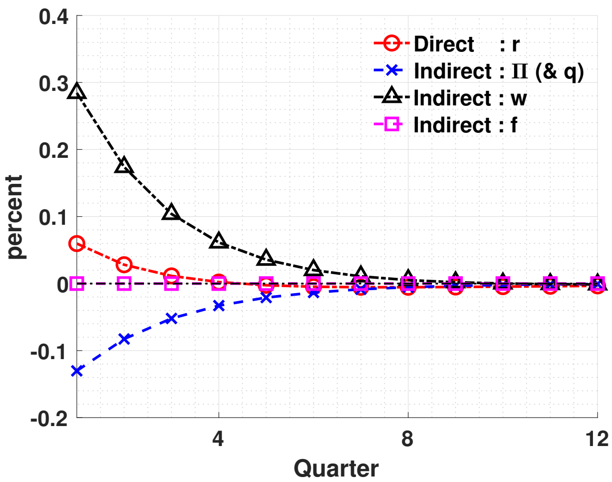


Table 11: Average response over the year

Wage setting	Rigid	Flexible
r change	-0.21	-0.30
Elasticities $(\epsilon_X = \frac{\Delta X}{X}/\Delta r)$		
Output (Y)	-4.21	-3.03
Investment (I)	-8.94	-6.56
Consumption (C)	-3.19	-1.55
Bond (B)	1.89	2.12
Capital (K)	-0.24	-0.07
Decomposition of C (%)		
Direct effect (r)	**24**	**24**
Indirect effect	**76**	**76**
Profit and equity price (Π & q)	48	-68
Wage and profit (w)	4	144
Job finding rate (f)	24	0

Notes: The figure shows the percentage deviations of consumption from its steady state level. Elasticities are shown as semi-elasticities. The decomposition into the direct and indirect effects is approximately exhaustive. Both effects add up to 100%. The fiscal authority is assumed to adjust lump-sum transfer for given bond-issuance rule in response to monetary policy shocks. Profit is the profit given to entrepreneurs as compensations. The profits given to the equity holders are reflected in the equity return.

substantially when wage-setting is flexible. Since firms are not allowed to adjust their inventory stock in the flexible wage model, they must expand their production more in response to an increase in aggregate demand. Such an increase in production drives up the real wage by raising the demand for labor. Even though extensive margin adjustment of labor supply is not allowed, employed households now supply more labor as the real wage has risen. Thus, output rises more on impact in the flexible wage model, by 0.5% in the flexible wage model compared to 0.3% in the baseline model. However, as the marginal cost of production has increased substantially due to a higher real wage, profits fall despite the increase in sales. Profits fall by 1% in the flexible wage model while they increase by 1% in the baseline model. Consequently, the dividend rate falls by 6 basis points per annum on impact in the flexible wage model, while increasing by roughly the amount in the baseline model.

Because of the differences in the response of aggregate variables, the transmission mechanisms of monetary policy shocks are different in the two models. In both models, the indirect effects are larger than the direct effect. However, unlike in the baseline model, the largest consumption responses come from an increase in the real wage. On impact, aggregate consumption increases by about 0.3% in the flexible wage model. Over the first year, the effect of changes in the real wage accounts for more than 100% of the overall response. In contrast, the aggregate consumption response from changes in profits is negative in the flexible wage model since profits fall substantially after the shock. The significant fall in the dividend rate induced by the decrease in profits causes aggregate consumption to fall by more than 0.1% in the flexible wage model. Over the first year, around 68% of the consumption increase due to changes in the real wage is offset by the fall in the dividend rate. These results are consistent

Figure 12: Decomposition across wealth groups

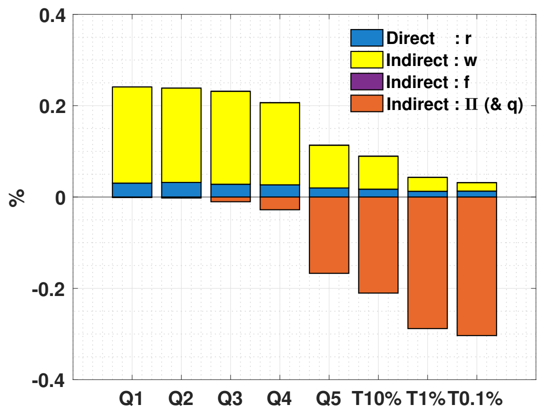


Table 12: Consumption equivalents

Wage setting	Rigid	Flexible
Q1	1.01	3.53
Q2	0.65	3.11
Q3	0.43	2.31
Q4	0.37	0.87
Q5	1.24	-1.68
Top 10 %	1.64	-2.85
Top 1 %	1.77	-4.70
Top 0.1 %	1.87	-4.63
Employed	0.61	1.71
Unemployed	2.26	1.33
Entrepreneur	11.9	-30.3
Avg.	0.73	1.61

Notes: The figure shows the decomposition of each group's average consumption response over the first year in terms of the percentage deviations from the steady state. The table shows welfare gain of each group from an expansionary monetary policy shock in terms of the consumption equivalent. The unit for consumption equivalent is basis point.

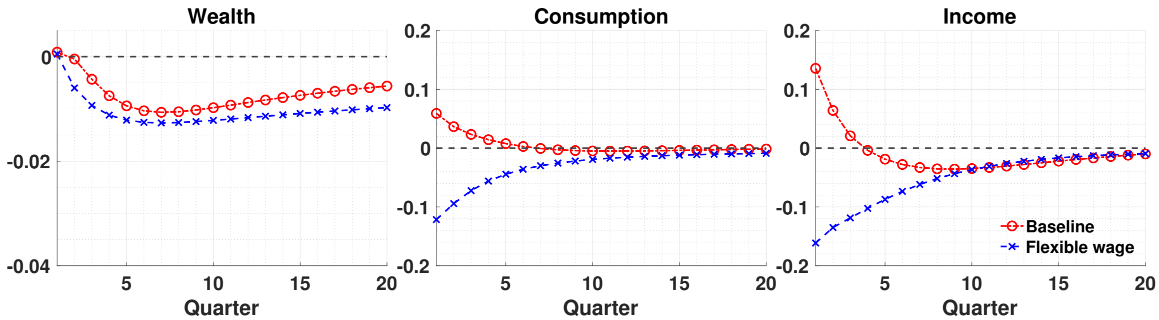
with Kaplan et al. (2018): the indirect effects are much larger than the direct effect, and most of the indirect effects are due to changes in the real wage.

5.2 Distributional consequences of monetary policy

The two models are in stark contrast regarding the distributional consequences of monetary policy shocks as well. Unlike in the baseline model, there are clear winners and losers from an expansionary monetary policy shock in the flexible wage model. As shown in Figure 12, the bottom 80% of households increase their consumption while the top 20% experience a fall in consumption in the first year after the shock. The decomposition shows that most of the increase in consumption for the bottom 80% is due to an increase in the real wage, while the fall in profits explains the fall in consumption for the top 20%. Contrary to the baseline model, the consumption equivalent welfare gain is decreasing in the amount of wealth in the flexible wage model. Indeed, the top 20% wealth group experiences welfare losses. Households in the first quintile would need to receive 3.53 basis points of their steady state consumption to forgo an expansionary monetary policy shock. In sharp contrast, top 0.1% households are willing to give up 4.63 basis points of their steady state consumption to avoid a decrease in the policy rate. Figuratively, an expansionary monetary policy shock is doves for the poor but hawks for the rich when the real wage is flexible.

The impulse responses of Gini indices also confirm the strong distributional consequences of monetary policy shocks in the flexible wage model. Figure 13 shows the responses of wealth, consumption, and income Gini indices both in the baseline and the flexible wage model. As shown in the figure, an expansionary monetary policy shock reduces wealth, consumption, and income inequality in the flexible wage model with the largest change for income inequality. This is due to drastic differences in the

Figure 13: Responses of Gini indices



Notes: Impulse responses of Gini indices are shown as the absolute differences in indices. Wealth consists of bond and equity holdings. Income includes all types of income, including labor earning, asset income, unemployment benefit and government transfer.

responses of profits and wages to monetary policy shocks in the flexible wage model. Since the income of poor households rises while wealthy households' income falls, income inequality substantially falls after an expansionary monetary policy shock. Only part of the lower income inequality translates into a decrease in consumption inequality, since the MPC is less than one. Finally, in both models, wealth inequality falls after an expansionary monetary policy shock. The magnitude of the fall is only slightly larger in the flexible wage model, however, because poor households have a much smaller marginal propensity to save than wealthy households.

The result presented in this section is consistent with the existing results showing that an expansionary monetary policy shock reduces inequality. However, in this paper, I show that, this result crucially depends on the responses of profits and the real wage in the model. Based on my results, I argue that expansionary monetary policy shocks leads to strong reduction in inequality when the real wage is flexible and profits fall after an expansionary monetary policy shock. If profits increase substantially, as in the data, an expansionary monetary policy shock can increase inequality. In other words, the distributional consequences of monetary policy shocks are the results of model features, not a robust feature of New Keynesian models.

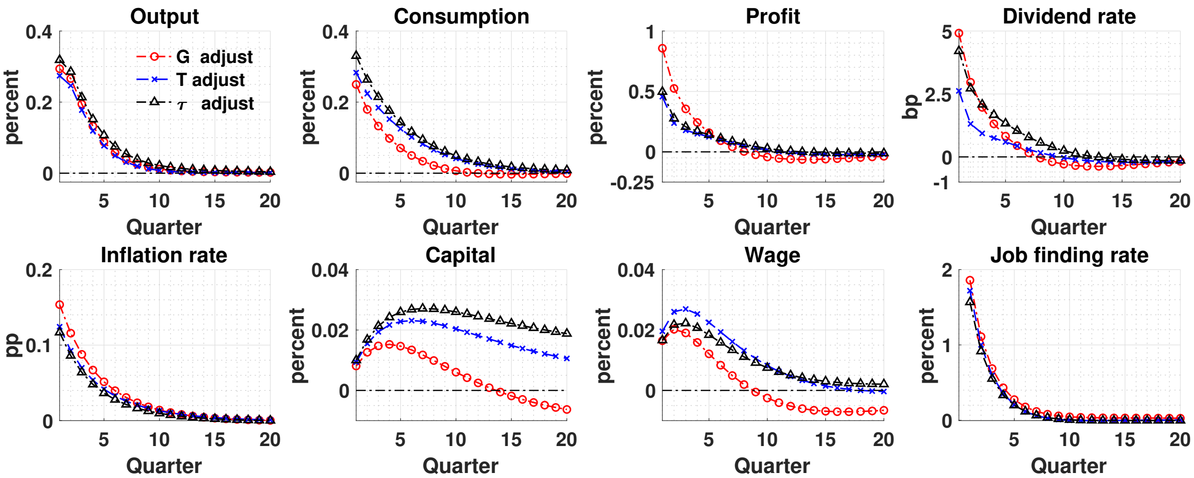
6 The role of fiscal policy

So far I have assumed that the fiscal authority adjusts government purchases when the value of real government debt changes due to monetary policy shocks. Alternatively, the fiscal authority could adjust lump-sum transfers or tax rates to meet the inter-temporal budget constraint. In this section, I examine the implications of alternative assumptions on fiscal policy and compare the results with the baseline model.[46]

Figure 14 shows impulse responses of aggregate variables under different assumptions on fiscal

[46]Regarding tax rates adjustment, I adjust both income and profit tax rate by the same amount.

Figure 14: IRFs with different fiscal policy responses



Notes: Impulses responses to annualized 25 bp expansionary monetary policy shock. G, T, and τ denote government purchase, lump-sum transfer, and tax rates respectively. Inflation rate, unemployment rate, job finding rate, and capital utilization rate are expressed in terms of percentage point deviations from steady state. Nominal and real rate, and dividend rate are shown as basis point deviations from steady state. Inflation rate, nominal and real rate, and dividend rate are annualized. All other variables are shows as percent deviations from steady state.

policy is conducted. Overall, the responses are similar across models with different assumptions. If the fiscal authority chooses to adjust government purchases, this leads to a one-to-one increase in the aggregate demand for goods. Thus, the initial increase in the aggregate demand is largest when the fiscal authority adjusts government purchases to respond to monetary policy shocks. Consequently, the increase in profits is largest in the baseline model. However, since higher demand leads to a higher inflation rate, the increase in the real wage is smallest in the baseline model as the wage is subject to nominal rigidity. Therefore, the consumption response is smallest in the baseline model as well.[47] In addition, the increase in public consumption crowds out private investment ant results in a faster depreciation of the aggregate capital stock. Overall, tax cuts lead to the biggest response of output, consumption, and capital accumulation though the differences are not so substantial.

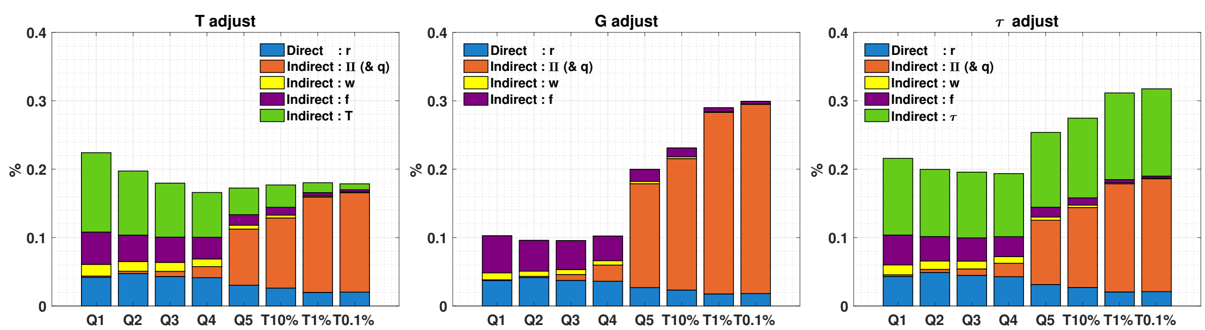
Turning to transmission mechanisms, Table 13 shows that the relative importance of direct and indirect effects is similar across different fiscal responses; the direct effect accounts for around 20% of the consumption response in the first year while the remaining 80% is accounted for by indirect effects. Depending on the fiscal policy, the decomposition of the indirect effects is quantitatively slightly different across models. The effect of the increase in profits is largest in the baseline model.

[47]This result makes a sharp contrast with the result in Kaplan et al. (2018), where the government purchase leads to larger responses not only in output but also in consumption. This is due to a different assumption on wage rigidity. When the wage is flexible, the benefit of expansion in output is distributed to the households in the form of an increase in the wage. Thus, an expansion triggered initially by government purchase also leads to a larger response in consumption and investment. However, if the wage is assumed to be rigid, the benefit of expansion is mostly given to the equity holders and the entrepreneurs. Since they are households with low MPC, further stimulus in the aggregate consumption is smaller. Consequently, the aggregate consumption and investment increase by less when the government purchase is adjusted.

Table 13: Transmission mechanisms under different fiscal policies

	T	G	τ
r change (pp annual)	-0.20	-0.21	-0.19
Elasticities ϵ_X			
Output (Y)	-4.18	-4.29	-5.03
Investment (I)	-11.48	-8.94	-12.98
Consumption (C)	-4.31	-3.19	-5.10
Bond B	1.85	1.89	1.84
Equity A	-0.34	-0.24	-0.38
Decomposition of C			
Direct effect (r)	21	24	18
Substitution effect	24	28	21
Income effect	-3	-4	-3
Indirect effect	79	76	82
Profits and equity price (Π & q)	21	48	20
Wage (w)	6	4	4
Job finding rate (f)	16	24	12
Lump-sum transfer (T) of tax rates (τ)	36	0	46

Figure 15: Consumption decomposition over wealth groups



Notes: The figure shows the decomposition of each group's average consumption response over the first year in terms of the percentage deviations from the steady state under different assumptions on fiscal responses.

When the fiscal authority boosts aggregate demand indirectly by increasing lump-sum transfers or reducing tax rates, the response of profits is smaller than in the baseline model, and, therefore, the contribution of profits to the increase in aggregate consumption is smaller; In the baseline model, about 50% of the consumption response is due to profits, while this contribution is about 20% in alternative models. Similarly, the effect of changes in the job-finding rate is smaller when the fiscal authority adjusts lump-sum transfers or tax rates instead of government purchases. Reducing tax rates leads to a larger response in consumption than increasing transfers, because lower taxes increase household labor supply and the dividend rate increase.[48]

The differences in households' consumption responses across wealth groups are more evident. As shown in Figure 15, the consumption response of bottom 80% households is much larger when the

[48]Note that, in the model, the dividend rate is proportional to after-tax profits.

Figure 16: Responses of Gini indices

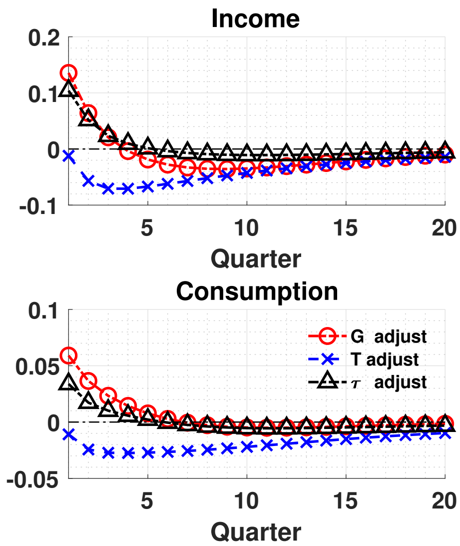


Table 14: Consumption equivalents

	T	G	τ
Q1	5.30	1.01	3.03
Q2	4.07	0.65	2.62
Q3	3.25	0.43	2.23
Q4	2.29	0.37	1.86
Q5	1.52	1.24	2.22
Top 10 %	1.21	1.64	2.39
Top 1 %	0.28	1.77	2.12
Top 0.1 %	0.25	1.87	2.18
Employed	3.16	0.61	2.28
Unemployed	4.77	2.26	3.59
Entrepreneur	2.30	11.9	9.77
Avg.	3.25	0.73	2.38

Notes: G, T, and τ denote government purchase, lump-sum transfer and tax rates respectively. The left figure shows impulse responses of Gini indices for income and consumption as the absolute difference from the steady state level. The right table shows welfare gain of each group from an expansionary monetary policy shock in terms of the consumption equivalent. The unit for consumption equivalent is basis point.

fiscal authority adjusts lump-sum transfers or tax rates, while the consumption response from wealthy households is much smaller when the government adjusts transfers than in the baseline model. These results show that the short-term distributional consequences from a monetary expansion are largely determined by the fiscal responses.

The responses of Gini indices, as well as the computation of consumption equivalents, confirm that the distributional consequences of monetary policy shocks depend on fiscal policy. An expansionary monetary policy shock has linear welfare effects across wealth distribution only when the fiscal authority responds to the shock by adjusting lump-sum transfer. Both income and consumption Gini indices decrease under the lump-sum transfer adjustment. By contrast, in the other two cases, both income and consumption inequality increase. Likewise, welfare gains measured as consumption equivalents are decreasing in wealth only when the lump-sum transfers are adjusted. If the government adjusts government purchases or tax rates instead, welfare gains tend to be higher for wealthier households than for middle class. If the government adjusts its purchase in response to an expansionary monetary policy shock, the welfare gains are the largest for top 0.1% households.

The results in this section show that, in a HANK model with realistic features, the aggregate and distributional effects of monetary policy shocks depend on fiscal policy responses.

7 Conclusion

In this paper, I build a heterogeneous agents New Keynesian model with two assets, wage rigidity, job market frictions, and inventory holding to study the effect of monetary policy on consumption and inequality. The most important innovation of the paper is that the model generates pro-cyclical responses of profits to monetary policy shocks, unlike many other standard New Keynesian models. Given that wages are the biggest component of income for most households while wealthy households are the major claimants to profits in the economy, the model developed in this paper provides a proper setting for studying the transmission mechanisms and the distributional consequences of monetary policy shocks.

Regarding the transmission mechanism of monetary policy shocks, indirect effects through changes in variables other than the real interest rate are stronger than the direct effect of the real interest rate as in Kaplan et al. (2018). However, the decomposition of the indirect effects provides a more detailed understanding of the mechanisms. First, the real wage has little effect on aggregate consumption since it responds little to monetary policy shocks while the job-finding rate has a much stronger impact. More importantly the increase in profits results in a substantial increase in consumption, particularly of wealthy households.

Consequently, the distributional consequences of monetary policy are much weaker or even work in the opposite direction compared to what is argued in existing work. An expansionary monetary policy shock reduces wealth inequality but increases consumption and income inequality in the baseline model. Welfare gains measured as consumption equivalents are also larger for wealthier households. This is because the increase in profits is substantial while the real wage responds little to an expansionary monetary policy shock. This result is in stark contrast with the existing results suggesting that an expansionary monetary policy shock lowers inequality.

Finally, I also show that the distributional consequences of monetary policy shocks crucially depend on fiscal policy. An expansionary monetary policy shock reduces inequality only when the government increases lump-sum transfers in response to a fall in the real value of government debt. In alternative cases where the fiscal authority adjusts government purchases or tax rates, welfare gains are larger for wealthier households than for middle class. Given that lump-transfers are progressive and tax cuts are regressive, it is not surprising that fiscal responses affect the distributional consequences of monetary policy.

In this paper, I provide alternative results on the distributional consequences of monetary policy shocks, and emphasize the importance of the interactions between monetary and fiscal policy in shaping not only the aggregate but also the distributional consequences of monetary policy. I leave the implications of these results for the conduct of optimal monetary and/or fiscal policy and the effects of unconventional monetary policy to future study.

References

[1] Akerlof, George A., William R. Dickens, and George L. Perry, "The Macroeconomics of Low Inflation", 1996, Brookings Papers on Economic Activity, 27(1), pp.1-76.

[2] Barattieri, Alessandro, Peter Gottschalk, and Susanto Basu, "Some Evidence on the Importance of Sticky Wages," 2010, Federal Reserve Bank of Boston Working Papers 10-11.

[3] Bayer, Christian, Lien Pham-Dao, Ralph Luetticke, and Volker Tjaden, "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk", 2018, *Econometrica*.

[4] Bayer, Christian and Ralph Luetticke,"Solving heterogeneous agent models in discrete time with many idiosyncratic states by perturbation methods," 2018, Centre for Economic Policy Research.

[5] Bils, Mark and James A. Khan, "What Inventory Behavior Tells Us about Business Cycles," 2000, *American Economic Review*, 90 (3), pp.458-481.

[6] Broer, Tobias, Niels-Jakob Harbo Hansen, Per Krusell, and Erik Öberg, "The New Keynesian Transmission Mechanism: A Heterogeneous-Agent Perspective.", 2016, Centre for Economic Policy Research Discussion Paper 11382.

[7] Carroll, Christopher D., "The method of endogenous gridpoints for solving dynamic stochastic optimization problems," 2006, *Economics Letters*, 91(3), pp.312-320.

[8] Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber, "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins", 2011, *American Economic Review*, 101 (3), pp.471-75.

[9] Christiano, Lawrence J., Martin Eichenbaum and Charles L. Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", 2005, *Journal of Political Economy*, 113 (1), pp.1-45.

[10] Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt, "Unemployment and Business Cycles," 2016, *Econometrica*, 84 (4), pp.1523-1569.

[11] Coibion, Olivier, Yuriy Gorodnichenko, Lorenz Kueng and John Silvia, "Innocent Bystanders? Monetary Policy and Inequality in the U.S.", 2017, *Journal of Monetary Economics*, 88 (June), pp.70-89.

[12] den Haan, Wouter J., Garey Ramey, and Joel Watson, "Job Destruction and Propagation of Shocks," 2000, *American Economic Review*, 90 (3), pp.482-498.

[13] , Elsby, Michael, "Evaluating the Economic Significance of Downward Nominal Wage Rigidty," 2009, *Journal of Monetary Economics*, 56 (2), pp.154-169.

[14] , Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin, "Optimal Monetary Policy with Staggered Wage and Price Contracts," 2000, *Journal of Monetary Economics*, 46, pp.281-313.

[15] Gornemann, Nils, Keith Kuester and Makoto Nakajima, "Doves for the Rich, Hawks for the Poor? Distributional Consequences of Monetary Policy", 2016, International Finance Discussion Papers, Board of Governors of the Federal Reserve System, No. 1167.

[16] Greenwood, Jeremy, Zvi Hercowitz and Gregory W. Huffman, "Investment, Capacity Utilization, and the Real Business Cycle," 1988, *American Economic Review*, Vol. 78, No. 3, pp. d402-417.

[17] Jung, Yongseung, and Tack Yun, "Monetary Policy Shocks, Inventory Dynamics, and Price-setting Behavior", 2005, Federal Reserve Bank of Snafrancisco Working Paper.

[18] Khan, Shulamit, "Evidence of Nominal Wage Stickiness from Microdata," 1997, *American Economic Review*, 87 (5), pp.993-1008

[19] Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante, "Monetary Policy According to HANK", 2018, *American Economic Review*, 108 (3), pp.697-743.

[20] Kurmann, Andre, and Erika McEntarfer, "Downward Nominal Wage Rigidity in the United States: New Evidence from Worker-Firm Linked Data," 2019, School of Economics Working Paper Series 2019-1, LeBow College of Business, Drexel University.

[21] McKay, Alisdair, Emi Nakamura, and Jón Steinsson, "The Power of Forward Guidance Revisited," 2016, *American Economic Review*, 106 (10), pp.3133-3158.

[22] Reiter, Michael, "Solving heterogeneous-agent models by projection and perturbation," 2009, *Journal of Economic Dynamics and Control*, 33(3), pp.649-665.

[23] Rotemberg, Julio J, "Sticky Prices in the United States," 1982, *Journal of Political Economy*, 90 (6), pp.1187-1211.

[24] Schmitt-Grohé, Stephanie and Martin Uribe, "Solving dynamic general equilibrium models using a second-order approximation to the policy function," 2004, *Journal of Economic Dynamics and Control*, 28 (4), pp.755-775.

[25] Schmitt-Grohé, Stephanie and Martin Uribe, "Comparing Two Variants of Calvo-Type Wage Stickiness," 2006, NBER Working paper 12740.

[26] Shimer, Robert, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," 2005, *American Economic Review*, 95 (1), pp.25-49.

[27] Young, Eric R., "Solving the incomplete markets model with aggregate uncertainty using the Krusell-Smith algorithm and non-stochastic simulations," 2009, *Journal of Economic Dynamics and Control*, 34 (1), pp.36-41.

[28] Winberry, Thomas, "A toolbox for solving and estimating heterogeneous agent macro models," 2016, mimeo, Chicago Booth.